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ABSTRACT

Optimal State-Contingent Regulation under Limited Liability

We consider an optimal regulation model in which the regulated firm's production cost is subject to random, publicly-observable shocks. The distribution of these shocks is correlated with the firm's cost type, which is private information. The regulator designs an incentive-compatible regulatory scheme that adjusts itself automatically *ex post* given the realization of the cost shock. We derive the optimal scheme, assuming that there is an upper bound on the financial losses that the firm can sustain in any given state. We first consider a two-types, two-states case, and then extend the results to the case of a continuum of firm types and an arbitrary finite number of states. We show that the first best allocation can be implemented if the state of nature conveys enough information about the firm's type and (or) the maximal loss that the firm can sustain is sufficiently large. Otherwise, the solution is characterized by classical second-best features.

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1 Introduction

The optimal regulation literature has seen major developments in the recent twenty years. This literature has mainly focused on the issue of how to regulate a firm when it has private information about its demand and cost functions. Most of this literature considers static models: the firm is assumed to face a given demand and cost functions and the question then is how to induce it to truthfully report its private information in order to enable the regulator to maximize a weighted average of consumers' surplus and firm's profits. In practice however, regulated rates are typically set for an extended period of time, typically few years. During this period, the demand and cost conditions may be subject to random shocks. Therefore, it is important to design flexible regulatory mechanisms that can respond to these shocks. This is evident from the California electricity crisis in 2000/1, where fixed regulated retail prices completely insulated consumers from fluctuations of prices in the deregulated wholesale market. These fixed retail prices effectively drove California's two largest utilities - Pacific Gas & Electric and Southern California Edison - to the point of insolvency after wholesale prices rose by 500% between the second half of 1999 and the second half of 2000.[1]

The present analysis differs from classical optimal regulation theory (e.g., Baron and Myerson, 1982, Laffont and Tirole, 1986, and Lewis and Sappington, 1988) because the firm's cost is subject to publicly observable random shocks that are correlated with the firm's type which is private information for the firm. For instance, the cost shocks could be fluctuations of the observable cost components of the firm; when the firm has high observable costs its unobserved cost components, which represent its type, are also likely to be high. Or, the shocks could be costly equipment failures: when the firm experiences a large number of such failures, its technology (i.e., its type) is likely to be inefficient. The regulator designs an incentive compatible regulatory scheme that adjusts itself automatically ex post given the realization of the publicly observed shock, before the firm produces. This regulatory scheme can be thought of as an "indexed," or state-contingent incentive scheme: the regulator does not have to redesign it after the realization of each cost shock.

[1] For details, see Joskow (2001).

Another important element of our model are limited liability constraints: there is an upper bound on the losses that the firm can sustain in any given state of nature (each corresponds to a specific cost shock). We find that whenever the realization of the random cots shocks conveys enough information about the firm's type and (or) the maximal deficit that the firm can sustain in any given state is sufficiently large, the regulator can design a scheme that implements the first-best allocation despite the fact that the firm's type, which determines the distribution of its costs in the various states of nature, is private information. The regulatory scheme that implements the first-best under the most stringent limited deficit constraints "rewards" the firm in exactly one state of nature and imposes the same "punishment" on the firm in all other states. When the conditions ensuring that the first-best allocation can be implemented fail, we find that the solution is characterized by classical second-best features, i.e., the production levels of inefficient types are distorted downwards to reduce the expected cost of informational rents, there is no distortion "at the top," and there is no (expected) rent "at the bottom." We obtain these results first in a simple, two-types, two-states case, and then extend them to the case of a continuum of firm types and an arbitrarily large, but finite, number of states of nature.

The idea that regulators can exploit the correlation between ex post public signals and the firm's type and design signal-dependent transfers that implement the first-best allocation was first explored by Riordan and Sappington (1988). Their results are analogous to those of Crémer and McLean (1985, 1988) in the context of auction theory - the main difference being that in the context of auctions, the reports of other bidders play the role that the ex post public signals play in the Riordan and Sappington model. Our paper differs from Riordan and Sappington (1988) in that in their model, the ex post signals are purely informational whereas in our paper, they are real cost shocks that affect not only the firm's transfers but also the firm's cost and hence its output. Another important difference is that Riordan and Sappington's methodology allows them to prove that the first-best solution can be implemented but does not provide a characterization of the optimal regulatory scheme. In contrast, we fully characterize the optimal regulatory scheme both when the first-best solution can be implemented as well as when it cannot be implemented.

It has long been recognized that limited liability constraints are important to assess the robustness of the first-best implementation results of Crémer and McLean (1985, 1988) and Riordan and Sappington (1988). Robert (1991) considers an auction problem in which each bidder can have finitely many possible types but the types of different bidders are correlated. He shows that if there are upper bounds on the payments that the bidders can make, then the auctioneer may not be able extract the full surplus from each bidder as in Crémer and McLean (1988). Kosmopoulou and Williams (1998) consider a related model of group decision-making although in their model, there is a continuum of agents' types. They show that it is impossible to implement the first-best allocation if agents' types are approximately independent and either the monetary transfers among agents or their ex post payoffs are subject to limited liability constraints. In this paper we therefore incorporate limited liability constraints into the analysis and show how the optimal solution to the regulator's problem changes with the size of the maximal loss that the firm can sustain in any given state of nature.

Closer to our paper, Demougin and Garvie (1991) were the first to study optimal regulation with a continuum of firm's types, correlated information, and limited liability constraints. We extend their analysis in several ways. First, as Riordan and Sappington (1988), the signals in Demougin and Garvie (1991) are purely informational. Hence, the firm's output in their paper is independent of the signals whereas in our paper it is state-contingent. Second, Demougin and Garvie consider either the case where the firm must earn nonnegative profits in each state of nature or the case in which the regulator is constrained to use nonnegative transfers in every state. In our model by contrast, the maximal loss that the firm can sustain in each state of nature is a parameter. In particular, we characterize the solution for the regulator's problem for various levels of this parameter. Third, the signal in Demougin and Garvie is binary whereas we consider an arbitrarily large (but finite) number of states of nature. We show that in order to implement the first-best solution under the most stringent limited deficit constraints, the regulator should use transfers that reward the firm in exactly one state and impose the same (minimal) punishment on the firm in all other states. Finally, while Demougin and Garvie rely on constrained calculus of variations techniques, our approach has the advantage of building on the by now familiar and relatively simple methodology of Baron and Myerson (1982) which we adapt to the case of ex post cost shocks and limited liability constraints.

The rest of the paper is organized as follows. Section 2 presents the simple case of two-types and two-states; Section 3 is devoted to the continuum of types, multiple signals case. Some proofs are relegated to an Appendix.

2 The two-types, two states case

Consider a regulated firm that produces a single product. The consumers' utility is,

$$U(q,t) = S(q) - t, \tag{1}$$

where q is the firm's output and t is the total transfer made to the firm. We assume that $S(\cdot)$ is twice continuously differentiable, strictly increasing and concave The function $S(\cdot)$ and the transfer t can have at least two interpretations. If the regulated firm produces a public good which the regulator procures, then q is simply the size or the quality of the public good, $S(q)$ is the gross aggregate utility that consumers derive from the public good, and t is the amount paid to the firm out of the state's budget. If the firm is a regulated monopoly producing a private good, then $S(q) = \int_0^q P(\xi)d\xi$ is the gross consumers' surplus and $P(\cdot)$ is the inverse demand function for the good. In that case, $t = P(q)q + A$ is the regulated firm's revenue, where $P(q)q$ is the aggregate sum of the usage fees that consumers pay, and A is either a subsidy paid to the firm out of the state's budget, or the aggregate sum of fixed fees paid by consumers.

The firm's cost of production depends on both the realization of a random but publicly observed state of nature (i.e., cost shock), s, and on the firm's type, θ, which we will assume to be private information for the firm. For instance, s could represent the observable components of the cost function while θ could represent the unobserved components. Alternatively, s could represent the number of costly equipment failures that the firm experiences and θ could represent the efficiency of the firm's technology. In this section we assume that there are only two possible states of nature, good (g) and bad (b), and that the firm could either be of a high (cost) type (h) or a low (cost) type (ℓ). The firm's cost in state $s = g, b$ when its type is $\theta = h, \ell$ is $C_{\theta s}(q)$, where $C_{hs}(q) > C_{\ell s}(q)$ and $C'_{hs}(q) > C'_{\ell s}(q)$ for $s = g, b$

and $C_{\theta b}(q) > C_{\theta g}(q)$ for $\theta = h, \ell$. That is, high type firms have higher total and marginal costs than low type firms in every state of nature and each type of firm has a higher total cost in bad states than in good states. We also assume that $C_{\theta s}(q)$ is increasing, weakly convex in q, and $C_{\theta s}(0) < S'(0)$ for $\theta = h, \ell$ and $s = g, b$. Recalling that t denotes the firm's revenue, the firm's profit is

$$\pi_{\theta s}(q,t) = t - C_{\theta s}(q). \tag{2}$$

Let ϕ_ℓ be the probability that the firm's type is low and $\phi_h = 1 - \phi_\ell$ the probability that its type is high. We denote by $p_{\theta s}$ the conditional probability of state s given that the firm's type is θ. Hence, $p_{hg} + p_{hb} = p_{\ell g} + p_{\ell b} = 1$. For future reference it is convenient to write the conditional probabilities in matrix form as,

$$H = \begin{pmatrix} p_{\ell g} & p_{\ell b} \\ p_{hg} & p_{hb} \end{pmatrix}, \tag{3}$$

and denote its determinant by J. Since in general, $p_{\ell g} \neq p_{hg}$, the two types of the firm differ from one another in two ways: (i) their cost of production in each state, and (ii) the likelihood that each state occurs. Put differently, firms' types in our model are multidimensional. In this respect, our model differs from most of the literature on adverse selection, where types differ from one another with respect to a single parameter (e.g., their marginal costs). Note that when $p_{\ell g} \neq p_{hg}$, the conditional probabilities matrix H has full rank, and thus, $J \equiv p_{\ell g}p_{hb} - p_{hg}p_{\ell b} \neq 0$. In what follows we will assume that $J \geq 0$. This assumption implies that $\frac{p_{hb}}{p_{hg}} \geq \frac{p_{\ell b}}{p_{\ell g}}$, so the a high (cost) type firm is more likely to draw a bad state than a low type firm. In the case where s represents the observable components of the cost function and θ the unobserved components, this means that when firms have higher observed costs they are also likely to have higher unobserved costs. And, when s represents the number of equipment failures and θ the efficiency of the firm's technology, then $J \geq 0$ means that when the firm uses a costly inefficient technology, it is also likely to experience a relatively large number of equipment failures.[2] In general, J can be viewed as a measure of the correlation between the firm's type and the realization of the state of nature: the

[2]The assumption that $J \geq 0$ seems like a natural assumption although it is not essential: the analysis in the case where $J < 0$ (the low cost type firm is more likely to draw a bad state) is completely analogous.

higher is J, the stronger is the correlation between the firm's type and the state of nature; consequently, the realization of the state s is more informative about the firm's type.

2.1 The regulator's problem under full information

The regulator chooses a vector $(q_{hg}, q_{\ell g}, q_{hb}, q_{\ell b}, t_{hg}, t_{\ell g}, t_{hb}, t_{\ell b})$ that specifies a production level and a transfer from consumers to the firm for each type of firm and each state of nature. This vector is chosen *before* the state of nature is realized. The regulator's objective is to maximize the expected welfare function

$$\sum_{\theta} \sum_{s} \phi_\theta p_{\theta s} \left[U(q_{\theta s}, t_{\theta s}) + \alpha \pi_{\theta s}(q_{\theta s}, t_{\theta s}) \right], \qquad 0 \leq \alpha < 1. \tag{4}$$

The parameter α captures the regulator's marginal rate of substitution between net consumers' surplus and firm's profits. Since $\alpha < 1$, the regulator will try to minimize the profits that accrue to the firm.

As usual, we assume that the regulator is constrained to select a mechanism which ensures that the firm must at least break even on average (otherwise, the regulatory scheme amounts to a confiscation of property). In addition, we also assume that the firm's profit in every state cannot fall below M, where $M \leq 0$. That is, M is the maximal loss that the firm can sustain. Given these assumptions, and using the definitions of U and $\pi_{\theta s}$ to rewrite the regulator's objective function in a more convenient form, the regulator's problem under full information is,

$$\max_{\substack{q_{hg}, q_{\ell g}, q_{hb}, q_{\ell b}, \\ t_{hg}, t_{\ell g}, t_{hb}, t_{\ell b}}} \sum_{\theta} \sum_{s} \phi_\theta p_{\theta s} \left[S(q_{\theta s}) - C_{\theta s}(q_{\theta s}) - (1-\alpha) \pi_{\theta s}(q_{\theta s}, t_{\theta s}) \right], \tag{RP}$$

subject to ex ante individual rationality constraints,

$$\sum_{s} p_{\theta s} \pi_{\theta s}(q_{\theta s}, t_{\theta s}) \geq 0, \qquad \theta = h, \ell, \tag{EIR_θ}$$

and state-by-state ex post individual rationality (or limited deficit) constraints,

$$\pi_{\theta s}(q_{\theta s}, t_{\theta s}) \geq M, \qquad \theta = h, \ell, \quad s = g, b. \tag{$IR_{\theta s}$}$$

Since $\alpha < 1$, it is optimal to set the transfers such that the EIR_θ constraints will be just binding. Substituting from the EIR_θ constraint into the objective function and recalling that $S'(q) > 0$, that $C_{\theta s}(q)$ is weakly convex in q, and $C_{\theta s}(0) < S'(0)$ for $\theta = h, \ell$ and $s = g, b$, it follows that the first-best production levels, $q^*_{\theta s}$, are defined implicitly by the first order conditions,

$$S'(q_{\theta s}) = C'_{\theta s}(q_{\theta s}), \qquad \theta = h, \ell, \quad s = g, b. \tag{5}$$

That is, at the optimum the regulator uses marginal cost pricing in each state of nature. Since by assumption, $C'_{hs}(q) > C'_{\ell s}(q)$ for $s = g, b$, it follows that $q^*_{\ell g} > q^*_{hg}$ and $q^*_{\ell b} > q^*_{hb}$. That is, the low-cost firm produces more than the high type firm in each state of nature. Given the optimal production levels, the optimal transfers are set such that,

$$\sum_s p_{\theta s}\pi_{\theta s}(q^*_{\theta s}, t^*_{\theta s}) = 0, \qquad \pi_{\theta s}(q^*_{\theta s}, t^*_{\theta s}) \geq M, \qquad \theta = h, \ell, \quad s = g, b. \tag{6}$$

When $M = 0$, (6) implies that $\pi_{\theta s}(q^*_{\theta s}, t^*_{\theta s}) = 0$ for $\theta = h, \ell$, and $s = g, b$, so $t^*_{\theta s}(\theta) = C_{\theta s}(q^*_{\theta s})$. However when $M < 0$, the regulator has many degrees of freedom in choosing transfers that will satisfy (6). In particular, the regulator can set transfers such that the firm will earn a positive profit in one state and will incur a loss (smaller than M) in the other state.

2.2 The regulator's problem under asymmetric information

We now turn to the case where the firm's type is not observed by the regulator. By the Revelation Principle, we can restrict attention, without a loss of generality, to direct revelation mechanisms in which the firm truthfully reports its type to the regulator, and, given a report $\widehat{\theta} = h, \ell$, the regulator requires the firm to produce $q_{\widehat{\theta} s}$ units in state s and gives the firm a transfer $t_{\widehat{\theta} s}$ in state s. The regulator's problem in that case is given by RP subject to the EIR_θ constraints, the $IR_{\theta s}$ constraints, and the following incentive compatibility constraints:

$$\sum_s p_{\theta s}\pi_{\theta s} \geq \sum_s p_{\theta s}\left[t_{js} - C_{\theta s}(q_{\widehat{\theta} s})\right], \qquad \theta = h, \ell, \quad \widehat{\theta} \neq \theta, \tag{IC_i}$$

where, to simplify notation, we write, $\pi_{\theta s} = \pi_{\theta s}(q_{\theta s}, t_{\theta s})$. Substituting for t from equation (2) into IC_ℓ and IC_h and simplifying, the two incentive constraints can be rewritten as

$$\sum_s p_{\ell s}\pi_{\ell s} \geq \sum_s p_{\ell s}\left[\pi_{hs} + \Delta_s(q_{hs})\right], \tag{IC_ℓ}$$

and

$$\sum_s p_{hs}\pi_{hs} \geq \sum_s p_{hs}\left[\pi_{\ell s} - \Delta_s(q_{hs})\right], \qquad (IC_h)$$

where $\Delta_s(q) \equiv C_{hs}(q) - C_{\ell s}(q)$ is the cost difference between the high and the low cost firms in state $s = g, b$.

To characterize the solution to the regulator's problem, we will first simplify it through a series of Lemmata. It should be noted that the problem cannot be simplified with the usual techniques of the mechanism design literature. For instance, since in general $p_{\ell g} \neq p_{hg}$ and $p_{\ell b} \neq p_{hb}$, it is not true that if EIR_h is binding then EIR_ℓ must be slack (as we shall see below, it is possible that at the optimum, both constraints are binding).

Lemma 1. *At the optimum, EIR_θ and IC_θ, $\theta = \ell, h$ cannot be both slack.*

Proof: Assume by way of negation that both EIR_ℓ and IC_ℓ are slack. Since EIR_ℓ is slack, either $IR_{\ell g}$ or $IR_{\ell b}$ or both are also slack, so it is possible to slightly lower $t_{\ell g}$ or $t_{\ell b}$ or both. This lowers the right-hand side of IC_h and hence relaxes it, without affecting IR_{hg} and IR_{hb}. At the same time, the value of the regulator's objective function is enhanced since $\alpha < 1$, thereby contradicting the assumption that the solution is optimal. The proof that EIR_h and IC_h cannot be both slack is analogous. ■

Lemma 1 implies that each type of the firm either breaks even in expectation, or has a binding incentive constraint (to prevent it from misreporting its type), or both. In the latter case, the solution would coincide with the first-best solution.

Lemma 2: *At the optimum, $t_{\ell b}$ and t_{hg} can be set such that $IR_{\ell b}$ and IR_{hg}, respectively, will be binding while $IR_{\ell g}$ and IR_{hb} are slack.*

Proof: Suppose that at the optimum $IR_{\ell b}$ is slack. Now consider an alternative allocation in which $t_{\ell b}$ is lowered by $\varepsilon_{\ell b} > 0$ until $IR_{\ell b}$ is just binding and $t_{\ell g}$ is increased by $\frac{p_{\ell b}\varepsilon_{\ell b}}{p_{\ell g}}$ to ensure that EIR_ℓ and IC_ℓ remain intact. These changes relax $IR_{\ell g}$ (since $t_{\ell g}$ is increased) but have no effect on EIR_h, IR_{hg}, IR_{hb}, and on the regulator's objective function. At the same time the right-hand side of IC_h changes by

$$p_{hg}\frac{p_{\ell b}\varepsilon_{\ell b}}{p_{\ell g}} - p_{hb}\varepsilon_{\ell b} = p_{hg}\varepsilon_{\ell b}\left[\frac{p_{\ell b}}{p_{\ell g}} - \frac{p_{hb}}{p_{hg}}\right] \leq 0,$$

where the inequality follows because the assumption that $J \equiv p_{hb}p_{\ell g} - p_{hg}p_{\ell b} \geq 0$ implies that $\frac{p_{\ell b}}{p_{\ell g}} \leq \frac{p_{hb}}{p_{hg}}$. Hence, IC_h is relaxed. Altogether, this implies that the new allocation also solves the regulator's problem. Since $IR_{\ell b}$ is binding, EIR_ℓ implies that $IR_{\ell g}$ must be slack. The proof concerning t_{hg} is completely analogous. ■

Lemma 2 is useful because it implies that at any optimal solution, the transfers can be set, without any loss of generality, such that $\pi_{\ell b} = \pi_{hg} = M$. This lemma therefore allows us, once the output levels are determined, to pin down the value of one transfer for each type of firm. What is then left is to pin down the values of the two remaining state-contingent transfers. In economic terms, note that M can be interpreted as the largest punishment that the regulator can impose on the firm. With this interpretation in mind, Lemma 2 implies that each type of firm receives the largest feasible punishment in the state of nature which it is less likely to draw. Given that $J \geq 0$, the high type firm is less likely to draw the good state and the low type firm is less likely to draw the bad state; hence the high type firm is punished in the good state while the low type firm is punished in the bad state.

Lemma 3. *If the optimal production levels are strictly monotonic with respect to the firm's type in each state of nature, i.e.,* $q_{\ell s} > q_{hs}$ *for* $s = g, b$*, then,* EIR_ℓ *and* EIR_h *cannot be both slack.*

Proof: Assume by way of negation that at the optimum, EIR_ℓ and EIR_h are both slack. Then, Lemma 1 implies that IC_ℓ and IC_h must be both binding, while Lemma 2 implies that $\pi_{\ell b} = \pi_{hg} = M$. Hence, we can write IC_ℓ and IC_h respectively, as

$$p_{\ell g}\pi_{\ell g} + p_{\ell b}M = p_{\ell g}\left[M + \Delta_g(q_{hg})\right] + p_{\ell b}\left[\pi_{hb} + \Delta_b(q_{hb})\right], \tag{7}$$

and

$$p_{hg}M + p_{hb}\pi_{hb} = p_{hg}\left[\pi_{\ell g} - \Delta_g(q_{\ell g})\right] + p_{hb}\left[M - \Delta_b(q_{\ell b})\right]. \tag{8}$$

Dividing equation (7) by $p_{\ell g}$, dividing equation (8) by p_{hg}, and adding the two yields,

$$\left[\frac{p_{hb}}{p_{hg}} - \frac{p_{\ell b}}{p_{\ell g}}\right](\pi_{hb} - M) + [\Delta_g(q_{\ell g}) - \Delta_g(q_{hg})] + \left[\frac{p_{hb}}{p_{hg}}\Delta_b(q_{\ell b}) - \frac{p_{\ell b}}{p_{\ell g}}\Delta_b(q_{hb})\right] = 0. \quad (9)$$

The first term on the left-hand side of (9) is nonnegative since $J \equiv p_{hb}p_{\ell g} - p_{hg}p_{\ell b} \geq 0$, and since by Lemma 2, IR_{hb} is slack, so that $\pi_{hb} > M$. The second term is strictly positive given the assumption that output is strictly monotonic (recall that $\Delta_s(q)$ is assumed strictly increasing with q). Finally, the third term is strictly positive as $J \geq 0$ implies that $\frac{p_{hb}}{p_{hg}} \geq \frac{p_{\ell b}}{p_{\ell g}}$ and as output is strictly monotonic. The left-hand side of (9) must therefore be strictly positive, a contradiction. We conclude that EIR_ℓ and EIR_h cannot be both slack. ■

Lemma 3 implies that the optimal mechanism does not give a positive expected rent to both types of the firm: at least one type must break even in expectation.

Lemma 4. *If the optimal production levels are strictly monotonic with respect to the firm's type in each state of nature, i.e., $q_{\ell s} > q_{hs}$ for $s = g, b$, then EIR_h is binding.*

Proof: Assume be way of negation that EIR_h is slack. Then IC_h is binding by Lemma 1 and EIR_ℓ is binding by Lemma 3. Since $\pi_{\ell b} = M$ by Lemma 2, EIR_ℓ implies that $\pi_{\ell g} = -\frac{p_{\ell b}}{p_{\ell g}}M$; hence, IC_h can be rewritten as

$$\begin{aligned} p_{hg}M + p_{hb}\pi_{hb} &= p_{hg}[\pi_{\ell g} - \Delta_g(q_{\ell g})] + p_{hb}[M - \Delta_b(q_{\ell b})] \\ &= p_{hg}\left[-\frac{p_{\ell b}}{p_{\ell g}}M - \Delta_g(q_{\ell g})\right] + p_{hb}[M - \Delta_b(q_{\ell b})] \qquad (10) \\ &= p_{hg}M\left[\frac{p_{hb}}{p_{hg}} - \frac{p_{\ell b}}{p_{\ell g}}\right] - p_{hg}\Delta_g(q_{\ell g}) - p_{hb}\Delta_b(q_{\ell b}). \end{aligned}$$

Since EIR_h is slack, the left-hand side of (10) is strictly positive. The right-hand side is strictly negative since $M < 0$, since $J \geq 0$ implies $\frac{p_{hb}}{p_{hg}} \geq \frac{p_{\ell b}}{p_{\ell g}}$, and since $\Delta_g(q_{\ell g})$ and $\Delta_b(q_{\ell b})$ are both positive. This contradicts the assumption that at the optimum EIR_h is slack. ■

Lemma 4 implies that if output is strictly monotonic with respect to the firm's type in every state of nature, then, the optimal mechanism is such that the high type firm breaks even in expectation. That is, in expectation there is no rent "at the bottom."

In order to characterize the optimal mechanism, let us first define the information rents of the low type firm under the first-best production plan. That is, the expected payoff of a low type firm from reporting that its type is high when the regulator requires the high type firm to produce the first-best output levels, q^*_{hg} and q^*_{hb}. As we shall see shortly, these information rents play an important role in the optimal solution. Recalling from Lemma 2 that $\pi_{\ell b} = \pi_{hg} = M$, it follows that the information rents of the low type firm are

$$\begin{aligned} R^*_\ell &\equiv p_{\ell g}\left[M + C_{hg}(q^*_{hg}) - C_{\ell g}(q^*_{hg})\right] + p_{\ell b}\left[-\frac{p_{hg}M}{p_{hb}} + C_{hb}(q^*_{hb}) - C_{\ell b}(q^*_{hb})\right] \\ &= \frac{p_{\ell g}p_{hb} - p_{\ell b}p_{hg}}{p_{hb}}M + p_{\ell g}\Delta_g(q^*_{hg}) + p_{\ell b}\Delta_b(q^*_{hb}) \qquad (11) \\ &= \frac{J}{p_{hb}}M + p_{\ell g}\Delta_g(q^*_{hg}) + p_{\ell b}\Delta_b(q^*_{hb}). \end{aligned}$$

The intuition behind R^*_ℓ is as follows. The high type firm's gets a payoff of M in the good state and $-\frac{p_{hg}M}{p_{hb}}$ in the bad state. The latter payoff arises since the output levels under the first-best production plan are strictly monotonic with respect to the firm's type; hence by Lemma 4, EIR_h is binding. Thus, if the low type firm reports that its type is high, its payoff is M with probability $p_{\ell g}$, and $-\frac{p_{hg}M}{p_{hb}}$ with probability $p_{\ell b}$. In addition, the low type firm enjoys a cost saving of $\Delta_g(q^*_{hg})$ in the good state and $\Delta_b(q^*_{hb})$ in the bad state due to its cost advantage over the high type firm.

We are now ready to characterize the optimal mechanism.

Proposition 1. *Let $q^*_{\ell g}$, $q^*_{\ell b}$, q^*_{hg} and q^*_{hb} be the first-best production levels and recall that at the first-best both types of firms break even in expectation. Then,*

(i) *The regulator can implement the first-best solution if and only if $R^*_\ell \leq 0$. At the optimum, $\pi^*_{\ell b} = \pi^*_{hg} = M < 0$, $\pi^*_{\ell g} = -\frac{p_{\ell b}}{p_{\ell g}}M > 0$, and $\pi^*_{hb} = -\frac{p_{hg}}{p_{hb}}M > 0$.*

(ii) *If $R^*_\ell > 0$, the regulator cannot implement the first-best solution. The optimal production levels are denoted $q^{**}_{\ell g}$, $q^{**}_{\ell b}$, q^{**}_{hg} and q^{**}_{hb} in this case, and are defined implicitly by the following first-order conditions:*

$$S'(q^{**}_{\ell s}) = C'_{\ell s}(q^{**}_{\ell s}), \qquad s = g, b, \qquad (12)$$

and

$$S'(q^{**}_{hs}) = C'_{hs}(q^{**}_{hs}) + (1-\alpha)\frac{\phi_\ell p_{\ell s}}{\phi_h p_{hs}}\Delta'_s(q^{**}_{hs}), \qquad s = g, b. \qquad (13)$$

Proof. *See the Appendix.*

Part (i) of Proposition 1 is closely related to Riordan and Sappington (1988) who show that the first-best solution can be implemented, provided that the regulator can condition the regulatory scheme on the realization of an ex post signal that is correlated with the firm's type and provided that the conditional probabilities matrix which specifies the likelihood of the various states of nature conditional on the firm's type has full rank (in our case this simply means that $J \neq 0$). There are two important differences however. First, unlike the ex post signals in Riordan and Sappington which affect are purely informational and only affect the firm's transfers, here the states of nature affect the firm's cost directly and therefore the firm's output.

Second and more importantly, in order to induce truthful reporting, the regulator needs to "punish" the low type firm in the bad state of nature which is less likely to be associated with the low type firm (the regulator does not "punish" the high type firm similarly since, given the first-best output levels, this firm does not wish to report that its type is low). But unlike in Riordan and Sappington, we assume that the firm needs to earn at least M in every state of nature. Hence, the regulator has only a limited ability to punish the low type firm for misreporting its type. Part (i) of the proposition shows that given this limitation, the first-best can be implemented if and only if $|MJ|$ is sufficiently large; that is, the maximal loss that the firm can sustain and/or the correlation between the likelihood of each state and the firm's type are sufficiently large. When these conditions hold, the low type firm cannot get positive information rents from misreporting its type.

When $|MJ|$ is small, the low type firm can get positive information rents when it produces the first-best output levels of the high type firm. Since $\alpha < 1$, leaving such information rents to the low type firm is costly from the regulator's perspective and hence, the first-best solution cannot be implemented. As part (ii) of the proposition shows, the regulator deals with this case by distorting the output of the high type firm downward in both states of nature. The optimal solution then has the familiar second-best features: there is no distortion but there are positive rents "at the top" (the low type firm produces its first-best output level in both states and get a positive expected profit) and there is a downward output distortion but full rent extraction "at the bottom" (the high type firm produces less than in the first-best solution in both states of nature and its expected profit is 0).[3] Moreover, the proposition shows that the distortion of the high type firm's output becomes larger as (i) the regulator places a smaller weight on the firm's profit, (ii) the relative probability that the firm's type is low, and (iii) the difference between the marginal costs of the low and high type firms is large.

3 The continuum of types, finitely many states, case

In this section we extend the preceding analysis by assuming that the regulated firm's type is drawn from the interval $\Theta = [\underline{\theta}, \overline{\theta}]$ and the set of states of nature is $\{1, ..., n\}$, with higher states representing higher cost shocks (i.e., "worse" states of nature). In the next subsection we will present our basic assumptions about the cost functions and the distributions of types and states of nature. In Section 3.2, we explore the conditions under which the regulator can implement the first-best solution and in Section 3.3 we will characterize the second-best solution to the regulator's problem when the first-best solution cannot be implemented.

3.1 Basic assumptions and notation

We assume that the firm's total cost function is linear and given by

$$C(q, s, \theta) = c_s(\theta)q,$$

where $0 < c_1(\theta) < c_2(\theta) < ... < c_n(\theta)$ for all $\theta \in \Theta$. That is, higher states are associated with higher marginal costs and therefore represent worse states of nature. In addition we assume

[3] Kessler, Lülfesmann, and Schmitz (2000) show, in the context of a similar two-types, two-states principal-agent model with adverse selection, that at the optimum, the principal may distort the action of the inefficient agent (the production level of the high cost firm in the context our model) upward rather than downward (Proposition 2 in their paper). Their result differs from ours because they impose an upper bound on the transfers from the agent to the principal, whereas we impose a bound on the agent's payoff. We believe that in the context of a regulation model, bounds on the firm's profit are more natural than bounds on the size of the transfers (which are only one component of the firm's profit).

that for all $s \in \{1, ..., n\}$, the marginal cost, $c_s(\theta)$, is a strictly positive, twice continuously differentiable, strictly increasing, and convex function of θ.

Let F denote the cumulative distribution function of the firm's type, θ, on the support Θ, and assume that the associated density function, f, is strictly positive and continuously differentiable on Θ. The conditional probability of state $s = \{1, ..., n\}$, given type θ, is denoted,

$$p_s(\theta) = \Pr(s \mid \theta).$$

A regulatory scheme is now a state-contingent array, $(q_s(\theta), t_s(\theta))_{s=1,\ldots,n}$, where $q_s(\theta)$ is the required production level of type θ in state s and $t_s(\theta)$ is the associated transfer from the regulator to the firm.[4] The profit of a type θ firm in state s is

$$\pi_s(\theta) = t_s(\theta) - c_s(\theta)q_s(\theta). \tag{14}$$

As before, we assume that the firm's profit in each state cannot fall below M, where $M \leq 0$.

The regulator's problem is the continuous analog of (RP):

$$\max_{(q_s(\theta),t_s(\theta))_{s=1,..,n}} \int_{\underline{\theta}}^{\overline{\theta}} \sum_s p_s(\theta) \left[S(q_s(\theta)) - c_s(\theta)q_s(\theta) - (1-\alpha)\pi_s(\theta)\right] f(\theta)d\theta, \qquad 0 \leq \alpha < 1, \tag{RP'}$$

subject to the ex ante participation constraints,

$$\sum_s p_s(\theta)\pi_s(\theta) \geq 0, \qquad \forall \theta \in \Theta, \tag{EIR_θ}$$

state-by-state limited-deficit constraints,

$$\pi_s(\theta) \geq M, \qquad \forall \theta \in \Theta, \quad \forall s \in \{1, ..., n\}, \tag{$IR_{\theta,s}$}$$

incentive compatibility constraints:

$$\sum_s p_s(\theta)\pi_s(\theta) \geq \sum_s p_s(\theta) \left[t_s(\widehat{\theta}) - c_s(\theta)\, q_s(\widehat{\theta})\right], \qquad \forall \theta, \widehat{\theta} \in \Theta, \tag{$IC_{\theta,\widehat{\theta}}$}$$

[4] As in Section 2, the transfer function, $t_s(\theta)$ includes the aggregate usage fees, $P(q_s(\theta))q_s(\theta)$), and the aggregate fixed fees (or subsidy), A.

and nonnegativity constraints:

$$q_s(\theta) \geq 0, \qquad \forall \theta \in \Theta, \quad \forall s \in \{1, ..., n\}.$$

The first-best solution to the regulator's problem is attained in the absence of private information, in which case the $IC_{\theta,\widehat{\theta}}$ constraints can be ignored. Since $\alpha < 1$, it is optimal to set the transfers as low as possible so that the EIR_θ constraint will be just binding. Substituting from the EIR_θ constraints into the objective function and maximizing with respect to $q_s(\theta)$ reveals that the first-best production level, $q_s^*(\theta)$, is implicitly defined by

$$S'\left(q_s^*(\theta)\right) = c_s(\theta), \qquad \forall \theta \in \Theta, \quad \forall s \in \{1, ..., n\}. \tag{15}$$

Since $S(\cdot)$ is increasing, concave, and continuously differentiable, and since $c_s(\theta) > 0$, $q_s^*(\theta)$ is positive and unique. By EIR_θ, the first-best transfers and expected profit of the firm must therefore be such that:

$$\sum_s p_s(\theta)\pi_s^*(\theta) \equiv \sum_s p_s(\theta)\left[t_s^*(\theta) - c_s(\theta)\, q_s^*(\theta)\right] = 0, \qquad \forall \theta \in \Theta. \tag{16}$$

As in the two-types, two states case, the regulator has in general many degrees of freedom in setting the transfers such that the $IR_{\theta,s}$ constraints will be satisfied. For instance, the regulator can set the transfers such that $t_s^*(\theta) = c_s(\theta)\, q_s^*(\theta)$ for all $s \in \{1, ..., n\}$, in which case the firm just breaks even in every state. However, if $M < 0$, the regulator can also set transfers such that the firm will earn positive profits in some states and will incur losses (smaller than $|M|$) in other states.

In the next subsection we show that the first-best solution can be implemented even if the firm's type is private information, provided that the maximum deficit $|M|$ is sufficiently large. In Section 3.3 we will consider the second-best solution when it is impossible to achieve the first-best solution.

3.2 Implementation of the first-best solution

To establish conditions under which the first-best solution can be implemented, we first replace the $IC_{\theta,\widehat{\theta}}$ constraints with the first-order necessary conditions for truthful revelation, assuming that the state-contingent regulatory scheme, $(q_s(\theta), t_s(\theta))_{s=1,\ldots,n}$, is differentiable.

We will then verify that the resulting solution is differentiable and will provide conditions ensuring that it is globally incentive compatible.[5]

Let $\widehat{\theta}$ be the report of a firm whose true type is θ. If all firms report their types truthfully, then, local incentive compatibility requires that,

$$\begin{aligned} 0 &= \frac{d}{d\widehat{\theta}} \left(\sum_s p_s(\theta) \left[t_s(\widehat{\theta}) - c_s(\theta) q_s(\widehat{\theta}) \right] \right) \Bigg|_{\widehat{\theta}=\theta} \\ &= \sum_s p_s(\theta) \left(t_s'(\theta) - c_s(\theta) q_s'(\theta) \right), \qquad \forall \theta \in \Theta. \end{aligned} \tag{17}$$

Differentiating (21) and using (22), we get

$$\sum_s p_s'(\theta) \pi_s^*(\theta) = \sum_s p_s(\theta) c_s'(\theta)\, q_s^*(\theta) \equiv B^*(\theta), \qquad \forall \theta \in \Theta. \tag{18}$$

Ignoring global incentive compatibility for the moment, the first-best solution can be implemented, provided that we can find transfers such that (17) and (18) hold simultaneously and the firm's profit, $\pi_s^*(\theta)$, is at least M in each state of nature.

In order to establish conditions under which we can find such transfers for the most stringent limited deficit constraints, we will solve the following constrained optimization problem; for each given θ:

$$\max \min \left\{ \pi_1^*(\theta), ..., \pi_n^*(\theta) \right\} \tag{19}$$

subject to (16) and (18). If the solution to this maxmin problem is above M for all $\theta \in \Theta$, then it is possible to find transfers that implement the first-best solution. Otherwise, any system of transfers that induces truthfully reports and leaves zero expected rents (i.e., satisfies equations (16) and (18)) will necessarily be such that the firm would lose more than $|M|$ in at least one state of nature. Such a system of transfers would then violate at least one of the $IR_{\theta,s}$ constraints.

Lemma 5: *The solution to the above maxmin problem must be such that the firm earns a profit in exactly one state of nature and incurs the same loss in all other states.*

Proof: Clearly, $\pi_s^*(\theta) = 0$ for all $s \in \{1, ..., n\}$ cannot be a solution since it violates (18). By (16) then, the firm necessarily earns a positive profit in at least one state of nature and incurs a loss in at least one other state of nature.

[5] On the differential approach to mechanism design, see Laffont and Maskin (1980).

Now fix θ and consider any solution to the maxmin problem. Let i be the state in which the firm's profit is highest and j be the state in which its loss is highest. That is, $\pi_j^*(\theta) \leq \pi_s^*(\theta) \leq \pi_i^*(\theta)$ for all $s \in \{1, ..., n\}$. To show that the firm earns a positive profit in exactly one state and makes the same loss in all other states, we prove that $\pi_s^*(\theta) = \pi_j^*(\theta)$ for all $s \neq \{i, j\}$. To this end, suppose by way of negation that there exists a state $k \neq \{i, j\}$ such that $\pi_j^*(\theta) < \pi_k^*(\theta) < \pi_i^*(\theta)$. Solving equation (16) for $\pi_i^*(\theta)$, substituting in (18) and simplifying, we get

$$\sum_{s \neq i} p_s(\theta) \gamma_s(\theta) \pi_s^*(\theta) = B^*(\theta), \qquad \forall \theta \in \Theta, \tag{20}$$

where $\gamma_s(\theta) \equiv \frac{p_s'(\theta)}{p_s(\theta)} - \frac{p_i'(\theta)}{p_i(\theta)}$. If we increase $\pi_j^*(\theta)$ slightly by ε and adjust $\pi_k^*(\theta)$ by $-\frac{p_j(\theta)\gamma_j(\theta)}{p_k(\theta)\gamma_k(\theta)}\varepsilon$, then the above equation continues to hold. By (16), the resulting change in $\pi_i^*(\theta)$ is $-\frac{p_j(\theta)}{p_k(\theta)}\left[1 + \frac{\gamma_j(\theta)}{\gamma_k(\theta)}\right]\varepsilon$. The profit levels in all other states remain unchanged. From these expressions it is clear that we can always choose ε small enough such that after we increase $\pi_j^*(\theta)$ slightly by ε, we still have $\pi_j^*(\theta) < \pi_k^*(\theta) < \pi_i^*(\theta)$. This contradicts the assumed optimality of the solution. Consequently, it must be the case that $\pi_s^*(\theta) = \pi_j^*(\theta)$ for all $s \neq \{i, j\}$. Since by definition $\pi_j^*(\theta) < 0 < \pi_i^*(\theta)$, it follows that for all $\theta \in \Theta$, the firm earns a profit only in state i and incurs the same loss in all other states. ■

Lemma 5 is a key step in characterizing the least restrictive set of conditions under which it is possible to implement the first-best solution to the regulator's problem because it says that the first-best can be implemented with a regulatory scheme that involves only two profit levels for the firm: a positive profit in one state and a loss in all other states. Moreover, this regulatory scheme involves minimal punishments in any given state of nature and hence has the "best shot" at satisfying the state-by-state limited-deficit constraints. Intuitively, in order to induce truth telling, the regulator needs to "punish" the firm whenever it misreports its type. Since the firm does not know in advance which state of nature will be realized, it takes into account only the expected level of the punishments. Hence, in order to satisfy the state-by-state limited-deficit constraints, it is optimal for the regulator to spread the punishments over as many states as possible. The regulator must then "reward" the firm in the remaining state of nature in order to ensure that it breaks even on average (otherwise its EIR_θ constraint is violated). From this intuition it is clear that we may also be able to implement the first-best solution to the regulator's problem with other types of regulatory schemes; for instance, we may be able to implement the first-best solution with schemes that involve more than just one level of reward, or more than just one level of punishment, or a scheme that rewards the firm in more than one state of nature. However, such regulatory schemes will be able to implement the first-best solution to the regulator's problem under more stringent limited deficit constraints than the maxmin scheme.

Given Lemma 5 we can now characterize the reward and punishment that the regulator uses in the maxmin scheme. Since $\pi_s^*(\theta) = \pi_j^*(\theta)$ for all $s \neq \{i, j\}$, and using the fact that $\sum_s p_s(\theta) = 1$ and $\sum_s p_s'(\theta) = 0$, it follows from equation (20) that

$$\pi_i^*(\theta) = \frac{(1 - p_i(\theta))\, B^*(\theta)}{p_i'(\theta)}, \qquad \pi_j^*(\theta) = -\frac{p_i(\theta)B^*(\theta)}{p_i'(\theta)}, \qquad \forall j \neq i. \tag{21}$$

Equation (21) shows that the reward state must be such that $p_i'(\theta) > 0$. This also insures that the profit levels in (21) are well-defined. Moreover, it is clear that in order to satisfy the state-by-state limited-deficit constraints, it must be the case that the punishment is such that $-\frac{p_i(\theta)B^*(\theta)}{p_i'(\theta)} \geq M$, or $\frac{p_i(\theta)B^*(\theta)}{p_i'(\theta)} \leq -M$ for all $\theta \in \Theta$. Since we are interested in a regulatory scheme that can implement the first-best solution to the regulator's problem under the most stringent limited deficit constraints, we should obviously pick the state with the highest $\frac{p_i'(\theta)}{p_i(\theta)}$ ratio in order to reward the firm.

Given (21), the transfers that implement the first-best solution for the widest set of values of M are such that

$$t_s^*(\theta) = \begin{cases} c_i(\theta)\, q_i^*(\theta) + \frac{(1-p_i(\theta))B^*(\theta)}{p_i'(\theta)}, & s = i \\ c_s(\theta)\, q_s^*(\theta) - \frac{p_i(\theta)B^*(\theta)}{p_i'(\theta)}, & \forall s \neq i. \end{cases} \tag{22}$$

These transfers are clearly differentiable (recall that $p_i'(\theta) > 0$ for all $\theta \in \Theta$). We must now check that these transfers satisfy the $IC_{\theta,\widehat{\theta}}$ constraints not only locally but for also globally, i.e., for all $\theta, \widehat{\theta} \in \Theta$. To this end, we first impose the following restrictions on the conditional probability system.

Assumption 1. *The conditional probability $p_s(\theta)$ is a continuously differentiable function of θ with $p_s(\theta) \geq \epsilon > 0$ for all $s \in \{1, ..., n\}$ and all $\theta \in \Theta$. Moreover, $p_n(\theta)$ is an increasing and concave function of θ for all $\theta \in \Theta$.*

Assumption 2. *The conditional probability distribution* $(p_s(\theta))_{s=1,\ldots,n}$ *has the first-order stochastic dominance (FOSD) property:* $\left(p_s(\widehat{\theta})\right)_{s=1,\ldots,n}$ *dominates* $(p_s(\theta))_{s=1,\ldots,n}$ *in the sense of strict first-order stochastic dominance if and only if* $\widehat{\theta} > \theta$.

Assumption 3. *The likelihood ratio,* $r_s(\theta) \equiv \frac{p_s(\theta)}{p_n(\theta)}$, *is decreasing with* θ *for all* $s \neq n$.

The first part of Assumption 1 ensures that all states of nature can be realized no matter what the firm's type is. Absent this assumption, say if $p_s(\theta)$ for some s and some θ then, after observing the state s the regulator would be able to rule out the possibility that the firm's type is θ. The first part of Assumption 1 then ensures that the regulator cannot rule out any type on the basis of the realized state of nature. The second part of Assumption 1 says that the probability of drawing the worst state, n, increases with the firm's type but at a decreasing rate. This implies in turn that less efficient types are more likely to draw state n than more efficient types. Corollary 1.4 in Riordan and Sappington (1988) shows that the concavity of the likelihood function for the signal (our Assumption 1 in essence) together with the convexity of the cost function with respect to θ (which we also assume), ensure the existence of transfers that implement the first-best. It is therefore not surprising that these properties will play a similar role in our setting. Assumption 2 implies that more efficient types have a higher probability of having good states (i.e., states with "small" index) than less efficient types. That is, $\sum_{s<t} p_s(\widehat{\theta}) < \sum_{s<t} p_s(\theta)$ for $\widehat{\theta} > \theta$ and $t < n$. Assumption 3 implies that $\frac{p_s'(\theta)}{p_s(\theta)} < \frac{p_n'(\theta)}{p_n(\theta)}$ for all $\theta \in \Theta$. It is therefore a form of the monotone likelihood ratio property (Milgrom, 1981) which is common in the mechanism design literature.

Proposition 2. *Suppose that Assumptions 1-3 hold and* $q_s^*(\theta)c_s'(\theta)$ *is nondecreasing with* s *for all* θ, *where* $q_s^*(\theta)$ *is defined by (15). Then, the maxmin transfers defined by (22) implement the first-best output levels if and only if*

$$\frac{p_n(\theta)}{p_n'(\theta)} B^*(\theta) < -M, \qquad \forall \theta \in \Theta. \tag{23}$$

Proof. *See the Appendix.*

Proposition 2 generalizes the first-best implementation Theorem of Riordan and Sappington (1988), and particularly Corollary 1.4 in their paper. The only somewhat unusual assumption in the statement of Proposition 2 is: "$q_s^*(\theta)c_s'(\theta)$ is nondecreasing with s". This assumption is only a sufficient condition for first-best implementation and only involves fundamental data of the model; in particular we can reformulate it as,

$$(S')^{-1}(c_s(\theta))\,c_s'(\theta) \leq (S')^{-1}(c_{s+1}(\theta))\,c_{s+1}'(\theta), \qquad \forall s < n, \forall \theta \in \Theta.$$

The following simple example will help to illustrate this assumption. Suppose that $S(q) = \frac{q^{1-\epsilon}}{1-\epsilon}$ (the inverse demand for the regulated good, P, has constant elasticity, ϵ) and $c_s(\theta) = \theta s^{\beta}$, with $\beta > 0$. Then, $q_s^*(\theta)\,c_s'(\theta) = \theta^{-\frac{1}{\epsilon}} s^{\beta(1-\frac{1}{\epsilon})}$ is increasing with s for all $\theta \in \Theta$ if and only if $\epsilon > 1$ (i.e., if the elasticity of the inverse demand function exceeds 1). Clearly, the assumption is not very restrictive and holds if the inverse demand for the regulated firm's good is elastic.

3.3 The second-best solution under ex post limited-deficit constraints

In this section we assume that condition (23) fails so that it is impossible to construct transfers that implement the first-best production level. We therefore characterize the second-best regulatory scheme that solves RP' subject to the $IC_{\theta,\widehat{\theta}}, EIR_{\theta}$ and $IR_{\theta,s}$ constraints. Our strategy for solving this constrained optimization problem will be to substitute the necessary condition for local incentive compatibility (17) in RP' and to ignore the ex ante participation constraints EIR_{θ} in a first step. We will then check that the resulting solution is differentiable, satisfies the EIR_{θ} constraints, and is globally incentive compatible.

Using (17) and the definition $r_s(\theta) \equiv \frac{p_s(\theta)}{p_n(\theta)}$, we can express $t_n'(\theta)$ as a function of the other $n-1$ transfers functions:

$$t_n'(\theta) = \sum_s r_s(\theta)c_s(\theta)q_s'(\theta) - \sum_{s \neq n} r_s(\theta)t_s'(\theta).$$

Integrating $t_n'(\theta)$ from $\overline{\theta}$ (the "worst" type) to θ yields,

$$\begin{aligned} t_n(\theta) &= t_n(\overline{\theta}) - \int_{\theta}^{\overline{\theta}} \left[\sum_s r_s(x)c_s(x)q_s'(x) - \sum_{s\neq n} r_s(x)t_s'(x) \right] dx \\ &= t_n(\overline{\theta}) - \sum_s r_s(\overline{\theta})c_s(\overline{\theta})q_s(\overline{\theta}) + \sum_{s\neq n} r_s(\overline{\theta})t_s(\overline{\theta}) + \sum_s r_s(\theta)c_s(\theta)q_s(\theta) \\ &\quad - \sum_{s\neq n} r_s(\theta)t_s(\theta) + \int_{\theta}^{\overline{\theta}} \left[\sum_s \left[r_s(x)c_s(x)\right]' q_s(x) - \sum_{s\neq n} r_s'(x)t_s(x) \right] dx, \end{aligned}$$

where the second equality follows from integration by parts. Since by definition, $\pi_s(\theta) = t_s(\theta) - c_s(\theta)q_s(\theta)$ and $r_n(\theta) = 1$, the first three terms in the above expression equal $\sum_s r_s(\overline{\theta})\pi_s(\overline{\theta})$. Moreover, since $r_n'(\theta) = 0$, $\sum_{s\neq n} r_s'(x)t_s(x) = \sum_s r_s'(x)t_s(x)$. Hence, the square bracketed expression equals

$$\sum_s r_s(x)c_s'(x)q_s(x) + \sum_s r_s'(x)c_s(x)q_s(x) - \sum_{s\neq n} r_s'(x)t_s(x) = \sum_s \left[r_s(x)c_s'(x)q_s(x) - r_s'(x)\pi_s(x)\right].$$

We therefore write the firm's transfer in state n as

$$\begin{aligned} t_n(\theta) &= \sum_s r_s(\overline{\theta})\pi_s(\overline{\theta}) + \sum_s r_s(\theta)c_s(\theta)q_s(\theta) - \sum_{s\neq n} r_s(\theta)t_s(\theta) \\ &\quad + \int_{\theta}^{\overline{\theta}} \sum_s \left[r_s(x)c_s'(x)q_s(x) - r_s'(x)\pi_s(x)\right] dx. \end{aligned} \tag{24}$$

Given (24), using the fact that $r_s(\theta) \equiv \frac{p_s(\theta)}{p_n(\theta)}$ and simplifying, the expected transfer of the firm, where the expectation is taken with respect to the firm's type and with respect to the state of nature, is given by

$$\begin{aligned} &\int_{\underline{\theta}}^{\overline{\theta}} \left[\sum_{s\neq n} p_s(\theta)t_s(\theta) + p_n(\theta)t_n(\theta) \right] f(\theta)d\theta \\ &= \int_{\underline{\theta}}^{\overline{\theta}} \left[\sum_s r_s(\overline{\theta})\pi_s(\overline{\theta}) + \sum_s r_s(\theta)c_s(\theta)q_s(\theta) \right] p_n(\theta)f(\theta)d\theta \qquad (25) \\ &\quad + \int_{\underline{\theta}}^{\overline{\theta}} \left[\int_{\theta}^{\overline{\theta}} \sum_s \left[r_s(x)c_s'(x)q_s(x) - r_s'(x)\pi_s(x)\right] dx \right] p_n(\theta)f(\theta)d\theta. \end{aligned}$$

After integration by parts, the expression in the last line of (25) becomes

$$\begin{aligned}
&\left.\int_{\theta}^{\overline{\theta}} \sum_s \left[r_s(x)c_s'(x)q_s(x) - r_s'(x)\pi_s(x)\right] dx \varphi_n(\theta)\right|_{\underline{\theta}}^{\overline{\theta}} \\
&+ \int_{\underline{\theta}}^{\overline{\theta}} \left[\sum_s \left[r_s(\theta)c_s'(\theta)q_s(\theta) - r_s'(\theta)\pi_s(\theta)\right] \varphi_n(\theta)\right] d\theta \qquad (26)\\
= &\int_{\underline{\theta}}^{\overline{\theta}} \left[\sum_s \left[r_s(\theta)c_s'(\theta)q_s(\theta) - r_s'(\theta)\pi_s(\theta)\right] \varphi_n(\theta)\right] d\theta,
\end{aligned}$$

where $\varphi_n(\theta) \equiv \int_{\underline{\theta}}^{\theta} p_n(x)f(x)dx$. Substituting from (25) and (26) into (RP') and rearranging, the regulator's problem, given local incentive compatibility, becomes

$$\begin{aligned}
&\max_{\substack{(q_s(\theta))_{s=1,..,n} \\ (t_s(\theta))_{s=1,..,n-1} \\ t_n(\overline{\theta})}} \int_{\underline{\theta}}^{\overline{\theta}} \sum_s \left[S\left(q_s(\theta)\right) - \alpha c_s(\theta)q_s(\theta)\right] p_s(\theta)f(\theta)d\theta \\
&- (1-\alpha)\int_{\underline{\theta}}^{\overline{\theta}} \left[\sum_s r_s(\overline{\theta})\pi_s(\overline{\theta}) + \sum_s r_s(\theta)c_s(\theta)q_s(\theta)\right] p_n(\theta)f(\theta)d\theta \qquad (RP")\\
&- (1-\alpha)\int_{\underline{\theta}}^{\overline{\theta}} \left[\sum_s \left[r_s(\theta)c_s'(\theta)q_s(\theta) - r_s'(\theta)\pi_s(\theta)\right] \varphi_n(\theta)\right] d\theta,
\end{aligned}$$

subject to the EIR_θ constraints, the $IR_{\theta,s}$ constraints, and subject to the constraints that $q_s(\theta) \geq 0$ for all $s = \{1, ..., n\}$ and all $\theta \in \Theta$.

To characterize the solution to the regulator's problem we shall study a relaxed version of RP" in which we ignore the EIR_θ constraints and maximize the regulator's objective function pointwise, subject to the $IR_{\theta,s}$ constraints and the nonnegativity constraint on $q_s(\theta)$. We will then verify that the solution to the relaxed problem is differentiable and will provide sufficient conditions for this solution to satisfy the EIR_θ constraints and to be globally incentive compatible.

Lemma 6: *The regulator's relaxed problem, RP", has a unique solution such that*

$$S'\left(q_s^{**}(\theta)\right) = c_s(\theta) + (1-\alpha)\, c_s'(\theta)\frac{F(\theta \mid n)}{f(\theta \mid n)}, \qquad (27)$$

$$t_s^{**}(\theta) = c_s(\theta)q_s^{**}(\theta) + M, \qquad \forall s \neq n, \forall \theta \in \Theta, \qquad (28)$$

$$t_n^{**}(\theta) = c_n(\theta)q_n^{**}(\theta) + \int_\theta^{\overline{\theta}} \sum_s r_s(x)c_s'(x)q_s^{**}(x)dx - \frac{M\left(1 - p_n(\overline{\theta})\right)}{p_n(\overline{\theta})}, \qquad \forall\theta \in \Theta. \tag{29}$$

This solution is differentiable in θ, and if the conditional hazard rate $\frac{f(\theta|n)}{F(\theta|n)}$ is nonincreasing with θ, then $q_s^{**}(\theta)$ is decreasing with θ for all s, implying that less efficient types produce less in every state of nature.

Proof: By Assumption 3, the coefficient of $\pi_s(\theta)$ in RP" is negative for all $s \neq n$ (since $r_n'(\theta) = 0$, the coefficient of $\pi_n(\theta)$ is 0). Hence, it is optimal for the regulator to set transfers such that $\pi_s(\theta)$ will be as low as possible; given the $IR_{\theta,s}$ constraints, it follows that at the solution to the relaxed problem, $\pi_s^{**}(\theta) \equiv t_s^{**}(\theta) - c_s(\theta)q_s^{**}(\theta) = M$, $\forall s \neq n$. Substituting this equality in RP" and recalling that $r_n'(\theta) = 0$, the first-order condition for $q_s(\theta)$ is

$$\begin{aligned} &\left[S'\left(q_s(\theta)\right) - \alpha c_s(\theta)\right] p_s(\theta) f(\theta) - (1-\alpha)\, c_s(\theta) p_s(\theta) f(\theta) \\ &- (1-\alpha)\, r_s(\theta) c_s'(\theta) \varphi_n(\theta) = 0, \qquad \forall s, \forall \theta \in \Theta. \end{aligned} \tag{30}$$

Now, note that the cumulative distribution of θ, conditional on state n being realized, is given by

$$F(\theta \mid n) = \frac{\int_{\underline{\theta}}^{\theta} p_n(x)f(x)dx}{\int_{\underline{\theta}}^{\overline{\theta}} p_n(x)f(x)dx} = \frac{\varphi_n(\theta)}{\int_{\underline{\theta}}^{\overline{\theta}} p_n(x)f(x)dx}.$$

Hence,

$$\varphi_n(\theta) = \frac{F(\theta \mid n)}{f(\theta \mid n)} p_n(\theta) f(\theta).$$

Substituting this equality in (30) and simplifying, yields equation (27). Since $S(\cdot)$ is increasing and concave, $q_s^{**}(\theta)$ is differentiable and unique. If $\frac{f(\theta|n)}{F(\theta|n)}$ is nonincreasing with θ (so $\frac{F(\theta|n)}{f(\theta|n)}$ is nondecreasing with θ), then, together with the assumption that $c_s(\theta)$ is increasing and convex, the right-hand side of (27) is increasing with θ, so $q_s^{**}(\theta)$ is decreasing with θ.

To characterize the transfers, note that (28) follows immediately from the fact that $\pi_s(\theta) = M$ for all $s \neq n$, and all $\theta \in \Theta$. As for (29), note that since by definition $r_n'(\theta) = 0$, $t_n(\overline{\theta})$ appears only in the second line of RP". Hence, it is clear that it is optimal to choose the transfers of type $\overline{\theta}$ such that $\sum_s r_s(\overline{\theta})\pi_s(\overline{\theta}) = 0$ Substituting this equality in (24), using

(28) and the fact that $r_n(\theta) = 1$ and $r_n'(\theta) = 0$, yields

$$\begin{aligned} t_n^{**}(\theta) &= c_n(\theta)q_n^{**}(\theta) + \sum_{s\neq n} r_s(\theta)\left[c_s(\theta)q_s^{**}(\theta) - t_s^{**}(\theta)\right] \\ &\quad + \int_{\theta}^{\overline{\theta}} \sum_{s} r_s(x)c_s'(x)q_s^{**}(x)dx - \int_{\theta}^{\overline{\theta}} \sum_{s\neq n} r_s'(x)Mdx \\ &= c_n(\theta)q_n^{**}(\theta) - \sum_{s\neq n} r_s(\theta)M + \int_{\theta}^{\overline{\theta}} \sum_{s} r_s(x)c_s'(x)q_s^{**}(x)dx - \sum_{s\neq n} r_n(\overline{\theta})M + \sum_{s\neq n} r_s(\theta)M. \end{aligned}$$

Noting that $\sum_{s\neq n} r_n(\overline{\theta}) = \frac{\sum_{s\neq n} p_s(\overline{\theta})}{p_n(\overline{\theta})} = \frac{1-p_n(\overline{\theta})}{p_n(\overline{\theta})}$, yields (29). From (28) and (29) it is clear that $t_s^{**}(\theta)$ is unique and differentiable for all s. ■

Equation (27) is the continuous analog of (12) and (13). Since $F(\underline{\theta} \mid n) = 0$, it follows that $S'\left(q_s^{**}(\underline{\theta})\right) = c_s(\underline{\theta})$: the regulator uses marginal cost pricing for the lowest possible type of the firm so there is no distortion "at the top." This is exactly as in the two-types, two-states case (see equation (12)). For higher cost types, equations (15) and (27) reveal that since $S(\cdot)$ is concave and $c_s'(\theta) > 0$, then $q_s^*(\theta) > q_s^{**}(\theta)$ for all $s = \{1, ..., n\}$ and all $\theta > \underline{\theta}$. Hence, unless $\theta = \underline{\theta}$, the regulator distorts the firm's output level downward in every state of nature. Noting that $c_s'(\theta)$ is the continuous analog of $\Delta_s'(\cdot)$ and $\frac{F(\theta|n)}{f(\theta|n)}$ is the continuous analog of $\frac{\phi_\ell p_{\ell s}}{\phi_h p_{hs}}$, it follows that the distortion term in equation (27) is the exact analog of the distortion term in equation (13). Hence, just like in the two-types, two-states case, the regulator distorts the firm's output to a larger extent when (i) he attaches a smaller weight to firm's profits, (ii) there is a bigger difference between the costs of different types of the firm, and (iii) there is a relatively low likelihood that the firm's cost is low. Moreover, since the right-hand side of (27) is increasing with s, the firm's output level is smaller in worse states of nature.

Equation (27) generalizes the results of Baron and Myerson (1982) and Demougin and Garvie (1991). To see how, suppose first that there is only one state of nature, i.e., $n = 1$, and let the marginal cost be given by $c(\theta) = \theta$. Then, (27) becomes,

$$S'\left(q^{**}(\theta)\right) = \theta + (1-\alpha)\frac{F(\theta)}{f(\theta)},$$

which is just Baron and Myerson's (1982) classic formula for second-best optimality.

Next, suppose that the regulator's objective is to maximize consumers' surplus so that $\alpha = 0$, and suppose that the marginal cost is state independent and given by $c(\theta) = \theta$. Then, (27) becomes

$$S'\left(q_s^{**}(\theta)\right) = \theta + \frac{F(\theta \mid n)}{f(\theta \mid n)},$$

which is just equation (2) in Demougin and Garvie (1991).[6]

Given the transfers in (28) and (29), the expected profit of a type $\overline{\theta}$ firm is $\sum_s p_s(\overline{\theta})\pi_s(\overline{\theta}) = 0$. Hence, the second-best solution features the familiar no (expected) rent "at the bottom" property. In the next proposition we establish sufficient conditions under which all types $\theta < \overline{\theta}$ earn positive expected rents due to their private information. This ensures in turn that all the EIR_θ constraints are satisfied. In addition, the conditions ensure that the solution to the regulator's relaxed problem is globally incentive compatible.

Proposition 3. *Suppose that Assumptions 1-3 hold and the conditional hazard rate* $\frac{f(\theta|n)}{F(\theta|n)}$ *is nonincreasing with* θ*. Then, for all* M*, there exists a* $\delta > 0$ *such that if* $|p_s'(\theta)| < \delta$ *for all* $s = \{1, ..., n\}$*, the solution characterized by (27)-(29) satisfies the* EIR_θ *constraints and is globally incentive compatible.*

Proof. *See the Appendix.*

4 Conclusion

In this paper we studied the design of optimal regulation schemes when the regulated firm is subject to ex post cost shocks. We showed that the regulator can design a regulatory scheme that adjusts itself automatically following the realization of each shock without having to

[6] It is also interesting to note that if we replace $c_s(\theta)$ in equation (27) with $\overline{c}(\theta) \equiv \sum_s p_s(\theta)c_s(\theta)$ which is the expected value of $c_s(\theta)$ over all state of nature, then we get equation (15b) in Baron and Besanko (1984). They also assume that the regulated firm is subject to cost shocks, but the regulator in their model cannot observe the states of nature as in our model and hence cannot condition the firm's output or transfers on these states. Instead, the regulator audits the firm's costs and penalize the firm if its reported costs are above its realized costs. Since the penalties are imposed after the firm has already produced, it is clear that the firm's output in their model depends on $\overline{c}(\theta)$ rather than on $c_s(\theta)$ as in our model.

renegotiate the entire scheme. Our model differs from most of the optimal regulation literature in that firms have multidimensional types: they differ from each other not only with respect to their production costs but also with respect to the likelihood of having a cost shock: some types are more susceptible to negative shocks than others.

We showed that under certain conditions, the regulator can exploit the correlation between firms' types and the likelihood of the various cost shocks and design a regulatory scheme that implements the first-best solution, despite the fact that the firm's type is private information. To implement the first-best, the regulator needs to punish the firm if the realized state of nature is relatively unlikely given the firm's reported type. And, in order to ensure that the firm breaks even on average (so the regulatory scheme does not amount to a confiscation of property), the regulator needs to reward the firm in the remaining states. In general, however, firms cannot sustain unlimited losses and this fact imposes a constraint on the regulatory scheme. Our analysis reveals that the scheme that implements the first-best solution under the most stringent limited-deficit constraints "rewards" the firm in exactly one state of nature and imposes the same "punishment" on the firm in all other states. This scheme is feasible provided that the correlation between the firm's type and the cost shocks is sufficiently strong and/or the firm can sustain sufficiently large deficits in any given state. When these conditions fail, we are able to fully characterize the solution to the regulator's problem and show that it has classical second-best features. One benefit of our approach is that we extend the well known Baron and Myerson methodology to the case of ex post cost shocks and limited liability constraints and can characterize the optimal solution using straightforward calculus.

5 Appendix: Proofs

Proof of Proposition 1: (i) Suppose that IC_ℓ and IC_h are both slack. Then, by Lemma 1, EIR_ℓ and EIR_h are binding. Since EIR_ℓ and EIR_h are binding while IC_ℓ and IC_h are slack, the solution to the regulator's problem is the first-best solution. Now it remains to show that at the first-best solution, IC_ℓ and IC_h are indeed slack. To this end, note that since EIR_ℓ and EIR_h are binding, the left-hand sides of IC_ℓ and IC_h vanish. Hence, we only need to show that the right-hand sides of IC_ℓ and IC_h are both negative.

Equation (10) reveals that since $J \geq 0$ and $M \leq 0$, the right-hand side of IC_h is strictly negative. Next, since EIR_h is binding, $\pi_{hb} = -\frac{p_{hg}}{p_{hb}}M$. Given this equation, the right-hand side of IC_ℓ becomes:

$$p_{\ell b}\left[\frac{p_{\ell g}}{p_{\ell b}} - \frac{p_{hg}}{p_{hb}}\right] M + p_{\ell g}\Delta_g(q^*_{hg}) + p_{\ell b}\Delta_b(q^*_{hb}) = R^*_\ell < 0,$$

as required.

(ii) If $R^*_\ell > 0$, the first-best solution cannot be implemented, since it violates IC_ℓ. Consequently, IC_ℓ must be binding at the optimum. Given that EIR_h is binding as well, $\pi_{hb} = -\frac{p_{hg}}{p_{hb}}M$. Hence IC_ℓ can be written as

$$p_{\ell g}\pi_{\ell g} + p_{\ell b}M = p_{\ell b}M\left[\frac{p_{\ell g}}{p_{\ell b}} - \frac{p_{hg}}{p_{hb}}\right] + p_{\ell g}\Delta_g(q_{hg}) + p_{\ell b}\Delta_b(q_{hb}), \tag{A-1}$$

where the left-hand side is just the expected profit of the low type firm.

The regulator's problem is given by RP subject to equation (A-1), $\pi_{\ell b} = \pi_{hg} = M$, $\pi_{hb} = -\frac{p_{hg}}{p_{hb}}M$, and subject to monotonicity of output. To characterize the solution, we shall relax the problem by ignoring the monotonicity conditions, obtain a solution, and then verify that at this solution, output is indeed monotonic in the firm's type. Recalling that since EIR_h is binding, the high type firm gets expected profit zero, while the expected profit of the low type is given by the right-hand side of (A-1), the regulator's problem can be rewritten as

$$\max_{q_{hg},q_{\ell g},q_{hb},q_{\ell b}} \sum_\theta \sum_s \phi_\theta p_{\theta s}\left[S(q_{\theta s}) - C_{\theta s}(q_{\theta s})\right]$$
$$-(1-\alpha)\phi_\ell\left[p_{\ell b}M\left[\frac{p_{\ell g}}{p_{\ell b}} - \frac{p_{hg}}{p_{hb}}\right] + p_{\ell g}\Delta_g(q_{hg}) + p_{\ell b}\Delta_b(q_{hb})\right]$$

The properties of S and $C_{\theta s}$ ensure that the solution to the regulator's problem is defined implicitly by the first order conditions (12) and (13). To verify that output is monotonic in the firm's type as we assumed above, note that the conditions in the proposition imply that $q^{**}_{\ell s} = q^*_{\ell s}$ for $s = g, b$. Now, since $\alpha < 1$ and $\Delta'_s(q) > 0$ for $s = g, b$, these conditions also imply that $q^{**}_{hs} < q^*_{hs}$ for $s = g, b$. Hence, $q^{**}_{\ell s} = q^*_{\ell s} > q^*_{hs} > q^{**}_{hs}$ for $s = g, b$, as required. ∎

Proof of Proposition 2. By Assumption 3,

$$r_s'(\theta) \equiv \frac{p_s'(\theta)p_n(\theta) - p_s(\theta)p_n'(\theta)}{(p_n(\theta))^2} = \frac{p_s(\theta)}{p_n(\theta)}\left[\frac{p_s'(\theta)}{p_s(\theta)} - \frac{p_n'(\theta)}{p_n(\theta)}\right] < 0, \qquad \forall s \neq n.$$

Hence $\frac{p_s'(\theta)}{p_s(\theta)} < \frac{p_n'(\theta)}{p_n(\theta)}$ for all $s \neq n$, implying that state n has the highest $\frac{p_i'(\theta)}{p_i(\theta)}$ ratio and hence should be the one in which the firm is rewarded.

Since the maxmin transfers defined by (22) were derived from (16), these transfers clearly satisfy the EIR_θ constraint for all $\theta \in \Theta$. We now check that these transfers ensure global incentive compatibility. To this end, note that if we substitute the maxmin transfers from (22) with $i = n$ into the $IC_{\theta,\widehat{\theta}}$ constraints and simplify, we get

$$0 \geq \frac{p_n(\theta) - p_n(\widehat{\theta})}{p_n'(\widehat{\theta})} B^*(\widehat{\theta}) + \sum_s p_s(\theta) q_s^*(\widehat{\theta}) \left(c_s(\widehat{\theta}) - c_s(\theta)\right), \qquad \forall \theta, \widehat{\theta} \in \Theta, \tag{A-2}$$

where the left hand side vanishes because the transfers were chosen such that $\sum_s p_s(\theta)\pi_s^*(\theta) = 0$. Assuming that $\widehat{\theta} > \theta$ and dividing by $\widehat{\theta} - \theta$, the right-hand side of (A-2) can be written as

$$\left(\frac{p_n(\theta) - p_n(\widehat{\theta})}{\widehat{\theta} - \theta}\right) \frac{B^*(\widehat{\theta})}{p_n'(\widehat{\theta})} + \sum_s p_s(\theta) q_s^*(\widehat{\theta}) \left(\frac{c_s(\widehat{\theta}) - c_s(\theta)}{\widehat{\theta} - \theta}\right).$$

Since $c_s(\theta)$ is increasing and convex, while by Assumption 1, $p_n(\theta)$ is increasing and concave

$$\frac{c_s(\widehat{\theta}) - c_s(\theta)}{\widehat{\theta} - \theta} \leq c_s'(\widehat{\theta}), \qquad \frac{p_n(\theta) - p_n(\widehat{\theta})}{\widehat{\theta} - \theta} \leq -p_n'(\widehat{\theta}).$$

Hence, using the definition of $B^*(\cdot)$ we get

$$\begin{aligned} & \left(\frac{p_n(\theta) - p_n(\widehat{\theta})}{\widehat{\theta} - \theta}\right) \frac{B^*(\widehat{\theta})}{p_n'(\widehat{\theta})} + \sum_s p_s(\theta) q_s^*(\widehat{\theta}) \left(\frac{c_s(\widehat{\theta}) - c_s(\theta)}{\widehat{\theta} - \theta}\right) \\ \leq \; & -B^*(\widehat{\theta}) + \sum_s p_s(\theta) q_s^*(\widehat{\theta}) c_s'(\widehat{\theta}) \\ = \; & \sum_s q_s^*(\widehat{\theta}) c_s'(\widehat{\theta}) \left[p_s(\theta) - p_s(\widehat{\theta})\right]. \end{aligned} \tag{A-3}$$

The expression in the last line of (A-3) is nonpositive since by assumption, $q_s^*(\cdot)\, c_s'(\cdot)$ is weakly increasing with s, and since by Assumption 2, $p_s(\cdot)$ satisfies FOSD. Hence, (A-1) is satisfied, implying that global incentive compatibility is ensured. The proof in the case of $\widehat{\theta} < \theta$ is analogous.

Finally we need to verify that the first-best production levels can be implemented with the maxmin transfers if and only if (23) holds. Since by construction, the maxmin transfers satisfy the EIR_θ constraints and since we already verified that they ensure global incentive compatibility, we only need to verify that the maxmin transfers satisfy the $IR_{\theta,s}$ constraints. The "if" part of the statement follows directly from the fact that if (23) holds, then the maxmin transfers ensure that firm's loss in states $1, ..., n-1$ is equal to or exceeds M. To prove the "only if" part, note that since by construction, the maxmin transfers ensure that the firm's loss is minimal in every given state, and since Assumption 3 ensures that state n has the highest $\frac{p_i(\theta)}{p_i'(\theta)}$ ratio, it is obvious that if (23) is violated for some θ, then under any system of transfers that is locally incentive compatible and leaves the firm no expected rent (i.e., satisfies equation (16) and (18)), at least one type firm would incur a loss greater than M in at least one state. That is, at least one of $IR_{\theta,s}$ constraints will be violated. ■

Proof of Proposition 3. We begin with the EIR_θ constraints. The proof of Lemma 6 shows that $\sum_s r_s(\overline{\theta})\pi_s(\overline{\theta}) = 0$. Hence, $EIR_{\overline{\theta}}$ is binding. To show that EIR_θ holds for $\theta < \overline{\theta}$, note from (29) that the profit of type $\theta < \overline{\theta}$ in state n is

$$\begin{aligned} \pi_n^{**}(\theta) &= t_n^{**}(\theta) - c_n(\theta)q_n^{**}(\theta) \\ &= \int_\theta^{\overline{\theta}} \sum_s r_s(x)c_s'(x)q_s^{**}(x)dx - \frac{M(1-p_n(\overline{\theta}))}{p_n(\overline{\theta})}. \end{aligned}$$

Since by Lemma 6, $\pi_s^{**}(\theta) = M$, $\forall s \neq n$, and recalling that $r_s(x) \equiv \frac{p_s(x)}{p_n(x)}$, the expected profit of type $\theta < \overline{\theta}$ is

$$\begin{aligned} \sum_s p_s(\theta)\pi_s^{**}(\theta) &= (1-p_n(\theta))\,M + p_n(\theta)\left[\int_\theta^{\overline{\theta}} \sum_s r_s(x)c_s'(x)q_s^{**}(x)dx - \frac{M(1-p_n(\overline{\theta}))}{p_n(\overline{\theta})}\right] \\ &= \left(1 - \frac{p_n(\theta)}{p_n(\overline{\theta})}\right) M + p_n(\theta)\int_\theta^{\overline{\theta}} \frac{B^{**}(x)}{p_n(x)}dx, \end{aligned}$$

where $B^{**}(x) \equiv \sum_s p_s(x)c_s'(x)q_s^{**}(x)$. Differentiating the expected profit expression, we get

$$\frac{d}{d\theta}\left(\sum_s p_s(\theta)\pi_s^{**}(\theta)\right) = -\frac{p_n'(\theta)}{p_n(\overline{\theta})}M + p_n'(\theta)\int_\theta^{\overline{\theta}} \frac{B^{**}(x)}{p_n(x)}dx - B^{**}(\theta). \qquad \text{(A-4)}$$

Since $EIR_{\overline{\theta}}$ is binding (so $\sum_s p_s(\overline{\theta})\pi_s^{**}(\overline{\theta}) = 0$), it is sufficient to show that the derivative in (A-4) is negative in order to establish that the EIR_θ constraints are satisfied for all $\theta \in \Theta$.

Our strategy will be to show that the right-hand side of (A-4) is bounded from above and its upper bound is negative for small enough δ.

To find an upper bound for the right-hand side of (A-4), note from the proof of Lemma 6 that,

$$\frac{F(\theta \mid n)}{f(\theta \mid n)} = \frac{\int_{\underline{\theta}}^{\theta} p_n(x) f(x) dx}{p_n(\theta) f(\theta)}.$$

But since by Assumption 1, $p_n(\theta)$ is increasing with θ,

$$\frac{F(\theta \mid n)}{f(\theta \mid n)} \leq \frac{\int_{\underline{\theta}}^{\theta} p_n(\theta) f(x) dx}{p_n(\theta) f(\theta)} = \frac{F(\theta)}{f(\theta)}.$$

On the other hand, since by Assumption 1, $p_s(\theta) \geq \epsilon > 0$ for all $s \in \{1, ..., n\}$ and all $\theta \in \Theta$,

$$\frac{F(\theta \mid n)}{f(\theta \mid n)} \geq \frac{\int_{\underline{\theta}}^{\theta} \epsilon f(x) dx}{f(\theta)} = \frac{\epsilon F(\theta)}{f(\theta)}.$$

Therefore, equation (27) implies that for all $s \in \{1, ..., n\}$ and all $\theta \in \Theta$, $\underline{q}_s^{**}(\theta) \leq q_s^{**}(\theta) \leq \overline{q}_s^{**}(\theta)$, where $\underline{q}_s^{**}(\theta)$ and $\overline{q}_s^{**}(\theta)$ are defined implicitly by

$$S'\left(\underline{q}_s^{**}(\theta)\right) = c_s(\theta) + (1-\alpha)\, c_s'(\theta) \frac{F(\theta)}{f(\theta)},$$

and

$$S'\left(\overline{q}_s^{**}(\theta)\right) = c_s(\theta) + (1-\alpha)\, c_s'(\theta) \frac{\epsilon F(\theta)}{f(\theta)}.$$

Using these expressions, the definition of $B^{**}(\cdot)$, and the assumption that $p_n'(\theta) < \delta$ for all $\theta \in \Theta$, we get

$$\begin{aligned} \frac{d}{d\theta}\left(\sum_s p_s(\theta)\pi_s^{**}(\theta)\right) &< \delta \left[\frac{-M}{p_n(\overline{\theta})} + \int_{\theta}^{\overline{\theta}} \frac{\sum_s p_s(x) c_s'(x) \overline{q}_s^{**}(x)}{p_n(x)} dx\right] - \sum_s p_s(x) c_s'(x) \underline{q}_s^{**}(x) \\ &\leq \delta \left[\frac{-M}{\epsilon} + \int_{\theta}^{\overline{\theta}} \frac{\sum_s c_s'(x) \overline{q}_s^{**}(x)}{\epsilon} dx\right] - \sum_s \epsilon c_s'(x) \underline{q}_s^{**}(x), \qquad \text{(A-5)} \end{aligned}$$

where the second line follows because $\epsilon \leq p_s(\theta) \leq 1$ for all $s \in \{1, ..., n\}$ and all $\theta \in \Theta$. For sufficiently low δ, the right-hand side of (A-5) is negative, so $\frac{d}{d\theta}\left(\sum_s p_s(\theta)\pi_s^{**}(\theta)\right) < 0$. Hence, for sufficiently low δ, the EIR_θ constraints are satisfied for all $\theta \in \Theta$.

It now remains to check that the solution to the regulator's relaxed problem satisfies $IC_{\theta,\widehat{\theta}}$ for all $\theta, \widehat{\theta} \in \Theta$. Substituting the transfers defined by (28) and (29) into the $IC_{\theta,\widehat{\theta}}$, recalling that $r_s(\theta) \equiv \frac{p_s(\theta)}{p_n(\theta)}$ and simplifying, the constraint becomes

$$\begin{aligned} &\int_{\theta}^{\overline{\theta}} \sum_s r_s(x) c_s'(x) q_s^{**}(x) dx - \int_{\widehat{\theta}}^{\overline{\theta}} \sum_s r_s(x) c_s'(x) q_s^{**}(x) dx \\ \geq\; & \sum_s r_s(\theta) \left[c_s(\widehat{\theta}) - c_s(\theta) \right] q_s^{**}(\widehat{\theta}), \qquad \forall \theta, \widehat{\theta} \in \Theta. \end{aligned} \tag{A-6}$$

Now suppose that $\widehat{\theta} \neq \theta$. Then, (A-6) becomes

$$\int_{\theta}^{\widehat{\theta}} \sum_s r_s(x) c_s'(x) q_s^{**}(x) dx \geq \sum_s r_s(\theta) \left[c_s(\widehat{\theta}) - c_s(\theta) \right] q_s^{**}(\widehat{\theta}), \qquad \forall \theta, \widehat{\theta} \in \Theta. \tag{A-7}$$

Integrating the left-hand side by parts, we get

$$\begin{aligned} &\sum_s \left[r_s(\widehat{\theta}) c_s(\widehat{\theta}) q_s^{**}(\widehat{\theta}) - r_s(\theta) c_s(\theta) q_s^{**}(\theta) \right] - \int_{\theta}^{\widehat{\theta}} \sum_s c_s(x) \left[r_s'(x) q_s^{**}(x) + r_s(x) q_s^{**\prime}(x) \right] dx \\ \geq\; & \sum_s r_s(\theta) \left[c_s(\widehat{\theta}) - c_s(\theta) \right] q_s^{**}(\widehat{\theta}), \qquad \forall \theta, \widehat{\theta} \in \Theta. \end{aligned}$$

Rearranging terms and using the fact that $r_s(\widehat{\theta}) - r_s(\theta) = \int_{\theta}^{\widehat{\theta}} r_s'(x) dx$ and $q_s^{**}(\widehat{\theta}) - q_s^{**}(\theta) = \int_{\theta}^{\widehat{\theta}} q_s^{**\prime}(x) dx$,

$$\begin{aligned} &\int_{\theta}^{\widehat{\theta}} \sum_s r_s'(x) c_s(\widehat{\theta}) q_s^{**}(\widehat{\theta}) dx + \int_{\theta}^{\widehat{\theta}} \sum_s r_s(\theta) c_s(\theta) q_s'^{**}(x) dx \\ \geq\; & \int_{\theta}^{\widehat{\theta}} \sum_s c_s(x) \left[r_s'(x) q_s^{**}(x) + r_s(x) q_s^{**\prime}(x) \right] dx, \qquad \forall \theta, \widehat{\theta} \in \Theta. \end{aligned}$$

Rearranging terms once again and multiplying both sides of the inequality by $\frac{2}{\left(\widehat{\theta}-\theta\right)^2}$,

$$\begin{aligned} &\frac{2}{\left(\widehat{\theta}-\theta\right)^2} \int_{\theta}^{\widehat{\theta}} \sum_s -q_s'^{**}(x) \left[r_s(x) c_s(x) - r_s(\theta) c_s(\theta) \right] dx \\ \geq\; & \frac{2}{\left(\widehat{\theta}-\theta\right)^2} \int_{\theta}^{\widehat{\theta}} \sum_s -r_s'(x) \left[c_s(\widehat{\theta}) q_s^{**}(\widehat{\theta}) - c_s(x) q_s^{**}(x) \right] dx, \qquad \forall \theta, \widehat{\theta} \in \Theta. \end{aligned} \tag{A-8}$$

We now establish that as δ goes to 0, the left-hand side of (A-8) converges to a strictly positive term while the right-hand side has an upper bound that converges to 0. We begin with the right-hand side of (A-8). Since $r_s(x) = \frac{p_s(x)}{p_n(x)}$, $-r_s'(x) = \frac{p_s(x)p_n'(x) - p_s'(x)p_n(x)}{(p_n(x))^2}$. But since by assumption, $|p_s'(\cdot)| < \delta$ and $\epsilon \leq p_s(\theta) \leq 1$, it follows that $-r_s'(x) \leq \frac{2\delta}{\epsilon^2}$. Using this inequality and the fact that $c_s'(\cdot) \geq 0$ and $q_s^{**\prime}(\cdot) \leq 0$, yields

$$\begin{aligned}
& \frac{2}{\left(\widehat{\theta}-\theta\right)^2} \int_\theta^{\widehat{\theta}} \sum_s -r_s'(x) \left[c_s(\widehat{\theta}) q_s^{**}(\widehat{\theta}) - c_s(x) q_s^{**}(x)\right] dx \\
= & \frac{2}{\left(\widehat{\theta}-\theta\right)^2} \int_\theta^{\widehat{\theta}} \sum_s -r_s'(x) \left[\int_x^{\widehat{\theta}} \left[c_s'(z) q_s^{**}(z) + c_s(z) q_s'^{**}(z)\right] dz\right] dx \\
\leq & \frac{2}{\left(\widehat{\theta}-\theta\right)^2} \int_\theta^{\widehat{\theta}} \sum_s \frac{2\delta}{\epsilon^2} \left[\int_x^{\widehat{\theta}} c_s'(z) q_s^{**}(z) dz\right] dx \\
\leq & \frac{2}{\left(\widehat{\theta}-\theta\right)^2} \int_\theta^{\widehat{\theta}} \sum_s \frac{2\delta}{\epsilon^2} \left[c_s'(\overline{\theta}) q_s^{**}(\underline{\theta}) \int_x^{\widehat{\theta}} dz\right] dx \\
= & \frac{2}{\left(\widehat{\theta}-\theta\right)^2} \int_\theta^{\widehat{\theta}} \sum_s \frac{2\delta}{\epsilon^2} \left[c_s'(\overline{\theta}) q_s^{**}(\underline{\theta}) \left(\widehat{\theta} - x\right)\right] dx \\
= & \frac{2}{\left(\widehat{\theta}-\theta\right)^2} \sum_s \frac{2\delta}{\epsilon^2} c_s'(\overline{\theta}) q_s^{**}(\underline{\theta}) \frac{\left(\widehat{\theta}-\theta\right)^2}{2} \\
= & \frac{2\delta}{\epsilon^2} \sum_s c_s'(\overline{\theta}) q_s^{**}(\underline{\theta}).
\end{aligned}$$

Clearly, this expression converges to 0 as δ goes to 0. As for the left-hand side of (A-8), recall that for all $s \in \{1, ..., n\}$ and all $\theta \in \Theta$, $\underline{q}_s^{**}(\theta) \leq q_s^{**}(\theta) \leq \overline{q}_s^{**}(\theta)$. Since $q_s^{**\prime}(\cdot) \leq 0$, it follows that $q \in \left[\underline{q}_s^{**}(\overline{\theta}), \overline{q}_s^{**}(\underline{\theta})\right]$. Let

$$k \equiv \max_s \max_q \left\{|S"(q)| \mid q \in \left[\underline{q}_s^{**}(\overline{\theta}), \overline{q}_s^{**}(\underline{\theta})\right]\right\},$$

be the upper bound on $|S"(q^{**}(\theta))|$. Then, equation (27) implies that

$$-q_s'^{**}(\theta) = \frac{c_s'(\theta) + (1-\alpha)\left(c_s"(\theta)\frac{F(\theta|n)}{f(\theta|n)} + c_s'(\theta)\frac{d}{d\theta}\left(\frac{F(\theta|n)}{f(\theta|n)}\right)\right)}{|S"\left(q_s^{**}(\theta)\right)|} \geq \frac{c_s'(\underline{\theta})}{k}.$$

Using this inequality, noting that $r_s(\cdot) \equiv \frac{p_s(\cdot)}{p_n(\cdot)} \geq \epsilon$, and recalling that $-r_s'(\cdot) \leq \frac{2\delta}{\epsilon^2}$ and

$c_s'(\cdot) > 0$, we get

$$\begin{aligned}
& \frac{2}{\left(\widehat{\theta}-\theta\right)^2}\int_\theta^{\widehat{\theta}}\sum_s -q_s'^{**}(x)\left[r_s(x)c_s(x)-r_s(\theta)c_s(\theta)\right]dx \\
= & \frac{2}{\left(\widehat{\theta}-\theta\right)^2}\int_\theta^{\widehat{\theta}}\sum_s -q_s'^{**}(x)\left[\int_\theta^x \left[r_s'(z)c_s(z)+r_s(z)c_s'(z)\right]dz\right]dx \\
\geq & \frac{2}{\left(\widehat{\theta}-\theta\right)^2}\int_\theta^{\widehat{\theta}}\sum_s -q_s'^{**}(x)\left[\int_\theta^x \left[-\frac{2\delta}{\epsilon^2}c_s(z)+\epsilon c_s'(z)\right]dz\right]dx \\
\geq & \frac{2}{\left(\widehat{\theta}-\theta\right)^2}\int_\theta^{\widehat{\theta}}\sum_s -q_s'^{**}(x)\left[\int_\theta^x \left[-\frac{2\delta}{\epsilon^2}c_s(\overline{\theta})+\epsilon c_s'(\underline{\theta})\right]dz\right]dx \\
= & \frac{2}{\left(\widehat{\theta}-\theta\right)^2}\int_\theta^{\widehat{\theta}}\sum_s -q_s'^{**}(x)\left[-\frac{2\delta}{\epsilon^2}c_s(\overline{\theta})+\epsilon c_s'(\underline{\theta})\right](x-\theta)dx \\
\geq & \frac{2}{\left(\widehat{\theta}-\theta\right)^2}\sum_s \frac{c_s'(\underline{\theta})}{k}\left[-\frac{2\delta}{\epsilon^2}c_s(\overline{\theta})+\epsilon c_s'(\underline{\theta})\right]\frac{\left(\widehat{\theta}-\theta\right)^2}{2} \\
= & \sum_s \frac{c_s'(\underline{\theta})}{k}\left[-\frac{2\delta}{\epsilon^2}c_s(\overline{\theta})+\epsilon c_s'(\underline{\theta})\right].
\end{aligned}$$

As δ goes to 0, this expression converges to $\sum_s \frac{(c_s'(\underline{\theta}))^2\epsilon}{k} > 0$. Hence, for a sufficiently small δ, (A-8) hold, implying that $IC_{\theta,\widehat{\theta}}$ holds for all $\theta, \widehat{\theta} \in \Theta$. ■

6 References

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