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ABSTRACT

Disruptive Technologies and the Emergence of Competition*

We formalize the phenomenon of disruptive technologies (Christensen, 1997) that initially serve isolated market niches and, as they mature, expand to displace established technologies from mainstream segments. Using a model of horizontal and vertical differentiation with discrete customer segmentation, we show how the threat of disruption varies with the rate of technological advance, the number of firms using each technology, segments sizes, marginal costs, and the ability of firms to price discriminate. We characterize the effect of disruption on prices, market shares, social welfare and innovation incentives. We show that a shift from isolation to disruption lowers prices and increases social welfare, but may either increase or decrease the profits of firms using the new technology.

By identifying the drivers and implications of technology competition, we contribute to debates about market definition that are often central in anti-trust deliberations. Moreover, we call into question standard results on the effects of mergers in Cournot models. Prior work finds that, absent efficiency gains, mergers among Cournot competitors lower welfare and are only profitable for the merging firms at high levels of concentration. We show that neither of these results need hold when mergers can alter the boundaries of technology competition.

JEL Classification: L10, L40 and M20
Keywords: market definition, mergers and threat of substitutes

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1. Introduction

New technologies are often commercialized in a specialized niche. Some stay in their niche, while others go on to penetrate mainstream segments and compete with incumbent technologies. Although this phenomenon has received little attention in formal theory, it has long been studied by historians of technology (e.g., Basalla, 1988), economic historians (e.g., Rosenberg, 1976), and business and marketing strategists (e.g., Foster, 1986; Moore, 1991). The emergence of technology competition has risen to particular prominence as a result of Christensen's inductive work (1997), which begins with his analysis of the hard disk drive industry. Consider the following characterization:

In 1985 the hard disk drive market for personal computers was divided into two segments: desktop computer users who cared about capacity, and portable computer users who cared about both capacity and portability. A 5.25-inch hard disk technology, which offered higher capacity than the 3.5-inch alternative, was used in the desktop segment.[1] The 3.5-inch hard drives, which were smaller and more energy efficient, served the emerging market for portable computers. Thus, the two technologies were initially isolated, each limited to serving consumers in a different market segment. With time, the performance of both technologies improved, but the 5.25-inch drives always offered significantly higher capacity than the 3.5-inch drives. By 1988, however, the 3.5-inch drives had expanded beyond the portable segment to capture the low-end of the desktop segment.

Christensen terms technologies like the 3.5-inch hard drives **disruptive technologies**. A disruptive technology offers a novel mix of attributes compared to the established technology, but is inferior to the established technology according to the needs of consumers in the primary (mainstream) market segment. The disruptive technology is, therefore, initially purchased by consumers in a secondary (niche) market segment who place high value on the new technology's attribute mix. As the new technology matures, its performance improves, but its perceived quality in the primary segment remains inferior to that of the established technology. Despite this performance inferi-

[1] Note that the product here is an internal hard disk drive, not a removable floppy disk drive.

ority, the new technology is able to enter the primary segment because the established technology is "over supplying" customer needs. Christensen (1997) documents similar dynamics with a variety of technologies including laser and inkjet printers, minimill and integrated steel plants and hydraulic and steam powered earthmoving equipment. For scholars, such disruptive technologies highlight the question of the boundaries of technology competition and how those boundaries change over time (Adner, 2001). For managers, disruptive technologies highlight the danger posed to incumbent firms from too quickly dismissing new technologies as inferior and irrelevant to their market positions.[2]

We address three main research questions. First, we explore whether or not there is competition between two technologies; that is, whether or not a new technology is disruptive at a point in time. Second, we explore whether or not disruption is triggered by technological progress, due perhaps to performance over supply, and if so, how long it takes a new technology to break out of its niche. In the initial excitement surrounding Christensen's work, analysts and firms tended to see disruptive threats everywhere.[3] That many of these threats did not materialize highlights the importance of distinguishing disruptive threats from technologies that will remain isolated in a niche.[4] Our third question is what are the effects of disruption on competitive outcomes such as prices, social welfare, firm market shares, profitability, and innovation incentives.[5]

The emergence of technology competition has not been the focus of formal economic theorizing. While one can interpret the vast economics literature on product differen-

[2]See for example the January 25, 1999 *Forbes* cover story "Danger: Stealth Attack," about Christensen's research and the challenge posed by disruptive technologies.

[3]For example, consider the following quote from *The Economist*: "Starting with this issue, *The Economist Technology Quarterly* will offer readers a foretaste of what new developments are threatening—no, guaranteeing—to disrupt the way business is done in the years ahead" (December 9, 2000).

[4]For example, Porter (2001) claims that incumbents were too quick to treat new internet-based business models as disruptive technologies.

[5]In addition to identifying the phenomenum of disruption, Christensen (1997) observes that established firms face an "Innovator's Dilemma" in that their internal resource allocation processes lead them to systematically underinvest in disruptive technologies. In this paper, our focus is on competitive dynamics rather than internal resource allocation. An exception is Section 9.1 where we extend our model to consider the incentives of established firms to use a disruptive technology.

tiation as being about technology competition—for example, different positions in a Hotelling model can be interpreted as arising from firms using different technologies—this literature takes as given the set of competing firms and relevant consumers. For example, the common assumption in spatial models that the market is covered implies that some consumers are always choosing between adjacent technologies and hence that the technologies are always in direct competition. In other words, the received literature assumes away the very question of whether or not two technologies compete.

We develop a model of vertical and horizontal differentiation that is well suited to studying the emergence of competition between distinct technologies. Consumers belong to one of two segments—a primary segment and a secondary segment. Segments differ in how they evaluate products and hence allow for horizontal product differentiation. Within a segment, products can be vertically differentiated. There are two product technologies, a new technology and an established technology. We consider Cournot competition in which there are an arbitrary number of firms using each of the two technologies and we allow firms to vary in their marginal costs.[6] The source of dynamics in the model is that consumers' willingness to pay for the products increases over time as the production technologies mature.

In considering our first question, whether there is competition between the two technologies, we show that the threat of disruption is increasing in the number of new technology firms, the relative size of the primary segment, the primary segment's utility from the new technology product, and the marginal costs of the established firms. We show that the threat of disruption is decreasing in the number of established technology firms, the secondary segment's utility from the new technology product, the primary segment's utility from the established technology product, and the marginal costs of the new technology firms. The disruptive threat is greater when firms can price discriminate across segments. The intuition for many of these results comes from the

[6] An advantage of Cournot over Bertrand is that it allows for multiple firms using a single technology. With Bertand competition, one can only have a single firm of a given type (as otherwise there is marginal cost pricing), and hence one can not readily address the effect of competition among firms using a given technology on the emergence of competition between technologies.

fact that disruption occurs when new technology firms pursue a high volume, low price strategy that allows them to break into the primary segment. Consider, for example, the intuition for the effect of marginal costs. The lower the marginal costs of new technology firms, the more attractive is a high volume strategy and hence the greater the threat of disruption by the new technology. On the other hand, the lower the marginal costs of the established technology firms, the greater their output and hence the lower the scope for new technology firms to increase their volumes by disrupting the primary market. Hence, the lower the costs of established technology firms, the lower the threat of disruption.

Despite the fact that disruption is associated with low costs for the new technology, we find that the lowest cost new technology firm is *not* necessarily the one that initiates disruption. On the one hand, the lowest cost firm has the highest margins among new technology firms, which favors output expansion and hence disruption. On the other hand, the lowest cost firm has the highest market share in the secondary segment and hence the most to loose from the fall in price that comes with disruption.

In considering the dynamics of disruption, we find that technology improvement can lead to disruption. We model performance over supply as reducing the established technology's rate of utility improvement relative to that of the new technology. We show that while performance over supply facilitates disruption, it is not necessary for it. We highlight the importance of market structure (i.e., the number of firms using each technology) as an important driver of the dynamics because it determines the extent to which consumer surplus from each technology increases over time. All the factors that make disruption more likely at a point in time, also serve to speed its arrival.

In considering the effects of disruption, we show that social welfare is unambiguously increasing because prices for both products fall with disruption. We show that the profits of new technology firms need not increase with disruption because their increased volumes can be more than offset by increased competition. Indeed, it is possible that established technology firms increase their output in response to disruption. We find that concentration tends to increase with disruption because the effect of cost asymmetries on market share is amplified by the increased number of competitors.

By identifying the drivers of technology competition this paper contributes to debates about market definition that are often central in antitrust deliberations. The standard approach to market definition is to focus on the degree of substitutability between products as measured by both own-price and cross-price elasticities. This approach has been criticized as being inherently static, and hence unable to accommodate the possibility of future competition from alternative technologies (Teece and Coleman, 1998).[7] Our theory explicitly accounts for the emergence of technology competition and characterizes its effect on competitive outcomes. Moreover, our model calls into question standard results on the effects of mergers in Cournot models (Salant et al., 1983; Farrel and Shapiro, 1990). Prior work finds that, absent efficiency gains, mergers among Cournot competitors lower social welfare and reduce the profits of the merging firms unless the post merger market structure is highly concentrated. In contrast, we show that neither of these results need hold when mergers can alter the boundaries of technology competition.

The paper proceeds as follows. Section 2 describes the model. Section 3 presents important preliminaries including the derivation of the demand function and the definition of the possible pure strategy Nash equilibria. Section 4 characterizes the drivers of disruption at a point in time, while Section 5 discusses issues of equilibrium existence and uniqueness. Section 6 examines the effect of technology improvement on the emergence of competition. Section 7 characterizes the effects of disruption on competitive outcomes and Section 8 looks at the implications for merger analysis. Section 9 extends the model to allow for multi-technology firms and price discrimination. Section 10 concludes. All proofs are in an appendix, as is discussion of mixed strategy equilibria.

[7] See also the antitrust case *Bourns, Inc v Raychem Corporation*, US District Court, Central District of California, Case No CV 98-1765 CM and the associated discussion in Pleatsikas and Teece (2001) for problems with using traditional approaches to market definition in settings characterized by multiple evolving technologies and multiple customer segments. See also Sutton (1998, chapter 6) for similar issues in the flow meter industry.

2. Model

There is a new technology ($\lambda = N$) and an established technology ($\lambda = N$), with $n_\lambda \geq 1$ firms using technology λ. We assume each firm uses only one technology. Section 9.1 relaxes this assumption and looks at multi-technology firms. Firms vary in their cost of production. The marginal cost of production for the ith firm using technology λ is denoted by $c_{\lambda i} \geq 0$. Denote the average cost of firms using technology λ by $\bar{c}_\lambda = \frac{1}{n_\lambda} \sum_{i=1}^{n_\lambda} c_{\lambda i}$.

There are two discrete market segments indexed by $m = 1, 2$. A consumer in segment m has a willingness to pay for the new product (i.e., a product made using the new technology) of $u_{Nm} + d$ and a willingness to pay for the established product (i.e., a product made using the established technology) of $u_{Em} + d$. The first component of willingness to pay, $u_{\lambda m}$, is the same for all consumers in a given segment and reflects product differentiation. The ordering of u_{Em} and u_{Nm} captures vertical differentiation within segment m. The ordering $u_{\lambda 1}$ and $u_{\lambda 2}$ captures horizontal differentiation (i.e., the extent to which consumers in different segments value the same product differently). We have in mind that these differences in willingness to pay arise because segments vary in the weights they place on different product attributes and products vary in their attribute levels depending on the underlying production technology.[8] Discrete segmentation is a good representation of horizontal differentiation in many settings such as when the product is a component used in multiple end products (e.g., hard disk drives, which are used in notebook, desktop and mainframe computers). Other examples of discrete consumer heterogeneity include personal versus professional users, industry segments (in business-to-business markets) and national markets.

The second component of willingness to pay, d, varies across consumers within a segment and is independent of the product. In segment m, we assume that d is uniformly distributed between $-\infty$ and D with a density $S_m > 0$. S_m parameterizes

[8] An example of such dependence is the following. Consumer choice depends on two attributes and technology λ's performance on these attributes is given by x_λ and y_λ. Utilities are $u_{\lambda m} = (x_\lambda)^{\alpha_m}(y_\lambda)^{1-\alpha_m}$ where α_m reflects the relative weight placed on attribute x by segment m.

the size of market segment m. We make no restriction on relative segment size (i.e., it need not be that $S_1 > S_2$). Without loss of generality, we assume that $D = 0$.[9]

We consider Cournot competition and denote the output of firm i using technology λ by $q_{\lambda i}$ and total outputs by $Q_\lambda = \sum_{i=1}^{n_\lambda} q_{\lambda i}$. Firm profits are $\pi_{Ni} = q_{Ni}(P_N(Q_N, Q_E) - c_{Ni})$ and $\pi_{Ei} = q_{Ei}(P_E(Q_E, Q_N) - c_{Ei})$ where $P_\lambda(\cdot, \cdot)$ are the inverse demand functions. Thus, there is a single price for each product; Section 9.2 extends the analysis to the case of price discrimination across segments. We focus on the pure-strategy Nash equilibria (PSNE) of the model.

We assume that as products mature over time their performance, and hence consumers' willingness to pay for them, is non-decreasing. That is, $u'_{\lambda m}(t) \geq 0$. Exogenous technological improvement, as in the literature on new technology adoption (e.g. Fudenberg and Tirole, 1985), can have several sources such as improving input quality and spillovers from R&D activities outside of the industry. In Section 7.2 we relax this assumption and explore innovation incentives within the model.

We make the following restrictions on the parameter space. Firms using the established technology only create value in segment 1: $u_{E1} > \max c_{Ei}$ and $u_{E2} \leq \min c_{Ei}$; while firms using the new technology create value in both segments: $u_{N1} > \max c_{Ni}$ and $u_{N2} > \max c_{Ni}$. We are interested in the case where segment 2 is relevant to the analysis (i.e., the new product is sold to segment 2) and where both technologies compete for segment 1 (i.e., where the new technology does not drive the output of established technology firms to zero). Lemma 3.4 identifies parameter restrictions such that these conditions are satisfied. We do *not* require that the new technology is inferior to the established technology in segment 1. We simplify the analysis by restricting attention to small cost asymmetries across firms using the same technology (i.e., $|\bar{c}_\lambda - c_{\lambda i}|$ not too large) such that all firms have positive output in equilibrium.

[9] Our specification of willingness to pay follows Katz and Shapiro (1985), but with two differences. First, whereas they study a single segment we consider two market segments. Second, whereas they consider $u_{\lambda m}$ to arise from network externalities, we assume that it arises from a segment's valuation of the product's attributes. We discuss extending our results to an alternative demand specification in Section 9.3.

3. Preliminaries

In this section we derive inverse demand functions, identify two possible pure strategy Nash equilibria, and introduce definitions that link parameters to equilibrium existence.

3.1. Demand Functions

We derive the inverse demand functions, $P_N(Q_N, Q_E)$ and $P_E(Q_E, Q_N)$, by solving for the market clearing prices for any given Q_N and Q_E.

Given the restriction $u_{E2} \leq \min c_{Ei}$, the established product is only bought by consumers in market segment 1. There are three possibilities for who buys the new product: only consumers in segment 2, only consumers in segment 1, and consumers in both segments. We start with the case where the new product is only bought by consumers in segment 2. Consumers in segment 2 with $d + u_{N2} - P_N \geq 0$ buy the new product and the measure of such consumers is $(u_{N2} - P_N)S_2$. Market clearing requires that $(u_{N2} - P_N)S_2 = Q_N$ and hence $P_N = u_{N2} - Q_N/S_2$ in this case. Similarly, the inverse demand curve for the established product in this case is $P_E = u_{E1} - Q_E/S_1$. Under what conditions is the new product only bought by consumers in segment 2? It must be that consumers in segment 1 get a higher surplus from the established product than the new product, that is $u_{E1} - P_E \geq u_{N1} - P_N$ or $\frac{Q_N}{S_2} \leq \frac{Q_E}{S_1} - (u_{N1} - u_{N2})$.

Now consider the case where the new product is only bought by consumers in segment 1. Since consumers in segment 1 buy both products, they must be indifferent between them and hence $u_{E1} - P_E = u_{N1} - P_N$. Total demand is then $(u_{N1} - P_N)S_1$, or equivalently $(u_{E1} - P_E)S_1$, and market clearing gives $P_N = u_{N1} - (Q_E + Q_N)/S_1$ and $P_E = u_{E1} - (Q_E + Q_N)/S_1$. In order for no consumers in segment 2 to buy the new product it must be that $u_{N2} - P_N \leq 0$, or equivalently $\frac{Q_N}{S_1} \leq u_{N1} - u_{N2} - \frac{Q_E}{S_1}$.

Consider the final case where consumers in both segments buy the new product. Since consumers in segment 1 buy both products, they must be indifferent between the two products and hence $u_{E1} - P_E = u_{N1} - P_N$. Total demand for both products is now the demand from segment 1, $(u_{N1} - P_N)S_1$, plus demand from segment 2, $(u_{N2} - P_N)S_2$ and market clearing yields $P_N = \frac{S_1 u_{N1} + S_2 u_{N2}}{S_1 + S_2} - \frac{Q_E + Q_N}{S_1 + S_2}$. The price of the established

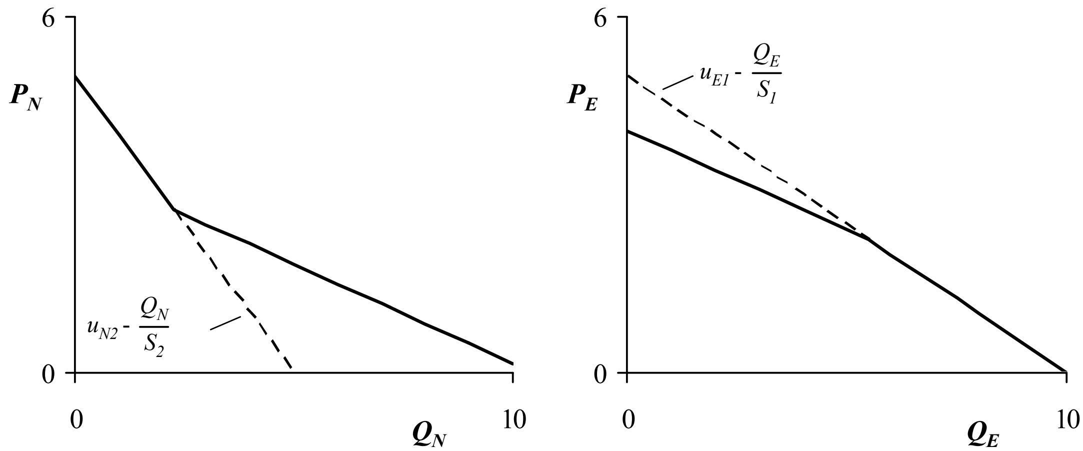

Figure 3.1: The inverse demand functions for products based on the new technology (left) and the established technology (right). Parameters are $S_1 = 2$, $S_1 = 1$, $u_{N2} = u_{E1} = 5$, $u_{N2} = 4.75$, and $Q_E = 4$ (left) and $Q_N = 3$ (right).

product is $P_E = u_{N1} - u_{E1} + P_N = u_{E1} - \frac{S_2}{S_1+S_2}(u_{N1} - u_{N2}) - \frac{Q_E+Q_N}{S_1+S_2}$. The conditions such that some consumers in segment 2 buy some but not all of the new product, $0 < (u_{N2} - P_N)S_2 < Q_N$, are just the converses of the prior two conditions. We collect the above results in the following lemma.

Lemma 3.1. *The inverse demand functions are*

$$P_N(Q_N, Q_E) = \begin{cases} u_{N2} - \frac{Q_N}{S_2} & \text{if } \frac{Q_N}{S_2} \leq \frac{Q_E}{S_1} - (u_{N1} - u_{N2}), \\ u_{N1} - \frac{Q_N+Q_E}{S_1} & \text{if } \frac{Q_N}{S_1} \leq (u_{N1} - u_{N2}) - \frac{Q_E}{S_1}, \\ \frac{S_1 u_{N1} + S_2 u_{N2}}{S_1+S_2} - \frac{Q_N+Q_E}{S_1+S_2} & \text{otherwise.} \end{cases}$$

$$P_E(Q_E, Q_N) = \begin{cases} u_{E1} - \frac{Q_E}{S_1} & \text{if } \frac{Q_E}{S_1} \geq \frac{Q_N}{S_2} + (u_{N1} - u_{N2}), \\ u_{E1} - \frac{Q_N+Q_E}{S_1} & \text{if } \frac{Q_E}{S_1} \leq (u_{N1} - u_{N2}) - \frac{Q_N}{S_2}, \\ u_{E1} - \frac{S_2(u_{N1}-u_{N2})}{S_1+S_2} - \frac{Q_E+Q_N}{S_1+S_2} & \text{otherwise.} \end{cases}$$

Note that both demand functions are piecewise linear, that established product demand is weakly concave, and that new product demand is weakly convex (See Figure 3.1). Because of the convexity in their demand function, equilibrium analysis needs to be concerned with both local and global optimality for new technology firms. The

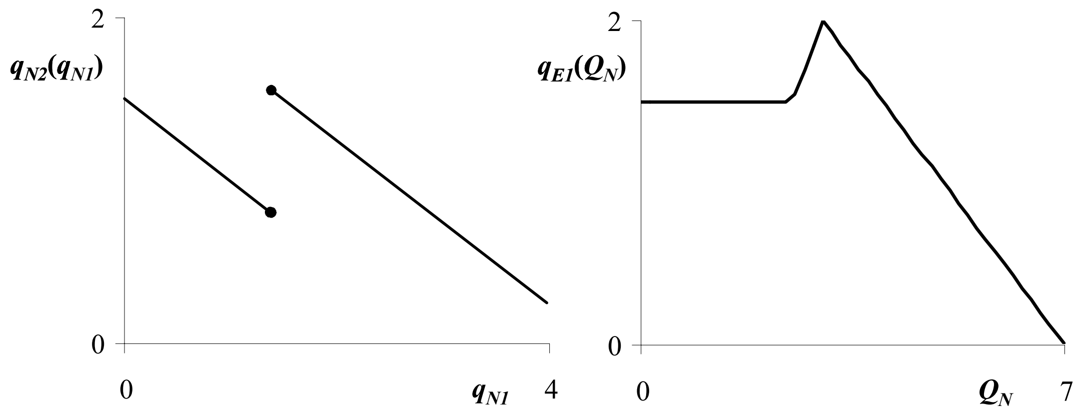

Figure 3.2: The best response function of one new technology firm to another (left) and of an established technology firm to the total output of new technology firms (right). Both are for $u_{E1} = 5$, $u_{N1} = 3$, $u_{N2} = 4$, $c_{Ei} = 2$, $c_{Ni} = 1$, $n_E = 1$, $n_N = 2$ and $S_2 = 1$, while $S_1 = 3$ (left) or $S_1 = 1$ (right).

emergence of competition between the technologies makes the demand curves for both types of product flatter. For new technology firms this is because additional new product output is spread over both segments. For established technology firms this is because additional established technology output shifts new product output into segment 2. With this flattening, best response functions are not everywhere downward sloping (see Figure 3.2). That is, in some regions outputs are strategic complements, rather than strategic substitutes. Finally, note that P_E is weakly decreasing in $(u_{N1} - u_{N2})$, which determines the ease with which established product output pushes the new product back into segment 2. Loosely, $(u_{N1} - u_{N2})$ reflects the extent to which new technology firms are committed to segment 1.

3.2. Pure Strategy Nash Equilibria

We distinguish between two different types of equilibria.

Definition 3.2. *The new technology is* ***disruptive*** *in some equilibrium if consumers in segment 1 and segment 2 buy the new product. The technologies are* ***isolated*** *in some equilibrium if consumers in segment 1 only buy the established product and* *consumers in segment 2 only buy the new product.*

Referring back to Lemma 3.1, disruption arises from new technology firms pursuing a high output, and hence low price, strategy; conversely, isolation involves these firms pursuing a low volume, high price strategy. Since the demand functions are piecewise linear and firm profits are $\pi_{\lambda i} = q_{\lambda i}(P_\lambda(Q_\lambda, Q_{-\lambda}) - c_{\lambda i})$, pure strategy Nash equilibria of our model are analogous to equilibria in a standard Cournot model with linear demand and asymmetric costs. Notice that in this type of model, differences in the demand intercept across types of firms, which arise from product differentiation, are equivalent to differences in marginal costs. Formally we have the following:

Lemma 3.3. *(i) There is a unique pure strategy Nash equilibrium (PSNE) in which the technologies are isolated. Equilibrium outputs and profits in this equilibrium are*

$$\begin{aligned} q^I_{Ni} &= \frac{S_2}{n_N+1}\left(u_{N2} - c_{Ni} + n_N(\bar{c}_N - c_{Ni})\right), \\ q^I_{Ei} &= \frac{S_1}{n_E+1}\left(u_{E1} - c_{Ei} + n_E(\bar{c}_E - c_{Ei})\right), \\ \pi^I_{Ni} &= \frac{\left(q^I_{Ni}\right)^2}{S_2} \text{ and } \pi^I_{Ei} = \frac{\left(q^I_{Ei}\right)^2}{S_1}. \end{aligned}$$

(ii) There is a unique PSNE in which the new technology is disruptive and all the established firms still have positive output. Equilibrium outputs and profits in this equilibrium are

$$\begin{aligned} q^D_{Ni} &= \frac{S_1+S_2}{n_E+n_N+1} \\ &\quad \left[\frac{S_1 u_{N1} + S_2 u_{N2}}{S_1+S_2} - c_{Ni} + n_N(\bar{c}_N - c_{Ni}) + n_E[(u_{N1} - c_{Ni}) - (u_{E1} - \bar{c}_E)]\right], \\ q^D_{Ei} &= \frac{S_1+S_2}{n_E+n_N+1} \\ &\quad \left[u_{E1} - c_{Ei} + n_E(\bar{c}_E - c_{Ei}) + n_N\left[(u_{E1} - c_{Ei}) - (u_{N1} - \bar{c}_N)\right] - \frac{S_2(u_{N1} - u_{N2})}{S_1+S_2}\right], \\ \pi^D_{Ni} &= \frac{\left(q^D_{Ni}\right)^2}{S_1+S_2} \text{ and } \pi^D_{Ei} = \frac{\left(q^D_{Ei}\right)^2}{S_1+S_2}. \end{aligned}$$

Henceforth we refer to the PSNE with (q_{Ni}^D, q_{Ei}^D) as the disruptive equilibrium ($\phi = D$) and the PSNE with (q_{Ni}^I, q_{Ei}^I) as the isolated equilibrium ($\phi = I$). It is useful to define total output in each of the equilibria by $Q_N^\phi = \sum_{i=1}^{n_N} q_{Ni}^\phi$ and $Q_E^\phi = \sum_{i=1}^{n_E} q_{Ei}^\phi$.

Consider the determinants of a new technology firm's output in the isolated equilibrium. Firm i's output q_{Ni}^I depends in part on the size of segment 2 (S_2) and the intensity of competition in the segment ($\frac{1}{n_N+1}$). In addition, it is useful to relate equilibrium output to a firm's value creation (Brandenburger and Stuart, 1996), which is the difference between a customer's willingness to pay for a product and the production cost. Output q_{Ni}^I depends on a measure of absolute value creation $(u_{N2} - c_{Ni})$ and of firm i's value creation relative to all new technology firms: $n_N(u_{N2} - c_{Ni}) - n_N(u_{N2} - \bar{c}_N) = n_N(\bar{c}_N - c_{Ni}))$.[10] The components of q_{Ei}^I have an analogous interpretation.

Now consider the disruptive equilibrium outputs, q_{Ni}^D and q_{Ei}^D. Because disruption effectively collapses the two segments into one market, equilibrium outputs depend on the size of the total market (S_1+S_2) and the overall intensity of competition ($\frac{1}{n_N+n_E+1}$). Because the established technology is only bought in segment 1, q_{Ei}^D naturally depends on that technology's value creation in segment 1 $(u_{Ei} - c_{Ei})$. Because the new technology is bought in both segments, q_{Ni}^D depends on a measure of absolute value creation across both segments $(\frac{S_1 u_{N1} + S_2 u_{N2}}{S_1+S_2} - c_{Ni})$. The term $n_N(\bar{c}_N - c_{Ni}) + n_E[(u_{N1} - c_{Ni}) - (u_{E1} - \bar{c}_E)] = (n_N + n_E)(u_{N1} - c_{Ni}) - n_N(u_{N1} - \bar{c}_N) - n_E(u_{E1} - \bar{c}_E)$ in q_{Ni}^D is a new technology firm's relative value creation for segment 1 consumers.[11] Similarly, q_{Ei}^D depends on the established technology firm's relative value creation in segment 1 $(n_E(\bar{c}_E - c_{Ei}) + n_N\left[(u_{E1} - c_{Ei}) - (u_{N1} - \bar{c}_N)\right])$. Finally, the output of established technology firms depends on the level of new technology firms' commitment to segment 1 $(u_{N1} - u_{N2})$.

[10] As a measure of absolute value creation, $u_{N2} - c_{Ni}$ abstracts from the consumer specific component of willingness to pay, d.

[11] Algebraically, q_{Ni}^D depends on relative value creation in segment 1 only arises because

$$\frac{S_1 u_{N1} + S_2 u_{N2}}{S_1 + S_2} - \frac{S_2(u_{N2} - u_{N1})}{S_1 + S_2} = u_{N1}.$$

We now restrict the parameter space so that the isolated and disruptive equilibria are the only two possible PSNE.

Lemma 3.4. *A sufficient condition for the new product to be bought by some consumers in segment 2 in any Nash equilibrium is*

$$u_{N2} > \frac{u_{N1} + n_N \bar{c}_N}{n_N + 1}. \tag{3.1}$$

Under this condition and $\min_i q_{Ei}^D \geq 0$*, the isolated and disruptive equilibria from Lemma 3.3 are the only possible PSNE.*

Henceforth, we assume that (3.1) holds and that $\min_i q_{Ei}^D \geq 0$. Because $u_{N2} > \bar{c}_N$, inequality (3.1) holds for n_N sufficiently large, as well as holding as long as u_{N1} is not too much greater than u_{N2}.[12]

3.3. Linking Model Parameters to Equilibrium Existence

In order to make precise statements about how the threat of disruption depends on parameters of the model, we introduce the following definitions.

Definition 3.5. *We say that* ***the set of parameters that support an equilibrium is increasing (decreasing) in parameter*** x *if existence for some* $x = x_1$ *implies existence for all* $x > x_1$ $(x < x_1)$*, holding fixed the value of all other parameters.*

Definition 3.6. *We say that* ***the threat of disruption is increasing (decreasing) in some parameter*** *if the set of parameters that support the disruptive equilibrium is increasing (decreasing) in the parameter and the set of parameters that support the isolated equilibrium is decreasing (increasing) in the parameter.*

By construction, the equilibrium outputs $q_{\lambda i}^{\phi}$ satisfy first order conditions and hence are locally optimal.[13] We now turn to global optimality. We denote the profit for the

[12] Note that inequality (3.1) is a sufficient but not necessary condition because the derivation assumes that $Q_E = 0$ and hence our analysis holds for even lower values of u_{N2}.

[13] Note that local optimality requires that total outputs lie on the appropriate part of the demand function. The condition for global optimality assures that this is the case.

ith new technology firm from its optimal deviation from the disruptive equilibrium onto the isolated part of the demand curve by

$$\hat{\pi}_{Ni}^{D} = \max_{q} q \left(u_{N2} - \frac{Q_N^D - q_{Ni}^D + q}{S_2} - c_{Ni} \right)$$

and we denote the optimal deviation by $\hat{q}_{Ni}^{D}$. Analogously, we denote the profit for the ith new technology firm from its optimal deviation from the isolated equilibrium onto the disruptive part of the demand curve by

$$\hat{\pi}_{Ni}^{I} = \max_{q} q \left(\frac{S_1 u_{N1} + S_2 u_{N2}}{S_1 + S_2} - \frac{Q_E^I + Q_N^I - q_{Ni}^I + q}{S_1 + S_2} - c_{Ni} \right)$$

and we denote the optimal deviation by $\hat{q}_{Ni}^{I}$. We denote the change in profit by

$$\Delta_{Ni}^{\phi} = \hat{\pi}_{Ni}^{\phi} - \pi_{Ni}^{\phi}.$$

Lemma 3.7. *A sufficient condition for the set of parameters that support equilibrium* $\phi = D, I$ *to be increasing (decreasing) in some parameter* x *is that the following hold for all* i: $\frac{\partial \Delta_{Ni}^{\phi}}{\partial x} \leq (\geq)\ 0$ *for for any set of parameters such that* $\Delta_{Ni}^{\phi} = 0$.

4. The Drivers of Disruption

The threat of disruption posed by the new technology varies as follows.

Proposition 4.1. *(i) The threat of disruption is increasing in* u_{N1}, c_{Ei} *and* S_1 *and decreasing in* u_{N2}, u_{E1} *and* S_2. *(ii) When* $c_{Ni} = c_N$ *for all* i, *the threat of disruption is increasing in* n_N; *when* $c_{Ei} = c_E$ *for all* i, *the threat of disruption is decreasing in* n_E.

The intuition for the results in Proposition 4.1 comes from the fact that disruption occurs when new technology firms find it profitable to pursue a low price, high volume strategy. The effect of each parameter comes from its impact on the profitability of such a strategy relative to the profitability of the lower volume, higher price strategy of isolation. Thus, for example, disruption is more likely the higher is u_{N1} because a higher utility of the new product in segment 1 means that the new-product price does not fall as much with disruption.

The effect of several of the parameters comes from their impact on the amount of additional volume generated by disruption. The bigger the size of segment 1 (S_1), the greater the potential to increase volume through disruption, and hence, the greater the threat of disruption. Similarly, the greater the output of the established technology firms (q_{Ei}), the less segment 1 demand is available for the new product and hence the lower the volume generated by disruption. From Lemma 3.3, we have that q_{Ei}^{ϕ} is decreasing in c_{Ei} and hence the threat of disruption increases with the cost of an established technology firm. On the other hand, q_{Ei}^{ϕ} is increasing in u_{E1} and hence the threat of disruption is decreasing. While the output of any individual established technology firm is falling in n_E, the total output of the established product, Q_E^{ϕ}, is increasing in n_E and hence the threat of disruption is decreasing in n_E.

The effects of the remaining parameters comes from their impact on the profitability of staying in segment 2. The larger the size of segment 2 (S_2), the more attractive it is for new technology firms to stay there. The higher the utility of the new product in the segment (u_{N2}), the greater the price premium for staying there. Finally, the more firms there are competing in segment 2 (n_N), the lower the price and the volume for any individual firm and the less attractive it is to stay there. Thus, the threat of disruption is increasing in S_2 and u_{N2} and decreasing in n_N.

Because market boundaries depend on the number of firms using each technology (i.e., on n_E and n_N), our theory has implications for merger analysis and we examine these more closely in Section 8. The effect of the costs of new technology firms on the threat of disruption is as follows.

Proposition 4.2. *(i) If $c_{Ni} = c_N + c_i$ for all i, the threat of disruption is decreasing in c_N. (ii) With multiple new technology firms and S_2 large relative to S_1, increases in own costs c_{Ni} increase firm i's incentive to disrupt. Formally, when $n_N > 1$, there exist critical values $s^I, s^D > 0$ such that $\frac{\partial \Delta_{Ni}^I}{\partial c_{Ni}} > 0$ when $\Delta_{Ni}^I = 0$ iff $S_1/S_2 < s^I$ and $\frac{\partial \Delta_{Ni}^D}{\partial c_{Ni}} < 0$ when $\Delta_{Ni}^D = 0$ iff $S_1/S_2 < s^D$. We have $\partial s^I/\partial n_N > 0$. (iii) Increases in new* *technology firm j's costs decrease the incentive of firm i to disrupt. Formally, $\frac{\partial \Delta^D_{Ni}}{\partial c_{Nj}} > 0$ when $\Delta^D_{Ni} = 0$ and $\frac{\partial \Delta^I_{Ni}}{\partial c_{Nj}} < 0$ when $\Delta^I_{Ni} = 0$. (iv) It is not necessarily the low cost firm which has the greatest incentive to deviate from the isolated equilibrium.*

According to part (i), uniform increases in the cost of new technology firms reduce the threat of disruption. This is because cost increases reduce margins and make the low price, high volume disruptive strategy less attractive.[14] Despite this result, we find in part (iv) that it is not necessarily the lowest cost firm that has the greatest incentive to deviate from the isolated equilibrium. On the one hand, the lowest cost firm has the highest margins among new technology firms, which favors output expansion and hence disruption. On the other hand, the lowest cost firm has the highest market share in the secondary segment and hence the most to loose from the fall in price that comes with disruption. In part (ii) we see that which effect dominates depends on segment size: the larger is segment 2, the less likely that the lowest cost firm initiates disruption and that this effect is greater the more new technology firms there are. Part (iii) shows that among the new technology firms an increase in a rival's cost decreases a firm's incentive to disrupt. This is because the firm's segment 2 market share is increasing in the costs of its rivals.

An important caveat to the results in this section are that we assume that all of the parameters are exogenous, while some parameters may affect the value of other parameters in some settings. For example, increasing the size of segment 2 might lower the costs of new technology firms (e.g., through experience effects) or increase the number of new technology firms (e.g., if the number of firms depends on the ability to cover fixed costs). Our results on the threat of disruption could be reversed if such indirect effects outweigh the direct effects. This offers an interesting avenue for future work, especially the possibility of endogenizing the number of firms using each technology.

[14] Our findings that the likelihood of disruption increases with the new technology's cost advantage $(\bar{c}_N - \bar{c}_E)$ and with the relative segment sizes (S_1/S_2) are consistent with Christensen's claims (1997, pages 15 and 81). Our finding is in contrast to the assumption in Klepper (1996) that cost reductions do not serve to attract new customers to a firm.

5. Existence and Uniqueness

We now turn to the question of whether PSNE exist and, given existence, whether they are unique. From Proposition 4.1, we know that critical values of u_{N2}^{D} and u_{N2}^{I} exist such that the disruptive equilibrium exists for $u_{N2} \leq u_{N2}^{D}$ and the isolated equilibrium exists for $u_{N2} \geq u_{N2}^{I}$. The questions of existence and uniqueness can be reduced to asking what is the is the ordering of u_{N2}^{D} and u_{N2}^{I}. For example, if $u_{N2}^{D} = u_{N2}^{I}$ then there is a unique PSNE for all values other than $u_{N2} = u_{N2}^{D}$. To characterize this ordering, we consider how these critical values depend on the parameter u_{N1}. We first define the critical values of u_{N2} such that firm i is indifferent about deviating from equilibrium ϕ:

$$u_{N2i}^{\phi}(x) = \{y : \Delta_{Ni}^{\phi} = 0 \text{ for } u_{N1} = x \text{ and } u_{N2} = y\}$$

Aggregating over all firms we then have

$$\begin{aligned} u_{N2}^{I}(x) &= \max_{i} u_{N2i}^{I}(x), \\ u_{N2}^{D}(x) &= \min_{i} u_{N2i}^{D}(x). \end{aligned}$$

These functions are linear and are illustrated in Figure 5.1. Note that the disruptive equilibrium exists to the right of the $u_{N2}^{D}(u_{N1})$ line and the isolated equilibrium exists to the left of the $u_{N2}^{I}(u_{N1})$ line. Where $u_{N2}^{I}(u_{N1}) > u_{N2}^{D}(u_{N1})$ there is a region with no PSNE and where $u_{N2}^{I}(u_{N1}) < u_{N2}^{D}(u_{N1})$ there is a region with multiple PSNE. Outside of these two regions there is a unique PSNE.[15]

The ordering depicted in the figure is general in that:

Lemma 5.1. *The linear functions $u_{N2}^{D}(u_{N1})$ and $u_{N2}^{I}(u_{N1})$ cross once, with $u_{N2}^{D}(u_{N1})$ cutting $u_{N2}^{I}(u_{N1})$ from below. The difference between the slope of $u_{N2}^{D}(u_{N1})$ and $u_{N2}^{I}(u_{N1})$ is increasing in n_E and n_N.*

[15] The points to the right of the dashed lines in Figure 5.1 fall outside of the boundaries specified in Lemma 3.1. The points with low u_{N2} are where the new technology bypasses segment 2 and just enters segment 1. The points with high u_{N1} are where output of the established technology firms is driven to zero.

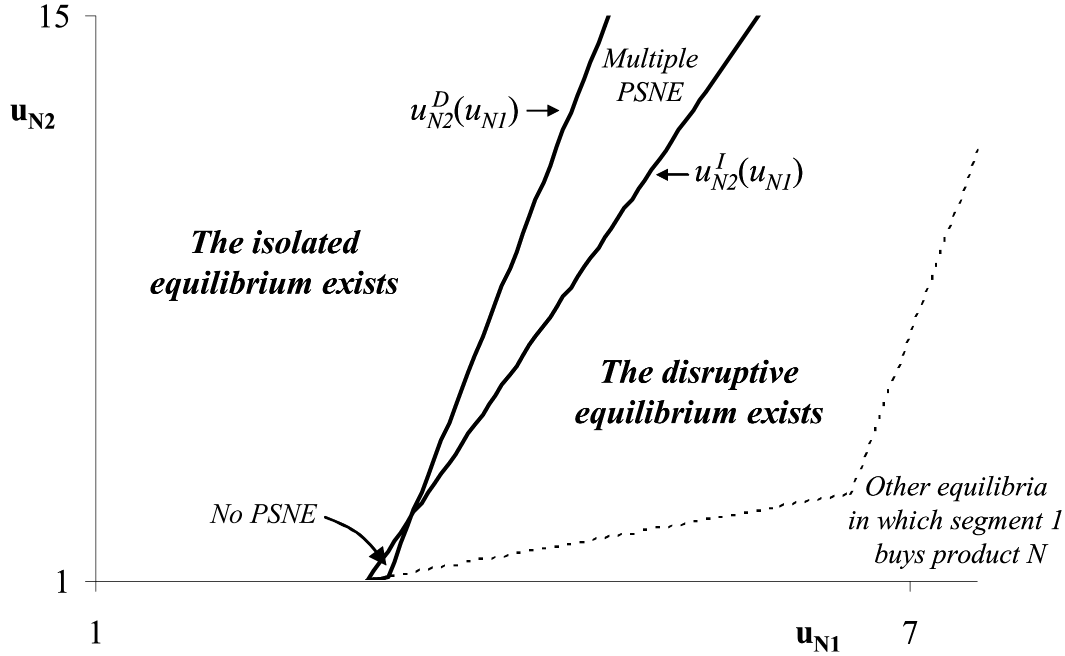

Figure 5.1: The set of pure strategy Nash equilibria as a function of (u_{N1}, u_{N2}) when $S_1 = 4S_2$, $n_E = n_N = 2$, $u_{E1} = 4$ and $c_{E1} = c_{E2} = c_{N1} = c_{N2} = 1$.

While there may be a unique PSNE, it is also possible to have either no PSNE or multiple equilibria. That the model exhibits both multiplicity and nonexistence is due to the possibility that outputs can be either strategic complements or strategic substitutes (see Section 3.1). Non existence of pure strategy equilibria can occur when the output of the new technology firms is a strategic complement for the established firms. Then, when established technology firms expect isolation, they have a low output and this can trigger disruption; when they expect disruption they have a high output and this can make disruption unattractive. When both occur, neither an expectation of disruption nor one of isolation can form the basis of a Nash equilibrium and a PSNE does not exist.

There are two sources of multiplicity in the model. When the output of the new technology firms is a strategic substitute for the established firms, then an expectation of disruption leads to a low output by the established technology firms and this can trigger disruption that would not occur if the established technology firms did not expect it. Second, when $n_N > 1$ the upward jump in the best response functions among new technology firms (see Figure 3.2) can give rise to multiple PSNE: the expectation that other new technology firms are playing a high-volume, disruptive strategy can cause a self-fulfilling escalation of production plans.

In Appendix II, we exhibit a mixed strategy equilibrium (for the case of $n_N = 1$) that exists when there is no PSNE. Numerical analysis indicates that the comparative statics on the threat of disruption in Proposition 4.1 holds for the probability of disruption in the mixed strategy equilibrium.

6. The Effect of Improving Technologies

Does the threat posed by a new technology increase as the technologies mature? Recall that utility levels in the model increase over time as the technologies mature, $u'_{\lambda m}(t) \geq 0$. Formally, we are interested in whether the threat of disruption is increasing t.

We know from Proposition 4.1 that the threat of disruption is increasing in u_{N1} and decreasing in u_{N2} and u_{E1} and hence results will depend critically on the relative rates at which the different utilities are increasing over time. Thus, the extent to which the established technology has exhausted its ability to increase consumer utility relative to the new technology ("performance over supply") does affect the threat posed by a new technology, but it not the only driver. Figure 6.1 illustrates how the new technology's **trajectory**—the relative rate at which u_{N1} and u_{N2} increase over time—influences whether or not there is disruption. As u_{N1} and u_{N2} increase along their trajectory, the solid lines in Figure 6.1, the set of PSNE change when the trajectory intersects with the critical thresholds $u^I_{N2}(u_{N1})$ and $u^D_{N2}(u_{N1})$.

We start with the limiting case of performance oversupply where the utility from the established technology is no longer increasing. Under what conditions does the disruptive threat increase over time? Does it increase even if $u'_{N2}(t) > u'_{N1}(t)$ and the new technology trajectory favors segment 2?

Proposition 6.1. *Suppose $u'_E(t) = 0$. There exists an $r \in (0, \frac{1}{n_N+1})$ such that the*

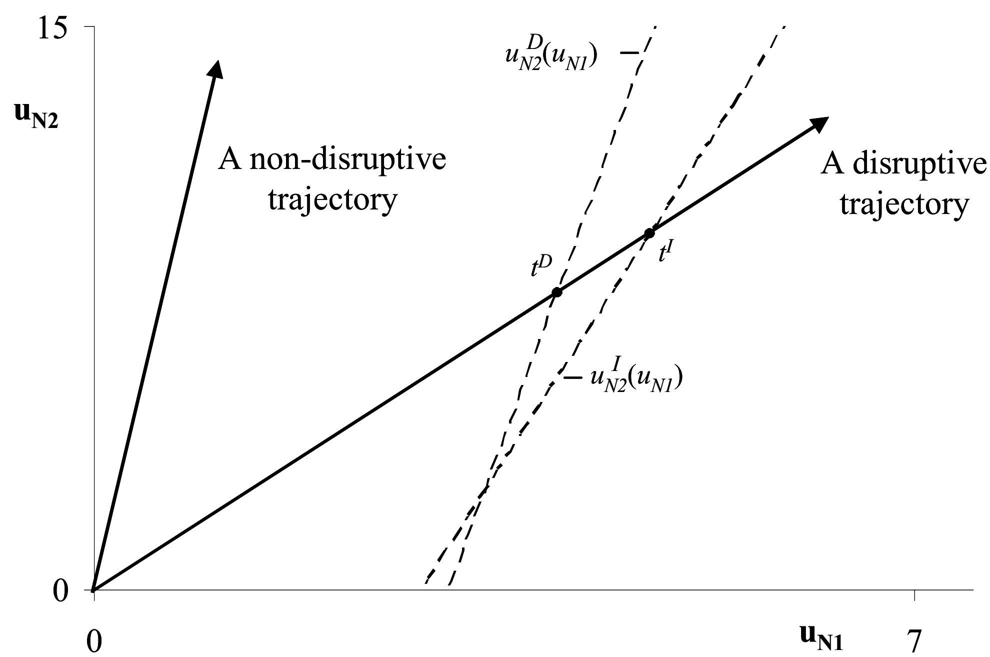

Figure 6.1: An illustaration of a disruptive and a non- disruptive new technology trajectory for the case where $u'_{E1}(t) = 0$ and hence $u^D_{N2}(u_{N1})$ and $u^I_{N2}(u_{N1})$ are constant over time.

threat of disruption is increasing in t iff $u'_{N1}(t) > ru'_{N2}(t)$. The parameter r is decreasing in n_N and S_1 and increasing in S_2.

We find that unless $u'_{N2}(t)$ is improving much faster than $u'_{N1}(t)$, the threat posed by the new technology does increase over time. For intuition as to why r, the critical ratio of improvement rates, is at most $\frac{1}{n_N+1}$, consider the breakdown of the isolated equilibrium. In this equilibrium, the surplus available to segment 1 consumers from buying the new product is $u_{N1}(t) - P_N(Q^I_N, Q^I_E)$ where $P_N(Q^I_N, Q^I_E) = \frac{u_{N2}(t)+n_N\bar{c}_N}{n_N+1}$ is the equilibrium price of the new product. This surplus is increasing over time as long as $u'_{N1}(t) > \frac{1}{n_N+1}u'_{N2}(t)$. Since the surplus from the established product is constant over time when $u'_{E1}(t) = 0$, the relative attractiveness of the new product to segment 1 is increasing over time. In general, we find that r is decreasing in n_N because more competition leads to more of the surplus going to consumers. The larger is S_1 relative to S_2 the more willing are new technology firms to give surplus to consumers in order to enter segment 1 and hence the lower is r.

Consider how Proposition 6.1 applies to the functional form, $u_{N1}(t) = b_{N1}t^{\beta}$ and $u_{N2}(t) = b_{N2}t^{\beta}$. The parameters b_{N1} and b_{N2} reflects both the rate of improvement in product performance and customers willingness to pay for those improvements. (The parameter $\beta \in (0,1]$ reflects both decreases in the rate of technological progress and the extent to which consumers exhibit decreasing marginal utility for product improvements.) The trajectory is then $u'_{N2}(t)/u'_{N1}(t) = b_{N2}/b_{N1}$ and disruption occurs for t sufficiently large if $b_{N1} > \frac{1}{n_N+1}b_{N2}$.

Now consider the possibility that the established technology is also improving, $u'_{E1}(t) > 0$. In Figure 6.1, this corresponds to a rightward shift over time in the $u^{\phi}_{N2}(u_{N1})$ functions. We focus on the case where all utilities are increasing at the same rate.

Proposition 6.2. *Suppose $u'_{E1}(t) = u'_{N1}(t) = u'_{N2}(t)$. (i) There exist critical values $n_2 > n_1 > 0$ such that for $n_E < n_1$ the threat of disruption is increasing in t and for $n_E > n_2$ the threat of disruption is decreasing in t. (ii) The critical values n_1 and n_2 are increasing in n_N and S_1/S_2.*

The intensity of rivalry among established technology firms as given by n_E matters because it effects the rate at which surplus from the established product is increasing over time. As long as established technology firms are appropriating a sufficient amount of the increasing utility (i.e. $n_E < n_1$), we find that the disruptive threat still increases over time even though the established technology matches the advances of the new technology.

We conclude that performance over supply facilitates disruption, but is not necessary for the dynamics that Christensen describes. Other factors that influence the dynamics are the new technology's trajectory $u'_{N2}(t)/u'_{N1}(t)$, the relative segment sizes (S_1/S_2), and the extent of rivalry amongst each group of firms (n_E and n_N).

Having explored the effects of time on the threat of disruption, we now suppose that there exist critical time thresholds t^D and t^I that determine whether or not these equilibria exist. See Figure 6.1 for an illustration. We now characterize the factors that determine these thresholds and hence the immediacy of any disruptive threat.

Proposition 6.3. *Suppose the disruptive (isolated) equilibrium exists* iff $t \geq t^D$ $(t \leq t^I)$. *Then* t^D (t^I) *is increasing (decreasing) in* S_2 *and* n_E *while it is decreasing (increasing) in* c_{Ei}, S_1, *and* n_N. *If* $u_\theta(t) = b_\theta t^\beta$ *for* $b_\theta > 0$ *and* $\theta \in \{E1, N1, N2\}$, *then* t^D (t^I) *is increasing (decreasing) in* b_{N1} *and decreasing (increasing) in* b_{E1} *and* b_{N2}.

Parameters have the same effect on the time to disruption as they had on the threat of disruption in Proposition 4.1. While u_{N1}, u_{N2} and u_{E1} matter at a point in time, it is the rates of increase in these utilities that matter over time.

It is possible that the threat of disruption is neither increasing nor decreasing for all t and hence it is possible for a technology to be only *temporarily* disruptive. This can occur if the relative curvatures of the $u_{\lambda m}(t)$ functions change over time. Consider, for example, $u_{N1}(t) = u_{N2}(t) = bt^\beta$, and $u_{E1}(t) = b(t+h)^\beta$ where the parameter h reflects a head start for the established technology. Then, for h large and t small, $u'_{E1}(t) = b\beta/(t+h)^{1-\beta}$ will be much smaller than $u'_{N1}(t) = u'_{N2}(t) = b\beta/t^{1-\beta}$ so that the threat of disruption initially increases over time, following the logic of Proposition 6.1. However, for sufficiently large t the effect of h fades and then, for n_E sufficiently small, the threat of disruption recedes, following the logic of Proposition 6.2.

7. The Effects of Disruption

We are interested in the effect of disruption on outputs, prices, social welfare, market shares, profitability and innovation incentives. Our approach is to compare these outcomes when the disruptive and the isolated equilibria both exist, which corresponds to a transition from isolation to disruption occurring as the new technology's trajectory moves through the multiple equilibrium region (see Figure 6.1).

Proposition 7.1. *Suppose both the isolated and the disruptive equilibrium exist. A shift from the isolated to the disruptive equilibrium lowers the price of both products* *and increases the output of the new technology firms. Social welfare is higher in the disruptive equilibrium and all consumers are weakly better off.*

Disruption is fundamentally an increase in rivalry as each firm faces more competitors. This leads to a fall in product prices towards marginal costs, which raises social welfare and leaves all consumers who purchase strictly better off.[16] Because of their non-monotonic best response function, the output of the established firms can either increase or decrease with disruption.[17]

Does disruption increase the profits of new technology firms at the expense of established technology firms? Not necessarily:

Proposition 7.2. *Suppose both the disruptive and the isolated equilibrium exist. (i) If $n_N = 1$, $\pi_{N1}^D \geq \pi_{N1}^I$. (ii) If $n_N > 1$, then for $S_2 u_{N2}$ sufficiently small and u_{N2} sufficiently large, $\pi_{N1}^D < \pi_{N1}^I$. (iii) If $c_{Ei} = \bar{c}_E$, $\pi_{Ei}^D < \pi_{Ei}^I$.*

When there is only one new technology firm, it can always guarantee itself at least its profits in the isolated equilibrium. Hence, its profits in the disruptive equilibrium must be at least as great. This is not the case when there are more than one new technology firm because they may face a coordination problem (due to the upward jump in their best response functions). When $S_2 u_{N2}$ is large, new technology firms will have high profits in the isolated equilibrium. However, when S_2 is sufficiently small, the output of the new technology firms is so great under an expectation of disruption that no single firm can (profitably) restore the isolated outcome by unilaterally reducing its output and hence the disruptive equilibrium will exist as well. Thus, a shift from isolation to disruption can lower the profits of new technology firms when they start in a small but high margin segment.

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[16] These results hold for the mixed strategy equilibrium with $n_N = 1$ analyzed in Appendix II.

[17] The ratio of outputs is

$$\frac{Q_E^D}{Q_E^I} = \left(\frac{n_E + 1}{n_E + n_N + 1}\right)\left(\frac{S_1 + S_2}{S_1}\right)\left(1 + \frac{R_E}{u_{E1} - \bar{c}_E}\right).$$

and established technology firms are more likely to increase output the larger is n_E and S_2 relative to n_N and S_1. The quantity R_E is defined later in this section. In the mixed strategy equilibirum, Q_E is increasing in the probability of disruption.

The increase in competition that comes with a shift from isolation to disruption reduces the profits of the established technology firms, at least if they have average costs. The reason for the restriction to firms with average costs is that disruption can either amplify or dampen the effects of cost asymmetries on market shares and relative profitability, as shown in Section 7.1.[18]

To facilitate the statement of results in the Sections 7.1 and 7.2, it is convenient to define the following:

$$\begin{aligned} R_N &= n_E \left[(u_{N1} - \bar{c}_N) - (u_{E1} - \bar{c}_E) \right] + \frac{S_1}{S_1 + S_2}(u_{N1} - u_{N2}), \\ R_E &= n_N \left[(u_{E1} - \bar{c}_E) - (u_{N1} - \bar{c}_N) \right] - \frac{S_2}{S_1 + S_2}(u_{N1} - u_{N2}). \end{aligned}$$

R_λ depends on a technology's relative value creation in segment 1 with an adjustment for the extent to which the new technology is committed to segment 1. Note that there is no *a priori* restriction on the sign of R_N or R_E, but there is a tendency for them to have the have the opposite sign (e.g., for $n_N = n_E$ and $S_1 = S_2$ we have $R_N = -R_E$).

7.1. The Effect of Disruption on Market Share

We now characterize the effect of a shift from isolation to disruption on firm market shares. We focus on a firm's share of a particular product market, that is on q_{Ni}/Q_N or q_{Ei}/Q_E.

Consider the market share of the ith new technology firm in the isolated equilibrium,

$$\frac{q^I_{Ni}}{Q^I_N} = \frac{1}{n_N} \frac{u_{N2} - c_{Ni} + n_N(\bar{c}_N - c_{Ni})}{u_{N2} - \bar{c}_N}.$$

First notice that a firm's market share is increasing in $n_N(\bar{c}_N - c_{Ni})$: firms with low costs have higher market share, and this effect is increasing in the number of rivals. Moreover, what matters are cost asymmetries relative to average value creation, $u_{N2} - \bar{c}_N$. That

[18] We have identified parameters for which disruption increases the profits of a low cost established technology firm. Numerical analysis of a large parameter space failed to turn up any parameters for which a high cost firm had an increase in profits from disruption.

is, the market share of low cost firms is falling as average value creation across new technology firms increases because their cost advantage looms less large. Conversely, the market share of firms with above average costs are strongly decreasing in the number of rivals n_N and increasing in $u_{N2} - \bar{c}_N$.

Now consider the effect of a shift from isolation to disruption. There are two effects. First, the number of competitors increases to $n_N + n_E$ and this will tend to increase the market share of low cost firms at the expense of their high cost rivals. Second, the effective value creation of new technology firms is now R_N. As long as R_N is not too large relative to $u_{N2} - \bar{c}_N$, the effect of cost asymmetries on market share are amplified by disruption:

Proposition 7.3. *(i) The shift in market share* $\frac{q^D_{Ni}}{Q^D_N} - \frac{q^I_{Ni}}{Q^I_N}$ *has the same sign as* $\bar{c}_N - c_{Ni}$ *iff* $R_N < \frac{n_E}{n_N+1}(u_{N2} - \bar{c}_N)$. *(ii) The shift in market share* $\frac{q^D_{Ei}}{Q^D_E} - \frac{q^I_{Ei}}{Q^I_E}$ *has the same sign as* $\bar{c}_E - c_{Ei}$ *iff* $R_E < \frac{n_N}{n_E+1}(u_{E1} - \bar{c}_E)$.

In part (ii) we see that there is an analogous result for established technology firms. Thus, barring a large shift in effective value creation (R_λ), we find that disruption increases standard measures of concentration such as the Herfindal index, $\sum_{i=1}^{n_\lambda}(q_{\lambda i}/Q_\lambda)^2$, or the n-firm concentration ratio.

7.2. The Effect of Disruption on Innovation Incentives

A first step towards endogenizing technology improvement in the model is to consider innovation incentives and how these change with disruption. A reduction in a firm's marginal cost is equivalent in our model to a corresponding increase in all of its utility levels. A firm's incentives to lower costs in Cournot models are increasing in the number of competitors and in the firm's output. In particular, we have

$$\begin{aligned}
\frac{\partial \pi^I_{Ni}}{\partial c_{Ni}} &= -2\frac{n_N}{n_N+1}q^I_{Ni}, \\
\frac{\partial \pi^D_{Ni}}{\partial c_{Ni}} &= -2\frac{n_N+n_E}{n_E+n_N+1}q^D_{Ni}.
\end{aligned}$$

There are two effects from a shift from isolation to disruption. First, the increase in the number of competitors increases the incentive to innovate (i.e. $\frac{n_N}{n_N+1} < \frac{n_N+n_E}{n_E+n_N+1}$). Second, an increase in output increases the incentive to innovate, and while not all new technology firms necessarily experience an increase in output, we do know from Proposition 7.1 that average output increases. Thus, we have shown the following:

Proposition 7.4. *Averaging across new technology firms, the incentive to lower costs is greater under disruption than under isolation.*

This suggests that the threat posed by a new technology increases after disruption because innovation incentives increase. While established technology firms also experience an increase in number of competitors, their output might fall sufficiently to offset that effect. Hence the analogous result does not hold from established technology firms. Moreover, one might expect that innovation incentives are more important for the new rather than the established technology because there is more scope for improvement.

While some innovative activity, such as cost reductions, have an equal impact on value creation in both segments, other innovative activity will disproportionately benefit one segment. For example, efforts to reduce the power consumption of a disk drive mostly benefit the portable segment. Such considerations are important for the new technology after disruption. Two comparisons are of interest. The first is the incentive to improve segment 1 utility, $\frac{\partial \pi_{Ni}^D}{\partial u_{N1}}$, relative to the incentive to improve segment 2 utility, $\frac{\partial \pi_{Ni}^D}{\partial u_{N2}}$. Relative incentives are particularly important when the firm must choose between different R&D projects because of limited resources. The second comparison of interest is the incentive to improve utility for segment 2 under isolation, $\frac{\partial \pi_{Ni}^I}{\partial u_{N2}}$, and under disruption, $\frac{\partial \pi_{Ni}^D}{\partial u_{N2}}$. We can address these issues for a monopolist new technology firm.[19]

Proposition 7.5. *Suppose $n_N = 1$. (i) $\frac{\partial \pi_{Ni}^D}{\partial u_{N1}} > \frac{\partial \pi_{Ni}^D}{\partial u_{N2}} > 0$. (ii) There exists some $k > 0$ such that $\frac{\partial \pi_{Ni}^I}{\partial u_{N2}} > \frac{\partial \pi_{Ni}^D}{\partial u_{N2}}$ iff $R_N \leq k$.*

[19] We restrict attention to $n_N = 1$ because our prior analysis can accomodate new technology firms increasing their utilities only if the improvement is the same across segments (and hence is equivalent to a reduction in costs).

Under disruption, a monopolist new technology firm always has a greater incentive to target innovation at segment 1 than at segment 2. This is true even if segment 2 is larger than segment 1. The reason for the strong bias towards segment 1 consumers is that increases in $(u_{N1} - u_{N2})$ serve to commit the firm to segment 1, which leads to greater accommodation by the established technology firms. Despite this commitment effect, it is still the case that $\frac{\partial \pi^D_{Ni}}{\partial u_{N2}} > 0$. The commitment effect lowers $\frac{\partial \pi^D_{Ni}}{\partial u_{N2}}$ relative to $\frac{\partial \pi^I_{Ni}}{\partial u_{N2}}$ and, as long as R_N is not too large, the absolute incentive to pursue improvements targeted at segment 2 falls with disruption. Proposition 7.5 suggests a caveat to our earlier welfare result. Although the price declines that accompany disruption make all consumers better off initially, consumers in segment 2 might possibility be worse off in the long-run because the focus of innovative activities shifts towards segment 1.

8. Merger Analysis

Although Section 6 focuses on disruption triggered by improving technologies, disruption can result from shifts in any of the parameters. Shifts in the number of firms, n_E and n_N, are of particular interest as they are often associated with mergers and acquisitions and hence are subject to regulatory approval. There is a prior literature on mergers in Cournot models (Salant et al, 1983; Perry and Porter, 1985; Farrell and Shapiro, 1990) that considers what happens to prices, profits and welfare when Cournot competitors merge. One of the central issues in that literature is whether mergers are profitable for the merging firms. While mergers raise total industry profits, it is quite possible that the profits of the merged entity is less than the sum of the profits of the merging firms prior to the merger. For example, Salant et al. (1983) show that a merger is only profitable if it involves at least 80% of the firms in an industry for the case of linear demand and constant marginal costs. Subsequent papers focus on how efficiency gains ("synergies") can increase the scope for profitable mergers. Absent such efficiency gains, mergers of Cournot competitors cause price to rise and welfare to fall.

Prior results on profits, prices and welfare need not hold in our model because we allow mergers to alter the boundaries of competition, a possibility not considered in the received literature. Because the threat of disruption is increasing in n_E (Proposition 4.1), a merger among established firms can lead to the emergence of technology competition. Conversely, because the threat of disruption is decreasing in n_N, a merger among new technology firms can eliminate technology competition:

Proposition 8.1. *(i) For any $n_N \geq 2$, there exist parameter values such that a merger by two of the new technology firms causes the unique PSNE to shift from disruption to isolation. (ii) For any $n_E \geq 2$, there exist parameter values such that a merger by two of the established technology firms causes the unique PSNE to shift from isolation to disruption.*

To see that changes in the boundaries of competition can reverse prior results on the effects of mergers, consider the following example. There are originally three firms of each type, $n_E = n_N = 3$, all of whom have the same constant marginal costs, $c_E = c_N = 1$. Both technologies give the same utility to consumers in segment 1, $u_{N1} = u_{E1} = 5$, while the new technology gives somewhat higher utility to consumers in segment 2, $u_{N2} = 6$. Segment 1 is twice the size of segment 1, $S_1 = 2S_2 = 2$. We denote the consumer surplus in segment m by $\mathcal{C}_m$. First, consider the effect of a merger between two of the new technology firms. The first two columns of Table 8.1 report the unique PSNE, prices, profits, consumer surplus and social welfare before and after the merger; values are rounded to one decimal place.

As in part (i) of Proposition 8.1, the merger of the new technology firms shifts play from the disruptive to the isolated equilibrium. As a result, the decrease in rivalry from the merger far exceeds that which occurs in a standard Cournot model and the post-merger profits of 2.8 for the combined entity are greater than the pre-merger profits of 2.4 for the two merging firms—the merger is profitable despite involving a small number of firms and there being no synergies.

We can contrast the effect of a merger in our model to that in a standard Cournot model that is calibrated to yield equivalent outcomes prior to the merger. Given that $c_N = c_E$ and $u_{N1} = u_{E1}$ in the example, the profits and prices under the disruptive

	Pre-Merger	**Post-Merger I**	**Post-Merger II**
	$n_E = n_N = 3$	$n_E = 3$, $n_N = 2$	$n_E = n_N = 2$
Unique PSNE	disruption	isolation	disruption
Prices P_N	1.6	2.7	1.9
P_E	1.6	2.0	1.9
Profits π_{Ni}	1.2	2.8	2.3
π_{Ei}	1.2	2.0	2.3
Surplus $\mathcal{C}_1$	11.4	9.0	9.8
$\mathcal{C}_2$	9.6	5.6	8.6
Social Welfare	27.9	26.1	27.4

Table 8.1: The effect of two sequential mergers when $u_{N1} = u_{E1} = 5$, $u_{N2} = 6$, $c_E = c_N = 1$, $S_1 = 2S_2 = 2$ and initially $n_N = n_E = 3$.

equilibrium in our model are exactly the same as the profits and prices in an n-firm Cournot model with $n = n_E + n_N$, demand given $P(Q) = \frac{S_1 u_{N1} + S_2 u_{N2}}{S_1+S_2} - \frac{Q}{S_1+S_2}$, and constant marginal costs of 1 for all firms.[20] In this standard Cournot model a reduction in n from 6 to 5 only raises prices from 1.6 to 1.7, while firm profits increase from 1.2 to 1.6, with both effects being considerably below what occurs in our model. In the standard Cournot model the merger reduces social welfare by less than one percent (from 27.6 to 27.4) whereas in our model it falls over six percent (from 27.9 to 26.1).

Now consider a subsequent merger by two of the established technology firms, a shift from column two to three of Table 8.1. As in part (ii) of Proposition 8.1, the merger of the established technology firms shifts play from the isolated to the disruptive equilibrium. Despite the decrease in the number of firms, there is a net increase in rivalry. As a result, prices fall and social welfare increases. Not only do the profits of the merging firms fall (from 4 to 2.3), but total industry profits fall as well (from 11.6 to 9.2). Finally, note the asymmetry across segments in the effect of mergers on consumer surplus.[21]

[20] The only difference is that consumer surplus is slightly higher in our model since for low output consumer willingness to pay is higher than $P(Q)$.

[21] The small net effect of the two mergers on social welfare (from 27.9 to 27.4) is approximately the same as the net effect of going from $n = 6$ to $n = 4$ in the standard Cournot model with $P(Q) = \frac{S_1 u_{N1} + S_2 u_{N2}}{S_1+S_2} - \frac{Q}{S_1+S_2}$ and constant marginal costs of 1.

9. Extensions

9.1. Multi-Technology Firms

In addition to describing the phenomena of disruptive technologies, Christensen (1997) seeks to explain the "innovator's dilemma," an observation that established firms often do not exploit a disruptive technology despite having the capability to do so. Indeed, in many of Christensen's examples, the new technology is developed by an established firm, which then chooses not to commercialize it. Christensen's explanation of the innovator's dilemma, which he bases on the sociology of resource dependence (Pfeffer and Salancik, 1978), is that a firm's technology decisions are inordinately influenced by the needs of the existing customers in the primary segment, who are not interested, at least initially, in the new technology.

To explore the incentives for established firms to use a disruptive technology, we extend our model to allow some firms to use both technologies. In particular, suppose that $n_M < \min(n_E, n_N)$ firms have the capability to use both technologies and let these be the first n_M firms of each type. We have that firm $i \leq n_M$ produces $q_{Ni} \geq 0$ units using the new technology at marginal costs c_{Ni} and $q_{Ei} \geq 0$ units using the established technology at marginal cost c_{Ei}. Thus, the maximization problem of a multi-technology firm is

$$\max_{q_{Ei}\geq 0,\ q_{Ni}\geq 0} q_{Ei}(P_E(Q_E, Q_N) - c_{Ei}) + q_{Ni}(P_N(Q_N, Q_E) - c_{Ni}).$$

The first order conditions in a disruptive equilibrium for multi-technology firm i are

$$\begin{aligned} P_E - c_{Ei} - \frac{q_{Ei} + q_{Ni}}{S_1 + S_2} &= 0, \\ P_N - c_{Ni} - \frac{q_{Ei} + q_{Ni}}{S_1 + S_2} &= 0. \end{aligned}$$

That is, optimal output trades off the margins, $P_\lambda - c_{\lambda i}$, earned from an additional sale with the decline in margins, $P_E' = P_N' = -\frac{1}{S_1+S_2}$, times total output, $q_{Ei} + q_{Ni}$. Since the decline in margins is the same across products in our model, there is a corner solution where the firm produces using *only* the technology that has the higher margin.

In a disruptive equilibrium consumers in segment 1 must be indifferent between the two products and hence we have $P_E - P_N = u_{E1} - u_{N1}$ so that the difference in margins is $(P_E - c_{Ei}) - (P_N - c_{Ni}) = (u_{E1} - c_{Ei}) - (u_{N1} - c_{Ni})$. We have shown the following:

Proposition 9.1. *In a disruptive equilibrium, multi-technology firm i only uses the production technology for which it has higher value creation, $u_{\lambda 1} - c_{\lambda i}$, in segment 1.*

Our model elucidates the innovator's dilemma. In cases where the established technology is better suited to segment 1, not using the new technology is profit maximizing after disruption. Echoing Christensen, we show that focusing on the technology that best suits the primary segment can be consistent with short-run profit maximization. Although an established firm will want to switch to the new technology if it progresses to a point where it has superior value creation, (unmodeled) early mover advantages for the new technology firms may keep it from doing so successfully.

9.2. Price Discrimination

Our base model assumes no price discrimination based on segment. This is consistent with most of the leading examples of disruptive technologies (e.g., disk drives, printers, earthmoving equipment and retail formats). Price discrimination, however, is important in some contexts. For example, if the segments are geographically distinct markets and there are transportation costs, then price discrimination is possible.

Formally, we assume that with price discrimination across segments, new technology firms can choose separate quantities $q_{Ni,1}$ and $q_{Ni,2}$ to sell into market segments 1 and 2, respectively. The model is considerably simplified by this assumption because each segment can be analyzed separately. We limit ourselves to generalizing the results on the drivers of disruption from Section 4.

Proposition 9.2. *(i) Suppose new technology firms can price discriminate across segments. There is a unique Nash equilibrium. The threat of disruption is increasing in u_{N1} and c_{Ei}, decreasing in n_E, u_{E1} and $\min_i c_{Ni}$, and independent of S_1, S_2, n_N and u_{N2}. (ii) If the technologies are isolated with price discrimination, they are necessarily isolated without price discrimination.*

In part (i) we see that the effects of u_{N1}, u_{E1}, n_E, and c_{Ei} on disruption are the same with and without price discrimination. With price discrimination the firms do not need to trade off profits in each segment and hence we find that u_{N2}, S_1, S_2 and n_N no longer affect disruption. Moreover, without this trade-off it is unambiguously the low cost firm that has the greatest incentive to disrupt. In part (ii) we see that the threat of disruption is greater with price discrimination. Thus, we find that the ability to price discriminate across market segments is a potentially important determinant of the boundaries of technology competition.

9.3. An Alternative Demand Specification

In an earlier version of the paper (Adner and Zemsky, 2001), we work with an alternative demand specification that builds on Shaked and Sutton (1982). Specifically, we assume that consumers in segment m have a taste for quality θ that is uniformly distributed over $[0, 1]$ with density $S_m > 0$ and that the willingness to pay of a consumer in segment m is $\theta u_{\lambda m}$ (rather than the additive specification $u_{\lambda m} + d$ we use in this paper). The prior specification is less tractable. In the earlier version, we only analyze the isolated equilibrium and we assume homogeneous costs (i.e., $c_{Ni} = c_N$ and $c_{Ei} = c_E$). However, we do study both the Cournot equilibrium with an arbitrary number of firms and a Bertrand duopoly (i.e., $n_N = n_E = 1$).[22] The results in Proposition 4.1 on the set of parameters that support the isolated equilibrium all hold in the alternative demand specification. We show that the set of parameters that support the isolated equilibrium is smaller under Bertrand than Cournot competition.[23]

[22] For the Cournot analysis we restrict attention to $u_{E1} > u_{N1}$ because the inverse demand function changes when this condition reverses.

[23] We expect that the results from Proposition 4.1 on the disruptive equilibrium would hold in the alternative specification as well. One result that might well be sensative to our modeling of demand is the stark result on multi-technology firms: in the alternative demand specification, there is not one best technology for all consumers in segment 1.

10. Conclusion

Despite its importance for technology competition and antitrust deliberations, market definition is usually taken as a purely empirical question. The standard approach in theoretical work in industrial organization is to take as given the set of competing firms and the set of relevant consumers. In contrast, we take the boundaries of competition as one of our central research question. Inspired by recent empirical observations of disruptive technologies, we use a model of horizontal and vertical differentiation to explore how market boundaries are shaped not just by consumer preferences, but also by the profit maximizing behavior of firms.

Our results hold a variety of implications for firm strategy. For established technology firms, we show that even if the new technology's trajectory is significantly better suited to its niche, the technology may still prove to be disruptive and we identify a variety of factors (costs, segments sizes, etc.) that must be weighed in assessing the likelihood of disruption. For new technology firms, we show that it is not necessarily the market leader in the niche that has the greatest incentive to pursue a disruptive strategy. Further, we show that disruption should not be blindly embraced by new technology firms because it can lead to a fall in their profits. Given the possibility of multiple equilibria, both types of firms may benefit from actions to shape industry expectations in order to coordinate play on their preferred outcome. While it is possible that all firms prefer the isolated equilibrium because there is less competition, it is also possible that new technology firms benefit from an expectation of disruption because established technology firms reduce their output in anticipation. Finally, we show that mergers among established technology firms can actually increase industry rivalry if they alter the existing boundaries of competition.

There are many directions in which the analysis could be extended. One could endogenize the number of firms using each technology and then study the effect of disruption on entry and exit of each type of firm. The current model is only weakly dynamic and there is ample room to develop more dynamic models in which there are inter-temporal linkages in firm profit functions. For example, one could allow firms to choose their technology trajectory as in the simulations of Adner (2002). In addition, one could study a variety of strategies that established firms could pursue in an effort to deter disruption such as making strategic commitments and investing in complementary assets (Teece, 1986).[24]

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Appendix I: Proofs

Proof of Lemma 3.3 It is useful to start by recalling the n-firm Cournot equilibrium with linear demand and homogeneous products. Suppose $P(Q) = u - Q/S$ and there are n firms indexed by i with marginal costs c_i. Each firm maximizes profits $\pi_i = q_i(u - c_i - \sum_{j=1}^{n} q_j/S)$ or equivalently

$$\pi_i = q_i(v_i - \sum_{j=1}^{n} q_j/S) \text{ where } v_i = u - c_i. \tag{10.1}$$

Let $\bar{v} = \sum_{i=1}^{n} v_i/n$. Assuming $|v_i - \bar{v}|$ sufficiently small, there is a unique Nash equilibrium with output and profits given by

$$\begin{aligned} q_i^* &= \frac{S}{n+1}(v_i + n(v_i - \bar{v})), \\ \pi_i^* &= (q_i^*)^2/S. \end{aligned}$$

Any PSNE in our model has the same form as q_i^* and π_i^*, once the appropriate values for S, n and v_i are substituted.

Consider first the output and profits of the new technology firms in a PSNE under a presumption of isolation. Given isolation, we have that $P_N(Q_N, Q_E) = u_{N2} - \frac{Q_N}{S_2}$ from Lemma 3.1. Firm i's profit function is then $\pi_{Ni} = q_{Ni}(u_{N2} - c_{Ni} - \sum_{j=1}^{n_N} q_{Nj}/S_2)$. Hence, this is equivalent to the objective in (10.1) when $S = S_2$, $n = n_N$ and $v_i = u_{N2} - c_{Ni}$. Since $v_i - \bar{v} = \bar{c}_N - c_{Ni}$ in this case, the expressions for q_{Ni}^I and π_{Ni}^I follow. An analogous argument establishes the expressions for q_{Ei}^I and π_{Ei}^I.

Now consider a PSNE under an assumption of disruption with all firm outputs positive. The profit function of the ith new technology firm is then $\pi_{Ni} = q_{Ni}(\frac{S_1 u_{N1} + S_2 u_{N2}}{S_1 + S_2} - c_{Ni} - \frac{Q_N + Q_E}{S_1 + S_2})$ and the profit function of the ith established technology firm is $\pi_{Ei} = q_{Ei}(u_{E1} - \frac{S_2}{S_1 + S_2}(u_{N1} - u_{N2}) - c_{Ej} - \frac{Q_E + Q_N}{S_1 + S_2})$. Hence, this is equivalent to the objective (10.1) for $S = S_1 + S_2$, $n = n_N + n_E$ and the first n_N firms having $v_i = \frac{S_1 u_{N1} + S_2 u_{N2}}{S_1 + S_2} - c_{Ni}$ and the next n_E firms having $v_i = u_{E1} - \frac{S_2}{S_1 + S_2}(u_{N1} - u_{N2}) - c_{Ei}$. Noting that for new technology firms we have $n(v_i - \bar{v}) = n_N(\bar{c}_N - c_{Ni}) + n_E(\bar{c}_E - c_{Ni}) + n_E(u_{N1} - u_{E1})$ and for established technology firms we have $n(v_i - \bar{v}) = n_E(\bar{c}_E - c_{Ei}) + n_N(\bar{c}_N - c_{Ei}) + n_N(u_{E1} - u_{N1})$, we get the stated expressions for q_{Ni}^D and q_{Ei}^D. ■

Proof of Lemma 3.4 Suppose the consumers in segment 2 do not buy the new product. From Lemma 3.1, it must be that $\frac{Q_N}{S_1} \leq u_{N1} - u_{N2} - \frac{Q_E}{S_1}$ and that $P_\phi = u_{\phi 1} - \frac{Q_N + Q_E}{S_1}$ for $\phi = N, E$. We can find a lower bound on Q_N by assuming that $Q_E = 0$, in which case the equilibrium total output of new technology firms is $Q_N^* = S_1 \frac{n_N}{n_N+1}(u_{N1} - \bar{c}_N)$. Consumers in segment 2 want to buy the new product if $\frac{Q_N^*}{S_1} > u_{N1} - u_{N2}$, which is equivalent to (3.1). If this condition and $\min q_{Ei}^D \geq 0$ are satisfied, then $\{q_{Ei}^I, q_{Ni}^I\}$ and $\{q_{Ei}^D, q_{Ni}^D\}$ are the only two sets of pure strategies that can be locally optimal. ■

Proof of Lemma 3.7 The proof is in two parts. Part i. In the first part we show that equilibrium $\phi = D, I$ exists *iff* $\Delta_{Ni}^\phi \leq 0$ for all i. Consider the disruptive equilibrium. First, suppose $\Delta_{Ni}^D \leq 0$ for all i. Because $\frac{Q_N^D}{S_2} \leq \frac{Q_E^D}{S_1} - (u_{N1} - u_{N2})$ implies that $\Delta_{Ni}^D > 0$ for all i, $\Delta_{Ni}^\phi \leq 0$ implies that q_{Ni}^D is locally as well as globally optimal. Second, suppose the disruptive equilibrium exists. Then q_{Ni}^D must be locally optimal and equilibrium profits for firm i are π_{Ni}^D. If $\frac{Q_N^D - q_{Ni}^D - \hat{q}_{Ni}^D}{S_1} < u_{N1} - u_{N2} - \frac{Q_E^D}{S_1}$, then there is another local optimum for firm i at $\hat{q}_{Ni}^D$ and global optimality requires $\Delta_{Ni}^D \leq 0$. If $\frac{Q_N^D - q_{Ni}^D - \hat{q}_{Ni}^D}{S_1} \geq u_{N1} - u_{N2} - \frac{Q_E^D}{S_1}$ then we have directly that $\Delta_{Ni}^D \leq 0$. The proof for $\phi = I$ is similar. This establishes part i.

Part ii. Let x be some parameter of the model such that $\frac{\partial \Delta_{Ni}^\phi}{\partial x} \leq 0$ for all i and for any set of parameters such that $\Delta_{Ni}^\phi = 0$. Suppose that equilibrium ϕ exists for $x = x_1$ but not for $x = x_2 > x_1$. From part i, $\Delta_{Ni}^\phi \leq 0$ at x_1 and $\Delta_{Ni}^\phi > 0$ at x_2. By the continuity of Δ_{Ni}^ϕ, there exists an $x_3 \in [x_1, x_2)$ such that $\Delta_{Ni}^\phi = 0$ and $\frac{\partial \Delta_{Ni}^\phi}{\partial x} > 0$, which is a contradiction. Hence, equilibrium ϕ is increasing in x. A similar argument holds for the decreasing case. ■

Proof of Proposition 4.1 (i) From their definition in the text we have

$$
\begin{aligned}
\hat{q}_{Ni}^D &= \frac{1}{2}\left(S_2(u_{N2} - c_{Ni}) - Q_N^D + q_{Ni}^D\right), \\
\hat{q}_{Ni}^I &= \frac{1}{2}\left((S_1 + S_2)(\frac{S_1 u_{N1} + S_2 u_{N2}}{S_1 + S_2} - c_{Ni}) - Q_E^I - Q_N^I + q_{Ni}^I\right),
\end{aligned}
$$

where

$$
\begin{aligned}
Q_N^I &= S_2 \frac{n_N}{n_N + 1}(u_{N2} - \bar{c}_N), \\
Q_E^I &= S_1 \frac{n_E}{n_E + 1}(u_{E1} - \bar{c}_E),
\end{aligned}
$$

and

$$
\begin{aligned}
Q_N^D &= (S_1+S_2)\frac{n_N}{n_E+n_N+1}(\frac{S_1u_{N1}+S_2u_{N2}}{S_1+S_2}-\bar{c}_N+n_E(u_{N1}-\bar{c}_N-u_{E1}+\bar{c}_E)),\\
Q_E^D &= (S_1+S_2)\frac{n_E}{n_E+n_N+1}\\
&\quad (u_{E1}-\frac{S_2}{S_1+S_2}(u_{N1}-u_{N2})-\bar{c}_E+n_N(u_{E1}-\bar{c}_E-u_{N1}+\bar{c}_N)).
\end{aligned}
$$

For any parameter x of the model other than S_1 and S_2 we have that

$$
\frac{\partial \Delta_{Ni}^D}{\partial x}=\frac{\hat{q}_{Ni}^D}{S_2}\frac{\partial \hat{q}_{Ni}^D}{\partial x}-\frac{2q_{Ni}^D}{S_1+S_2}\frac{\partial q_{Ni}^D}{\partial x}
$$

and when $\Delta_{Ni}^D=0\Leftrightarrow \hat{q}_{Ni}^D=q_{Ni}^D\sqrt{\frac{S_2}{S_1+S_2}}$ we have

$$
\frac{\partial \Delta_{Ni}^D}{\partial x}=\Phi_i^D\left(\sqrt{S_1+S_2}\frac{\partial \hat{q}_{Ni}^D}{\partial x}-\sqrt{S_2}\frac{\partial q_{Ni}^D}{\partial x}\right)
$$

where $\Phi_i^D=\frac{2q_{Ni}^D}{\sqrt{S_2}(S_1+S_2)}$. Similarly, for all parameters other than $S_1$and S_2 we have that

$$
\frac{\partial \Delta_{Ni}^I}{\partial x}=\Phi_i^I\left(\sqrt{S_2}\frac{\partial \hat{q}_{Ni}^I}{\partial x}-\sqrt{S_1+S_2}\frac{\partial q_{Ni}^I}{\partial x}\right)
$$

where $\Phi_i^I=\frac{2\hat{q}_{Nj}^I}{\sqrt{S_2}(S_1+S_2)}$ when $\Delta_{Ni}^I=0$. Henceforth, we assume that $\Delta_{Ni}^{\phi}=0$ for $\phi=D,I$.

Consider first the effect of u_{N1}. We have

$$
\begin{aligned}
\frac{\partial \Delta_{Ni}^I}{\partial u_{N1}} &= -\Phi^I\sqrt{S_2}S_1/2<0,\\
\frac{\partial \Delta_{Ni}^D}{\partial u_{N1}} &= \Phi^D\left(\sqrt{S_2}+\sqrt{S_1+S_2}(n_N-1)/2\right)\frac{(n_E+1)S_1+n_ES_2}{n_E+n_N+1}>0.
\end{aligned}
$$

Hence, by Lemma 3.7, the threat of disruption is increasing in u_{N1}. The results for u_{N2} follow from

$$
\begin{aligned}
\frac{\partial \Delta_{Ni}^I}{\partial u_{N2}} &= \Phi_i^I\frac{S_2}{n_N+1}\left(\sqrt{S_2}-\sqrt{S_1+S_2}\right)<0,\\
\frac{\partial \Delta_{Ni}^D}{\partial u_{N2}} &= \Phi_i^D\frac{S_2}{n_E+n_N+1}\left(\sqrt{S_1+S_2}\,(n_E+2)\,/2-\sqrt{S_2}\right)>0.
\end{aligned}
$$

The effect of the established technology utility is given by

$$
\begin{aligned}
\frac{\partial \Delta_{Ni}^{I}}{\partial u_{E1}} &= -\Phi_i^I \left(\frac{1}{2} S_1 \sqrt{S_2} \frac{n_E}{n_E+1} \right) < 0, \\
\frac{\partial \Delta_{Ni}^{D}}{\partial u_{E1}} &= \Phi_i^D \frac{(S_1+S_2)n_E}{n_E+n_N+1} \left(\sqrt{S_2} + \sqrt{S_1+S_2} \frac{n_N-1}{2} \right) > 0,
\end{aligned}
$$

and the effect of changing the cost of one established technology firm is given by

$$
\begin{aligned}
\frac{\partial \Delta_{Ni}^{I}}{\partial c_{Ek}} &= \Phi_i^I \left(\frac{1}{2} \frac{\sqrt{S_2} S_1}{n_E+1} \right) > 0, \\
\frac{\partial \Delta_{Ni}^{D}}{\partial c_{Ek}} &= -\Phi_i^D \frac{(S_1+S_2)}{n_E+n_N+1} \left(\sqrt{S_2} + \sqrt{S_1+S_2} \frac{n_N-1}{2} \right) < 0.
\end{aligned}
$$

Note that the first order condition for $\hat{q}_{Ni}^I$ can be written as $\hat{P}_{Ni}^I - c_{Ni} - \hat{q}_{Ni}^I/(S_1+S_2) = 0$ where $\hat{P}_{Ni}^I$ is the price when firm i deviates from the isolated equilibrium. For the new technology to be disruptive, it must be that this price is below $u_{N1} - Q_E^I/S_1$, the highest valuation of a customer not served by the established technology. The effect of S_1 on the isolated equilibrium is given by

$$
\begin{aligned}
\frac{\partial \Delta_{Ni}^{I}}{\partial S_1} &= -\frac{\hat{q}_{Ni}^I}{S_1+S_2} \left(2 \frac{\partial \hat{q}_{Ni}^I}{\partial S_1} - \frac{\hat{q}_{Ni}^I}{S_1+S_2} \right). \\
&= \frac{\hat{q}_{Ni}^I}{S_1+S_2} \left(u_{N1} - \frac{Q_E^I}{S_1} - \hat{P}_{Ni}^I \right) > 0.
\end{aligned}
$$

Similarly, one can show that $\frac{\partial \Delta_{Ni}^D}{\partial S_1} < 0$. Equilibrium existence depends on relative market size S_1/S_2 and hence it must be that $\frac{\partial \Delta_{Ni}^I}{\partial S_2} < 0$ and $\frac{\partial \Delta_{Ni}^D}{\partial S_2} > 0$.

(ii) Assume that $c_{Ei} = c_E$ and $c_{Ni} = c_N$. Then all firms using a technology are homogeneous and $\Delta_{Ni}^{\phi} = \Delta_N^{\phi}$ for $\phi = D, I$. Although we are only interested in integer value of n_ϕ, we can still proceed as before using Lemma 3.7. The result for n_E follows from

$$
\begin{aligned}
\frac{\partial \Delta_{N}^{I}}{\partial n_E} &= -\Phi^I \left(\sqrt{S_2} S_1 \frac{D + u_{E1} - c_E}{2(n_E+1)^2} \right) < 0, \\
\frac{\partial \Delta_{N}^{D}}{\partial n_E} &= \Phi^D \left(\sqrt{S_2} + \sqrt{S_1+S_2} \frac{n_N-1}{2} \right) \frac{q_{Ei}^D}{n_E+n_N+1} > 0.
\end{aligned}
$$

The result for n_N follows from

$$\begin{aligned}\frac{\partial \Delta_N^I}{\partial n_N} &= \Phi^I \frac{q_{Ni}^I}{n_N+1}\left(\sqrt{S_1+S_2}-\sqrt{S_2}\right) > 0,\\ \frac{\partial \Delta_{Nj}^D}{\partial n_N} &= \Phi^D \frac{q_{Ni}^D}{n_E+n_N+1}\left(\sqrt{S_2}-\sqrt{S_1+S_2}\frac{n_E+2}{2}\right) < 0.\end{aligned}$$

■

Proof of Proposition 4.2 (i) Suppose $c_{Ni} = c_N + c_i$ for all i. The set of parameters that support the isolated equilibrium is increasing in c_N since

$$\begin{aligned}\frac{\partial \Delta_{Ni}^I}{\partial c_N} &= \Phi^I \frac{\sqrt{S_2}}{n_N+1}\left(\sqrt{S_1+S_2}\sqrt{S_2}-\left((S_1\frac{n_N+1}{2}+S_2\right)\right)\\ &< \Phi^I \frac{\sqrt{S_2}\sqrt{S_1+S_2}}{n_N+1}\left(\sqrt{S_2}-\sqrt{S_1+S_2}\right) < 0.\end{aligned}$$

The set of parameters that support the disruptive equilibrium is decreasing in c_N since

$$\begin{aligned}\frac{\partial \Delta_{Ni}^D}{\partial c_N} &= \Phi^D \frac{\sqrt{S_1+S_2}}{n_E+n_N+1}\\ &\quad \left(\sqrt{S_2}\sqrt{S_1+S_2}(n_E+1)-\frac{1}{2}\left(S_2(n_E+n_N+1)-(S_1+S_2)(n_E+1)(n_N-1)\right)\right)\\ &> \Phi^D \frac{\sqrt{S_1+S_2}}{n_E+n_N+1}\\ &\quad \left(\sqrt{S_2}\sqrt{S_2}(n_E+1)-\frac{1}{2}\left(S_2(n_E+n_N+1)-(S_2)(n_E+1)(n_N-1)\right)\right)\\ &= \Phi^D \frac{\sqrt{S_1+S_2}}{n_E+n_N+1}\frac{1}{2}S_2 n_E n_N > 0.\end{aligned}$$

We conclude that the threat of disruption is decreasing in c_N.

(ii) Suppose $n_N > 1$. For $\Delta_{Ni}^I = 0$ we have that the effect of own costs is

$$\frac{\partial \Delta_{Ni}^I}{\partial c_{Ni}} = \Phi_i^I \frac{\sqrt{S_2}}{n_N+1}\left(\sqrt{S_1+S_2}\sqrt{S_2}n_N-\left(S_1\frac{n_N+1}{2}+S_2\right)\right).$$

Letting $s = S_1/S_2$, we have that $\frac{\partial \Delta_{Ni}^I}{\partial c_{Ni}} > 0$ iff $\chi^I(s, n_N) = \sqrt{s+1}n_N - \left(s\frac{n_N+1}{2}+1\right) > 0$, which is satisfied for s sufficiently small. Define s^I by $\chi^D(s^I, n_N) = 0$ and then from the implicit function theorem we have

$$\begin{aligned}\frac{\partial s^I}{\partial n_N} &= -\frac{2\sqrt{s^I+1}-s^I}{n_N/\sqrt{s^I+1}-(n_N+1)}\\ &= \frac{s^I+2}{n_N(1+n_N-n_N/\sqrt{s^I+1})} > 0\end{aligned}$$

where the second equality follows from $\sqrt{s^I+1} = \left(s^I\frac{n_N+1}{2}+1\right)/n_N$.

For $\Delta^D_{Ni}=0$ we have

$$\begin{aligned}\frac{\partial \Delta^D_{Ni}}{\partial c_{Ni}} &= \Phi^D_i \frac{\sqrt{S_1+S_2}}{n_E+n_N+1}\\ &\quad \left(\frac{1}{2}\left(S_2(n_E+2n_N)+S_1(n_N-1)\right)-\sqrt{S_2}\sqrt{S_1+S_2}(n_E+n_N)\right).\end{aligned}$$

Letting $s=S_1/S_2$, we have that $\frac{\partial \Delta^D_{Ni}}{\partial c_{Ni}} < 0$ *iff* $\chi^D(s,n_N) = \frac{1}{2}\left((n_E+2n_N)+s(n_N-1)\right) - \sqrt{s+1}(n_E+n_N) < 0$, which is satisfied for s sufficiently small. Define s^D by $\chi(s^D,n_N)=0$.

(iv) When $\Delta^\phi_{Ni}=0$ we have for $j\neq i$

$$\begin{aligned}\frac{\partial \Delta^I_{Ni}}{\partial c_{Nj}} &= \Phi^I_i \frac{S_2}{n_N+1}\left(\sqrt{S_2}-\sqrt{S_1+S_2}\right) < 0,\\ \frac{\partial \Delta^D_{Ni}}{\partial c_{Nj}} &= \Phi^D_i \frac{(S_1+S_2)}{n_E+n_N+1}\left(\sqrt{S_1+S_2}\frac{n_E+2}{2}-\sqrt{S_2}\right) > 0.\end{aligned}$$

(v) Suppose firm 1 and firm 2 of type N have the same cost. Hence $\Phi^I_1=\Phi^I_2$. Now consider the effect of a small increase in firm 1's costs. We have that

$$\frac{\partial \Delta^I_{N2}}{\partial c_{N1}} - \frac{\partial \Delta^I_{N1}}{\partial c_{N1}} = \Phi^I_1 \frac{\sqrt{S_2}}{n_N+1}\left(1-\sqrt{s+1}-\sqrt{s+1}n_N+\left(s\frac{n_N+1}{2}+1\right)\right) < 0$$

for $s=S_1/S_2$ sufficiently small. Hence, firm 1 has a greater incentive to deviate for s sufficiently small. This contradicts the low cost firm necessarily having the greatest incentive to deviate from the isolated equilibrium. ■

Proof of Lemma 5.1 The functions $u^\phi_{N2i}(x)$ are all linear. The slope of $u^I_{N2i}(x)$ is given by

$$\frac{\partial u^I_{N2i}}{\partial u_{N1}} = -\frac{\frac{\partial \Delta^I_{Ni}}{\partial u_{N1}}}{\frac{\partial \Delta^I_{Ni}}{\partial u_{N2}}} = \frac{(n_N+1)S_1}{2\left(\sqrt{S_1+S_2}-\sqrt{S_2}\right)\sqrt{S_2}} \tag{10.2}$$

As (10.2) is independent of i, this gives the slope of the function $u^I_{N2}(u_{N1})$ as well. The slope of $u^D_{N2}(x)$ is given by

$$\frac{\partial u^D_{N2i}}{\partial u_{N1}} = -\frac{\frac{\partial \Delta^D_{Ni}}{\partial u_{N1}}}{\frac{\partial \Delta^D_{Ni}}{\partial u_{N2}}} = \frac{\left(\sqrt{S_2} + \sqrt{S_1+S_2}(n_N-1)/2\right)\left((n_E+1)S_1 + n_E S_2\right)}{S_2\left(\sqrt{S_1+S_2}\,(n_E+2)\,/2 - \sqrt{S_2}\right)}. \tag{10.3}$$

To show that $u^D_{N2}(u_{N1})$ crosses $u^I_{N2}(u_{N1})$ once from below it suffices to show that (10.3) is greater that (10.2). Note that (10.3) is increasing in n_E since

$$\frac{\partial^2 u^D_{N2i}}{\partial u_{N1}\partial n_E} = \frac{\left(\sqrt{S_2} + \sqrt{S_1+S_2}(n_N-1)/2\right)}{S_2}\frac{\sqrt{S_1+S_2}\left((S_1+2S_2) - 2\sqrt{S_2}\sqrt{S_1+S_2}\right)}{4\left(\sqrt{S_1+S_2}\,(n_E+2)\,/2 - \sqrt{S_2}\right)^2} > 0$$

and hence it is sufficient to show the result for $n_E = 0$, in which case

$$\frac{\partial u^D_{N2}}{\partial u_{N1}} - \frac{\partial u^I_{N2}}{\partial u_{N1}} = \frac{S_1(n_N-1)}{2\left(\sqrt{S_1+S_2} - \sqrt{S_2}\right)S_2}\left(\sqrt{S_1+S_2} - \sqrt{S_2}\right) > 0.$$

We have already established that the difference between the slope of $u^D_{N2}(u_{N1})$ and $u^I_{N2}(u_{N1})$ is increasing in n_E. Turning to the effect of n_N, we have

$$\begin{aligned}\frac{\partial^2 u^D_{N2i}}{\partial u_{N1}\partial n_N} - \frac{\partial^2 u^D_{N2i}}{\partial u_{N1}\partial n_N} \quad = \quad & \frac{1}{2\sqrt{S_2}} \\ & \left[\frac{\left(\sqrt{S_1+S_2}\right)}{\sqrt{S_2}}\frac{n_E(S_1+S_2)+S_1}{\sqrt{S_1+S_2}\,(n_E+2)\,/2 - \sqrt{S_2}} - \frac{S_1}{\left(\sqrt{S_1+S_2} - \sqrt{S_2}\right)}\right],\end{aligned}$$

which is in increasing in n_E and positive for $n_E = 0$. ■

Proof of Proposition 6.1 Suppose $u'_E(t) = 0$. (i) We are interested in conditions under which the disruptive equilibrium is increasing in t, that is under which $\partial\Delta^D_{Ni}/\partial t < 0$ for all i. We have that

$$\frac{\partial \Delta^D_{Ni}}{\partial t} = \frac{\partial \Delta^D_{Ni}}{\partial u_{N1}}u'_{N1}(t) + \frac{\partial \Delta^D_{Ni}}{\partial u_{N2}}u'_{N2}(t).$$

From Proposition 4.1, we have that $\partial\Delta^D_{Ni}/\partial u_{N1} < 0$ and $\partial\Delta^D_{Ni}/\partial u_{N2} > 0$ and hence we have $\partial\Delta^D_{Ni}/\partial t > 0$ *iff* $u'_{N1}(t)/u'_{N2}(t)$ greater than a critical value $r^D > 0$. Specifically, letting $s = S_1/S_2$ we have

$$r^D = -\frac{\frac{\partial \Delta^D_{Ni}}{\partial u_{N2}}}{\frac{\partial \Delta^D_{Ni}}{\partial u_{N1}}} = \frac{\sqrt{s+1}\,(n_E+2) - 2}{\left(2 + \sqrt{s+1}(n_N-1)\right)\left((n_E+1)s + n_E\right)}.$$

The isolated equilibrium is decreasing in t when $\partial\Delta^I_{Ni}/\partial t > 0$ for all i. We have that

$$\frac{\partial\Delta^I_{Ni}}{\partial t} = \frac{\partial\Delta^I_{Ni}}{\partial u_{N1}} u'_{N1}(t) + \frac{\partial\Delta^I_{Ni}}{\partial u_{N2}} u'_{N2}(t).$$

From Proposition 4.1, we have that $\partial\Delta^I_{Ni}/\partial u_{N1} > 0$ and $\partial\Delta^I_{Ni}/\partial u_{N2} < 0$ and hence we have $\partial\Delta^D_{Ni}/\partial t > 0$ *iff* $u'_{N1}(t)/u'_{N2}(t)$ greater than a critical value $r^I > 0$. Specifically, letting $s = S_1/S_2$ we have

$$r^I = -\frac{\frac{\partial\Delta^I_{Ni}}{\partial u_{N2}}}{\frac{\partial\Delta^I_{Ni}}{\partial u_{N1}}} = \frac{2\left(\sqrt{s+1}-1\right)}{s(n_N+1)},$$

We have $\lim_{s\to 0} r^\phi = \frac{1}{n_N+1}$ and $\lim_{s\to\infty} r^\phi = 0$. We have that the disruptive threat is increasing in t if $u'_{N1}(t)/u'_{N2}(t) > r$, where $r = \min\{r^I, r^D\}$.

It is straigh forward to show that $\partial r^D/\partial n_N < 0$, $\partial r^I/\partial n_N < 0$ and $\partial r^I/\partial s < 0$. To show that $\partial r^D/\partial s < 0$. It is sufficient to show the result for $n_N = 1$, in which case $\partial^2 r^D/\partial s^2 < 0$ and it is sufficient to show the result from $s = 0$. When $n_N = 1$ and $s = 0$, we have $\partial r^D/\partial s = -1/4$. ■

Proof of Proposition 6.2 Suppose $u'_{E1}(t) = u'_{N1}(t) = u'_{N2}(t)$. We have

$$\begin{aligned}\frac{\partial\Delta^D_{Ni}}{\partial t} &= \left(\frac{\partial\Delta^D_{Nj}}{\partial u_{E1}} + \frac{\partial\Delta^D_{Nj}}{\partial u_{N1}} + \frac{\partial\Delta^D_{Nj}}{\partial u_{N2}}\right) u'_{N1}(t) \\ &= \Phi^D_i \frac{\sqrt{S_1+S_2}}{n_E+n_N+1}\left(\sqrt{S_2}\sqrt{S_1+S_2} - \frac{1}{2}\left(S_2(n_E+2) - S_1(n_N-1)\right)\right) u'_{N1}(t).\end{aligned}$$

Solving $\frac{\partial\Delta^D_{Ni}}{\partial t} = 0$ for n_E yields

$$n^D_E = 2(\sqrt{s+1}-1) + s(n_N-1),$$

which is strictly positive for $n_N \geq 1$ and increasing in $s = S_1/S_2$ and n_N. Thus, $\frac{\partial\Delta^D_{Ni}}{\partial t} > 0$ *iff* $n_E < n^D_E$. A similar argument gives

$$n^I_E = \frac{s(n_N+1)}{2\left(\sqrt{s+1}-1\right)} - 1,$$

which is strictly greater than n_N and increasing in s and n_N. The Proposition then holds for $n_1 = \min\{n^D_E, n^I_E\}$ and $n_2 = \max\{n^D_E, n^I_E\}$. ■

Proof of Proposition 6.3 We prove the result for the disruptive equilibrium; the proof for isolation is similar. Suppose the disruptive equilibrium exists *iff* $t \geq t^D$. At $t = t^D$, $\min_i \Delta_{Ni}^D = 0$. From Proposition 4.1, $\partial \Delta_{Ni}^D / \partial x > 0$ for $x \in \{S_2, u_{N2}, u_{E1}, n_E\}$. Hence, increasing the value of parameter S_2, u_{N2}, u_{E1} or n_E implies that $\min_i \Delta_{Ni}^D > 0$ at $t = t^D$ and that disruption occurs for a value of t greater than t^D. Conversely, $\partial \Delta_{Ni}^D / \partial y < 0$ for $y \in \{c_{Ei}, S_1, u_{N1}, n_N\}$. Hence increasing the value of c_{Ei}, S_1, u_{N1} or n_N implies that $\Delta_{Ni}^D < 0$ and hence disruption occurs for a value of t less than t^D. If $u_\theta(t) = b_\theta t^\beta$, then changes in b_θ are equivalent to changes in u_θ. ■

Proof of Proposition 7.1 Note that existence of the disruptive equilibrium assures that $\frac{Q_N^D}{S_2} \geq \frac{Q_E^D}{S_1} - (u_{N1} - u_{N2})$ and the existence of the isolated equilibrium assures that $\frac{Q_N^I}{S_2} \leq \frac{Q_E^I}{S_1} - (u_{N1} - u_{N2})$ because otherwise the firms will not be on the segment of the demand curve that makes their equilibrium outputs locally optimal. Let P_N^ϕ and P_E^ϕ denote the price of the output for the new and established products, respectively, in equilibrium $\phi = D, I$. These prices satisfy the following first order conditions

$$
\begin{aligned}
P_N^I - \frac{q_{Ni}^I}{S_2} &= c_{Ni}, && (10.4)\\
P_N^D - \frac{q_{Ni}^D}{S_1 + S_2} &= c_{Ni}, && (10.5)\\
P_E^I - \frac{q_{Ei}^I}{S_1} &= c_{Ei}, && (10.6)\\
P_E^D - \frac{q_{Ei}^D}{S_1 + S_2} &= c_{Ei}. && (10.7)
\end{aligned}
$$

Suppose $P_N^D \geq P_N^I$. From the demand function in Lemma 3.1 this implies that $Q_N^D \leq Q_N^I$. However, from equations (10.4) and (10.5), $P_N^D \geq P_N^I$ implies that $q_{Ni}^D > q_{Ni}^I$ for all i and hence $Q_N^D > Q_N^I$, which is a contradiction. We conclude that $P_N^D < P_N^I$ and from Lemma 3.1 this implies that $Q_N^D > Q_N^I$.

Now suppose $P_E^D \geq P_E^I$. Then equations (10.6) and (10.7) imply that $Q_E^D > Q_E^I$. However, this and $Q_N^D > Q_N^I$ imply that $P_E^D < P_E^I$. By contradiction, we conclude that $P_E^D < P_E^I$. Since the prices of both products fall with disruption (but are still above marginal cost), we conclude that social welfare increases as does the consumer surplus of all consumers that purchase one of the products in the disruptive equilibrium. ■

Proof of Proposition 7.2 Suppose both the disruptive and the isolated equilibrium exist. (i) Multiplicity requires that $\pi^I_{Ni} \geq \hat{\pi}^I_{Ni}$ and $\pi^D_{Ni} \geq \hat{\pi}^D_{Ni}$. If $n_N = 1$, then $q^I_{Ni} = \hat{q}^D_{Ni}$, the optimal deviation from disruption is the same as the optimal output under isolation. Hence, the profits are the same as well, $\pi^I_{Ni} = \hat{\pi}^D_{Ni}$. Thus multiplicity with $n_N = 1$ requires that $\pi^D_{Ni} \geq \hat{\pi}^D_{Ni} = \pi^I_{Ni} \geq \hat{\pi}^I_{Ni}$. (ii) Since $\hat{q}^D_{Ni} = \frac{1}{2}\left(S_2(u_{N2} - c_{Ni}) - Q^D_N + q^D_{Ni}\right)$, for $S_2(u_{N2} - c_N) < Q^D_N + \max_i q^D_{Ni}$ we have $\pi^D_{Ni} > \hat{\pi}^D_{Ni}$ since $\hat{q}^D_{Ni} \leq 0$. For u_{N2} sufficiently large we have $\pi^I_{Ni} > \max\{\hat{\pi}^I_{Ni}, \pi^D_{Ni}\}$ even holding $S_2(u_{N2} - c_N)$ constant and the isolated equilibrium is preferred even though the disruptive equilibrium exists as well. (iii) For $c_{Ei} = \bar{c}_E$ we have $q^\phi_{Ei} = Q^\phi_E / n_E$ for $\phi = D, I$. We know from Proposition 7.1 that $P^D_E < P^I_E$. Hence, if $Q^D_E \leq Q^I_E$ then $q^D_{Ei} \leq q^I_{Ei}$ and with both output and price reduced it must be that $\pi^D_{Ei} < \pi^I_{Ei}$. Suppose $Q^D_E > Q^I_E$. Then

$$
\begin{aligned}
\pi^I_{Ei} &= \frac{Q^I_E}{n_E}\left(P_E(Q^I_E, Q^I_N) - c_{Ei}\right) \\
&\geq \frac{Q^D_E}{n_E}\left(P_E(\frac{Q^D_E}{n_E} + \frac{(n_E - 1)Q^I_E}{n_E}, Q^I_N\right) - c_{Ei}) \\
&> \frac{Q^D_E}{n_E}\left(P_E(Q^D_E, Q^D_N) - c_{Ei}\right) \\
&= \pi^D_{Ei}.
\end{aligned}
$$

■

Proof of Proposition 7.3 (i) The within-group market share of new technology firms under disruption is

$$
\frac{q^D_{Ni}}{Q^D_N} = \frac{1}{n_N} \frac{n_E(\bar{c}_N - c_{Nj}) - n_N(c_{Nj} - \bar{c}_N) + \frac{S_1 u_{N1} + S_2 u_{N2}}{S_1 + S_2} + n_E((u_{N1} - \bar{c}_N - (u_{E1} + \bar{c}_E)) - c_{Nj}}{\frac{S_1 u_{N1} + S_2 u_{N2}}{S_1 + S_2} + n_E((u_{N1} - \bar{c}_N) - (u_{E1} - \bar{c}_E)) - \bar{c}_N}. \tag{10.8}
$$

We have $\frac{q^D_{Ni}}{Q^D_N} - \frac{q^I_{Ni}}{Q^I_N}$ has the same sign as $(\bar{c}_N - c_{Ni})\left(n_E(u_{N2} - \bar{c}_N) - (n_N + 1)R_N\right)$. That is $\frac{q^D_{Ni}}{Q^D_N} - \frac{q^I_{Ni}}{Q^I_N}$ is increasing in $(\bar{c}_N - c_{Ni})$ *iff* $R_N < \frac{n_E}{n_N+1}(u_{N2} - \bar{c}_N)$. (ii) We have

$$
\begin{aligned}
\frac{q^I_{Ei}}{Q^I_E} &= \frac{1}{n_E}\frac{u_{E1} - c_{Ei} - n_E(c_{Ei} - \bar{c}_E)}{u_{E1} - \bar{c}_E} \\
\frac{q^D_{Ej}}{Q^D_E} &= \frac{1}{n_E}\frac{u_{E1} - c_{Ei} - n_E(c_{Ej} - \bar{c}_E) - n_N(\bar{c}_E - c_{Ei}) + R_E}{u_{E1} - \bar{c}_E + R_E}
\end{aligned}
$$

and hence $\frac{q^D_{Ei}}{Q^D_E} - \frac{q^I_{Ei}}{Q^I_E}$ has the same sign as $(\bar{c}_E - c_{Ei})\left(n_N(u_{E1} - \bar{c}_E) - (n_E + 1)R_E\right)$. ■

Proof of Proposition 7.5 Suppose $n_N = 1$. We have

$$\begin{aligned}
\frac{\partial \pi^D_{Ni}}{\partial u_{N2}} &= \frac{2}{n_E+2}\left(\frac{S_2}{S_1+S_2}\right) q^D_{Ni}, \\
\frac{\partial \pi^D_{Ni}}{\partial u_{N1}} &= \frac{2}{n_E+2}\left(\frac{S_1}{S_1+S_2}+n_E\right) q^D_{Ni}, \\
\frac{\partial \pi^I_{Ni}}{\partial u_{N2}} &= q^I_{Ni}.
\end{aligned}$$

For part (i), $\frac{\partial \pi^D_{Ni}}{\partial u_{N1}} > \frac{\partial \pi^D_{Ni}}{\partial u_{N2}}$ follows from $\frac{S_2}{S_1+S_2} < 1 \leq n_E$. For part (ii), since $\frac{S_2}{S_1+S_2} q^D_{Ni} - q^I_{Ni} = R_N$, we have that $\frac{\partial \pi^I_{Ni}}{\partial u_{N2}} > \frac{\partial \pi^D_{Ni}}{\partial u_{N2}}$ *iff* R_N is not too positive. ■

Proof of Proposition 8.1 (i) Fix the values of all parameters except u_{N1} and u_{N2} such that $n_E \geq 2$ and $c_{Ei} = c_E$ for all i. From Lemma 5.1 there exists a $\hat{u}_{N1}$ such that $u^I_{N2}(\hat{u}_{N1}) = u^D_{N2}(\hat{u}_{N1})$. Note that reducing n_N by 1 causes the functions $u^I_{N2}(u_{N1})$ and $u^D_{N2}(u_{N1})$ to shift to the right. Hence, for $\varepsilon > 0$ sufficiently small, the point $(u_{N1}, u_{N2}) = (\hat{u}_{N1} + \varepsilon, u^I_{N2}(\hat{u}_{N1}))$ has disruption as the unique PSNE prior to the reduction of n_N and isolation as the unique PSNE after the reduction in n_N. (ii) Similar to the proof of part (i). ■

Proof of Proposition 9.2 (i) With price discrimination, each segment is an independent Cournot game with linear demand and hence a unique equilibrium. It is straight foward to show that the new product is sold to segment 1, i.e., $\max_i q_{Ni,1} > 0$, *iff*

$$u_{N1} - \min_i c_{Ni} - \frac{n_E}{n_E+1}(u_{E1} - \bar{c}_E) > 0. \tag{10.9}$$

Hence, the set of parameters that support a disruptive equilibrium (i.e., one in which $\max_i q_{Ni,1} > 0$) are increasing in u_{N1} and c_{Ei}, decreasing in n_E, u_{E1} and $\min_i c_{Ni}$, and independent of S_1, S_2, n_N and u_{N2}.

(ii) The technologies are isolated with price discrimination *iff* condition (10.9) is not satisfied. In the limit as $S_2 \to 0$, the condition for the existence of the disruptive equilibrium converges to inequality (10.9). Since the set of parameters that support the disruptive equilibrium are decreasing in S_2, if condition (10.9) is not satisfied then the disruptive equilibrium does not exist without price discrimination. A similar argument establishes that if condition (10.9) is not satisfied, then the isolated equilibrium exists without price discrimination. Hence, if the technologies are isolated with price discrimination, then the isolated equilibrium is the unique PSNE without price discrimination. ■

Appendix II: Mixed Strategy Equilibrium

If a pure strategy Nash equilibrium does not exist, we expect there to be a mixed strategy equilibrium of the following form. The established technology firms expand output to the point where the new technology firm with the greatest incentive to deviate from the isolated equilibrium is indifferent between disruption and isolation and disrupts with a probability that makes the output of the established firms optimal. In this section we give details for the case where $n_N = 1$ and report on numerical analyses that show that the factors that increase the threat of disruption in Section 4 increase the probability of disruption in the mixed strategy equilibrium as well.

Restrict attention to $n_N = 1$ and let $\delta \in (0,1)$ be the probability that the new technology firm disrupts. That is, with probability $1-\delta$ the new technology firm produces its isolated output $Q_N^I = S_2(u_{N2} - c_{N1})/2$ and with probability δ it produces some greater disruptive output $\bar{Q}_N$. Given this strategy, the objective of the established firms is

$$\max_{q_{Ei}} q_{Ei}\left[(1-\delta)(u_{E1} - \frac{Q_E}{S_1}) + \delta(u_{E1} - \frac{S_2(u_{N1}-u_{N2})}{S_1+S_2} - \frac{Q_E + \bar{Q}_N}{S_1+S_2}) - c_{Ej}\right].$$

and their total output is the following function of δ

$$\bar{Q}_E(\delta) = \left(\frac{n_E}{n_E+1}\right)\left(\frac{S_1(S_1+S_2)}{S_1+S_2(1-\delta)}\right)\left(u_{E1} - \bar{c}_E - \delta(\frac{\bar{Q}_N}{S_1+S_2} + \frac{S_2(u_{N1}-u_{N2})}{S_1+S_2})\right).$$

The disruptive output for the new technology firm comes from

$$\max_{Q_N} Q_N(\frac{S_1 u_{N1} + S_2 u_{N2}}{S_1+S_2} - \frac{\bar{Q}_E(\delta) + Q_N}{S_1+S_2} - c_N)$$

and hence

$$\bar{Q}_N = \frac{1}{2}(S_1+S_2)\left(\frac{S_1 u_{N1} + S_2 u_{N2}}{S_1+S_2} - c_N - \frac{\bar{Q}_E(\delta)}{S_1+S_2}\right).$$

The new technology firm must be indifferent between its two output levels, i.e.

$$\frac{(\bar{Q}_N)^2}{S_1+S_2} = \frac{(Q_N^I)^2}{S_2}. \tag{10.10}$$

Solving equation (10.10) for $\bar{Q}_E(\delta)$ yields

$$\bar{Q}_E(\delta) = S_1(u_{N1} - c_N) - (\sqrt{(S_1 + S_2)S_2} - S_2)\,(u_{N2} - c_N)\,.$$

There are now two expressions for $\bar{Q}_E(\delta)$ from which one can solve for a closed form expression for δ.

Relative to the isolated equilibrium, the mixed strategy has higher output of the established product, $\bar{Q}_E > Q_E^I$, and higher expected output of the new product because $\bar{Q}_N > Q_N^I$. Hence, the prices of the established product falls, $P_E(\bar{Q}_E, \bar{Q}_N) < P_E(\bar{Q}_E, Q_N^I) < P_E^I$, as does the expected price of the new product because $P_N(\bar{Q}_N, \bar{Q}_E) < P_N^I$. Hence, social welfare is higher. These results are consistent with the effects of disruption in Proposition 6.2.

Because the closed form expression for δ is very complex, we resort to numerical analysis to characterize comparative statics on δ. We created $2.62 * 10^{10}$ parameter combinations within the following bounds: S_1 and S_2 between 0 and 1, c_N and $\bar{c}_E$ between 0 and 2, u_{N1} and u_{N2} between c_N and 10, u_{E1}between $\bar{c}_E$ and 10 and n_E between 1 and 4. Within these bounds we allowed the parameters to vary by increments of .1 (except for n_E which we restricted to whole numbers) and considered every possible combination of parameters. We identified $8.53 * 10^8$ parameter combinations for which the mixed strategy equilibrium exists. In all cases, we found that the probability of disruption δ is increasing for small changes in S_1, $\bar{c}_E$ and u_{N1} and decreasing for small changes in S_2, c_N, u_{N2}, u_{E1} and n_E. These results are consistent with the results in Proposition 4.1 and Proposition 4.2 part (i).