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ABSTRACT

Precautionary Bidding in Auctions

We analyse bidding behaviour in auctions when risk-averse buyers bid for a good whose value is risky. We show that when risk in the valuations increases, DARA bidders will reduce their bids by more than the appropriate increase in the risk premium. Ceteris paribus, buyers will be better off bidding for a more risky object in first-price, second-price, and English auctions with affiliated common (interdependent) values. This 'precautionary bidding' effect arises because the expected marginal utility of income increases with risk, so buyers are reluctant to bid so highly. We also show that precautionary bidding behaviour can make DARA bidders prefer to bid in a common values setting than in a private values one when a risk-neutral or CARA bidder would be indifferent. Thus the potential for a 'winners curse' can be a blessing for rational DARA bidders.

JEL Classification: D44 and D81
Keywords: English auctions, first-price auctions, prudence, risk, risk-aversion, second-price auctions and winner's curse

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1 Introduction

In many real world auctions the value of the goods for sale is subject to ex post risk. At the time of the sale, buyers can only estimate the value of the good and they are well aware that the true value to them will be revealed only some time after the sale. Part of this risk is what might be called "winner's curse" risk: uncertainty about other buyers' (or the seller's) information which is not revealed in the course of the auction. However, there is also almost invariably *pure risk* in the valuations, arising from information that none of the buyers (nor the seller) can obtain, which will be resolved after the good has been allocated. The sale of oil tracts, art, antiques, wine, and procurement contracts provide obvious examples which exhibit both types of risk. In each case, there is something about the future resale price, authenticity, quality, and so on, of these goods which cannot be perfectly foreseen, and which from the buyers' point of view is purely random.

Despite the ubiquity of pure ex post risk, there has to date been no analysis of its effect on the bidding behavior of risk-averse agents.[2] The core of this paper is devoted to providing such an analysis in a framework similar to the general symmetric interdependent-values model of Milgrom and Weber (1982).

Our main result is that in common auction forms (first price, second price, and English auctions), symmetric DARA bidders (buyers whose utility functions are the same and exhibit decreasing absolute risk aversion) reduce their bids by *more* than the corresponding increase in the risk premium when pure risk is added to their values. Therefore, holding the number and the ex ante characteristics of the participants fixed, buyers will be better off bidding for a more risky object.

In the first price auction, this result follows from an effect we may call *precautionary bidding*. As with precautionary saving, when agents face a risk, their marginal utility of income rises.[3] This causes buyers to bid less aggressively because they value more highly each extra dollar of income, as compared to the increased probability of winning the good. In the case of DARA preferences, this effect is so strong that the buyers end up with higher expected utilities when the noise is present in their valuations. This result is surprising for the following reason. Under general conditions, DARA individuals become more risk-averse in facing one risk (i.e., losing the object) when they are forced to face an independent risk (i.e., object value).[4] Since increasing the degree of risk aversion leads to more aggressive bidding in a first price auction, we might therefore expect that increasing the riskiness of the good would make buyers more risk-averse and so raise bids and make them worse off.[5] However, this latter effect turns out to be smaller than the precautionary effect.

In the second price and English auctions, as valuations become noisy, the buyers also reduce their bids by more than the corresponding increase in the risk premium. The necessary and sufficient conditions for the effect to occur are the same (i.e., DARA), although the intuition behind the result is slightly different. Recall that in these auctions, the buyers submit bids assuming that they will receive zero surplus from winning whenever they win. Therefore, in the presence of noise, DARA bidders will reduce their bids by the large risk premium that would be required if their surplus were zero. But when they actually win, the buyers will have a positive surplus on average and their expected payment will have been reduced by a risk premium that was "too large." So overall they will be better off.

Our finding that the seller would like to reduce the pure risk faced by buyers is distinct from the linkage principle (due to Milgrom and Weber (1982)). This principle implies that the seller should commit to reveal any information affiliated with the buyers' signals because the commitment reduces the potential winner's curse that the buyers face. Note, however, that the winner's curse arises because winning provides information about the value of the good, not because of the pure risk in the good's value. Conversely, it is completely possible for the private value of a good to an individual to be risky without any winner's curse implications for bidding. An obvious distinction between the linkage principle and the effects of white noise is that precautionary bidding will arise only when buyers are decreasingly risk-averse (DARA), whereas the linkage principle will hold even when buyers are risk-neutral, but have affiliated common values.

The behavior of risk-averse buyers in an environment with affiliated common values has to our knowledge been hardly studied at all. We use our analysis of precautionary bidding to throw more light on this topic. We show that DARA buyers engage in precautionary bidding in response to the risk inherent in other buyers' signals not revealed in the course of the auction. Because of this, DARA buyers may prefer an interdependent values auction to a private values setting that would be equally attractive for risk-neutral buyers.

The paper is laid out as follows. In Section 2, we consider the consequences of adding pure noise to the prize in a symmetric auction model with affiliated signals and interdependent valuations. We prove that DARA buyers engage in precautionary bidding in first price, second price, and English auctions, and will benefit from more risk in the good's value. We also discuss some implications of our result, including for the revenue ranking of auctions by the seller. In Section 3, we show that even in the absence of additional noise, pure common-value components alone suffice to generate the precautionary bidding effect. In Section 4, we offer concluding remarks.

2 Precautionary Bidding

2.1 General Symmetric Model with Noisy Valuations

We assume that there are n potential buyers for a given good. The seller's reservation value for the good is zero. Buyer i receives a private signal (type), $s_i \in [\underline{s}, \bar{s}]$. The joint distribution of the signals has a positive, twice-differentiable density, which is symmetric and affiliated.[6] We will denote the vector of signals of buyers other than i by s_{-i}, the highest among the signals of buyers other than i by $s_{-i}^{\max} \equiv \max_{j \neq i}\{s_j\}$, and the joint density of $s_{-i}^{\max}$ and s_i by $f(s_{-i}^{\max}, s_i)$. Since all signals are affiliated, $s_{-i}^{\max}$ and s_i are also affiliated, that is, $\partial^2 \ln f(y,x)/\partial x \partial y \geq 0$.

The ex post monetary value of the good for buyer i is

$$v_i = v(s_i, s_{-i}) + z_i,$$

where $v : [\underline{s}, \bar{s}]^n \rightarrow \mathbb{R}$ is a continuous, weakly increasing function, which is strictly increasing in its first argument and invariant to permutations of its last $n-1$ arguments (i.e., for all $\hat{s}_{-i}$ permutations of s_{-i}, $v(s_i, s_{-i}) = v(s_i, \hat{s}_{-i})$); the additional term, z_i, is the realization of a zero-mean random variable $\tilde{z}_i$. Note that the specification of the valuation functions is symmetric, and a buyer's valuation depends only on the collection of signals of the other buyers (besides his own), not on the identities of the other buyers.[7]

We assume that the $\tilde{z}_i$'s come from a symmetric joint distribution and that each $\tilde{z}_i$ is independent of $(s_1, ..., s_n)$.[8] The noise is interim unobservable and uninsurable. When the $\tilde{z}_i$'s are degenerate, $\tilde{z}_i \equiv 0$, we say that the buyers have *deterministic valuations*, and when the $\tilde{z}_i$'s are non-degenerate, we say that they have *noisy valuations*. We can interpret this "noise term" affecting the buyers' values in either of two ways. First, it could be a result of common shocks (such as a change in the oil price or the amount of oil underground); or second, it could be buyer-specific symmetrically distributed shocks (such as unforeseen production costs).

The buyers evaluate their monetary surplus (consisting of their initial wealth minus the transfer paid to the seller, plus the good's value when they win) according to a strictly concave and thrice differentiable utility function, u, and they are expected utility maximizers. We normalize their initial wealth and $u(0)$ to zero, and assume that the good is valuable to them for all realizations of the signals, that is, $E[u(v_i + \tilde{z}_i) | \forall j, s_j = \underline{s}] > 0$. We will use the notions of decreasing and constant absolute risk aversion (DARA and CARA, respectively), defined the standard way as $-(\partial^2 u/\partial x^2)/(\partial u/\partial x)$ being strictly decreasing and constant, respectively. From now on we will assume that u belongs to either the DARA or CARA family.

2.2 Main Results

We now analyze how ex ante symmetric DARA or CARA buyers' behavior and indirect utility changes as a result of more noise being added to their valuations. In particular, we will compare two situations, one where $\tilde{z}_i \equiv 0$ (deterministic valuations), and another where $\tilde{z}_i$ is an independent random variable with zero mean and finite variance (noisy valuations). Holding everything else the same, we show that DARA buyers have higher indirect utilities (while CARA buyers are indifferent) when noise is present in their valuations in the English (button-), the first price, and the second price auctions.[9]

As a preparation for the proofs, define

$$\bar{u}(w\,;\,x, y) = E[u(v(s_i, s_{-i}) + w)\,|\,s_i = x,\, s_{-i}^{\max} = y]. \tag{1}$$

This is the (expected) utility of buyer i when he gets the noise-free good (whose value is still risky due to interdependent values), given that his wealth level is w, his own signal is x, and the highest of the other buyers' signals is y. We will use three key properties of $\bar{u}$; a short explanation of each property (with references) is provided below.

Property 1. The function $\bar{u}$ is strictly increasing in x, weakly increasing in y; and for all x and y, $\bar{u}$ is a concave, strictly increasing utility function in w.

This property follows because v is weakly increasing (strictly increasing in s_i), and the monotonicity and concavity of u in w are preserved under expectation.

Property 2. If u is DARA then for $x' > x$ and all y and w, $\bar{u}(w\,;\,x', y)$ is strictly less risk-averse in w than $\bar{u}(w\,;\,x, y)$ is; similarly, for $y' > y$ and all x and w, $\bar{u}(w\,;\,x, y')$ is weakly less risk-averse in w than $\bar{u}(w\,;\,x, y)$ is. On the other hand, if u is CARA then for all x, y and w, $\bar{u}$ is also CARA in w with the same degree of absolute risk aversion.

This property follows because, by affiliation, the random variable $v(s_i, s_{-i})$ given s_i and $s_{-i}^{\max}$ increases in s_i in the monotone likelihood ratio sense, and therefore, when u is DARA, the resulting expected utility function, $\bar{u}$, will exhibit a lower level of risk aversion in w for a higher s_i (this result is due to Jewitt (1987); see also Eeckhoudt et al. (1996) and Athey (2000)). The same holds for an increase in $s_{-i}^{\max}$, except that $v(s_i, s_{-i})$ increases in $s_{-i}^{\max}$ weakly, and hence the decrease in the level of risk aversion will also be weak. The CARA property (and the level of absolute risk aversion) are preserved when a background risk is added.

Property 3. The functions $-\partial\bar{u}/\partial x$ and $-\partial\bar{u}/\partial y$ are increasing and concave functions of w as well. If u is DARA then, holding x and y fixed, $-\partial\bar{u}/\partial x$ exhibits a strictly higher and $-\partial\bar{u}/\partial y$ a weakly higher level of risk aversion in w than $\bar{u}$ does. If u is CARA then all three functions exhibit the same degree of absolute risk aversion in w.

This last property easily follows from Property 2 combined with the observations that (i) u is strictly concave with a positive third derivative and (ii) decreasing absolute risk aversion of a utility function is equivalent to the negative of the marginal utility being more risk-averse than the utility function (Kimball (1990)).

Now we are ready to prove,

Theorem 1 *Consider the general symmetric affiliated model with DARA buyers. The buyers' utilities are strictly higher with noisy rather than deterministic valuations in the symmetric equilibria of the second price and the English auctions.*

Proof. First, assume that valuations are deterministic. Let

$$\bar{v}(x,y) \equiv E[v(s_i, s_{-i}) \,|\, s_i = x,\, s_{-i}^{\max} = y].$$

Define $\pi(s_i)$ solving

$$\bar{u}(-\bar{v}(s_i, s_i) + \pi(s_i)\,;\, s_i, s_i) = 0, \tag{2}$$

which means, by (1), that $\pi(s_i)$ compensates buyer s_i for the common value risk conditional on $s_{-i}^{\max} = s_i$ at zero expected surplus.

We claim that a symmetric increasing equilibrium exists, with bid functions

$$b(s_i) = \bar{v}(s_i, s_i) - \pi(s_i). \tag{3}$$

First, note that $b(s_i)$ is strictly increasing. By differentiating identity (2) in s_i,

$$\frac{\partial}{\partial w}\bar{u}(-b(s_i); s_i, s_i)(-b'(s_i)) + \frac{\partial}{\partial x}\bar{u}(-b(s_i); s_i, s_i) + \frac{\partial}{\partial y}\bar{u}(-b(s_i); s_i, s_i) = 0,$$

where $\partial\bar{u}/\partial w > 0$, $\partial\bar{u}/\partial x > 0$, and $\partial\bar{u}/\partial y \geq 0$, therefore $b' > 0$. In order to establish that bidding $b(s_i)$ by buyer i of type s_i is a best response to b played by all $j \neq i$, suppose towards contradiction that i bids $\hat{b} > \bar{v}(s_i, s_i) - \pi(s_i)$ instead of (3) while all others play according to b. This makes a difference only if, for $s_{-i}^{\max} = y$, $\hat{b} > b(y) > \bar{v}(s_i, s_i) - \pi(s_i)$. Then i will receive, instead of 0,

$$\bar{u}(-\bar{v}(y, y) + \pi(y))\,;\, s_i, y) < \bar{u}(-\bar{v}(y, y) + \pi(y))\,;\, y, y) = 0,$$

where the inequality follows from Property 1 of $\bar{u}$ and $s_i < y$. Hence bidding $\hat{b} > \bar{v}(s_i, s_i) - \pi(s_i)$ is not profitable. An analogous argument rules out bidding $\hat{b} < \bar{v}(s_i, s_i) - \pi(s_i)$. Therefore (3) is a symmetric increasing equilibrium bid function.[10]

If valuations are risky then in the symmetric equilibrium buyer s_i bids

$$\beta(s_i) = \bar{v}(s_i, s_i) - \pi(s_i) - \pi_z(s_i), \tag{4}$$

where $\pi_z(s_i)$ solves

$$E_z\bar{u}(-\bar{v}(s_i, s_i) + \pi(s_i) + \tilde{z}_i + \pi_z(s_i)\,;\, s_i, s_i) = 0. \tag{5}$$

That is, type s_i further reduces his bid by the compensating risk premium for $\tilde{z}_i$ at the risky initial wealth (risky due to the common value risk) that gives him zero surplus. The derivation is identical to that of the equilibrium under deterministic valuations, (3), and therefore is omitted.

By (2) and (5), $\bar{u}(w\,;\,y,y) = E_z\bar{u}(w+\tilde{z}_i+\pi_z(y)\,;y,y)$ at $w=-\bar{v}(y,y)+\pi(y)$. Therefore, by Property 2 of $\bar{u}$, for all $s_i > y$,

$$\bar{u}(-\bar{v}(y,y)+\pi(y)\,;\,s_i,y) < E_z\bar{u}(-\bar{v}(y,y)+\pi(y)+\tilde{z}_i+\pi_z(y)\,;\,s_i,y).$$

Taking expectations over $s_{-i}^{\max} \equiv y \leq s_i$, and using the definiton of $\bar{u}$, we obtain

$$\begin{aligned} & E\left[\left(u(v(s_i,s_{-i})-\bar{v}(s_{-i}^{\max},s_{-i}^{\max})+\pi(s_{-i}^{\max}))\right)\mathbf{1}_{\left\{s_{-i}^{\max}\leq s_i\right\}}\right] \\ < \;\; & E\left[\left(u(v(s_i,s_{-i})-\bar{v}(s_{-i}^{\max},s_{-i}^{\max})+\pi(s_{-i}^{\max})+\tilde{z}_i+\pi_z(s_{-i}^{\max}))\right)\mathbf{1}_{\left\{s_{-i}^{\max}\leq s_i\right\}}\right]. \end{aligned}$$

This means that buyer i with type s_i is strictly better off in the equilibrium with noisy valuations than in the equilibrium with deterministic valuations.

The argument is similar, although somewhat simpler, in the case of the English auction. In the efficient symmetric equilibrium of this auction with deterministic values, buyer i plans to quit at $v(s_i,s_{-i})$, such that for all active $j \neq i$, $s_j = s_i$, and for all inactive $j \neq i$, s_j equals j's true type. By strict monotonicity of v_i in s_i, lower types plan to quit earlier. When a buyer drops out, the other buyers infer his type and repeat the above calculation until only one buyer remains active. The winner will therefore pay $v(x,s_{-i})$ at $x = s_{-i}^{\max}$, which is (weakly) less than his actual valuation because $s_i \geq s_{-i}^{\max}$. With noisy valuations, buyer i plans to quit at $v(s_i,s_{-i})-\pi_0$ such that for all active $j \neq i$, $s_j = s_i$, for all inactive $j \neq i$, s_j

equals j's true type, and π_0 solves $u(0) = Eu(\tilde{z}_i + \pi_0)$. If i is the winner then he pays $v(x, s_{-i}) - \pi_0$ at $x = s_{-i}^{\max}$. Since the deterministic part of his surplus is non-negative, the compensating risk premium for noise $\tilde{z}_i$ is less than π_0, therefore the buyer prefers the equilibrium under noisy valuations. ■

The same result holds for the first price auction as well.

Theorem 2 *Consider the general symmetric affiliated model with DARA buyers. The buyers' utilities are strictly higher with noisy rather than deterministic valuations in the symmetric equilibrium of the first price auction.*

Proof. Let the symmetric equilibrium bid functions under deterministic and noisy values be $b(s_i)$ and $\beta(s_i)$, respectively.[11] Under deterministic values, in equilibrium, the utility of buyer i with signal s_i from pretending to have type $\hat{s}_i$ is

$$U(s_i, \hat{s}_i) = \int_{\underline{s}}^{\hat{s}_i} \bar{u}(-b(\hat{s}_i)\,;\, s_i, y)\, f(y \,|\, s_i) dy,$$

where $f(y \,|\, s_i)$ is the probability density function of $y = s_{-i}^{\max}$ conditional on s_i. Let $V(s_i) = U(s_i, s_i)$. By incentive compatibility and the Envelope Theorem, for all $s_i \in [\underline{s}, \bar{s})$, the derivative of V (from the right, in case $s_i = \underline{s}$) must be

$$V'(s_i) = \int_{\underline{s}}^{s_i} \frac{\partial}{\partial x} \bar{u}(-b(s_i); s_i, y)\, f(y \,|\, s_i) dy + \int_{\underline{s}}^{s_i} \bar{u}(-b(s_i); s_i, y)\, \frac{\partial}{\partial s_i} f(y \,|\, s_i) dy. \tag{6}$$

Similarly, if $\tilde{V}(s_i)$ denotes s_i's indirect expected utility under noisy values in the equilibrium of the FPA then we have,

$$\begin{aligned} \tilde{V}'(s_i) = &\int_{\underline{s}}^{s_i} \frac{\partial}{\partial x} E_z \bar{u}(-\beta(s_i) + \tilde{z}_i\,; s_i, y)\, f(y \,|\, s_i) dy \\ &+ \int_{\underline{s}}^{s_i} E_z \bar{u}(-\beta(s_i) + \tilde{z}_i\,; s_i, y)\, \frac{\partial}{\partial s_i} f(y \,|\, s_i) dy. \end{aligned} \tag{7}$$

Note that buyer i with type s_i wins with probability 0 in either setup, therefore $V(\underline{s}) = \tilde{V}(\underline{s}) = 0$. Moreover, both V and $\tilde{V}$ are continuous (by incentive compatibility). We will now show that if $V(s_i) = \tilde{V}(s_i)$ for some $s_i \in [\underline{s}, \bar{s})$ then $V'(s_i) < \tilde{V}'(s_i)$, which implies that $V(s_i) < \tilde{V}(s_i)$ for all $s_i \in (\underline{s}, \bar{s}]$, as claimed.

Suppose that $V(s_i) = \tilde{V}(s_i)$ for some $s_i \in [\underline{s}, \bar{s})$. Then,

$$\int_{\underline{s}}^{s_i} \left[E_z \bar{u}(-\beta(s_i) + \tilde{z}_i\,; s_i, y) - \bar{u}(-b(s_i)\,; s_i, y) \right] f(y, s_i) dy = 0. \tag{8}$$

The expression in square brackets is weakly increasing and continuous in $y = s_{-i}^{\max}$ whenever it is non-negative by Property 2 of $\bar{u}$, while its expected value with respect to $s_{-i}^{\max}$ (given s_i) is 0; therefore the integrand switches sign at most once, from negative to positive. By affiliation, $\frac{\partial}{\partial s_i} f(y, s_i)/f(y, s_i)$ is increasing in y. Therefore the product of the integrand in (8) and $\frac{\partial}{\partial s_i} f(y, s_i)/f(y, s_i)$ has a non-negative integral (a simple proof can be given along the lines of Lemma 1, Persico (2000)). That is,

$$\int_{\underline{s}}^{s_i} \left[E_z \bar{u}(-\beta(s_i) + \tilde{z}_i \,; s_i, y) - \bar{u}(-b(s_i)\,; s_i, y)\right] \frac{\partial}{\partial s_i} f(y, s_i) dy \geq 0,$$

and so the second term in (7) is not less than the second term in (6).

By Property 3 of $\bar{u}$, which is preserved under integration,

$$\begin{aligned} &\int_{\underline{s}}^{s_i} \frac{\partial}{\partial x} \bar{u}(-b(s_i)\,; s_i, y)\, f(y \,|\, s_i) dy \\ &< \int_{\underline{s}}^{s_i} E_z \frac{\partial}{\partial x} \bar{u}(-\beta(s_i) + \tilde{z}_i \,; s_i, y)\, f(y \,|\, s_i) dy, \end{aligned} \tag{9}$$

and therefore $V'(s_i) < \tilde{V}'(s_i)$. This completes the proof. ■

Remark. It is clear from the proofs of Theorem 1 and Theorem 2 that in the general symmetric model with CARA preferences, all types of every buyer are indifferent between playing the equilibrium with deterministic or noisy valuations in the first price, second price, and English auctions.

The intuition underlying precautionary bidding in the first price auction is the following. For DARA (CARA) individuals, the precautionary premium—the amount required to compensate the *marginal* utility of income for an increase in risk—exceeds (equals) the risk premium (Kimball 1990). Thus, if a DARA buyer reduces his bid by the amount of the compensating risk premium, his marginal utility of income is still higher than in the absence of noise, so he will reduce his bid still further to save income in case he wins and must bear the risk of the good.[12] (For CARA buyers, marginal and total utility are both exactly compensated by the risk premium.)

This intuition applies to some other auction formats as well, so the phenomenon of precautionary bidding is more general than what is demonstrated in Theorems 1 and 2. In the working paper version of this article (Eső and White 2001), we show that DARA bidders will be better off if a risk is added to their values in an all-pay auction with interdependent values and independent signals, or with private values and affiliated signals (provided the all-pay auction has an equilibrium in the latter case). In Section 3 below, we will examine another incidence of precautionary bidding where the added noise is not independent of buyers' signals. In particular, we show that common value components in the buyers' valuations suffice to generate the precautionary bidding effect.

2.3 Implications of the Results

Up until now, the auction literature has abstracted from the issue of pure ex post risk. Our results imply that this abstraction is fully justified only in the case of CARA preferences as these buyers would reduce their bids by exactly the risk premium. Since DARA buyers benefit from the introduction of risk in their valuations, they may attempt collectively to commit *not* to acquire information about shocks that affect their payoffs (e.g., not doing test drilling on a tract of oil for sale), if it is possible to make such actions publicly observable. Further, it is not clear whether even an individual buyer would wish to learn his valuation more precisely. One may conjecture that while more information about his valuation should benefit the buyer, it might draw forth a more aggressive response from rival bidders as well.

The results also imply that a seller who is as risk-averse as the buyers will wish to provide insurance against the noise. This is not simply because the risk reduction directly increases the buyers' willingness to pay, but also because it limits precautionary bidding and thus intensifies competition. By contrast, if the auction-designer's profit depends not only on the expected revenue from the auction itself, but also on the number of bidders attending the auction (as is for example the case for competing internet auction websites) then it may be a good idea for him to auction goods whose value is uncertain. Auctions with risky objects (such as online auctions, where unseen goods are bought from complete strangers) may be very popular with DARA buyers who anticipate a large surplus to be made. It is important to see that this effect arises with risk-averse buyers rather than risk-loving ones, and that the popularity of risky auctions may be completely rational.

Finally, let us remark on the empirical testability of our model of precautionary bidding. In order to run a direct empirical test, one would need an accurate estimate of both the degree of the buyers' risk aversion and the riskiness of the good, and estimating these parameters is a difficult econometric exercise (see Campo et al. (2000)). An indirect but simpler test would be to see whether auctions of riskier items attract more buyers (controlling for the type of the auction, the ex ante distribution of the expected value of the good, and the characteristics of the buyers). One way of measuring the "riskiness" of the good's value might be the level of detail in the seller's description of the good, or the extent to which potential buyers can examine the good before the auction (for example, the ease of access to a forest or oil tract for pre-bid investigation).

2.4 Ranking of Auctions with Noisy Private Valuations

In this subsection, we explore some issues that arise when the buyers' baseline valuations, to which the noise is added, are *private* (rather than interdependent).

The proof of Theorem 1 reveals that in the second price or English auction, as the risk increases, bids are reduced by the amount of the risk premium that would obtain if buyers had zero surplus. In the first price auction (Theorem 2), we do not have such a simple interpretation. Our result thus appears to raise a question regarding the ranking of auctions: does the differential reduction in the bids alter the preference ordering of the seller (or the buyers) over the different auction forms, relative to the situation when noise is not present?

The answer to this question turns out to be straightforward. Under independent private values and with risk-averse (not necessarily DARA) buyers, Maskin and Riley (1984) show that the seller prefers the first price auction over the second price auction. Under independent private values but with DARA buyers, Matthews (1987) shows that the buyers prefer the second price auction over the first price auction.[13] When an independent noise is added to the independent private valuations in these models, both results remain true. This is so because both risk aversion and the DARA property are preserved after the introduction of noise and taking expectations (Pratt (1964)). In Eső and White (2001), we complete the preference ordering of auctions by DARA buyers (with independent, private, and possibly noisy) valuations by showing that they prefer the first price auction to the all-pay auction.

Our result does call into question the well-known result that the seller's revenue in a first price auction (with independent signals and private values) will increase as the buyers become more risk-averse.[14] It remains true that if the good's value is risky then the fear of losing it will motivate more risk-averse buyers to bid more highly. But against this, they will bid less highly because they dislike the riskiness of the good, and even less highly because of the precautionary effect. So it is an open question under exactly which circumstances the seller will earn higher expected revenue from more risk-averse buyers.

3 Comparison of Common- and Private-Value Auctions: When the “Winner’s Curse” is a Blessing

In this section, we show that the precautionary bidding effect is not limited to exogenous, mean-zero risks, and that other forms of risk are likely to cause the same type of behavior. In particular, in interdependent-value contexts, the good’s value is risky for the buyer because when he wins, he does not necessarily know the other buyers’ signals affecting his valuation. As a result of this, DARA buyers will prefer certain common values environments to “comparable” private values environments.[15] In other words, the common-value risk (associated with the potential “winner’s curse”) may in fact be a blessing for DARA buyers.

In order to simplify the analysis, let us assume that the buyers’ signals are independent draws from the same distribution, and that the valuations, though interdependent, can be written as

$$v_i = s_i + h(s_{-i}), \tag{10}$$

where $h : \mathbb{R}^{n-1} \to \mathbb{R}$ is weakly increasing and invariant to permutations of its arguments (i.e., for all $\hat{s}_{-i}$ permutations of s_{-i}, $h(s_{-i}) = h(\hat{s}_{-i})$). The advantage of this specification is that it will simplify the task of finding comparable interdependent- and private-value environments.

In Proposition 1, we compare an environment where the buyers' valuations are private and equal to their signal with another one where their valuations are interdependent as in (10). The two environments are the same in all other respects, including the number of buyers and the (i.i.d.) ex ante distribution of their signals.

Interestingly, if the selling mechanism is the English (button-) auction, then the buyers obtain the same indirect utilities in the two environments no matter what their risk preferences are (as long as the model is symmetric).[16] However, if the selling mechanism is either a first price or a second price auction, then only risk-neutral or CARA buyers will be indifferent between these private- and interdependent-values environments; DARA buyers will strictly prefer the latter. This, as we establish, is a consequence of precautionary bidding.

Proposition 1 *Assume that the buyers' types are independently and identically distributed. If the buyers are risk-neutral or CARA then they receive the same utilities in the first price and second price auctions when either $v_i = s_i$ for all i, or $v_i = s_i + h(s_{-i})$ for all i, where h is weakly increasing and invariant to permutations of its arguments. If buyers have the same DARA preferences and the mechanism is either a first price or second price auction, then they prefer the environment where v_i has a common component as in (10) to the environment where $v_i = s_i$.*

Proof. First, consider the indirect utility of risk-neutral buyers obtained in an efficient mechanism in an ex ante symmetric environment with independent signals and interdependent valuations. By incentive compatibility, using well-known arguments (the Envelope Theorem or following Myerson (1981)), the payoff of a buyer with type s_i will be

$$V^{RN}(s_i) = V^{RN}(\underline{s}) + \int_{\underline{s}}^{s_i} \Pr(s_{-i}^{\max} \leq x) E\left[\frac{\partial}{\partial s_i} v(x, s_{-i}) \,|\, s_{-i}^{\max} \leq x\right] dx.$$

In this environment, both the first price and second price auctions are efficient and yield zero surplus to the lowest type, $V^{RN}(\underline{s}) = 0$. Moreover, when (10) holds, the second term of the above expression (the integral) is the same no matter whether or not $h \equiv 0$. Therefore, given that the selling mechanism is either a first price or a second price auction, risk-neutral buyers receive the same payoff in the two environments under consideration.

Second, suppose that the buyers have the same risk-averse utility functions (either CARA or DARA). The additively separable interdependent values environment specified in (10) can be thought of as one with noisy private values where the "noise" comes from the signals of the other buyers. While the hypotheses of Theorems 1 and 2 do not hold (the "noise" is not independent of s_{-i}, nor is it independent of s_i conditional on winning), their proofs go through with some modifications.

In the second price auction, under private values, buyer i with type s_i bids $b(s_i) = s_i$ and his equilibrium expected utility is $E\left[u(s_i - s_{-i}^{\max})\mathbf{1}_{\{s_{-i}^{\max} \leq s_i\}}\right]$. Under interdependent values of the form (10), buyer i with signal s_i bids $\beta(s_i) = s_i +$

$\bar{h}(s_i) - \pi_h(s_i)$, where $\bar{h}(y) = E\left[h(s_{-i}) \,|\, s_{-i}^{\max} = y\right]$ and $\pi_h(y)$ is defined implicitly by $E\left[u(h(s_{-i}) - \bar{h}(y) + \pi_h(y)) \mid s_{-i}^{\max} = y\right] \equiv 0$. Note that the random variable $h(s_{-i}) - \bar{h}(y)$ given $s_{-i}^{\max} = y$ is independent of s_i. The equilibrium payoff of buyer i with type s_i conditional on winning against a given $s_{-i}^{\max} = y < s_i$, is

$$E\left[u(s_i + h(s_{-i}) - y - \bar{h}(y) + \pi_h(y)) \mid s_{-i}^{\max} = y\right] \geq u(s_i - y), \tag{11}$$

where the inequality holds as equality for CARA, and strict inequality for DARA preferences. The inequality follows because the two sides are equal at $s_i = y$, that is, u is exactly compensated by the risk premium $\pi_h(y)$ for the added risk, $h(s_{-i}) - \bar{h}(y)$ given $s_{-i}^{\max} = y$. When $s_i > y$, by the CARA (DARA) property, this risk premium, $\pi_h(y)$, exactly (strictly more than) compensates the buyer's utility for the same risk. By taking expectations of (11) over $s_{-i}^{\max} \equiv y \leq s_i$, we complete the proof for the second price auction.

In the first price auction, let the symmetric equilibrium bid function be b under private values, and β under interdependent values as in (10). The utilities of type s_i pretending $\hat{s}_i$ under private values and, respectively, interdependent values as in (10), can be written as

$$\begin{aligned} U^{PV}(s_i, \hat{s}_i) &= \Pr(s_{-i}^{\max} \leq \hat{s}_i) u(s_i - b(\hat{s}_i)), \\ U^{CV}(s_i, \hat{s}_i) &= \Pr(s_{-i}^{\max} \leq \hat{s}_i) E[u(s_i + h(s_{-i}) - \beta(\hat{s}_i)) \,|\, s_{-i}^{\max} \leq \hat{s}_i]. \end{aligned}$$

Denote $V(s_i) = U^{PV}(s_i, s_i)$ and $\tilde{V}(s_i) = U^{CV}(s_i, s_i)$. Using standard arguments (the Envelope Theorem), for all $s_i \in [\underline{s}, \bar{s})$, the (right-hand) derivatives are,

$$\begin{aligned} V'(s_i) &= \Pr(s_{-i}^{\max} \le s_i) u'(s_i - b(s_i)), \\ \tilde{V}'(s_i) &= \Pr(s_{-i}^{\max} \le s_i) E[u'(s_i + h(s_{-i}) - \beta(s_i)) \,|\, s_{-i}^{\max} \le s_i]. \end{aligned}$$

If $V(s_i) = \tilde{V}(s_i)$ for some $s_i \in [\underline{s}, \bar{s})$, that is,

$$u(s_i - b(s_i)) = E[u(s_i + h(s_{-i}) - \beta(s_i)) \,|\, s_{-i}^{\max} \le s_i],$$

then, by the CARA or DARA property of u,

$$u'(s_i - b(s_i)) \le E[u'(s_i + h(s_{-i}) - \beta(s_i)) \,|\, s_{-i}^{\max} \le s_i],$$

where the inequality holds as equality under CARA, and strict inequality under DARA preferences. Therefore, for all $s_i \in [\underline{s}, \bar{s})$, $V(s_i) = \tilde{V}(s_i)$ implies $V'(s_i) < \tilde{V}'(s_i)$. Since both functions are continuous and $V(\underline{s}) = \tilde{V}(\underline{s})$, we obtain $V(s_i) < \tilde{V}(s_i)$ for all $s_i \in (\underline{s}, \bar{s}]$. This concludes the proof. ∎

The result of Proposition 1 is of practical importance because in some settings bidders may be able to choose between entering auctions where they will face significant common-values risks and those where they will not. In other cases, whether the auction of a given good is largely a private or common values affair may be determined by prior moves taken by the bidders. For example, consider two firms which will later compete in a procurement auction. If—prior to the auction—these two firms choose similar production technologies, then the subsequent auction will have a strong common-value component: one firm's estimate of the likely cost of fulfilling the contract is likely to be important information for the other firm. But if the two firms choose production technologies which are very different from one another, then information about one firm's production costs may not be at all useful in estimating the other firm's likely cost, and the auction will take place in a private values environment. The results of this section suggest that if the firms are DARA,[17] they may be better off choosing technologies which are "too correlated" (from the seller's and perhaps the social point of view) in order to benefit from the softened (precautionary) bidding.[18]

4 Conclusions

We have shown that in a general symmetric model with affiliated signals and interdependent values, risk-averse DARA buyers are better off when the value of the good auctioned becomes more risky. This research can be thought of as extending Matthews' (1987) comparison of auction environments from the buyers' perspective in a new direction. Instead of comparing different auction formats with the same information structure, we have compared different information structures holding the auction format fixed. We have shown that when the object's value is subject to an additional independent risk, buyers behave less aggressively in the first price, second price, and English auctions, reducing their bids by more than the amount of the appropriate increase in the risk premium. We call this effect *precautionary bidding*.

We have shown that the same effect occurs when the risk associated with the value of the good arises because buyers care about each others' signals (i.e. "winner's curse risk"). Thus in first and second price auctions, DARA buyers are better off bidding in a common values setting than a private values one, where risk-neutral buyers would be indifferent. Thus environments which allow for a potential winner's curse may in fact be a "blessing" to DARA buyers.

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Notes

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[2]Buyers display risk aversion in a variety of auction scenarios. For a survey of the experimental evidence see Kagel (1995). Econometric evidence based on data from timber auctions is provided by Paarsch (1992) and Athey and Levin (2001).

[3]See Kimball (1990) and the literature dating back to Leland (1968), Drèze and Modigliani (1972) and Sandmo (1970).

[4]See Eeckhoudt et al (1996) and the references cited therein.

[5]Auctions with risk-averse buyers have been studied by Holt (1980), Riley and Samuelson (1981), Maskin and Riley (1984), and others.

[6]For the definition and properties of affiliation see Milgrom and Weber (1982).

[7]An alternative notation would be to write the valuation function as $v(X_1, Y_1, ..., Y_{n-1})$,

where X_1 stands for i's own signal (s_i), and Y_k stands for the kth highest among the other buyers' signals. Hence, the deterministic part of the valuation in our model is equivalent to the buyer's (expected) valuation in the general symmetric affiliated model of Milgrom and Weber (1982).

[8]Note that we allow the $\tilde{z}_i$'s to be correlated, or even $\tilde{z}_i \equiv \tilde{z}$ for all i.

[9]The results are confined to comparing noisy and deterministic valuations, but immediately extend to situations where another independent noise is added to make already noisy valuations still riskier. This is so because the DARA property is preserved under addition of independent mean-zero noise (see Kihlstrom, Romer, and Williams, 1981).

[10]Under risk neutrality the equilibrium bid is $\bar{v}(s_i, s_i)$. We have shown that in the case of DARA bidders it is reduced by the risk premium $\pi(s_i)$, which compensates the buyer for the risk of the others' signals at zero expected surplus.

[11]The existence of a symmetric equilibrium in our symmetric, affiliated environment with risk-averse buyers follows from standard arguments (see Milgrom and Weber (1982), section 6, for risk neutral buyers).

[12]Evidently, the same feature of the utility function causes precautionary saving in response to income risk, hence the name for our phenomenon: "precautionary

bidding."

[13]Under independent private values the English and second price auctions are outcome-equivalent. Therefore both the seller and the buyers are indifferent between these two formats.

[14]For references, see footnote 5.

[15]We will define the private values environments "comparable" to certain common values environments such that risk-neutral buyers will be exactly indifferent between participating in either of the two auctions.

[16]When only two buyers remain in the English auction, say, i and j with signals s_i and s_j, they both know all the other buyers' signals (the true s_k, for all $k \neq i, j$) from the drop-out prices. In the interdependent values case, the buyers build this information into their valuations and bid $s_i + h(s_i, (s_k)_{k \neq i,j})$ and $s_j + h(s_j, (s_k)_{k \neq i,j})$, respectively. Suppose the winner is i, then his utility is $u(s_i + h(s_j, (s_k)_{k \neq i,j}) - s_j - h(s_j, (s_k)_{k \neq i,j}) = u(s_i - s_j)$. In the private values case, if s_i wins then he pays s_j yielding a utility of $u(s_i - s_j)$. Hence if i wins then his utility is the same in the common and private values setups for all realizations of s_j.

[17]For a model of why firms in imperfect capital markets will tend to display

decreasing absolute risk aversion, see Froot, Scharfstein and Stein (1993).

[18]Similar remarks could apply to the choice of customer base by car, art, wine and antique retailers who buy their product in wholesale auctions: if they choose to serve customers with similar tastes, the common-value risk will be larger, because when bidding in auctions they will all be interested in estimating the same properties of the objects for sale. This reduction in competition effect would counteract the bidders' desire to differentiate themselves to avoid excessive competition in the retail market.