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AGING POPULATION AND EDUCATION FINANCE

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ABSTRACT

Aging Population and Education Finance

Conventional wisdom suggests that an aging of population will increase political pressure to tilt the composition of social spending in favour of the elderly, while potentially sacrificing other publicly provided goods such as education. This view seems to be supported by recent empirical findings that per child public education spending tends to be lower in US jurisdictions with a higher fraction of elderly residents. Do these cross-sectional findings also carry the dynamic implication that longevity will lead over time to waning political support for funding of public education? This Paper challenges such an implication. We present a model that is consistent with the aforementioned cross-sectional regressions yet predicts an overall positive impact of increasing longevity on public education funding and economic growth.

JEL Classification: D99, H52, H73 and I22
Keywords: local public funding of education, overlapping generations and political equilibrium

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1. Introduction

The populations of Western democracies are rapidly aging. In the US, for example, the share of people over the age of 65 was 9.8% in 1970, 11.3% in 1980, 12.5% in 1990, and 12.8% in 2000; it is expected to increase to more than 22% in 2030 (see US Bureau of Census, 2002). Similarly, in OECD countries this share was 18% in 1990, projected to grow to around 31% respectively, according to the World Bank Development Report (2002) The situation is no different in developing countries; globally, in the past 50 years the average life expectancy has increased from 46 to 66 years (UN Population Fund, 2002).

It has been recognized that such a sharp increase in expected longevity in the near future may have profound policy effects. Attention has mostly focused on the need to reform the social security system, whose current arrangement, pay-as-you-go, has been viewed as unsustainable in view of the population aging – see OECD report (1998). In developed countries with the safety net extending "from cradle to death", aging of the population may also have adverse effect on economic growth. Moreover, there are concerns about political effects of aging: as the voting population becomes older, the political pressure to tilt the composition of social spending in favor of the elderly may increase. The voting models of Browning (1975) and Boadway and Wildasin (1989), have precisely this flavor. A by-product of this political pressure could be a decrease in the spending on the young, specifically, on their education. It stands to reason, however, that the elderly may not in fact object to education spending altogether, either because of altruistic concerns for the young, or because of considerations of property value capitalization (disregarded in the paper's model), or because such spending may be seen as the means to enhance productivity thus ensuring a higher return on savings for retirement, which is modeled here. In any case, the intensity of potential political pressures to reduce education spending as a result of demographic trends is a matter for a theoretical as well as an empirical exploration (see, e.g., Poterba, 1998).

Recent empirical research, e.g., Rubinfeld (1977) and Poterba (1997), has yielded some evidence of weakening political support for public education. In particular, Poterba (1997) argues based on a study of US states that the increasing fraction of the elderly population results in a decrease in education spending, after controlling for other basic demographic variables and fixed effects. Ladd and Murray (2001) and Harris, et al. (2001) qualify these conclusions, claiming that the use of states' data may have biased Poterba's results. With county data (as in the former paper) and school district data (as in Harris, et al.) these authors fail to uncover a significant effect of the elderly on public education support. Further, Ladd and Murray (2001) discern a pattern where the elderly tend to reside in communities with fewer children.

Casual observation also does not appear to suggest waning of support for public education. Consider the period 1970-90, which is covered by the above-referred empirical studies. In that period, while the share of individuals aged 65 and higher increased by almost 25%, the per-pupil spending in public elementary and secondary schools soared by 75%, from $4000 to $7000 (measured in 2000 dollars) in an almost linear fashion.[1] This amounts to an average growth rate in per-pupil spending of more than 2.8%, at the time when the average growth rate in GNP per capita was around 1.6%. However, Razin et al. (2002) present data on labor tax rates and overall social transfers in a selected group of rich countries, arguing that they are, if at all, negatively related to aging; they suggest that political forces may, indeed, work to reduce labor taxation and the overall amount of transfers in an aging economy.

In this paper, we undertake a close examination of the possible effects of aging on education spending. Specifically, stipulating that human capital promotes growth, we model the attitudes of the population of voters towards public spending on education and analyze how these attitudes, and their political balance, may be affected by increased longevity. To examine this

[1] See the Digest of Educational Statistics (2000). Note also that Fernandez and Rogerson (2001) who studied a panel data set of US states over 1950-1990 have found that the expenditure on public primary and secondary education outpaced the growth of personal income devoted to education in 1950-70, and grew at approximately equal rate in 1970-90.

issue we introduce in Section 2 a simple OLG model where the population consists in each period of working adults and retirees. These two groups form the population of active voters who decide on the amount of spending on education of children, which will enhance their productivity when the children reach a working age. The two groups of voters in our model have opposing preferences with regard to education spending. While the retirees would like to see it decreased to a minimum (since they will not get to enjoy the future benefits of a productivity increase), the working adults are interested in significant education spending in anticipation that this will ensure higher returns on their savings when they retire; moreover, an increase in their longevity increases the incentive of this group to support education spending.[2] Thus, aging has two contradictory effects: on the one hand, it increases the proportion of voters opposed to education spending, but on the other hand, it increases the willingness of supporting coalition of voters to spend on education. Where the actual amount of spending is politically determined, the equilibrium hinges both on the relative political power (size) of the two groups and on the strength of their respective willingness (or reluctance) to spend on education. The comparative dynamics of the equilibrium amounts of spending with respect to expected longevity depends on the balance of the two opposing effects. With our specification of the model, we find that the second effect (an increased incentive to ensure growth of future productivity) dominates, so that aging should, in fact, result in increasing education tax rates.

We first demonstrate this effect in Section 2 for the benchmark case of a single jurisdiction economy, where the entire population of voters determines unified consumption tax rates to finance education. The main analysis, undertaken in Section 3, considers a two-district economy where a fixed fraction of retirees migrate in each period from one of the jurisdictions to another. Education is locally financed through locally determined consumption taxes. The

[2] Our assumption that individuals rationally expect the aggregate future effect of public spending they vote on is crucial. Some results on the negative impact of aging on public programs and economic growth, such as in Holtz-Eakin et al. (2002) and Zhang et al. (2003), hinge on the alternative assumption that voters have no such foresight.

benefits from productivity, that is enhanced by education, are economy-wide since physical capital is perfectly mobile. Thus, the working residents of a district are unable to fully appropriate those benefits resulting from their spending on education. This free riding effect somewhat decreases their willingness to vote for education spending, relative to the single-jurisdiction case. Nevertheless, we show that increasing longevity still has a positive dynamic effect, i.e., it leads to higher funding of education in both districts, similarly to the single jurisdiction case. However we demonstrate that such funding is higher in the district with lower (due to migration) proportion of retirees, while the district with a larger share of the elderly will have a relatively lower education tax rate. Thus the cross-sectional effect of increasing population share of retirees is negative, consistent with the major findings in Poterba (1997).

The important lesson we learn from the model is that, while a cross-section analysis may well reveal the empirically obtained negative relationship between the fraction of the elderly and the spending on education, this does not necessarily imply that the same should hold true in a time series, i.e., that this fraction should decrease over time. In other words, while districts with a larger fraction of the elderly may be spending on education less than districts with a smaller fraction of the elderly, it may still be the case that increased longevity would increase the overall spending on education. Thus, public education may not be doomed in an aging economy. More generally, on the methodological side, this exercise illustrates the pitfalls of making long run, general equilibrium inferences from partial equilibrium sources of variation.

Another interesting lesson is that the method of education finance may have important implications for political incentives to support education in an aging economy. In particular, local financing creates free riding problems, thus diminishing the support for education among the working adults, whereas this issue does not exists when education is financed nationally. While local financing of education has been traditionally viewed as inequitable, our analysis reveals that it may also inhibit incentives for intergenerational transfers.

2. The benchmark model of a single jurisdiction economy

We consider an OLG economy where the economic lifespan of all individuals consists of two periods of equal lengths--a working period and retirement. Time t is measured in discrete units equal to the lengths of working (as well as retirement) period. We identify "generation t" as all individuals whose working period occurs at time t; they are assumed identical, born in period t-1 and, subject to survival, spend their retirement during t+1. Prior to entering the working period, i.e., during childhood, all individuals receive publicly provided education but do not make any economic decisions. During the working period, each individual inelastically supplies one unit of labor time and gives birth to one child. At the very end of the working period, everyone faces a lottery: dying immediately, or living through the entire retirement period. The probability of survival ρ is the same for all individuals. Since working population is constant, its measure can be normalized to one. Hence at any time t there is a unit measure of workers, a unit measure of children (students), and, by the law of large numbers, measure ρ of retirees. Thus, ρ has two interpretations here: it captures both the longevity of a given individual, and it is also directly proportional to the fraction of retirees alive in a given period. All individuals within a cohort are identical and are characterized by their human capital, $h(t)$.[3] Let w_t denote the wage rate per unit of human capital, then $w_t h(t)$ is an individual's labor income. All adults, working and retired, pay a consumption tax at the rate of η_t, used to fund education. The tax rate is determined in each period t via a political process to be described later. Individuals take the tax rates as given in their consumption-saving decisions.

Decisions of individual agents. Working adults of generation t individuals allocate their labor income between current consumption, $c^y(t)$, and saving, $s(t)$, according to the budget constraint

$$(1 + \eta_t)\, c^y(t) + s(t) = w_t\, h(t) \tag{1}$$

In order to void the model of the effects of accidental bequests, the issue tangential to the focus

[3] For brevity we omit individual-specific subscripts since the individuals within each generation are identical.

of this paper, we assume that agents make savings in the form of an actuarially fair annuity, purchased at a price $q > 1$ that covers transaction costs. Let R_{t+1} denote the gross rate of return on private savings at a time t. Then the retirement period budget constraint faced by a generation t individual is given by:

$$(1 + \eta_{t+1})\, c^o(t) = R_{t+1}\, s(t)/\rho q \tag{2}$$

where $c^o(t)$ is the consumption level of a retired individual of generation t. Lifetime preferences of each agent derive from his consumption levels during working life and retirement. Namely, a generation t individual maximizes the expected utility, assumed logarithmic (the significance of this simplification will be discussed later), subject to the budget constraints (1)-(2):

$$max\ U(c^y(t), c^o(t)) = ln(c^y(t)) + \rho\beta\, ln(c^o(t)) \tag{3}$$

where $0 < \beta < 1$ is the discount factor.

Education sector. Education in our model is financed by consumption taxes levied on all adults and provided uniformly to all children. In period t education is provided to individuals of generation t+1 and is funded through taxes paid at time t by generation t as well as by surviving retired members of generation t-1. For simplicity, the amount of human capital acquired is assumed linearly related to (hence measured by) per child spending:

$$h(t+1) = \eta_t\, [c^y(t) + \rho\, c^o(t\text{-}1)] \tag{4}$$

Production sector. The economy's production function is of the Cobb-Douglas type:

$$Y(t) = F(K(t), H(t)) = A(K(t))^{\delta}\, (H(t))^{1\text{-}\delta} \tag{5}$$

where $Y(t)$ is output in period t, $K(t)$ and $H(t)$ are aggregate inputs of physical and human capital, respectively, $A > 0$ is a given parameter, and $\delta \in (0, 1)$, physical capital income share.[4]

The dynamic competitive equilibrium in the economy is defined as a sequence of wage rates w_t, gross rates of interest R_t, individual decisions on consumption levels $c^y(t)$, $c^o(t)$ and

[4] Note that due to the normalization of population measure to one, the aggregate $H(t)$ equals the individual human capital level $h(t)$; likewise, aggregate savings $S(t)$ in period t are equal to individual savings $s(t)$.

saving $s(t)$, and the levels of human capital attained by children $h(t+1)$, such that, given the initial endowments $h(0)$, $K(0)$ and the sequence of tax rates η_t, $t = 0,1,\ldots,$

(i) the consumption and saving decisions are solutions of problem (1)-(3) for any t, where wage w_t, interest rate R_t, and the individual level of human capital $h(t)$ are given;

(ii) the levels of human capital are determined according to education sector equation (4), where consumption levels are given (as determined in (i));

(iii) the wage and interest rates are determined competitively by respective marginal products:

$$w_t = F_2(K(t), H(t)) \;; \qquad R_t = F_1(K(t), H(t))$$

where $H(t) = h(t)$.

(iv) capital markets clear at all times: aggregate input of physical capital $K(t)$ at time t equals to previous period's aggregate savings $S(t\text{-}1)$.

We'll first analyze the dynamic competitive equilibrium subject to a given sequence of tax rates and will then consider political determination of the tax rates. For given tax rates, the equilibrium values are as follows[5] (recall that working population size is normalized to one):

$$s(t) = \rho\beta(1-\delta)Y(t) \,/\, (1 + \rho\beta), \quad c^y(t) = (1-\delta)\, Y(t) / \,[(1 + \rho\beta)(1+\eta_t)],$$

$$c^o(t) = \delta Y(t+1) \,/\, \rho q\, (1+\eta_{t+1}), \quad c^o(t\text{-}1) = \delta Y(t) \,/\, \rho q\, (1+\eta_t), \quad \text{and} \qquad (6)$$

$$h(t+1) = \eta_t Y(t)[1\text{-}\delta + \delta q^{-1}\,(1+\rho\beta)] / \,[(1+\eta_t)(1+\rho\beta)]$$

Substituting these values in (5) we obtain:

$$Y(t+1) = AY(t)\; [\eta_t/(1+\eta_t)]^{1-\delta}\, [\rho\beta\,(1\text{-}\delta)]^{\delta}\, [1\text{-}\delta + \delta q^{-1}\,(1+\rho\beta)]^{1-\delta} \,/\, (1+\rho\beta) \qquad (7)$$

Note that according to (7) higher education tax rate η_t would translate, other things being equal, into higher growth rate.

Political equilibrium. According to the expressions in (6), the welfare level of a retiree in a

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[5] This follows directly from the first order condition for (1)-(3), $-1/(w_t h(t) - s(t)) + \rho\beta/s(t) = 0$, which implies that the optimal level of savings is $s(t) = \rho\beta\, w_t h(t)/\,(1 + \rho\beta)$; and from the competitive equilibrium conditions in the factor markets, $w_t = (1\text{-}\delta) A(S(t\text{-}1))^{\delta}(H(t))^{-\delta} = (1\text{-}\delta)\, Y(t)(H(t))^{-1}$ and $R_t = \delta A(S(t\text{-}1))^{\delta-1}(H(t))^{1\text{-}\delta} = \delta\, Y(t)S((t\text{-}1))^{-1}$

period t is given by

$$V^o(\text{t}) = ln\ (c^o(t\text{-}1)) = ln\ (\delta Y(t)/\rho q\ (1+\eta_t)) \qquad (8)$$

while the lifetime expected welfare of a contemporary working adult is:

$$V^y(\text{t}) = ln(c^y(t)) + \rho\beta\ ln(c^o(t)) = ln[(1\text{-}\delta)Y(t)/\rho q\ (1+\rho\beta)(1+\eta_t)] + \rho\beta\ ln[\delta Y(t+1)/\rho q\ (1+\eta_{t+1})] \quad (9)$$

Thus it is clear that, in any period t, the preferences for education financing of the working adults and the retirees differ. Recall that the consumption tax is used to finance education of future workers, so it has no effect on current production $Y(t)$. Therefore (8) implies that the retirees prefer not to finance education at all. In contrast, differentiating (9) with respect to η_t we obtain that the tax rate most preferred by a working adult is $\eta_t^y(\rho) = \rho\beta(1\text{-}\delta)$ which is obviously increasing in the longevity parameter ρ.[6] The intuition here is that, when working adults expect to live longer, they want to save more, and – since human and physical capital are complements – they desire a higher future level of the former so as to increase the return on the latter.

Because of the divergent policy preferences of the two politically active population groups, the workers and the retirees, policy choices are determined through a political process. Specifically, building on micro-founded political models such as in Lindbeck and Weibull, (1987) (see also Persson and Tabellini, 2000, ch. 3) we suppose that two political parties in a representative democracy choose a platform on issues affecting consumption of the voters. In addition, the parties' programs have another, ideological, dimension, unrelated to consumption welfare, where their positions are different, fixed, and are known and matter to voters. It is assumed that parties know the distribution of voters' preferences on fiscal policy but can only rely on estimates, such as poll data, of the overall distribution of the electorate's ideological biases. The parties commit themselves to platforms under uncertainty about the extent of the ideological bias. In such a model, the choice of platforms affects the (expected) vote shares, thus

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[6] Noting that the next-period tax rate η_{t+1} is taken as given here, differentiation reveals that $\partial V^y(t)\ /\ \partial\eta_t = (c^y(\text{t}))^{-1}$ $(\partial c^y(\text{t})/\ \partial\eta_t) + \rho\beta\ (c^o(\text{t}))^{-1}\ (\partial c^o(\text{t})/\ \partial\eta_t) = 0$. It can be easily verified that the resulting tax rate $\eta_t^y(\rho)$ is indeed optimal: $V^y(\text{t})$ increases for $\eta < \eta_t^y(\rho)$ and decreases for $\eta > \eta_t^y(\rho)$.

potentially altering the identity of a swing voter in each group. The implication of the political equilibrium analysis then is that, provided that the ideological positions are distributed independently of preferences toward consumption (applied to our purposes, this means that ideological biases have the same distribution among workers and retirees), the parties converge to platforms that maximize the aggregate consumption welfare of the electorate.[7]

An important, and arguably realistic, feature of this model of political process is that the influence of a group of voters depends on how much it stands to gain or lose (in marginal utility terms) from a change in government policy. This is a salient feature of voting models such as Lindbeck and Weibull (1987) (sometimes referred to as probabilistic), which distinguishes them from the median voter type of models. Indeed, straightforward application of a majority voting mechanism is problematic in an OLG framework, where the old are always in the minority. Their interests and increasing numbers (as long as they remain a minority) will therefore have no impact on political outcomes if age is the only determinant of political choices. Probabilistic voting models circumvent the problem by implicitly introducing another, statistically independent, dimension of voter characteristics (such as ideology). Similar in spirit are the models of voting in OLG framework (e.g., Razin et al., 2002) where the second independent characteristic of agents is introduced explicitly, so that the number of the old agents will affect the identity of a (young) median voter.

Given the sizes of the two constituent age groups in our model, the aggregate welfare at time t is defined by $V(t) = V^y(t) + \rho V^o(t)$; it follows then that the period t's tax rate chosen in political equilibrium is determined as

$$\eta_t(\rho) = argmax\ [ln(c^y(t)) + \rho\beta\ ln(c^o(\mathrm{t})) + \rho\ ln(c^o(t\text{-}1))] \qquad (10)$$

[7] Qualitatively similar outcome is produced by other models of political process.. Suppose, for example, that the old and the working adults are organized into two distinct interest groups which offer contributions to the parties in order to influence political outcomes. If the extent of the political influence of a lobby is proportionately related to the lobby's size then, at equilibrium, the outcome again maximizes the average voters' welfare (see Persson and Tabellini, Chs. 3, 7, and Dixit and Londregan, 1996). Thus, the precise specification of the nature of the political

where the current income $Y(t)$ and future tax rate η_{t+1} are taken as given while the consumption levels that enter this expression are functions of η_t defined by equations (6) i.e., they represent the agents' optimal choices in the dynamic competitive equilibrium framework defined above.

The role of our assumption of logarithmic preferences can now be appreciated. Besides its usual convenience in obtaining closed form solutions, it provides a more critical simplification in this framework. Namely, it leads to political equilibrium solutions, as in (10), that are not only unique, but also independent of future political outcomes, as can be seen in expression (9). This eliminates a strategic motive to influence the outcomes of future votes, hence they can be taken as given. While the issues of intertemporal strategy in political process and multiplicity of resulting equilibria are important, they go beyond the scope of this paper. In fact, our assumption that policy makers can ignore the effect of their choices on future political environment follows much of the literature in the field, see e.g., Verbon and Verhoeven (1992). Holtz-Eakin et al. (2002) and Zhang et al ((2003) achieve this in related OLG models by using logarithmic preferences while Razin et al (2002) assume linear technology which renders the form of preferences irrelevant in their framework.

We can now introduce *the dynamic political equilibrium* (DPE) in the economy. It is defined by a sequence of tax rates η_t, wage rates w_t, gross rates of interest R_t, individual decisions on consumption levels $c^y(t)$, $c^o(t)$ and saving $s(t)$, As well as the levels of human capital attained by the next generation $h(t+1)$, for $t = 0,1,\ldots$ such that (i) the sequence of economic variables w_t, R_t, $c^y(t)$, $c^o(t)$, $s(t)$, $h(t+1)$, $y(t+1)$ for $t = 0,1,\ldots$ constitutes a dynamic competitive equilibrium for given initial $h(0)$, $K(0)$, and the sequence of tax rates η_t ; and (ii) given the choices of $c^y(t)$, $c^o(t)$, $s(t)$, defined as functions of η_t, the DPE values of tax rates are optimal solutions of corresponding problem (10).

The first order condition for problem (10) is

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process may not be crucial for the basic results.

$$0 = \partial V(t)/\partial \eta_t = (c^y(\text{t}))^{-1} (\partial c^y(\text{t})/ \partial \eta_t) + \rho\beta (c^o(\text{t}))^{-1} (\partial c^o(\text{t})/ \partial \eta_t) + \rho (c^o(\text{t-1}))^{-1} (\partial c^o(\text{t-1})/ \partial \eta_t)$$

$$= - [1 + \rho + \rho\beta(1-\delta)]/(1+\eta_t) + \rho\beta(1-\delta)/\eta_t$$

which implies that the optimal solution is

$$\eta_t(\rho) \equiv \eta(\rho) = \rho\beta(1-\delta)/ (1 + \rho) \quad (11)$$

Indeed, $\partial V(t)/\partial \eta$ is positive when $\eta < \eta(\rho)$, and negative when $\eta > \eta(\rho)$. Thus $\eta_t(\rho)$ defined in (11) is the political equilibrium tax rate at time t, which happens to be stationary. Observe that $\eta_t(\rho)$ is an increasing function of longevity ρ, so that an increase in dependency ratio results in higher education tax rates. The reason for this is that, while old adults inelastically (regardless of expected longevity) prefer minimum education spending, working adults' preference for such spending increases with longevity, as explained before.

Moreover, the following result states that, under a reasonable simplifying restriction that the capital income share $\delta \geq 0.2$ (recent US estimates, see e.g., Laitner, 2000, put it at 0.33), the amount of education funding (aggregate and per child) will also rise.[8]

<u>Proposition 2.1</u>. *Assuming that $\delta \geq 0.2$, a marginal increase in the longevity parameter ρ at the start of period t_0 will cause an increase in the DPE levels of education funding $h(t)$ and output $Y(t)$ in all subsequent periods, $t \geq t_0+1$.*

It is important to note that the upward shock to longevity parameter assumed in the Proposition means that the number of retirees (and hence the dependency ratio) will increase from period t_0 on, but it also implies that the expected duration of future retirement will be higher for the working adults. These two factors obviously have mutually opposing effects on political equilibrium. Proposition 2.1 shows that, in our model, the interest of future retirees to ascertain higher future returns outweighs the stronger political representation of the current retirees (who would prefer to annul education taxes); as a result, not only the equilibrium tax rates increase,

[8] All proofs are relegated to the appendix.

but so do the absolute amounts of education funding per student. The intuition here is that, because of the complementarity between human and physical capital, the increase in education taxes because of increased longevity will result in larger output only when the marginal productivity of the latter is sufficiently high, which holds when δ is not too small.[9]

3. Local financing of education

3.1. The model

We will now modify the benchmark model to analyze the education funding system separated into districts which individually determine the level of funding and procure it. To keep the model as simple as possible, suppose that the economy consists of two districts, indexed i =1, 2. The overlapping generations structure of the population is the same as in the benchmark model, and so is the structure of agents' decisions. The only difference is that the political decisions concerning education funding are made separately in each district. Recall that the measure of total working population was normalized, so that it equals 1 at all times. Without loss of generality we assume that the districts are of equal size, i.e., the measure of working population in each district equals 0.5 in each period t, and so does the measure of the student population.

As before, we assume that each member of generation t faces, at the end of period t, the probability ρ of surviving for an additional period t+1, the retirement; thus the population of retirees has measure ρ. Since our goal is to account for cross-district variation of the age profile of the population, we now introduce the assumption that upon retirement some individuals migrate to a different community. For simplicity, we assume that migration only takes place from district 2 into district 1, and that a constant fraction Δ of district 2's residents move into district 1 when retired. Note that this migration pattern is assumed to be driven by exogenous differences between the districts (such as climate, amenities, etc.), not modeled here. We also

[9] Soares (2003) obtains a similar effect in a related setting but using majority voting to define political equilibrium. .

assume that each individual makes the retirement location decision in advance, contingent only on survival. Finally, we assume that capital is perfectly mobile across the districts.

Local government in each district finances education in period t via consumption tax on all residents of the district in this period, so that districts' tax revenues are affected by the interdistrict migration of retirees introduced above. To account for respective tax flows in the model we need to introduce variables $c_{ij}^{o}(t)$ which stand for the old-age consumption planned by an individual of generation t who works in district i and expects to retire in district j.[10]

With this notation, the individual budget constraints of an individual, who works in district i and contemplates to retire in district j, can be stated as follows:

$$(1 + \eta_{t,i})\, c_i^{y}(t) + s_i(t) = w_t h_i(t) \qquad \text{and} \qquad (1 + \eta_{t+1,j})\, c_{ij}^{o}(t) = R_{t+1}\, s_i(t) / \rho q \tag{12}$$

where $\eta_{t,j}$ denotes the education tax rate in district j politically determined in period t while possible combinations of indices $_{ij}$ in the second relationship in (12) are $_{11}$, $_{21}$, and $_{22}$.

The expected lifetime utility of this individual is given by

$$U(c_i^{y}(\text{t}), c_{ij}^{o}(\text{t})) = ln(c_i^{y}(\text{t})) + \rho\beta\, ln(c_{ij}^{o}(\text{t})) \tag{13}$$

The per child amounts of education spending in the two districts are equal to respective tax revenues, and hence are expressed as follows:

$$h_1(t+1) = \eta_{t,1}\, [c_1^{y}(\text{t}) + \rho\, c_{11}^{o}(\text{t-1}) + \Delta\rho\, c_{21}^{o}(\text{t-1})]$$

$$h_2(\text{t+1}) = \eta_{t,2}\, [c_2^{y}(\text{t}) + (1-\Delta)\rho\, c_{22}^{o}(\text{t-1}))$$

The economy's production sector and the structure of dynamic competitive equilibrium are as defined in Section 2, therefore in equilibrium

$$Y(t) = A\, (S(\text{t-1}))^{\delta}\, (H(\text{t}))^{1-\delta}$$

where $S(t\text{-}1)$ and $H(t)$ are the economy's aggregate levels of private savings and education, respectively, in period t, i.e.:

[10] Note that, as it will be seen later, the levels of young-age consumption and saving are not affected by retirement location under our model specifications.

$$S(t\text{-}1) = [s_1(t\text{-}1) + s_2(t\text{-}1)]/2$$

$$H(t) = [h_1(t) + h_2(t)]/2 \tag{14}$$

using the assumption that the population mass of working adults in each district equals 0.5.

The period t tax rates are determined in each district by its government, which seeks to maximize the aggregate expected utility of the voters residing in the district in period t, similar to the government policy of the single district economy of the previous section. Note, however, the strategic interdependence between the respective policy choices in the two districts: the level of education funding chosen in a district affects next period's aggregate productivity and hence the rate of return on savings throughout the economy. We assume that each district government views tax rates chosen in the other district as given, and consider the Nash equilibrium outcomes of the tax rate determination. We define, therefore, the equilibrium of the tax-setting game between the two districts in period t, as a pair of tax rates $\eta_{t,1}(\rho)$ and $\eta_{t,2}(\rho)$, such that

(i) they form within-district political equilibria, i.e.,

$$\eta_{t,1}(\rho) = argmax\{V_1(t) = [ln(c_1^y(t)) + \rho\beta\, ln(c_{11}^o(t)) + \rho\, ln(c_{11}^o(t\text{-}1)) + \Delta\rho\, ln(c_{21}^o(t\text{-}1))]/2\} \tag{15}$$

$$\eta_{t,2}(\rho) = argmax\{V_2(t) = [ln(c_2^y(t)) + (1\text{-}\Delta)\rho\beta\, ln(c_{22}^o(t)) + \Delta\rho\beta\, ln(c_{21}^o(t)) + + (1\text{-}\Delta)\rho\, ln(c_{22}^o(t\text{-}1))]/2\} \tag{16}$$

where in each of the problems the tax rate chosen in the other district is taken as given;

(ii) they constitute a Nash equilibrium vis-à-vis each other.

With this additional element of interdistrict strategic interaction, the definition of the *dynamic political equilibrium (DPE) in the multi-district economy* is otherwise identical to the one introduced in Section 2. Thus the sequence of events in every period forming the DPE in the multi-district economy is as follows. Each district determines the education tax rate through the political mechanism based on the interests of its welfare maximizing constituents, as defined in (15)-(16). Then the individuals make their consumption-investment decisions, based in part on the tax rates set in districts; these decisions determine the factor prices through competitive market mechanism. The intertemporal equilibrium consists of sequences of all such decisions over time, so that in particular, no district government is interested to unilaterally change its tax rate, the factor prices are competitively determined, and the individual consumption decisions are optimal. An important assumption underlying this definition of equilibrium is that current district governments (and hence the constituent groups they represent) take the tax rates set by future governments as given. In other words, although future tax rates, in equilibrium, are history dependent, current governments do not consider the possibility of manipulating the decisions of future governments.

3.2. Equilibrium analysis

Let $\{\eta_{t,1}, \eta_{t,2}\}$ be a sequence of education tax rates in the districts for $t = 0,1,\ldots$. Taking them, as well as wage rates w_t and gross rates of interest R_{t+1} as given, working adults of generation t residing in district i maximize their utility (13) subject to constraints (12). Similarly to the analysis in the previous section, the first order conditions determine the levels of savings as

$$s_i(t) = \rho\beta\, w_t h_i(t)/\,(1+\rho\beta)$$

Applying the factor markets equilibrium conditions we then obtain:

$$s_i(t) = [\rho\beta(1-\delta)Y(t)\,/\,(1+\rho\beta)]\; h_i(t)/H(t)$$

$$c_i^y(t) = \{(1-\delta)\,Y(t)/\,[(1+\rho\beta)(1+\eta_{t,i})]\}\,h_i(t)/H(t)$$

and since $s_i(t)/S(t) = h_i(t)/H(t)$, we write :

$$c_{ij}^{o}(t) = [\delta Y(t+1)/\rho q\;(1+\eta_{t+1,j})]s_i(t)/S(t) = [\delta Y(t+1)/\rho q\;(1+\eta_{t+1,j})]h_i(t)/H(t),$$

$$c_{ij}^{o}(t\text{-}1) = [\delta Y(t)/\rho q\;(1+\eta_{t,j})]s_i(t\text{-}1)/S(t\text{-}1) = [\delta Y(t)/\rho q\;(1+\eta_{t,j})]h_i(t\text{-}1)/H(t\text{-}1)$$

This, in turn, implies the following relationships determining per child spending on education in each district $i = 1,2$ in period t:

$$h_i(t+1) = B_i(t)Y(t)\eta_{t,i}/(1+\eta_{t,i}) \tag{17}$$

where $B_1(t) = (1-\delta)h_1(t)/(H(t)(1+\rho\beta)) + \delta q^{-1}[h_1(t\text{-}1) + \Delta h_2(t\text{-}1)]/H(t\text{-}1)$ and

$B_2(t) = (1-\delta)h_2(t)/(H(t)(1+\rho\beta)) + (1-\Delta)\delta q^{-1} h_2(t\text{-}1)/H(t\text{-}1)$.

Therefore, according to (14), $H(t+1) = Y(t)D(t)/2$, which leads to the expression for the aggregate output dynamics:

$$Y(t+1) = 2^{\delta-1} A Y(t)[(1\text{-}\delta)\rho\beta/(1 + \rho\beta)]^{\delta} [D(t)]^{1-\delta}$$

where $D(t) = \eta_{t,1} B_1(t)/(1+\eta_{t,1}) + \eta_{t,2} B_2(t)/(1+\eta_{t,2})$.

Further analysis, details of which are presented in the appendix, leads to the following characterization of the equilibrium:

Proposition 3.1. *For any values of parameters $\rho \in [0, 1]$ (longevity) and $\Delta \in [0, 1]$ (the rate of cross-district migration of retirees), the two-district model possesses unique stationary DPE characterized by constant district tax rates $\eta_1(\rho, \Delta)$, $\eta_2(\rho, \Delta)$ and constant cross-district ratio of education funding levels. When $\Delta = 0$, the tax rates in the districts are identical:*

$$\eta_1(\rho, 0) = \eta_2(\rho, 0) = (1\text{-}\delta)\rho\beta[2(1+\rho)]^{-1}$$

The levels and growth rates of education funding in districts 1 and 2 are also equal at all times.[11]

3.3. Comparative dynamics

We shall now study the effects of longevity, ρ, and of the parameter of cross-district distribution of the retirees, Δ, on the dynamic political equilibrium (DPE). We'll limit the analysis to the case when the value of Δ is small. This will prove to be sufficient for the paper's purposes, which is to demonstrate the possibility of divergence between cross-district and dynamic effects of an increasing population share of retirees. We next show this in two sets of results. First, we consider the effects of the marginal shocks to the values of parameters Δ and ρ on the stationary

[11] Note that in case $\Delta = 0$, i.e., when there is no migration, the above result is readily comparable to the results for the benchmark single jurisdiction model where the equilibrium tax rate is, according to expression (11), $\eta(\rho) = (1\text{-}\delta)\rho\beta(1+\rho)^{-1}$. The comparison implies that, in our model, decentralization of education funding leads to lower equilibrium tax rates. This also translates into relatively lower levels and growth rates of education funding under the decentralized funding system.

tax rates in a stationary DPE and then address the same questions in a (non-stationary, or transitional) comparative dynamics framework, evaluating the direct effects on tax rates of one-time marginal shocks to the values of Δ and ρ .

Consider a stationary DPE that, according to Proposition 3.1, is uniquely defined by the values of the parameters ρ and Δ and is characterized by intertemporally constant tax rates $\eta_1(\rho, \Delta)$, $\eta_2(\rho, \Delta)$ and constant cross-district ratio of education funding levels. First, we analyze the effect of an increase in the longevity parameter ρ obtaining the following

Proposition 3.2. *Compare stationary DPEs corresponding to different values of longevity* ρ*. The DPE corresponding to a higher* ρ *is characterized by relatively higher rates of education taxes in all districts. Moreover, given the same level of aggregate income Y(t) in a period t, the DPE corresponding to higher longevity allocates relatively larger funds toward education in period t.*

Recall that $\eta_1 = \eta_2$ when $\Delta=0$. The following result characterizes the effect of a marginal increase in the value of Δ., i.e., the divergence between equilibrium tax rates of the districts that differ in the share of retirees.

Proposition 3.3. *Assume that* Δ*, the net rate of migration of retirees from district 2 to district 1, is small but positive. Then for any* $\rho \in (0, 1]$*, the stationary DPE education tax rate is lower in the district with a relatively larger share of retirees,* $\eta_1(\rho, \Delta) < \eta_2(\rho, \Delta)$.[12]

We now consider a comparative dynamics experiment with respect to Δ and ρ for a non-stationary DPE supposing that longevity ρ marginally increases at the start of period τ. The following result characterizes the direct effect on the DPE variables within period τ.

[12] The effect on the relation between the absolute levels of education funding in the districts is not clear cut, since while higher number of retirees in district 1 drives tax rates down, it also increases the tax base. However, under realistic restrictions $\beta < 0.5$ (Holtz-Eakin et al., 2002 argue that $\beta = 0.308$ is an appropriate parametrization) and $\delta \geq 0.25$ it is possible to show that DPE level of education funding in district 1 indeed falls relative to district 2 as a result of increasing Δ.

Proposition 3.4. *A positive shock to the aggregate longevity ρ at the start of period* τ , *while Δ stays constant and small, results in higher education tax rates in both districts of the two-district economy in period* τ.

Consider now, similarly to the above experiment the effect of a change in Δ . Suppose that Δ, while equal to 0 for $t \leq \tau - 1$, marginally increases at the start of period τ, and consider the adjustment in DPE pertaining to this period only. Under an additional restriction on the model's parameters we obtain the following result.

Proposition 3.5. *Assume that the following condition is satisfied:* $\delta(1-\rho^2\exists) < (1-\delta)\rho q$.[13] *Then the education tax rate $\eta_{\tau,1}$ chosen in district 1, which has a relatively larger share of retirees, is lower than the tax rate $\eta_{\tau,2}$ in district 2 where population of retirees is relatively smaller.*

Compare the results of this section with those obtained in Section 2 for the model of a single jurisdiction economy. In both cases, the elderly prefer to eliminate any spending on education; and in both cases the working adults support positive levels of education spending so as to increase productivity of future workers. In the two-district economy, however, these productivity gains will not be fully appropriated by a district's residents. Indeed, the resulting gains in returns on savings will spill over to benefit residents of the other district. Taking this into account, the working adults in each district are less eager to vote on education spending than they were in the single jurisdiction economy of Section 2. Therefore, as shown by Proposition 3.1, the stationary equilibrium education tax rates are lower, *ceteris paribus*, in a two-district economy than in a single jurisdiction case. Nevertheless, Propositions 3.2 and 3.4 demonstrate that increasing longevity (hence, increasing share of retirees in the population) has a positive aggregate dynamic effect in a two-district economy thus reinforcing the Proposition 2.1, where

[13] This condition is consistent with standard calibrations used in the literaturec: using the earlier cited estimates $\beta = 0.308$, $\delta = 0.33$, it requires that $\rho > 0.46$. Note that given the demographic confines of OLG models the

the same was shown to hold in a single-district case. This is contrasted by the fact, proved by Propositions 3.3 and 3.5, that cross-sectional effect of a higher proportion of retirees on education funding is negative. This demonstration of a possible divergence of dynamic and cross-sectional effects of increasing population shares of retirees on education funding accomplishes the goal of this paper.

Finally, we note that while the analysis throughout the paper assumes an exogenous shock to longevity, its main insights should hold true under an endogenous secular increase in longevity. Thus, suppose, in line with the empirically observed regularity, that next-period longevity is an increasing function of current output. It appears likely that this in fact should only reinforce our above results.

5. Conclusion

Demographic transition characterized by increasing longevity is expected to have a great deal of political and economic implications. In particular, it has been suggested that the political support for public education will wane as a result of a growing political influence of the elderly. Indeed, some recent empirical work seems to detect an inverse relationship between the fraction of the elderly and the amount of spending on public education across the states and the communities in the United States. Nonetheless, casual empirical evidence does not suggest a reduction in public financing of education over time, neither in the US nor in the OECD countries.

This paper is an attempt to reconcile the above evidence within a simple theoretical framework. In the presented OLG economy, education financing is determined by governments representing the interests of the currently living young and old agents. While the latter may have no stake in education, the younger adults realize their collective interest in future productivity of the economy (via returns to savings) and hence are supportive of education. An increase in longevity affects the political equilibrium in two ways. First, there is a negative effect, because

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benchmark values used for ρ are above 0.5 for developed countries. See, e.g., Holtz-Eakin et al. (2002) and Razin of an increase in the fraction of the elderly. This is countervailed, however, by the increase in the support for education funding among younger adults who face the prospect of longer retirement. For a single jurisdiction economy we have shown that the second effect dominates, so that an increase in longevity has a positive overall effect on education funding. Moreover, this aggregate result is sustained in a multi-district economy where education is financed locally. We show, however, that this result can well co-exist with the opposite cross-sectional relation: the jurisdiction characterized by higher fraction of the elderly provides relatively lower education funding. In sum, perhaps with a self-serving bias, we present a more optimistic assessment of the consequences of aging for public education than featured in the earlier studies.

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Appendix

Proof of Proposition 2.1. First consider the equilibrium growth rate

$$g(t) = Y(t+1)/Y(t) = A[\eta(\rho)/(1+\eta(\rho))]^{1-\delta}\,[\rho\beta\,(1\text{-}\delta)]^{\delta}\,[(1\text{-}\delta) + \delta\,\mathrm{q}^{-1}\,(1+\rho\beta)]^{1-\delta}\,/\,(1+\rho\beta)$$

Straightforward analysis shows that $\partial ln\ g(t)/\partial\rho > 0$ for any t if $\delta \geq 0.2$. This proves the part of the Proposition concerning output levels. Next consider the expression in (6) for the level of human capital/education funding $h(t+1)$. Let $\eta_t = \eta(\rho)$ and observe that $\partial h(t+1)/\partial\rho > 0$ for $\delta =$ 0.2, when $Y(t)$ is considered given. It is also straightforward to show that $\partial^2 h(\mathrm{t}+1)/\partial\rho\,\partial\delta > 0$ whenever $\delta > 0$. Combined with the established fact that output levels $Y(t)$ grow for all $t \geq t_0+1$, this proves the remaining part of the Proposition concerning the levels of human capital, whenever $\delta \geq 0.2$. ■

Details of proof of Proposition 3.1.

Using the relationships in the text, we obtain the following expressions for aggregate welfare functions for the districts (recalling that the measure of working adult population equals 0.5 in each district):

$$\mathrm{V}_1(\mathrm{t}) = [-ln(1+\eta_{t,1}) + \rho\beta\, lnY(t+1) - \rho\, ln(1+\eta_{t,1}) - \Delta\rho\, ln(1+\eta_{t,1})]/2 + \text{terms independent of } \eta_{t,1}$$

$$= [-(1+\rho+\Delta\rho)\, ln(1+\eta_{t,1}) + (1\text{-}\delta)\rho\beta\, lnD(t))/2 + \text{terms independent of } \eta_{t,1} \qquad \text{(A1)}$$

$$\mathrm{V}_2(\mathrm{t}) = [-ln(1+\eta_{t,2}) + \rho\beta\, lnY(t+1) - (1\text{-}\Delta)\rho\, ln(1+\eta_{t,2}))/2 + \text{terms independent of } \eta_{t,2}$$

$$= [-(1+\rho-\Delta\rho)\, ln(1+\eta_{t,2}) + (1\text{-}\delta)\rho\beta\, lnD(t))/2 + \text{terms independent of } \eta_{t,2} \qquad \text{(A2)}$$

Differentiating welfare functions $V_1(t)$ and $V_2(t)$ with respect to $\eta_{t,1}$ and $\eta_{t,2}$, respectively, and bearing in mind that expressions $B_1(t)$ and $B_2(t)$ defined in in the text do not depend on either $\eta_{t,1}$ or $\eta_{t,2}$, we obtain the first order condition for problem (15):

$$(1+\rho+\Delta\rho)D(t) = (1-\delta)\rho\beta\, B_1(t)/\,(1+\eta_{t,1}) \tag{A3}$$

and the first order condition for problem (16):

$$(1+\rho-\Delta\rho)D(t) = (1-\delta)\rho\beta\, B_2(t)/\,(1+\eta_{t,2}) \tag{A4}$$

Straightforward analysis shows that $\partial^2 V_1(t)/\partial(\eta_{t,1})^2 < 0$ at the point where the first order condition (A3) holds. Therefore the tax rate $\eta_{t,1}$ satisfying (A3) is indeed the optimal solution of the district 1 tax rate determination problem (15). Similarly, the tax rate $\eta_{t,2}$ satisfying (A4) is the optimal solution of problem (16).

Together, equations (A3) and (A4) imply that

$$(1+\rho+\Delta\rho)\eta_{t,1} + (1+\rho-\Delta\rho)\eta_{t,2} = (1-\delta)\rho\beta \tag{A5}$$

and

$$(1+\eta_{t,1})\,(1+\rho+\Delta\rho)/\,B_1(t) = (1+\eta_{t,2})\,(1+\rho-\Delta\rho)/\,B_2(t) \tag{A6}$$

Thus, the equilibrium tax rates $\eta_{t,i}$, i= 1,2, in the two-district tax-setting game in period t are jointly determined by (A5)-(A6), the solution of which is:

$$\eta_{t,1}(\rho, \Delta) = (1+\rho+\Delta\rho)^{-1}\,(1 + B_2(t)/\,B_1(t))^{-1}\,[(1-\delta)\rho\beta + 1+\rho-\Delta\rho - (1+\rho+\Delta\rho)B_2(t)/\,B_1(t)] \tag{A7}$$

$$\eta_{t,2}(\rho, \Delta) = (1+\rho-\Delta\rho)^{-1}\,(1 + B_1(t)/\,B_2(t))^{-1}\,[(1-\delta)\rho\beta + 1+\rho+\Delta\rho - (1+\rho-\Delta\rho)B_1(t)/\,B_2(t)] \tag{A8}$$

According to their definitions (see the relationships (17)), the expressions $B_i(t)$, i= 1,2, depend on human capital levels attained by individuals in generations t and t-1, hence on the tax rates chosen in periods t-1 and t-2. Therefore, the relationships (A7)-(A8) constitute, in general, a system of second-order difference equations with respect to the tax rates. The sequence of tax rates $\eta_{t,i}$, i= 1,2 solving this dynamical system determines the DPE in the two-district economy.

Observe that when there is no interdistrict migration of retirees, i.e., if $\Delta = 0$ at all times, then $B_1(t) = B_2(t) = (1-\delta)(1+\rho\beta)^{-1} + q^{-1}\delta$. Therefore the system (A7)-(A8) is reduced in this case to the *stationary equilibrium sequence*:

$$\eta_{t,1}(\rho, 0) = \eta_{t,2}(\rho, 0) \equiv (1-\delta)\rho\beta[2(1+\rho)]^{-1} \tag{A9}$$

Consider now $z_{t+1} = h_1(t+1)/h_2(t+1)$, the ratio between per child levels of education spending in the districts. Relationships (A5), and (A6) yield the following expression for this ratio:

$$z_{t+1} = \eta_{t,1}\,(1+\rho+\Delta\rho)/[\eta_{t,2}\,(1+\rho-\Delta\rho)] =$$

$$= \eta_{t,1}\,(1+\rho+\Delta\rho)/[(1-\delta)\rho\beta - \eta_{t,1}\,(1+\rho+\Delta\rho)] \qquad \text{(A10)}$$

This implies

$$\eta_{t,1} = z_{t+1}\,(1-\delta)\rho\beta\ /\ [(1+z_{t+1})(1+\rho+\Delta\rho)],\ \ \eta_{t,2} = (1-\delta)\rho\beta\ /\ [(1+z_{t+1})(1+\rho-\Delta\rho)] \qquad \text{(A11)}$$

It also follows from the above that

$$h_1(t+1)/\,H(t+1) = 2\eta_{t,1}\,(1+\rho+\Delta\rho)/(1-\delta)\rho\beta\,,\ \ h_2(t+1)/\,H(t+1) = 2\eta_{t,2}\,(1+\rho-\Delta\rho)/(1-\delta)\rho\beta \qquad \text{(A12)}$$

We can now use these relationships to explore the existence of a stationary equilibrium sequence of tax rates when $\Delta > 0$, i.e., the existence of a stationary DPE, such that

$$\eta_{t,1}\,(\rho, \Delta) \equiv \eta_1\ ,\quad \eta_{t,2}\,(\rho, \Delta) \equiv \eta_2 \qquad \text{(A13)}$$

for some positive numbers η_1, η_2.

Assume that the stationary DPE exists for a given $\Delta > 0$. Then relationships (A10), (A12) show that the ratios $h_1(t)/h_2(t)$ and $h_1(t)/H(t)$ also should be constant over time, and so should the values $B_1(t)$, $B_2(t)$, $D(t)$. Substitutions reveal that the system of second order difference equations (A5)-(A6) indeed possesses unique stationary solution, specifically, the system can be reduced to an equation for η_1:

$$\{2(1+\rho)\,[(1-\delta)/(1+\rho\beta\,)+q^{-1}\delta] - \Delta q^{-1}\delta[2+2\rho+\rho\beta(1-\delta)]\}\,\eta_1 =$$

$$\rho\beta(1-\delta)\{[(1-\delta)/(1+\rho\beta\,)+q^{-1}\delta] - \Delta q^{-1}\delta\,[2+2\rho+\rho\beta(1-\delta)]/(1+\rho+\Delta\rho)\} \qquad \text{(A14)}$$

whereas η_2 is uniquely defined by equation (A5), given η_1. Since (A14) defines η_1 uniquely, this proves the existence and uniqueness of stationary DPE. ■

<u>Proof of Proposition 3.2</u>.

We now differentiate the expressions (A14) and (A5) with respect to the longevity parameter ρ. Differentiation (A14) at $\Delta = 0$ yields

$$\partial\eta_1(\rho, \Delta)/\partial\rho = \beta(1-\delta)/\ 2(1+\rho)^2 \qquad \text{(A15)}$$

This along with (A5) implies that $\partial\eta_i(\rho, \Delta)/\partial\rho > 0$ for i=1,2 and any value of $\rho \in [0, 1]$ when Δ is close to 0.

Recall now the composition of aggregate funding of education in period t in a stationary DPE,

$$\mathrm{H(t+1)} = 0.5\mathrm{Y(t)}[\eta_{t,1}B_1(t)/(1+\eta_{t,1}) + \eta_{t,2}B_2(t)/(1+\eta_{t,2})]$$

Applying first relationship (A6) and then (A5) to the above equality, we obtain:

$$\mathrm{H(t+1)} = 0.5\mathrm{Y(t)}\beta(1-\delta)\rho B_1(t)/(1+\rho+\Delta\rho) \qquad \text{(A16)}$$

Further, since the ratios $h_i(t)/H(t)$ are constant in a stationary DPE and are characterized by expressions (A12) where $\eta_{t,1} \equiv \eta_1(\rho, \Delta)$, $\eta_{t,2} \equiv \eta_2(\rho, \Delta)$, substituting them in equation defining $B_1(t)$ we can rewrite (A16) as

$$\mathrm{H(t+1)} = \mathrm{Y(t)}[(1-\delta)(1+\rho\beta)^{-1}\eta_1 + \delta\rho\eta_1 + \Delta\delta\rho\eta_2] \qquad \text{(A17)}$$

We then calculate $\partial\mathrm{H(t+1)}/\partial\rho$ for $\Delta = 0$ using (A19) as well as the expression (A15) for $\partial\eta_1(\rho, \Delta)/\partial\rho$ and show that $\partial\mathrm{H(t+1)}/\partial\rho$ is positive for any value of $\rho \in [0, 1]$ when $\Delta = 0$, hence by continuity it is also positive when Δ is sufficiently close to 0. ■

Proof of Proposition 3.3.

Recall that the stationary equilibrium tax rates $\eta_1(\rho, \Delta)$, $\eta_2(\rho, \Delta)$ are determined by equation (A14) along with (A5). Differentiation of (A14) at Δ=0 yields

$$\partial\eta_1(\rho, 0)/\partial\Delta = -\Delta \mathrm{q}^{-1}\delta[2+2\rho+\rho\beta(1-\delta)]\rho\beta(1-\delta)/\{4(1+\rho)^2[(1-\delta)/(1+\rho\beta)+\mathrm{q}^{-1}\delta]\}$$

It follows that $\partial\eta_1(\rho, \Delta)/\partial\Delta < 0$ for any value of $\rho \in [0, 1]$ whenever Δ is close to 0. Differentiate now both sides of equation (A5) with respect to Δ at $\Delta = 0$ to obtain (after substituting $\Delta = 0$):

$$\partial\eta_{t,1} / \partial\Delta + \partial\eta_{t,1} / \partial\Delta = 0 \qquad \text{(A18)}$$

Combined with the fact $\partial\eta_1(\rho, \Delta)/\partial\Delta < 0$, this implies that $\partial\eta_2(\rho, \Delta)/\partial\Delta > 0$. To complete the proof, recall again that according to Proposition 3.1 $\eta_1 = \eta_2$ when $\Delta = 0$. ■

Proof of Proposition 3.4.

We'll differentiate the tax rates with respect to ρ, assuming that Δ is small, using their expressions in (A7) and (A8). Using the fact that $\partial[B_1(t)/B_2(t)]/\partial\rho = 0$ when $\Delta = 0$, it is easy to obtain the expressions for the derivatives, subject to $\Delta = 0$:

$\partial\eta_{\tau,1} / \partial\rho = [1 + B_1(t)/B_2(t)]^{-1} (1+\rho)^{-1} (1-\delta) [1 - \rho(1+\rho)]$

$\partial\eta_{\tau,2} / \partial\rho = [1 + B_2(t)/B_1(t)]^{-1} (1+\rho)^{-1} (1-\delta) [1 - \rho(1+\rho)]$

Since both expressions are positive, and due to continuity of derivatives, the Proposition's claim is valid for sufficiently small values of Δ . ■

Proof of Proposition 3.5.

Rewrite relationship (A6) as

$$(1 + \eta_{t,1})/ (1 + \eta_{t,2}) = [(1+\rho-\Delta\rho)/(1+\rho+\Delta\rho)]\, B_1(t)/ B_2(t)$$

and differentiate this expression with respect to Δ in a neighborhood of 0. Using the definitions of $B_i(t)$, it is straightforward to show that at Δ=0 this derivative has the same sign as the expression $\delta(1-\rho^2\exists) - (1-\delta)\rho q$. Since q>1 by its definition, the condition on parameters stated in the Proposition ascertains that the expression is negative. By continuity, the value of the derivative remains negative for any Δ sufficiently close to 0. Thus, when the rate of migration Δ marginally rises then in the new DPE taxes in period τ will be relatively lower in district 1, where the proportion of retirees is higher, than in district 2. Now recall the relationship (A18):

$$\partial \eta_{t,1} / \partial \Delta \; + \; \partial \eta_{t,1} / \partial \Delta = 0$$

Combined with the fact $\eta_1(\rho, 0) = \eta_2(\rho, 0)$ in Proposition 3.1 and with the fact stated above that $\eta_{\tau,1}$ decreases relative to $\eta_{\tau,2}$, this yields the stated result. ■