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ABSTRACT

Endogenous Contracts Under Bargaining in Competing Vertical Chains*

We investigate the endogenous determination of contracts in competing vertical chains where upstream and downstream firms bargain first over the type of contract and then over the contract terms. Upstream firms always opt for non-linear contracts, which specify the input quantity and its total price. Downstream firms also opt for non-linear contracts, unless their bargaining power is low, in which case they prefer wholesale price contracts. While welfare is maximized under two-part tariffs, these are dominated in equilibrium by non-linear contracts.

JEL Classification: L13, L14, L22, L42 and L81
Keywords: bargaining, non-linear contracts, strategic contracting, two-part tariffs, vertical chains and wholesale prices

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1. Introduction

'Vertical contracts' signed among firms that operate at different stages of a supply chain - "upstream" and "downstream" firms – (such as input producers and final good manufacturers, or wholesalers and retailers, respectively) can take and do take in reality a variety of forms. There is an increased theoretical and practical interest in the nature of these contracts, justified not only by the fact that the specific terms of vertical contracts can have a strong impact on firm behavior and welfare, but also by the fact that their impact can crucially depend on the specific form that they take. Thus, it is not surprising that a number of important papers examining strategic aspects of vertical contracting already exists (see Vickers, 1985; Rey and Tirole, 1986; Fershtman and Judd, 1987; Bonanno and Vickers, 1988; Gal-Or, 1991; Rey and Stiglitz, 1995; Martimort, 1996; Kühn, 1997, to mention just a few).[1] While the focus of each of these papers may be on different manifestations of firms' behavior, all of them are developed around a common general theme: how the type and/or the terms of contracts are determined within a vertical chain, and how these affect the behavior of the downstream firms of the same and of rival chains, and thus the firms profits and the consumers' surplus.

Our paper also deals with the important issue of vertical contracting. In particular, we investigate the endogenous determination of *both* the type and the terms of vertical contracts when *both* the upstream and downstream firms in competing vertical chains possess some power over these two contracting aspects. A key innovative feature of our modeling approach is that both upstream and downstream firms are considered to be large players, who act strategically, during the determination of both the type and the specific terms of vertical contracts. This feature is consistent with many real world cases in which the contracts are determined through bilateral negotiations.[2] In such cases, an assumption that the power to set the vertical contracts rests exclusively with the upstream firms (as in most of the existing literature), or alternatively, exclusively with the downstream firms, is extreme and not innocuous. In our analysis, we allow both upstream and downstream firms to get involved in the contracting procedure by

[1] See literature reviews e.g. by Tirole (1988, ch.4), Katz (1989), Dobson and Waterson (1996), Rey and Tirole (1997), and Irmen (1998). Early studies have examined the behavior of a single upstream firm, see e.g. Dixit (1983), Mathewson and Winter (1984), and Bresnahan and Reiss (1985).

[2] See e.g. recent European Commission (1999) and OECD (1999) reports concerning increased power of retailers in the food sector, as well as Dobson and Waterson (1996).

assigning some bargaining power over the choice of both the type and the terms of contracts to each side of each vertical chain. Our analysis, thus, nests as special cases situations where all the power is either upstream or downstream.

We consider three different types of vertical contracts. The first is a standard *wholesale price contract*, that is, a linear contract specifying a per-unit of input price that the downstream firm has to pay to the upstream. The second is a *two-part tariff contract* that involves, in addition to the wholesale price, a fixed fee - transfer. Finally, the third is a *nonlinear contract* that specifies both the total quantity of input and the respective total payment that the downstream firm has to make to its upstream supplier.[3] This last type of contract is clearly extremely nonlinear as it specifies a 'finite' price only for a certain quantity choice and corresponds to a 'price-quantity package' or 'quantity bundle'.[4]

In the main body of our analysis, we assume that there is 'quantity commitment' in the case of a nonlinear contract. In other words, we assume that the downstream firm's final output is equal to the input quantity specified in the nonlinear contract. In an extension of the basic model, we relax this assumption and assume instead that there is 'free disposal', that is, the downstream firm is free to produce a lower final product quantity than the input quantity specified in the nonlinear contract. It is important to note that, even though under this alternative formulation there is no direct downstream quantity commitment, there is another type of commitment. There is commitment to a maximum capacity up to which the downstream firm will produce the final product at zero marginal cost in the last stage of the game. This is due to the fact that the total input price is a *sunk cost* for the downstream firm during the market competition stage.

We consider the following model. There are two vertical chains, each consisting of one upstream and one downstream firm – there is exclusive dealing between the upstream and the downstream firm that constitute each vertical chain. We analyze a

[3] Björnerstedt and Stennek (2001) also consider the same type of nonlinear contracts in bilateral oligopoly. However, they do not allow for any other type of contract to be signed between upstream and downstream firms.

[4] One should distinguish between 'price-quantity' package contracts and the nonlinear contract schedules, i.e. contracts that specify a total payment for each input quantity delivered downstream, which are considered by Kühn (1997). Note that a two-part tariff is a special case of a nonlinear contract schedule. Kühn (1997) shows that, under certainty, nonlinear contract schedules lead to equilibria that are Pareto inferior to the equilibrium that arises under two-part tariffs and are, thus, not expected to be chosen by vertical chains. Having these observations in mind and in order to be more concise, we use the term 'nonlinear contracts' in order to refer to the 'price-quantity' packages described above.

three-stage game. In the first stage, within each vertical chain, the upstream and the downstream firm negotiate over the contract type. The bargaining process is the simplest possible one, postulating that with some probability (reflecting its bargaining power) the upstream firm makes a "take-it-or-leave-it" proposal of a contract type (among the three possible ones), while with the remaining probability the proposal is made by the downstream firm. After each vertical chain has determined its contract type (and that has been observed by the rival chain), in the second stage, the upstream and the downstream firm in each vertical chain bargain over the contract terms. The bargaining procedure is modeled by evoking the generalized Nash bargaining solution, with the bargaining powers of the firms being the same across the two bargaining stages. Finally, in the third stage, the two downstream firms (after having observed the contract terms of the two chains) produce differentiated products and compete in quantities.[5]

By considering oligopolistic markets in both the upstream and downstream sector, we are able to explore how strategic considerations arising in both sides of the market - upstream and downstream - affect both the vertical contract type choice and the contracts' terms. In addition, by assuming that the contract type is negotiated before the contract terms, we capture the idea that the type of contract is a choice with "longer-run" characteristics than the choice of its exact terms.

The main results are as follows. First, regardless of the bargaining power of the upstream firm relative to that of the downstream firm, an upstream firm would prefer to operate with a nonlinear contract rather than a wholesale price or a two-part tariff contract. The reason is that, for any configuration of contract types specified in the rival chain, a nonlinear contract allows both maximization of the vertical chain's (expected) joint profits and the vertical chain to act as a Stackelberg leader vis-à-vis the rival chain by committing to a specific final good quantity level. Second, a downstream firm would also opt for a nonlinear contract (for the same reasons with the upstream firm), but only if its bargaining power is not too low. Otherwise, it would prefer a wholesale price contract, through which it can keep a larger portion of the lower – due to double marginalization – chain's (expected) joint profits. The downstream firm can keep a larger share of the joint profits under a wholesale price contract than under a nonlinear contract, because in the former the input price is the

[5] Section 7 includes a discussion on the observability of the contract terms before the downstream competition stage and also on price competition in the final goods market.

only instrument with which the upstream firm can transfer profits upstream. We show that the higher is the product differentiation in the final product market, the more prone is a downstream firm to choose a wholesale price contract. This occurs because when the products are not close substitutes, the role of strategic commitment vis-à-vis the rival chain becomes secondary and that of intra-chain bargaining dominates. Third, welfare would be maximized under two-part tariff competition. Since such contracts do not arise in equilibrium, we find that the market does not deliver the "optimal" contract arrangement.

Further, we show that our results hold true independently of whether the nonlinear contracts are with or without final output quantity commitment, as long as the marginal production cost of the input is not too low. The reason for this latter result is the following. In the case that the nonlinear contracts are without downstream quantity commitment, when a vertical chain applies such a contract, it is able to commit to an aggressive downstream behavior. This is so because as we mentioned earlier it induces its downstream firm to act as a zero marginal cost competitor (up to the input quantity – capacity – specified by the contract terms). This commitment then to an aggressive downstream behavior is more valuable when the input marginal cost is high.

Finally, considering in an extension the case in which the firms can choose only among two-part tariffs and wholesale price contracts, we find that while a two-part tariff is always preferred by an upstream firm, a downstream firm chooses a wholesale price contract when its bargaining power is not sufficiently high. We also find that, removing the nonlinear contract from the feasible set of contracts makes it more likely that a downstream firm opts for a wholesale price contract.

As discussed above, the literature includes a number of studies on strategic vertical contracting, in some of which the contract types are chosen endogenously by the firms (see e.g. Gal-Or, 1991, Rey and Stiglitz, 1995).[6] However, the literature has largely ignored the role that downstream firms play in this contractual procedure.[7] We

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[6] See Section 6 for a comparison of our results with those of Gal-Or (1991) and Rey and Stiglitz (1995).

[7] There are now several papers that examine situations where there is market power both upstream and downstream, see e.g. Horn and Wolinsky (1988), Dobson and Waterson (1997), Chen (2001), Inderst and Wey (2003). However, these papers have a different focus from ours, that is, they do not deal with the choice of contract type in competing rival chains.

believe that rather than being passive, downstream firms play an important role in many vertically linked oligopolistic industries. Such a consideration may be emerging as increasingly important as there is increased concentration in the retail level of many sectors. Our model, thus, involves the downstream firms both in the contract type selection process, as well as in the determination of the contract terms.

The remainder of the paper is structured as follows. In Section 2, we set up the basic model. In Section 3, we examine the final two stages of the game: downstream competition and equilibrium contract terms, for given choices of contract types. In addition, we emphasize the main strategic characteristics of the possible contractual configurations. In Section 4, we analyze the first stage equilibrium, that is, the contract types that prevail in equilibrium. In Section 5, we extend the analysis to the case of nonlinear contracts without downstream quantity commitment. In Section 6, we restrict the choice of contracts among wholesale prices and two-part tariffs. In Section 7, we discuss some of the model's assumptions, as well as some possible extensions of our analysis. In Section 8, we conclude.

2. The Basic Model

We consider a two-tier industry consisting of two upstream firms - input suppliers and two downstream firms - final good producers.[8] Each upstream firm, denoted by U_i, i=1,2, produces an input facing a constant marginal cost equal to c. Each downstream firm, denoted by D_i, i =1,2, produces a final good transforming one unit of input into one unit of final product. Each downstream firm has an exclusive relationship with one of the two upstream firms, and thus it obtains its input only from that particular upstream firm. In terms of notation, we assume that U_i has an exclusive relationship with D_i, i =1,2, and refer to each (U_i, D_i) pair as a vertical chain. We assume that a downstream firm faces no other costs than the total cost of obtaining the input from its upstream supplier.

The inverse demand function for the final product of each downstream firm D_i is:

$$p_i = a - q_i - \gamma q_j; \quad i \neq j; \quad i, j = 1,2; \quad a > c \geq 0; \quad 0 < \gamma \leq 1 \tag{1}$$

where q_i and p_i are respectively the output and the price of firm D_i's final product, and γ represents the degree of product differentiation. The closer to one is γ, the closer

[8] Or wholesalers and retailers, respectively.

substitutes the two final products are. When γ tends to zero, the goods produced by the two downstream firms are (almost) independent and thus, the two firms are operating as monopolists in separate downstream markets.

The terms of trade within each vertical chain are determined by a contract, prior to any productive activity. Each vertical chain can select among three different contract types. The first, denoted by W, is a linear pricing contract, consisting simply of a wholesale price w_i that D_i has to pay for each unit of input it obtains from U_i. The second, denoted by T, is a two-part tariff contract, consisting of a wholesale price and a fixed fee - transfer, (w_i, F_i). The third type, denoted by N, is a nonlinear contract that specifies the total input quantity and its respective total price, (q_i, T_i). This last type of contract corresponds to a so called 'price-quantity package' or 'quantity bundle'.[9]

We assume that both upstream and downstream firms possess some power over setting both the type and the terms of the vertical contracts. We restrict attention to the cases where the distribution of power is identical both across vertical chains and across the bargaining over the contract type and the contract terms within each vertical chain. In particular, we assume that the bargaining power of each upstream firm is β and of each downstream firm 1-β, with $0 \leq \beta \leq 1$.

In the first stage of the game, the *type* of contract that will be subsequently signed within each vertical chain is chosen. The simplest way to capture the firms' relative power while bargaining over the type of contract within each vertical chain is to assume that, with probability β the contract is chosen by the upstream firm and with probability 1-β by the downstream firm, with the probability β being independent across the two vertical chains.[10]

In the second stage, bargaining takes place within each vertical chain regarding the *terms* of the selected contracts. For instance, if the contract type employed by the (U_i, D_i) chain is a two-part tariff, then in the second stage U_i and D_i negotiate both over the value of the wholesale price w_i and the value of the fixed fee F_i. A standard way to capture negotiations over the contract terms within each vertical chain is to evoke the generalized Nash bargaining solution, with the Nash product powers β and 1-β representing again the upstream and downstream bargaining power.[11] Note that,

[9] For a description of these types of contracts, see also Tirole (1988), p. 134, 148-149, and 153-158.

[10] For a similar way of modeling the bargaining see Rey and Tirole (1997), and Chemla (2003).

[11] In principle, the power that a firm possesses in setting the contract type does not have to be equal to its bargaining power over the contract terms. Although this assumption is adopted here for simplicity (its generalization is straightforward), it can be justified on the basis that the relative bargaining power

as we are dealing with a compound problem that encompasses two synchronous bargaining processes, applying the Nash bargaining solution is not entirely straightforward, since one should account for the dynamic interdependencies between the simultaneous bargaining sessions. We assume that while bargaining in this case, each vertical chain takes as given the values of the negotiated variables of the rival chain. That is, the solution concept employed is the Nash equilibrium between the two Nash Bargaining problems.[12]

In the third stage of the game, after both the type and the terms of the vertical contracts have been determined, downstream firms choose the quantities of their final products in order to maximize their profits. In the case that a downstream firm is engaged in a nonlinear contract, we assume, in our basic model, that its final product quantity is directly determined (through the input quantity specified) by the contract. Thus technically, in this case, the quantity of the downstream firm has been already determined in the previous stage. In Section 5, we extend our analysis to the case where such final product quantity commitment is not feasible and the vertical chain can only commit to a maximum capacity up to which its downstream firm produces the final good at zero marginal cost. In this case, the final output cannot exceed the total input quantity specified by the contract.

Summarizing, the timing of the game is:

Stage 1: Within each vertical chain, "nature" simultaneously and independently grants the choice of contract type with probability β to the upstream firm and with probability 1-β to the downstream firm. The selected by "nature" agents choose among three different types of contracts: a wholesale price contract *W*; a two-part tariff contract *T*; and a nonlinear contract *N*.

Stage 2: Within each vertical chain, simultaneously and independently, the upstream firm and the downstream firm bargain over the terms of the contract chosen

of the firms cannot differ too much between the two negotiation stages, i.e. that over the contract type and that over the contract terms. Note also that the results would be qualitatively similar if instead of modeling the bargaining in stage 2 as a Nash bargaining game, we modeled it in a way similar to that of the bargaining in stage 1. That is, if "take-it-or-leave-it" offers regarding the contract terms were made with probability β by the upstream firms and with probability 1-β by the downstream firms.

[12] One can show that, if there is no exchange of information among sessions while negotiations last and if downstream competition occurs only after bargaining has been terminated in both sessions, Binmore's (1982) observation that the Nash solution is essentially implemented by non-cooperative, alternating offers and counteroffers bargaining games (à la Rubinstein, 1982) can be extended to this case too.

in the previous stage. During the negotiations, the bargaining power of the upstream firm is β and that of the downstream firm is 1-β.

Stage 3: Each downstream firm D_i transforms the input into a final good and competes in the final goods market in quantities.

We derive the subgame perfect Nash equilibria of the above three-stage game.

3. Contract Terms Choice and Downstream Competition

- **Third Stage: Downstream Competition**

If neither vertical chain has signed an *N* contract, i.e. in cases [*T*, *T*], [*W*, *W*], [*W*, *T*] and [*T*, *W*] with the first and second entry denoting the contract type employed by the (U_1, D_1) and (U_2, D_2) chain respectively, the last stage of the game corresponds to a standard Cournot game. Each downstream firm D_i chooses q_i to maximize its gross profits, given its input price w_i and the quantity of its rival q_j,

$$\pi_{D_i}(q_i, q_j) = (a - q_i - \gamma q_j)q_i - w_i q_i, \quad i \neq j, \quad i, j = 1,2 \tag{2}$$

The downstream quantity reaction functions are:

$$R_i(w_i, q_j) = \frac{a - \gamma q_j - w_i}{2} \tag{3}$$

Note from (3) that a decrease in the wholesale price charged to D_i shifts the reaction function of D_i upwards. Solving the system of equations (3), we obtain the Cournot equilibrium quantities:

$$q_i(w_i, w_j) = \frac{a(2-\gamma) - 2w_i + \gamma w_j}{4 - \gamma^2} \tag{4}$$

If a vertical chain has signed an *N* contract, the quantity produced by its downstream firm has been determined in the previous stage during the chains' negotiations over the contracts terms. If only one vertical chain employs an *N* contract, i.e. cases [*N*, *W*], [*W, N*], [*T, N*] and [*N*, *T*], its downstream firm simply transforms all the purchased quantity of input to output, while the rival chain's downstream firm, employing either a *T* or a *W* contract, reacts optimally to that quantity in the way described by equation (3). This case corresponds to a standard Stackelberg leader-follower game.

Finally, if both chains have signed an *N* contract, case [*N*, *N*], since the terms of the *N* contract dictate the quantity of the final product, it follows that in the last stage downstream firms simply produce the quantities that have been specified in the previous stage. Thus, in the third stage of the game, downstream firms do not make any strategic decisions.

- **Second Stage: Contract Terms**

In the second stage, within each vertical chain, the upstream and downstream firm negotiate simultaneously and independently over the terms of the contracts chosen in the previous stage. Instead of analyzing meticulously all the nine possible subgames, in this subsection we concentrate on providing crucial arguments that lead to our main results that are stated in the next section. We start our analysis with the following Lemma.

Lemma 1: *Whenever a vertical chain employs either an N contract or a T contract, bargaining over the contract terms leads to the maximization of the joint profits of the chain's upstream and downstream firm,* $\pi_{U_i-D_i} = \pi_{U_i} + \pi_{D_i}$*, given the contract terms of the rival chain. Moreover, through the transfer (the fixed fee* F_i *or the total input price* T_i*), the chain's maximum joint profits are distributed to the upstream and downstream firm according to their respective bargaining powers, β and 1- β.*

Proof: This is a consequence of the fact that the gross profits of the upstream firm π_{U_i} and downstream firm π_{D_i} are independent of the transfer specified in the contract (say A_i). Hence, maximization of the generalized Nash product with respect to the transfer A_i,

$$\max_{A_i} [\pi_{U_i} + A_i]^{\beta}[\pi_{D_i} - A_i]^{1-\beta} \tag{5}$$

leads to an optimal transfer $A_i^* = \beta(\pi_{U_i} + \pi_{D_i}) - \pi_{U_i} = \beta\pi_{U_i-D_i} - \pi_{U_i}$. As a result, the net profits of the upstream and downstream firm become respectively, $\pi_{U_i} + A_i^* = \beta\pi_{U_i-D_i}$ and $\pi_{D_i} - A_i^* = (1-\beta)\pi_{U_i-D_i}$. Substituting these into (5), it is easy to see that the generalized Nash product reduces to an expression proportional to the chain's joint profits $\pi_{U_i-D_i}$. □

Based on Lemma 1, we observe that if both vertical chains have chosen an *N* contract in the first stage, i.e. in the subgame [*N*, *N*], each chain maximizes its joint profits, taking as given the quantity chosen by the rival chain:

$$\max_{q_i} \pi^{NN}_{U_i-D_i} = (a - q_i - \gamma q_j)q_i - cq_i \tag{6}$$

It is clear from (6) that, in the [*N*, *N*] case, the two vertical chains play a standard Cournot game with marginal costs equal to the input marginal cost *c*. The equilibrium input, and thus final good, quantities are then given by $q_i^{NN} = v/(2+\gamma)$, where $v \equiv a - c$.

In the [*N*, *T*] case, the (U_1, D_1) chain chooses its input quantity q_1 and simultaneously the (U_2, D_2) chain selects its wholesale price w_2, in order each to maximize its joint profits. Since the (U_1, D_1) chain, through its input quantity choice, can commit to an equal final good production by its downstream firm, it acts as a Stackelberg leader in setting its quantity, to which the rival chain's downstream firm will react as Stackelberg follower in the following stage according to (3). Formally, the two chains solve the following maximization problems:

$$\max_{q_1} \pi_{U_1-D_1}^{NT}(q_1, w_2) = (a - q_1 - \gamma R_2(q_1, w_2) - c)q_1 \qquad (7)$$

$$\max_{w_2} \pi_{U_2-D_2}^{NT}(q_1, w_2) = (a - R_2(q_1, w_2) - \gamma q_1 - c)R_2(q_1, w_2) \qquad (8)$$

By inspection of (8), the higher the negotiated wholesale price w_2 is, the lower is the downstream D_2's output (since from (3), $\partial R_2 / \partial w_2 < 0$) and the lower are the joint profits of the (U_2, D_2) chain. Therefore, the vertical chain that employs the *T* contract sets its wholesale price equal to marginal cost, $w_2^{NT} = c$. The reason is that, as the output of the rival chain's downstream firm is determined at the same stage as its wholesale price, the (U_2, D_2) chain has no incentive to manipulate the wholesale price for its downstream firm. If the rival's quantity was not fixed, the chain may have an incentive to choose w_2 in such way as to commit its downstream firm to a more aggressive behavior in the final goods market. In the case currently under analysis, there are no such considerations and the chain simply gives its downstream firm incentives reflecting the true input cost. As a consequence, the [*N*, *T*] case reduces to a standard Stackelberg game in which the marginal costs are *c* and the equilibrium quantities are $q_1^{NT} = v(2-\gamma)/2(2-\gamma^2)$ and $q_2^{NT} = v(4 - 2\gamma - \gamma^2)/4(2-\gamma^2)$.

In contrast, the negotiated wholesale price cannot be equal to the marginal input cost *c* when both vertical chains have chosen a *T* contract in the previous stage, i.e. in the [*T*, *T*] case. In this case, each chain chooses w_i to maximize its joint profits, taking as given the wholesale price of its rival, i.e.

$$\max_{w_i} \pi_{U_i-D_i}^{TT}(w_i, w_j) = (a - q_i(w_i, w_j) - \gamma q_j(w_i, w_j) - c)q_i(w_i, w_j) \qquad (9)$$

where $q_i(w_i, w_j)$ and $q_j(w_i, w_j)$ are given by (4). Solving for the equilibrium, we find that the wholesale prices are such that $w_i^{TT} = [2c(2+\gamma) - a\gamma^2]/(4+2\gamma-\gamma^2) < c$. Thus, in the [*T*, *T*] case, wholesale prices take the form of subsidies from the upstream firms to their respective downstream firms. The intuition is that, by decreasing its wholesale price, a chain allows its downstream firm to commit to a more aggressive behavior. Technically, it shifts its downstream reaction curve out and, as reaction curves are downward-sloping, this results in lower quantity for the rival downstream firm, and higher quantity and profit for the own firm. A similar result has been obtained in the "strategic delegation" literature, where the upstream firms make "take-it-or-leave-it" two-part tariff offers (see, e.g., Vickers, 1985, Fershtman and Judd, 1987, Sklivas, 1987). Here we extend this result to the case where both the upstream and the downstream firms participate in the determination of the contract terms. In this regard, notice that the equilibrium level of w_i^{TT} is independent of β. This is because, by Lemma 1, the chain's behavior when a *T* contract is employed corresponds to maximization of the joint profits. Note that while each vertical chain chooses to unilaterally commit to more aggressive behavior, in equilibrium, the two chains are trapped into a prisoners' dilemma situation. Their profits are lower than those in the [*N*, *N*] case in which the chains maximize joint profits on the basis of their true marginal input cost *c*. This is also reflected by the fact that the equilibrium output under *T* contracts is larger than under *N* contracts, i.e. $q_i^{TT} = 2v/(4+2\gamma-\gamma^2) > q_i^{NN}$.

An implication of the above analysis is the following Lemma, which will be useful for the sequel:

Lemma 2: *The equilibrium joint profits of the vertical chains can be ranked as:*

(i) $\pi_{U_i-D_i}^{NN} > \pi_{U_i-D_i}^{TT}$, *and*

(ii) $\pi_{U_1-D_1}^{NT} > \pi_{U_1-D_1}^{NN} > \pi_{U_1-D_1}^{TN} (= \pi_{U_2-D_2}^{NT})$

Proof: Part (i) is an immediate consequence of our analysis in the previous paragraph. Part (ii) is based on the following two arguments. First, as we saw above, in the cases [*N, T*], [*N, N*], and [*T, N*], any output decision maker (i.e. the downstream firm or the vertical chain) faces the same marginal cost *c*. Second, it is well known that for the symmetric cost case, the Stackelberg leader's profits, (U_1, D_1)'s profits under [*N, T*], are larger than the profits of the Cournot competitors, profits under [*N, N*], and those are larger than the profits of a Stackelberg follower, (U_1, D_1)'s profits under [*T, N*]. □

We next turn to the cases in which at least one vertical chain employs a *W* contract. The negotiated wholesale price of that chain stems from the maximization of the chain's Nash product, and in contrast to the cases of a *T* or an *N* contract, it does *not* correspond to the chain's maximization of joint profits. For instance, the (U_2, D_2) vertical chain solves:

$$\max_{w_2} \ [\pi_{U_2}]^{\beta}[\pi_{D_2}]^{1-\beta} = [(w_2 - c)q_2(.)]^{\beta}[(a - \gamma q_1(.) - q_2(.) - w_2)q_2(.)]^{1-\beta} \qquad (10)$$

where $q_i(.) = q_i(w_1, w_2)$ (see (4)), in case that the rival chain (U_1, D_1) employs a *W* or a *T* contract, i.e. in the [*W*, *W*] and [*T*, *W*] cases. In contrast, if the (U_1, D_1) chain employs an *N* contract, i.e. in the [*N*, *W*] case, $q_2(.) = R_2(q_1, w_2)$ (see (3)), while $q_1(.) = q_1$ is taken as given by (U_2, D_2). Note further that in the [*W*, *W*] case, the (U_1, D_1) chain solves a problem similar to (10), while in the [*T*, *W*], and [*N*, *W*], cases its problem is described by (9), and (7), respectively.

It is now convenient to state the following Lemma:

Lemma 3: *Whenever a vertical chain employs a W contract, the chain's joint profits are not maximized and are distributed in such a way that the ratio of the upstream firm's to downstream firm's profits is always lower than their relative bargaining power,* $\beta/(1-\beta)$.

Proof: The first part follows immediately from (10). In order to prove the second part, we first note that after taking the logarithm of (10), the first order condition becomes,

$$\frac{\beta(\partial \pi_{U_2}/\partial w_2)}{\pi_{U_2}} + \frac{(1-\beta)(\partial \pi_{D_2}/\partial w_2)}{\pi_{D_2}} = 0 \ \text{ or, } \ \frac{\pi_{U_2}}{\pi_{D_2}} = \frac{\beta}{1-\beta}\left[-\frac{(\partial \pi_{U_2}/\partial w_2)}{(\partial \pi_{D_2}/\partial w_2)}\right] \qquad (11)$$

It remains to show that the term in square brackets is smaller than 1, i.e. that an increase in wholesale price increases the upstream profits by less than it decreases the downstream profits in equilibrium. Note first that,

$$\frac{\partial \pi_{U_2}}{\partial w_2} = q_2(.)[1 + \frac{w_2 - c}{q_2(.)}\frac{\partial q_2(.)}{\partial w_2}] < q_2(.) \qquad (12)$$

because $w_2 > c$ and by (3) and (4), $\partial q_2(.)/\partial w_2 < 0$ in all cases. Second, by using the envelope theorem, we get

$$\frac{\partial \pi_{D_2}}{\partial w_2} = q_2(.)[-1-\gamma \frac{\partial q_1(.)}{\partial w_2}] \leq -q_2(.) \tag{13}$$

because $\partial q_1(.)/\partial w_2 > 0$ in the [*W, W*] and [*T, W*] cases and $\partial q_1(.)/\partial w_2 = 0$ in the [*N, W*] case. Combining (12) and (13), we prove our result. □

Intuitively, an increase in the wholesale price raises the upstream profits by less than $q_2(.)$, because such an increase in the marginal output cost has a negative effect on the final good production and thus on the input quantity demanded by its downstream firm. On the other hand, the downstream profits decrease by more than $q_2(.)$, because an increase in its marginal cost makes the rival downstream firm more aggressive and this negative strategic effect adds up with the (negative) own-costs effect. Further, maximization of the chain's Nash product implies that the optimal wholesale price is such that the weighted - by the respective bargaining powers - percentage decrease in upstream profits and percentage increase in downstream profits are equal (see (11)). Therefore, the ratio of upstream to downstream profits under the optimal wholesale price is lower than their relative bargaining power.

The first order condition for the (U_2, D_2) chain in the [*W, W*] and [*T, W*] cases can be written as:

$$\frac{\beta}{w_2 - c} + \frac{\partial q_2/\partial w_2}{q_2} - \frac{(1-\beta)(\partial q_2/\partial w_2 + 1)}{a - q_2 - \gamma q_1 - w_2} = \frac{(1-\beta)\gamma\, \partial q_1/\partial w_2}{a - q_2 - \gamma q_1 - w_2} \tag{14}$$

where $q_i = q_i(w_1, w_2)$, *i*=1,2, are given by (4). Similarly, the first order condition for the (U_2, D_2) chain in case [*N, W*] can be written as:

$$\frac{\beta}{w_2 - c} + \frac{\partial R_2/\partial w_2}{q_2} - \frac{(1-\beta)(\partial R_2/\partial w_2 + 1)}{a - q_2 - \gamma q_1 - w_2} = 0 \tag{15}$$

where $q_2 = R_2(q_1, w_2)$ is given by (3). Note further that, from the first order conditions for D_2 in the last stage, we have that $q_2 = a - q_2 - \gamma q_1 - w_2$ in the all cases in which the (U_2, D_2) chain applies a *W* contract.

In the [*N, W*] case, the (U_2, D_2) chain's downstream firm sets its output as a Stackelberg follower in the last stage, responding optimally to the rival chain's quantity that has been determined during the contract terms negotiations stage. Therefore, the strategic effect $\partial \mathrm{q}_1/\partial w_2 > 0$, i.e. the ($U_2$, D_2) chain's ability to commit its downstream firm to a more aggressive final good market behavior through a reduction of its wholesale price, that exists in the [*T*, *W*] case, is absent here. This can be seen by comparing the RHS of (14) and (15), with the strategic effect appearing only in the former. As the strategic effect is absent in the [*N*, *W*] case, the wholesale price negotiated by the (U_2, D_2) chain is expected to be higher than the respective input price in the [*T*, *W*] case. This is further reinforced by the fact that an increase in w_2 has a stronger negative effect on D_2's output in the [*T*, *W*] than the [*N*, *W*] case, i.e. $\partial q_2/\partial w_2 < \partial R_2/\partial w_2$. This is so, because in the [*N*, *W*] case the (U_2, D_2) chain takes the quantity of its rival chain's downstream firm D_1 as given, while in the [*T*, *W*] case it expects D_1 to optimally adjust its quantity along its downwards sloping reaction function. The above observations, in conjunction with the first order conditions (14) and (15), imply that $w_2^{TW} < w_2^{NW}$.

The following Lemma is a consequence of the above observations:

Lemma 4: *The joint profits of a vertical chain are higher under an N than under a T contract, whenever the rival chain employs a W contract,* $\pi_{U_1-D_1}^{NW} > \pi_{U_1-D_1}^{TW}$.

Proof: This is due to two reasons. First, since $w_2^{NW} > w_2^{TW}$, $R_2(q_1, w_2^{NW}) < R_2(q_1, w_2^{NW})$. Hence, the ($U_1$, D_1) chain that employs an *N* contract can achieve higher joint profits as a Stackelberg leader when its rival chain has selected a higher wholesale price for its (Stackelberg follower) downstream firm, i.e. $\pi_{U_1-D_1}^{NW}(w_2^{NW}) > \pi_{U_1-D_1}^{NW}(w_2^{TW})$. Second, the latter ($U_1$, D_1)'s joint profits are higher than when that chain employs a *T* contract – in which case its downstream firm acts as a Cournot competitor in the final good market - and w_2^{TW} is charged by the rival chain, i.e. $\pi_{U_1-D_1}^{NW}(w_2^{TW}) > \pi_{U_1-D_1}^{TW}(w_2^{TW})$.[13] □

It should be noticed that in all cases in which at least one vertical chain employs a *W* contract, i.e. cases [*W, W*], [*W, T*], [*T, W*], [*W, N*], and [*N, W*], the equilibrium outcome depends on the bargaining power distribution. For instance, in the [*W, W*] case, the solution of the system of equations (14) leads to the equilibrium wholesale price and output,

$$w_i^{WW} = \frac{2(2-\beta)c + a\beta(2-\gamma)}{4-\beta\gamma}; \qquad q_i^{WW} = \frac{2v(2-\beta)}{(2+\gamma)(4-\beta\gamma)} \tag{16}$$

[13] These can be seen from a diagram in the (q_1, q_2) space where we draw the isoprofit curves of the (U_1, D_1) vertical chain. Clearly, the higher the wholesale price that the rival chain charges, the more its downstream firm's reaction function shifts in, the looser is thus the constraint that the (U_1, D_1) chain faces and the higher are the joint profits it can achieve. Moreover, when the (U_1, D_1) chain switches to a *T* contract, its isoprofit curve is not tangent anymore to the given constraint and thus its profit level is lower than under an *N* contract.

Clearly, as the bargaining power of the upstream firm β tends to zero, the wholesale price tends to the marginal input cost *c*. Moreover, the higher β is, the higher is the wholesale price and the lower the final good quantity. This is in sharp contrast to other subgames where maximization of the chains' joint profits implies that the only role that the bargaining power of the upstream and the downstream firm play is to distribute the chain's maximum joint profits among those firms accordingly.

The remaining subgames can be solved following the procedure described above and for brevity are not analyzed here. Tables 1 and 2 summarize the equilibrium outcomes (wholesale prices, final good quantities and upstream and downstream firms' profits) for all possible second stage subgames. Finally, Proposition 1 compares wholesale prices across the various subgames.

Proposition 1: *The equilibrium wholesale prices can be ranked as follows*

(i) $w_i^{WW} > w_1^{WN} > w_1^{WT} > w_1^{TN} > w_i^{TT} > w_1^{TW}$ *for all* $\beta > \beta_{cr}(\gamma)$,

(ii) $w_1^{WN} > w_i^{WW} > w_1^{WT} > w_1^{TN} > w_i^{TT} > w_1^{TW}$ *for all* $\beta < \beta_{cr}(\gamma)$

where $\beta_{cr}(\gamma) = 2\gamma(1-\gamma)/(2-\gamma^2)$.

Proof: See Appendix.

As expected, under *T* contracts wholesale prices are always lower than under *W* contracts. Moreover, the wholesale price of the chain that employs a *T* contract is the lowest when the rival chain employs a *W* contract and the highest when the rival chain employs an *N* contract. Finally, the wholesale price of the chain that employs a *W* contract is the lowest when a rival chain employs a *T* contract, while it is the highest when the rival chain employs an *N* contract (*W* contract) for sufficiently low (high) upstream bargaining power.

4. Equilibrium Contractual Configurations

In this section we determine the contract types that emerge in equilibrium. In other words, we determine the equilibrium of the first stage of the game, where upstream and downstream firms bargain over the contract type within each vertical chain.

Since the strategy space of either U_i or D_i is {*W*, *T*, *N*}, there are nine possible contractual configurations within each vertical chain: (*W*, *W*); (*W*, *T*); (*W*, *N*); (*T*, *W*);

(T, T); (T, N); (N, W); (N, T); (N, N). For instance, (N, W) means that the U_i's proposed contract type is a nonlinear contract, while the D_i's proposal is a wholesale price contract. As the contract type proposed is always accepted in equilibrium, we can (loosely speaking) say that, within each vertical chain, with probability β (1-β) the upstream (downstream) firm chooses the contract type.

The following result facilitates the rest of our analysis, since it reduces significantly the number of candidate equilibria of the game:

Proposition 2: *For both the upstream firm U_i and the downstream firm D_i, $i=1,2$, the two-part tariff contract T is strictly dominated by the nonlinear contract N, for all values of β and γ.*

Proof: This is an immediate consequence of Lemmas 1-3. Lemma 1 tells us that the upstream and downstream firm share the same interests regarding the comparison between an N contract and a T contract. Hence, it is sufficient to rank a vertical chain's joint profits under these two contracts. Combining the inequalities stated in Lemmas 2 and 3 we have $\pi^{NN}_{U_1-D_1} > \pi^{TN}_{U_1-D_1}$, $\pi^{NT}_{U_1-D_1} > \pi^{TT}_{U_1-D_1}$ and $\pi^{NW}_{U_1-D_1} > \pi^{TW}_{U_1-D_1}$. That is, an N contract leads to strictly higher joint profits than a T contract for the vertical chain (U_1, D_1), independently of whether the rival chain employs an N, a T or a W contract (or any convex combination of them). Thus, the T contract is strictly dominated by the N contract for both the upstream and the downstream firm. □

The intuition behind Proposition 2 is as follows. We know from Lemma 1 that both the N and the T contract lead to maximization of the joint profits of the chain's upstream and downstream firm. This holds independently of the outcome of the first stage negotiations over the contract type in the rival vertical chain. The N contract, however, has an additional commitment value that is absent in the case of a T contract. To see this, consider the following two situations. First, when the rival chain's first stage negotiations have resulted in either a T or a W contract, a vertical chain has incentives to switch from a T to an N contract because by doing so it can commit to a certain output level in the second stage, and thus it can transform its downstream firm to a Stackelberg leader in the final goods market. Second, when the rival chain's negotiations have resulted in an N contract, the vertical chain has again incentives to switch from a T to an N contract, because while with a T contract its downstream firm is a Stackelberg follower, with an *N* contract it is a Cournot competitor. Therefore, in the first stage the vertical chain 'expects' to obtain higher joint profits with the adoption of an *N* contract than with a *T* contract.[14] Finally, as the upstream and downstream firm's incentives are aligned with the incentives of the vertical chain, the *N* contract strictly dominates the *T* contract for both.

Proposition 2 permits us to eliminate the dominated *T* contract from the strategy space of the upstream and downstream firms and, thus restrict their contract type choice between contracts *N* and *W*. [15] The next Proposition states that the *W* contract is also dominated for the upstream firms.

Proposition 3: *The nonlinear contract N is a strictly dominant strategy for each upstream firm* U_i*, i=1,2, for all values of* β *and* γ*.*

Proof: This is a consequence of Proposition 2 and Lemmas 1 and 3. From Lemmas 1 and 3 it follows that a vertical chain attains lower joint profits under a *W* contract than an *N* contract. It also follows that the upstream firm's share of those profits is lower under *W* than under *N*, independently of the rival chain's choice of contract in the first stage. As a result, the *W* contract is always dominated by the *N* contract for the upstream firm. □

The intuition is straightforward. The upstream firm will never opt for a *W* contract, because such a contract implies not only "leaving money on the table", but also a relatively smaller share of the lower joint profits for the upstream firm. In particular, in the case of an *N* contract, the upstream firm can transfer, through the total input price T_i, the corresponding to its bargaining power portion of the maximum joint profits upstream. In the case of a *W* contract, due to the lack of transfer fee, the upstream firm cannot even transfer the same portion out of the lower joint profits that result from the *W* contract.

[14] The expectation here refers to the uncertain outcome of the negotiations over the contract type in the rival vertical chain. The vertical chain's upstream and downstream firm rationally expect that negotiations over the contract type in the rival chain will lead with probability β to the contract preferred by the rival upstream firm and with probability 1- β to the contract preferred by the rival downstream firm. Of course, the joint profits that a vertical chain can achieve in the second stage differ according to the agent that "nature" selects to make the proposal in the rival chain in the first stage.

[15] In a different context, without strategic interactions between vertical chains, Iyer and Villas-Boas (2003) show that two-part tariffs do not emerge in equilibrium. Under demand uncertainty and ex-post contract terms negotiations between a single upstream and a single downstream firm, they find that two-part tariffs lead to lower profits for both firms.

Propositions 2 and 3 imply that the number of first-stage "candidate" equilibria is reduced to the following four: [(*N*, *N*), (*N*, *N*)]; [(*N*, *W*), (*N*, *W*)]; [(*N*, *N*), (*N*, *W*)]; [(*N*, *W*), (*N*, *N*)]. Note, that the first entry within each bracket refers to the contractual configurations proposed by the upstream and downstream firm, respectively, within the (U_1, D_1) chain and the second entry to the respective ones within the (U_2, D_2) chain. The equilibrium contractual configurations and the conditions under which each of them arises are stated in Proposition 4.

Proposition 4:

(i) The contractual configuration [(N, N), (N, N)] is an equilibrium for all β *such that* $\beta \leq \beta_M(\gamma)$, *with* β_M *increasing in* γ, $\lim_{\gamma \to 0} \beta_M(\gamma) = 0$, *and* $\beta_M(1) = 0.882$.

(ii) The contractual configuration [(N, W), (N, W)] is an equilibrium for all β *such that* $\beta \geq \beta_m(\gamma)$, *with* β_m *increasing in* γ, $\lim_{\gamma \to 0} \beta_m(\gamma) = 0$, *and* $\beta_m(1) = 0.7911$.

(iii) $\beta_m(\gamma) < \beta_M(\gamma)$ *for all values of* γ.

Proof: See Appendix.

Corollary 1 follows from Proposition 4 and indicates that there are no asymmetric equilibria, that is, there are no equilibria in which the contractual configurations (i.e. the contracts chosen by the upstream and the downstream firm, respectively) differ across the competing vertical chains.

Corollary 1: *The contractual configurations [(N, N), (N, W)] and [(N, W), (N, N)] never arise in equilibrium.*

While no asymmetric equilibria exist, we should note that our analysis does allow for the possibility that we observe asymmetric behavior: in two identical vertical chains one may be employing an *N* contract and the other a *W* contract. The reason is that, in the first stage of the game, nature randomly selects whether the upstream or the downstream firm is choosing the contract type in each vertical chain and this randomization is independent across chains. Thus, when [(*N*, *W*), (*N*, *W*)] is the equilibrium contractual configuration, the two vertical chains may be observed to employ different contracts.

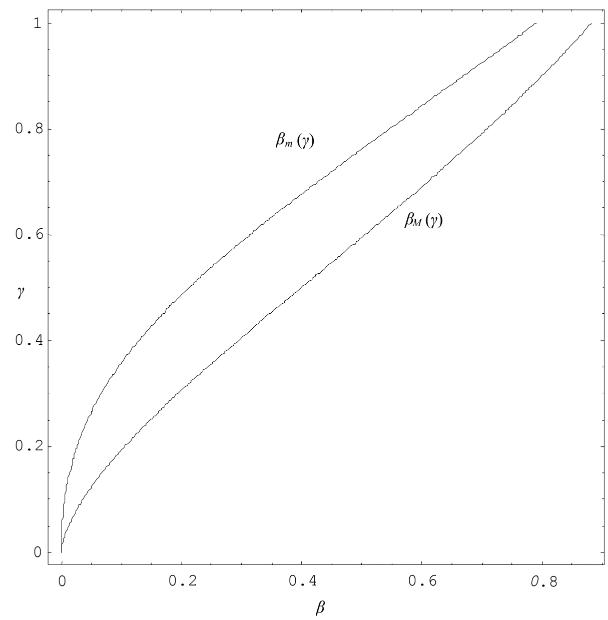

Fig.1: Contractual Configurations Equilibria

Summarizing, only two contractual configurations can arise in equilibrium. First, the configuration in which both vertical chains select nonlinear contracts, independently of whether the upstream or the downstream firm is the agent who proposes the type of contract. And second, the configuration in which, within each vertical chain, a nonlinear contract is proposed by the upstream firm and a wholesale price contract by the downstream firm. In this case, the frequency with which the nonlinear, or the wholesale price, contract is selected within each chain is proportional to β, and 1-β, i.e. the bargaining power of the upstream, and downstream firm, respectively.

Moreover, the [(*N*, *N*), (*N*, *N*)] configuration is the unique equilibrium if, given γ, the upstream power is sufficiently small, i.e. $\beta < \beta_m(\gamma)$; while [(*N*, *W*), (*N*, *W*)] is the unique equilibrium if the upstream power is sufficiently large, i.e. $\beta > \beta_M(\gamma)$. Finally, for intermediate values of β, $\beta_m(\gamma) \leq \beta \leq \beta_M(\gamma)$, these two equilibrium configurations co-exist. Fig.1 illustrates the respective regions in the (β, γ)-space.

The intuition of the above results is as follows. From the upstream firm's point of view, an *N* contract has three desirable features: (i) downstream quantity commitment, (ii) maximization of the vertical chain's joint profits, and (iii) the ability to transfer upstream a portion of the joint profits that is proportional to its bargaining power - and has no drawbacks. As a result, an upstream firm would always opt for such a contract. From a downstream firm's point of view, an *N* contract has only the first two desirable features. A downstream firm will then opt for an *N* contract, only if the share (as reflected in its power, $1-\beta$) that it could capture out of the chain's joint profits is large enough. Otherwise, a *W* contract may become more desirable, because it guarantees to the downstream firm a larger share of the lower joint profits. In fact, we know from Lemmas 1 and 4 that when β is arbitrarily close to 1, while the downstream profits are infinitesimal under an *N* contract, they are strictly positive under a *W* contract. This holds, despite the fact that since a wholesale price contract does not include a fixed payment, there is double marginalization in the case of a *W* contract, and thus the total "pie" is lower under a *W* than under an *N* contract. Finally, as the upstream power increases (higher β), it becomes more likely that an *N* contract is the outcome of the negotiations of the rival chain in the first stage and, thus, the joint profits that the vertical chain under consideration expects in the second stage are reduced. This may provide an additional incentive to the downstream firm to opt for a wholesale price contract when its claims over the joint profits are relatively small (high values of β).

Further, analyzing the role of product differentiation is also instructive. For low values of γ (close to zero) the two products are relatively independent and strategic considerations relative to the rival chain are insignificant. As a result, the *N* contract is not very attractive because the strategic value of downstream quantity commitment is low. For low γ, a downstream firm is simply concerned with capturing some of the profit of its own vertical chain (note though that the downside of a *W* contract is double marginalization that leads to a lower joint profit for the chain). On the other hand, as γ increases and the products become closer substitutes, downstream market competition is intensified and strategic downstream quantity commitment becomes more valuable. As a result, in such cases (and for given bargaining power) a downstream firm would tend to prefer an *N* contract.

It is also interesting to discuss what happens in the extreme cases, i.e. (i) when the final goods are independent and (ii) if one of the sides has all the bargaining power. When the two downstream firms behave (almost) as local monopolists i.e. $\gamma \to 0$, the equilibrium contractual configuration is [(*N*, *W*), (*N*, *W*)] regardless of the value of β. Further, if the upstream firm within each of the two vertical chains has all the power, $\beta = 1$, it follows from Proposition 4 that, independently of the degree of product differentiation, the equilibrium contractual configuration is [(*N*, *W*), (*N*, *W*)].

Moreover, as $\beta = 1$, it is the upstream firm that selects always the contract type in the first stage, and as a result, only nonlinear contracts would be observed in equilibrium. In the opposite extreme, that is when $\beta = 0$, the unique equilibrium is [(*N*, *N*), (*N*, *N*)]. Thus, in both extreme cases, when the upstream firms or the downstream firms have all the power, only nonlinear contracts would be selected in equilibrium. The following Remark summarizes:

Remark 1: *Nonlinear contracts in both vertical chains is the equilibrium contractual configuration when β=1 or β=0, for all values of γ.*

Let us now turn to the analysis of welfare issues. Measuring welfare as the sum of consumers' and producers' surplus, as Proposition 5 states below, two-part tariff contracts are preferable compared to any other type of contract. In particular, welfare takes its highest value under two-part tariff contracts and its lowest value under wholesale price contracts, with the nonlinear contracts being in between.

Proposition 5: *Welfare takes its highest value when both vertical chains employ two-part tariff contracts, while it takes its lowest value when both chains employ wholesale prices. The complete welfare ranking of the contractual configurations, for given β, is as follows:*

(i) $W_{TT} > W_{TN} > W_{NN} > W_{WT} > W_{WN} > W_{WW}$ *whenever the goods are sufficiently poor substitutes*

(ii) $W_{TT} > W_{TN} > W_{WT} > W_{NN} > W_{WN} > W_{WW}$ *when the degree of product differentiation is in some intermediate range*

(iii) $W_{TT} > W_{TN} > W_{WT} > W_{WN} > W_{NN} > W_{WW}$ *whenever the goods are sufficiently close substitutes*

Proof: See Appendix.

This is a consequence of the fact that the equilibrium wholesale prices are higher than the marginal input cost *c*, in the [*W, W*] case and lower than *c* in the [*T, T*] case. Moreover, it is a consequent of the fact that in the [*N, N*] case, the vertical chains' quantity decisions are based on their "true" marginal input cost *c*. Therefore, equilibrium output quantities are the largest under [*T, T*] and the smallest under [*W, W*], with those in the [*N, N*] case lying in between. From our previous analysis we know that the preferable type of contracts from the welfare point of view, the two-part tariff contracts, will not be observed in equilibrium. Thus, the firms' choices do not coincide with the choice of a "social planner."

5. Nonlinear Contracts without Downstream Quantity Commitment

In this section, we relax our assumption that there is 'quantity commitment' in the case of nonlinear contracts. That is, we relax the assumption that the final good's quantity is equal to the total input quantity specified in the nonlinear contract. Instead, we assume that there is 'free disposal' under nonlinear contracts, that is, a downstream firm is free to produce any final product quantity up to the input quantity specified in the contract terms. Hence, the input quantity specified in the nonlinear contract acts merely as a capacity constraint for the downstream firm. Since the total input price has been paid in the second stage of the game, it is a *sunk cost* for the downstream firm in the market competition stage. As a result, in the market competition stage, the downstream firm faces a *zero* marginal production cost up to the specified capacity (and infinite thereafter). This reveals an alternative commitment mechanism inherent in the nonlinear contract. The vertical chain, through the use of a nonlinear contract, can commit to *an aggressive downstream competition up to the capacity level specified during the contract terms negotiations*.

Whether or not a contract between an upstream and a downstream firm can directly dictate the quantity to be supplied in the downstream market depends on the details of the specific market under consideration. In some cases, technological, legal or other institutional factors imply that a downstream retailer automatically forwards to the final consumers the quantity of the final good that it receives from an upstream manufacturer. In some other cases, the downstream firm may be receiving intermediate inputs from an upstream supplier and after making the total payment required for all the input units may have the option to simply not use some of them ('free disposal').

We show that under 'free disposal', if the marginal production cost of the input c is not too low, our previous analysis holds with no need for any modification. Thus, our results turn out to be (to a major extent) robust with respect to the nature of commitment inherent in the nonlinear contracts. In fact, the marginal input cost c can be thought of as a measure of the effectiveness of the alternative commitment mechanism. The higher c is, the more valuable is for the vertical chain to be able to commit to an aggressive downstream behavior by inducing its downstream firm to act as a zero marginal cost competitor. In contrast, when the marginal input cost is low, the nonlinear contract loses a great part of its commitment value. A modified analysis would be required in order to determine the equilibrium contractual configurations in this case, a task that is out of the scope of the present paper. The following Proposition states our main result in this Section:

Proposition 6: *Propositions 1-5 hold also in case that a vertical chain, through a nonlinear contract, can commit only to a specific downstream capacity, if the marginal production cost of input c is not too low, i.e. if* $c \geq a\hat{c}_n(\gamma)$*, where*

$$\hat{c}_n(\gamma) = \frac{(2-\gamma)(8-3\gamma^2)\sqrt{4-\gamma^2} - 2(4-\gamma^2)(4-2\gamma-\gamma^2)}{2(8-3\gamma^2)\sqrt{4-\gamma^2} - 2(4-\gamma^2)(4-2\gamma-\gamma^2)}$$

with $\hat{c}_n$ *increasing in γ,* $\lim_{\gamma \to 0} \hat{c}_n(\gamma) = 0$*, and* $\hat{c}_n(1) = 0.235$*, independently of the distribution of power between the upstream and the downstream firm* $(\beta, 1-\beta)$.

Proof: See Appendix.

The intuition behind Proposition 6 is as follows. The vertical chain (U_i, D_i), through the negotiations over the nonlinear contract's terms, can strategically induce a capacity constraint, say K_i, to its downstream firm D_i up to which the latter produces at zero marginal cost. In this case, the reaction function of D_i becomes kinked at K_i, i.e. from (3) $R_i(q_j, K_i) = \max[K_i, (a-\gamma q_j)/2]$. Further, in equilibrium, a chain employing an N contract never selects an input quantity in excess of the output that its downstream firm will actually produce in the final good market. By eliminating the excess downstream capacity, the vertical chain saves on input production costs and thus increases its joint profits. Given this, in the [N, N] case, for any input, and thus output quantity of the rival chain $K_j = q_j$, the vertical chain (U_i, D_i) will optimally set the input quantity along its own reaction function $R_i^{U_i - D_i}(K_j) = (a - K_i - \gamma K_j - c)$. As a result, the equilibrium in the [N, N] subgame is the standard Cournot with marginal costs equal to c just like in the case of nonlinear contracts with downstream quantity commitment.

In the [*N, T*] case, the chain (U_1, D_1) has a clear incentive to induce a capacity constraint to its downstream firm in order to transform D_1 to a Stackelberg leader in the final goods market, with unit costs equal to the chain's marginal cost *c*. The rival chain (U_2, D_2) could respond by setting a wholesale price equal to *c* and thus obtain joint profits equal to the Stackelberg follower's profits in the standard Stackelberg game with marginal costs *c*. This is so, because in this case (U_2, D_2) maximizes its joint profits over the residual demand and, in the absence of any strategic effects, it follows the marginal cost pricing rule. Alternatively, it could induce an asymmetric costs Cournot downstream game (where D_1's marginal cost is zero) by optimally setting a wholesale price far below the marginal input cost *c.* It turns out that this latter option cannot be profitable unless *c* is too low. The reason is that a low *c* implies a fierce downstream competition, which results in overproduction and thus in lower joint profits for the (U_2, D_2) chain than those obtained by "accepting" a Stackelberg follower role based on "true" production costs. In this sense, the higher the marginal input cost *c* is, the more significant are the consequences from switching to an asymmetric Cournot downstream game. Indeed, if *c* is high enough, the (U_2, D_2) chain is unable to realize positive profits by inducing a downstream Cournot game and the (unique) equilibrium is the same as under downstream quantity commitment. In other words, the higher *c* is, the stronger is the threat from not conforming to the role of the follower; hence, *c* is a measure of the commitment to the leader's role facilitated by the *N* contract.

A similar reasoning applies in the [*N, W*] case. One difference is that, since the upstream and the downstream firm of the rival chain negotiate only over the wholesale price, the wholesale price of (U_2, D_2) is always higher than *c* (and also increases with the upstream power β). As a result, the follower chain's joint profits are lower (and the leader chain's higher) than under the [*N, T*] case in which the wholesale price is always equal to *c.* A second difference is that the (U_2, D_2) chain's objective is now to maximize its Nash product, instead of its joint profits. The (U_2, D_2) chain will not switch to a Cournot asymmetric costs downstream game unless its gain is higher than conforming to the Stackelberg follower role. As previously, it turns out that, if *c* is not too low, the equilibrium of the [*N, W*] subgame is the same as under downstream quantity commitment, independently of the distribution of power between the upstream and the downstream firm.

6. Wholesale Prices vs. Two-Part Tariffs

In some markets, technological or institutional considerations may make nonlinear contracts of the type discussed above (with or without final output commitment) non-feasible. Further, the previous literature has often focused on the choice among wholesale price and two-part tariffs contracts. Accordingly, and for the completeness of the analysis, in this subsection we constrain the set of vertical contracts to include only two types: wholesale price contracts, and two-part tariff contracts.

The following Lemma summarizes our findings regarding the equilibrium behavior of the upstream firms:

Lemma 5: *The two-part tariff contract T is a strictly dominant strategy for U_i, i=1,2, for all values of β and γ.*

Proof: From Lemmas 1 and 3 it follows that a vertical chain attains lower joint profits under a *W* contract than a *T* contract. It also follows that the upstream firm's share of those profits are lower under *W* than under *T*, independently of the rival chain's choice of contract in the first stage. Hence, the *T* contract strictly dominates the *W* contract for the upstream firm. □

Note that, although it is always a dominant strategy for the upstream firms to choose two-part tariff contracts, under some circumstances the upstream firms are trapped into a prisoners' dilemma situation. That is, although in some cases both upstream firms would be better off when they both choose wholesale price contracts, each of them individually has a dominant strategy to choose a two-part tariff contract. More precisely, it can be seen that the profits of each upstream firm under wholesale price contracts exceed those under two-part tariffs for all $0 \leq \beta < \beta_{cr}^{0}(\gamma)$, where

$$\beta_{cr}^{0}(\gamma) = \max[0, \frac{2(4\gamma - \gamma^3 + 6\gamma^2 - 8)}{\gamma^2(2+\gamma)}] \quad (17)$$

with $\beta_{cr}^{0}(\gamma)$ (weakly) increasing in γ, $\beta_{cr}^{0} = 0$ for γ=0.921 and $\beta_{cr}^{0}(1) = 2/3$. Gal-Or (1991) and Rey and Stiglitz (1995) have also shown that two-part tariff contracts are dominant strategies for the upstream firms in a framework in which upstream firms have all the bargaining power and downstream firms compete in a Bertrand fashion. Here, we confirm that this result also holds under Cournot competition and we also extend it to the cases in which the downstream firms possess some bargaining power

(β<1). Further, while in Gal-Or (1991) and Rey and Stiglitz (1995), the upstream firms, under some conditions, find themselves in a prisoners' dilemma setting similar to the one described above, in our model we find that under Cournot competition the prisoners' dilemma situation is present only when β<1, and in particular only when $\beta < \beta_{cr}^{0}(\gamma)$, and not when the upstream firms have all the bargaining power.

Lemma 5 implies that the only remaining candidate equilibria are [(*T, T*), (*T, T*)], [(*T, W*), (*T, W*)], [(*T, W*), (*T, T*)] and [(*T, T*), (*T, W*)]. The following Proposition presents the equilibrium contractual configurations in the two-type contract framework.

Proposition 7: *When only T and W types of contracts are feasible:*

(i) The contractual configuration [(T, T), (T, T)] is an equilibrium for all $\beta < \beta_{cr}^{1}(\gamma)$*, with* β_{cr}^{1} *increasing in γ,* $\lim_{\gamma \to 0} \beta_{cr}^{1}(\gamma) = 0$*, and* $\beta_{cr}^{1}(1) = 0.694$.

(ii) The contractual configuration [(T, W), (T, W)] is an equilibrium for all $\beta > \beta_{cr}^{2}(\gamma)$*, with* β_{cr}^{2} *increasing in γ,* $\lim_{\gamma \to 0} \beta_{cr}^{2}(\gamma) = 0$*, and* $\beta_{cr}^{2}(1) = 0.495$. *Moreover,* $\beta_{cr}^{2}(\gamma) < \beta_{cr}^{1}(\gamma) < \beta_{m}(\gamma)$ *for all values of γ.*

(i) There are no asymmetric contractual configuration equilibria.

Proof: See Appendix

The results of Proposition 7 are qualitatively similar to the results of Proposition 4. In particular, the downstream firms will opt for a two-part tariff contract only if their bargaining power is sufficiently low. Otherwise, they prefer wholesale price contracts. The only difference that we observe is that the emergence of wholesale price contracts is more likely when nonlinear contracts are non-feasible. More precisely, the threshold values of β are now lower than when upstream and downstream firms can choose among a larger set of contract types. This is so, because nonlinear contracts not only lead to joint profit maximization (like two-part tariffs) but they also have a commitment value that prevents vertical chains from acting aggressively through setting wholesale prices below marginal input costs. In other words, they prevent the vertical chains from being trapped into a prisoners' dilemma situation. Thus, when the firms have the option to employ a nonlinear contract, wholesale price contracts are less desirable and their emergence is less likely than in the case that nonlinear contracts are non-feasible.

7. Discussion and Possible Extensions

We discuss briefly here some of the underlying assumptions of the model, highlighting the role they play in the analysis. At the same time, we suggest some directions for possible future research.

- *Type of contract chosen before the contract terms.* Assuming that the type of the contract is selected, in each vertical chain, before the specification of the exact contract terms, captures the idea that the type of the contract is a choice with "longer-run" characteristics than the choice of its exact terms. The type of contract employed by each vertical chain can be thus observable by its rivals and involves a sort of commitment. While this assumption (which has also been adopted by Gal-Or, 1991 and Rey and Stiglitz, 1995) may not hold in some real world cases, we believe that in others it represents an essential feature of firms' behavior. The exact terms of trade are typically easier to change (perhaps as responding to marginal variations in market conditions – although our analysis does not explicitly incorporate uncertainty or dynamics) whereas shifting from one contract type to another may require a more complicated procedure, involvement of firms' more senior management and legal departments, or changes in the monitoring and trading technology. Thus, it seems reasonable to assume that a given firm may know a rival chain's type of contract even when it does not know the exact terms of trade. In our setting, if this assumption was violated, bargaining would have to take place simultaneously over both the type and the terms of the contract. That would be a non-trivial complication, as the outcome of bargaining over the type, a discrete choice, may be very sensitive to the exact modeling details of the bargaining process.

- *Uncertainty in the downstream market.* Our analysis shows that a nonlinear contract is always desirable by the upstream firm and possibly by the downstream firm (if its bargaining power is not too low) as it represents joint profit maximization and a direct or indirect commitment to downstream quantity. More generally, what may make a contract specifying a certain quantity level not an optimal choice is the presence of uncertainty. In particular, if at the time the contract is signed there is demand (or cost) uncertainty, a more flexible contract that involves a marginal price may be preferable. Introducing uncertainty in such a fashion into the model is thus expected to generate additional equilibria and to make a nonlinear contract less likely to occur.

- *Multiple downstream firms.* In our model each upstream firm signs a contract with only one downstream firm. Even without violating the assumption of exclusive dealing, we could extend the model to examine the role of multiple independent downstream firms. What is, then, expected to be a key consideration is that an upstream firm will have to be concerned not only with the marginal incentives of its own downstream firms relative to the rivals', but also relative to other own downstream firms (in this context Baye *et al.*, 1996, find that there is unilaterally a strategic incentive to have a large number of downstream firms if only wholesale prices can be charged, whereas Saggi and Vettas, 2002, find that the strategic incentives get reversed when two-part tariffs are used).

- *Asymmetric bargaining power.* Our analysis assumes that the relative bargaining power of the upstream and downstream firm in the two vertical chains is identical (as captured by β). The problem could be re-worked under an alternative assumption that the relative bargaining powers are different ($\beta_1 > \beta_2$). An interesting question then would be if a firm (upstream or downstream) could increase its equilibrium profit by having a lower bargaining power, as a result of the strategic interaction.

- *Bertrand competition.* Modeling the downstream (final) market as a homogenous or differentiated Bertrand (rather than Cournot) would lead to a modified analysis. One key factor here is that there is no longer a unilateral strategic incentive for a firm to commit itself to more aggressive behavior (see Gal-Or, 1991, and Rey and Stiglitz, 1997, for analyses of downstream price competition). On the other hand, commitment to a specific final good quantity may be profitable for both the upstream and the downstream firm and, thus, the preponderance of nonlinear contracts may turn out to be robust to the type of downstream competition.[16]

- *Unobservable contracts and renegotiation.* It is important in our analysis that firms know the choices made in the previous stages before making their subsequent decisions. This is a central assumption in the main body of the literature on strategic

[16] There is at least one complication one would have to face when attempting to "transfer" our analysis to a setting with downstream price competition: under nonlinear contracts that specify a given quantity of input, the final stage of the game would involve price competition under capacity constraints and, in such situations, there are typically complications with the derivation of (pure strategy) equilibria.

vertical contracting. It may be important to assess the implications of assuming that contracts are not perfectly observable (see e.g. Fershtman and Kalai, 1997) or that they can be renegotiated at a later stage. Regarding this last point, there is recent work showing that, even when there is no full commitment (in the sense that renegotiation is possible), the strategic effects are still present. This happens when there is downstream Cournot competition and commitment to more aggressive behavior is unilaterally desirable (see Caillaud *et al.*, 1995).

8. Conclusions

In this paper we analyze the strategic characteristics of different types of vertical contractual arrangements employed in competing vertical chains and examine which contracts may emerge in equilibrium when both the upstream and downstream firms have bargaining power over both the type and the terms of contracts.

We show that, regardless of their bargaining power, upstream firms always prefer nonlinear contracts that specify the total input quantity and its respective price. This is due to the fact that, given any contract type configuration in the rival chain, a nonlinear contract leads to maximization of the vertical chain's joint profits and at the same time allows for quantity pre-commitment relative to the rival chain. Downstream firms also opt for nonlinear contracts (with the same rationale), but only if their bargaining power is not too low. Otherwise they prefer linear (wholesale price) contracts that allow them to enjoy a larger share of an otherwise smaller "pie" for the vertical chain. In any case, while social welfare would be maximized under two-part tariff competition, such contracts never arise endogenously. We should note that while some of our results have been derived in the context of a linear demand model, the intuition behind them appears robust and of more general applicability.

We have also extended the analysis by modifying the set of feasible contracts in two directions. First, we considered an alternative formulation where nonlinear contracts do not directly imply downstream quantity commitment and showed that all our results hold true as long as the marginal cost of the upstream production is not too low. The analysis of this alternative formulation offers some additional interesting insights. Even when downstream quantity commitment is not feasible, a contract specifying the total input quantity (and its total price) is characterized by commitment to zero downstream marginal cost, up to the specified input quantity. The (strategic) value of this commitment depends on the level of the upstream production's marginal cost. This is why, when this cost is low, a nonlinear contract of this type may be less attractive to the firms.[17] Second, considering the case in which nonlinear contracts are not feasible, we obtain results similar in spirit to our main results. That is, upstream firms prefer two-part tariffs, while downstream firms prefer linear contracts if their bargaining power is low and two-part tariffs otherwise.

While this is, to the best of our knowledge, the first paper that examines the relation between firms' bargaining power in vertical chains and the strategic choice of vertical contracts, more work needs to be done on the topic. This work, in addition to the extensions mentioned in the previous section, will hopefully include an empirical study of how the types of contracts are influenced by the relative bargaining power of firms in oligopolistic industries.[18]

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[17] The case of low cost, with this alternative formulation of the contract terms, is beyond the scope of this paper but is still of independent interest and the object of some parallel research of ours.

[18] See Lafontaine and Slade (1997), Slade (1998), and Villas-Boas (2002) for an empirical examination of vertical contracts.

Appendix

Proof of Proposition 1

Simple comparison of the equilibrium wholesale prices included in Table 1 leads to the results included in parts (i) and (ii). □

Proof of Proposition 4

(i) Since by Proposition 1, we have that T is dominated by N, for D_i, i=1,2, in order for the contractual configuration [(N, N), (N, N)] to be an equilibrium, it is sufficient to show that one of the downstream firms, e.g. D_1 has no incentive to deviate from N to W, given that the chain (U_2, D_2) chooses (N, N). To do so, we have to compare its profits with a wholesale price contract to its profits with a nonlinear contract, given that its competing vertical chain selects a nonlinear contract with probability 1. Setting the difference $\pi_{D_1}^{WN} - \pi_{D_1}^{NN}$ equal to zero, we find that there exists a unique critical value, $\beta_M(\gamma)$ in terms of γ, such that:

$$\pi_{D_1}^{WN} - \pi_{D_1}^{NN} > 0 \text{ for } \beta > \beta_M(\gamma); \quad \pi_{D_1}^{WN} - \pi_{D_1}^{NN} < 0 \text{ for } \beta < \beta_M(\gamma)$$

We then establish that $\partial\beta_M(\gamma)/\partial\gamma>0$, $lim_{\gamma\to 0}\beta_M(\gamma)=0$, $\beta_M(1)=0.882$. It follows immediately that the contractual configuration [(N, N), (N, N)] is an equilibrium for $\beta\leq\beta_M(\gamma)$.

(ii) Given Proposition 2, in order for the contractual configuration [(N, W), (N, W)] to be an equilibrium, it is sufficient to show that one of the downstream firms, e.g. D_1 has no incentives to deviate from W to N, given that the chain (U_2, D_2) chooses (N, W). To do so, we have to compare its profits when it chooses a nonlinear contract with its profits when it chooses a wholesale price contract, given that the outcome of the negotiations in the competing vertical chain leads to a nonlinear contract with probability β and to a wholesale price contract with probability 1-β. Taking the profit difference and setting it equal to zero, we find that there exists a unique critical value $\beta_m(\gamma)$ in terms of γ, such that:

$$\beta\pi_{D_1}^{NN} + (1-\beta)\pi_{D_1}^{NW} - \beta\pi_{D_1}^{WN} - (1-\beta)\pi_{D_1}^{WW} > 0 \text{ for } \beta<\beta_m(\gamma)$$

$$\beta\pi_{D_1}^{NN} + (1-\beta)\pi_{D_1}^{NW} - \beta\pi_{D_1}^{WN} - (1-\beta)\pi_{D_1}^{WW} < 0 \text{ for } \beta>\beta_m(\gamma)$$

We then establish that $\partial\beta_m(\gamma)/\partial\gamma>0$, $lim_{\gamma\to 0}\beta_m(\gamma)=0$, $\beta_m(1)=0.7911$. Thus, it follows immediately that the contractual configuration [(N, W), (N, W)] is an equilibrium for $\beta\leq\beta_m(\gamma)$.

(iii) From part (ii) we know that $\Delta = \beta\pi_{D_1}^{NN} + (1-\beta)\pi_{D_1}^{NW} - \beta\pi_{D_1}^{WN} - (1-\beta)\pi_{D_1}^{WW} < 0$ for all $\beta > \beta_m(\gamma)$. If this expression is negative when evaluated at $\beta = \beta_M(\gamma)$, $0 < \gamma \le 1$, then we can infer that $\beta_M(\gamma)$ lies to the right of $\beta_m(\gamma)$ for $0 < \gamma \le 1$. Since along the $\beta_M(\gamma)$ line $\pi_{D_1}^{WN} = \pi_{D_1}^{NN}$, we obtain that $\Delta = (1-\beta)(\pi_{D_1}^{NW} - \pi_{D_1}^{WW})$. From Table 2, one can check that $\pi_{D_1}^{NW} < \pi_{D_1}^{WW}$ for all $\beta > \widehat{\beta}(\gamma)$ where $\widehat{\beta}(\gamma) < \beta_M(\gamma)$; hence, $\Delta < 0$ when evaluated at $\beta = \beta_M(\gamma)$ and thus, $\beta_m(\gamma) < \beta_M(\gamma)$ for all $0 < \gamma \le 1$. □

Proof of Proposition 5

We define welfare as the sum of consumers' and producers' surplus. Note that in our case producers' surplus refers both to the surplus of the upstream and the downstream firms. Based on the construction of Singh and Vives (1984), and after some manipulation, it follows that total welfare is given by:

$$W(q_1, q_2) = (a-c)(q_1+q_2) - \frac{1}{2}(q_1^2 + q_2^2 + 2\gamma q_1 q_2) \qquad \text{(A1)}$$

We calculate the welfare level that corresponds to each of the six second stage subgames by substituting into (A1) the respective equilibrium quantities included in Table 1. Table 3 includes the welfare values for all the cases.

We start from the comparison of the symmetric cases: W_{WW}, W_{TT}, W_{NN}. Calculating the respective differences, it follows that: $W_{TT} > W_{NN} > W_{WW}$. Similarly in the asymmetric cases, we have: $W_{TN} > W_{WT} > W_{WN}$. Next we find that $W_{WN} > W_{WW}$, $W_{TT} > W_{TN}$, and $W_{NN} < W_{TN}$. Calculating the difference $W_{NN} - W_{WN}$, we find that, for given β, there exists a critical value of γ, $\gamma'(\beta)$, such that $W_{NN} > W_{WN}$ for $\gamma < \gamma'(\beta)$, and vice versa. The critical value of γ is $\gamma'(\beta) = [\beta'(\gamma)]^{-1}$, where

$$\beta'(\gamma) = 2\frac{32\gamma - 64 - \gamma^6 - 4\gamma^4 - 8\gamma^3 + 48\gamma^2 + 4\gamma^5 + 2\sqrt{K}}{64 + 12\gamma^4 + 8\gamma^5 - 32\gamma^3 - 48\gamma^2 + \gamma^6} \qquad \text{(A2)}$$

with $K = (\gamma^6 + 2\gamma^5 - 6\gamma^4 + 4\gamma^2 - 16\gamma + 16)(2+\gamma)^2(4-\gamma^2-2\gamma)^2$

Similarly, calculating the difference $W_{NN} - W_{WT}$, we find that for given β, there exists a critical value of γ, $\gamma''(\beta)$, such that $W_{NN} > W_{WT}$ for $\gamma < \gamma''(\beta)$, and vice versa. The critical value of γ is $\gamma''(\beta) = [\beta''(\gamma)]^{-1}$, where

$$\beta''(\gamma) = 2\frac{128\gamma^2 - 128 + 4\gamma^5 - 48\gamma^4 - 32\gamma^3 - \gamma^8 - 2\gamma^7 + 10\gamma^6 + 64\gamma + \sqrt{L}}{128 - 224\gamma^2 - 32\gamma^3 + 144\gamma^4 - 36\gamma^6 + 32\gamma^5 + \gamma^9 - 12\gamma^7 + 3\gamma^8} \qquad \text{(A3)}$$

with $L = (\gamma^6 + 2\gamma^5 - 6\gamma^4 + 4\gamma^2 - 16\gamma + 16)(2-\gamma)^2(4-\gamma^2-2\gamma)^2(2+\gamma)^4$

Comparing (A2) to (A3), it follows that: $\gamma'(\beta) > \gamma''(\beta)$ Thus, for $\gamma < \gamma''(\beta)$, W_{NN} > W_{WT} >W_{WN}. While for $\gamma > \gamma'(\beta)$, we have that W_{WT} > W_{WN} >W_{NN} and for $\gamma''(\beta) < \gamma < \gamma'(\beta)$, W_{WT} > W_{NN} >W_{WN}. □

Proof of Proposition 6

To prove this we need to show that in all the subgames where an *N* contract is employed by at least one vertical chain, the equilibrium when the chain is unable to commit to a specific downstream quantity during the contract terms negotiations stage remains the same with the equilibrium in the case that when the chain can commit to a specific downstream quantity. In the former case, the chain can instead commit to a capacity (equal to the input quantity specified by the nonlinear contract) up to which its downstream firm produces at zero marginal cost, since the total input price is a sunk cost for the downstream firm at the downstream competition stage. We will consider the cases [*N, N*], [*N, T*] and [*N, W*] separately in order to find sufficient conditions for the equilibrium to be robust under the alternative commitment assumption.

The [*N, N*] case: Let K_i be the input quantity specified by the (U_i, D_i) chain's contract terms negotiations. W.l.o.g. we can restrict attention to $K_i < \overline{K} = a - c$, where $\overline{K}$ is an input quantity so large that even if the rival chain's capacity is zero, the profits of the (U_i, D_i) chain are nil when its downstream firm D_i produces at capacity. Indeed, as the (U_i, D_i) chain's profits are negative for all $q_i = K_i > \overline{K}$, the chain cannot credibly commit to a downstream production equal to capacity in this case. Now since D_i's marginal cost equals zero, it is easy to see from (3) that its reaction function is given by $R_i(K_i, q_j) = \min[K_i, (a - \gamma q_j)/2]$, *i,j*=1,2. That is, the D_i's reaction function is kinked at its capacity level K_i, after which it becomes perpendicular to the q_i axis. Clearly, if K_1 and K_2 are large enough, i.e. $K_1 \geq a/(2+\gamma)$ and $K_2 \geq a/(2+\gamma)$, the third stage equilibrium is $q_1^* = q_2^* = a/(2+\gamma)$. On the other hand, if K_i is small enough and K_j is large enough, i.e. $2K_i + \gamma K_j < a$ and $2K_j + \gamma K_i > a$, the equilibrium is $q_i^* = K_i$ and $q_j^* = (a - \gamma K_i)/2$. Finally, if K_1 and K_2 are small enough, i.e. $2K_1 + \gamma K_2 \leq a$ and $2K_2 + \gamma K_1 \leq a$, the equilibrium is $q_1^* = K_1$ and $q_2^* = K_2$. The latter implies that, for any permissible K_j, the (U_i, D_i) chain can induce, if it wishes, a two-sided capacity constraint equilibrium, i.e. $q_i^* = K_i$ and $q_j^* = K_j$. Since in this case the (U_i, D_i) chain's profits are maximized along its reaction function, $R_i^{U_i - D_i}(K_j) = (a - \gamma K_j - c)/2$, it is clear that the ($U_i$, D_i) chain has an incentive to induce such an equilibrium by properly selecting its input quantity. An immediate consequence is that both vertical chains have incentives to induce the capacity constraint equilibrium and by doing so we end up in the standard Cournot equilibrium where $q_i^* = K_i^* = (a - c)/(2 + \gamma)$.

<u>The [*N, T*] case</u>: Let K_1 be the input quantity specified by the (U_1, D_1) chain's contract terms negotiations and w_2 the wholesale price specified by the (U_2, D_2) chain's negotiations. As D_1's marginal cost is zero, its reaction function from (3) is $R_1(K_1, q_2) = \min[K_1, (a - \gamma q_2)/2]$, while the reaction function of the rival firm D_2 is $R_2(q_1, w_2) = (a - \gamma q_1 - w_2)/2$. For small K_1, i.e. $K_1 < [a(2-\gamma) + \gamma w_2]/(4 - \gamma^2)$, the third stage equilibrium is $q_1^* = K_1$ and $q_2^*(w_2) = (a - \gamma K_1 - w_2)/2$; otherwise, the third stage equilibrium is an asymmetric Cournot, $q_1^C(0, w_2) = [a(2-\gamma) + \gamma w_2]/(4 - \gamma^2)$ and $q_2^C(0, w_2) = [a(2-\gamma) - 2w_2]/(4 - \gamma^2)$. Now for any given K_1, the (U_2, D_2) chain has two options. First, to induce a one-sided capacity constrained third stage equilibrium, in which case (U_2, D_2) will optimally set a wholesale price $w_2 = c$ in order to maximize the chain's joint profits $\pi_{U_2 - D_2}(K_1, w_2) = (a - \gamma K_1 - q_2^*(w_2) - c)q_2^*(w_2)$. And second, to induce an asymmetric Cournot equilibrium by setting a low enough wholesale price, i.e. $w_2 < \widetilde{w}_2(K_1) = [(2-\gamma)/\gamma][K_1(2+\gamma) - a]$, in which case the chain's profits will be $\pi_{U_2 - D_2}^C(0, w_2) = [a - \gamma q_1^C(0, w_2) - q_2^C(0, w_2) - c]q_2^C(0, w_2)$, or else

$$\pi_{U_2 - D_2}^C(0, w_2) = \frac{[a(2-\gamma) - 2w_2][a(2-\gamma) - c(4-\gamma^2) + w_2(2-\gamma^2)]}{(4-\gamma^2)^2} \qquad \text{(A4)}$$

Note further that, if $w_2 = c$, the (U_1, D_1) chain can induce its most-preferred equilibrium (i.e. the equilibrium that maximizes the chain's joint profits given the reaction function of the rival downstream firm D_2, $R_2(q_1, c)$) by selecting $K_1^S = (a-c)(2-\gamma)/2(2-\gamma^2)$, provided that its downstream firm D_1 will do produce at capacity at the third stage, that is, if $q_1^C(0, c) \geq K_1^S$. It is easy to check that this occurs if $c/a > \hat{c}_n^1(\gamma) \equiv (2-\gamma)\gamma^2/(8-2\gamma^2+\gamma^3)$, with $\lim_{\gamma\to 0}\hat{c}_n^1(\gamma) = 0$, $\hat{c}_n^1(1) = 0.2$ and $\partial\hat{c}_n^1/\partial\gamma > 0$.

Let $c/a > \hat{c}_n^1(\gamma)$ and $K_1 = K_1^S$. If the (U_2, D_2) chain's joint profits are not higher when it follows its second option (i.e. to induce an asymmetric Cournot game in the third stage) then the equilibrium in the [*N, T*] case coincides with that under commitment to downstream quantity. This would occur if there does not exist a $w_2 < \tilde{w}_2(K_1^S) = (2-\gamma)[a\gamma^2 - c(4-\gamma^2)]/2\gamma(2-\gamma^2)$ s.t. $\pi_{U_2-D_2}^C(0, w_2) > \pi_{U_2-D_2}(K_1^S, c)$, where $\pi_{U_2-D_2}(K_1^S, c) = (a-c)^2(4-2\gamma-\gamma^2)^2/16(2-\gamma^2)^2$ are the profits of the Stackelberg follower. Note first from (A4) that, for the (U_2, D_2) chain's price-cost margin and $\pi_{U_2-D_2}^C(0, w_2)$ to be positive, $w_2 > \underline{w} \equiv (2-\gamma)[c(2+\gamma)-a]/(2-\gamma^2)$. However, $\tilde{w}_2(K_1^S) > \underline{w}$ only if $c/a > \gamma/(2+\gamma) > \hat{c}_n^1(\gamma)$, in which case the ($U_2$, D_2) chain has no incentive to induce an asymmetric Cournot downstream game.

Further, maximizing (A4) w.r.t. w_2 we obtain the (unrestricted) optimal wholesale price for the (U_2, D_2) chain, $w_2^u = (2-\gamma)[2c(2+\gamma) - a\gamma^2]/4(2-\gamma^2)$, in which case its (unrestricted) maximum profits are $\pi_{U_2-D_2}^u = [2c - a(2-\gamma)]^2/8(2-\gamma^2)$. However, we have $\tilde{w}_2(K_1^S) > w_2^u$ only if $c/a < \gamma^2/4 < \gamma/(2+\gamma)$. Moreover, $\pi_{U_2-D_2}^u \le \pi_{U_2-D_2}(K_1^S, c)$ if

$$c/a > \hat{c}_n^2(\gamma) \equiv \frac{8-4\gamma-\gamma^3-(4-2\gamma-\gamma^2)\sqrt{2(2-\gamma^2)}}{16-\gamma(2+\gamma)^2} \qquad \text{(A5)}$$

with $\lim_{\gamma\to 0}\hat{c}_n^2(\gamma) = 0$, $\hat{c}_n^2(1) = 0.2265$ and $\partial\hat{c}_n^2/\partial\gamma > 0$. It can be also checked that $\hat{c}_n^1(\gamma) < \hat{c}_n^2(\gamma) < \gamma^2/4$. Clearly, $\pi_{U_2-D_2}^C(w_2 < w_2^u) < \pi_{U_2-D_2}^u \le \pi_{U_2-D_2}(K_1^S, c)$ for all $\gamma^2/4 < c/a < \gamma/(2+\gamma)$. Therefore, if $c/a \ge \hat{c}_n^2(\gamma)$, the ($U_2$, D_2) chain has no incentive to induce an asymmetric Cournot downstream game. Finally, as $\hat{c}_n^2(\gamma) > \hat{c}_n^1(\gamma)$, we conclude that the equilibrium in the [*N, T*] subgame coincides with that under no commitment to downstream quantity if $c/a \ge \hat{c}_n^2(\gamma)$.

An implication of the above analysis is that in the [*N, T*] subgame, for all $c/a > \gamma/(2+\gamma)$, there exists a unique equilibrium which is equivalent to a standard Stackelberg equilibrium with both marginal equal costs equal to *c*. In contrast, for all $c/a < \hat{c}_n^2(\gamma)$, there exists also a unique equilibrium which is equivalent to a Cournot asymmetric costs equilibrium with downstream costs zero for D_1 and w_2^u for D_2. For all $\hat{c}_n^2(\gamma) < c/a < \gamma/(2+\gamma)$, the above two equilibria coexist and are Pareto ranked with the Stackelberg equilibrium leading to higher profits for both chains than the Cournot one. A focal point argument can be used in the latter case for selecting the Pareto superior Stackelberg equilibrium.

The [*N, W*] case: As in the [*N,* T] case, the downstream reaction functions are $R_1(K_1, q_2) = \min[K_1, (a-\gamma q_2)/2]$ and $R_2(q_1, w_2) = (a - \gamma q_1 - w_2)/2$; hence, for small K_1, i.e. $K_1 < [a(2-\gamma) + \gamma w_2]/(4-\gamma^2)$, the third stage equilibrium is $q_1^* = K_1$ and $q_2^*(w_2) = (a - \gamma K_1 - w_2)/2$; otherwise, it is $q_1^C(0, w_2) = [a(2-\gamma) + \gamma w_2]/(4-\gamma^2)$ and $q_2^C(0, w_2) = [a(2-\gamma) - 2w_2]/(4-\gamma^2)$. Again, for any given K_1, the (U_2, D_2) chain can induce (i) a capacity constrained equilibrium, in which case it will optimally set a wholesale price $w_2^S(K_1) = [a\beta + (2-\beta)c - \beta\gamma K_1]/2 > c$ to maximize the chain's Nash product $B_2^S(K_1, w_2) = [(w_2 - c)q_2^*(w_2)]^\beta [(a - \gamma K_1 - q_2^*(w_2) - w_2)q_2^*(w_2)]^{1-\beta}$; or (ii) an asymmetric Cournot equilibrium by setting $w_2 < \tilde{w}_2(K_1) = [(2-\gamma)/\gamma][K_1(2+\gamma) - a]$, in which case the chain's Nash product, after substituting $q_i^C(0, w_2), i = 1,2$, becomes:

$$B_2^C(0, w_2) = \frac{(w_2 - c)^\beta [a(2-\gamma) - 2w_2]^{2-\beta}}{(4-\gamma^2)^{2-\beta}} \tag{A6}$$

Note further that, for any $w_2 > c$, the (U_1, D_1) chain can induce the equilibrium that maximizes the chain's joint profits given D_2's reaction function $R_2(q_1, w_2) < R_2(q_1, c)$ by selecting $K_1^S(w_2) = [a(2-\gamma) - 2c + \gamma w_2]/2(2-\gamma^2)$, provided that its downstream firm D_1 will do produce at capacity at the third stage, that is, if $q_1^C(0, w_2) \geq K_1^S(w_2)$. From the reaction functions in the (K_1, w_2)-space, i.e. $K_1^S(w_2)$ and $w_2^S(K_1)$, we obtain the (candidate) one-sided capacity constrained equilibrium,

$$K_1^S = \frac{(a-c)[4-(2-\beta)\gamma]}{8-(4-\beta)\gamma^2}; w_2^S = \frac{(4-2\gamma-\gamma^2)a\beta + 2c[4-2\gamma^2 - \beta(2-\gamma-\gamma^2)}{8-(4-\beta)\gamma^2} \tag{A7}$$

Note from (A7) that if $\beta = 0$, $w_2^S = c$ and $K_1^S = (a-c)(2-\gamma)/2(2-\gamma^2)$, which are the same as in the [*N, T*] case. Moreover, that $\partial w_2^S/\partial\beta > 0$ and $\partial K_1^S/\partial\beta > 0$. Finally, it is can be checked that $q_1^C(0, w_2^S) \geq K_1^S$ if

$$c/a > \hat{c}_n^3(\gamma, \beta) \equiv [4-(2-\beta)\gamma]\gamma^2/[16-(2-\beta)(2\gamma^2+\gamma^3)]$$

with $\partial\hat{c}_n^3/\partial\gamma > 0$, $\partial\hat{c}_n^3/\partial\beta > 0$, $\lim_{\gamma\to 0}\hat{c}_n^3(\gamma,\beta) = 0$, $\hat{c}_n^3(1,0) = 0.2$ and $\hat{c}_n^3(1,1) = 0.23077$.

Let $c/a > \hat{c}_n^3(\gamma,\beta)$ and $K_1 = K_1^S$. If the (U_2, D_2) chain's Nash product is not higher when it induces an asymmetric Cournot game in the third stage, then the equilibrium in the [*N, W*] case coincides with that under commitment to downstream quantity. This would occur if there does not exist a

$$w_2 < \tilde{w}_2(K_1^S) = \frac{(2-\gamma)[2a\gamma(\beta+\gamma) - c(2+\gamma)\{4-(2-\beta)\gamma\}]}{\gamma[8-(4-\beta)\gamma^2]} \tag{A8}$$

s.t.

$$B_2^C(0,w_2) > B_2^S(K_1^S, w_2^S) = 2^{-2+\beta}\beta^\beta(2-\beta)^{2-\beta}(a-c)^2(4-2\gamma-\gamma^2)^2/[8-(4-\beta)\gamma^2]^2$$

Note first from (A6) that, for the (U_2, D_2) chain's Nash product $B_2^C(0,w_2)$ to be positive, $w_2 > c$. However, $\tilde{w}_2(K_1^S) > c$ only if $c/a > \dfrac{(2-\gamma)\gamma(\beta+\gamma)}{8+2\beta\gamma-2\gamma^2-\gamma^3} \equiv \sigma_n^h(\gamma,\beta)$, with $\sigma_n^h(\gamma,\beta) > \hat{c}_n^3(\gamma,\beta)$, in which case the ($U_2$, D_2) chain has no incentive to induce an asymmetric Cournot downstream game.

Further, maximizing (A6) w.r.t. w_2 we obtain the (unrestricted) optimal wholesale price for the (U_2, D_2) chain, $w_2^u = [2c(2-\beta) + a\beta(2-\gamma)]/4$, in which case its (unrestricted) maximum Nash product is

$$B_2^{Cu} = (2-\beta)^{2-\beta}\beta^\beta[2c-a(2-\gamma)]^2/2^{2+\beta}(4-\gamma^2)^{2-\beta}$$

However, $\tilde{w}_2(K_1^S) > w_2^u$ only if $c/a < \dfrac{(2-\gamma)\gamma^2(8-4\beta\gamma-\beta^2\gamma)}{2(32-8\gamma^2-(2-\beta)^2\gamma^3} \equiv \sigma_n^\ell(\gamma,\beta)$, with $\sigma_n^\ell(\gamma,\beta) < \sigma_n^h(\gamma,\beta)$. Moreover, $B_2^{Cu} \le B_2^S(K_1^S, w_2^S)$ if

$$c/a > \hat{c}_n^4(\gamma,\beta) \equiv \frac{2^\beta(4-\gamma^2)(4-2\gamma-\gamma^2) - (2-\gamma)(4-\gamma^2)^{\beta/2}[8-(4-\beta)\gamma^2]}{2^\beta(4-\gamma^2)(4-2\gamma-\gamma^2) - 2(4-\gamma^2)^{\beta/2}[8-(4-\beta)\gamma^2]} \tag{A9}$$

with $\partial\hat{c}_n^4/\partial\gamma > 0$, $\partial\hat{c}_n^4/\partial\beta > 0$, $\lim_{\gamma\to 0}\hat{c}_n^4(\gamma,\beta) = 0$, $\hat{c}_n^4(1,0) = 0.2$ and $\hat{c}_n^4(1,1) = 0.235$. It can be further checked that $\hat{c}_n^3(\gamma,\beta) < \hat{c}_n^4(\gamma,\beta) < \sigma_n^\ell(\gamma,\beta)$ for all (γ,β). Clearly, $B_2^C(w_2 < w_2^u) < B_2^{Cu} \le B_2^S(K_1^S, w_2^S)$ for all $\sigma_n^\ell(\gamma,\beta) < c/a < \sigma_n^h(\gamma,\beta)$. Therefore, if $c/a \ge \hat{c}_n^4(\gamma,\beta)$, the ($U_2$, D_2) chain has no incentive to induce an asymmetric Cournot downstream game. Finally, as $\hat{c}_n^4(\gamma,\beta) > \hat{c}_n^3(\gamma,\beta)$, we conclude that the equilibrium in the [*N, W*] subgame coincides with that under commitment to downstream quantity if $c/a \geq \hat{c}_n^4(\gamma, \beta)$.

An implication of the above analysis is that in the [*N, W*] subgame, for all $c/a > \sigma_n^h(\gamma, \beta)$, there exists a unique equilibrium which is equivalent to a Stackelberg equilibrium. In contrast, for all $c/a < \hat{c}_n^4(\gamma, \beta)$, there exists also a unique equilibrium which is equivalent to a Cournot asymmetric costs equilibrium with downstream costs zero for D_1 and w_2^u for D_2. While for all $\hat{c}_n^4(\gamma, \beta) < c/a < \sigma_n^h(\gamma, \beta)$, the above two equilibria coexist and are Pareto ranked with the Stackelberg equilibrium leading to higher surplus for both chains than the Cournot one. A focal point argument can be used in the latter case for selecting the Pareto superior Stackelberg equilibrium.

Finally, let $\hat{c}_n(\gamma) = \max[\hat{c}_n^2(\gamma), \max_\beta \hat{c}_n^4(\gamma, \beta)]$. It can be checked that $\hat{c}_n(\gamma) = \hat{c}_n^4(\gamma, 1)$ for all γ. The previous analysis implies that for all $c/a \geq \hat{c}_n(\gamma)$ all three subgames have the same equilibrium as under downstream quantity commitment. □

Proof of Proposition 7:

The proof is along the lines of Proposition 4 with the nonlinear contract *N* replaced by a two-part tariff contract *T* everywhere. □

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Table 1: Equilibrium Contract Terms: Wholesale Prices and Final Good Quantities

	W	T	N
W	$w_i^{WW} = \frac{2(2-\beta)c + a\beta(2-\gamma)}{4-\beta\gamma}$ $q_i^{WW} = \frac{2v(2-\beta)}{(2+\gamma)(4-\beta\gamma)}$	$w_1^{WT} = \frac{a\beta(16-8\gamma-8\gamma^2+2\gamma^3+\gamma^4) + 2c\left(8(2-\gamma^2)-\beta(8-4\gamma-4\gamma^2+\gamma^3)\right)}{32-16\gamma^2+\beta\gamma^4}$ $w_2^{WT} = \frac{a(\gamma-2)\gamma^2(4+\beta\gamma)+2c\left(16-4\gamma^2-(2-\beta)\gamma^3\right)}{32-16\gamma^2+\beta\gamma^4}$ $q_1^{WT} = \frac{2v(4-\gamma^2-2\gamma)(2-\beta)}{32-16\gamma^2+\beta\gamma^4}$ $q_2^{WT} = \frac{2v(2-\gamma)(4+\beta\gamma)}{32-16\gamma^2+\beta\gamma^4}$	$w_1^{WN} = \frac{a\beta(4-2\gamma-\gamma^2)+2c(4-2\gamma^2-\beta(2-\gamma-\gamma^2))}{8-(4-\beta)\gamma^2}$ $q_1^{WN} = \frac{(2-\beta)v(4-2\gamma-\gamma^2)}{2(8-(4-\beta)\gamma^2)}$ $q_2^{WN} = \frac{v(4-(2-\beta)\gamma)}{8-(4-\beta)\gamma^2}$
T	See [W,T]	$w_i^{TT} = \frac{2c(2+\gamma)-a\gamma^2}{4+2\gamma-\gamma^2}$ $q_i^{TT} = \frac{2v}{4+2\gamma-\gamma^2}$	$w_1^{TN} = c$ $q_1^{TN} = \frac{v(4-2\gamma-\gamma^2)}{4(2-\gamma^2)}$ $q_2^{TN} = \frac{v(2-\gamma)}{2(2-\gamma^2)}$
N	See [W,N]	See [T,N]	$q_i^{NN} = \frac{v}{2+\gamma}$

Table 2: Second Stage Equilibrium Profits

	W	T	N
W	$\pi_{D_i}^{WW}=\frac{4(2-\beta)^2v^2}{(2+\gamma)^2(4-\beta\gamma)^2}$ $\pi_{U_i}^{WW}=\frac{2(2-\beta)\beta v^2(2-\gamma)}{(2+\gamma)(4-\beta\gamma)^2}$	$\pi_{D_1}^{WT}=\frac{4(2-\beta)^2v^2(4-2\gamma-\gamma^2)^2}{(32-16\gamma^2+\beta\gamma^4)^2}$ $\pi_{D_2}^{WT}=\frac{2(1-\beta)v^2(2-\gamma)^2(4+\beta\gamma)^2(2-\gamma^2)}{(32-16\gamma^2+\beta\gamma^4)^2}$ $\pi_{U_1}^{WT}=\frac{2v^2(2-\beta)\beta(4-2\gamma-\gamma^2)(16-8\gamma-8\gamma^2+2\gamma^3+\gamma^4)}{(32-16\gamma^2+\beta\gamma^4)^2}$ $\pi_{U_2}^{WT}=\frac{2\beta v^2(2-\gamma)^2(4+\beta\gamma)^2(2-\gamma^2)}{(32-16\gamma^2+\beta\gamma^4)^2}$	$\pi_{D_1}^{WN}=\frac{(2-\beta)^2v^2(4-2\gamma+\gamma^2)^2}{4(8-(4+\beta)\gamma^2)^2}$ $\pi_{D_2}^{WN}=\frac{(1-\beta)v^2\left(4-(2-\beta)\gamma\right)^2(2-\gamma^2)}{2\left(8-(4-\beta)\gamma^2\right)^2}$ $\pi_{U_1}^{WN}=\frac{(2-\beta)\beta v^2(4-2\gamma-\gamma^2)^2}{2\left(8-(4-\beta)\gamma^2\right)^2}$ $\pi_{U_2}^{WN}=\frac{(1-\beta)v^2\left(4-2(2-\beta)\gamma\right)^2(2-\gamma^2)}{2\left(8-(4-\beta)\gamma^2\right)^2}$
T	$\pi_{D1}^{TW}=\frac{2(1-\beta)v^2(2-\gamma)^2(4+\beta\gamma)^2(2-\gamma^2)}{(32-16\gamma^2+\beta\gamma^4)^2}$ $\pi_{D_2}^{TW}=\frac{4(2-\beta)^2v^2(4-2\gamma-\gamma^2)^2}{(32-16\gamma^2+\beta\gamma^4)^2}$ $\pi_{U_1}^{TW}=\frac{2\beta v^2(2-\gamma)^2(4+\beta\gamma)^2(2-\gamma^2)}{(32-16\gamma^2+\beta\gamma^4)^2}$ $\pi_{U_2}^{TW}=\frac{2(2-\beta)\beta v^2(4-2\gamma-\gamma^2)(16-8\gamma-8\gamma^2+2\gamma^3+\gamma^4)}{(32-16\gamma^2+\beta\gamma^4)^2}$	$\pi_{D_i}^{TT}=\frac{2(1-\beta)v^2(2-\gamma^2)}{(4+2\gamma-\gamma^2)^2}$ $\pi_{U_i}^{TT}=\frac{2\beta v^2(2-\gamma^2)}{(4+2\gamma-\gamma^2)^2}$	$\pi_{D_1}^{TN}=\frac{(1-\beta)v^2(4-2\gamma-\gamma^2)^2}{16(2-\gamma^2)^2}$ $\pi_{D_2}^{TN}=\frac{(1-\beta)v^2(2-\gamma)^2}{8(2-\gamma^2)}$ $\pi_{U_1}^{TN}=\frac{\beta v^2(4-2\gamma-\gamma^2)^2}{16(2-\gamma^2)^2}$ $\pi_{U_2}^{TN}=\frac{\beta v^2(2-\gamma)^2}{8(2-\gamma^2)}$
N	$\pi_{D_1}^{NW}=\frac{(1-\beta)v^2\left(4-(2-\beta)\gamma\right)^2(2-\gamma^2)}{2\left(8-(4-\beta)\gamma^2\right)^2}$ $\pi_{D_2}^{NW}=\frac{(2-\beta)^2v^2(4-2\gamma+\gamma^2)^2}{4(8-(4+\beta)\gamma^2)^2}$ $\pi_{U_1}^{NW}=\frac{(1-\beta)v^2\left(4-2(2-\beta)\gamma\right)^2(2-\gamma^2)}{2\left(8-(4-\beta)\gamma^2\right)^2}$ $\pi_{U_2}^{NW}=\frac{(2-\beta)\beta v^2(4-2\gamma-\gamma^2)^2}{2\left(8-(4-\beta)\gamma^2\right)^2}$	$\pi_{D_1}^{NT}=\frac{(1-\beta)v^2(2-\gamma)^2}{8(2-\gamma^2)}$ $\pi_{D_2}^{NT}=\frac{(1-\beta)v^2(4-2\gamma-\gamma^2)^2}{16(2-\gamma^2)^2}$ $\pi_{U_1}^{NT}=\frac{\beta v^2(2-\gamma)^2}{8(2-\gamma^2)}$ $\pi_{U_2}^{NT}=\frac{\beta v^2(4-2\gamma-\gamma^2)^2}{16(2-\gamma^2)^2}$	$\pi_{D_i}^{NN}=\frac{(1-\beta)v^2}{(2+\gamma)^2}$ $\pi_{U_i}^{NN}=\frac{\beta v^2}{(2+\gamma)^2}$

Table 3: Welfare Levels

	W	T	N
W	$W_{WW} = 4v^2(2-\beta)\dfrac{6-\beta\gamma+2\gamma-\beta\gamma^2+\beta}{(2+\gamma)^2(4-\beta\gamma)^2}$	$W_{WT} = \dfrac{4v^2[128\gamma+32\beta-48\beta\gamma+8\beta\gamma^2+96\gamma^2-6\gamma^4 + 3\beta^2\gamma^4+\beta\gamma^6+6\beta\gamma^5+12\beta\gamma^3-3\beta^2\gamma^5-8\gamma^2\beta^2 - 8\gamma\beta^2-192++8\beta^2\gamma^3-56\gamma^3+8\beta^2-14\beta\gamma^4]}{-(32-16\gamma^2+\beta\gamma^4)^2}$	$W_{WN} = \dfrac{v^2[384-256\gamma-64\beta+96\beta\gamma - 16\beta^2+64\beta\gamma^2-192\gamma^2+16\beta^2\gamma+4\beta^2\gamma^3 - \beta^2\gamma^4-64\beta\gamma^3+112\gamma^3+12\gamma^4-4\beta\gamma^4]}{8(8-4\gamma^2+\beta\gamma^2)^2}$
T	$W_{TW} = \dfrac{4v^2[128\gamma+32\beta-48\beta\gamma+8\beta\gamma^2+96\gamma^2-6\gamma^4 + 3\beta^2\gamma^4+\beta\gamma^6+6\beta\gamma^5+12\beta\gamma^3-3\beta^2\gamma^5-8\gamma^2\beta^2 - 8\gamma\beta^2-192++8\beta^2\gamma^3-56\gamma^3+8\beta^2-14\beta\gamma^4]}{-(32-16\gamma^2+\beta\gamma^4)^2}$	$W_{TT} = 4v^2\dfrac{3+\gamma-\gamma^2}{(4+2\gamma-\gamma^2)^2}$	$W_{TN} = v^2\dfrac{96-48\gamma^2+3\gamma^4-64\gamma+28\gamma^3}{32(2-\gamma^2)^2}$
N	$W_{NW} = \dfrac{v^2[384-256\gamma-64\beta+96\beta\gamma - 16\beta^2+64\beta\gamma^2-192\gamma^2+16\beta^2\gamma+4\beta^2\gamma^3 - \beta^2\gamma^4-64\beta\gamma^3+112\gamma^3+12\gamma^4-4\beta\gamma^4]}{8(8-4\gamma^2+\beta\gamma^2)^2}$	$W_{NT} = v^2\dfrac{96-48\gamma^2+3\gamma^4-64\gamma+28\gamma^3}{32(2-\gamma^2)^2}$	$W_{NN} = v^2\dfrac{3+\gamma}{(2+\gamma)^2}$