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No. 3986

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LABOUR ECONOMICS

Centre for Economic Policy Research

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ISSN 0265-8003

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Discussion Paper No. 3986
July 2003

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CEPR Discussion Paper No. 3986

July 2003

ABSTRACT

Equilibrium Search Unemployment, Endogenous Participation and Labour Market Flows*

The sustainability of a welfare state requires high employment/high participation to raise the tax base and avoid distortions. To analyse labour market participation decisions in a world with market frictions, we propose and solve a three-state macro model of the labour market. We show that workers' decisions about entering into and exiting from the labour market are fundamentally different in the presence of frictions: irreversible costs paid by workers at the entry level imply that labour supply is determined by two margins, the entry and exit margins. From the normative point of view, we show that the existence of two margins alters significantly the conventional effects of payroll taxes and unemployment benefits. From the positive point of view, our model rationalizes the existence of most labour market flows and of 'marginally attached workers'. Furthermore, a calibration improves the usual representations of labour markets.

JEL Classification: J20 and J30
Keywords: labour supply, search frictions and welfare state

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*This Paper is a revised version of the paper 'Labor Market Flows and Equilibrium Unemployment'. We thank M Burda, P Cahuc, M Dewatripont, C Wyplosz, G Violante, F Panunzi, and particularly E Faraglia, C Michelacci and C Pissarides, for helpful comments, as well as seminar participants at ECARES-ULB, Bank of Italy (Ente Einaudi), Humboldt Universität, Berlin, Lausanne (DEEP-HEC), Geneva (IUHEI) Séminaire Fourgeaud in Paris, CEMFI (CEPR Workshop on Unemployment, Redistribution and Inequality), LSE, the CEPR European Summer Symposium in Labour Economics (ESSLE) and the EEA and ESEM 2002 meetings in Venice. We are also indebted to Robert Shimer who kindly provided us with the gross US flows data and François Marque for his decisive contribution to a proof in Section 6. This Paper is produced as part of a CEPR Research Network on 'New Techniques for the Evaluation of European Labour Market Policies', funded by the European Commission under the Research Training Network Programme (Contract No: HPRN-CT-2000-00071).

Submitted 23 April 2003

1 Introduction

The fervid debate over the sustainability of many welfare state programs suggests that the size of the active population matters. In light of the recent demographic trends, only economies with high employment and high participation can maintain actual levels of social security. But most economic analysis understands labor supply only in a world without search frictions, where workers' participation to the labor market is described by a neoclassical labor supply function. In the four chapters of the Handbook of Labor Economics devoted to labor supply, there are very few references to the role of search frictions[1].

It has to be acknowledged that participation decisions in imperfect labor markets are not fully understood. Further, in a macroeconomic perspective, we know little about the interactions between the participation decisions and ¯rms' incentives to create jobs. In other words, few works allow for a rigorous analysis of the macroeconomic equilibrium with endogenous participation and unemployment. This theoretical weakness prevents policy actions (payroll taxes, unemployment insurance, employment subsidies) from being fully understood. Our paper is, to our knowledge, the ¯rst one that attempts to do so, in investigating a three-state macro model of the labor market, in which the following decisions by agents are endogenous: job creation decisions by ¯rms; job destruction by pairs worker/employer; entry and exit decisions in the labor market; and in extensions, search e®ort margins.

Our modelling approach is based on the observation that people spend simultaneously a large amount of time in both market and home production, a feature of the data that has been already exploited in the macroeconomic literature. By enriching the time allocation problem on the part of the household, so as to explicitly consider the choice between leisure, home production and market work, the business cycle literature has improved the calibration of various aspects of the data, such as output volatility, the correlation between hours and productivity and the correlation between investments in home and market capital (Rios-Rull (1993), Mc Grattan et al. (1991), Gomme et al. (2001)). But the existing business cycle literature studies home production within frictionless labor markets. Our goal, conversely, is to study the border between market and home production in an imperfect labor market.[2] In our world, heterogenous workers face idiosyncratic shocks to home productivity, but market frictions impose a cost to labor market participation. Since we work with a technologically ¯xed number of hours, our analysis abstracts from the intensive margin of labor supply, and concentrates to the extensive margin. In the paper, we explore in details the e®ects of time consuming search, a market friction into employment that has attracted a great deal of attention in the macro literature (Hall, 1999 and Mortensen and Pissarides, 1999).

A key question in this setting is what frictions do to a very simple home production

[1] The most advanced paper in this direction is Blundell and MacCurdy (1999) who discuss intertemporal decisions (pp.1672+), notably human capital decisions and habit persistence, with no explicit role for stochastic arrival of job o®ers. They conclude page 1686 in saying that "There remain a number of big issues that we have not touched on in this chapter but that are important for labour market analysis. (...) Another issue relates to the process of job search and job matching."

[2] An important exception is Nosal et al. (1992), who show that in an indivisible labor model with home production, involuntary unemployment arises in equilibrium without assuming that leisure is an inferior good.

model. Since labor market participation decision is associated with paying an irreversible entry cost, the paper shows that the decisions to participate and stop participating di®er, and that labor supply is described by two margins: an entry margin and an exit margin. Both coincide when the irreversible sunk cost vanishes.

But how important is the existence of these two labor market margins, and what frictional models tell us vis-a-vis baseline models? The paper argues that there are both positive and normative considerations. From the normative standpoint, we show that policy design must be set according to the frictional models, and properly considering the two di®erent labor supply margins. When taxes can not be levied to home production, the employment rate is ine±ciently low in a world with and without frictions. Moreover, in contrast with the existing literature, when the quit margin is properly taken into account, the decentralized vacancy unemployment ratio is ine±ciently low even when wages internalize search frictions. Further, the paper mitigates the relevance of the entitlement e®ects of unemployment bene¯t, the fact that an increase in unemployment insurance increases the attractiveness of market participation among non eligible non-employed (Mortensen, 1977 and Fredriksson and Holmlund, 2001). When the quit margin is accounted for, an increase in unemployment bene¯ts has both incentives and disincentives e®ects on market participation, even at constant search intensity and in a partial equilibrium context, when the job ¯nding rate is constant.

>From the positive standpoint, there are several implications. First, we argue that the two labor supply margins readily rationalize a labor market with three states, whereby people spend time in employment, unemployment and full time home production. Second, a simple version of the model can quantitatively account for the large °ows between the three labor market states. Indeed, a calibration of the model ¯ts the monthly °ows for the U.S. The paper also discusses a version of the model in which individuals endogenously choose their search intensity, so that unemployed with low search intensity are classi¯ed as out of the labor force. Note that our framework is also fully consistent with the recent work of Jones and Ridell (1999) who emphasize the di±culty to de¯ne the frontier between non-participation and unemployment. Notably, they show that there exist agents reporting that they would like a job but do not search, which we account for.

Our work is not the ¯rst attempt to incorporate endogenous labor market participation features to standard models of search. On the microeconomic side, Seater (1977), Burdett-Mortensen (1978), Burdett (1979), Burdett-Kiefer-Mortensen-Neuman (1984), Swaim-Podgursky (1994) have successfully investigated the relations between search frictions and labor supply, with a ¯xed supply of jobs. Our theoretical distinction between inactivity and unemployment, empirically consistent with Flinn and Heckman (1983), is inspired by Burdett and Mortensen (1978). In the macro-search literature, Bowden (1980), Mc Kenna (1987); Pissarides (1990), chap. 6; Sattinger (1995) have introduced a labor demand side and endogenous participation, in a way that brings few new insights as compared to the standard (two state) model of matching. Individuals have a heterogenous value of non-market time and decide in a static (though intertemporal) way about their participation to the labor market. It follows that the °ows between activity and inactivity are driven by macroeconomic changes (in productivity, in unemployment) and are thus mainly cyclical or conjunctural °ows. In contrast, our theory, building on both macroeconomic factors and individual (household) shocks, is able to account for permanent, structural flows between activity and inactivity, even when macro-conditions are unchanged.

The paper proceeds as follows. Section 2 develops a baseline model of home production. Section 3 describes the analytical framework of the model when irreversible entry costs take the form of search frictions. It defines the main properties of the labor supply margins in a partial equilibrium context, when the job finding rate is exogenously fixed. Section 4 derives the general equilibrium of the model and proves its existence. Section 5 analyses the normative implications of the model in the context of taxation while section 6 focuses on the entitlement effects of unemployment benefits. Section 7 presents a calibration of the model, and shows how our framework can rationalize most flows across the three labor market states, with a brief extension of the model to endogenous search effort.

2 Heterogeneous Home Production and Indivisible Labor

Let us first illustrate how intertemporal labor supply decisions are taken when the valuation of leisure is stochastic and when participation to the labor market requires entry costs. Assume first that there is a mass 1 of individuals who derive utility from home production (leisure) and from market activity. Individuals are allocated one unit of time

$$\begin{aligned} 1 &= h_w + h_h \\ \text{with } h_w &\in \{0; e\} \end{aligned}$$

where h_w is time spent in the market and h_h is time spent in home production/leisure. The time spent on the market is technologically fixed. We consider a given skill segment of the labor force in which the marginal productivity is homogenous, at a level y. Information is perfect and individuals are paid a wage w equal to their marginal product in market activity. Utility is linear in home production and in w. Thus the per period utility function is, in each activity W and H (employment and full-time home production),

$$\begin{aligned} v^W &= (1 - e)x + y \\ v^H &= x \end{aligned}$$

We assume that there is some heterogeneity in the valuation of non-market activities. Concretely, home productivity x is heterogeneous and stochastic, and its value changes according to a Poisson process at rate λ. Conditional on the arrival rate of a shock; the value of home productivity takes a value from a continuous distribution $f(x)$ defined over the support $x \in [x^{min}; x^{max}]$: The key worker decision is whether to work 0 or e hours in the labor market, so that our model is an extensive margin model.[3] It is important to note that, following

[3] This is why we ignore hereafter issues such as the intertemporal elasticity of substitution, bargaining over hours or work sharing.

Becker (1965), home production or leisure consumption are formally expressed in the same way (raising individual's utility).[4] Hereafter, we keep the home production interpretation of x but interpretations in terms of time-varying marginal utility of leisure are possible.

In a world without market frictions, the participation decision is described by a single cut-off point x^* so that

$$x^* = y/e$$

In words, this condition implies that the marginal benefit of market activity is equal to its opportunity cost in terms of foregone home productivity. Since there is a mass 1 of individuals, in the baseline model a fraction $F(x^*)$ is employed while the remaining fraction is engaged full-time in home production.

Assume now that an irreversible fixed cost C has to be paid each time the worker enters or re-enters into market activity. The market friction C, the specific nature of which will be specified later, introduces a dynamic choice in the market participation decision. The entry and the exit decisions will necessarily differ. One can formally derive this intuition with some algebra, to account for the intertemporal structure of the model which plays a crucial role here. Let $H(x)$ be the value function for being full time in home production and $W(x)$ the value of being employed and engaged part-time in home production. The Bellman equations are derived in Appendix 9.1 as well as all details of this Section. The participation decisions are described by two cut-off points x^γ and x^q defined respectively by the following equations

$$\text{entry:} \qquad W(x^\gamma) - H(x^\gamma) = C \tag{1}$$

$$\text{quit:} \qquad W(x^q) - H(x^q) = 0 \tag{2}$$

where x^γ is the entry cut-off point and x^q is the quit cut-off point.[5] Developing the algebra, the entry and exit margins read respectively:

$$\frac{e(x^q - x^\gamma)}{r+\lambda} = C \tag{3}$$

$$x^q = x^* + \frac{\lambda}{r+\lambda}\int_{x^\gamma}^{x^q} F(x)dx \tag{4}$$

The entry margin (3) is clear: the entry cost has to equal the surplus from a job at x^γ, which is given by $(W - H)(x^\gamma) = \frac{e(x^q - x^\gamma)}{r+\lambda}$.[6] The quit margin (4) is also interesting to

[4] Gronau (1977, p. 1100) states that "[the distinction between home production and leisure], so common in everyday language disappeared in Becker's more general formulation. The omission is partly due (...) to the large number of borderline cases (eg. is playing with a child leisure or work at home?)".

[5] If we label emp the share of employed individual and n the share of individual involved full time in home production, their expressions read $emp = \frac{F(x^\gamma)}{1+F(x^\gamma)-F(x^q)}$ and $n = 1 - emp$ where emp increases with both cut-off points.

[6] Even without explicit search frictions, there is indeed an inframarginal surplus of a job, since only workers at x^q are indifferent between W and H. Here, the surplus truly comes from C. It is determined from Bellman equations (18) and (19) in Appendix 9.1, in noticing that $\partial(W - H)(x)/\partial x = -e/(r+\lambda)$ and that $(W - H)(x^q) = 0$.

understand: the sacrifice of home production for the marginally indifferent worker is above market productivity by a quantity reflecting future, anticipated gains of being on the job, given that quitting involves repaying the cost C. We label this effect the employment-hoarding. It formally corresponds to the integral term in equation (4).

Proposition 1 With $C > 0$, we have $x^o < x^* < x^q$ and when $C \to 0$, x^q and x^o converge to x^*.

Proposition 1 implies that the employment-hoarding effects disappear when entry costs are equal to zero. At this stage, several additional insights have emerged from the introduction of the entry cost C. First, on a positive viewpoint, the model rationalizes the existence of a well identified group of agents in labor statistics: there are indeed persons willing to work, but not ready to pay the entry cost, i.e. non-employed agents whose home productivity belongs to the interval $[x^o; x^q]$. In Jones and Ridell (1999), those workers are called 'marginally attached' to the labor market. In light of our modelling choice, Jones and Ridell's results reveal the importance of irreversible entry costs into the labor market.

Second, on a more normative and policy viewpoint, policy design should not overlook the fact that individuals adjust over two labor supply margins. To the best of our knowledge, existing models of labor market participation ignore the quit margin, and focus on the entry margin only. In our model, the quit margin is activated thanks to the assumption of stochastic utility of non-participation.[7] This is why in what follows, we will focus on a dynamic model with $\lambda \geq 0$ and discuss the policy implications in terms of taxes and unemployment benefits.

One can now establish a bridge between the findings of this section and conventional search theory: entry costs C can be thought as the costs paid by job seekers (time, money) in an imperfect labor market. Those costs are larger, the more time consuming is the job finding process. To go further, we will thus now introduce three states (employment, unemployment and fully engaged in home production, which we call for simplicity non-participants). This will endogenize the entry cost in the most natural way, and bring a third set of insights: we will account for five of the six flows between those three states, the sixth transition being subject to discussion and accounted for in Section 7.

3 A model with search frictions and rent sharing

3.1 Framework

In this section, workers wanting to participate to the labor market undertake time-consuming search. The time allocation problem of the worker is modified as follows: h_w is the number of hours actually worked, h_s is the search intensity necessary to obtain a job and h_h is the

[7] To see this point better, assume that home production is time invariant ($\lambda = 0$); in this case, the quit margin is inactive (or better, latent), since all employed workers are such that $x \leq x^o < x^q$: the quit margin never strikes, although the quit cut-off point is still well defined at $x^q = x^*$. The only wedge between the entry and the exit margin is given by the interest rate lost on the participation cost C.

choice of hours spent in leisure/home production. The time constraint is thus

$$\begin{aligned} 1 &= h_w + h_s + h_h \\ \text{with } h_w &\in \{0, e\} \\ h_s &\in \{0, s\} \end{aligned}$$

where e is the inelastic number of hours worked and s is the inelastic number of hours spent to find a job (we discuss and relax this assumption later on). It follows that in the three states W, U, H, where W is employment, U is unemployment and H is full-time home production, the flow utility of agents is given by

$$\begin{aligned} v^W &= (1 - e)x + w \\ v^U &= (1 - s)x \\ v^H &= x \end{aligned}$$

where x is home productivity and w is the total wage received for the e hours worked.[8] Throughout the analysis, we assume $e \geq s$. As a limit case, assuming $e = s$ is inconsistent with empirical works indicating that job search activity is a small fraction of the hours worked.[9] On the other hand, we may view such an assumption as an extreme form of indivisibility of labor: anyone seriously interested in getting a paid activity on the market has to organize one's life in a way that may be inconsistent with most activities out of the market (child care, benevolent activities, etc...). So, we define a fully indivisible economy as one in which entering the labor market involves a sacrifice of home production regardless of the employment status: $e = s$. This assumption is however not necessary until Section 4.3 where the labor demand side of the model is introduced.

Labelling W, U, H the present-discounted value of the utility of workers in each state and using W for $W(x)$, $W' = W(x')$ etc... for simplicity of exposition, the Bellman recursive equations in the three states read:

$$\begin{aligned} (r + \lambda)W &= v^W + \lambda \int_{x^{min}}^{x^{max}} Max(W', U', H')dF(x') + \delta[Max(U, H) - W] \\ (r + \lambda)U &= v^U + \lambda \int_{x^{min}}^{x^{max}} Max(U', H')dF(x') + p(W - U) \\ (r + \lambda)H &= v^H + \lambda \int_{x^{min}}^{x^{max}} Max(U', H')dF(x') \end{aligned}$$

where δ is an exogenous destruction rate of the job and p is the job finding rate for workers (treated as a parameter in partial equilibrium and endogenized in general equilibrium).

To solve for wages, we need to introduce firms. A firm has either 0 or one worker. With a finite value of p successful matches enjoy a pure economic rent. As is conventional in the search-matching literature, those rents are split in fixed proportion between firms and

[8] In absence of firm's heterogeneity, on-the-job search does not occur in equilibrium.
[9] For instance, Layard et al. (1991), pages 237-41.

workers. Formally, the value of a filled position for the firm depends on x if the wage does depend on x. We have

$$(r+\lambda)J(x) = y - w(x) + \delta(V_V - J) + \lambda \int_{x^{min}}^{x^{max}} Max(J'; V_V)dF(x') \tag{5}$$

where y is the marginal product of the worker and V_V is the value of a job vacancy (treated in partial equilibrium as a parameter).

Nash-bargaining over w follows the usual rule

$$w = ArgMax[W - Max(U; H)]^{\beta}[J - V_V]^{1-\beta} \tag{6}$$

and it follows that wages split the surplus in shares β and $1-\beta$. It can be guessed that there are two wage rules, depending on the sign of $U - H$ if $U > H$, then workers hit by an exogenous destruction shock (δ) look for another job. If $U < H$, then workers hit by this type of shock exit from the labor market and are engaged full-time in home production. The expression for wages is in Appendix 9.2, in equations (22) and (23). They conventionally appear as a weighted average (with weights respectively β and $1-\beta$) of the marginal product net of the firm's outside option (in equity value) and of a term reflecting the threat point of workers, i.e. either U or H:

3.2 Reservation strategies and definitions

Let us re-introduce the cut-off points x^{γ} and x^{q}, now defined by

$$U(x^{\gamma}) = H(x^{\gamma}) \tag{7}$$
$$W(x^{q}) = H(x^{q}) \tag{8}$$

This leads to the Bellman equations in Appendix 9.3. We can then derive the slopes of the value functions W, U and H with respect to x in the general case $e \geq s$. With linear utility, the value functions are piecewise linear functions of x. In addition, the ordering of the slopes implies the following ordering of intersections: $x^{q} > x^{\gamma}$. This is always the case in a viable labor market with $W > U$.

Note also that (7) mirrors (1) if it is rewritten as $W(x^{\gamma}) = H(x^{\gamma}) + C$ where $C = W(x^{\gamma}) - U(x^{\gamma})$ is the endogenous entry cost. Figure 1 shows these value functions as a function of x. $W(x)$ has a kinked point at the cut-off value of home production x^{γ}, corresponding to the change in the outside option of workers.

We can clarify a few concepts by introducing some definitions. Above x^{q} one finds only workers engaged in full-time home production, or non-participants. Between the two cut-off points x^{γ} and x^{q}, one finds two categories of workers. First, some of them are non-participants but do not search for a job (Jones and Ridell, 1999). These workers would accept a job if offered one, but do not wish to pay the search cost. Second, there are employed workers. As in Section 2, those workers had at a time a low value of $x < x^{\gamma}$ in their individual history, and searched and found a job in the past. We call them unattached employed workers since they would leave the labor market after a job destruction shock δ. They get a wage $w^{na}(x)$

standing for non-attached (equation (23) in Appendix 9.2). Finally, below x^{γ}, one finds both unemployed job seekers and employed workers. We label the latter attached employed workers; since they are willing to search for a new a job if hit by a job destruction shock δ; and get a wage $w^a(x)$ displayed in equation (22) in Appendix 9.2.

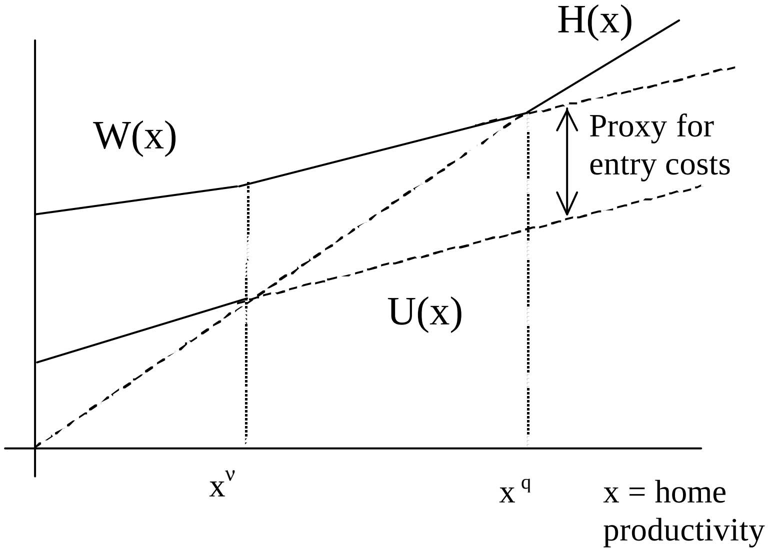

Figure 1: Value functions

3.2.1 Entry margin

The first indifference condition (7) defines an **entry-margin**, a level of home productivity in which the worker is indifferent between being full time in home production or being in search unemployment. Formally, the entry margin reads $sx^{\gamma} = p(W - U)(x^{\gamma})$. This states that the forgone value of home production in the job search activity sx^{γ} has to be compensated by an equivalent gain in expected surplus $p(W - U)$ given search frictions. In using equations (30) in Appendix 9.4, we obtain something similar to equation (3):

$$\beta\frac{e(x^q - x^{\gamma})}{r + \lambda + \delta} = \frac{sx^{\gamma}}{p} \tag{9}$$

The term $\frac{sx^{\gamma}}{p}$ stands as the expected value of forgone home production during search, which is precisely equal to the (endogenized) entry cost $C = (W - U)(x^{\gamma})$. The parameter β reflects here the existence of bargaining over the surplus, while wages were equal to marginal product (i.e. $\beta = 1$) in the benchmark model.

3.2.2 Quit margin

The second indifference condition (8) defines a **quit-margin**, a level of home productivity in which a worker is just indifferent between working in the market or being full time in home production. Let us introduce $\overline{S} = \int_{x_{min}}^{x^q} [J' - V_V + W - Max(U'; H')] dF(x')$ the average value of a match net of the firm and the worker's outside option. Formally, the quit margin writes, using the Bellman equations in Appendix 9.3: $ex^q = w^{na}(x^q) + \beta\lambda\overline{S}$. This states that the forgone value of home production on the job has to be compensated by the surplus of the job, itself being the sum of the flow wage and of future expected surplus of the job given stochastic transitions in x. Using the wage of the unattached (23) and the surplus (31) in Appendices 9.2 to 9.4, we then obtain

$$x^q = y/e + \frac{\lambda}{r+\lambda+\delta} \int_{x^\gamma}^{x^q} F(x)dx + O(e-s) - (r/e)V_V \qquad (10)$$

where $O(e-s) = \frac{\lambda(e-s)/e}{r+\lambda+\delta+\beta p} \int_{x_{min}}^{x^\gamma} F(x)dx$. This equation is very similar to (4), and simply appears as augmented by $-(r/e)V_V$ and by $O(e-s)$. Equation (10) says that the sacrifice of home production on the job is equal to the marginal product plus two employment-hoarding effects coming from the surplus on-the-job: the first effect is identical to the employment-hoarding effect of (4), while the second term is specific to the presence of unemployment, and disappears in a fully indivisible economy ($O(0) = 0$).[10] The last term in the quit margin refers to the outside option of the firm, and reflects the joint nature of the quitting decision.

4 Participation decisions in partial and general equilibrium

We are now able to derive some useful properties of the participation strategies of workers. We proceed in the following steps. First, we establish some existence results in the general case with $e \geq s$ and $\lambda \geq 0$. Then we explain the role of dynamic participation ($\lambda > 0$). Finally we determine the general equilibrium properties of the model with $\lambda \geq 0$ in the fully indivisible economy $e = s$.

4.1 Existence in partial equilibrium

In partial equilibrium, p is treated as a parameter as well as V_V.[11] The two labour supply margins can be usefully analyzed in the space $(x^q; x^\gamma)$. The left panel of Figure 2 illustrates. It shows that there is a unique equilibrium in $(x^q; x^\gamma)$. The proof of this statement is in Appendix 9.5.1.

4.2 The emergence of the quit margin

When $\lambda = 0$, i.e. when there is no stochastic change in x, we have a 'static participation model', as opposed to dynamic participation when $\lambda > 0$. In the former case, the quit margin

[10] See equation (31) in Appendix 9.4 for an expression of the expected surplus of employment.

[11] We study x^γ and x^q for a given worker in a given firm, while rV_V, explicited later on, may be a function of $x^{\gamma'}$ and $x^{q'}$ in other firms.

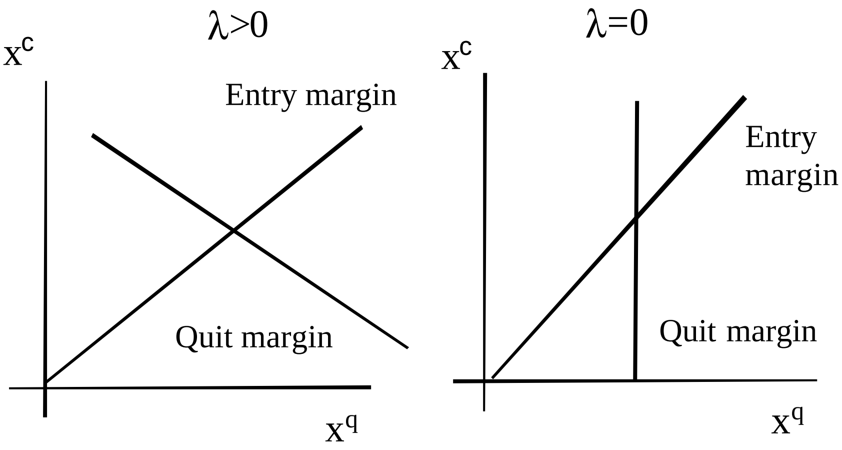

Figure 2: Entry and Exit Margins in Partial Equilibrium

is latent, while it is activated in the latter case. Indeed, as in Section 2, if $\lambda = 0$, people are permanently, either in or out the labor force. The quit cut-off point is still defined at $ex^q = y - rV_V$, but above x^c there are only non-employed workers, and since $x^q > x^c$, the quit margin is not active. The quit margin is thus latent. In the right panel of Figure 2, the quit margin is vertical. In contrast, when $\lambda > 0$, x evolves in time, and as a result, employed workers quit the labor force when $x > x^q$: the quit margin is active.

As we will argue throughout the paper, the activation of the latent quit margin is crucial in order to understand welfare, the impact of taxation and the role of unemployment benefits. To see this, denote by Z an arbitrary shift parameter raising the entry curve only. This may feature either policies (higher unemployment benefits) or technology (lower search frictions), in the spirit of respectively Section 6 and 4.7. In partial equilibrium with fixed p:

Proposition 2 Ceteris paribus, any increase in Z has smaller effects on the entry cut-off point x^c when $\lambda > 0$ than when $\lambda = 0$. Further, when $\lambda > 0$, the rise in x^c is partially offset by a decline of the quit cut-off point x^q.

To see this, imagine the same shift of the entry margin in both panels of Figure 2. In the left panel, x^q decreases and x^c increases less than in the right panel. The exact intuition will be explained more precisely in Section 6. Briefly, what is at work here is that, in the left, the shift $dx^c > 0$ at constant x^q implies a decline in the surplus of the job. This reduces the size of the employment-hoarding effect, so that workers quit more frequently, i.e. x^q falls. The logic of this proposition will be applied to several policy tools in next sections.

4.3 Labor demand

The general equilibrium is derived by adding a free-entry condition on firms which endogenizes V_V and p. In line with the traditional matching literature, an additional vacant position for a firm is established at no fixed cost, but at a flow cost c. It thus writes

$$rV_V = -c + \chi(J^e - V_V)$$

where χ is the job contact intensity for the firm and J^e is the expected value of the job given wage bargaining, and taking into account the density of workers actively looking for a job in the market.

In principle, thanks to the assumption of inelastic search effort s, $J^e = \left(\int_{x^{min}}^{x^o} J' dF(x')\right) / F(x^v)$, or in other words, workers actively looking for a job, in the interval $[x^{min}; x^o]$, are met by firms with identical probabilities, and the density of those workers is the conditional density of x's in the population[12].

Further, investigation of the wage equations in Appendix 9.2 shows that whenever $s = e$, the wage of the attached workers no longer depends on x. It follows that $J(x)$ and $W(x) - U(x)$ are constant over the interval of integration $[x^{min}; x^o]$. Then, by assuming $e = s$, $J(x)$ is constant for all newly hired workers, i.e. for $x < x^o$, and is equal, by continuity to $J^e = J(x^v) = (1-\beta)\frac{x^q - x^o}{r+\lambda+\delta}e$. The latter equality determines χ by the job-creation margin $V_V = 0$, implying

$$\frac{c}{\chi} = (1-\beta)\frac{e(x^q - x^o)}{r+\lambda+\delta} \tag{11}$$

It simply says that the surplus from a job for the firm is equal to the expected search/recruitment costs. Given the simplification brought by this assumption, the general equilibrium is solved in assuming $e = s$ and further, to avoid unimportant constant terms but without implication for the results, $e = 1$.

The model is then simply closed by the assumption of a matching process: the total number of contacts per unit of time is denoted by $M(u; v)$ where u is the number of unemployed job seekers in the population and v is the number of job vacancies. We denote by $\theta = v/u$ their ratio, traditionally called market tightness. We have, under the usual constant returns to scale assumption in M, that $\chi = M/v = \chi(\theta)$ with $\chi' < 0$: In addition the meeting probability p becomes also endogenous, and can be expressed as a simple function of market tightness θ. Formally, p is defined as a function $p = M/u = p(\theta)$ with $p' > 0$; and thus p is uniquely obtained from χ by the job-creation margin.

4.4 General equilibrium

Denote by n the non-participation rate, i.e. the ratio of the number of inactive workers to the total population (normalized to 1). Then,

Definition: A market equilibrium is a n-uple $(x^o; x^q; \theta; u; n)$ and two wage rules (one for the attached, one for the unattached workers) satisfying: the entry margin for workers; the quit margin for workers; the job creation margin for firms; the steady-state condition for unemployment flows; the steady-state condition for inactivity flows.

[12] This is not assumed, but was proved in Garibaldi-Wasmer (2001). The proof is ommitted here but can be supplied on request.

The derivation of the general equilibrium involves the three equations solving for three endogenous variables: $x^q; x^\gamma; \phi$. Then comes the derivation of the stocks (unemployment and non-participation) from steady-state conditions on flows.

The three equations are the following:

$$\frac{c}{\chi(\phi)} = (1-\beta)\frac{x^q - x^\gamma}{r+\lambda+\delta} \tag{JC}$$

$$\frac{x^\gamma}{p(\phi)} = \beta\frac{x^q - x^\gamma}{r+\lambda+\delta} \tag{Entry}$$

$$x^q = y + \frac{\lambda}{r+\lambda+\delta}\int_{x^\gamma}^{x^q} F(z)dz. \tag{Quit}$$

Equation (JC) was obtained from the labor demand equation (11). Equations (Entry) and (Quit) are just identical to the definitions of entry and quit in equations (9) and (10) when $e = s = 1$ and $V_V = 0$. We now show the existence (and uniqueness) of such an equilibrium.

4.5 Existence in general equilibrium

One can easily eliminate x^γ from those equations in noting that (Entry) and (JC) imply

$$x^\gamma = \frac{\beta}{1-\beta}c\phi. \tag{12}$$

This states, in a reduced form, that higher shares of the surplus and better labor market prospects induce further entry of workers into the labor market. Using (12), one obtains two relations between ϕ and x^q that have opposite slopes. One thus can express the equilibrium in the space $[\phi; x^q]$. In Figure 3 we label the modified equations with a star indicating that they are distinct from equations (JC) and (Quit) because of the elimination of x^γ. The modified JC curve is positively sloped and states that more stable workers (higher x^q) raise job creation. The modified (Quit) equation is downward sloping and states that, with better labor market tightness, the surplus of a job is lower and the capital loss of quitting is lower, reducing x^q.

Proposition 3 A sufficient condition for existence and uniqueness is $y > 0$.

This is proved by seeing that the intercept of the JC curve is 0 and the intercept of the quit margin curve, denoted by q_0 is defined by $q_0 = y + \frac{\lambda}{r+\lambda+\delta}\int_0^{q_0} dF(x) > y$.

4.6 Stocks and flows

In this subsection, we first calculate the equilibrium stock of workers in different states. With the steady-state assumption, one obtains the unemployment rate $u_r = u/(1-n)$ (the proof

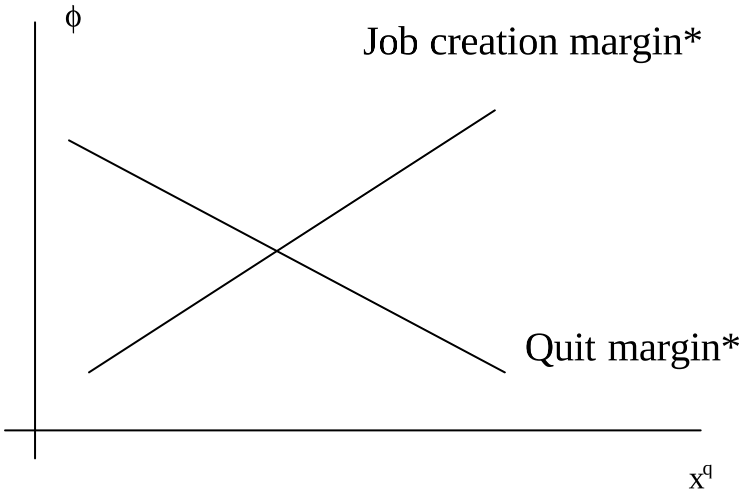

Figure 3: General Equilibrium

is in Appendix 9.6)

$$u_r = \frac{\delta + q}{p + \delta + q} \tag{13}$$

with $q = \lambda[1 - F(x^q)]$. The steady-state stocks of the other states are too complicated for being reported here, but can be calculated (see a sketch of the calculation in Appendix 9.6).

In the general case, equilibrium unemployment is determined through q by a whole new set of parameters linked to inactivity and non-market production. Those parameters are absent from the classical two state analysis of the labor market. Second, the effect of the quit rate is in steady-state exactly the same as an increase in the job destruction rate: it increases the inflows into unemployment, because the number of people leaving a job for inactivity will be matched by an equivalent number of workers entering activity through unemployment.

We can further comment on (13): unemployment is also affected by q through an indirect effect of en: the quit rate is anticipated by firms along the job creation margin, through a reduction in vacancy posting. There is thus an adverse effect on p and thus on unemployment.

Our model endogenizes five out of the six flows. In addition, there are internal flows between attached and unattached workers. We will calibrate an economy based on the five flows in Section 7 and then discuss the existence in data of a 6^{th} flows between out of the labor force and employment.

4.7 The role of frictions

For our purpose (understanding the role of frictions on labor market participation), it is very important to analyze the e®ect of p on the two participation cut-o® points. Assume that there is a shift parameter a®ecting p and Â, i.e. a scale parameter in the matching function, denoted by $Â_0$. A frictionless labor market is reached when $Â_0$ and thus p is in¯nite. Let us ¯rst investigate the limit cases of (Quit) and (Entry). This leads to the equivalent of Proposition 1: the di®erence x^q ¡ $x^°$ is a measure of market frictions, and tends to zero in a frictionless labor market. The common reservation values of x^q and $x^°$ is then the market productivity y as in the neoclassical case. Conversely, the distinction between the entry and the quit cut-o® point is all the more important in more imperfect labor markets. Indeed, when frictions are very high ($Â_0 = 0$), the entry cut-o® point is zero and nobody wishes to enter the labor market.

Implicitly in the above statements, the (JC) curve was ignored. Once (JC) is taken into account, p is no longer exogenous and a shift in $Â_0$ has an impact on p itself. At constant x^q and constant $x^°$, a shift in $Â_0$ implies that Á increases since ¯rms pay lower hiring costs. This additionally raises p.[13] Other results can be found in Appendix 9.5.3.

The e®ect of a decline in frictions can also be seen as an application of Proposition 2: a decline in frictions acts as a shift parameter Z. Indeed, when frictions decrease, i.e. when p increases, the entry curve is shifted up (people are more induced to enter the labor market) but the quit curve is una®ected when e = s. The quit cut-o® point is thus reduced. The intuition is that a larger p is associated with a lower surplus of employment, and so quitting has a lower opportunity cost.

5 Taxation and welfare

5.1 The social planner problem

We consider a central planner maximizing the sum of market and non-market production plus a non linear term in the level of public spending G. G has to be ¯nanced by a tax t on a base B. The level of public spending G yields a level of utility P(G) with $P^0 > 0$, $P^{00} < 0$. The tax base B by de¯nition cannot include non-market production (it is impossible to tax home cooking or newspaper reading). Thus, B is equal to market production. Home production is denoted by H. If z is a vector of control variables of the policy maker, the general program of the central planner reads

$$\underset{t;z}{\mathrm{Max}}\, -(z;t) = B(1 ¡ t) + H + P(Bt) \text{ under some constraint } F(z) = 0$$

Let us state a useful proposition for the analysis of welfare (see Appendix 9.7.1 for a proof).

Proposition 4 The optimal taxation problem does not a®ect the choice of the policy maker with respect to other variables z:

[13] Mathematically, at constant $x^°$ and x^q, we have $dp=dÂ_0 = p=Â_0 + 1=Á ¤ (1 ¡ ´)@Á=@Â_0$.

We can thus hereafter simplify the derivation of the social optimum by ignoring taxation.[14] Using this result, we can study the program of the central planner in the context of our search economy, and show that the optimal values of $\phi; x^q; x^{\circ}$ are jointly determined by the following expressions (see Appendix 9.7.1 for a proof).

$$\frac{c}{\chi(\phi)} = (1-\eta)\frac{x^q - x^{\circ}}{\lambda+\delta} \quad \text{(JC*)}$$

$$\frac{x^{\circ}}{p(\phi)} = \eta\frac{x^q - x^{\circ}}{\lambda+\delta} \quad \text{(Entry*)}$$

$$x^q = y + \frac{\lambda}{\lambda+\delta}\int_{x^{\circ}}^{x^q} F(x)dx \quad \text{(Quit*)}$$

where $-\eta$ is the elasticity of χ with respect to ϕ.[15]

Comparing the results of the previous equations with the decentralized equilibrium without taxation, described by equations (JC), (Entry) and (Quit), we immediately obtain the Hosios condition $\eta = \beta$. In this case, the decentralized equilibrium without taxation is efficient: it reaches a labor market allocation that is identical to the social planner allocation with optimal taxation. This result might have been expected, since it is a synthesis of the efficiency results obtained in Pissarides (2000, § 6&8) with either endogenous destruction but fixed participation or exogenous destruction but endogenous participation (though static, with $\lambda = 0$). In what follows, we assume that the Hosios condition is satisfied.

5.2 Taxation, search unemployment and the Quit Margin

The conventional wisdom on payroll taxation in a search unemployment world is summarized by the work of Pissarides (1988): a proportional tax on wages has no effect on unemployment as long as the tax affects the replacement rate of the unemployed.[16] Few scholars, and Holmlund (2002) in particular have challenged this result. They have argued that an increase in payroll taxes raises unemployment in search equilibrium models when non-taxable home production is introduced.[17] Holmlund's results are obtained under two set of assumptions differing from our approach. First, labor market participation is static, and thus does not include an active quit margin. And second, the economy is partially indivisible, in the sense that the fraction of time devoted to home production differs when employed and unemployed: $e > s$ where s is search intensity, endogenous in Holmlund (2002). There are two natural questions to raise here. First, what does the existence of two separate entry and exit margins add to the analysis? Second, do these results still hold in a fully indivisible economy $e = s$?

[14] Note however, that the reciprocal of the proposition is not true, since optimal taxation is affected by the optimal choice of variables $z = z^*$, given that $P'(B(z^*)t) = 1$.

[15] Formally, $\eta = \eta(\phi) = -\chi'(\phi)\phi/\chi(\phi)$.

[16] There exists also an extensive empirical literature on the effects of taxes on labor costs. The results of that literature are mixed. See, for example, Tryvainen (1995), Gruber and Nymeen and Rodseth (1999).

[17] Other papers, such as Sandmo (1990) Frediksen et al. (1995), Sorensen (1997) Kolm (2000) have studied taxation with home production, but these latter models do not focus on job search, and are mainly concerned with tax differentials between market an home production.

Formally, if one introduces a proportional tax on wages at rate t in an economy with full indivisibility ($e = s \leq 1$) and dynamic participation ($\lambda > 0$), the reduced form of the model reads (Appendix 9.7.2):

$$\frac{c(1-t)}{\chi(\phi)} = (1-\beta)e\frac{x^q - x^\gamma}{r+\lambda+\delta} \qquad \text{(JC(t))}$$

$$\frac{x^\gamma}{p(\phi)} = \beta\frac{x^q - x^\gamma}{r+\lambda+\delta} \qquad \text{(Entry(t))}$$

$$x^q = y/e(1-t) + \frac{\lambda}{r+\lambda+\delta}\int_{x^\gamma}^{x^q} F(x)dx \qquad \text{(Quit(t))}$$

Inspecting equations (JC(t)), (Entry(t)) and (Quit(t)) it immediately follows that payroll taxes influence the three cut-off points with $\lambda > 0$.

Proposition 5 When the quit margin is active ($\lambda > 0$), a marginal increase in payroll taxation reduces the three cut-off points: $\frac{\partial\phi}{\partial t} < 0$; $\frac{\partial x^q}{\partial t} < 0$ and $\frac{\partial x^\gamma}{\partial t} < 0$. Further, the proportional tax rate raises the unemployment rate and reduces the employment rate:

While the proof is left to Appendix 9.7.2, it is important to add that the effect of taxation on market tightness is proportional to the difference between x^q and x^γ.[18] Using the results of Section 4.7, this implies that taxation reduces job creation quantitatively all the more, the more frictions there are in the economy. A contrario, the adverse effect disappears in a frictionless economy.

This is a positive results, and does not yet convey any normative implication. To understand the welfare effects of taxation we need to compare the solution of the social planner with the decentralized equilibrium with taxes, as we do next.

Remark 1 When the Hosios condition is satisfied and participation is dynamic ($\lambda > 0$), a proportional taxation on wages leads to inefficiently low cut-off points. In addition, the employment rate is inefficiently low and the unemployment rate inefficiently high.

In other words, with positive t, firms create too few jobs, non-participants do not enter enough in the market and employed workers quit too frequently. To properly compare our results to those in the literature, and to show that our new results are mainly due to the emergence of the quit margin, we compare the decentralized and the social planner equilibrium when participation is static. Indeed, when the Hosios condition is satisfied and participation to the labor market is static ($\lambda = 0$) a proportional tax is consistent with an efficient level of unemployment and an optimal vacancy unemployment ratio. The entry margin, conversely, is inefficiently low. To summarize our findings, we can say that in a fully indivisible economy ($e = s$) there are three key cases: i) if the entry margin is not active ($x^\gamma > x^{max}$) and participation is static ($\lambda = 0$), than proportional taxation is consistent

[18] $\frac{\partial\phi}{\partial t}$ is proportional to $-\lambda\int_{x^\gamma}^{x^q} zf(z)dz$.

with an efficient unemployment rate and market tightness; ii) if the entry margin is active ($x^{\gamma} < x^{max}$) and participation is static ($\lambda = 0$), than the decentralized unemployment and market tightness are efficient but the entry rate is too low; iii) if participation is dynamic ($\lambda > 0$), proportional taxation distorts all the three margins in the labor market with frictions; further, when frictions decrease, i.e. when x^{γ} and x^{q} converge, ϕ is less and less distorted even with $\lambda > 0$ as footnote 18 shows.

Our analysis further suggests that Holmlund's result on the link between proportional taxation and unemployment critically hinges on the partial indivisibility of labor: with $e > s$, the job creation decision by firms is critically affected by taxes, since the wage of the attached workers they will hire in equation (22) depends on $(e - s)x$. Taxation of market activities raises the labor costs to firms and thus distorts the economy. Note that our results would hold if in our set-up we added up constant and linearly taxed unemployment benefits, as in Pissarides (1998). The role of unemployment benefits is explored in the next section.

6 Unemployment benefits and the entitlement effect

The economics of the unemployment benefits has recently attracted a great interest among academic economists. While unemployment benefits are typically justified in terms of their insurance component, they also reduce search effort on the part of the unemployed, with additional adverse effects on wages. Overall, unemployment is increased by a lower labor demand. Beyond such disincentive effects on the insured workers, the existing literature has also emphasized a positive link between unemployment benefits and market participation, since an increase in unemployment insurance reinforces the attractiveness of market participation among non eligible non-employed in general, and among people who are out of the labor force in particular. This is called the entitlement effect.[19]

The analysis of the entitlement effect has however always been performed within static participation models, with no role left for the quit margin. In this section, in the spirit of Proposition 1, we argue that once the individual reaction over the quit margin is properly taken into account, an increase in unemployment benefits may lead to both higher entry and higher exit with ambiguous effect on market participation. This is shown to be true in the most unfavorable case, in partial equilibrium context with a constant job finding rate, so as to control for the adverse effect of unemployment benefits on the vacancy unemployment ratio.

For analytical simplicity, we study these questions in a fully indivisible economy, with $e = s = 1$. We assume that employed workers are covered by unemployment benefits if they lose their job. Along the line of Fredriksson and Holmlund (2001), we also assume that benefits eligibility at a level b is gained at the beginning of the employment relationship.[20] Conversely, unemployed workers who were previously full time in home production are not

[19] It was firstly pointed out by Mortensen (1977), it was mentioned by Atkinson (1991) in an influential survey, and has recently received a lot of attention. Notably, Fredriksson and Holmlund (2001) studied such effects in their analysis of optimal sequencing of unemployment benefit. Related papers are Cahuc and Lehmann (2000) and Lehmann and Vanderlinden (2002).

[20] This assumption simplifies drastically the derivation of wages.

covered by benefits, but may receive an assistance $b_0 \leq b$. Workers out of the labor force do not receive any income, because this is equivalent to a pure shift of the distribution $F(x)$.[21] The model thus features two type of unemployed workers, which in this section we label covered and uncovered unemployed. Covered unemployed workers are denoted with a superscript c; so that the flow utility in the four states read $v^W = w$; $v^{Uc} = b$; $v^U = b_0$ and $v^H = x$.

One can show that the time allocation problem is still described by a set of reservation strategies, i.e. three threshold values $x^q; x^{oc}$ and x^o. At these cut-off values, workers respectively quit their job because they prefer inactivity to employment, leave the labor market even when being eligible to benefits, and leave the labor market in which they are not eligible. In addition, we assume that $x^q > x^{oc} > x^o$; which will be true in equilibrium in a viable market. If we let U and U^c be the value functions for covered and uncovered unemployment, the three cut-off points are the solution to $H(x^o) = U(x^o)$; $H(x^{oc}) = U^c(x^{oc})$; $H(x^q) = W(x^q)$. The value functions and a graphical representation of the asset values can be seen in Figure 9 in Appendix 9.8.

After few steps of algebra (described in Appendix 9.8.1 to 9.8.4), we obtain:

$$\frac{c}{q(\theta)} = (1-\beta)\frac{x^q - x^{oc}}{r+\lambda+\delta} \tag{JC}$$

$$\frac{x^{oc}}{p} = \beta\frac{x^q - x^{oc}}{r+\lambda+\delta} + \frac{b+\Psi}{p} \tag{Entry^c(b)}$$

$$\frac{x^{o}}{p} = \beta\frac{x^q - x^{oc}}{r+\lambda+\delta} + \frac{b_0}{p} + \frac{(b+\Psi)-b_0}{r+\lambda+p} \tag{Entry(b)}$$

$$x^q = y + \frac{\lambda}{r+\delta+\lambda}\int_{x^{oc}}^{x^q} F(z)dz + \Psi \tag{Quit(b)}$$

where $\Psi = \frac{\lambda}{r+\lambda}\int_{x^o}^{x^{oc}} F(x)dx$: The endogenous value Ψ is the loss of surplus for an unemployed workers when eligibility to benefits is lost (see Appendix 9.8) and its effect can be interpreted as participation-hoarding.

Definition 1 The participation-hoarding effect is the additional incentive to participate to the labor market induced by conditional eligibility to benefits and is accounted for by the term Ψ, i.e. the loss of eligibility in case of a withdrawal from market activity.

Eligible unemployed individuals and employed workers, in order to keep eligibility, hold on to market participation in anticipation of future changes in the value of home production. Note that all three cut-off points are affected: Ψ directly raises x^{oc} and x^q by one to one, but it also affects intertemporally by $p/(r+\lambda+p)$ the cut-off point x^o. Note also that when $\lambda = 0$ this effects does not exists, since $\Psi = 0$. We now study the properties of the participation margins in partial equilibrium under two alternative assumptions about eligibility, and then discuss the general equilibrium properties of the model.

[21] See Appendix 9.8 for this point.

6.1 The full eligibility case

As a benchmark case, we assume that $b = b_0$, so that unemployed workers obtain the same level of benefits regardless of whether they were previously employed or non-employed. In such a case, the entry margins of both covered and uncovered workers coincide and $x^{\gamma c} = x^{\gamma}$; and $\tilde{b} = 0$. Thus, both (JC) and (Quit) become identical to the benchmark equations of Section 4. The equations (Entry^c(b)) and (Entry(b)) converge to the same equation:

$$\frac{x^{\gamma}}{p} = -\frac{x^q - x^{\gamma}}{r + \lambda + \delta} + \frac{b}{p} \tag{14}$$

When p is fixed, the partial equilibrium effects of the entitlement effect can be summarized as follows. When participation is static ($\lambda = 0$) an increase in unemployment benefits at constant job finding rate increases the entry cut-off point, increases the employment rate and has no effect on the unemployment rate. To prove this result, note that when $\lambda = 0$ an increase in b has no impact on x^q (which is indeed inactive) while it increases the entry cut-off point x^{γ}. Since the unemployment rate is $u_r = \delta/(\delta + p)$ and the participation rate is $n = 1 - F(x^{\gamma})$, the unemployment rate, at a given p, is independent of b, while through x^{γ} an increase in b raises both participation and employment. The effects described above can also be looked upon in the left panel of Figure 4, where we report the shift of the entry margin along the constant and latent quit margin. As the following proposition shows, the activation of the quit margin modifies the economics of the entitlement effect.

Proposition 6 Entitlement Effect and the Quit Margin. When $\lambda > 0$, an increase in unemployment benefits increases the entry cut-off point, reduces the quit cut-off point and increases the unemployment rate.

The formal proof of this remark can be obtained by totally differentiating equations (14) and (Quit) with respect to a change in b so as to show that $\frac{\partial x^{\gamma}}{\partial b} > 0$ and $\frac{\partial x^q}{\partial b} < 0$. The economics of this latter effect ($\frac{\partial x^q}{\partial b} < 0$) was ignored by the existing literature. It is best seen by looking at equation (Quit). At constant x^q the traditional entitlement effect ($\frac{\partial x^{\gamma}}{\partial b} > 0$) reduces the surplus. This reduces the size of the employment-hoarding effect (already discussed in Section 2) in equation (Quit), so that x^q must necessarily fall. Note also that this latter effect is absent when $\lambda = 0$. Such result can also be illustrated in the second panel of Figure 4, where we show that the entry margin shifts along a downward sloping quit margin, leading to a reduction of the quit cut-off point and an increase in the entry cut-off point. Since the unemployment rate in this case is $u_r = (\delta + q)/(\delta + p + q)$; with $q = \lambda[1 - F(x^q)]$, the unemployment rate increases in response to an increase in b, even at constant p. This clearly mitigates the 'entitlement effect of benefits'.

6.2 The conditional eligibility case

While the modeling of unemployment benefit carried out in the previous sub-section neatly illustrates the effect of the quit margin on the entitlement effect, it remains unsatisfactory in other dimensions. Arguably, the reduction in the quit margin in response to the increase in b

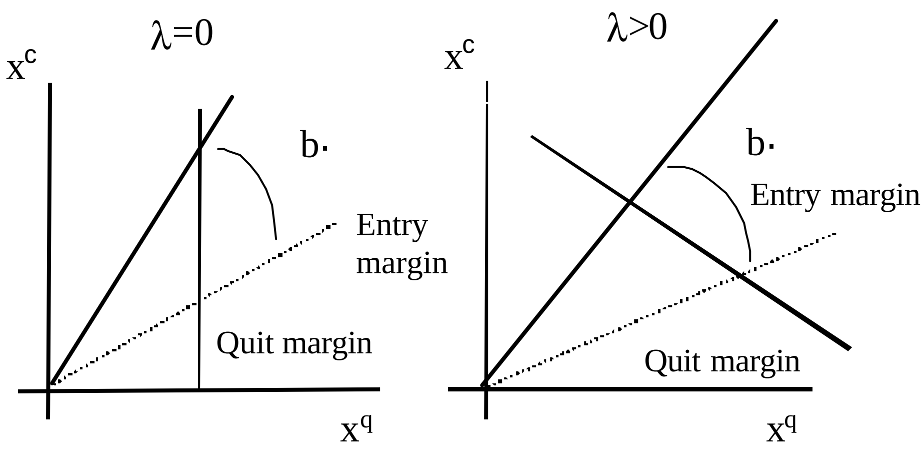

Figure 4: The Entitlment E®ect with Static and Dynamic Participation (full eligibility)

may be overestimated, since in reality individuals who quit their job enter the unemployment pool only after a period of full time home production, and thus lose eligibility. Let us assume that $b_0 = 0$ and $b > 0$, which implies that $x^{o} < x^{oc}$, i.e. that $\hat{b} > 0$ and thus that the participation-hoarding e®ect is active.

One can formally establish the partial equilibrium comparative static of the model over the three labor supply cut-o® points $x^q; x^{oc}$ and x^{o}, holding ¯xed market tightness θ (see Appendix 9.8.5).

Proposition 7 At $\bar{x}$ p(θ), the e®ect of bene¯ts is such that $\partial x^{oc}/\partial b > 0$, $\partial x^{o}/\partial b > 0$ and if δ is su±ciently low $\partial x^q/\partial b < 0$

This proposition suggests that the e®ect of an increase in b on x^{oc} and x^{o} is the standard eligibility e®ects of bene¯ts, which induces an increase in the entry cut-o® points of both eligible and non-eligible unemployed. The e®ect of an increase in b on the quit cut-o® point is now more complicated. In the quit margin there are now both the employment-hoarding e®ect and the participation-hoarding e®ect. While the increase in b reduces the employment-hoarding e®ect, causing a potential reduction in x^q, it also increases the participation-hoarding e®ect, since with larger bene¯ts workers lose more from a voluntary quit into inactivity. The overall e®ect is then ambiguous, and depends on the size of δ. For low values of δ the employment-hoarding e®ect prevails, and a larger b reduces the quit margin. For su±ciently large values of δ the second e®ect dominates, since larger δ reduces the size of the employment-hoarding e®ect. To sum up, in the four state model, the presence of the quit margin mitigates the entitlement e®ect, however only at low values of δ.

6.3 General equilibrium results of bene¯ts

In a conventional two-state model, the main e®ect of unemployment bene¯ts is to raise wages because of better outside options of workers a®ecting the outcome of bargaining. This discourages job creation through a general equilibrium feedback. Thus, higher bene¯ts raise unemployment and reduce employment. In our three-state model ($b = b_0$), the general equilibrium e®ect is clearly there: at the time of job creation, ¯rms face workers covered by bene¯ts and thus make lower pro¯ts. A full di®erentiation of the system ($Á; x^°; x^q$) shows that, as before, $@x^°=@b > 0$, $@x^q=@b < 0$ and $@Á=@b < 0$ (Garibaldi Wasmer, 2001). Thus, the entitlement e®ect of bene¯ts is still mitigated, and unemployment raises for two reasons, because of a higher quit rate q and because of a lower job ¯nding rate p. Finally, when new entrants do not fully bene¯t from b, i.e. in the four state model above, the general equilibrium involves four equations to di®erentiate and we cannot provide analytical results. Numerical simulations suggest that an increase in b leads to a reduction of $p(Á)$ and $x^°$ and an increase in $x^{°c}$. The e®ect on x^q can also be positive, depending on the size of the shift in x^q. The four state model will be used for the calibration exercise of the next section.

7 A quantitative analysis

7.1 The stylized facts

Let us ¯rst start with a description of the facts we want to illustrate here. Following Abraham and Shimer (2001) and Faraglia (2003), we use the gross monthly °ows of workers between the three states E, U and N, applying the Abowd and Zellner (1985) correction for mis-reported labor market status. We only consider the post-June 1995 period (since there are missing data between June and September 1995) for two groups of workers: the 15-64 population (hereafter referred to as `total') and the 25-54 population (`prime-age'). When the °ows are de°ated by the origin population, they are called transition rates. When they are de°ated by the total (or prime-age) population, they are called `°ow rates'. Table 1 indicates the sample averages for the di®erent °ows and stocks.

It is remarkable to observe that, even when taking out the extreme of the age distribution, as we do in considering the sample 25-54, one still have large °ows from and to inactivity. It is notably the case that exits from employment to unemployment are less frequent than exits from employment to inactivity. The other °ows have standard values. Note that there are also important direct °ows from inactivity to employment. In this section we argue this is due to non-corrected mis-classi¯cation: any person having a job had to make a minimal e®ort (going to an interview or negotiating the wage or working conditions), which cannot be detected by labor force surveys. This well known phenomenon is a time aggregation bias (Petrongolo Pissarides, 2002). To be consistent our theoretical model, ne °ows should instead add-up to both nu and ue °ows. In section 7.3, we consider an alternative interpretation of these °ows.

7.2 Calibration

We calibrate the model on the basis of the °ow rate, knowing that the stocks are determined in steady-state from these °ows. As it is obvious from Tables 1, there are six °ows to consider, while our model can account only for ¯ve of them. So, consistent with the model and the discussion above on the infra-month transitions, our calibration will be based on the modi¯ed rates: $\overline{nu} = nu + ne$ and $\overline{ue} = ue + ne$.

Table 2 reports the calibration exercise for the three state ($b = b_0$) and the four state models ($b > b_0$) for the 25-54 population in the second half of the nineties. This implies an unemployment rate of 3:5 percent, a non-participation rate of 15:4 percent, and a market tightness of 0:5, the latter being a reference value for most of the matching literature. Further, in the four state model we also calibrated the eu °ow rate, which is 0:68 percent in Table 2. The calibration code searches the parameter space for values of y; b; ±; c, x_0[22].

The results are reported in the lower part of Table 2. The four state model provides a better description of the US labor market than the three state model, and in the rest of this section we discuss the result of the latter calibration Household production is approximately 40 percent of market production (GNP), a statistic which appears to be in line with existing estimates on the size of the informal sector (Eisner 1988). It also appears that 3% of the total population is between x° and x^{q}, among which 2.23% are employed unattached. This means that 0.77% of the population corresponds to the marginally attached de¯ned by Jones and Ridell (1999). Including those in a broader de¯nition of unemployment would raise the unemployment rate by 0.91 percentage points, or a 23% increase, which is consistent with Jones and Ridell's evaluation for Canada.[23]

Table 2 shows that the model economy we constructed not only mimics the calibration statistics, but seems also capable of matching the relative ranking of all the labor market °ows, even though the actual degree of resemblance of the various statistics varies across °ows. In particular, our model economy displays larger en and ue °ows than those experienced by the US economy, and lower un and nu °ows. Such di®erences are likely due to two simplifying assumptions of our model: the absence of the discouraged worker e®ect and the homogeneity of labor force types. In reality, it is well documented that unemployment is a state dependent phenomenon and that long spell of unemployment are associated with heterogeneity across workers. All in all, this generates additional °ows out of the labor force.[24]

[22] The total number of contacts is $x_0 \tilde{A}^{i\,´} V$ (i.e. x_0 is a scale parameter; then ´ is the elasticity of $\hat{A}(\tilde{A})$, the ¯nding rate of workers by ¯rms. The pure monthly discount rates r is 0:005, ¯ = ´ = 0:5 so as to satisfy the Hosios with standard values. The distribution of home productivity is negative exponential ($f(\mu) = Be^{i B\mu}$) with parameter $B = 0{:}5$, while the arrival rate of the idiosyncratic shock ¸ is set to 0:14 in the three state model and 0:1 in the four state model.

[23] i.e. p. 149, "In our data, such individuals constitute 25% to 35% of U so that including the marginally attached among the unemployed would raise the current Canadian unemployment rate from about 9 to 11 or 12 percent."

[24] Accounting for these phenomena would certainly reduce the model generated ue °ow, and would simultaneously improve the match of the un °ow. The too large en °ow calibrated by our model suggests that in reality the share of unattached worker is probably less than the estimated one. Another way to improve the calibration exercise would be to solve the model with two types of workers: one type always attached to the labor force and another type that behaves in a way consistent with our model.

Our model is however a good first pass for rationalizing the large US. flows.

7.3 Further issues

Our model does a good job at rationalizing five of the six flows in the labor market, but fails to explain flows directly from non-employment to employment without a transition to unemployment, i.e. in our theoretical definition, a state in which workers actively look for a job. A part of the answer is in our model and the calibration strategy: as said above, en flows may be a pure mis-classification problem, due to infra-monthly transition.

A similar interpretation is that search effort is continuous, and that some workers are mis-classified as inactive while their search effort is strictly positive but below the detection point of the statisticians. These people get jobs, despite low search effort, and the transition is recorded as part of the ne flows. The last interpretation, which we did not allow for so far, is that 'jobs bump into people', even though they make no search effort, so that truly inactive workers obtain job offers.

As we argued in the previous section, our model already rationalizes the existence of such 'marginally attached workers'. To go beyond however, one can relax the assumption that s, the fraction of time devoted to search, is inelastic. This will make the model consistent with all possible interpretations of data and offer a synthesis between the empirical findings of Jones and Ridell (1999) and our model. Indeed, even with endogenous search effort, and except in specific cases, the two margins still emerge in equilibrium.

With one additional variable in the model, i.e., how much effort is made in equilibrium by workers, we need to simplify wage determination: wage are assumed to be constant over time, and posted by firms so that they maximize the value of a job vacancy. This notably implies that inefficient separation will occur. However, as we will show, the structure of the model, namely the existence of two separate margins, will be preserved.

Let us denote by $\overline{w}$ the value of the wage. The asset values of the state employment and non-employment (unemployment no longer exists) is as follows:

$$
\begin{aligned}
(r+\lambda)W(x) &= \overline{w} + ex + \delta(N(x) - W(x)) + \lambda \int Max(W(x'); N(x'))dF(x') \\
(r+\lambda)N(x;s) &= x(1-s) + p(\phi;s)[Max(W(x) - N(x;s); 0] + \lambda \int N(x';s')dF(x')
\end{aligned}
$$

with $p(\phi;s) = \phi\chi(\phi)\sigma(s)$ is the product of an aggregate component and of the efficiency of search time, with $\sigma' > 0$ and $\sigma'' < 0$. We make the following assumptions: $\sigma'(0) < +\infty$ and $\sigma(0) \geq 0$.[25]

Workers' search efforts are determined such as to maximize $N(x;s)$: the first order condition states that the marginal cost of search, namely home productivity, has to equal the

[25] The assumption $\sigma(0) = 0$ means that getting a job is impossible without a minimum search effort, while $\sigma(0) > 0$ implies that some jobs are offered to individuals. In both cases, when hit by a job offer, workers decide whether to accept the job offers. They do so only when the value of the job exceeds the value of non-employment.

marginal return in terms of expected surplus gained:

$$x = \sigma'(s)\phi\chi(\phi)(W(x) - N(x;s)) \text{ for } s > 0$$

It is easy to show that the optimal search effort, $s^* = s(x)$ is decreasing with x. At some point, s^* is at a corner solution zero. Hereafter, we denote by $N(x)$ the indirect value of non-employment. We can formally define $\tilde{x}^q$ and $\tilde{x}^o$ in this context:

$$\begin{aligned} N(\tilde{x}^q) &= W(\tilde{x}^q) \\ s(\tilde{x}^o) &= 0 \end{aligned}$$

In words, $\tilde{x}^q$ is the value of home production leading workers to quit, while $\tilde{x}^o$ is the value of home production making workers indifferent between full-time home production and marginal search effort. Formally, $\tilde{x}^o$ is the solution to:

$$\tilde{x}^o = \sigma'(0)\phi\chi(\phi)(W(\tilde{x}^o) - N(\tilde{x}^o)) \tag{15}$$

This equation implies that, for finite $\sigma'(0)$, $W(\tilde{x}^o) - N(\tilde{x}^o) > 0$, implying $\tilde{x}^o < \tilde{x}^q$. In other words, we still have two distinct entry and exit margins.[26]

Now, the important parameter here is $\sigma(0)$: when this quantity is equal to zero, only active job seekers (a statistician would call them unemployed) access to jobs, and ne flows are only a matter of statistical illusion. On the other hand, when $\sigma(0) > 0$, there are truly non-active individuals that get job offers. Among them, as explained above, only those between $\tilde{x}^o$ and $\tilde{x}^q$ would accept the offers, consistent with Jones and Ridell's findings. Workers with x above $\tilde{x}^q$ would instead reject them. Each alternative assumption about $\sigma(0)$ rationalizes one aspect of the discussion on ne flows. Equation (65) in Appendix 9.9 uses this distinction to derive the job creation margin in such a world.

8 Conclusions

Our model allows for a rather precise description of the labor market. It includes several categories of individuals: attached employed workers, unattached employed workers, unemployed workers, marginally attached non-employed workers and true non-participants. All this is delivered with a tractable model of endogenous job creation and the solution is characterized with three equations only, solving for two reservation values for workers and one job creation rate. Five of the six usual labor market flows are accounted for in the benchmark model, the sixth requires an extension to treat endogenous search effort. Several insights of the literature on taxation and unemployment benefits are either clarified, mitigated or generalized, at little additional cost.

[26] Above $\tilde{x}^o$, workers would like to make negative search efforts, i.e. to raise home production. At the extreme, when $x > \tilde{x}^q$, workers reject any job offer and would like to have a zero arrival rate of offers $\sigma(s) = 0$, which happens with negative search. However, since time cannot be borrowed, these individuals hit the corner solution $s = 0$.

Extensions of this work include policy simulations of the impact of workfare policies and subsidies towards activity, a better accounting for firms' heterogeneity and the introduction of several classes of workers. The present paper is a first step in the direction of an accurate calibration of frictional labor markets.

Table 1: Average Monthly Flows in the US Labor Market.

	Flows [a]					
	eu	en	ue	un	nu	ne
15-64 Population						
Transitions	1.02	1.62	25.90	16.59	3.46	4.43
Flow Rates	0.74	1.18	0.90	0.58	0.81	1.04
Stocks [b]	E=P	U=P	N=P	U=L		
	72.90	3.50	23.60	4.58		
25-54 Population						
Transitions	0.83	1.01	25.61	13.28	4.61	3.38
Flow Rates	0.68	0.82	0.76	0.40	0.71	0.52
Stocks	E=P	U=P	N=P	U=L		
	81.58	3.00	15.42	3.55		

[a] The first (second) letter refers to the source (destination) population e.g. eu is the employment unemployment flow.
[b] E is employment, N is out of the labor force, U is unemployment and L is the labor force.
Averages 1995:10 2001:12. Abowd Zellner correction (Abowd and Zellner (1985), Table 5).
Source: Gross CPS data provided by Robert Shimer and Elisa Faraglia and Authors' calculation.

Table 2: Calibration to the US Labor Market

Parameters	Notation	3 states	4 states	US Economy
Fixed Parameters				
Matching Elasticity	´	0.50	0.50	
Discount Rate	r	0.005	0.005	
Idiosyncratic Shock Rate	¸	0.140	0.10	
Workers' Surplus Share	¯	0.500	0.50	
Time in Market Activity	e	0.880	0.80	
Distribution [a]	B	0.500	0.50	
Code Determined Parameters				
Matching Function Constant	x_0	1.19	0.87	
Separation Rate	±	0.011	0.01	
Productivity	y	4.058	2.68	
Search Costs	c	2.283	2.07	
Eligible Unemployed Income	b	2.288	2.22	
Non-Eligible Unemployed Income	b_0	2.288	0.00	
Equilibrium Values				
Uncovered Entry Margin	$x^°$ $F(x^°)$	3.43 0.82	3.17 0.80	
Covered Entry Margin	$x^{°c}$ $F(x^{°c})$		3.54 0.83	
Quit Margin	x^q $F(x^q)$	3.85 0.85	3.92 0.86	
Calibrated Statistics				
Unemployment Rate	u=(1 ¡ n)	3.55	3.55	3.55
Non Participation Rate	n	15.40	15.40	15.42
Market Tightness	Á	0.50	0.50	
eu °ow rate	eu	0.83	0.68	0.68
Implied statistics				
share household gdp		0.18	0.38	0.33
en °ow rate	en	1.69	1.17	0.82
ue °ow rate	ue	2.98	1.85	1.28
un °ow rate	un	0.09	0.06	0.40
nu °ow rate	nu	0.39	0.31	1.23
share covered unemployed		1.00	0.36	0.33
Attached Employed	E_a	78.99	79.37	
Non-Attached Employed	E_{na}	2.60	2.23	
(a); Distribution is Negative Exponential with parameter B:				
Source: Authors' calculation				

9 Appendix

9.1 Benchmark model

The value functions when the worker is engaged in market production W and in full-time home production H read

$$rW(x) = y + (1 - e)x + \lambda \int Max[W(x'); H(x')]dF(x') - \lambda W(x); \qquad (16)$$

and

$$rH(x) = x + \lambda \int Max[W(x'); H(x')]dF(x') - \lambda H(x) \qquad (17)$$

The value functions (16) and (17) imply that $W'(x) = (1 - e)/(r + \lambda)$ and $H'(x) = 1/(r + \lambda)$. They have thus constant slopes, and are represented as two straight lines on Figure 5. They intersect once at x^*. The PDV of non-market activity has obviously a larger slope, since market participation allows people to enjoy the value of home production only for a fraction of time $1 - e < 0$:

When workers have to pay an entry cost C, the value function for full-time home production is modified as follows

$$rH(x) = x + \lambda \int Max[W(x') - C; H(x')]dF(x') - \lambda H(x)$$

while the function for market activity is identical to equation (16). The emergence of C implies that the value function H intersects both with the value function W and with the value function $W - C$, respectively at x° and another time at x^{q}, as represented in Figure 6. The value functions then rewrite as

$$(r + \lambda)W(x) = y + (1 - e)x + \lambda \int_{x^{min}}^{x^{q}} W(x')dF(x') + \lambda \int_{x^{q}}^{x^{max}} H(x')dF(x') \qquad (18)$$

and

$$(r + \lambda)H(x) = x + \lambda \int_{x^{min}}^{x^{\circ}} W(y; x')dF(x') + \lambda \int_{x^{\circ}}^{x^{max}} H(x')dF(x') - \lambda F(x^{\circ})C \qquad (19)$$

To sum up, with positive entry costs C, the entry decision and the exit decision now differ: being indifferent between working and quitting simply requires the value of employment to be equal to non-participation, while indifference between entering the labor market and staying out requires the value of employment net of C to be equal to non-participation.

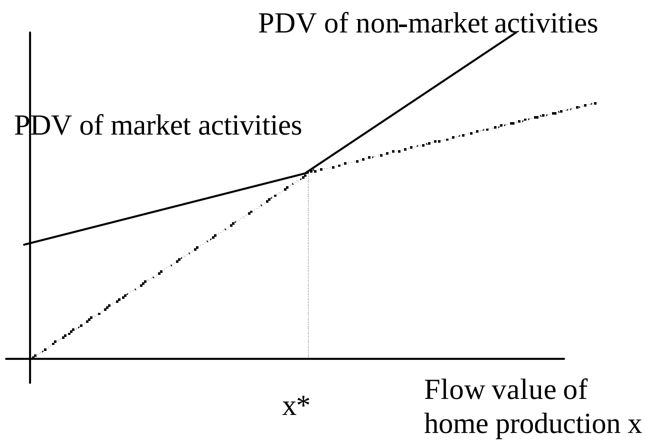

Figure 5: (Neoclassical) Participation decisions without entry cost

In terms of algebra, the quit margin $H(x^{q}) = W(x^{q})$ implies that

$$y + \lambda F(x^{\circ})C + \lambda \int_{x^{\circ}}^{x^{q}} [W(x') - H(x')]dF(x') = ex^{q} \qquad (20)$$

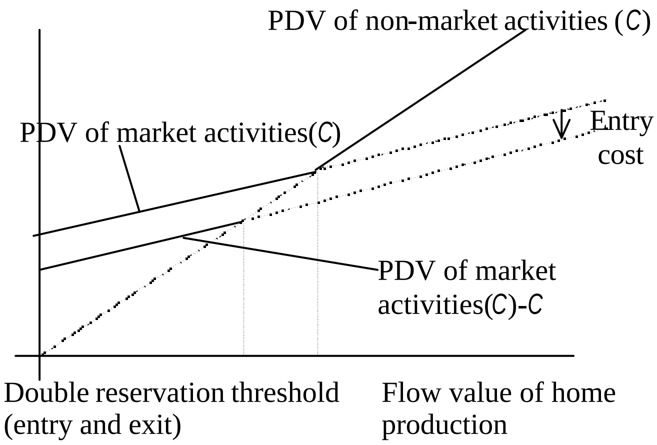

Figure 6: Participation decisions with irreversible entry cost

or, after integrating by part,

$$\lambda \int_{x^{\gamma}}^{x^{q}} [W(x') - H(x')]dF(x') = -\lambda F(x^{\gamma})[W(x^{\gamma}) - H(x^{\gamma})] - \frac{\lambda e}{r+\lambda} \int_{x^{\gamma}}^{x^{q}} F(x)dx$$

and combining (20) with (1) provides (4). The entry margin $H(x^{\gamma}) + C = W(x^{\gamma})$ implies

$$y - ex^{\gamma} + \lambda \int_{x^{\gamma}}^{x^{q}} [W(x') - H(x')]dF(x') = -\lambda F(x^{\gamma})C + C(r+\lambda) \tag{21}$$

Using (20) above to replace the integral term above, we obtain right away (3).

The proof of proposition 1 comes next.

Proof. From (3), $x^{q} - x^{\gamma} = (r+\lambda)C/e > 0$. Then, from (4), $x^{q} > y/e = x^{*}$. Further, $x^{\gamma} = x^{q} - (r+\lambda)C/e$. Since $\int_{x^{\gamma}}^{x^{q}} F(x)dx < x^{q} - x^{\gamma} = (r+\lambda)C/e$, we have that $x^{\gamma} < x^{*} - rC/e$. Finally, $C \to 0$ implies from (3) that $x^{q} = x^{\gamma}$ which leads from (4) to $x^{q} = x^{*}$. ■

9.2 Wage determination

Denote by $\overline{S}_w = \int_{x_{min}}^{x_{max}} Max(W'; U'; H') - Max(U'; H')dF(x')$, by $\overline{S}_f = \int_{x_{min}}^{x_{max}} Max(J' - J^V; 0)dF(x')$ and by $\overline{S} = \overline{S}_f + \overline{S}_w$. Thanks to (6), we have that $\overline{S}_f = (1-\beta)\overline{S}$ and $\overline{S}_w = \beta\overline{S}$. Taking differences of the Bellman equations, we obtain:

$$\begin{aligned}
(r+\lambda+\delta)(W-U)(x) &= w(x) + (s-e)x - p(W-U)(x) + \lambda\beta\overline{S} \\
(r+\lambda+\delta)(W-H)(x) &= w(x) - ex + \lambda\beta\overline{S} \\
(r+\lambda+\delta)(J-J^V)(x) &= y - w(x) + \lambda(1-\beta)\overline{S} - rJ^V
\end{aligned}$$

Using (6) and simplifying for discount factors $(r+\delta+\lambda)$, we have the sharing rule

$$(1-\beta)(W - Max(U; H)) = \beta(J - J^V)$$

Further noting that that terms in $\overline{S}$ cancel out in the above equality, the expression for wages comes easily: we obtain

$$w^a(x) = \beta(y - rV_V) + (1-\beta)[(e-s)x + p(W-U)(x)] + \text{ if } U > H \quad (22)$$

$$w^{na}(x) = \beta(y - rV_V) + (1-\beta)ex \text{ if } H > U \quad (23)$$

where a refers to attached workers while na refers to unattached workers. The threat point differs according to attachment to the labor market. For the attached workers, the outside option is U, i.e. to be unemployed, since they wish to look for another job in case of job loss. They consider the gain in home production out of a job $(e-s)x$ and the expected gain of search $p(W-U)$. For the unattached workers, the outside option is to be full time in home production, i.e. H, since they leave the labor market in case of job loss. They consider the gain in home production out of a job ex and expect no gain from search since they are inactive on the job market.

9.3 Bellman equations with search frictions

Introducing x^{o} and x^{q}, the Bellman equations become

$x \leq x^{o}$) (24)

$$(r+\lambda)W(x) = w + (1-e)x + \delta(U-W)(x) + \lambda\int_{x^{min}}^{x^{q}} W(x')dF(x') + \lambda\int_{x^{q}}^{x^{max}} H(x')dF(x')$$

$x^{o} \leq x \leq x^{q}$) (25)

$$(r+\lambda)W(x) = w + (1-e)x + \delta(H-W)(x) + \lambda\int_{x^{min}}^{x^{q}} W(x')dF(x') + \lambda\int_{x^{q}}^{x^{max}} H(x')dF(x')$$

$x \leq x^{o}$) (26)

$$(r+\lambda)U(x) = (1-s)x + p(W-U)(x) + \lambda\int_{x^{min}}^{x^{o}} U(x')dF(x') + \lambda\int_{x^{o}}^{x^{max}} H(x')dF(\theta')$$

$x^{q} \leq x$) (27)

$$(r+\lambda)H(x) = x + \lambda\int_{x^{min}}^{x^{o}} U(x')dF(x') + \lambda\int_{x^{o}}^{x^{max}} H(x')dF(x')$$

9.4 Determination of the surplus $S(x)$ and $\overline{S}$

Let us first define $S(x) = J(x) + V_V + W(x) - Max(U(x); H(x))$. One remarks that

$$\frac{\partial S}{\partial x} = \frac{-e}{r+\lambda+\delta} \text{ for } x^{q} \geq x \geq x^{o}$$

$$\frac{\partial S}{\partial x} = \frac{-e+s}{r+\lambda+\delta+\beta p} \text{ for } x^{o} \geq x \geq x^{Min}$$

$$S(x^{q}) = 0$$

$S(x)$ is continuous in x^{γ} with a discontinuity in slopes. As a result, in the general case, we have

$$S(x) = \frac{e(x^q - x)}{r + \lambda + \delta} \text{ for } x^q \geq x \geq x^{\gamma}$$
$$\text{and notably } S(x^{\gamma}) = \frac{e(x^q - x^{\gamma})}{r + \lambda + \delta}$$

By Nash-bargaining, we obtain where we used the property that for any $x \geq x^{\gamma}$ the surplus of the worker and the firm can be written as

$$J(x) - V_V = (1 - \beta) e \frac{x^q - x}{r + \lambda + \delta} \tag{28}$$
$$W(x) - H(x) = \beta e \frac{x^q - x}{r + \lambda + \delta} \tag{29}$$
$$(W - U)(x^{\gamma}) = \frac{\beta (x^q - x^{\gamma}) e}{r + \lambda + \delta} \tag{30}$$

We can also determine the value of $\overline{S}$ defined in Appendix 9.2. An integration by part leads to

$$\begin{aligned} \overline{S} &= \int_{x^{min}}^{x^{\gamma}} S(x)dF(x) + \int_{x^{\gamma}}^{x^q} S(x)dF(x) \\ &= F(x^{\gamma})S(x^{\gamma}) - 0 + \frac{e - s}{r + \lambda + \delta + \beta p} \int_{x^{min}}^{x^{\gamma}} F(x)dx \\ &\quad + F(x^q)S(x^q) - F(x^{\gamma})S(x^{\gamma}) + \frac{e}{r + \lambda + \delta} \int_{x^{\gamma}}^{x^q} F(x)dx \end{aligned}$$

which brings equation (31):

$$\overline{S} = \frac{e - s}{r + \lambda + \delta + \beta p} \int_{x^{min}}^{x^{\gamma}} F(x)dx + \frac{e}{r + \lambda + \delta} \int_{x^{\gamma}}^{x^q} F(x)dx \tag{31}$$

Under the assumption that $e = s$, $\overline{S}$ has the following simple expression

$$\overline{S} = e \frac{\int_{x^{\gamma}}^{x^q} F(x)dx}{r + \lambda + \delta} \tag{32}$$

9.5 Partial equilibrium with finite p

9.5.1 Existence and uniqueness

The proof for uniqueness of $x^{\gamma}; x^q$ for a given p is simple to obtain. First, the expression of the quit margin in equation (10) is downward sloping, the expression for the entry margin in equation (9) is upward sloping. The intuition is as follows: along the entry margin, a higher x^q means a higher surplus from participation since jobs will last longer. This raises x^o i.e. the incentives to participate. Along the quit margin, a higher x^o has an a priori ambiguous impact on $\overline{S}$, as (10) indicates. There is a positive effect due to $O(e - s)$, i.e. the part of the expected surplus reflecting the states in which the worker is attached (a higher x^o increases the mass of those workers), and a negative effect due to $\frac{\lambda}{r+\lambda+\delta}\int_{x^o}^{x^q} F(x)dx$ reflecting the states in which the worker is unattached (a higher x^o decreases the mass of those workers). The latter dominates.[27] It is also easy to see that the intersections with the horizontal axis ($x^o = x^{min}$) are such that the intercept of the entry margin is below x^{min} while the intercept of the quit margin is given implicitly by $x^q = y/e - (r/e)V^V + \frac{\lambda}{r+\lambda}\int_{x^{min}}^{x^q} F(x)dx$. A sufficient condition for uniqueness is that the latter is above x^{min}. Finally note that the intersection of both margins is below the 45 degree line, indicating $x^q > x^o$.

9.5.2 Effects of frictions

Now, we can analyze the effect of frictions on both margins. The entry margin is shifted leftward by a larger p, while the quit margin shifts leftward too. The intuitions for these results is the following. At the entry margin, the larger p, the easier it is to find a job, and thus the larger the incentive to participate in the labor market (higher x^o at a given x^q). Put otherwise, the opportunity cost of searching, sx^o/p is lower, raising incentives to participate.

The quit margin depends on p only when $e > s$: a larger p shifts the quit margin to the left, i.e. reduces x^q for a given x^o. If $e = s$, the quit margin does not depend on p. In fact, a larger p raises the rate at which workers exit from unemployment. Since workers devote more time to home production when unemployed than when employed, a larger p implies that a marginal gain of home productivity x is less valuable when p is large. In other words, the slopes of W and U in Figure 1 become closer to each other when p is larger. The difference between the two slopes is precisely the term $\frac{\lambda(e-s)/e}{r+\lambda+\delta+\beta p}$ which enter in the surplus after an integration by part. Note that, when $e = s$ the curves W and U are parallel and in fact horizontal on that graph and the effect of p vanishes. Similarly, when $\beta = 0$, we have $W \equiv U$ and a fortiori have identical slopes, which is why, in such a case, the effect of p disappears. To sum up, a larger p reduces the expected surplus of employment conditional on being unattached and reduces the gap between x^q and y/e. Under the assumption that $e = s$, the quit margin can be compactly written as

$$x^q = y/e - (r/e)V_V + \frac{\lambda}{r+\lambda+\delta}\int_{x^o}^{x^q} F(x)dx \tag{33}$$

and does not depend on p. We view the dependence of the quit margin to p as a second order effect, compared to the direct, intuitive effect of p on the entry margin: larger p reduce the entry costs and thus raise x^o.

[27] In Figure 1, one indeed could see that the slope of the surplus (the area between $W(x)$ and $Max[U(x); H(x)]$ is larger when $H > U$). Since the surplus from participation is lowered by higher x^o, this implies more frequent quits i.e. a lower x^q.

9.5.3 Comparative statics

Take the total differentials of the two equations so that

$$
\begin{aligned}
F^e(x^o; x^q; p) &= 0 \\
F^q(x^o; x^q; p) &= 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial F^e}{\partial x^o}dx^o + \frac{\partial F^e}{\partial x^q}dx^q + \frac{\partial F^e}{\partial p}dp &= 0 \\
\frac{\partial F^q}{\partial x^o}dx^o + \frac{\partial F^q}{\partial x^q}dx^q + \frac{\partial F^q}{\partial p}dp &= 0
\end{aligned}
$$

Taking the partial derivatives we find that

$$
\begin{aligned}
Adx^o - Bdx^q &= Hdp \\
Cdx^o + Ddx^q &= -Kdp
\end{aligned}
$$

where

$$
\begin{aligned}
B &= \frac{p\beta e}{r+\lambda+\delta} & A &= s + B & H &= \beta e\frac{x^q - x^o}{r+\lambda+\delta} \\
D &= \frac{e(r+\delta+\lambda(1-F^q))}{r+\delta+\lambda} & C &= F(x^o)\frac{\lambda s(r+\delta+\lambda)+\lambda e\beta p}{(r+\delta+\lambda+\beta p)(r+\lambda+\delta)} & K &= \frac{\beta\lambda(e-s)\int_{x_{min}}^{x^o} F(x)dx}{(r+\delta+\lambda+\beta p)^2}
\end{aligned}
$$

Applying Kramer's rule it follows that

$$
\begin{aligned}
\frac{dx^q}{dp} &= \frac{-AK - HC}{AD + BC} < 0 \\
\frac{dx^o}{dp} &= \frac{HD - BK}{AD + BC} = ?
\end{aligned}
$$

Proposition 8 In a fully indivisible economy (e=s), a decrease in market frictions (a rise in p) reduces the quit cutoff point x^q and increases the entry cutoff point x^o.

Figure 7 summarizes this result. Note that x^o increases with p not only when $e = s$, but also when $e > s$, provided that one of the following conditions is true: p is close enough to zero, or p is large enough, or when the bargaining power of workers β is low enough.

Remark 2 With $e = s$ $K = 0$ so that $\frac{dx^q}{dp} = \frac{-HC}{AD+BC} < 0$ and $\frac{dx^o}{dp} = \frac{HD}{AD+BC} > 0$. Further, as p tends to infinite, $H \to 0$ since $x^q \to x^o$; so that $\frac{dx^o}{dp} = \frac{dx^q}{dp} = 0$. Further, when $p \to 0$ we have that $\left|\frac{dx^o}{dp}\right| > \left|\frac{dx^q}{dp}\right|$ since D>C as long as $e(r+\delta+\lambda(1-F(x^q)) > e\lambda F(x^o)$ which is always true as p goes to zero (since x^o goes to zero)

Remark 3 In a partially indivisible economy, with $e > s$, the ambiguity of $\frac{dx^o}{dp}$ is entirely due to term $K = \frac{\beta\lambda(e-s)\int_{x_{min}}^{x^o} F(x)dx}{(r+\delta+\lambda+\beta p)^2}$. But we can say three things: i) $K = 0$ if e=s; $K = 0$ as p tends to infinite and $K = 0$ as p tends to to zero (since x^o) is zero. We can basically ignore the term K and work with e = s

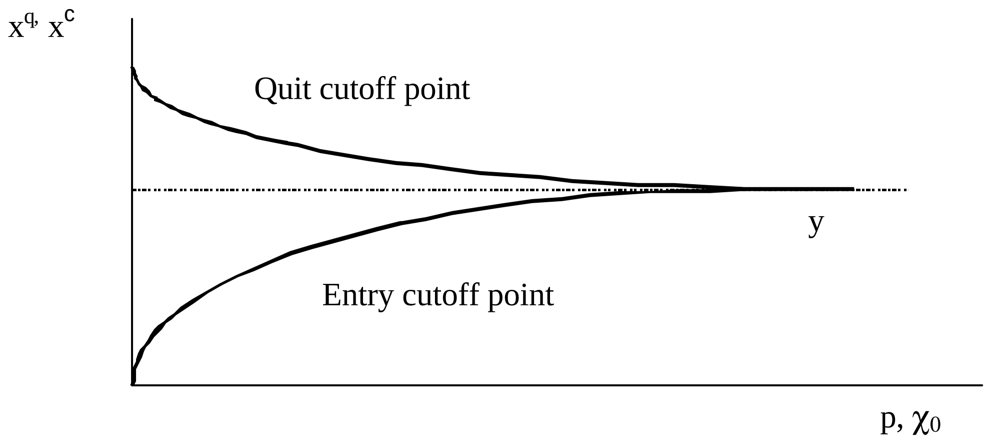

Figure 7:

9.6 Stocks and °ows

We ¯rst de¯ne a few notations: let's denote with capital letters the stocks of workers E_a, E_{na}, N_U and N respectively the employed attached, employed unattached, unemployed and the non-participants (in full-time home production), and by small letters the °ows between those stocks. All the °ows in the model are represented in Figure 8.

One can write the evolution of the stocks of workers in the three categories by:

$$dE_a{=}dt = ¡\,(e_a n + e_a u + e_a e_{na})E_a + ue_a N_U + e_a e_{na} E_{na} \quad (34)$$

$$dE_{na}{=}dt = ¡\,(e_{na} n + e_{na} u + e_{na} e_a)E_{na} + e_a e_{na} E_{na} \quad (35)$$

$$dN_U{=}dt = ¡\,(ue_a + un)N_U + e_a u E_a + nuH \quad (36)$$

$$dN{=}dt = ¡\,nuN + e_{na} n E_{na} + unN_U + e_a n E_a \quad (37)$$

In steady-state and replacing the rates of transition by their values, one obtains:

$$N\,¸F(x^°)\ ¡\ ¸N_U(1\ ¡\ F(x^°)) = ¸E_a(1\ ¡\ F(x^q)) + E_{na}(± + ¸\ ¡\ ¸F(x^q)) \quad (38)$$

$$N_U(¸\ ¡\ ¸F(x^°) + p) = N\,¸F(x^°) + E_a ± \quad (39)$$

$$E_a(± + ¸\ ¡\ ¸F(x^°)) = pN_U + E_{na}\,¸F(x^°) \quad (40)$$

$$E_{na}(± + ¸\ ¡\ ¸F(x^q) + ¸F(x^°)) = ¸E_a(F(x^q)\ ¡\ F(x^°)) \quad (41)$$

$$E_{na} + E_a + N_U + N = 1 \quad (42)$$

Take (40) + (41), and denote by $E = E_a + E_{na}$. One gets:

$$E(± + ¸\ ¡\ ¸F(x^q)) = pN_U$$

which immediately leads to

$$u_r = \frac{N_U}{E} = \frac{± + q}{± + p + q}$$

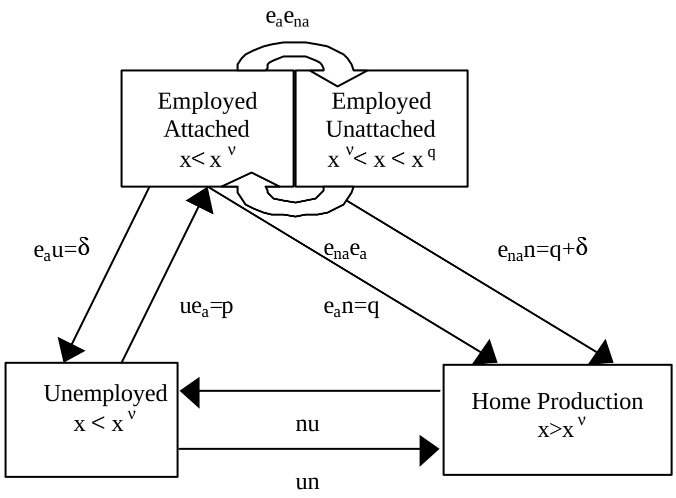

Figure 8:

where

$$q = \lambda(1 - F(x^q))$$

Then, using (41), one has that

$$\frac{E_a}{E} = \frac{\delta + q + \lambda F(x^\gamma)}{\delta + q + \lambda F(x^q)} = \frac{\delta + q + \lambda F(x^\gamma)}{\delta + \lambda}$$

and thus

$$\frac{E_{na}}{E} = \frac{\lambda(F(x^q) - F(x^\gamma))}{\delta + \lambda}$$

Finally, using (39), one has:

$$N \lambda F(\theta^\gamma) = (\lambda - \lambda F(x^\gamma) + p)N_U - \delta \left(\frac{\delta + q + \lambda F(x^\gamma)}{\delta + \lambda} E \right)$$

which, dividing by $E + N_U = 1 - n$ where n is the non-participation rate, leads to

$$\frac{N}{E + N_U} \lambda F(x^\gamma) = (\lambda - \lambda F(x^\gamma) + p)u_r - \delta \left(\frac{\delta + q + \lambda F(x^\gamma)}{\delta + \lambda} \right) (1 - u_r) \qquad \text{or} \qquad (43)$$

$$\frac{n}{1 - n} \lambda F(x^\gamma) = (\lambda - \lambda F(x^\gamma) + p)\frac{\delta + q}{\delta + p + q} - \delta \left(\frac{\delta + q + \lambda F(x^\gamma)}{\delta + \lambda} \right) \frac{p}{\delta + p + q} \qquad (44)$$

which cannot be easily simplified.

9.7 Taxes

9.7.1 Welfare analysis

The proof the neutrality result is a follows

Proof. The first order condition on t implies that $P'(Bt) = 1$ or $P'(G) = 1$. In words, the marginal utility of public spending must be equal to its unit marginal cost. In turn, the first order conditions on variables z imply

$$\frac{\partial B}{\partial z}(1-t) + \frac{\partial H}{\partial z} + t\frac{\partial B}{\partial z}P'(Bt) - \upsilon\frac{\partial F}{\partial z}(z) = 0$$

where υ is the Lagrange multiplier associated to the constraints $F(z) = 0$. This condition implies, using the first order condition on t, that

$$\frac{\partial(B+H)}{\partial z} = \upsilon\frac{\partial F}{\partial z}(z)$$

■

The proof of the social planner problem is as follows. The social planner maximizes over $u; x^\gamma; x^q$

$$\Omega = y(1-n-u) - c\phi u + H$$

where H is home production of inactive workers and $u = N_U$ is the mass of unemployed workers (total population is normalized to 1). NB, $u \neq u_r = N_U/E$. Denoting by $f^H(x)$ the density of inactive workers and n their total number, home production is

$$H = n\int_{x^\gamma}^{+\infty} xf^H(x)d\theta$$

We can prove that f^H is proportional to f and the problem thus rewrites

$$\begin{aligned}\underset{u;\theta^\gamma;\theta^q}{Max}\ \Omega &= y(1-u-n) - c\phi u + \int_{x^\gamma}^{+\infty} xf(x)dx \\ &\quad -(1-u-n)\frac{\lambda}{\lambda+\delta}\int_{x^\gamma}^{x^q} xf(x)dx\end{aligned}$$

, subject to constraints

$$pu - (\delta+q)(1-u-n) = 0$$

$$(1-u-n)\left[\frac{\delta}{\delta+\lambda}(\delta+q) - \frac{\lambda}{\lambda+\delta}\lambda F(x^\gamma)\right] - u(\lambda+p) + \lambda F(x^\gamma) = 0$$

The first two immediately lead to:

$$\begin{aligned}\text{Entry} &: x^\gamma = p\eta\frac{x^q - x^\gamma}{\lambda+\delta} \\ \text{JC} &: \frac{x^q - x^\gamma}{\lambda+\delta}(1-\eta) = \frac{c}{\chi(\phi)} \\ \text{Quit} &: x^q = y + \frac{\lambda}{\lambda+\delta}\int_{x^\gamma}^{x^q} F(x)dx\end{aligned}$$

9.7.2 Taxation in the decentralized economy

Proofs are derived with $e = 1$, without implication. If t is the marginal tax rate on wages the labor cost can be found to be:

$$
\begin{aligned}
w^{a} &= \beta(y + c\phi) \\
w^{na}(x) &= \beta y + x(1 - \beta)/(1 - t)
\end{aligned}
$$

The model with taxes can be written as

$$
\begin{aligned}
\text{Entry} &: \quad x^{\gamma} = \frac{\beta\phi c(1 - t)}{1 - \beta} \\
\text{Quit} &: \quad x^{q} = y(1 - t) + \frac{\lambda}{\lambda + r + \delta}\int_{x^{\gamma}}^{x^{q}} F(z)dz \\
\text{JC} &: \quad \frac{x^{q} - x^{\gamma}}{r + \lambda + \delta}(1 - \beta) = \frac{c(1 - t)}{\chi(\phi)}
\end{aligned}
$$

From the first equation we have that $x^{\gamma} = x^{\gamma}(\phi; t)$ with $\frac{\partial x^{\gamma}}{\partial t} = -\frac{\beta\phi c}{1-\beta} < 0$ and $\frac{\partial x^{\gamma}}{\partial \phi} = \frac{\beta c(1-t)}{1-\beta} > 0$ so that the equations become

$$
\begin{aligned}
x^{q} - y(1 - t) - \frac{\lambda}{\lambda + r + \delta}\int_{x^{\gamma}(\phi;t)}^{x^{q}} F(z)dz &= 0 \\
\frac{x^{q} - x^{\gamma}(\phi; t)}{r + \lambda + \delta}(1 - \beta) - \frac{c(1 - t)}{\chi(\phi)} &= 0
\end{aligned}
$$

Taking the total differential we have

$$
\begin{aligned}
A dx^{q} + B d\phi &= -H dt \\
C dx^{q} - D d\phi &= -K dt
\end{aligned}
$$

with $A = [\frac{r + \lambda(1 - F^{q}) + \delta}{r + \lambda + \delta}]$; $B = \frac{\lambda F(x^{\gamma})}{\lambda + r + \delta}\frac{\partial x^{\gamma}}{\partial \phi}$; $H = [y + \frac{\lambda F(x^{\gamma})}{\lambda + r + \delta}\frac{\partial x^{\gamma}}{\partial dt}]$ $C = \frac{1-\beta}{r + \lambda + \delta}$; $D = -[-\frac{1-\beta}{r + \lambda + \delta}\frac{\partial x^{\gamma}}{\partial \phi} + \frac{c(1-t)\chi'(\phi)}{\chi^{2}(\phi)}]$; $K = [-\frac{1-\beta}{r + \lambda + \delta}\frac{\partial x^{\gamma}}{\partial t} + \frac{c}{\chi(\phi)}]$. Let us assume that $H > 0$; Applying Kramer's rule it follows that

$$
\frac{dx^{q}}{dt} = \frac{HD + BK}{-AD - BC} < 0
$$

while

$$
\frac{d\phi}{dt} = \frac{-AK + HC}{-AD - BC} < 0.
$$

To see the latter effect note that substituting from the definition of $A; K; H; C$ one obtains that

$$
\begin{aligned}
-AK + HC &= -\lambda\int_{x^{\gamma}}^{x^{q}} F(z)dz + \lambda(F(x^{q})x^{q} - F(x^{\infty})x^{\infty}) \\
&= \lambda\int_{x^{\gamma}}^{x^{q}} zf(z)dz > 0.
\end{aligned}
$$

Further note that with $\lambda = 0$; $\frac{d\phi}{dt} = 0$, since $A = 1$; $K = \frac{\beta\phi c}{r+\delta} + \frac{c}{\chi(\phi)}$; $H = y$ and $C = \frac{1-\beta}{r+\delta}$ so that

$$(-AD - BC)\frac{d\phi}{dt} = \frac{y(1-\beta)}{r+\delta} - \frac{c}{\chi(\phi)} - \frac{\beta\phi c}{r+\delta} = 0$$

9.8 Unemployment benefits

Consider the most general case where $v^W = w$; $v^{Uc} = b$; $v^U = b_0$ and $v^H = b_H + x$: compared to Section 6, even non-active workers get an income b_H which may be either 0 or b_0. The present discounted value of utility W writes as a function of the value of covered unemployment:

$$rW(x) = w(x) + \delta\{Max[U^c(x); H(x)] - W(x)\} + \lambda\int_{x^{min}}^{x^{max}} \{Max[W'; U^{c\prime}; H'] - W\} dF(x'); \tag{45}$$

As before, we shall assume that, for every x such that $W(x) > H(x)$, then $W(x) > U^c(x)$. This corresponds to a restriction on parameters denoted by R, which ensures that the market is viable and is simply, as it will be shown ex-post, $y > b$. The value of covered unemployment writes:

$$rU^c(x) = b + p[W(x) - U^c(x)] + \lambda\int_{x^{min}}^{x^{max}} \{Max[U'; H'] - U^c\} dF(x'); \tag{46}$$

while the value of uncovered unemployment writes:

$$rU(x) = b_0 + p[W(x) - U(x)] + \lambda\int_{x^{min}}^{x^{max}} \{Max[U'; H'] - U\} dF(x'); \tag{47}$$

The value of home production writes

$$rH(x) = b_H + x + \lambda\int_{x^{min}}^{x^{max}} \{Max[U'; H'] - H\} dF(x'); \tag{48}$$

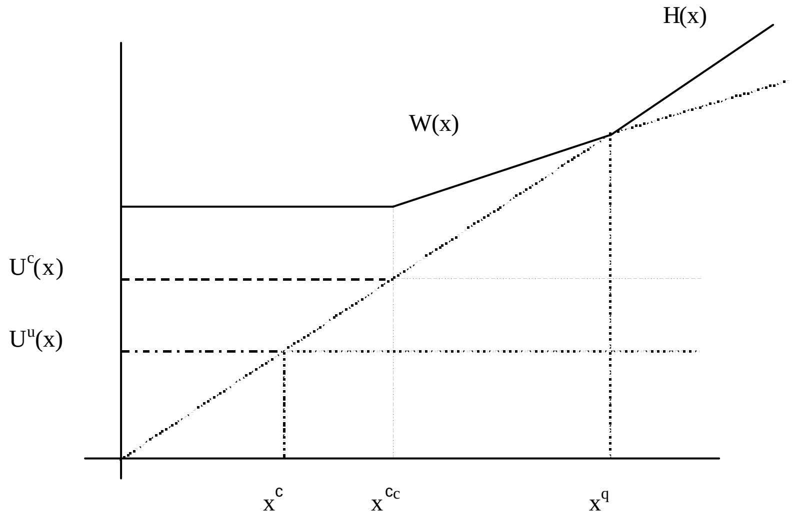

Figure 9:

With the use of the threshold values x^{cv}, x^{o} and x^{q}, one can rewrite the value functions as:

$x \le x^{oc}$)
$$(r+\lambda)W^{a}(x) = w^{a} + \delta(U^{c} - W)(x) + \lambda\int_{x^{min}}^{x^{q}} W(x')dF(x') + \lambda\int_{x^{q}}^{x^{max}} H(x')dF(x') \quad (49)$$

$x^{oc} \le x \le x^{q}$)
$$(r+\lambda)W^{na}(x) = w^{na} + \delta(H - W)(x) + \lambda\int_{x^{min}}^{x^{q}} W(x')dF(x') + \lambda\int_{x^{q}}^{x^{max}} H(x')dF(x') \quad (50)$$

$x \le x^{oc}$)
$$(r+\lambda)U^{c}(x) = b + p(W - U^{c})(x) + \lambda\int_{x^{min}}^{x^{oc}} U^{c}(x')dF(x') + \lambda\int_{x^{oc}}^{x^{max}} H(x')dF(x') \quad (51)$$

$x \le x^{c}$)
$$(r+\lambda)U(x) = b_{0} + p(W - U)(x) + \lambda\int_{x^{min}}^{x^{o}} U(x')dF(x') + \lambda\int_{x^{o}}^{x^{max}} H(x')dF(x') \quad (52)$$

$x^{q} \le x$)
$$(r+\lambda)H(x) = b_{H} + x + \lambda\int_{x^{min}}^{x^{o}} U(x')dF(x') + \lambda\int_{x^{o}}^{x^{max}} H(x')dF(x') \quad (53)$$

We need to introduce some notations first:

$$\overline{S_{w}} = \int_{x^{min}}^{x^{q}} \{W(x') - Max[U^{c}(x'); H(x')]\}dF(x') \ge 0: \quad (54)$$

$$\overline{S_{w}} = \beta\overline{S} \quad (55)$$

Note that $\bar{S} = \int_{x^{min}}^{x^{\gamma c}} S^a(x)dF(x) + \int_{x^{\gamma c}}^{x^q} S^{na}(x)dF(x)$ where

$$\begin{aligned} S^a(x) &= W^a(x) - U^c(x) + J^a(x) \\ S^{na}(x) &= W^{na}(x) - H(x) + J^{na}(x) \end{aligned}$$

Finally, let's define Φ as

$$\Phi = \lambda \int_{x^{min}}^{x^{\gamma}} (U^c - U)(x')dF(x') + \lambda \int_{x^{\gamma}}^{x^{\gamma c}} (U^c - H)(x')dF(x') \tag{56}$$

The surplus of attached, unattached workers and firms is

$$\begin{aligned} (r + \lambda + \delta)(W - U^c) &= w^a - b - p(W - U^c) + \lambda\beta\bar{S} \\ (r + \lambda + \delta)(W - H) &= w^{na} - x - b_H + \Phi + \lambda\beta\bar{S} \\ (r + \lambda + \delta)J &= y - w + \lambda(1 - \beta)\bar{S} \end{aligned}$$

It comes trivially that

$$\begin{aligned} w^a(x) &= w^{\alpha} = (1 - \beta)b + \beta y + \beta p\bar{S} \\ w^{na}(x) &= (1 - \beta)(x + b_H - \Phi) + \beta y \end{aligned}$$

This implies that S^a, J^a, U^c and W^a do not depend on x, which brings notably $\bar{S}$ as

$$\begin{aligned} \bar{S} &= \int_{x^{min}}^{x^{\upsilon c}} S^a dF(z) + \int_{x^{\upsilon c}}^{x^q} S^{na} dF(z) \\ &= S^a(x^{\gamma c})F(x^{\gamma c}) + S^{na}(x^q)F(x^q) - S^{na}(x^{\gamma c})F(x^{\gamma c}) - S'^{na} \int_{x^{\gamma c}}^{x^q} F(z)dz \\ &\quad - S'^{na} \int_{x^{\gamma c}}^{x^q} F(z)dz \end{aligned}$$

Thus to get the surplus we just need the derivative of S^{na} with respect to x: First to get S^{na} recall that

$$(r + \delta + \lambda)S^{na}(x) = y - x - b_H + \lambda \int_{x^{min}}^{x^{\upsilon c}} S^a(z)dF(z) + \int_{x^{\upsilon c}}^{x^q} S^{na}(z)dF(z)$$

so that

$$(r + \delta + \lambda)S^{na}(x) = y - x + K$$

with K is a constant of x, so that

$$S'^{na}(x) = -\frac{1}{r + \delta + \lambda}$$

and

$$\overline{S} = \frac{1}{r+\delta+\lambda}\int_{x^{\gamma c}}^{x^{q}} F(z)dz$$

Further, using the Bellman equations for the covered and uncovered unemployment, we further show that, for all $x^{\gamma} < x < x^{\gamma c}$,

$$(r+\lambda+p)(U^{c}-U) = b - b_{0} + \Theta \tag{57}$$

and so, we have that

$$\Theta = (U^{c}-U)\lambda F(x^{\gamma}) - [U^{c}-H](x^{\gamma})\lambda F(x^{\gamma}) + \frac{\lambda}{r+\lambda}\int_{x^{\gamma}}^{x^{\gamma c}} F(x^{0})dx^{0}$$
$$= \frac{\lambda}{r+\lambda}\int_{x^{\gamma}}^{x^{\gamma c}} F(x^{0})dx^{0}$$

9.8.1 Entry

The new entry margin is defined by $H(x^{\gamma}) = U(x^{\gamma})$ and implies:

$$\frac{x^{\gamma}+b_{H}-b_{0}}{p} = [W(x^{\gamma})-U(x^{\gamma})] \tag{58}$$
$$= (W-U^{c})(x^{\gamma c}) + (U^{c}(x^{\gamma c})-U(x^{\gamma})) \tag{59}$$
$$= \beta\frac{x^{q}-x^{\gamma c}}{r+\lambda+\delta} + \frac{b-b_{0}+\Theta}{r+\lambda+p} \tag{60}$$

the last line using $W - U^{c}(x^{\gamma}) = \beta S^{a}(x^{\gamma})$. Note that there is also an alternative expression for the Entry margin, which reads

$$\frac{x^{\gamma}+b_{H}-b_{0}}{p} = [W(x^{\gamma})-U(x^{\gamma})]$$
$$= (W-U^{c})(x^{\gamma c}) + (U^{c}(x^{\gamma c})-U(x^{\gamma}))$$
$$= \beta\frac{x^{q}-x^{\gamma c}}{r+\lambda+\delta} + H(x^{\gamma c}) - H(x^{\gamma})$$
$$= \beta\frac{x^{q}-x^{\gamma c}}{r+\lambda+\delta} + \frac{x^{\gamma c}-x^{\gamma}}{r+\lambda}$$

9.8.2 Entry covered

It is determined by $H(x^{\gamma c}) = U^{c}(x^{\gamma c})$ and implies:

$$\frac{x^{\gamma c}+b_{H}-b-\Theta}{p} = [W(x^{\gamma c})-U^{c}(x^{\gamma c})] \tag{61}$$
$$= \beta\frac{x^{q}-x^{\gamma c}}{r+\lambda+\delta} \tag{62}$$

9.8.3 Quit

The equality $W(x^q) = H(x^q)$ implies the new quit margin:

$$\lambda\beta S - \Theta = x^q + b_H - w^{na}(x^q) \tag{63}$$

which, using the expression for the wage, is

$$x^q = y - b_H + \frac{\lambda}{r+\delta+\lambda}\int_{x^{\gamma c}}^{x^q} F(z)dz + \Theta$$

9.8.4 Job creation

The value of a vacancy is

$$rV_V = -c + \chi(\phi)[J^e - V_V]; \tag{64}$$

where

$$J^e = p_c\frac{\int_{x_{min}}^{x^{\gamma c}} J(x')dF(x')}{F(x^{\gamma c})} + (1-p_c)\frac{\int_{x_{min}}^{x^{\gamma}} J(x')dF(x')}{F(x^{\gamma})}$$

with p_c is the probability to meet a covered employed worker. From the perspective of the firm, there are two different types of job seekers, the new entrant ones whose $x < x^{\gamma}$ and the laid-off unemployed workers, whose $x < x^{\gamma c}$, as displayed in Figure 10.

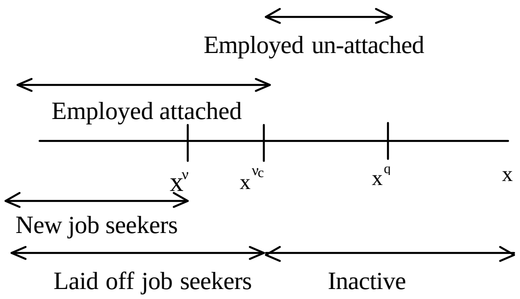

Figure 10:

With the assumption that wages are all negotiated with as a threat point of workers the value of covered unemployment, $J(x)$ is constant for all J below $x^{\gamma c}$, and thus the job creation margin is:

$$\frac{c}{\chi(\phi)} = (1-\beta)\frac{x^q - x^{\gamma c}}{r+\lambda+\delta}$$

To sum up:

$$
\begin{aligned}
JC' &: \quad \frac{c}{q(\theta)} = (1-\beta)\frac{x^{q}-x^{oc}}{r+\lambda+\delta} \\
Entry^{c'} &: \quad \frac{x^{oc}+b_H-b-\tilde{b}}{p} = \beta\frac{x^{q}-x^{oc}}{r+\lambda+\delta} \\
Entry' &: \quad \frac{x^{o}+b_H-b_0}{p} = \beta\frac{x^{q}-x^{oc}}{r+\lambda+\delta} + \frac{b-b_0+\tilde{b}}{r+\lambda+p} \\
Quit' &: \quad x^{q} = y - b_H + \frac{\lambda}{r+\delta+\lambda}\int_{x^{oc}}^{x^{q}} F(z)dz + \tilde{b}
\end{aligned}
$$

As can be seen from the equations above, b_H only appears as a pure shift parameter of the distribution of x and is thus mostly nominal. We loose nothing by normalizing b_H at 0, which is thus assumed now. Let's write the model holding constant $p(\theta)$.

$$
\begin{aligned}
Entry^{c'} &: \quad \frac{x^{oc}}{p} = \beta\frac{x^{q}-x^{oc}}{r+\lambda+\delta} + \frac{b+\tilde{b}}{p} \\
Entry' &: \quad \frac{x^{o}}{p} = \beta\frac{x^{q}-x^{oc}}{r+\lambda+\delta} + \frac{b+\tilde{b}}{r+\lambda+p} \\
Quit' &: \quad x^{q} = y + \frac{\lambda}{r+\delta+\lambda}\int_{x^{oc}}^{x^{q}} F(z)dz + \tilde{b}
\end{aligned}
$$

9.8.5 Comparative statics of the participation margins

We ignore in this part the fourth equation, namely (JC) and treat both p and V_V as parameters.[28] From ($Entry(b)$) and ($Entry^{C}(b)$), we have:

$$
Diff: x^{oc} - x^{o} = (b+\tilde{b})(1-B)
$$

where $B = \frac{p}{r+\lambda+p} > 0$. Further note

$$
\begin{aligned}
A &= \frac{\lambda}{r+\lambda}F(x^{o}) < 1; \quad A^{c} = \frac{\lambda}{r+\lambda}F(x^{oc}) < 1 \\
A^{q} &= \frac{\lambda}{r+\lambda+\delta}F(x^{q}) < 1; \quad R^{q} = 1 - A^{q} > 0 \\
C &= 1/(1-B) = 1 + \frac{p}{r+\lambda} > 1 \\
Z &= -\frac{p}{r+\delta+\lambda}; \quad \delta' = \frac{\delta}{r+\lambda+\delta}
\end{aligned}
$$

[28] This makes sense, since we study the role of unemployment benefits on the three participation margins x^{o}, x^{oc} and x^{q} for a given worker in a given firm, while rV_V, different from 0 in partial equilibrium, is a function of x^{o}, x^{oc} and x^{q} in other firms.

and differentiating at constant p the system

$$
\begin{aligned}
Entry^c(b) &: \quad x^{oc} = \beta p \frac{x^q - x^{oc}}{r+\lambda+\delta} + b + \tilde{b} \\
Diff(b) &: \quad x^{oc} - x^o = (b+\tilde{b})(1-B) \\
Quit(b) &: \quad x^q = y + \tilde{b} + \frac{\lambda}{r+\delta+\lambda}\int_{x^{oc}}^{x^q} F(z)dz
\end{aligned}
$$

we obtain:

$$
\begin{aligned}
A dx^o + (1 - A^c + \beta B)dx^{oc} - Z dx^q &= db \\
dx^o(C - A) - dx^{oc}(C - A^c) + 0 dx^q &= -db \\
dx^o A - dx^{oc}(\delta' A^c) + R^q dx^q &= 0
\end{aligned}
$$

Denote by

$$
W = \begin{pmatrix} A & 1 - A^c + Z & -Z \\ C - A & -(C - A^c) & 0 \\ A & -\delta' A^c & R^q \end{pmatrix}
$$

the matrix. Note that all coefficients are signed: $C > A$ and $C > A^c$ and $1 - A^c > 0$. The determinant of the matrix of the associated system has sign of

$$
\det(MAT) = (C - A)[A^c \delta' Z - R^q(1 - A^c + Z)] + (C - A^c)A(-R^q - Z)
$$

All terms are negative except the very first one, $A^c \delta' Z$. The determinant is negative either when $b = 0$ (i.e. $A = A^c$) or when $\delta = 0$ (i.e. $\delta' = 0$). However, we can formally prove that the determinant is always negative[29]. It is sufficient to demonstrate that $R^q - \delta' A^c \geq 0$. Let us denote by

$$
G(\delta; z) = 1 - \frac{\lambda F(x^q(\delta))}{r+\lambda+z} - \frac{\lambda}{r+\lambda}\frac{zF(x^{oc}(\delta))}{r+\lambda+z}
$$

where it is clear that δ affects x^q and x^{oc} through an effect that we want to eliminate, and that $G(\delta;\delta) = R^q - \delta' A^c$. Noticing that i) $G(\delta; 0) = 1 - \frac{\lambda F(x^q(\delta))}{r+\lambda} \geq 0$; ii) $G(\delta; +\infty) = 1 - A^c \geq 0$; and iii) $\partial G / \partial z = \frac{\lambda[F(x^q(\delta)) - F(x^{oc}(\delta))]}{(r+\lambda+z)^2} \geq 0$, it is clear that $G(\delta; z) \geq 0$ for all z, notably for $z = \delta$. The determinant of W is thus negative.

Then, the adj. matrix of W is defined by

$$
W^{adj} = \begin{pmatrix}
\begin{vmatrix} -(C - A^c) & 0 \\ -\delta' A^c & R^q \end{vmatrix} & -\begin{vmatrix} C - A & 0 \\ A & R^q \end{vmatrix} & \begin{vmatrix} C - A & -(C - A^c) \\ A & -\delta' A^c \end{vmatrix} \\
-\begin{vmatrix} 1 - A^c + Z & -Z \\ -\delta' A^c & R^q \end{vmatrix} & \begin{vmatrix} A & -Z \\ A & R^q \end{vmatrix} & -\begin{vmatrix} A & 1 - A^c + Z \\ A & -\delta' A^c \end{vmatrix} \\
\begin{vmatrix} 1 - A^c + Z & -Z \\ -(C - A^c) & 0 \end{vmatrix} & -\begin{vmatrix} A & -Z \\ C - A & 0 \end{vmatrix} & \begin{vmatrix} A & 1 - A^c + Z \\ C - A & -(C - A^c) \end{vmatrix}
\end{pmatrix}
$$

[29] For this proof, we are grateful to François Marque.

Thus, since $W^{-1} = \left(W^{adj}\right)' / \det W$, we have $\frac{\partial}{\partial b}\begin{pmatrix} x^o \\ x^{oc} \\ x^q \end{pmatrix} = W^{-1}\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$. We have then

$$\frac{dx^o}{db} = \frac{\begin{vmatrix} -(C - A^c) & 0 \\ -\delta' A^c & R^q \end{vmatrix} + \begin{vmatrix} 1 - A^c + Z & -Z \\ -\delta' A^c & R^q \end{vmatrix}}{\det(MAT)} = \frac{num^o}{\det(MAT)}$$

$$\frac{dx^{oc}}{db} = \frac{-\begin{vmatrix} C - A & 0 \\ A & R^q \end{vmatrix} - \begin{vmatrix} A & -Z \\ A & R^q \end{vmatrix}}{\det(MAT)} = \frac{num^{oc}}{\det(MAT)}$$

$$\frac{dx^q}{db} = \frac{\begin{vmatrix} C - A & -(C - A^c) \\ A & -\delta' A^c \end{vmatrix} + \begin{vmatrix} A & 1 - A^c + Z \\ A & -\delta' A^c \end{vmatrix}}{\det(MAT)} = \frac{num^q}{\det(MAT)}$$

which simplifies as

$$\begin{aligned} num^o &= R^q\left(\frac{-p}{r+\lambda} + \frac{\beta p}{r+\lambda+\delta}\right) - \delta' A^c Z < 0 \Rightarrow \frac{dx^o}{db} \geq 0 \\ num^{oc} &= -CR^q - AZ < 0 \Rightarrow \text{sign}\frac{dx^{oc}}{db} \geq 0 \\ num^q &= -\delta' A^c \left(1 + \frac{p}{r+\lambda}\right) + \frac{Ap}{r+\lambda} - \frac{A\beta p}{r+\lambda+p} \Rightarrow \text{sign}\frac{dx^q}{db}? \end{aligned}$$

When $\delta' = 0$ (i.e. $\delta = 0$), num^q is positive so that $\frac{dx^q}{db} < 0$. When $\delta' = 1$ ($\delta \to +\infty$) then $num^q = -(A^c - A)\frac{p}{r+\lambda} - \frac{A\beta p}{r+\lambda+p} < 0$ so that $\frac{dx^q}{db} > 0$. Note that A^c and A depend on δ through the cut-off points x^o and x^{oc}, so that, despite the monotonicity of num^q with respect to δ we cannot go further than to say that in the neighborhood of $\delta = 0$, $\frac{dx^q}{db} < 0$ and in the neighborhood of $\delta = \infty$, $\frac{dx^q}{db} > 0$.

9.9 Endogenous search effort

Once we have defined and calculated the margins as in Section 7.3, we can rewrite the Bellman equations, leading to

$$\begin{aligned} (r+\lambda)W(x) &= \overline{w} + (1-e)x + \delta(N(x) - W(x)) + \lambda\int_{x^{min}}^{\tilde{x}^q} W(x')dF(x') + \int_{\tilde{x}^q}^{x^{max}} N(x')dF(x') \\ \text{for } x &< \tilde{x}^o,\ (r+\lambda)N(x;s) = x(1-s^*) + p(\theta;s^*)(W(x) - N(x;s^*)) + \lambda\int_{\tilde{x}^q}^{x^{max}} N(x';s')dF \\ \text{with } s^* &> 0 \\ \text{for } \tilde{x}^o &< x < \tilde{x}^q,\ (r+\lambda)N(x;0) = x + p(\theta;0)(W(x) - N(x;0)) + \lambda\int_{\tilde{x}^q}^{x^{max}} N(x';s')dF(x') \\ \text{for } x &> \tilde{x}^q,\ (r+\lambda)N(x;0) = x + \lambda\int_{\tilde{x}^q}^{x^{max}} N(x';s')dF(x') \end{aligned}$$

while the job creation side is determined by a free-entry condition:

$$\frac{c}{q(\theta)} = \left(\int_{x^{min}}^{\varepsilon^{o}} \sigma(s(x))dF(x) + \int_{\varepsilon^{o}}^{\varepsilon^{q}} \sigma(0)dF(x) \right) J \tag{65}$$

where $J = \frac{y - \overline{w}}{r+\delta+\lambda F(\varepsilon^{q})}$ is the value of a job, constant of x for all x below ε^{q}.

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