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ABSTRACT

Business Cycles with Free Entry Ruled by Animal Spirits*

The Paper approaches business cycles in terms of extrinsic uncertainty related, not to dynamic indeterminacy of intertemporal equilibria (in the neighborhood of an attractor) or to multiplicity of steady states (in non-linear models), but to static indeterminacy of free-entry oligopolistic equilibria within each period. We consider an OLG economy in which firms, supplying differentiated goods within each one of many sectors, and producing under increasing returns-to-scale, compete in prices in perfectly contestable markets. The number of active firms is shown to vary across sectoral equilibria, depending upon the (correct) producers' conjectures on the actions of their competitors. These conjectures are assumed to be coordinated by some extrinsic Markov chain, thus generating endogenous shocks in both the mark-up factor and productivity, and resulting in perturbations of the dynamic system (as in the case of exogenous random shocks). Consumers' expectations may magnify the extrinsic uncertainty characterizing producers' conjectures. Since the source of fluctuations does not rely on dynamic indeterminacy, we can weaken the condition on the degree of increasing returns, which may be arbitrarily small (with a moderate positive elasticity of labour supply), provided goods' substitutability within each sector becomes arbitrarily large.

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1 Introduction

The study of business cycles has been pursued in the two last decades according to two distinct approaches. The first assumes that cycles are driven by exogenous random shocks on fundamentals, in particular, as in the Real Business Cycle literature, productivity disturbances, the propagation of which reflects the structure of the economy. By contrast, the second supposes a deterministic environment with equilibrium indeterminacy, accounting for the emergence of endogenous, possibly stochastic, fluctuations associated with changing but self-fulfilling expectations ("animal spirits" or "sunspots").[1] The two approaches should in principle be seen as complementary rather than substitutable, since spontaneous variations in expectations may well coexist with shocks on fundamentals, modulating their propagation and virtually amplifying their effects. However, the two approaches have been linked up with two conflicting views of the market system, assumed in the first to be a complete system of perfectly competitive markets, and supposed in the second to be stricken with some market failure, typically due to the presence of externalities or to the exercise of market power.

Net business formation, that is, the variation in the number of active firms, may play a significant role in the latter approach, because of the existence of network externalities between firms in different sectors, and also because of the dependence of economies of scale as well as of markups on the degree of concentration. Clearly, the number of active firms must be viewed as endogenous, since it results from individual decisions to participate in the production process. Usually, this number is supposed to be uniquely determined by the zero profit condition, considered to be implied by the free entry assumption. More precisely, a *sequential symmetric game* is assumed, in which firms freely decide at the first stage whether to participate and, if they do, choose market strategies (prices or quantities) at the second stage. Under the implicit condition that all participating firms benefit from an *equal treatment* at the equilibrium of this stage, their number must be such that profits (net of participating costs) be zero, at least approximately because of the so-called integer number problem. Perturbations of market demand, resulting from shocks either on fundamentals or on expectations and acting upon individual scales, will then induce variations in the equilibrium number of active firms.[2]

By introducing heterogeneity inside each generation of consumers-investors-producers (in the form of a non-degenerate distribution of participation costs), together with strategic complementarity due to the presence of network externalities across sectors, Chatterjee and Cooper [10] and Chatterjee, Cooper and Ravikumar [12] obtain multiplicity of steady state equilibria. This allows them to construct sunspot equilibria, such that the state variables randomly jump from the neighborhood of some steady state to the neighborhood of another.

[1] Surveys of these two approaches are given by King and Rebelo [20] and Benhabib and Farmer [7], respectively.

[2] See for instance Chatterjee and Cooper [11], Portier [22] or Dos Santos Ferreira and Lloyd-Braga [15].

As stressed by the authors, the source of multiplicity is here to be found in the static *asymmetric* structure of the model, independently of the presence of intertemporal linkages between the variables. In other words, what is crucial is the *static indeterminacy* of the equilibrium of the game played at any period by young producers, rather than any specifically dynamic property of the model.

In the present paper, we also adopt the convenient overlapping generations setting, while reverting to the more conventional distinction between consuming and investing households with a multi-period horizon on the one hand, and producing firms that rationally refer to a one-period horizon on the other hand. We further admit firms to be involved in any period and in any sector in a *simultaneous symmetric game*, an option that appears more natural in this context than assuming a sequential game resulting from the artificial subdivision of the period into different stages. Under *free entry* (resulting from the existence of an unbounded set of identical players, together with the absence of participation costs), and because of internal economies of scale, there will be both active and inactive firms at any Nash equilibrium of that game. By definition, both categories rationally decide, according to their specific mutually consistent conjectures, either to participate in the production process, at the activity level leading to the maximum attainable non-negative profit, or to stay inactive as the only way for them to avoid negative profits.[3] Multiplicity of such Nash equilibria, with an indeterminate number of active firms, is not difficult to obtain in spite of the complete game symmetry, as shown by d'Aspremont, Dos Santos Ferreira and Gérard-Varet [1]. The zero profit condition thus appears as just a particular criterion of equilibrium selection. Any other equilibrium is nevertheless legitimate. As a consequence, the *equilibrium* number of active firms is prone to undergo variations in response to changing self-fulfilling producers' conjectures on their competitors' decisions, conjectures which play a role similar to that of consumers-investors' expectations in the usual "animal spirits" story. One may notice that our story comes in fact even closer than the usual one to the original Keynes' [19] view of animal spirits as "a spontaneous urge to action rather than inaction" (p. 161).

Formally, our approach amounts to imposing random shocks on the number of active firms as if it were exogenous, thus somewhat reproducing the Real Business Cycle approach, in spite of a quite different interpretation. Of course, as that number is in fact endogenous, it must be compatible with equilibrium conditions, in order for such random shocks to be admissible. Those conditions will however appear to be weaker than those which allow to derive existence of sunspot equilibria either from *dynamic* indeterminacy in the neighborhood of some attractor, or from multiplicity of steady states.[4] They involve only an

[3] This quite standard concept of Nash equilibrium *with free entry* has been introduced by Novshek [21] in the context of Cournot competition.

[4] For an introduction to the study of stochastic endogenous fluctuations, in non-linear two-dimensional models in particular, see Grandmont [17], opening the symposium on this topic in the *Journal of Economic Theory.* Benhabib [5], introducing the symposium on sunspots in macroeconomics in the same review, comments on the importance of weakening the conditions for dynamic indeterminacy assumed in the earlier models of stochastic endogenous

arbitrarily small degree of internal economies of scale (although in the form of increasing marginal returns) and a moderate (positive) elasticity of labor supply, and do not depend upon non-linearities. They are compatible with dynamic determinacy, as sunspots result in our model from perturbations of the (log-linear) dynamic system itself, due to spontaneous variations of the *static* conjectures ruling the producers' activity level, their participation in particular. Also, given the possibility of such variations, volatile consumers' expectations can play their usual part, adding to the extrinsic uncertainty related to producers' conjectures.

We use an overlapping generations model, where finance-constrained consumers live for an arbitrary finite number of periods (allowing for a period length that makes sense in the business cycle context). The final good is competitively produced from intermediate goods supplied by price-setting firms, which specialize in one variety of some class. We thus assume multisector production of intermediate goods, with a differentiated oligopoly in prices within each sector.[5] Section 2 presents the model, and section 3 defines and characterizes the ensuing intertemporal equilibria. Section 4 establishes conditions for existence of stationary sunspot equilibria ruled by finite Markov chains or, more precisely, for existence of an equilibrium associated with any given Markov chain, and for existence of a set of Markov chains supporting a given configuration of stationary random values of the endogenous variables. Our model has been designed for establishing essentially qualitative results, and we did not attempt to evaluate its performance on the basis of a calibrated version. However, we briefly discuss in section 5 its capacity to reproduce at least the typically observed signs of the correlations between endogenous variables. We conclude in section 6.

2 The model

We use an overlapping generations model for convenience: With some frequent simplifying assumptions, like strongly separable preferences, Cobb-Douglas technologies and complete capital depreciation, this framework leads to a log-linear dynamic system in the deterministic case. No specific properties of overlapping generations economies are however involved in our results. By assuming that consumers live for many periods, being finance constrained within each period, we get rid of the of the well-known problem associated with two-period-lived agents, that of obtaining existence of fluctuations with a too low frequency (see Woodford [26]). Admittedly, the assumption of complete capital depreciation may then appear inadequate, and is only introduced for easiness of exposition. Without it we lose log-linearity, but our argument may be transposed to the log-linearized system around the steady state.

The structure of our economy has been recurrently used in the literature (since the Symposium on Growth, Fluctuations, and Sunspots, in the *J. Econ.*

fluctuations.

[5] The oligopolistic structure is crucial, not the form of competition. Dos Santos Ferreira and Dufourt [14] build on the same kind of indeterminacy assuming Cournotian competition, in a calibrated model with infinitely lived consumers.

Theory **63** (1994): see Benhabib and Rustichini [8]). It consists of a consumer sector, composed here by a finite number $T+1$ of coexisting generations of representative consumers, a competitive sector producing one final good, and a large number m of imperfectly competitive sectors producing intermediate goods. In each one of these sectors, a small endogenous number n of active firms produce highly substitutable goods under moderately increasing returns, and compete in prices in contestable markets.

2.1 Producers

One final good, used both for consumption and investment, is produced out of mn intermediate goods. These are produced by monopolistic firms, each one supplying a specific variety, among N potential varieties, of one of m classes of intermediate goods.

The final sector may be described by the perfectly competitive behavior of a representative firm, with the following program:

$$\max_{(y,Y)\in[0,\infty)^{mN+1}} \left\{ PY - \sum_{i=1}^{m}\sum_{j=1}^{N} p_{ij}y_{ij} \mid Y \leq m \prod_{i=1}^{m} \left(\sum_{j=1}^{N} y_{ij}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}\frac{1}{m}} \right\}, \tag{1}$$

where Y is the output of final good (sold at price P), y_{ij} is the input of the j-th variety of the i-th class of intermediate goods (purchased at price p_{ij}), and $\sigma \in]1,\infty[$ is the constant elasticity of substitution between any two varieties of the same class of intermediate goods (the corresponding elasticity between classes of goods being 1). By convention, we admit that $p_{ij}=\infty$ whenever the the j-th variety of the i-th class of intermediate goods is actually not produced and, accordingly, that $p_{ij}y_{ij}=0$ even with an infinite price, as soon as the quantity is zero. As well known, the solution to the above program (when prices are compatible with positive production) is:

$$y_{ij} = \left(\frac{p_{ij}}{\mathcal{P}(p_i)}\right)^{-\sigma} \frac{PY}{m\mathcal{P}(p_i)}, \text{ with} \tag{2}$$

$$\mathcal{P}(p_i) = \left(\sum_{j=1}^{N} p_{ij}^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \text{ and } P = \prod_{i=1}^{m} \mathcal{P}(p_i)^{\frac{1}{m}}. \tag{3}$$

Because of the constancy of returns to scale, the scale Y is determined only at the economy's level.

Intermediate goods are produced out of capital and labor according to a technology displaying increasing returns:

$$y_{ij} \leq \left[F(k_{ij}, l_{ij})\right]^{\frac{1}{\gamma}}, \tag{4}$$

where F is a neoclassical production function, and $0<\gamma<1$. Also, the assumptions of homogeneity of degree 1 of F and of perfect competition in factor markets, entailing constancy of the average cost, allow us to approach the problem of the firm producing the j-th variety of the i-th class of intermediate goods by first considering the cost minimization of one unit of output:

$$\min_{(k_{ij},l_{ij})\in[0,\infty)^2} \{Rk_{ij} + Wl_{ij} \mid F(k_{ij}, l_{ij}) \geq 1\} \equiv c(R, W), \tag{5}$$

where c is the unit cost function, with arguments R and W, the rental of capital and the wage, respectively.

For simplicity, we assume a Cobb-Douglas production function: $F(k_{ij}, l_{ij}) = ak_{ij}^{\alpha} l_{ij}^{1-\alpha}$ (with $a > 0$ and $0 < \alpha < 1$). By first order conditions, we then obtain:

$$c(R, W) = \frac{1}{a}\left(\frac{R}{\alpha}\right)^{\alpha}\left(\frac{W}{1-\alpha}\right)^{1-\alpha} \text{ and } \frac{R}{W} = \frac{\alpha}{1-\alpha}\frac{l_{ij}}{k_{ij}}. \tag{6}$$

By contrast with the final good and factor markets, oligopolistic price competition prevails in the intermediate sector: Firms behave in their output markets as monopolists facing the objective demand of the final sector, contingent upon the prices set by the other producers in the same sector (because of the unit elasticity of substitution between classes of goods, prices in other sectors are irrelevant). Hence, firm j's program in the i-th sector is:

$$\max_{p_{ij}\in[0,\infty]} \left\{ \left(\frac{p_{ij}}{\mathcal{P}\left(p_{ij}, p_{i(-j)}\right)}\right)^{1-\sigma} b - c(R, W)\left[\left(\frac{p_{ij}}{\mathcal{P}\left(p_{ij}, p_{i(-j)}\right)}\right)^{1-\sigma} \frac{b}{p_{ij}}\right]^{\gamma} \right\}, \tag{7}$$

with the standard notation $p_{i(-j)} = (p_{i1}, ..., p_{ij-1}, p_{ij+1}, ..., p_{iN})$, and with $b = PY/m$ if firm j correctly perceives the objective demand.

2.2 Consumers

We assume that each generation is formed by a continuum, of mass normalized to $1/T$, of identical households living for $T+1$ periods. In period t, any household is assumed to face two constraints. The first is the standard *budget constraint*:

$$C_t + K_t \leq w_t L_t + d_t + (r_t + 1 - \delta) K_{t-1}, \tag{8}$$

where C_t denotes consumption, K_t capital at the end of period t, $w_t \equiv W_t/P_t$ the real wage, L_t employment, d_t real dividends (assumed to be received up to age $T-1$), $r_t \equiv R_t/P_t$ the real rental of capital, and δ the rate of depreciation. The second is a *finance constraint*:

$$C_t \leq (r_t + 1 - \delta) K_{t-1}, \tag{9}$$

imposing (in the spirit of Woodford [26]) that consumption spending be financed by income guaranteed by physical capital (as opposed to human capital) in the beginning of the period. We further assume strong separability, with respect to states of nature, dates and goods (final consumption and labor), of the expected utility of the representative household of age $\tau \in \{0, 1, ..., T\}$, at period t_0:

$$E_{t_0}\left[\sum_{t=t_0}^{t_0+T-\tau}\beta^{t-t_0}\left(U\left(C_t\right)-V\left(L_t\right)\right)\right], \tag{10}$$

where E_{t_0} denotes the mathematical expectation operator, conditional on information available at t_0, and U is assumed to be increasing and concave, V to be increasing and strictly convex, and the subjective discount factor β to belong to the interval $]0, 1[$.

Using the Lagrangian

$$E_{t_0}\left[\sum_{t=t_0}^{t_0+T-\tau}\beta^{t-t_0}\left(\begin{array}{c} U\left(C_t\right)-V\left(L_t\right) \\ -\lambda_t^b\left(C_t+K_t-w_tL_t-d_t-\left(r_t+1-\delta\right)K_{t-1}\right) \\ -\lambda_t^f\left(C_t-\left(r_t+1-\delta\right)K_{t-1}\right)\end{array}\right)\right], \tag{11}$$

we can formulate the following first order conditions for optimizing choices in period t_0:

$$\begin{aligned} U'\left(C_{t_0}\right) &= \lambda_{t_0}^b+\lambda_{t_0}^f,\ V'\left(L_{t_0}\right)/w_{t_0}=\lambda_{t_0}^b, \text{ and} \\ \lambda_{t_0}^b &= \beta E_{t_0}\left[\left(\lambda_{t_0+1}^b+\lambda_{t_0+1}^f\right)\left(r_{t_0+1}+1-\delta\right)\right]. \end{aligned} \tag{12}$$

Assuming for simplicity isoelastic sub-utility functions: $U(C) = (1/\rho)\,C^\rho$ and $V(L) = (v/\psi)\,L^\psi$, with parameters $\rho \in (0, 1]$, $\psi \in (1, \infty)$, and $v \in (0, \infty)$, the former conditions can be more explicitly formulated, in the case of binding financial constraints in periods t_0 and t_0+1 (the budget constraints being always binding because of an increasing V):

$$C_{t_0} = \left(r_{t_0}+1-\delta\right)K_{t_0-1} < K_{t_0}\left(\beta E_{t_0}\left[\left(r_{t_0+1}+1-\delta\right)^\rho\right]\right)^{\frac{1}{\rho-1}}, \tag{13}$$

$$L_{t_0} = \left(\frac{\beta}{v}w_{t_0}E_{t_0}\left[\left(r_{t_0+1}+1-\delta\right)^\rho\right]K_{t_0}^{\rho-1}\right)^{\frac{1}{\psi-1}}, \text{ and} \tag{14}$$

$$K_{t_0} = w_{t_0}L_{t_0}+d_{t_0}. \tag{15}$$

These conditions suppose a high enough degree of time preference $1/\beta$, in order for the finance constraints to be binding, and also a high enough degree of disutility of labor in terms of future expected consumption, as expressed by v/β, in order for L_{t_0} to be an interior solution, smaller than the labor endowment. This will always be assumed in the following.

The preceding conditions are independent upon the age τ, except that $C_{t_0} = 0$ if $\tau = 0$, as $K_{t_0-1} = 0$, and that $L_{t_0} = 0$ if $\tau = T$, since the household has then reached the last year of its life (so that $K_{t_0} = 0$), and C_{t_0} is assumed to be finance constrained. Thus, given that consumption decisions are the same for consumers of any age higher than 0, and that labor supply decisions are also the same for consumers of any age up to $T-1$, we must multiply individual quantities by T (the number of generations involved) times $1/T$ (the mass of each generation), making aggregates coincide with individual decisions. Notice also that, as all the profits are generated in the intermediate goods sectors and assumed to be equally distributed to consumers up to age $\tau = T-1$, we obtain as a condition for general equilibrium: $K_{t_0} = w_{t_0} L_{t_0} + d_{t_0} = Y_{t_0} - r_{t_0} K_{t_0 - 1}$.

3 Equilibrium

In order to characterize the intertemporal general equilibrium of this economy, we consider successively as its component blocks partial equilibrium in each sector and general equilibrium at any isolated period.

3.1 Sectoral equilibrium

We begin by defining an equilibrium in sector i:

Definition 1 *An equilibrium in the i-th sector, given the common conjecture on aggregate final expenditure b and the competitive prices R and W in the factor markets, is a vector $(p^*_{i1}, ..., p^*_{iN}) \in [0, \infty]^N$ such that p^*_{ij} is a solution to producer j's program (7), under the conjectures $p^*_{i(-j)}$.*

This is a standard concept of Nash equilibrium in prices. By extending the strategy spaces to infinite prices, we allow firms to rationally choose to be inactive, according to their (correct) conjectures on other firms' choices. If at least one such firm exists (the equilibrium number n of active firms is smaller than N), we may speak of a *sectoral free entry equilibrium*, since inactive firms are not hindered by any cost or product differentiation disadvantage with respect to active firms.

In the following, we shall limit our analysis to sectorial equilibria that are symmetric with respect to active firms. In other words, there are n firms setting the same finite price p leading to positive profits, and $N-n$ firms setting an infinite price and getting zero profits. We establish conditions for existence of such a sectoral free entry equilibrium in the following Proposition.

Proposition 1 *Under the condition: $1 < 1/(1-\gamma) < \sigma < (1+\gamma)/(1-\gamma)$, there exists a sectoral free entry equilibrium, such that n active firms set the price*

$$p_n = \left[\mu(n)\gamma c(R,W) b^{\gamma-1} n^{1-\gamma}\right]^{1/\gamma}, \text{ with} \tag{16}$$

$$\mu(n) = \frac{1+(n-1)\sigma}{(n-1)(\sigma-1)}, \tag{17}$$

the markup factor on marginal cost $\gamma c(R,W)(b/np)^{\gamma-1}$, provided n belongs to some (non degenerate) interval $[\max\{2, \underline{n}\}, \min\{N, \overline{n}\}]$, verifying:

$$\overline{n} = 1 + \frac{\gamma}{\sigma(1-\gamma)-1} \text{ and } (\underline{n}/\overline{n})^{\sigma(1-\gamma)-1}\left[\mu(\underline{n})/\mu(\overline{n})\right]^{\sigma-1} = (1-1/\overline{n})^{-\gamma}. \tag{18}$$

Proof. See Appendix A. ■

The restriction upon the admissible number of active firms stems first from profits becoming negative as soon as n is larger than $\overline{n}$, so that it would no more be optimal for anyone of n active firms to set the price p_n, as given by (16). Second, it would cease to be optimal for an inactive firm to set an infinite price and abstain from producing, if n were smaller than $\underline{n}$.

Clearly, if the interval $[\max\{2, \underline{n}\}, \min\{N, \overline{n}\}]$ is large enough to contain more than one integer, the symmetric sectoral free entry equilibrium is indeterminate, and we must rely on an explicit or implicit selection mechanism, coordinating the producers on some specific conjectures of their competitors' choices. When characterizing an equilibrium by the pair (n, p), we assume that each one of n active (resp. $N - n$ inactive) firms rightly conjectures that $n - 1$ (resp. n) competitors set the price p, whereas $N - n$ (resp. $N - n - 1$) others choose an infinite price.

3.2 General temporary equilibrium

We now consider a general temporary equilibrium, by imposing, in addition to conditions for free entry equilibria in all sectors, that monopolistic producers' conjectures on the final demand be correct, and that the competitive prices clear the final good and factor markets, given individually rational consumers' choices. These are made on the basis of (uncertain) expectations of the future real rental of capital which are represented by the random variable $\widetilde{r}_{t+1}$, associated with some probability distribution conditional on the information available at t. Again, we limit our analysis to equilibria that are symmetric with respect to active monopolistic firms, both inside each sector and across sectors.

Definition 2 *A symmetric general temporary equilibrium at period t, given the inherited aggregate capital resources K_{t-1} and consumers' uncertain expectations (conditional on information at t) of the future real rental price of capital $\widetilde{r}_{t+1}$, is a configuration of symmetric free entry equilibria (n_t, p_t), identical for each sector, at final expenditure $b_t = P_t Y_t / m$ (with the competitive price P_t and aggregate output Y_t) and factor prices (R_t, W_t), leading to employment L_t, such that:*

$$\frac{p_t}{P_t} = n_t^{1/(\sigma-1)}, \tag{19}$$

$$Y_t = A(n_t) K_{t-1}^{\alpha/\gamma} L_t^{(1-\alpha)/\gamma}, \text{ with } A(n_t) \equiv a^{1/\gamma} m^{1-1/\gamma} n_t^{\sigma/(\sigma-1)-1/\gamma}, \tag{20}$$

$$\frac{R_t}{P_t} = \frac{\alpha}{\gamma} \frac{Y_t}{\mu(n_t) K_{t-1}}, \tag{21}$$

$$\frac{W_t}{P_t} = \frac{1-\alpha}{\gamma} \frac{Y_t}{\mu(n_t) L_t} \text{ and} \tag{22}$$

$$L_t = \left[\frac{\beta}{v} \frac{W_t}{P_t} \left(Y_t - \frac{R_t}{P_t} K_{t-1} \right)^{\rho-1} E_t \left[(\widetilde{r}_{t+1} + 1 - \delta)^{\rho} \right] \right]^{\frac{1}{\psi-1}}. \tag{23}$$

Equations (19) and (20), stemming from (1), (3) and (4), express the conditions for equilibrium in the final good market: equality of price and cost and equality of demand and output. Equations (21) and (22), derived from (6) and (16), express conditions for equilibrium in the factor markets. Finally, equation (23) is simply condition (14) for an individually rational choice of labor supply.

3.3 Intertemporal equilibrium

Intertemporal equilibrium requires that, in every period, conditions for a symmetric general temporary equilibrium be satisfied, that capital be accumulated according to consumers' saving decisions, and finally that consumers' expectations be rational.

Definition 3 *A symmetric intertemporal equilibrium is a stochastic process* $\left\{\left(\widetilde{z}_t, \widetilde{K}_t, \widetilde{r}_t\right)\right\}_{t\in\mathbb{N}^*}$ *such that, for any* $t \in \mathbb{N}^*$ *and any state of nature, the corresponding realization* (z_t, K_t, r_t) *satisfies:*

(i) $z_t = ((n_t, p_t), P_t, Y_t, R_t, W_t, L_t)$ *is a symmetric general temporary equilibrium, given the realized capital resource* K_{t-1} *and consumers' expectations* $\widetilde{r}_{t+1}$ *(conditional on information at* t*);*

(ii)

$$K_t = Y_t - r_t K_{t-1}; \tag{24}$$

(iii)

$$r_t = R_t / P_t. \tag{25}$$

By using conditions (19) to (23) for a temporary equilibrium, the rule (24) of capital accumulation and the condition (25) for rational expectations can be written:

$$K_t = (1 - \alpha/\gamma\mu(n_t)) A(n_t) K_{t-1}^{\alpha/\gamma} L_t^{(1-\alpha)/\gamma} \tag{26}$$

$$K_t^{\rho\alpha/\gamma} E_t\left[\left(\frac{\alpha}{\gamma}\frac{A(n_{t+1})}{\mu(n_{t+1})} L_{t+1}^{(1-\alpha)/\gamma} + 1 - \delta\right)^{\rho}\right] = \frac{v}{\beta(1-\alpha)}\left[\gamma\mu(n_t) - \alpha\right] L_t^{\psi}, \tag{27}$$

Notice that an increase in n_t has two kinds of effects on capital accumulation. The first one works through the (decreasing) function A, and itself results from two opposite effects: more product variety enhances productivity (the analogue of the "taste for variety" effect in Dixit and Stiglitz [13]), and a higher number of firms determines a loss in economies of scale. One of the assumptions required for indeterminacy of the sectoral equilibrium ($\sigma > 1/(1-\gamma)$) is precisely that the second, negative, effect dominates. The second kind of effects, also negative on capital accumulation, works through the decreasing markup function μ: a rise in n_t diminishes the markup factor, increasing the capital share of income, the one devoted to consumption (because of the finance constraint), and thus reducing saving.

4 Existence of stationary sunspot equilibria

Equations (26) and (27) constitute a two-dimensional dynamic system with one pre-determined variable K. For simplicity, we will in the following assume complete capital depreciation ($\delta = 1$), leading to log-linearity of the dynamic system in the deterministic, autonomous case (with constant $n_t = n$). The corresponding Jacobian matrix

$$J = \begin{bmatrix} 1 & 0 \\ -\alpha/(1-\alpha) & \gamma/\rho(1-\alpha) \end{bmatrix} \begin{bmatrix} \alpha/\gamma & (1-\alpha)/\gamma \\ 0 & \psi \end{bmatrix} \tag{28}$$

has determinant $D = \alpha\psi/\rho(1-\alpha)$, trace $T = \gamma\psi/\rho(1-\alpha)$, and eigenvalues

$$\lambda_1 = \frac{\psi}{2(1-\alpha)\rho}\left(\gamma - \sqrt{\gamma^2 - 4\alpha(1-\alpha)\rho/\psi}\right) \text{ and} \tag{29}$$

$$\lambda_2 = \frac{\psi}{2(1-\alpha)\rho}\left(\gamma + \sqrt{\gamma^2 - 4\alpha(1-\alpha)\rho/\psi}\right). \tag{30}$$

In order to clearly differentiate our analysis from contributions resorting to indeterminacy of the steady state to establish existence of sunspot equilibria, we assume in the following that the steady state of this system is a saddle: $0 < \lambda_1 < 1 < \lambda_2$, implying $1 - T + D < 0$, or $\gamma > \alpha + (1-\alpha)\rho/\psi$. The right-hand side of the last inequality is smaller than 1, so that the inequality is satisfied for arbitrarily small economies of scale, whatever the value of ρ/ψ. By contrast, relying on the standard argument of indeterminacy of the steady state requires either a large enough degree $1/\gamma - 1$ of scale economies[6] or a ratio ρ/ψ close to 1, *i.e.* an elasticity of labor supply close to infinity,[7] as in Cazzavillan [9] and Dos Santos Ferreira and Lloyd-Braga [15].[8]

[6] For instance, Farmer and Guo [16] use $1/\gamma = 1.61$ (with a markup factor $\mu = 1.72$). These values are usually considered excessive, taking into account observed data (see for instance Basu and Fernald [3] and [4]). One of the explicit objectives of part of the literature building upon the foundations laid by the seminal Farmer and Guo paper has been to construct calibrated models where indeterminacy arises with empirically plausible values of $1/\gamma$: see Benhabib and Farmer [?], (1.08, assuming sector specific externalities in a model with two market sectors), Perli [23] (1.1, assuming sector specific externalities in a model with a market sector and a non-market sector), Barinci and Chéron [2] (1.3, assuming externalities in a one-sector finance constrained economy), Schmitt-Grohé [24] (1.1, assuming internal increasing returns and markup variability along the cycle), Weder [25] (1.07, assuming internal increasing returns in the investment sector only).

[7] From (14) and (15) the elasticity of labor supply with respect to the real wage is given by

$$\epsilon_w L = \psi / [\psi - 1 + (1-\rho) wL/(wL + d)] - 1 \in [(\rho/\psi)/(1-\rho/\psi), 1/(\psi - 1)].$$

Linearity of labor disutility ($\psi = 1$) or, equivalently, labor indivisibility (Hansen [18]), leading to infinite elasticity of labor supply, has been an almost universal specification since Farmer and Guo [16]. Exceptions are, for part of their simulations, Perli [23] and Weder [25].

[8] As economies of scale become arbitrarily small (as γ tends to 1), indeterminacy of sectoral equilibrium admittedly requires in our model that substitutability between varieties of the same good become arbitrarily high (that σ tend to ∞). This condition of intense competition inside each sector seems however less restrictive than the condition of (almost) infinite labor supply elasticity.

We now proceed to a change in variables, in order to obtain an autonomous one-dimensional sub-system in a new variable, corresponding to trajectories along the unstable branch of the saddle (the line generated by the eigenvector associated with λ_2). This is a preliminary step to introducing stationary stochastic trajectories of this new variable. To be explicit, we introduce the variable Λ_t, such that

$$\Lambda_t^{1/\rho} = \frac{\alpha}{\gamma} \frac{A(n_t)}{\mu(n_t)} K_{t-1}^{\alpha/\gamma - \lambda_1} L_t^{(1-\alpha)/\gamma}. \tag{31}$$

By denoting

$$H(n_t) = \frac{v}{\beta} \frac{\alpha}{1-\alpha} \left[(\gamma/\alpha) \mu(n_t) - 1 \right]^{1-\rho\lambda_1} \left[\frac{\alpha}{\gamma} \frac{A(n_t)}{\mu(n_t)} \right]^{-\rho(\lambda_1+\lambda_2)}, \tag{32}$$

we then obtain the following decomposable dynamic system:

$$\begin{aligned} K_t &= \left[(\gamma/\alpha) \mu(n_t) - 1 \right] \Lambda_t^{1/\rho} K_{t-1}^{\lambda_1} & (33) \\ E_t \left[\Lambda_{t+1} \right] &= H(n_t) \Lambda_t^{\lambda_2}. & (34) \end{aligned}$$

We want to establish conditions for existence of stochastic stationary solutions of equation (34). In the absence of intrinsic uncertainty, we assume occurrence of random shocks in the endogenous variables n_t and Λ_t, which are transmitted to the variable K_t, that would otherwise converge to a steady state value, according to equation (33).[9] To be precise, we look for a stationary Markovian process $\left(\widetilde{n}, \widetilde{\Lambda} \right)$ with a finite number S of states. With each state s, occurring with transition probability $\pi_{s's}$ conditional on the previous realization of state s', we associate a couple $(n_s, \Lambda_s) \in \mathsf{N}^* \times \mathsf{R}_+$, satisfying, for any s, both $\max\{2, \underline{n}\} \leq n_s \leq \min\{N, \overline{n}\}$ and

$$\sum_{s'=1}^{S} \pi_{ss'} \Lambda_{s'} = H(n_s) \Lambda_s^{\lambda_2}. \tag{35}$$

If the interval $[\max\{2, \underline{n}\}, \min\{N, \overline{n}\}]$ contains a single integer, or if we admit that some mechanism always selects the same n (for instance the largest integer in the admissible interval, as when the zero profit condition applies), existence of a non-degenerate stochastic equilibrium (with different values of $\widetilde{\Lambda}$) requires:

$$\min\{\Lambda_1, ..., \Lambda_S\} \equiv \underline{\Lambda} \leq H(n) \underline{\Lambda}^{\lambda_2} < H(n) \overline{\Lambda}^{\lambda_2} \leq \overline{\Lambda} \equiv \max\{\Lambda_1, ..., \Lambda_S\}, \tag{36}$$

implying: $\underline{\Lambda}^{1-\lambda_2} \leq H(n) \leq \overline{\Lambda}^{1-\lambda_2}$. This is possible only if $\lambda_2 \leq 1$, a case we have excluded, since it is already well known that stationary sunspot equilibria

[9] Notice however that shocks in n_t would have such an effect through the markup factor, even in presence of a stationary non-stochastic Λ_t.

exist in this case. Thus, we do need multiplicity of admissible values of n. But, in that case, non-degenerate stationary stochastic equilibria always exist, whatever the transition matrix $[\pi_{ss'}]$:

Proposition 2 *Assume the condition of Proposition 1. Take any finite Markov chain, with transition probabilities $\pi_{ss'}$ $(s, s' = 1, ..., S)$. Consider any associated random variable $\widetilde{n}$, taking at least two different values n_s and $n_{s'}$ in $\mathsf{N}^* \cap [\max\{2, \underline{n}\}, \min\{N, \overline{n}\}]$, such that $H(n_s) \neq H(n_{s'})$. Then, there exists a unique associated non-degenerate random variable $\widetilde{\Lambda}$, satisfying equation (35) for every $s = 1, ..., S$.*

Proof. See Appendix B. ■

Notice that, if the transition matrix has identical rows for all states leading to the same n_s (in particular, if the random variable $\widetilde{n}$ is i.i.d.), $H(n_s)\Lambda_s^{\lambda_2}$ must be constant as long as n_s does not vary across states, so that Λ_s is in fact a function of n_s. In that case, uncertainty is exclusively extrinsic (there are no technological shocks on a, for instance), but it works only through the coordination mechanism that selects the number of active firms at each period. However, this limitation does not apply anymore, as soon as we admit that the probability of occurrence of state s does depend upon the previous realized state, whether s' or s'', even if $n_{s'} = n_{s''}$. Then, $\widetilde{\Lambda}$ can take any number of different values, larger than the number of integers in the admissible interval $[\max\{2, \underline{n}\}, \min\{N, \overline{n}\}]$, so that consumers' beliefs may add extrinsic uncertainty to producers' animal spirits. The following proposition makes things precise, by stating the possibility of constructing a stationary stochastic equilibrium with any finite number of states and arbitrary values of $\widetilde{\Lambda}$ (in some admissible interval), with just two different values of $\widetilde{n}$, for instance $\lceil \underline{n} \rceil$ (the smallest integer larger than or equal to $\underline{n}$, assumed at least equal to 2) and $\lfloor \overline{n} \rfloor$ (the largest integer lower than or equal to $\overline{n}$, assumed larger than $\lceil \underline{n} \rceil$).

Proposition 3 *Assume again the condition of Proposition 1, leading moreover to the inequalities: $2 \leq \lceil \underline{n} \rceil < \lfloor \overline{n} \rfloor \leq N$, and $\underline{H} \equiv \min\{H(\lceil \underline{n} \rceil), H(\lfloor \overline{n} \rfloor)\} < \max\{H(\lceil \underline{n} \rceil), H(\lfloor \overline{n} \rfloor)\} \equiv \overline{H}$. Also, assume: $1 < \lambda_2 < 2$. Take any family of S different values of Λ, say $\Lambda_1 < \Lambda_2 < ... < \Lambda_S$, belonging to the interval $\left(\overline{H}^{-1/(\lambda_2 - 1)}, \underline{H}^{-1/(\lambda_2 - 1)}\right)$ and such that $\Lambda_1/\Lambda_S \leq \left(\underline{H}/\overline{H}\right)^{2/\lambda_2}$. Then there exist positive transition probabilities $\pi_{ss'}$ (with $s, s' = 1, ..., S$) and a mapping $\{1, ..., S\} \to \{\lceil \underline{n} \rceil, \lfloor \overline{n} \rfloor\}$, satisfying equation (35) for every $s = 1, ..., S$. The vector of transition probabilities can be chosen in an open convex subset of $\mathsf{R}_+^{S^2}$ of dimension $S(S-2)$.*

Proof. See Appendix B. ■

5 Business cycle properties

The properties of the business cycles generated in our model result in part from the extrinsic stochastic process imposing shocks in both the number of active firms and consumers' expectations of the rental of capital, and in part from the intrinsic transient dynamics adjusting the variables to their steady state values. In order to understand the co-movements of these variables, we begin with the second aspect, and consider the response to a switch in period t from state s to state s', assumed to persist in the subsequent periods.

We will refer to the case of a positive multiplicative shock in the autonomous dynamic system

$$K_t = [(\gamma/\alpha)\,\mu\,(n_{s'}) - 1]\,\Lambda_{s'}^{1/\rho} K_{t-1}^{\lambda_1} \equiv M_{s'} K_{t-1}^{\lambda_1} \tag{37}$$

associated with equation (33)), meaning that $M_{s'} > M_s$. Taking the stationary equilibrium value K_s corresponding to state s as the initial condition ($K_{t-1} = K_s$), we see that the new stationary equilibrium value $K_{s'}$ is higher (since $\lambda_1 < 1$), so that capital increases monotonically to $K_{s'}$. By equation (31), employment also increases monotonically to its new stationary equilibrium value $L_{s'}$, at least from period $t+1$ on (since $\Lambda_\tau = \Lambda_{s'}$ and $n_\tau = n_{s'}$ for any $\tau \geq t$, and since $\alpha/\gamma < \lambda_1$, as it can readily be checked from (29)). The same is clearly true as concerns output Y (by (20)), consumption C (by (13) and (21), total factor productivity $Y/K^{1-\alpha}L^\alpha$ (because of increasing returns to scale), and also labor productivity[10] Y/L and hence the real wage w (by (22)). Thus, all these variables are positively correlated, as we should expect, and converge monotonically to their steady state values from period $t+1$ on and as long as state s' persists.

Now, consider how the shock introduced by the jump from M_s to $M_{s'}$ comes about, by referring to two polar cases. This shock may be entirely ascribable to more optimistic consumers' expectations as regards capital rental in period $t+1$, with $n_{s'} = n_s$ and $\Lambda_{s'} > \Lambda_s$ (implied by $E_{s'}\left[\widetilde{\Lambda}\right] > E_s\left[\widetilde{\Lambda}\right]$, as can be seen from (34)). Such expectations lead to larger labor supply and employment: by (31), L_t is unambiguously larger than its initial value L_s, and so is of course the new stationary equilibrium value $L_{s'}$. The argument sketched above for all correlated variables for any period later than period t, can now be extended to period t, as soon as state s switches to state s'.

The other polar case is the one arising when the probabilities assigned to the different states are state independent ($\pi_{s'} = \pi_s$), so that $E_{s'}\left[\widetilde{\Lambda}\right] = E_s\left[\widetilde{\Lambda}\right] = \widehat{\Lambda}$: there is no shock on consumers' expectations. Obviously, $n_{s'} \neq n_s$ in that case, and we may then use equation (34) to determine $\Lambda(n) = \left(\widehat{\Lambda}/H(n)\right)^{1/\lambda_2}$. Conformity with standard observations would require $n_{s'} > n_s$, in order to have business formation and markup contraction associated with an expansionary shock. This is not guaranteed, and sufficient conditions are in fact difficult to establish, because the transition to a state with a higher number of active firms

[10] Notice that the stationary equilibrium value of labor productivity $A(n_{s'})\,K_{s'}^{\alpha/\gamma} L_{s'}^{(1-\alpha)/\gamma-1}$ is equal, by (31), to its value at period $t+1$, $A(n_{s'})\,K_t^{\alpha/\gamma} L_{t+1}^{(1-\alpha)/\gamma-1}$, multiplied by $(K_{s'}/K_t)^{[\alpha+(1-\alpha)\lambda_1-\gamma\lambda_1]/(1-\alpha)}$, which is larger than 1, since $K_{s'} > K_t$ and $\alpha + (1-\alpha)\,\lambda_1 > \gamma\lambda_1$.

triggers opposite effects, as can be seen from (33):[11] a negative effect through the markup factor $\mu(n)$ and a positive effect through Λ, if H is decreasing in n. A sufficient condition for this function to be decreasing is, by (32), that the ratio $A(n)/\mu(n)$ be increasing in n, which happens if $\sigma > 1/(1-\gamma)+1$ (a mild strengthening of the assumption of Proposition 1 for values of γ close to 1, that is, for moderate economies of scale).[12] This means that the negative effect of a higher number of firms on real factor prices through lower productivity is dominated by the corresponding positive effect through lower markups (see equations (20) to (22)).

In order to determine the ultimate dominating effect on M of a change in n, the one through μ or the one through Λ, we may calculate the sign of the elasticity of function $M(n) \equiv [(\gamma/\alpha)\mu(n)-1]\left[H(n)/\widehat{\Lambda}\right]^{-1/\rho\lambda_2}$:

$$\operatorname{sign}\{\epsilon M(n)\} = \operatorname{sign}\left\{\left(\frac{\gamma}{\alpha}\mu(n)-1\right)\epsilon A(n) - \left(\frac{1-\alpha}{\gamma\psi}\frac{\gamma}{\alpha}\mu(n)-1\right)\epsilon\mu(n)\right\}. \tag{38}$$

Under the condition $\sigma > 1/(1-\gamma)+1$, $\epsilon\mu(n) < \epsilon A(n) < 0$, so that M is an increasing function if $\gamma \leq (1-\alpha)/\psi$. For this sufficient condition to be satisfied, one would however have to assume an implausibly high degree of increasing returns. For moderately increasing returns (a high γ), the sign of $\epsilon M(n)$ will crucially depend upon the number n of active firms or, more exactly, upon the specific jump from n_s to $n_{s'}$ we are considering, because of the lack of monotonicity of M. If an increase in n is associated with an expansionary shock (an increase in M), it is easy to establish that employment L_t is higher already in period t than its value L_s before the shock: K_t is larger and the terms in n_t in equation (26) are smaller, so that L_t must be larger. Labor productivity Y_t/L_t must on the contrary be smaller in period t (before beginning to rise at $t+1$), at least for moderately increasing returns to scale, entailing a decreasing labor marginal productivity ($\gamma > 1-\alpha$). Finally, the decrease in the markup factor $\mu(n_t)$ curbs or reverses the impact on the real wage of the fall in productivity.

In order to show that the hypothesis of an expansionary shock with simultaneous business formation ($M_{s'} > M_s$ and $n_{s'} > n_s$) is not to be rejected as soon as we assume a small degree of scale economies, we just use a numerical example. We conform to custom, by making the technology parameters equal or close to their usual values in the recent literature: $\alpha = 0.3$ and $1/\gamma = 1.1$. By Proposition 1, the value chosen for γ constrains the elasticity σ of substitution between varieties inside each sector to lie in the interval $[11, 21]$, in order to obtain multiplicity of sectoral free entry equilibria. Also, by the same proposition,

[11] Notice that, in addition to the direct effects of a jump at t from n_s to $n_{s'}$ through the functions A and μ, which are patent in equation (26), we must take into account its indirect effect through the adjustment of L_t.

[12] Indeed, the absolute value of the elasticity of the markup factor with respect to the number of active firms $-\epsilon\mu(n) = n/(n-1)(1+(n-1)\sigma)$ is decreasing in n, so that it is lower bounded, for $n \leq \overline{n} = 1+\gamma/(\sigma(1-\gamma)-1)$, by $-\epsilon\mu(\overline{n}) = (1-\gamma)(\sigma(1-\gamma)-1)/\gamma$, a bound which is larger than $-\epsilon A(n) = 1/\gamma - \sigma/(\sigma-1)$ if $\sigma > 1/(1-\gamma)+1$.

setting $\sigma = 13$, the equilibrium number n of active firms is any integer belonging to the interval $[3, 6]$, leading to values of the markup factor μ in the interval $[1.1, 1.125]$. The value 1.1 of the markup factor is the one which corresponds to the zero profit condition. Finally, from section 4 we know that the ratio ρ/ψ must be smaller than $(\gamma - \alpha) / (1 - \alpha) = 0.87$ in order for the steady state to be a saddle. We accordingly choose the values $\psi = 1.2$ and $\rho = 0.96$, which lead to a high, but not infinite elasticity of labor supply, belonging to the interval $[4, 5]$. With these parameter values, we obtain the eigenvalues $\lambda_1 = 0.46$ and $\lambda_2 = 1.16$, and the values for M (up to a multiplicative constant):

$$M(3) = 1.031,\ M(4) = 1.040,\ M(5) = 1.042,\ M(6) = 1.041.$$

We see that any increase in n, except from 5 to 6, corresponds to an expansionary shock.

The small value of λ_1, partly due to the simplifying assumption of complete capital depreciation, makes this version of our model unfit to endogenously generate high persistence, independently of the characteristics of the driving random process, one of the interesting consequences of the animal spirits hypothesis, as stressed by Farmer and Guo [16]. As it is, the model must rely upon persistence built in the Markov chain which rules firms' effective participation, that is, in relatively high probabilities of keeping the same state next period, as in Chatterjee, Cooper and Ravikumar [12]. However, this is a natural characteristic to assume, since we do not expect the number of active firms to be highly volatile.

6 Conclusion

Business cycles have mainly been explained either by exogenous random shocks on the fundamentals (as in the RBC literature), or by extrinsic uncertainty due to indeterminacy occurring in an autonomous dynamic system which characterizes intertemporal equilibria (as in the endogenous fluctuations literature). Such indeterminacy is usually assigned either to the asymptotic local stability of a steady state (or of another attracting orbit), or to the multiplicity of steady states. In this paper we have proposed a different explanation, in which indeterminacy is a property of the static equilibrium prevailing in each period and in each oligopolistic production sector. This (free entry) equilibrium is characterized by a number of active firms that varies according to the (correct) competitors' conjectures. These are supposed to be coordinated by reference to some extrinsic stationary Markovian process which, by randomly selecting sectoral equilibria, generates endogenous shocks both on the markup factor and, because of increasing returns, on total factor productivity. Multiplicity of steady states thus emerges as a consequence not of the characteristics of the dynamic system describing intertemporal equilibria, but of stochastic perturbations of the system itself, as in RBC models. The difference with these models lies in the fact that such perturbations are entirely ruled by animal spirits, in a completely deterministic environment.

Because of the indeterminacy of the number of active firms, consumers' expectations may add extrinsic uncertainty to the system, in spite of the saddle property of the steady state. A significant point is that, as the source of fluctuations does not rely on local dynamic steady state indeterminacy, the degree of increasing returns can be kept arbitrarily small (even without an implausibly large elasticity of labor supply), provided the substitutability between goods within each sector can be made arbitrarily high.

The model in this paper has been designed essentially for an expository purpose, even if it reproduces from a qualitative point of view some of the main empirical features of business cycles. A quantitative approach of the mechanism here suggested should be explored by further research. This task is currently being undertaken, although with respect to a different model, by Dos Santos Ferreira and Dufourt [14].

7 Appendix

A Conditions for symmetric sectoral equilibrium

We follow d'Aspremont, Dos Santos Ferreira and Gérard-Varet [1] (example 2) in this Appendix. Consider the individual producer's objective function (7):

$$\Pi\left(p,\overline{P}\right)=b\left(\frac{p^{1-\sigma}}{p^{1-\sigma}+\overline{P}^{1-\sigma}}\right)-c\left[\left(\frac{p^{1-\sigma}}{p^{1-\sigma}+\overline{P}^{1-\sigma}}\right)\frac{b}{p}\right]^{\gamma}, \tag{39}$$

with simplified notations: $p=p_{ij}$, $\overline{P}^{1-\sigma}=\sum_{j'\neq j}p_{ij'}^{1-\sigma}$, $b=PY/m$ and $c=c\left(R,W\right)$. We shall also use the notation $\xi\left(p,\overline{P}\right)\equiv p^{1-\sigma}/\left(p^{1-\sigma}+\overline{P}^{1-\sigma}\right)$ for the share of variety j in the budget allocated to good i by the final sector. Denoting respectively by ∂_p and ϵ_p the partial derivative and elasticity operators with respect to p ($\epsilon_p\xi\left(p,\overline{P}\right)\equiv\partial_p\xi\left(p,\overline{P}\right)p/\xi\left(p,\overline{P}\right)$), the first and second order necessary conditions for an interior maximum of this function can be written:

$$\partial_p\Pi\left(p,\overline{P}\right)=\left[\epsilon_p\xi\left(p,\overline{P}\right)+\left(1-\epsilon_p\xi\left(p,\overline{P}\right)\right)\frac{\gamma c}{b}\xi\left(p,\overline{P}\right)^{\gamma-1}\left(\frac{b}{p}\right)^{\gamma}\right]b\frac{\xi\left(p,\overline{P}\right)}{p}=0, \tag{40}$$

$$\begin{aligned}\partial_{pp}^2\Pi\left(p,\overline{P}\right)\ &\mid_{\partial_p\Pi\left(p,\overline{P}\right)=0}= \\ \left[\frac{\epsilon_{pp}^2\xi\left(p,\overline{P}\right)}{1-\epsilon_p\xi\left(p,\overline{P}\right)}+\left(1-\gamma\right)\epsilon_p\xi\left(p,\overline{P}\right)+\gamma\right]b\frac{\partial_p\xi\left(p,\overline{P}\right)}{p}\ &\leq\ 0.\end{aligned} \tag{41}$$

Given that

$$\epsilon_p\xi\left(p,\overline{P}\right)=-\left(\sigma-1\right)\left(1-\xi\left(p,\overline{P}\right)\right)<0\text{ and} \tag{42}$$

$$\epsilon_{pp}^2\xi\left(p,\overline{P}\right)=-\epsilon_p\xi\left(p,\overline{P}\right)\frac{\xi\left(p,\overline{P}\right)}{1-\xi\left(p,\overline{P}\right)}=\left(\sigma-1\right)\xi\left(p,\overline{P}\right), \tag{43}$$

the second order condition is equivalent to

$$\frac{\xi\left(p,\overline{P}\right)}{\xi\left(p,\overline{P}\right)+\left(1-\xi\left(p,\overline{P}\right)\right)\sigma}-(1-\gamma)\left(1-\xi\left(p,\overline{P}\right)\right)+\frac{\gamma}{\sigma-1}\geq 0. \tag{44}$$

The left-hand side of this inequality is clearly increasing in $\xi\left(p,\overline{P}\right)$, hence decreasing in p, so that it can change signs only once, implying that the objective function has at most an interior maximum. Also, $\lim_{p\to 0}\Pi\left(p,\overline{P}\right)=-\infty$ and $\lim_{p\to\infty}\Pi\left(p,\overline{P}\right)=0$. Hence, the function $\Pi\left(\cdot,\overline{P}\right)$ either has no positive values ($p=\infty$ being a solution to the firm's maximization program), or has a unique positive maximum, so that the two conditions $\partial_p\Pi\left(p,\overline{P}\right)=0$ and $\Pi\left(p,\overline{P}\right)>0$ together are sufficient to a (positive) profit maximum $\Pi^*\left(\overline{P}\right)$ with finite price $p^*\left(\overline{P}\right)$ (and positive production). These conditions can be expressed as follows:

$$1-\frac{1}{\xi\left(p,\overline{P}\right)+\left(1-\xi\left(p,\overline{P}\right)\right)\sigma}=\gamma cb^{\gamma-1}\xi\left(p,\overline{P}\right)^{\gamma-1}p^{-\gamma}<\gamma, \tag{45}$$

stating that the reciprocal of the markup factor is equal to the ratio of marginal cost to price, and that average cost must be smaller than price. By the envelope theorem, the derivative of Π^* is equal to $\partial_{\overline{P}}\Pi\left(p^*\left(\overline{P}\right),\overline{P}\right)>0$, so that the firm will choose $p=\infty$ and stay inactive whenever it has a conjectured value of $\overline{P}$ smaller than the one satisfying: $cb^{\gamma-1}\xi\left(p^*\left(\overline{P}\right),\overline{P}\right)^{\gamma-1}p^*\left(\overline{P}\right)^{-\gamma}=1$, that is, leading to zero profits.

In a symmetric equilibrium with n active firms ($2\leq n\leq N$), setting the equilibrium price p_n, any such firm has a conjecture $\overline{P}_n=\left[(n-1)\,p_n^{1-\sigma}\right]^{1/(1-\sigma)}$, entailing $\xi\left(p_n,\overline{P}_n\right)=1/n$, and hence (using (45)):

$$\begin{aligned} p_n &= \left[\mu\left(n;\sigma\right)\gamma cb^{\gamma-1}n^{1-\gamma}\right]^{1/\gamma}\text{, with} && (46)\\ \mu\left(n;\sigma\right) &= \frac{1+(n-1)\,\sigma}{(n-1)\,(\sigma-1)}>\frac{1}{\gamma}, && (47)\end{aligned}$$

$\mu\left(n;\sigma\right)$ being the markup factor on marginal cost, which is decreasing in n. The last inequality (expressing the condition for positive profits) requires that the number n of active firms be smaller than some value $\overline{n}$, such that

$$\begin{array}{ll} \overline{n}=1+\frac{\gamma}{\sigma(1-\gamma)-1}\in(2,\infty) & \text{if } \frac{1}{1-\gamma}<\sigma<\frac{1+\gamma}{1-\gamma}\\ \overline{n}=\infty & \text{if } 1<\sigma\leq\frac{1}{1-\gamma}\end{array} \tag{48}$$

Notice that, when finite, the upper bound $\overline{n}$ is decreasing in σ and increasing in γ.

Besides, any inactive firm (if $n<N$) has a conjecture $\overline{P}=\left[np_n^{1-\sigma}\right]^{1/(1-\sigma)}$ such that its objective function has no positive values:

$$cb^{\gamma-1}\xi\left(p,\left[np_n^{1-\sigma}\right]^{1/(1-\sigma)}\right)^{\gamma-1}p^{-\gamma}\geq 1\text{, for any }p, \tag{49}$$

or equivalently that, for any p,

$$\left(c^{-1/(1-\gamma)}bp^{\gamma/(1-\gamma)}-1\right)p^{1-\sigma}\leq np_n^{1-\sigma}, \tag{50}$$

requiring $\sigma>1/\left(1-\gamma\right)$. Taking the maximum of the left-hand side of inequality (50), we get:

$$\left(\overline{n}-1\right)\overline{n}^{(1-\sigma)(1-\gamma)/\gamma}c^{(1-\sigma)/\gamma}b^{(\sigma-1)(1-\gamma)/\gamma}\leq n\left[\mu\left(n;\sigma\right)\gamma cb^{\gamma-1}n^{1-\gamma}\right]^{(1-\sigma)/\gamma}, \tag{51}$$

or

$$\left(1-1/\overline{n}\right)^{-\gamma}\geq\left(n/\overline{n}\right)^{\sigma(1-\gamma)-1}\left[\mu\left(n;\sigma\right)/\mu\left(\overline{n};\sigma\right)\right]^{\sigma-1}. \tag{52}$$

It is easy to check that this inequality is satisfied (strictly) for $n=\overline{n}$, and that its right-hand side is decreasing in n if $n\leq\overline{n}$. Hence, this inequality requires that the number n of active firms be no smaller than some value $\underline{n}<\overline{n}$. As a consequence, there is indeterminacy in the number of active firms if the interval $\left[\max\left\{2,\underline{n}\right\},\min\left\{N,\overline{n}\right\}\right]$, to which n must belong, contains more than one integer.

B Existence of non-degenerate stationary sunspot equilibria

Beginning with Proposition 2, we first prove existence of a solution $\Lambda=\left(\Lambda_1,...,\Lambda_S\right)$ to equation (35):

$$\sum_{s'=1}^{S}\pi_{ss'}\Lambda_{s'}=H\left(n_s\right)\Lambda_s^{\lambda_2}\qquad(s=1,...,S),$$

by using Brouwer's fixed-point theorem. In order to apply this theorem, it is enough to show that the continuous function $\mathcal{L}$, with

$$\mathcal{L}_s\left(\Lambda\right)=\left(\frac{1}{H\left(n_s\right)}\sum_{s'=1}^{S}\pi_{ss'}\Lambda_{s'}\right)^{1/\lambda_2},\text{ for }s=1,...,S, \tag{53}$$

defined on $\left[\overline{H}^{-1/(\lambda_2-1)},\underline{H}^{-1/(\lambda_2-1)}\right]^S$, with $\underline{H}\equiv\min_s\left\{H\left(n_s\right)\right\}$ and $\overline{H}\equiv\max_s\left\{H\left(n_s\right)\right\}$ takes all its values in this domain. Indeed, if $\overline{H}^{-1/(\lambda_2-1)}\leq\Lambda_s\leq\underline{H}^{-1/(\lambda_2-1)}$ for any s,

$$\begin{aligned}\overline{H}^{-1/(\lambda_2-1)}&\leq\left(\frac{\overline{H}}{H\left(n_s\right)}\right)^{1/\lambda_2}\overline{H}^{-1/(\lambda_2-1)}\leq\mathcal{L}_s\left(\Lambda\right)\\&\leq\left(\frac{\underline{H}}{H\left(n_s\right)}\right)^{1/\lambda_2}\underline{H}^{-1/(\lambda_2-1)}\leq\underline{H}^{-1/(\lambda_2-1)}.\end{aligned} \tag{54}$$

Now, if there is a couple (s, s') such that $H(n_s) \neq H(n_{s'})$, then the random variable $\widetilde{\Lambda}$, as determined by equation (53), is non-degenerate. Suppose indeed that $\Lambda_s = \Lambda_1$ for any s. Then, we obtain from this equation that $H(n_s) = \Lambda_1^{1-\lambda_2}$ for every s, contradicting the assumption.

In order to prove uniqueness of the solution to equation (53), we show that the determinant of the Jacobian matrix $I - \partial\mathcal{L}(\Lambda^*)$ of the function $\Lambda - \mathcal{L}(\Lambda)$, at any fixed-point Λ^* of $\mathcal{L}$, has always the same sign. Indeed, the generic term of $\partial\mathcal{L}(\Lambda^*)$ is

$$\partial_{s'}\mathcal{L}_s(\Lambda^*) = \frac{1}{\lambda_2}\frac{\pi_{ss'}\Lambda_s^*}{\sum_{s'=1}^{S}\pi_{ss'}\Lambda_{s'}^*} \geq 0, \tag{55}$$

and, for $s = 1, ..., S$,

$$\sum_{s'=1}^{S}\partial_{s'}\mathcal{L}_s(\Lambda^*) = \frac{1}{\lambda_2} < 1, \tag{56}$$

so that $I - \partial\mathcal{L}(\Lambda^*)$ is a Leontief matrix, and hence has a positive determinant. By the index theorem, Λ^* is unique. ¥

We proceed to the proof of Proposition 3. Transition probabilities satisfying equation (35) for every $s = 1, ..., S$ are largely indeterminate. Take, for any s, arbitrary positive probabilities $\pi_{ss'}$ with $s' \neq 1, S$, such that $\sum_{s'=2}^{S-1}\pi_{ss'} = \varepsilon_s < 1$. Then, for every s, we have:

$$\pi_{s1}\Lambda_1 + \sum_{s'=2}^{S-1}\pi_{ss'}\Lambda_{s'} + (1 - \varepsilon_s - \pi_{s1})\Lambda_S = H(n_s)\Lambda_s^{\lambda_2}, \tag{57}$$

or (denoting the mean of $\Lambda_2, ..., \Lambda_{S-1}$ at state s: $\widehat{\Lambda}_s \equiv \sum_{s'=2}^{S-1}(\pi_{ss'}/\varepsilon_s)\Lambda_{s'}$)

$$\pi_{s1} = \frac{(1-\varepsilon_s)\Lambda_S + \varepsilon_s\widehat{\Lambda}_s - H(n_s)\Lambda_s^{\lambda_2}}{\Lambda_S - \Lambda_1}. \tag{58}$$

We must show that, for any s and an adequate choice of the mapping $s \longmapsto n_s$, we get $0 < \pi_{s1} < 1 - \varepsilon_s$, or

$$\Lambda_1 + \varepsilon_s\left(\widehat{\Lambda}_s - \Lambda_1\right) < H(n_s)\Lambda_s^{\lambda_2} < \Lambda_S - \varepsilon_s\left(\Lambda_S - \widehat{\Lambda}_s\right). \tag{59}$$

Now, for any s such that $\Lambda_s < \sqrt{\Lambda_1\Lambda_S}$ (resp. $\Lambda_s > \sqrt{\Lambda_1\Lambda_S}$), choose n_s satisfying $H(n_s) = \overline{H}$ (resp. $H(n_s) = \underline{H}$), the choice being indifferent if $\Lambda_s = \sqrt{\Lambda_1\Lambda_S}$ for some s. Thus, if $\Lambda_s \leq \sqrt{\Lambda_1\Lambda_S}$,

$$\Lambda_1 < \overline{H}\Lambda_1^{\lambda_2} \leq H(n_s)\Lambda_s^{\lambda_2} \leq \overline{H}(\Lambda_1\Lambda_S)^{\lambda_2/2} \leq \underline{H}\Lambda_S^{\lambda_2} < \Lambda_S, \tag{60}$$

since, by assumption, $\Lambda_1 > \overline{H}^{-1/(\lambda_2-1)}$, $\Lambda_S < \underline{H}^{-1/(\lambda_2-1)}$ and $\Lambda_1/\Lambda_S \leq \left(\underline{H}/\overline{H}\right)^{2/\lambda_2}$. Similarly, if $\Lambda_s \geq \sqrt{\Lambda_1\Lambda_2}$,

$$\Lambda_1 < \overline{H}\Lambda_1^{\lambda_2} \leq \underline{H}(\Lambda_1\Lambda_S)^{\lambda_2/2} \leq H(n_s)\Lambda_s^{\lambda_2} \leq \underline{H}\Lambda_S^{\lambda_2} < \Lambda_S, \tag{61}$$

by the same assumption. Notice that the assumption on the admissible values of Λ implies $\left(\underline{H}/\overline{H}\right)^{1/(\lambda_2-1)} < \Lambda_1/\Lambda_S \leq \left(\underline{H}/\overline{H}\right)^{2/\lambda_2}$, which requires $\lambda_2 < 2$.

To conclude the proof it is enough to notice that, by the former inequalities and since $\Lambda_1 < \widehat{\Lambda}_s < \Lambda_S$, inequalities (59) are always satisfied provided ε_s is small enough. More precisely, given the conditions on the family $(\Lambda_1, ..., \Lambda_S)$, if we choose $H(n_s)$ as indicated above, there are, for any s, positive probabilities π_{s1} and π_{sS}, such that equation (35) is satisfied, for every possible choice of positive probabilities $\pi_{s2}, ..., \pi_{sS-1}$ adding up to ε_s such that

$$\varepsilon_s < \min\left\{1, \frac{H(n_s)\Lambda_s^{\lambda_2} - \Lambda_1}{\widehat{\Lambda}_s - \Lambda_1}, \frac{\Lambda_S - H(n_s)\Lambda_s^{\lambda_2}}{\Lambda_S - \widehat{\Lambda}_s}\right\}. \tag{62}$$

This means that, for $s = 1, ..., S$, the vector $(\pi_{s2}, ..., \pi_{sS-1})$ may be any point belonging to a (non-empty) open set of dimension $S-2$, which is the contraction (because $\widehat{\Lambda}_s$ depends upon $(\pi_{s2}, ..., \pi_{sS-1})$) of the set formed by the intersection of a half-space and the positive orthant of R^{S-2}. ¥

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