

DISCUSSION PAPER SERIES

No. 3917

LOAN PRICING UNDER BASEL CAPITAL REQUIREMENTS

Rafael Repullo and Javier Suárez

FINANCIAL ECONOMICS



Centre for **E**conomic **P**olicy **R**esearch

www.cepr.org

Available online at:

www.cepr.org/pubs/dps/DP3917.asp

LOAN PRICING UNDER BASEL CAPITAL REQUIREMENTS

Rafael Repullo, Centre for Monetary and Financial Studies (CEMFI) and CEPR
Javier Suárez, Centre for Monetary and Financial Studies (CEMFI) and CEPR

Discussion Paper No. 3917
May 2003

Centre for Economic Policy Research
90–98 Goswell Rd, London EC1V 7RR, UK
Tel: (44 20) 7878 2900, Fax: (44 20) 7878 2999
Email: cepr@cepr.org, Website: www.cepr.org

This Discussion Paper is issued under the auspices of the Centre's research programme in **FINANCIAL ECONOMICS**. Any opinions expressed here are those of the author(s) and not those of the Centre for Economic Policy Research. Research disseminated by CEPR may include views on policy, but the Centre itself takes no institutional policy positions.

The Centre for Economic Policy Research was established in 1983 as a private educational charity, to promote independent analysis and public discussion of open economies and the relations among them. It is pluralist and non-partisan, bringing economic research to bear on the analysis of medium- and long-run policy questions. Institutional (core) finance for the Centre has been provided through major grants from the Economic and Social Research Council, under which an ESRC Resource Centre operates within CEPR; the Esmée Fairbairn Charitable Trust; and the Bank of England. These organizations do not give prior review to the Centre's publications, nor do they necessarily endorse the views expressed therein.

These Discussion Papers often represent preliminary or incomplete work, circulated to encourage discussion and comment. Citation and use of such a paper should take account of its provisional character.

Copyright: Rafael Repullo and Javier Suárez

ABSTRACT

Loan Pricing Under Basel Capital Requirements*

We analyse the implications for the pricing of bank loans of the reform of capital regulation known as Basel II. We consider a perfectly competitive market for business loans where, as in the model underlying the internal ratings based (IRB) approach of Basel II, a single risk factor explains the correlation in defaults across firms. Our loan-pricing equation implies that low-risk firms will achieve reductions in their loan rates by borrowing from banks adopting the IRB approach, while high-risk firms will avoid increases in their loan rates by borrowing from banks that adopt the less risk-sensitive standardized approach of Basel II. We also show that only an extremely high social cost of bank failure might justify the proposed IRB capital charges for high-risk loans, partly because the margin income from performing loans is not counted as a buffer against credit losses, and we propose a margin income correction for IRB capital requirements.

JEL Classification: E43, G21 and G28

Keywords: bank regulation, capital requirements, credit risk, internal ratings, loan defaults and margin income

Rafael Repullo
CEMFI
Casado del Alisal 5
28014 Madrid
SPAIN
Tel: (34) 914290551
Fax: (34) 914291056
Email: repullo@cemfi.es

Javier Suárez
CEMFI
Casado del Alisal, 5
28014 Madrid
SPAIN
Tel: (34) 914290551
Fax: (34) 914291056
Email: suarez@cemfi.es

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=107018

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=126530

*We are grateful to Carlos Trucharte for his comments, and to Aitor Lacuesta for his help with the numerical computations.

Submitted 08 May 2003

1 Introduction

The Basle Accord of 1988 consolidated capital requirements as the cornerstone of bank regulation. It required banks to hold a minimum overall capital equal to 8% of their risk-weighted assets and included all business loans into the full weight category, so the flat 8% became the universal capital charge for corporate lending. Following widespread criticism about the risk-insensitiveness of these requirements, as well as recent advances in risk measurement, the Basel Committee on Banking Supervision (BCBS) is now close to finalize a reform, known as Basel II, whose primary goal is “to align capital adequacy assessment more closely with the key elements of banking risks” (BCBS, 2001, p. 1).

Basel II introduces a menu of approaches for determining capital requirements. The *standardized* approach contemplates the use of external ratings to refine the risk weights of the 1988 Accord (henceforth, Basel I), but leaves the capital charges for unrated companies essentially unchanged. The *internal ratings based* (IRB) approach allows banks to compute the capital charges for each exposure from their own estimate of the probability of default (PD) and, possibly, the loss given default (LGD).¹

This paper provides an analysis of this reform along the lines that would first come to the mind of an economist or a financial analyst. How will the new rules alter the pricing of bank loans? Will the volumes of bank lending be affected? How will the effects be distributed across credit risk categories? Will banks be safer under the new regulation? Does the new regulation reasonably trade off the benefits and costs of capital requirements?

We address these questions in the context of a perfectly competitive market for business loans. Importantly, we assume that loan default rates and, thus, banks’ credit losses are determined by the same asymptotic *single risk factor model* that is

¹Specifically, two variants of the IRB approach are proposed. In the foundation IRB banks provide an estimate of the PD of each borrower and a formula gives the corresponding capital charge. In the advanced IRB, banks also input their own estimate of the LGD.

used for the computation of capital charges in the IRB approach of Basel II.² Banks have zero intermediation costs, are funded with fully insured deposits and equity capital, and supply loans to a large number of unrated firms with risky investment projects. Although bank shareholders are risk-neutral, the cost of capital is assumed to be greater than the cost of deposits.³ A single factor of systematic risk explains the correlation in defaults across firms and, hence, the proportion of bank loans that default and the probability of bank failure. By limited liability, the final payoff of a bank's shareholders is equal to the bank's net worth if it is positive, and zero otherwise. The competitive equilibrium interest rate for each class of loans is determined by a zero net (marginal) value condition that makes each loan's (marginal) contribution to the expected discounted value of shareholders' final payoff equal to the (marginal) initial equity contribution that the loan requires.

There are a number of reasons to argue that our setup constitutes an adequate benchmark with which to start. The assumption of perfect competition allows us to abstract from the important but rather tangential discussion on what model of imperfect competition is most reasonable in banking. Also it allows us to make the best case for capital requirements, since banks with market power get rents that provide a buffer against failure and, in a multiperiod setting, might give banks an additional incentive to remain solvent.⁴ By examining an economy that conforms to the single risk factor model embedded in the new regulation, we give this regulation the best chance to demonstrate its internal consistency. Finally, the single risk factor model is good for tractability: in fact, it is the only model with a bottom-up approach to credit risk that yields simple closed-form solutions for the distribution of credit

²This model is credited to Michael Gordy; see Gordy (2000, 2002).

³We can rationalize this assumption by reference to explicit agency problems as in Holmstrom and Tirole (1997) or Diamond and Rajan (2000). The same assumption is made by Bolton and Freixas (2000), and Hellmann, Murdock and Stiglitz (2000), among others.

⁴In contrast, under perfect competition there are no such rents, so focusing on a two-period model implies no loss of generality in this dimension. Of course, in this setup we cannot capture frictions that are dynamic in nature, such as costs of issuing equity following the accumulation of credit losses. Modeling these frictions seems the natural next step in the analysis.

losses.⁵

Unlike in models where the distribution of the returns of bank assets has an unbounded support,⁶ in our setting the support is realistically bounded by the principal and interest payments established in loan contracts. Moreover, the variability of the returns comes from credit losses that can be directly related to the PD, the LGD, and the exposure to systematic risk of the corresponding loans. Thus, our loan pricing equations allow us to derive analytically the dependence of equilibrium loan rates on these parameters as well as on the capital requirement and the cost of bank capital.

These equations are used to assess the qualitative and quantitative implications of the move from Basel I to Basel II. We predict that low risk firms will concentrate their borrowing in banks that adopt the IRB approach and will enjoy lower loan rates. This follows immediately from the fact that, for these firms, the IRB capital charges are lower than the 8% of both Basel I and the standardized approach of Basel II. In contrast, high risk firms may find more attractive loan rates at the banks that adopt the standardized approach, in which case their interest rates will not change relative to the situation under Basel I. Somewhat paradoxically, these predictions imply that the banks specialized in low risk lending will not become safer since, first, the IRB approach will allow them to work with a lower capital buffer than under Basel I and, second, the subsequent fall in the interest rate of low risk loans will reduce their net interest income (or *margin income*) buffer, which constitutes an additional protection against insolvency.

At the quantitative level, our simulations (based on a cost of bank capital of 6% per annum over the risk free rate) show that adopting the IRB approach may imply a reduction in loan rates (relative to Basel I) of almost 50 basis points for loans with a PD of 0.03%, and an increase of 80 to 200 basis points for loans with a PD of 10%. Under the IRB approach, banks' probabilities of failure are extremely low no matter

⁵See Gordy (2000).

⁶For example, the geometric brownian motion process in Merton (1977), the normal distribution in Rochet (1992), and the lognormal distribution in Marshall and Prescott (2001).

the risk class of loans in which they could specialize. More surprisingly, the lowest probabilities of failure correspond to the banks whose lending is concentrated in high risk loans. The reason for this result is that, on top of their capital buffer, these banks enjoy a greater margin income buffer which is not credited for when the capital requirement is computed, but clearly reduces the probability of failure.

Our simulations also show that, under the IRB approach, the probabilities of bank failure are so low that the equilibrium rates for each class of loans are very close to the corresponding actuarially fair rates. In other words, the easy-to-compute rate that equates the expected payments of a loan to its weighted marginal funding cost (from deposits and capital, depending on how much of the latter is required by regulation) provides a very precise approximation to the solution of our pricing equations.

Finally, we examine whether the cost of the IRB capital requirements of Basel II may be justified in terms of a reduction in the social cost of bank failures. We construct a social welfare function by adding the expected payoffs of the four types of agents in the economy: entrepreneurs, bank shareholders, depositors, and the government. For simplicity, the government bears the deposit insurance liabilities as well as an additional social cost of bank failure which we assume proportional to the initial assets of the failed banks. Our welfare measure turns out to be equal to the expected net return of firms' investment projects minus the cost of the capital required for providing their loans and the corresponding expected social cost of bank failure. We characterize the socially optimal capital requirement for banks specialized in different classes of loans, and then we ask for what level of the social cost of bank failure the charges of the IRB approach would be optimal. We show that this cost is remarkably increasing in the PD, reaching implausibly high values for high PD loans. This suggests that the IRB charges for these loans are too high. We briefly discuss possible causes for this apparent flaw in the new regulation and derive a closed-form solution for a capital charge that, by introducing a margin income correction in the IRB capital requirement, would partly alleviate the problem.

The paper is organized as follows. Section 2 presents the model and derives the

main results on equilibrium loan pricing. Section 3 uses these results to discuss the qualitative and quantitative implications of the transition from Basel I to Basel II. Section 4 presents our welfare analysis of capital requirements, and Section 5 offers some concluding remarks. Appendix A extends the analysis to the case of positive intermediation costs, Appendix B discusses the approximation of equilibrium rates by actuarially fair rates, and Appendix C contains the proofs of the results stated in the main text.

2 The Model

Consider a risk-neutral economy with two dates, $t = 0, 1$, and a single *factor of systematic risk*, $z \sim N(0, 1)$. There is a continuum of measure one of firms, indexed by $i \in [0, 1]$, and a large number of banks. Each firm i has a project that requires a unit of investment at $t = 0$ and yields a gross return at $t = 1$ which is $1 + a$ if the project succeeds and $1 - \lambda$ if it fails. The success ($d_i = 0$) or failure ($d_i = 1$) of the project of firm i is determined by a (latent) random variable

$$x_i = \mu_i + \sqrt{\rho} z + \sqrt{1 - \rho} \varepsilon_i, \quad (1)$$

such that

$$d_i = \begin{cases} 0, & \text{if } x_i \leq 0, \\ 1, & \text{if } x_i > 0, \end{cases}$$

where $\varepsilon_i \sim N(0, 1)$ is independently distributed across firms and independent of z . Parameter $\mu_i \in \mathbb{R}$ is a measure firm i 's financial vulnerability while parameter $\rho \in [0, 1]$ captures its exposure to the systematic risk factor.⁷

Each firm i is owned by a penniless entrepreneur who finances the required investment with a bank loan. Bank loans are supplied by perfectly competitive banks that are funded with deposits and equity capital, and for simplicity have zero intermediation costs.⁸ Bank deposits are insured by a government-funded deposit insurance

⁷Notice that ρ is also the correlation between the latent variables of any two firms.

⁸We relax this assumption in Appendix A.

scheme, and they are in perfectly elastic supply at an interest rate which is normalized to zero.⁹ Banks' equity capital is provided by a special class of agents, called bankers, who require an expected rate of return $\delta \geq 0$ on their investment. A strictly positive value of δ captures either the scarcity of bankers' wealth or, perhaps more realistically, the existence of a premium for the agency and asymmetric information problems faced by the banks' shareholders.¹⁰

Because of limited liability, at $t = 1$ bankers receive their banks' net worth (that is, gross loan returns minus gross deposit liabilities) if it is positive, and zero otherwise. Bankers maximize the expected value of this payoff discounted at rate δ and net of their initial contribution of capital. Prudential regulation may require banks to hold some minimum equity capital, according to schemes that will be specified below.

From (1) we have that $x_i \sim N(\mu_i, 1)$, so the unconditional *probability of default* (PD) of firm i is given by

$$\bar{p}_i = \Pr(x_i > 0) = \Pr(x_i - \mu_i > -\mu_i) = \Phi(\mu_i), \quad (2)$$

where Φ denotes the cumulative distribution function of a standard normal random variable. Clearly, this probability is increasing in the financial vulnerability parameter μ_i , which, if convenient, can be expressed as a simple non-linear transformation of the PD, $\Phi^{-1}(\bar{p}_i)$.

We consider two observable classes of firms. For firms with $i \in [0, l]$ the financial vulnerability parameter μ_i takes the value μ_l , and for firms with $i \in (l, 1]$ it takes the value μ_h , where $\mu_l < \mu_h$. Thus, by (2), the PD of the firms in the first class, $\bar{p}_l = \Phi(\mu_l)$, is smaller than the PD of the firms in the second class, $\bar{p}_h = \Phi(\mu_h)$, so we will call them low risk and high risk firms, respectively. Parameter $l \in (0, 1)$ measures the proportion of low risk firms in the economy.

⁹Introducing a positive deposit insurance premium will not alter our main results insofar as the premium is insensitive to the composition of each bank's loan portfolio.

¹⁰See Holmstrom and Tirole (1997) and Diamond and Rajan (2000) for explicit models of why δ might be positive.

2.1 Banks' objective function

Consider a bank with a loan portfolio of size one at $t = 0$, and let $\gamma \in [0, 1]$ denote the proportion of its lending that is allocated to low risk firms. Since each firm's class is observable, the bank charges a loan rate r_l to low risk firms and a loan rate r_h to high risk firms. When the project of a low risk (high risk) firm succeeds the bank gets $1 + r_l$ ($1 + r_h$), while when it fails, the firm defaults on its loan and the bank gets $1 - \lambda$, so parameter λ measures the *loss given default* (LGD).¹¹

From (1) we have $x_i | z \sim N(\mu_i + \sqrt{\rho} z, 1 - \rho)$, so the probability of default of firm i conditional on the realization of the systematic risk factor z is

$$p_i(z) = \Pr(x_i > 0 | z) = \Pr\left(\varepsilon_i > -\frac{\mu_i + \sqrt{\rho} z}{\sqrt{1 - \rho}} \mid z\right) = \Phi\left(\frac{\mu_i + \sqrt{\rho} z}{\sqrt{1 - \rho}}\right). \quad (3)$$

This probability is increasing in z as well as in the financial vulnerability parameter μ_i . On the other hand, increasing the parameter ρ of exposure to the systematic risk factor decreases $p_i(z)$ for low values of z and increases it for high values of z , making the conditional probability of default more sensitive to the realization of z .¹²

Let $p_l(z)$ and $p_h(z)$ denote the functions $p_i(z)$ for $\mu_i = \mu_l$ and $\mu_i = \mu_h$, respectively. By the law of large numbers, $p_l(z)$ and $p_h(z)$ are also the *default rates* of low and high risk loans (that is, the proportion of these loans that default) when the systematic risk factor takes the value z .

If k is the fraction of the bank's portfolio that is funded with equity capital, then the bank's net worth at $t = 1$ conditional on the realization of the systematic risk factor z is given by

$$\begin{aligned} \pi(z) = & \gamma[(1 - p_l(z))(1 + r_l) + p_l(z)(1 - \lambda)] \\ & + (1 - \gamma)[(1 - p_h(z))(1 + r_h) + p_h(z)(1 - \lambda)] - (1 - k). \end{aligned} \quad (4)$$

¹¹We are implicitly assuming that the firms' net success return a is sufficiently large, so $a > r_j$ for $j = l, h$.

¹²In the limit $\rho = 0$ the conditional probability $p_i(z)$ equals $\bar{p}_i = \Phi(\mu_i)$ and is therefore independent of z , while in the limit $\rho = 1$ it is a discontinuous function, with $p_i(z) = 0$ for $z \leq -\mu_i$ and $p_i(z) = 1$ for $z > -\mu_i$.

The first term is the expected payment from low risk firms, which is equal to the payment from the firms in this class that do not default plus the payment from those that default. Similarly, the second term is the expected payment from high risk firms. The third term is the amount owed to the depositors, which is just $1 - k$ because the deposit rate is normalized to zero.

By limited liability, the bankers' payoff at $t = 1$ conditional on the realization of z is $\max\{\pi(z), 0\}$. The bank's objective function is to maximize the expected discounted value of this payoff net of bankers' initial infusion of capital, that is,

$$V = -k + \frac{1}{1 + \delta} E[\max\{\pi(z), 0\}], \quad (5)$$

where the discount rate used is the bankers' required rate of return δ . If \hat{z} denotes the critical value of z for which $\pi(z) = 0$ (or ∞ , if $\pi(z)$ is positive for all z), we can write

$$V = -k + \frac{1}{1 + \delta} \int_{-\infty}^{\hat{z}} \pi(z) d\Phi(z).$$

From here it follows that

$$\frac{\partial V}{\partial k} = -1 + \frac{\Phi(\hat{z})}{1 + \delta} < 0,$$

so the bank will not want to hold any capital above the minimum required by regulation.¹³ For this reason, from now onwards, k will denote the minimum capital requirement.

2.2 Capital requirements

Under Basel I the capital requirement applicable to all business loans is 8% so k is a constant. This is also the case for loans to unrated firms under the standardized approach of Basel II, while under the internal ratings based (IRB) approach of Basel II, bank capital must be sufficient to cover credit losses with a given confidence level α , which implies a direct linkage between k and the characteristics of each bank's loan portfolio.

¹³Notice that, if $\hat{z} < \infty$, then $\partial V/\partial k < 0$ obtains even when $\delta = 0$, that is, when bankers do not require a higher rate of return than depositors. This is due to the fact that deposits would still be a cheaper source of finance, since they are covered by deposit insurance in case of bank failure.

Specifically, if we let $z_\alpha = \Phi^{-1}(\alpha)$ denote the α -quantile of the distribution of the systematic risk factor z , then the default rate for the loans of class $j = l, h$ that leaves below a cumulative probability α is, by (3),

$$p_j(z_\alpha) = \Phi\left(\frac{\Phi^{-1}(\bar{p}_j) + \sqrt{\rho} \Phi^{-1}(\alpha)}{\sqrt{1 - \rho}}\right). \quad (6)$$

The IRB capital charge for each unit of loans of class j is

$$k_j = \lambda p_j(z_\alpha). \quad (7)$$

Maturity adjustments aside, equation (7) coincides with the Basel II formula for the computation of the IRB capital requirement on loans with an estimated PD of \bar{p}_j .¹⁴ Clearly, $\bar{p}_l < \bar{p}_h$ implies $k_l < k_h$, so the capital charge for lending to low risk firms is lower than the charge for lending to high risk firms. On the other hand, the capital requirement implied by (7) is directly proportional to the LGD λ , and is increasing in the confidence level α . Finally, one can show that the derivative of k_j with respect to parameter ρ is positive whenever $\Phi(\mu_j \sqrt{\rho}) > 1 - \alpha$, a condition that is easily satisfied for high values of the confidence level α .

The capital requirement for a bank that invests a proportion γ of its portfolio on loans to low risk firms has the additive form:

$$k(\gamma) = \gamma k_l + (1 - \gamma) k_h, \quad (8)$$

where k_j denotes the requirement per unit of loans of class $j = l, h$. Clearly, under Basel I and the standardized approach of Basel II, k_l and k_h equal the same constant k .

2.3 A specialization result

The analysis of equilibrium loan pricing is simplified by a result that states that under zero intermediation costs the bank's portfolio problem always has a corner

¹⁴In the latest Basel II proposals, the PD also determines the value to be imputed to the parameter ρ of exposure to systematic risk. This is based on empirical studies (for example, Lopez (2002)) which suggest the existence of a negative relationship between PDs (typically larger for small and medium sized firms) and the exposure to the risk factor z (typically smaller for those firms).

solution, so there will be banks specialized in high risk lending ($\gamma = 0$) and banks specialized in low risk lending ($\gamma = 1$). With positive intermediation costs that imply some complementarity in the provision of the various classes of loans, the problem may have an interior solution ($0 < \gamma < 1$), but we show in Appendix A that our equilibrium analysis remains essentially unchanged.

Substituting the capital requirement (8) into (4) and letting

$$\pi_j(z) = k_j + r_j - p_j(z)(\lambda + r_j), \quad (9)$$

we can more briefly write

$$\pi(z) = \gamma\pi_l(z) + (1 - \gamma)\pi_h(z). \quad (10)$$

The bank's objective function (5) then becomes

$$V = -[\gamma k_l + (1 - \gamma)k_h] + \frac{1}{1 + \delta} E[\max\{\gamma\pi_l(z) + (1 - \gamma)\pi_h(z), 0\}]. \quad (11)$$

Using this expression one can prove the following result.

Lemma 1 *With additive capital requirements and zero intermediation costs, it is optimal for banks to specialize in either high risk lending ($\gamma = 0$) or low risk lending ($\gamma = 1$).*

This result is due to the convexity introduced in the bank's objective function by limited liability, which implies that bankers appropriate the bank's net worth only when it is positive. Banks specialized in either high risk lending ($\gamma = 0$) or low risk lending ($\gamma = 1$) take advantage of limited liability whenever the systematic risk factor z is high enough to make negative the net worth $\pi_j(z)$ associated with the corresponding form of lending. In contrast, with a mixed loan portfolio ($0 < \gamma < 1$), there will generally be a range of realizations of z for which one of the loan classes makes a positive contribution to the bank's net worth, while the other makes a negative contribution. Clearly, bankers would prefer to hold each loan class as a separate corporate entity rather than netting the profits of the first class with the losses of the second.

2.4 Equilibrium loan pricing

To lighten the notation, let $p_l = p_l(z)$ and $p_h = p_h(z)$ denote the default rates of low and high risk loans, and let $F_l(p_l)$ and $F_h(p_h)$ denote the corresponding cumulative distribution functions. Using the expression of $p_j(z)$ derived from (3), it is immediate to show that for $j = l, h$ we have

$$F_j(p_j) = \Pr[p_j(z) \leq p_j] = \Phi \left(\frac{\sqrt{1-\rho} \Phi^{-1}(p_j) - \Phi^{-1}(\bar{p}_j)}{\sqrt{\rho}} \right). \quad (12)$$

The mean of the distribution of the default rate p_j is the PD \bar{p}_j , while the dispersion around the mean is entirely determined by (and increasing in) the exposure to systematic risk ρ .¹⁵

Now, from (9) and (11), the objective function of a bank specialized in loans of class j can be written as

$$V_j = -k_j + \frac{1}{1+\delta} \int_0^{\hat{p}_j} [k_j + r_j - p_j(\lambda + r_j)] dF_j(p_j),$$

where

$$\hat{p}_j \equiv \min \left\{ \frac{k_j + r_j}{\lambda + r_j}, 1 \right\}. \quad (13)$$

Under perfect competition, the *equilibrium rate* r_j^* for loans of class j is determined by the condition $V_j = 0$. Otherwise, the market for this class of loans would not clear, since banks would like either to infinitely expand their loan portfolio (if $V_j > 0$) or not to lend at all (if $V_j < 0$).¹⁶

The critical value \hat{p}_j is equal to 1 when $k_j \geq \lambda$, that is when the bank's capital can cover credit losses even when all its loans default, in which case

$$V_j = -k_j + \frac{1}{1+\delta} [k_j + r_j - \bar{p}_j(\lambda + r_j)] = \frac{1}{1+\delta} [(1 - \bar{p}_j)r_j - \bar{p}_j\lambda - \delta k_j].$$

The bank's objective function is then the discounted value of the expected net income from its loans, $(1 - \bar{p}_j)r_j - \bar{p}_j\lambda$, minus the opportunity cost of the capital invested by

¹⁵In fact, since $\partial F_j(p_j)/\partial \rho \geq 0$ if and only if $p_j \leq \Phi(\mu_j \sqrt{1-\rho})$, increasing ρ produces a mean-preserving spread in the distribution of the default rate p_j .

¹⁶Notice that, unlike in adverse selection models of the credit market (e.g., Stiglitz and Weiss (1981)), the risk class of each firm is observable, so loan rates do not have any impact on default rates.

the bankers, δk_j . In this case the equilibrium loan rate r_j^* is equal to the *actuarially fair rate*

$$\bar{r}_j = \frac{\bar{p}_j \lambda + \delta k_j}{1 - \bar{p}_j}, \quad (14)$$

defined as the rate that equals the expected net income on each loan to the opportunity cost of the required capital.¹⁷

When $k_j < \lambda$ the critical value \hat{p}_j is the default rate for which the bank's net worth at $t = 1$ equals zero. In this case, integrating by parts and using the fact that the integrand is zero for $p_j = \hat{p}_j$, we have

$$V_j = -k_j + \frac{\lambda + r_j}{1 + \delta} \int_0^{\hat{p}_j} F_j(p_j) dp_j.$$

The equilibrium loan rate r_j^* is then implicitly defined by the zero net value condition:

$$-k_j + \frac{\lambda + r_j}{1 + \delta} \int_0^{\hat{p}_j} F_j(p_j) dp_j = 0. \quad (15)$$

In what follows we will assume that the minimum capital required by regulation satisfies $0 < k_j < \lambda$, so the equilibrium loan rate r_j^* is determined by condition (15).¹⁸ The following proposition summarizes the properties of r_j^* .

Proposition 1 *Under Basel I (or the standardized approach of Basel II), the equilibrium loan rate r_j^* satisfies $0 < r_j^* < \bar{r}_j$ and is increasing in the capital requirement k_j , the PD \bar{p}_j , the LGD λ , and the cost of capital δ , and decreasing in the exposure to systematic risk ρ .*

Not surprisingly, r_j^* increases with the PD and the LGD of the loan, which increase expected credit losses, as well as with the cost and the level of the capital requirement.¹⁹ The effect of ρ is somewhat more intriguing, but it is explained by the

¹⁷The actuarially fair rate is also the loan rate that would prevail if bankers had unlimited liability, or if depositors were not insured and demanded proper compensation for the losses in case of bank failure, or if the government charged actuarially fair deposit insurance premia.

¹⁸When $k_j = 0$ we have $\hat{p}_j > 0$ for all $r_j > 0$, so the zero net value condition (15) can only be satisfied for $r_j^* = 0$. In this case $\hat{p}_j = 0$, and the bank fails with probability one.

¹⁹Interestingly, k_j has a positive impact on r_j^* even when $\delta = 0$. This is because requiring capital reduces the subsidization of credit losses by the deposit insurance system.

incidence of deposit insurance: when the variability of bank profits rises (as it is the case when ρ increases), the subsidization coming from the deposit insurance system increases. Under perfect competition, this increased subsidy is passed on to firms in the form of cheaper loans.

The pricing formula (15) implies that if the capital requirement $k_j = \lambda p_j(z_\alpha)$ resulting from the application of the IRB approach of Basel II coincides with the constant capital requirement of Basel I, then both regulatory regimes lead to the same equilibrium loan rate r_j^* . But under the IRB approach loan rates respond differently to changes in some parameters. Specifically, changes in the PD, the LGD, and the exposure to systematic risk alter the distribution of credit losses and thus change the capital requirement k_j , producing indirect effects on r_j^* which add to the (direct) effects described in Proposition 1. The implications are summarized in the following proposition.

Proposition 2 *Under the IRB approach of Basel II, the equilibrium loan rate r_j^* is more sensitive to changes in the PD \bar{p}_j and the LGD λ than under an initially equivalent Basel I capital requirement. Moreover, if $\Phi(\mu_j\sqrt{\rho}) > 1 - \alpha$, the equilibrium loan rate r_j^* may be increasing in the exposure to systematic risk ρ*

For the PD and LGD parameters, the properties of the IRB capital requirement defined by (7) together with the fact that k_j has a positive effect on r_j^* (Proposition 1) imply that the indirect effects reinforce the direct effects. However, this is not so for changes in the exposure to systematic risk. As noted above, if $\Phi(\mu_j\sqrt{\rho}) > 1 - \alpha$, an increase in ρ increases the IRB requirement k_j , so ρ will have a positive indirect effect on r_j^* , partially or even totally offsetting its negative direct effect on r_j^* .²⁰

To conclude this section it is interesting to note that the actual solvency probability implied by the IRB capital requirement is greater than the target confidence level α . To see this, notice that the definition (13) of \hat{p}_j together with the fact that

²⁰Numerical simulations show that, for realistic parameter values, the indirect effect dominates.

$r_j^* > 0$ implies

$$\widehat{p}_j = \frac{\lambda p_j(z_\alpha) + r_j^*}{\lambda + r_j^*} > p_j(z_\alpha),$$

so

$$\Pr(\pi_j(z) \geq 0) = \Pr(p_j \leq \widehat{p}_j) = F_j(\widehat{p}_j) > F(p_j(z_\alpha)) = \alpha.$$

This result follows from the fact that the net interest income earned on performing loans helps to (partially) compensate the losses incurred on defaulting loans, an effect that is not taken into account in the computation of the IRB capital requirement. We will come back to this issue in the following sections of the paper.

3 Implications of Basel II

This section uses the analytical framework developed above in order to discuss the qualitative and quantitative effects of the adoption of the Basel II reform of bank capital regulation.

3.1 Qualitative effects

As we have already pointed out, Basel I established a common capital requirement for all business loans, k^I (specifically, 8%), while Basel II allows banks to choose between the standardized approach, in which all loans to unrated firms carry a constant capital charge, k^S , and the IRB approach under which each class of loans j carries a different capital charge, k_j^{IRB} , computed using (7). Thus, our discussion must start analyzing which capital requirement will effectively determine the equilibrium interest rate for each class of loans. In this regard, a principle that immediately derives from our previous results is that, for each class of loans, the approach associated with the minimum capital charge will be the one permitting banks to offer the most attractive loan rate.

When comparing Basel II with its predecessor, the Basel Committee states that “the new framework should at least maintain the current overall level of capital in the system” (BCBS, 2001, p. 6). However, accomplishing such goal across the two

alternative approaches requires further clarification. In particular, it is argued that “in connection with the standardized approach, the Committee desires neither to produce a net increase nor a net decrease -on average- in minimum regulatory capital” (BCBS, 2001, p. 9). In our setup, as the two classes of (unrated) loans will carry the same standardized charge, k^S , this objective would translate into

$$k^S = k^I. \quad (16)$$

Hence if all banks were to adopt the standardized approach, moving from Basel I to Basel II would produce no change in equilibrium loan rates.

On the other hand, “in respect to the IRB approaches, the Committee’s ultimate goals are to ensure that the overall level of regulatory capital (...) provides capital incentives relative to the standardized approach (e.g. for the foundation IRB approach in the aggregate, a reduction in risk-weighted assets of 2% to 3%)” (BCBS, 2001, p. 9). This implies that, in our setup, the confidence level α for the IRB approach would be chosen so that

$$lk_l^{\text{IRB}} + (1 - l)k_h^{\text{IRB}} = (1 - \eta)k^S, \quad (17)$$

where η is a small number such as 0.02 or 0.03. For those values of η and two sufficiently different and sizeable classes of loans, (17) implies

$$k_l^{\text{IRB}} < k^S < k_h^{\text{IRB}}. \quad (18)$$

Hence banks adopting the IRB (standardized) approach of Basel II would be able to offer better rates to low risk (high risk) firms than banks adopting the standardized (IRB) approach. This allows us to state the following result.

Proposition 3 *Under Basel II, the equilibrium rates of low risk loans will be determined by the capital charges of the IRB approach and will be lower than under Basel I, while the equilibrium rates of high risk loans will be determined by the capital charges of the standardized approach and will be same as under Basel I.*

This result is due to the combination of a regulation calibrated to match, on an approach by approach basis, the capital charges of an “average” bank under Basel I, and the relatively advantageous (disadvantageous) treatment that low risk (high risk) lending receives in the IRB approach. The implication under specialization is that banks that lend to low risk firms will adopt the IRB approach, while banks that lend to high risk firms will adopt the standardized approach.²¹

The asymmetric effects on the equilibrium rates of low risk and high risk loans should not be read as a reflection of distortions introduced by Basel II. Rather, they reflect the correction of (possibly more worrying) distortions that prevailed under Basel I. A reform that allows banks to save capital on low risk loans may be justified if the previous regulation could not discriminate between different classes of loans and was conservatively targeted to guarantee a minimum degree of solvency for the banks specialized in the riskiest loans. According with this view, the main defect of Basel I would have been the excessive capital charges (and consequently excessively high interest rates) on low risk loans.

An interesting side effect of the correction of Basel I distortions is that the solvency probability of the banks specialized in low risk lending will fall. As noted above, this probability is given by $F_l(\hat{p}_l)$, where \hat{p}_l is increasing in the capital requirement and the equilibrium loan rate. As such banks will adopt the IRB approach, we have $k_l^{\text{IRB}} < k^{\text{I}}$ and $r_l^{\text{IRB}} < r_l^{\text{I}}$, and the result follows. Intuitively, after adopting the IRB approach, these banks will have a lower capital buffer and will charge lower rates, so their margin income buffer will also be lower. Both effects imply a higher probability of failure.²²

²¹If intermediation costs like those in Appendix A made banks non-specialized, and all banks were identical, then under (17) they would all have an incentive to adopt the IRB approach. However, if they were not identical, banks with a higher proportion of low risk (high risk) loans would adopt the IRB (standardized) approach of Basel II, so Proposition 3 would still hold.

²²Notice that despite the reduction in the solvency of the banks specialized in low risk lending, if low and high risk loans are sufficiently distinct, Basel II will probably keep them safer than the banks specialized in high risk lending that adopt the standardized approach. This is confirmed in the simulations below.

3.2 Quantitative effects

In order to assess the quantitative importance of our results, we now consider a number of realistic parameterizations of the model. In particular, we look at the equilibrium pricing under Basel I and Basel II of various classes of (uncollateralized) corporate loans that differ in their PDs, and we compute the levels of bank solvency to which they lead, measured by the probability of failure of banks specialized in each of them.

Two economies are considered. The first one corresponds to the parameters for corporate loans set in the 2001 Consultative Document of the Basel Committee, which are a LGD λ of 50% and an exposure to the systematic risk factor ρ of 20%. The second economy corresponds to the parameters for corporate loans set in the 2003 Consultative Document, which are a LGD λ of 45% and an exposure to the systematic risk factor which is decreasing in the PD according to the function

$$\rho(\bar{p}_j) = \frac{12}{100} \left(2 - \frac{1 - \exp(-50 \bar{p}_j)}{1 - \exp(-50)} \right), \quad (19)$$

which yields $\rho(0) = 0.24$ and $\rho(1) = 0.12$. In both economies, the required return on bank capital δ is set equal to 6%.²³

For each of these economies, and for PDs \bar{p}_j in a range from 0.03% (which is the minimum contemplated in Basel II) to 10%, we compute the equilibrium loan rates r_j^* and the probabilities of bank failure $1 - F_j(\hat{p}_j)$ under three different capital requirements. The first one corresponds to Basel I (or the standardized approach of Basel II for unrated firms) so $k_j^I = 0.08$. The second one corresponds to the 2001 IRB proposal for corporate loans with maturity of one year (IRB'01) which sets $k_j^{\text{IRB}'01} = s\lambda p_j(z_\alpha)$, where $s = 1.5624$ is a scaling factor,²⁴ $\lambda = 0.5$, and $p_j(z_\alpha)$ is the

²³Most bankers would consider this number rather conservative. We take it from the evidence on the equity premium (for an overview, see Siegel and Thaler (1997)). Such premium is computed from excess returns in equity markets and is partly explained by risk aversion. In our risk-neutral economy, however, a positive δ should reflect contracting costs due to agency and/or asymmetric information problems, and might take the form of price discounts in equity sales (for evidence on them, see Asquith and Mullins (1986)).

²⁴This factor is calibrated so that, once the proposed maturity adjustment is taken into account, the capital charge for a three year loan with a PD of 0.7% equals 8%.

α -quantile of $F_j(p_j)$ for $\alpha = 0.995$ and $\rho = 0.2$. The third one corresponds to the 2003 IRB proposal for corporate loans with maturity of one year (IRB'03) which sets $k_j^{\text{IRB'03}} = \lambda p_j(z_\alpha)$, where $\lambda = 0.45$, and $p_j(z_\alpha)$ is the α -quantile of $F_j(p_j)$ for $\alpha = 0.999$ and $\rho(\bar{p}_j)$ given by (19).

The results are shown in Table 1. The panels denoted Economy 1 and Economy 2 correspond, respectively, to the economies of reference of the IRB'01 and the IRB'03 proposals. Notice that, since we have normalized to zero the interest rate on (fully insured) deposits, all interest rates in these simulations should be interpreted as spreads over a risk-free rate.²⁵ Moreover, these spreads do not incorporate any component of intermediation or origination costs, since we have assumed them to be zero.

In both economies, for PDs of about 1%, the three capital requirements imply very similar capital charges and hence very similar loan rates. Yet, as stated in Proposition 2, loan rates are more sensitive to PDs under IRB capital requirements than under Basel I requirements, so for smaller (larger) PDs the rates implied by the former are smaller (larger) than those implied by the latter. Our analysis identifies two reasons for this different behavior. First and foremost, IRB capital requirements are increasing in the PD and banks pass the corresponding additional financing cost on to the borrowers in the form of higher loan rates. Second, under Basel I the banks' probability of failure and hence the implied deposit insurance subsidy are increasing in the PD, and banks pass this value on to the borrowers in the form of lower rates, partly offsetting the direct positive effect of PDs on loan rates.

According to Table 1, adopting the IRB approach may imply a reduction in loan rates of almost 50 basis points for loans with a PD of 0.03%, and an increase of 80 to 200 basis points for loans with a PD of 10%. These numbers illustrate the quantitative significance of the interest rate savings that, as predicted by Proposition

²⁵In reality there could be a positive spread between the risk-free rate and the deposit rate, reflecting either monopolistic rents in the deposit market or charges due to the costs of the liquidity and payment services associated with deposits. Yet, if there is a (collateralized) interbank market, then under certain conditions banks' deposit taking and lending activities would be separable, and the interbank repo rate would be the appropriate reference rate for the pricing of bank loans.

3, will make low risk (high risk) firms prefer to borrow from banks that adopt the IRB (standardized) approach of Basel II.

Table 1
Equilibrium loan rates and probabilities of bank failure
(all variables in per cent)

Economy 1						
$\lambda = 50\%$ and $\rho = 20\%$						
\bar{p}_j	r_j^*			$1 - F_j(\hat{p}_j)$		
	Basel I	IRB'01	IRB'03	Basel I	IRB'01	IRB'03
0.03	0.50	0.04	0.05	0.00	0.15	0.06
0.05	0.51	0.06	0.08	0.00	0.14	0.06
0.10	0.53	0.12	0.14	0.00	0.13	0.06
0.20	0.58	0.23	0.25	0.00	0.11	0.06
0.50	0.73	0.51	0.52	0.01	0.08	0.08
1.00	0.99	0.95	0.89	0.04	0.06	0.11
2.00	1.50	1.77	1.54	0.26	0.04	0.20
4.00	2.55	3.31	2.78	1.27	0.02	0.35
7.00	4.13	5.57	4.73	3.72	0.01	0.45
10.00	5.77	7.86	6.77	6.72	0.00	0.47

Economy 2						
$\lambda = 45\%$ and $\rho(\bar{p}_j) = 12\% \times \left(2 - \frac{1 - \exp(-0.5 \bar{p}_j)}{1 - \exp(-50)}\right)$						
\bar{p}_j	r_j^*			$1 - F_j(\hat{p}_j)$		
	Basel I	IRB'01	IRB'03	Basel I	IRB'01	IRB'03
0.03	0.49	0.04	0.05	0.00	0.19	0.08
0.05	0.50	0.06	0.08	0.00	0.18	0.08
0.10	0.53	0.12	0.14	0.00	0.16	0.08
0.20	0.57	0.21	0.24	0.00	0.13	0.08
0.50	0.71	0.49	0.49	0.00	0.07	0.07
1.00	0.94	0.90	0.84	0.02	0.03	0.06
2.00	1.41	1.66	1.44	0.07	0.01	0.05
4.00	2.37	3.10	2.59	0.26	0.00	0.03
7.00	3.88	5.19	4.37	0.96	0.00	0.02
10.00	5.47	7.30	6.24	2.23	0.00	0.02

The flat 8% capital requirement of Basel I translates into a probability of failure of virtually zero for banks specialized in low PD loans, while it leads to a probability of

some full percentage points for banks specialized in high PD loans. The latter effect is particularly sizeable in Economy 1 where the exposure of firms to the systematic risk factor does not fall as the PD increases. As predicted, when the IRB'01 and IRB'03 requirements are applied to their economies of reference, the probabilities of bank failure are lower than the benchmarks of 0.5% and 0.1% associated with the underlying confidence levels (99.5% and 99.9%, respectively). This is explained by the fact that IRB capital requirements do not allow the deduction of the net interest income of non-defaulting loans from the losses associated with defaulting loans (despite that the former clearly contributes to absorb the latter and hence reduces the probability of bank failure).²⁶ Notice that this effect is more significant when loan rates are high, which explains why, in the reference economy of each IRB requirement, the banks specialized in high risk loans exhibit lower probabilities of failure.²⁷

4 Optimal Capital Requirements

Requiring banks to hold capital increases their funding costs. Under perfect competition, these additional costs are transferred to the borrowers in the form of higher loan rates. To justify this social cost of regulation one needs to introduce some social benefit, for example in the form of a reduction in the probability and hence the expected cost of bank failures. In what follows we assume that the failure of a bank entails a social cost $c > 0$ per unit of loans. We consider a regulatory system that allows to impose a different capital requirement k_j to each loan class j , and we compute the level of the cost c for which the optimal capital requirement and the Basel II IRB requirement coincide, so the latter would be optimal.

²⁶Obviously, the scaling factor s in the IRB'01 case further contributes to reduce the probabilities of bank failure.

²⁷The probabilities of failure in Economy 1 under IRB'03 are increasing in \bar{p}_j because the capital requirement is set on the assumption that ρ is decreasing in the PD, while it is constant in this economy. Similarly, the probabilities of failure in Economy 2 under IRB'01 are decreasing in \bar{p}_j because the capital requirement is defined on the assumption that ρ is constant, while it is decreasing in this economy.

4.1 A social welfare function

In our risk-neutral economy, social welfare may be evaluated by simply adding the expected payoffs of the four classes of agents: entrepreneurs, bankers, depositors, and the government. For convenience, we will express these payoffs in $t = 1$ terms. Since bankers and depositors get expected returns that just cover the opportunity cost of their funds, their net expected payoffs are zero.

The entrepreneurs of each class j appropriate their projects' returns in excess of equilibrium loan repayments in the event of success, $a - r_j^*$, and get zero in the event of failure, so their expected payoff is

$$U_j^* = (1 - \bar{p}_j)[(1 + a) - (1 + r_j^*)] = (1 - \bar{p}_j)(a - r_j^*). \quad (20)$$

Since a measure l of entrepreneurs are of class $j = l$ and a measure $1 - l$ are of class $j = h$, in aggregate terms they get

$$U^* = lU_l^* + (1 - l)U_h^*. \quad (21)$$

Finally, under bank specialization, the expected payoff of the government is

$$G^* = lG_l^* + (1 - l)G_h^* \quad (22)$$

where

$$G_j^* = E[\min\{\pi_j(z), 0\}] - c[1 - F_j(\hat{p}_j)] \quad (23)$$

measures the contribution of the lending to firms of class $j = l, h$. The first term in (23) is the expected liability of the deposit insurance system for a bank of size one specialized in such lending (the expected value of the negative part of the bank's net worth), while the second is the expected social cost of bank failure (c times the corresponding probability). Using the definition (9) of $\pi_j(z)$ and the properties of the $\min\{\pi_j(z), 0\}$ function we have

$$E[\min\{\pi_j(z), 0\}] = E[\pi_j(z) - \max\{\pi_j(z), 0\}] = k_j + r_j^* - \bar{p}_j(\lambda + r_j^*) - E[\max\{\pi_j(z), 0\}].$$

But in equilibrium the bank's zero net value condition implies that $E[\max\{\pi_j(z), 0\}] = (1 + \delta)k_j$, so we can simply write

$$G_j^* = (1 - \bar{p}_j)r_j^* - \bar{p}_j\lambda - \delta k_j - c[1 - F_j(\hat{p}_j)]. \quad (24)$$

Social welfare is then measured by the sum of the expected payoffs of the entrepreneurs and the government, $W^* = U^* + G^*$. From (21) and (22), it is clear that W^* can be additively decomposed into the contribution of the lending to each class of firms, that is $W^* = lW_l^* + (1 - l)W_h^*$, where using (20) and (24) we have

$$W_j^* = U_j^* + G_j^* = (1 - \bar{p}_j)a - \bar{p}_j\lambda - \delta k_j - c[1 - F_j(\hat{p}_j)], \quad (25)$$

for $j = l, h$. Thus, the contribution to social welfare associated to each class of firms equals the expected net returns of their projects, $(1 - \bar{p}_j)a - \bar{p}_j\lambda$, minus the cost of the capital required for providing their loans, δk_j , and the expected social cost of the corresponding bank failures, $c[1 - F_j(\hat{p}_j)]$.

The optimal capital requirement for each loan class j can be obtained by maximizing W_j^* in (25) with respect to k_j . An interior solution is characterized by the first order condition

$$cF_j'(\hat{p}_j)\frac{d\hat{p}_j}{dk_j} = \delta, \quad (26)$$

where $F_j'(\hat{p}_j)$ is positive since it is the density function of the default rate p_j , and from (13) we have

$$\frac{d\hat{p}_j}{dk_j} = \frac{1}{\lambda + r_j^*} \left[1 + (1 - \hat{p}_j) \frac{\partial r_j^*}{\partial k_j} \right],$$

which is also positive since $\partial r_j^*/\partial k_j > 0$ by Proposition 1. Condition (26) simply equates the marginal social benefit of bank capital (increasing k_j increases the bankruptcy default rate \hat{p}_j both directly and through r_j^* , and thus reduces the probability of bank failure) to its marginal cost (increasing k_j increases the cost of financing firms' projects).

4.2 Quantifying the trade-off

Condition (26) implicitly defines the level of the social cost of bank failure c for which any given capital requirement k_j would be optimal.²⁸ Table 2 shows this implicit social cost for the two economies considered in Section 3.2 and the capital requirements specified in the IRB'01 and IRB'03 proposals. As in Table 1, the required return on bank capital is set equal to 6%, and PDs range from 0.03% to 10%.

Table 2
Implicit social cost of bank failure
(all variables in per cent)

\bar{p}_j	Economy 1		Economy 2	
	IRB'01	IRB'03	IRB'01	IRB'03
0.03	7.09	23.75	6.25	18.88
0.05	11.16	33.69	9.74	26.82
0.10	20.65	51.73	18.03	42.09
0.20	38.39	73.13	34.77	63.69
0.50	88.75	92.73	98.42	102.83
1.00	173.09	86.39	303.20	140.82
2.00	360.83	64.57	1.9×10^3	194.33
4.00	878.14	47.21	3.9×10^4	300.66
7.00	2.4×10^3	44.26	1.3×10^6	480.22
10.00	6.6×10^3	47.08	4.2×10^7	664.74

Table 2 shows that when the IRB'01 and IRB'03 requirements are applied to their economies of reference (that is, Economy 1 and Economy 2, respectively), the implicit social cost of bank failure is remarkably increasing in the PD. While it is moderate for low PDs, it becomes implausibly large for PDs above 1%, exceeding several times the size of the bank's balance sheet, which suggests that IRB capital charges are too high for high risk loans.²⁹

To interpret the results in Table 2, notice that by (26) the implicit social cost of bank failure is inversely proportional to the marginal reduction in the probability of

²⁸Obviously, one needs to check that the solution corresponds to a maximum.

²⁹Notice, however, that this problem may have little practical incidence if high PD firms turn to banks that adopt the standardized approach of Basel II.

bank failure that can be achieved by increasing k_j at the required levels of capital. It turns out that, with the confidence levels of Basel II, the marginal effect of k_j on bank solvency is tiny for high PDs, and only a huge social cost c may justify the size of these capital requirements.

The striking results in Table 2 are partly explained by the fact that equilibrium loan rates and hence banks' net interest income from non-defaulting loans (which provides them with a buffer in addition to the capital required by regulation) are increasing in the PD. As Table 1 illustrates, this effect reduces the probability of bank failure to negligible levels. The results may also reflect that the model that the regulator uses for setting IRB capital requirements is not (and is not intended to be) a description of the economy to which the regulation will apply, but just an instrument for implementing the capital requirements that the regulator considers adequate. Although a discussion along these lines opens the difficult and, in the end, empirical question of what model would then properly describe the economy, it may be interesting to elaborate briefly on it. Suppose, in particular, that Economy 1 (the model underlying the IRB'01 proposal) provides the right description of the actual economy, and that the parameters of IRB'03 were introduced just as a tool for defining the desired requirements for such economy. Then the social cost of bank failure that would justify the latest Basel proposals would be given by the IRB'03 column of Economy 1 in Table 2. It is apparent that those numbers are more sensible in size and homogeneous across PDs than the numbers in any of the other columns. So it is possible that the corrections introduced in 2003 did not respond to a change in the regulator's view about the primitive parameters of the economy, but to the desire to approach the capital requirements imposed on such an economy to the socially optimal ones.

4.3 A margin income correction

Correcting the problem of excessive capital charges for high risk loans is in principle straightforward. It simply requires deducting the net interest income of non-defaulting

loans from the losses associated with defaulting loans. In particular, one could require banks to hold a minimum capital k_j such that their net worth is positive with a target confidence level α , that is

$$\Pr(\pi_j(z) \geq 0) = \Pr(p_j \leq \hat{p}_j) = F_j(\hat{p}_j) = \alpha,$$

or, equivalently, $\hat{p}_j = F_j^{-1}(\alpha) = p_j(z_\alpha)$. Substituting the definition (13) of \hat{p}_j then gives

$$k_j = \lambda p_j(z_\alpha) - r_j^*[1 - p_j(z_\alpha)]. \quad (27)$$

The first term in (27) is the IRB capital requirement of Basel II, and the second is the proposed margin income correction. This correction is based on the α -quantile of the distribution of the default rate, $p_j(z_\alpha)$, because what matters for ensuring the confidence level α is the margin income when no more than such a fraction of loans default.

Since the equilibrium loan rate r_j^* depends on the capital requirement k_j , obtaining a closed-form expression for k_j requires solving simultaneously (27) and the zero net value condition (15). But under the proposed capital requirement, the critical default rate \hat{p}_j equals $p_j(z_\alpha)$. Hence we can explicitly solve for r_j^* and k_j , which gives

$$k_j = \frac{\lambda \int_0^{p_j(z_\alpha)} F_j(p_j) dp_j}{(1 + \delta)[1 - p_j(z_\alpha)] + \int_0^{p_j(z_\alpha)} F_j(p_j) dp_j}. \quad (28)$$

In order to avoid the numerical computation of the integral in (28), we can obtain an approximation to the proposed k_j by noting that for $p_j > p_j(z_\alpha)$ we have $F_j(p_j) > F_j(p_j(z_\alpha)) = \alpha$. Thus, for high values of the confidence level α , we have $\int_{p_j(z_\alpha)}^1 F_j(p_j) dp_j \simeq 1 - p_j(z_\alpha)$, so we can write

$$\int_0^{p_j(z_\alpha)} F_j(p_j) dp_j = \int_0^1 F_j(p_j) dp_j - \int_{p_j(z_\alpha)}^1 F_j(p_j) dp_j \simeq p_j(z_\alpha) - \bar{p}_j.$$

Substituting this approximation into (28) then implies

$$k_j \simeq \frac{\lambda[p_j(z_\alpha) - \bar{p}_j]}{\delta[1 - p_j(z_\alpha)] + 1 - \bar{p}_j}. \quad (29)$$

The same approximation can be obtained from (27) if the equilibrium loan rate r_j^* is replaced by the actuarially fair rate \bar{r}_j defined in (14). This is explained by the fact that, for high values of the confidence level α , the equilibrium and the fair rates are almost identical (see Appendix B).

Interestingly, the corrected IRB requirement (28) (as well as its approximation (29)) is decreasing in the cost of capital δ . This is explained by the fact that, under perfect competition, a higher cost of capital is borne by the borrowers in the form of higher loan rates, which add to the net interest income buffer. Thus, market conditions leading to a high cost of capital, such as imperfect capital markets or economic recessions, would *ceteris paribus* lower the corrected IRB requirements.

Computations parallel to those described in Section 4.2 show that the implicit social cost of bank failure that would justify the adoption of the corrected capital requirements in Economies 1 and 2 under confidence levels of 99.5% and 99.9%, respectively, is substantially smaller than under the IRB requirements of Basel II, especially for high risk loans (although the cost is still increasing in the PD). Perhaps more importantly, these computations reveal that the margin income correction may have an impact on equilibrium loan rates (relative to the rates obtained under the IRB requirements) as high as 25 basis points for high risk loans.

5 Concluding Remarks

We have analyzed the loan pricing implications of capital requirements in a credit market where, as in the model underlying the internal ratings based (IRB) approach of Basel II, loan default rates are driven by a single factor of systematic risk. We have focused on the effects for unrated firms of the transition from Basel I, with a common capital charge for all business loans, to Basel II, which allows banks to choose between a standardized approach (which treats all loans to unrated firms essentially as in Basel I) and an IRB approach (which makes capital charges a function of the bank's estimate of the PD).

Our predictions build on two important features of the new regulation. First, its calibration to match, on an approach by approach basis, the capital charges of an “average” bank under Basel I. Second, the relatively advantageous (disadvantageous) treatment that low risk (high risk) lending receives in the IRB approach. In these circumstances, we predict that banks specialized in low risk (high risk) lending will tend to adopt the IRB (standardized) approach. Accordingly, the equilibrium rates of low risk loans will be lower than under Basel I, while the equilibrium rates of high risk loans will be roughly the same as under Basel I.

We have computed the level of the social cost of bank failure that could justify the IRB capital requirements of Basel II. The variation in this cost across PDs and its implausibly large size for high PD loans suggests that the current design implies too high charges for such loans. We speculate that this result might reflect a discrepancy between the model that the regulator uses for the quantification of capital charges and the model that the regulator considers a proper description of the actual economy (which, perhaps, generates distributions of credit losses with fatter tails). The result is certainly related to the fact that Basel II does not take into account the net interest income from performing loans, which provides a buffer, in addition to capital, against credit losses. We have derived a simple closed-form formula that incorporates a margin income correction in IRB capital requirements.

An interesting quantitative finding (confirmed by the result in Appendix B) is that, with the levels of solvency implied by IRB capital requirements, the deposit insurance subsidy is very small, and hence has a negligible effect on loan pricing.³⁰ This is surprising in view of the vast literature on the risk-shifting incentives of banks under deposit insurance.³¹ In our economy, the distortions to the allocation of credit that such subsidy may cause are virtually zero (actually, they are replaced by distortions of an opposite sign due to the cost of bank capital). Of course, IRB requirements

³⁰This also implies that the actuarially fair deposit insurance premia for banks adopting the IRB approach would be very small.

³¹This literature includes Furlong and Keeley (1989), Genotte and Pyle (1991), and Hellmann, Murdock and Stiglitz (2000), among many others.

rely quite crucially on attributing to each loan an unbiased estimate of its PD. Our results suggest that the literature on moral hazard in banking should now focus on the incentives for banks to properly estimate and truthfully report the risk of their loans, that is, on the system of penalties and/or rewards that, in the context of a (possibly repeated) relationship between banks and their supervisors, ensures the compliance of the former. Precisely, this is the subject of the supervisory review process of Basel II, and would be a topic for another paper.

We close the paper by commenting on two simple extensions that expand the set of predictions that can be deduced from the analysis and, after proper calibration, would allow a finer quantification of the effects of Basel II. First, assuming that the demand for each class of loans is inelastic implies that changes in regulation only have an effect on loan rates, leaving the volume and composition of lending unchanged. Implications for quantities could be easily derived by introducing heterogeneity in the reservation utilities of the entrepreneurs. Specifically, if $H_j(U_j)$ denotes the measure of potentially borrowing entrepreneurs of class j whose reservation utility is less than or equal to U_j , then the market demand for loans of class j is given by $L_j(r_j^*) = H_j[(1 - \bar{p}_j)(a - r_j^*)]$, since only the entrepreneurs with reservation utilities below the expected payoff $U_j^* = (1 - \bar{p}_j)(a - r_j^*)$ derived in (20) will want to undertake their projects. Since $L_j(r_j^*)$ is decreasing, it follows that changes in parameters that affect the equilibrium loan rate r_j^* will produce variations of the opposite sign in the corresponding volume of lending.³² Accordingly, by Proposition 1, under Basel I (or the standardized approach of Basel II), equilibrium lending will be decreasing in the PD and the LGD of the corresponding class of loans, as well as in the capital requirement and the cost of capital. And, by Proposition 3, moving to Basel II will increase the volume of low risk lending, leaving high risk lending unchanged.

Second, taking the cost of bank capital δ as an exogenous parameter is equivalent to assuming a perfectly elastic supply of bank capital at such rate. In this context,

³²Quantitatively, the importance of these effects would depend on the elasticity of the demand for loans, which would be proportional to the density of entrepreneurs at the reservation utility U_j^* .

shocks to the different parameters of the model may induce fluctuations in the aggregate demand for bank capital but there are no feedback effects on loan rates (or loan volumes). Yet, these feedback effects are a great concern in the discussions about the potential *procyclicality* of Basel II.³³ A simple way to accommodate them is to introduce an increasing supply of bank capital, $K(\delta)$. With inelastic demands for each class of loans, the aggregate demand for bank capital is simply $lk_l + (1-l)k_h$, which does not depend on δ . The equilibrium cost of capital δ^* is then determined by the market clearing condition $K(\delta^*) = lk_l + (1-l)k_h$, and its variations recursively affect the pricing of bank loans according to the results in Proposition 1.³⁴

Thus, under Basel I (or the standardized approach of Basel II), the cost of capital would be increasing in the capital requirement and decreasing in the shocks to the supply of bank capital, inducing variations of the same sign in loan rates. And under the IRB approach of Basel II, the cost of capital would be decreasing in the shocks to the supply of bank capital and increasing in the target confidence level of the regulator. In this setting, if there is positive correlation between the factors that stimulate aggregate economic activity and bank capital, and negative correlation between these factors and capital requirements, then (unless there is a fully offsetting cyclical pattern in the demand for loans) the cost of bank capital would tend to be high in recessions and low in expansions. Obviously, moving to Basel II may exacerbate this procyclicality since its capital requirements are more sensitive to risk than those of Basel I.³⁵ On the other hand, according to Proposition 3, Basel II will reduce the overall demand for bank capital and, consequently, its cost, leading to lower average rates for both high and low risk firms.

³³See, for example, Lowe (2002).

³⁴With elastic loan demands, the recursivity of the system breaks down. An increase in δ increases the rates applied to each class of loans. If, consequently, the demand for loans decreases, so does the capital required by banks, introducing a further equilibrating force in the market for bank capital. Clearly, this mechanism would imply translating part of the adjustment to the equilibrium volumes of lending.

³⁵Notice that our margin income correction would partly compensate this effect, since the resulting IRB requirements are less sensitive to risk and decrease when the cost of capital increases.

Appendices

A The case of non-specialized banks

This Appendix extends our results to the case where the portfolio problem of the representative bank has an interior solution, so the bank simultaneously makes low and high risk loans. We first relax the assumption of zero intermediation costs, and show how the presence of complementarities in the bank's cost function may counterbalance the convexity that limited liability introduces in the bank's objective function. Then, assuming that the result is a bank that makes both classes of loans, we show that the comparative statics summarized in Proposition 1 would still hold.

Let $M(L_l, L_h)$ denote the intermediation costs that the representative bank incurs at $t = 0$ when it lends an amount L_l to low risk firms and an amount L_h to high risk firms. Assume that $M(L_l, L_h)$ is linearly homogeneous, increasing, and convex. By homogeneity we can write $M(L_l, L_h) = (L_l + L_h)m(\gamma)$, where $m(\gamma)$ is a function of the ratio $\gamma \equiv L_l/(L_l + L_h)$. In this case, the marginal costs of low risk and high risk lending satisfy

$$M_l(\gamma) = \frac{\partial M(L_l, L_h)}{\partial L_l} = m(\gamma) + (1 - \gamma)m'(\gamma)$$

$$M_h(\gamma) = \frac{\partial M(L_l, L_h)}{\partial L_h} = m(\gamma) - \gamma m'(\gamma),$$

which imply

$$m(\gamma) = \gamma M_l(\gamma) + (1 - \gamma)M_h(\gamma) \quad \text{and} \quad m'(\gamma) = M_l(\gamma) - M_h(\gamma).$$

Also, the convexity of $M(L_l, L_h)$ implies $m''(\gamma) > 0$.

For a loan portfolio of size one (that is, $L_l + L_h = 1$), the objective function of the representative bank becomes

$$V(\gamma) = -[\gamma k_l + (1 - \gamma)k_h] - m(\gamma) + \frac{1}{1+\delta} \int_{-\infty}^{\hat{z}} [\gamma \pi_l(z) + (1 - \gamma)\pi_h(z)] d\Phi(z), \quad (30)$$

where the critical value \hat{z} is implicitly defined by

$$\gamma\pi_l(\hat{z}) + (1 - \gamma)\pi_h(\hat{z}) = 0. \quad (31)$$

The first component in (30) is linear in γ , the second is concave (since $m''(\gamma) > 0$), and the third is convex (see the proof of Lemma 1). Hence we may have corner solutions (like in our basic model with no intermediation costs) or interior solutions. In what follows, we assume that the concavity of the intermediation cost term dominates and there is an interior solution characterized by the first order condition

$$V'(\gamma) = (k_h - k_l) - m'(\gamma) + \frac{1}{1+\delta} \int_{-\infty}^{\hat{z}} [\pi_l(z) - \pi_h(z)] d\Phi(z) = 0. \quad (32)$$

In this situation, a competitive equilibrium would be characterized by (32) together with the zero net value condition, $V(\gamma) = 0$, and the market clearing condition, $\gamma = l$, that equates the supply of low risk loans to the proportion of low risk firms in the economy.

Substituting $m'(\gamma) = M_l(\gamma) - M_h(\gamma)$ into $V'(\gamma) = 0$, setting $\gamma = l$, and rearranging gives

$$-k_l - M_l(l) + \frac{1}{1+\delta} \int_{-\infty}^{\hat{z}} \pi_l(z) d\Phi(z) = -k_h - M_h(l) + \frac{1}{1+\delta} \int_{-\infty}^{\hat{z}} \pi_h(z) d\Phi(z).$$

On the other hand, substituting $m(\gamma) = \gamma M_l(\gamma) + (1-\gamma)M_h(\gamma)$ into $V(\gamma) = 0$, setting $\gamma = l$, and rearranging gives

$$\gamma \left[-k_l - M_l(l) + \frac{1}{1+\delta} \int_{-\infty}^{\hat{z}} \pi_l(z) d\Phi(z) \right] + (1-\gamma) \left[-k_h - M_h(l) + \frac{1}{1+\delta} \int_{-\infty}^{\hat{z}} \pi_h(z) d\Phi(z) \right] = 0.$$

These two equations are equivalent to

$$-k_j - M_j(l) + \frac{1}{1+\delta} \int_{-\infty}^{\hat{z}} \pi_j(z) d\Phi(z) = 0, \quad (33)$$

for $j = l, h$. The pricing equation (33) is identical to that of the specialization case in the text, except for the fact that (i) it includes the equilibrium marginal cost term $M_j(l)$, and (ii) the critical value \hat{z} is defined by condition (31) instead of $\pi_j(\hat{z}) = 0$. Its interpretation is straightforward: the marginal benefit of making one additional

loan to a firm of class j must compensate the bank for the required capital and the marginal intermediation cost.

The comparative statics of the equilibrium loan rate r_j^* may be obtained by differentiating (33). Specifically, we have

$$\frac{\partial r_j^*}{\partial k_j} = \frac{1 - \frac{1}{1+\delta}[\Phi(\hat{z}) + \pi_j(\hat{z})\phi(\hat{z})\frac{\partial \hat{z}}{\partial k_j}]}{\frac{1}{1+\delta}[\int_{-\infty}^{\hat{z}} [1 - p_j(z)] d\Phi(z) + \pi_j(\hat{z})\phi(\hat{z})\frac{\partial \hat{z}}{\partial r_j}]},$$

where ϕ denotes the probability density function of a standard normal random variable. The problem in signing this expression is that $\pi_j(\hat{z})$ may, in principle, be positive or negative: we only know that $\pi_l(\hat{z}) \geq 0$ if and only if $\pi_h(\hat{z}) \leq 0$. However, for the confidence levels implicit in the current and the proposed Basel regulation, $\phi(\hat{z})$ is very small, so we have

$$\frac{\partial r_j^*}{\partial k_j} \simeq \frac{1 - \frac{1}{1+\delta}\Phi(\hat{z})}{\frac{1}{1+\delta} \int_{-\infty}^{\hat{z}} [1 - p_j(z)] d\Phi(z)} > 0.$$

Alternatively, when $\hat{z} \rightarrow \infty$, equation (33) becomes

$$(1 - \bar{p}_j)r_j - \bar{p}_j\lambda - \delta k_j - (1 + \delta)M_j(l) = 0,$$

which, solving for r_j , yields the actuarially fair rate

$$\bar{r}_j = \frac{\bar{p}_j\lambda + \delta k_j + (1 + \delta)M_j(l)}{1 - \bar{p}_j}.$$

As in the model with no intermediation costs, for large \hat{z} the equilibrium rate r_j^* is arbitrarily close to \bar{r}_j , and

$$\frac{\partial \bar{r}_j}{\partial k_j} = \frac{\delta}{1 - \bar{p}_j} > 0,$$

so we conclude that r_j^* must also be increasing in k_j . The rest of the comparative statics may be obtained in a similar way, replicating the results in Proposition 1.

B Equilibrium and actuarially fair rates

In this Appendix we show that the difference between the actuarially fair rate \bar{r}_j and the equilibrium loan rate r_j^* satisfies

$$0 < \bar{r}_j - r_j^* < \frac{(\lambda - k_j)[1 - F_j(\hat{p}_j)]}{(1 - \bar{p}_j)}. \quad (34)$$

To prove this, notice that the fact that $\max\{\pi, 0\} = \pi - \min\{\pi, 0\}$ allows us to rewrite the zero net value condition $V_j = 0$ as

$$-k_j + \frac{1}{1+\delta}E[k_j + r_j^* - p_j(\lambda + r_j^*)] - \frac{1}{1+\delta}E[\min\{k_j + r_j^* - p_j(\lambda + r_j^*), 0\}] = 0,$$

which implies

$$r_j^* - \bar{p}_j(\lambda + r_j^*) - \delta k_j = E[\min\{k_j + r_j^* - p_j(\lambda + r_j^*), 0\}] < 0.$$

On the other hand, integrating by parts, and using the definition (13) of \hat{p}_j we have

$$\begin{aligned} E[\min\{k_j + r_j^* - p_j(\lambda + r_j^*), 0\}] &= k_j - \lambda + (\lambda + r_j^*) \int_{\hat{p}_j}^1 F_j(p_j) dp_j \\ &> k_j - \lambda + (\lambda + r_j^*)(1 - \hat{p}_j)F_j(\hat{p}_j) = (k_j - \lambda)[1 - F_j(\hat{p}_j)]. \end{aligned}$$

Putting together the two inequalities implies

$$(k_j - \lambda)[1 - F_j(\hat{p}_j)] < r_j^* - \bar{p}_j(\lambda + r_j^*) - \delta k_j < 0,$$

which, given the definition (14) of the actuarially fair rate \bar{r}_j , proves the result.

Intuitively, the positive difference between \bar{r}_j and r_j^* is due to the fact that, under perfect competition, the deposit insurance subsidy is transferred to the borrowers in the form of lower equilibrium rates. The upper bound in (34) provides a first order approximation to the pricing error incurred if this effect is ignored. Clearly, for most values of $1 - F_j(\hat{p}_j)$ in Table 1, this upper bound is very small. In the Basel I case, the bound is effectively zero for low PDs. For the IRB requirement that corresponds to each of the two simulated economies (IRB'02 in Economy 1 and IRB'03 in Economy 2), the margin income buffers imply that $1 - F_j(\hat{p}_j)$ is strictly smaller than $1 - \alpha$, so the difference between \bar{r}_j and r_j^* is also tiny.³⁶

It should be noticed that computing the upper bound in (34) requires knowledge of the critical value \hat{p}_j and hence the equilibrium rate r_j^* . An alternative less tight bound can be derived by noting that $\hat{p}_j > k_j/\lambda$ so

$$\frac{(\lambda - k_j)[1 - F_j(\hat{p}_j)]}{(1 - \bar{p}_j)} < \frac{(\lambda - k_j)[1 - F_j(k_j/\lambda)]}{(1 - \bar{p}_j)}.$$

³⁶In the IRB simulations in Table 1, the difference between \bar{r}_j and r_j^* never exceeds 10 basis points.

Moreover, in the IRB approach we have $k_j = \lambda p_j(z_\alpha)$, so $F_j(k_j/\lambda) = F_j(p_j(z_\alpha)) = \alpha$, and $p_j(z_\alpha) > \bar{p}_j$, so the bound simplifies to

$$\frac{\lambda[1 - p_j(z_\alpha)](1 - \alpha)}{(1 - \bar{p}_j)} < \lambda(1 - \alpha),$$

which, for the values of the confidence level α considered in Basel II, is already a very small number.

C Proofs

Proof of Lemma 1 Since $\max\{\pi, 0\}$ is a convex function, while both the expectations operator and the capital requirement are linear, the bank's objective function $V(\gamma)$ in (11) is also convex and hence satisfies

$$V(\gamma) \leq \gamma V(1) + (1 - \gamma)V(0) \leq \max\{V(0), V(1)\},$$

which proves the result. ■

Proof of Proposition 1 To prove that for $0 < k_j < \lambda$ equation (15) has a unique solution that satisfies $0 < r_j^* < \bar{r}_j$, observe that for $r_j = 0$ we have

$$-k_j + \frac{\lambda}{1+\delta} \int_0^{\hat{p}_j} F_j(p_j) dp_j < -k_j + \lambda < 0,$$

while for $r_j = \bar{r}_j$, by the definition (14) of the actuarially fair rate \bar{r}_j and integrating by parts we have

$$\begin{aligned} 0 &= -k_j + \frac{1}{1+\delta} \int_0^1 [k_j + \bar{r}_j - p_j(\lambda + \bar{r}_j)] dF_j(p_j) \\ &< -k_j + \frac{1}{1+\delta} \int_0^{\hat{p}_j} [k_j + \bar{r}_j - p_j(\lambda + \bar{r}_j)] dF_j(p_j) = -k_j + \frac{\lambda + \bar{r}_j}{1+\delta} \int_0^{\hat{p}_j} F_j(p_j) dp_j. \end{aligned}$$

Since V_j is continuous and increasing in r_j the result follows.

Next, differentiating the condition (15) that implicitly defines the equilibrium loan rate r_j^* , and using the definitions (12) and (13) of $F_j(p)$ and \hat{p}_j , gives:

$$\frac{\partial V_j}{\partial r_j} = \frac{1}{1+\delta} \left[\frac{\lambda - k_j}{\lambda + r_j} F_j(\hat{p}_j) + \int_0^{\hat{p}_j} F_j(p_j) dp_j \right] > 0,$$

$$\frac{\partial V_j}{\partial k_j} = -1 + \frac{1}{1+\delta} F_j(\widehat{p}_j) < 0,$$

$$\frac{\partial V_j}{\partial \mu_j} = -\frac{\lambda+r_j}{(1+\delta)\sqrt{\rho}} \int_0^{\widehat{p}_j} \phi\left(\frac{\sqrt{1-\rho} \Phi^{-1}(p_j) - \mu_j}{\sqrt{\rho}}\right) dp_j < 0,$$

$$\frac{\partial V_j}{\partial \lambda} = -\frac{1}{1+\delta} [\widehat{p}_j F_j(\widehat{p}_j) - \int_0^{\widehat{p}_j} F_j(p_j) dp_j] < 0,$$

$$\frac{\partial V_j}{\partial \delta} = -\frac{1}{1+\delta} \left[\frac{\lambda+r_j}{1+\delta} \int_0^{\widehat{p}_j} F_j(p_j) dp_j \right] = -\frac{1}{1+\delta} k_j < 0,$$

which implies $\partial r_j^*/\partial k_j > 0$, $\partial r_j^*/\partial \bar{p}_j > 0$ (recall that $\bar{p}_j = \Phi(\mu_j)$), $\partial r_j^*/\partial \lambda > 0$, and $\partial r_j^*/\partial \delta > 0$. Finally, since an increase in ρ induces a mean-preserving spread on the distribution of p_j , and the upper bound \widehat{p}_j does not depend on ρ , the characterization of second-degree stochastic dominance (Rothschild and Stiglitz (1970)) immediately implies

$$\frac{\partial V_j}{\partial \rho} = \frac{\lambda+r_j}{1+\delta} \frac{\partial [\int_0^{\widehat{p}_j} F_j(p_j) dp_j]}{\partial \rho} > 0,$$

so $\partial r_j^*/\partial \rho < 0$. ■

Proof of Proposition 2 By the chain rule, the total effect of any parameter y on the equilibrium loan rate r_j^* will be:

$$\frac{dr_j^*}{dy} = \frac{\partial r_j^*}{\partial y} + \frac{\partial r_j^*}{\partial k_j} \frac{\partial k_j}{\partial y},$$

where the signs of $\partial r_j^*/\partial y$ and $\partial r_j^*/\partial k_j$ are obtained from Proposition 1, and the sign of $\partial k_j/\partial y$ from the comparative statics of the IRB capital requirement given by (7). ■

Proof of Proposition 3 The result follows immediately from the fact that by (16) and (18) we have $k_l^{\text{IRB}} < k^{\text{S}} = k^{\text{I}} < k_h^{\text{IRB}}$. ■

References

- [1] Asquith, P., and D. Mullins (1986): “Equity Issues and Offering Dilution,” *Journal of Financial Economics*, 15, 61-89.
- [2] Basle Committee on Banking Supervision (2001): *The New Basel Capital Accord*.
- [3] Basle Committee on Banking Supervision (2003): *The New Basel Capital Accord*.
- [4] Bolton, P., and X. Freixas (2000): “Equity, Bonds, and Bank Debt: Capital Structure and Financial Market Equilibrium under Asymmetric Information,” *Journal of Political Economy*, 108, 324-351.
- [5] Diamond, D. W., and R. G. Rajan (2000): “A Theory of Bank Capital,” *Journal of Finance*, 55, 2431-2465.
- [6] Furlong, F., and M. Keeley (1989): “Capital Regulation and Bank Risk-Taking: A Note,” *Journal of Banking and Finance*, 13, 883-891.
- [7] Genotte, G., and D. Pyle (1991): “Capital Controls and Bank Risk-Taking,” *Journal of Banking and Finance*, 15, 805-824.
- [8] Gordy, M. B. (2000): “Credit VAR and Risk-Bucket Capital Rules: A Reconciliation,” in *Proceedings of the 36th Annual Conference on Bank Structure and Competition*, Federal Reserve Bank of Chicago, 406-415.
- [9] Gordy, M. B. (2002): “A Risk-Factor Model Foundation for Ratings-Based Bank Capital Rules,” Federal Reserve Board, Finance and Economics Discussion Paper No. 55.
- [10] Hellmann, T. F., K. C. Murdock, and J. E. Stiglitz (2000): “Liberalization, Moral Hazard in Banking, and Prudential Regulation: Are Capital Requirements Enough?” *American Economic Review*, 90, 147-165.

- [11] Holmstrom, B., and J. Tirole (1997): “Financial Intermediation, Loanable Funds, and the Real Sector,” *Quarterly Journal of Economics*, 112, 663-691.
- [12] Lopez, J. (2003): “The Empirical Relationship between Average Asset Correlation, Firm Probability of Default and Asset Size,” *Journal of Financial Intermediation*, forthcoming.
- [13] Lowe, P. (2002): “Credit Risk Measurement and Procyclicality,” BIS Working Paper No. 116.
- [14] Marshall, D. A., and E. S. Prescott (2001): “Bank Capital Regulation with and without State-Contingent Penalties,” *Carnegie-Rochester Conference Series on Public Policy*, 54, 139-184.
- [15] Merton, R. C. (1977): “An Analytic Derivation of the Cost of Deposit Insurance and Loan Guarantees: An Application of Modern Option Pricing Theory,” *Journal of Banking and Finance*, 1, 3-11.
- [16] Rochet, J.-C. (1992): “Capital Requirements and the Behaviour of Commercial Banks,” *European Economic Review*, 36, 1137-1170.
- [17] Rothschild, M., and J. E. Stiglitz (1970): “Increasing Risk I: A Definition,” *Journal of Economic Theory*, 2, 225-243.
- [18] Siegel, J. J., and R. H. Thaler (1997): “Anomalies: The Equity Premium Puzzle,” *Journal of Economic Perspectives*, 11, 191-200.
- [19] Stiglitz, J. E., and A. Weiss (1981): “Credit Rationing in Markets with Imperfect Information,” *American Economic Review*, 71, 393-410.