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ABSTRACT

Taylor Rules in Practice: How Central Banks can Intercept Sunspot Expectations*

This Paper derives new results on the effects of employing Taylor rules in economies that are subject to real-market imperfections such as production externalities. It suggests that rules that should be avoided (chosen) in perfect-markets environments do in fact ensure (yield) unique (multiple) rational expectations solutions in alternative settings. Therefore, exact knowledge on the degree of market imperfection is pivotal for robust policy advice.

JEL Classification: E32 and E52
Keywords: cash-in-advance economies, increasing returns-to-scale, indeterminacy and Taylor rules

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Therefore, exact knowledge on the degree of market imperfection may be pivotal for robust policy advice.

1 Introduction

The Taylor-rule (1993) provides a good description of how many central banks attempt to set interest rates in order to achieve stable prices while avoiding large fluctuations in output and employment. There is evidence, however, that Taylor-rules can be a source of economic instability by themselves. For example, Benhabib et al. (2001) demonstrate that steering under such policy may introduce real indeterminacy in an otherwise determinate economy. As a consequence, the Taylor-rule debate frequently advises the monetary authority to assign aggressive backward-looking principles in which interest rates respond to predetermined variables, in particular to inflation (see for example Carlstrom and Fuerst, 1999, and Benhabib et al., 2003).

The present paper qualifies this assertion by suggesting that before spelling out concrete policy rules, the monetary authority must first be *au courant* with the specific economic environment. Carlstrom and Fuerst (2000, 2001) have demonstrated how monetary policy should depend on the timing convention in monetary models. The present paper adds another dimension to the discussion: the real side of the economy. In particular, it demonstrates that the presence of mild forms of market imperfections – modelled as arising from production externalities – may have fundamental consequences on how monetary policy should be conducted.

The motivation for the current research stems from the insights of recently formulated non-monetary dynamic general equilibrium models with sunspot equilibria and self-fulfilling prophecies.[1] In these models the possibility of a continuum of equilibria is the consequence of empirically plausible market imperfections – therefore, sunspot equilibria are more than theoretical curiosities. The present paper combines the two branches of the indeterminacy literature by money-augmenting these real models in order to examine the effects of monetary policy on indeterminacy as well as to assess monetary policy recommendations in suboptimal economies. The framework I will draw on is Wen (1998) which is currently the most successful attempt in terms

[1] A partial list includes Benhabib and Nishimura (1998), Christiano and Harrison (1999), Farmer and Guo (1994), Perli (1998), Schmitt-Grohé and Uribe (1997), Weder (1998, 2001), and Wen (1998).

of obtaining sunspot equilibria at small increasing returns and in generating realistic business cycles (see also Benhabib and Wen, 2003).

1.1 Main results

Given its relation to recent policy-proposals (i.e. Carlstrom and Fuerst, 1999, 2000, and Benhabib et al., 2003), the current paper is primarily concerned with backward-looking Taylor-rules. The main findings can be stated as follows: by responding sufficiently to (past) output movements in setting the nominal interest rate, the monetary authority can stabilize sunspot-driven economies. The reasoning is that – in a cash-in-advance environment – the nominal interest rate operates like an inflation tax on holding money. Now, suppose buoyant expectations about future output (or wealth) seep through the economy and economic activity rises simply because everybody believes so. Given some degree of increasing returns these expectations may be self-fulfilling. However, when monetary policy is set such to increase the nominal interest rate in response to the output fluctuations, the sunspot blips can be dimmed: policy has real effects because real consumption and labor demand become more costly. Consequently, the higher nominal interest rates build up the costs of the economic expansion and therefore of the optimistic expectations; the optimistic expectations will no longer be sustainable in the first place. Phrased alternatively, the inflation tax-distortions operate by preempting the effects of high increasing returns to scale and non-fundamental equilibria can be removed.

I also find that Taylor-rules work quite differently depending on the fundamentals of the economy. In fact, it appears to be the case that strategies which should be chosen (avoided) in perfect markets environments do in fact yield multiple (unique) rational expectations solutions in alternative settings. For example, backward-looking policy settings that ensure unique rational expectations in cases of constant returns to scale (aggressive with respect to inflation and somewhat passive with respect to output) are connected to indeterminacy at moderate imperfections and *vice versa*. On the other hand, current-looking rules which always create indeterminacy under constant returns are a vehicle to remove technology-based sunspot equilibria. Consequently, the central bank should have a clear picture of market imperfections before setting policy rules. Unfortunately, existing empirical studies do not provide an unambiguous answer on the extent of the imperfections.

Moreover, I find that output-targeting in perfectly-flexible pricing models does not necessarily increase the determinacy space. For example, for the constant returns to scale environment I find that an aggressive backward-looking inflation-targeting is sufficient for determinacy. However, if policy also targets output then a simple aggressive policy does no longer guarantee uniqueness. The policy must be sufficiently aggressive. This contrasts with the New Keynesian wisdom that output-targeting enlarges the determinacy region *ceteris paribus*.

1.2 Related work

The argument which is developed in the current paper is framed within a fully specified cash-in-advance environment which has been shown by Carlstrom and Fuerst (2000) to have fundamentally different policy implications than New-IS-LM or versions of money-in-utility-frameworks.[2] My study differs from theirs in three key aspects, however. First, their production technology is constant returns to scale. By contrast, I allow empirically plausible production externalities which lead to increasing returns. Second, in their model the central bank's nominal interest rate targets inflation only. The current paper considers versions of the original Taylor-rule in which the interest rate is increased or decreased according to what is happening to both real GDP and to inflation – as it turns out, output-targeting becomes essential in eliminating sunspot equilibria in imperfect economies. Furthermore, I present new results on interest smoothing.

The work here also shares similarities with Christiano (2000). Christiano introduces money into the Christiano and Harrison (1999) model which assumes increasing returns to scale equal the inverse of the capital share. As a consequence, his model has two stationary states. The monetary authority sets the nominal interest rate proportional to the contemporaneous employment level and is thereby able to eliminate endogenous sunspot cycles in the neighborhood of one of the steady states. In a sense, the economic mechanisms in Christiano's and my paper parallel. However, several important differences distinguish his work from mine. First, the present paper does not rest on scale economies that fault on empirical findings. Second, my economy does not move between two steady states. Third, as a consequence the choice and the optimal set of policy differ. More concretely, the monetary rule which is proposed near the high-level steady state by Christiano does

[2]King (2000) is a good review of the New-IS-LM model.

not generally deliver determinacy at modest increasing returns. My findings put a much more stringent cap on policy. Specifically, when assuming a current-looking rule as in Christiano, a mix of inflation and output targeting is needed to eliminate endogenous cycles. Fourth, the current work experiments with more general versions of the Taylor-rule (i.e. backward and forward looking and interest rate smoothing).

The remainder of the paper is organized as follows. The next Section presents the model economy. Section 3 discusses the connection of monetary policy, market imperfections and sunspot equilibria. Section 4 addresses interest rate smoothing. Section 5 concludes.

2 The economy

The physical setup of the economy's real part is a standard real business cycle model augmented by production complementarities. Currency is introduced by imposing restrictions on the timing of exchanges.

2.1 Preferences and technologies

The representative household seeks at time $t = 0$ to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t (\ln c_t - \eta l_t) \qquad 0 < \beta < 1,\ \eta > 0$$

where β, c_t and l_t are the discount factor, consumption and labor during t. The household rents labor and capital services to firms. All markets are perfectly competitive. The household's budget constraint can be stated as

$$M_{t+1} + P_t k_{t+1} = M_t + \Pi_t + P_t w_t l_t + P_t(r_t + 1 - \delta_t)k_t - P_t c_t + N_t(R_t - 1)$$

where P_t is the price level, M_t are the cash balances at the beginning of t, w_t is the real wage and r_t is the real rental rate of capital, k_t. The variable u_t denotes the degree of capital utilization. The depreciation rate of installed capital, δ_t, is increasing in utilization

$$\delta_t = \frac{1}{\theta} u_t^{\theta} \qquad \theta > 1.$$

N_t stands for one-period bank deposits which pay a short-term nominal interest given by R_t. Π_t is the profit flow from firms and intermediaries. A positive value is assigned to the inconvertible currency by assuming that during the shopping session the household is subject to the cash-in-advance-restriction

$$M_t + P_t w_t l_t \geq P_t c_t + N_t$$

that is, households circulate all their money (plus wage payments) to firms by consumption purchases and loans to the financial intermediaries.[3]

Output is produced by a large number of competitive firms with identical technologies. The economy as a whole is affected by organizational synergies that cause the output of an individual firm to be higher if all other firms in the economy are producing more. The term A_t stands for these aggregate externalities. The production complementarities are taken as given for the individual optimizer and they cannot be priced or traded. Departures from constant returns to scale are measured by $\gamma > 1$.[4] A firm of type i has technology

$$y_{i,t} = A_t^{\gamma-1}(u_t k_{i,t})^{\alpha} l_{i,t}^{1-\alpha} \qquad A_t = (u_t k_t)^{\alpha} l_t^{1-\alpha} \quad \text{and} \quad 0 < \alpha < 1.$$

Here, k_t (l_t) denotes – by way of normalization – the economy-wide average capital (labor) input. Before hiring workers, firms must borrow cash at the short term rate from the financial intermediaries. This is because they start the period without money balances to finance their wage bills. This is the second source of money demand.

2.2 Intermediaries, the central bank and government

The monetary branch of the economy comes in two parts: the intermediary sector and the central bank. The perfectly competitive intermediaries have two sources of cash. They accept loans from the households, N_t, which are repaid at the gross rate of interest R_t. Intermediaries also receive new cash

[3]A previous version of this paper has discussed a model with money-in-utility – very similar insights did arise.

[4]See Benhabib and Farmer (1994) and others for an alternative (and in reduced-form equivalent) formulation that incorporates internal increasing returns at the intermediate-firm level in an imperfectly competitive market structure without free entry. In that case, the externality parameter would (also) relate to the monopoly markup.

injections, $M^s_{t+1} - M^s_t$, from the economy's monetary authority. This money is loaned to firms. The intermediaries' constraint is

$$N_t + M^s_{t+1} - M^s_t \geq P_t w_t l_t.$$

Firms' loans must be repaid at the end of the period, for the financial intermediaries to use the proceeds to repay the households.

Most central banks implement monetary policy by controlling a short-term nominal interest rate. Accordingly, it has become standard to represent monetary policy in terms of commitment to a rule for the nominal rate of interest. In the present paper, the monetary authority sets the short-run nominal interest rate based on what is happening to both real GDP and inflation. For example, a backward-looking rule is given by

$$R_{t+1} = R\left(\frac{\pi_t}{\pi}\right)^{\tau}\left(\frac{y_t}{y}\right)^{\omega} \qquad \tau \geq 0 \quad \omega \geq 0$$

or in linearized from

$$\widehat{R}_{t+1} = \tau\widehat{\pi}_t + \omega\widehat{y}_t$$

in which the variables appear as percentage deviations from their stationary states R, π and y. We denote rules with $\tau < 1$ ($\tau > 1$) as passive (aggressive) since the nominal interest rate moves less (more) than one-for-one with past inflation. The term ω refers to the respective weight given to deviations of real GDP from the target level. Since the general equilibrium setting imposes a money demand relationship (that is, the cash-in-advance setup), the interest rate rule implies that the money supply is endogenous. Fiscal policy is passive in Leeper's (1991) sense.

2.3 Dynamics and calibration

In what follows, I restrict the analysis to a symmetric equilibrium in which $u_{i,t} = u_t$, $k_{i,t} = k_t$ and $l_{i,t} = l_t$. The aggregate production technology becomes

$$y_t = (u_t k_t)^{\alpha\gamma} l_t^{(1-\alpha)\gamma} \tag{1}$$

which exhibits returns to scale equal to γ. The economy's dynamics are given by

$$\eta c_t = \frac{w_t}{R_t} = \frac{\alpha y_t l_t^{-1}}{R_t} \tag{2}$$

$$u_t^\theta = \alpha \frac{y_t}{k_t} \tag{3}$$

$$E_t \frac{\beta}{c_{t+1}\pi_{t+1}} = \frac{1}{c_t R_t} \tag{4}$$

$$\frac{1}{c_t R_t} = E_t \frac{1}{c_{t+1}R_{t+1}} \beta[\alpha \frac{y_{t+1}}{k_{t+1}} + 1 - \frac{1}{\theta} u_{t+1}^\theta] \tag{5}$$

$$c_t + k_{t+1} = y_t + (1 - \frac{1}{\theta} u_t^\theta) k_t. \tag{6}$$

Equation (2) describes the leisure-consumption trade-off and (3) pins down the optimal utilization rate of capital. Equations (4) and (5) are the usual Fisher and Euler conditions. (6) repeats the intertemporal constraint.

No closed-form solution exists, thus the model must be approximated. In log-linearized form, the dynamics reduce to

$$\begin{bmatrix} E_t \widehat{c}_{t+1} \\ \widehat{R}_{t+1} \\ \widehat{k}_{t+1} \\ E_t \widehat{R}_{t+2} \end{bmatrix} = \mathbf{M} \begin{bmatrix} \widehat{c}_t \\ \widehat{R}_t \\ \widehat{k}_t \\ \widehat{R}_{t+1} \end{bmatrix}.$$

The dynamical system contains two non-predetermined (or jump) variables: $\widehat{c}_{t+1}$ and $\widehat{R}_{t+2}$. Determinacy requires that exactly two eigenvalues of the 4×4-matrix $\mathbf{M}$ are outside the unit circle. On the other hand, indeterminacy arises in situations in which at most one root has modulus greater than one. If, say, exactly three eigenvalues have modulus less than one, then there exist multiple rational expectations solutions which take on the form

$$\begin{bmatrix} \widehat{c}_{t+1} \\ \widehat{R}_{t+1} \\ \widehat{k}_{t+1} \end{bmatrix} = \widetilde{\mathbf{M}} \begin{bmatrix} \widehat{c}_t \\ \widehat{R}_t \\ \widehat{k}_t \end{bmatrix} + \begin{bmatrix} \zeta_{t+1} \\ 0 \\ 0 \end{bmatrix}. \tag{7}$$

Here ζ_{t+1} is an arbitrary random variable with $E_t \zeta_{t+1} = 0$.

Table 1: Calibration

α	β	δ
0.30	0.99	0.025

I will now assign parameter values and demonstrate the empirical plausibility of sunspot equilibria. The time unit is taken to be a quarter of a year. The calibration is based on empirical observations on post-war U.S. data. The capital share, α, is set equal to 30 percent, the discount factor, β, is chosen to be 0.99 and the steady state rate of capital depreciation, δ, is 2.5 percent. The parameter θ can then be derived from steady state conditions:

$$\theta = \frac{1/\beta - 1 + \delta}{\delta} = 1.40404.$$

When abstracting from the monetary side, the calibration implies a minimum degree of externalities, $\gamma^{\min}$, needed for indeterminacy that amount to 1.103681742. The value is reasonable given empirical findings. For example, Caballero and Lyons (1992) obtain increasing returns estimates in the order of 1.26 to 1.56. Baxter and King (1991) find returns to scale of 1.53, however, combined with a standard error of 0.56. Burnside, Eichenbaum and Rebelo (1995) report a point estimate of 0.98. Again their standard error of 0.34 is large. Basu and Fernald (1997) also find close to constant returns, however, the imparted estimation-uncertainty is significant again. Laitner and Stolyarov (2004) suggest point estimates ranging from 1.09 to 1.1, however, confidence intervals essentially include constant returns.

The reasoning for multiplicity in the real economy is as follows. Equations (1) and (3) entail the reduced form-output

$$y_t = \text{const} * k_t^{\frac{\alpha\gamma(\theta-1)}{\theta-\alpha\gamma}} l_t^{\frac{(1-\alpha)\gamma\theta}{\theta-\alpha\gamma}}.$$

Thus, the effective labor-output elasticity is larger than unity for[5]

$$\gamma > \gamma^{up} = \frac{\theta}{\alpha + (1-\alpha)\theta} > 1.$$

Accordingly, the reduced-form labor demand curve is upward sloping at mild increasing returns.[6]

[5] If the depreciation costs are high ($\theta \to \infty$) and accordingly capital utilization is set constant by agents, the condition reduces to that found in Harrison and Weder (2002); the minimum increasing returns fall outside of the plausible region. However, the condition also implies an upward-sloping labor demand curve and all of the below qualitative results can be replicated in such an environment.

[6] Upward sloping labor demand arises for $\gamma \geq \gamma^{\min} \geq \gamma^{up} = 1.0945$. The reason for

Now, how do sunspot equilibria come about? Suppose people suddenly have pessimistic expectations and expect lower future income. The permanent income motive will reduce today's consumption. The static-first order condition (2) implies that the labor supply schedule moves outwards. Given the upward-sloping equilibrium labor demand curve, employment and investment will actually both fall today. As a consequence, the future capital stock, output and consumption will all be low, and in sum, the initially pessimistic expectations will be self-fulfilled. The sunspot circle is completed.

The determinacy properties of the model do not change when money is introduced and the central bank pegs the nominal rate ($\tau \to 0$ and $\omega \to 0$). The specific interest rate policy causes the economy to behave identically to the model that only includes the real sector.

3 How should monetary policy be conducted?

This Section discusses the effects of various versions of the Taylor-rule on the qualitative dynamics of the artificial economy. It opens by assuming that the central bank sets nominal interest rates after having observed (past) inflation and output (Section 3.1.). This is followed by discussions of forward-looking and current-looking rules (Sections 3.2. and 3.3.).

3.1 Indeterminacy zones with backward-looking rules

This Subsection will examine various versions of backward-looking Taylor-rules in economies with constant and increasing returns to scale. I will start combing for parametric indeterminacy zones by considering a constant returns to scale technology ($\gamma = 1$) which will help in understanding the increasing returns cases.

the gap between $\gamma^{\min}$ and γ^{up} and the gap's positive dependence on the time period's length comes from discounting future benefits (costs) at the relevant intertemporal margins. When formulating the model in continuous time, the two thresholds are identical – the same sort of gap arises in the original Benhabib and Farmer (1994) model (see Salyer, 1995).

3.1.1 Constant returns to scale and inflation-targeting

When I set $\omega = 0$, the four eigenvalues of $\mathbf{M}$ are

$$\left\{ \frac{1-\beta(1-\alpha)(1-\delta)}{\alpha\beta}, \frac{\alpha}{1-\beta(1-\alpha)(1-\delta)}, \tau, 0 \right\}.$$

The eigenvalues are the same as those reported in Carlstrom and Fuerst (2000) despite the absence of variable capital utilization and variable capital depreciation in their model. The third eigenvalue exposes the policy's direct impact on dynamics: policy-induced indeterminacy can be avoided simply by responding aggressively to past inflation whereas passive responses lead to multiplicity. The reasoning for the occurrence of sunspot equilibria can be understood as follows: suppose that current inflation increases by one percent, which given an aggressive policy, implies that the $t+1$ nominal rate goes up by more than one percent. The inflation tax depresses future consumption and lifts current consumption – the real rate goes down. This, however, is only possible when the rate of inflation, π_{t+1}, increases. As a result, the central bank's target R_{t+2} rises by even more and policy induces an unsustainable explosive inflation-pattern. The initial rise in inflation is not supported and consequently the sunspot cycle is stopped. In contrast, if the bank follows a passive policy, the chain of events remains stationary and sunspot expectations are self-fulfilled.

3.1.2 Constant returns to scale and output-targeting

Carlstrom and Fuerst (2000) do not consider the effects of output-targeting on the economy's dynamics. This will be done here next as a further step towards an understanding of Taylor-rules in suboptimal equilibria. Suppose that the central bank reacts in part to past movements in output, $\omega > 0$.

Dynamics can be derived analytically within special cases. Thus, let us assume a simplified economy without capital and in which output is produced with the linear technology

$$y_t = Al_t$$

or simply $\alpha \to 0$. The model reduces to the scalar equation

$$E_t\widehat{R}_{t+2} = (\tau - \omega)\widehat{R}_{t+1}.$$

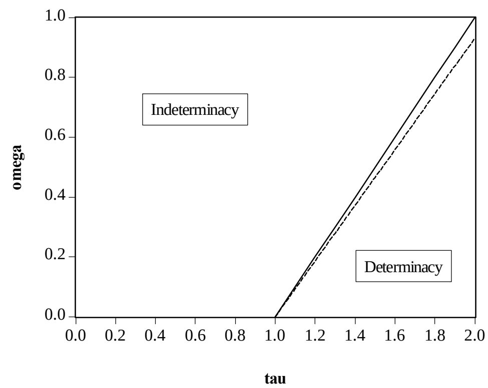

Figure 1: *Sunspot zones when returns to scales are constant; backward-looking rules. Dashed line corresponds to the border while assuming* $\gamma = 1.05$.

Indeterminacy arises for parameter constellations that satisfy

$$\omega - 1 < \tau < 1 + \omega. \tag{8}$$

The right-hand-side inequality in (8) repeats the aforementioned result. Namely that active inflation-fighting policies eliminate sunspot equilibria. More generally, the central bank must simultaneously select both policy parameters because an intermediate range of τ values that generates indeterminacy for given ω exists. Put in another way, letting for example $\tau = 1.5$ as in Taylor's original formulation, then the bank's output-coupled response must fall outside $1/2 < \omega < 2.5$ otherwise policy-induced cycles crop up.[7] In sum, policy should be a combination of active inflation targeting and a not too aggressive response to output otherwise sunspot cycles arise. The same results apply for the complete model. There is a discontinuity at $\tau = \omega = 0$ at which the model undergoes real determinacy but nominal indeterminacy.[8] Figure 1 summarizes numerical solutions. The dashed line in Figure 1 suggests that the dynamical structure stays broadly unaffected when very small increasing returns are introduced.

[7] The Appendix reports the analytical dynamics for an alternative special case which includes endogenous capital.

[8] I would like to thank one referee for pointing this out to me.

Indeterminacy from output-targeting arises as follows. Suppose again that current inflation increases. Future nominal interest rates will rise which will increase today's consumption. Therefore, the labor supply curve shifts inward which, since the capital stock is given, lowers current output. Accordingly, the future interest rate will rise by less than under the pure inflation-peg:

$$\widehat{R}_{t+1} = \tau \underset{(\uparrow)}{\widehat{\pi}_t} + \omega \underset{(\downarrow)}{\widehat{y}_t}.$$

As a consequence, stationary sunspot sequences become more likely when the central bank targets output. These sunspot sequences are only possible when the ω-weights fall into a certain range: only relatively small values of ω – in combination with aggressive inflation-targeting – deliver determinacy.[9]

3.1.3 Increasing returns to scale

Next, I turn to cases in which the economy is imperfect and subject to real indeterminacy stemming from production externalities. That is, I will discuss how the determinacy properties of the Wen (1998) model change when a monetary policy rule is introduced. The question I ask is: can the central bank stamp out externality-generated sunspot equilibria by choosing an appropriate Taylor-design?

When the central bank reacts to inflationary movements only, indeterminacy arising from mild production externalities cannot be eliminated. This can easily be seen by numerically checking for the minimum increasing returns for alternative τ-values (other parameters are as in Table 1). These returns to scale are 1 at $\tau \to 0$, they jump to 1.103708061 at $\tau = 1$ and are 1.103734395 at $\tau = 1000000$. I therefore shift attention to cases involving $\omega > 0$.

Generally, once non-convexities in the production function are present, analytical versions of the eigenvalues become highly non-linear and offer only limited insights. Thus, I will employ numerical solutions.[10] Let us step up γ to 1.11 so that production complementarities induce sunspot equilibria in environments without money (the other parameters are as in Table 1). The

[9] For very large ω-values, the output-related movements will be too strong to preserve stationarity and sunspots can be ruled out again. Numerically, ω must be larger than 30 which is implausibly high (not reported in Figure 1).

[10] Analyical solutions can be found in the Appendix.

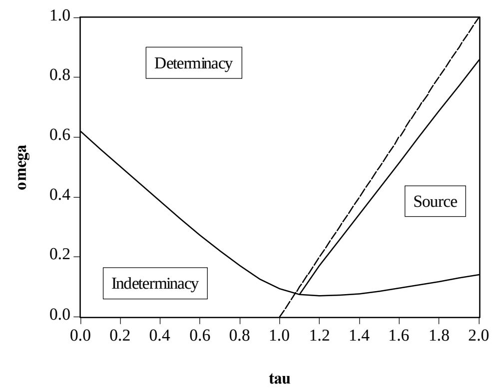

Figure 2: *Inflation and output targeting under increasing returns to scale* ($\gamma = 1.11$)*; backward-looking rules. To the right of the dashed line, the constant returns to scale model has a unique equilibrium.*

specific value draws on Benhabib and Wen (2003) who propose it by observing that the model's cycle frequency matches that of U.S. output. Furthermore, the magnitude of scale economies is at the low end of what is needed to generate indeterminacy and it falls into the scope of the studies mentioned in Section 2.3.

Figure 2 pins down the policy advice under the assumption of $\gamma = 1.11$. The only way for the central bank to tackle endogenous cycles that arise from the production externalities is by working against output movements: if it does not ($\omega = 0$), indeterminacy always occurs. The Figure also shows that for any given τ, the output-related response must be sufficiently large to obtain unique solutions: even when ω takes on strictly positive values, the specific choice of inflation-targeting may imply any of the three possible regimes (indeterminacy, determinacy or source – the latter region may imply endogenous cycles on its own). In agreement with the above analytical experiment, monetary policies that are very offensive with respect to output fluctuations (or even $\omega \to \infty$) are generally a good insurance against indeterminacy. Lastly, the dashed line in Figure 2 repeats the border between determinacy and indeterminacy for the no-externalities case: there exists a small region for which determinacy is possible at both constant and increasing returns.

Figure 3 shows that an economy with constant capital utilization displays

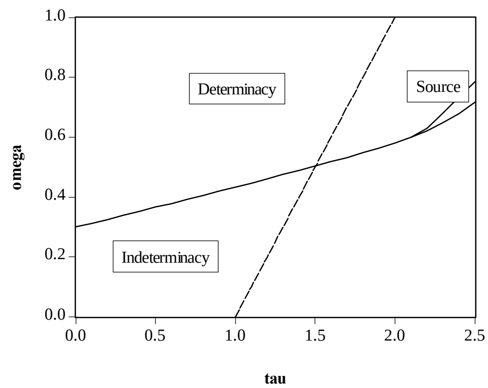

Figure 3: *Inflation and output targeting under increasing returns to scale* ($\gamma = 1.50$)*, backward-looking rules. Source denotes triangle region on upper right hand side. To the right of the dashed line, the constant returns to scale model has a unique equlibrium.*

the same sort of problem for the central bank. The model is identical to Benhabib and Farmer (1994) augmented by the same monetary superstructure which was outlined in Section 2. I have set the returns to scale at which is an empirically unrealistic value, however, it is needed to generate indeterminacy. The other parameters are set as in Table 1. Under constant returns to scale the dynamical properties of the models with and without variable capital utilization are identical as can be seen from the dashed line. Once again, output-targeting is key to pre-empt sunspot equilibria under increasing returns. There is a small parametric region in which determinacy arises with and without scale economies. Overall, the (qualitative) results parallel those of the non-constant utilization model – the general policy implications are not restricted to Section 2's model.

The result that output-targeting eliminates sunspot equilibria is reminiscent of Guo and Lansing (1998) and Christiano and Harrison (1999) who find that progressive tax systems effectively eliminate indeterminacy by taxing away increasing returns in real economies. The Taylor policy suggested here generates a similar distortionary effect. The economic reasoning is easy to see: if ω is sufficiently large, then output fluctuations (i.e. production bunching) that arise from believing in them simply become too costly. Let us walk through a sunspot sequence that is stopped by the central bank for further understanding of the result. Suppose that people embellish optimistic expectations without any fundamental cause. By projecting high future income, they will ratchet up today's consumption expenditures. The high increasing returns will increase today's employment and output as a result of the upward sloping equilibrium labor demand curve. Now, if the output response of the central bank is strong enough, then R_{t+1} will increase and the initially sanguine expectations will be preempted (i) by increasing the costs of hiring labor (Equation 2) and (ii) by reducing $t+1$ consumption demand (Equation 4) which will lower $t+1$ employment and therefore output. The initially optimistic expectations are not fulfilled and the sunspot cycle is broken.

To conclude, the analysis of backward-looking rules suggests that central banks should be careful about the specific economic environment when setting policy rules. I find that Carlstrom and Fuerst's (2000) advocated backward-looking inflation-targeting rule no longer guarantees uniqueness. It is imperative for the interest rate to intercept output fluctuations. To offer a concrete example, my numerical analysis suggests that Taylor's (1993) original principle ($\tau = 1.5$ and $\omega = 0.5$) constitutes a successful code of stabilizing sunspot fluctuations at $\gamma = 1.11$. However, Figure 1 also shows that if increasing returns are slightly smaller – say, at $\gamma = 1.05$ – then the economy skids into a regime under the clout of sunspots.

Proposition 1 *(Backward-looking rules) The central bank can rule out indeterminacy by aggressively targeting inflation when returns to scale are constant. However, the presence of mild production externalities requires a tough stand on output in order to eliminate indeterminacy.*

3.2 Indeterminacy zones with forward-looking rules

It may be suspected that the result is dependent on assuming a backward-looking policy rule. Yet, the general picture that Taylor-rule settings should consider technology does not change when the central bank pays attention to expected values such as in

$$\widehat{R}_t = \tau E_t \widehat{\pi}_{t+1} + \omega E_t \widehat{y}_{t+1} \qquad \tau \geq 0 \quad \omega \geq 0. \tag{9}$$

With forward-looking rules, the model boils down to

$$\begin{bmatrix} E_t\widehat{c}_{t+1} \\ E_t\widehat{R}_{t+1} \\ \widehat{k}_{t+1} \end{bmatrix} = \mathbf{J} \begin{bmatrix} \widehat{c}_t \\ \widehat{R}_t \\ \widehat{k}_t \end{bmatrix}$$

which involves the two jump-variables $\widehat{c}_{t+1}$ and $\widehat{R}_{t+1}$.

3.2.1 Constant returns to scale and inflation-targeting

Beginning with the case of inflation targeting and no externalities ($\omega = 0$ and $\gamma = 1$), the three eigenvalues of $\mathbf{J}$ are

$$\left\{ \frac{1-\beta(1-\alpha)(1-\delta)}{\alpha\beta}, \frac{\alpha}{1-\beta(1-\alpha)(1-\delta)}, \frac{1}{\tau} \right\}.$$

This reverses the result obtained under backward-looking rules: all policy responses $\tau > 1$ are precluded, so it is necessary for policy to be passive in Taylor's sense. The underlying economics are easy to grasp. The real rate is given by (using equation 9)

$$r_t = R_t - E_t\pi_{t+1} = \frac{\tau - 1}{\tau} R_t.$$

It decreases when current (sunspot-driven) consumption rises. For $\tau > 1$, the fall in the real rate translates into a fall of the nominal rate. Thus, the inflation-consumption distortion declines and the sunspot cycle is completed.

3.2.2 Constant returns to scale and output-targeting

Let us turn to cases involving output-targeting. Resorting again to a linear production technology without capital leads to the simplified model

$$E_t\widehat{R}_{t+2} = \frac{1}{\tau - \omega} \widehat{R}_{t+1}$$

from which more general Taylor formulas can be discussed analytically. Determinacy requires

$$\tau > \omega - 1 \qquad \text{for} \qquad \tau < \omega$$

or

$$\tau < \omega + 1 \qquad \text{for} \qquad \tau > \omega.$$

Now there are two zones of indeterminacy. Again, if $\omega = 0$, the monetary authority should be passive when responding to expected inflation. However, in the case of little or no inflation-targeting ($\tau \to 0$), the monetary authority should be passive when responding to expected output. Moreover, monetary policy should respond in tendency equally to output and to inflation. Asymmetric responses will likely drive the economy into sunspot districts. Numerical solutions lead to parallel results for the complete model: in terms of Figure 1, the two zones are simply swapped.[11]

In sum, policy should not be too aggressive. The economic reasoning for this indeterminacy is easy to understand. Suppose policy is aggressive. A sunspot-driven increase in consumption lowers the real rate and given $\tau > 1$ it also lowers the nominal rate. The rise in consumption will decrease labor supply and therefore output. Current investment falls and next period's capital stock will be smaller. Thus, it is likely that future output is lower as well – there will be downward pressure on the current nominal rate. If the fall of R_t is sufficient because policy reacts sharply to the output fluctuations, the sunspot cycle cannot be broken.

3.2.3 Increasing returns to scale

Let us now go through the increasing returns to scale scenario. Suppose that production externalities are $\gamma = 1.11$. Figure 4 plots determinacy regions in the $\tau - \omega$-space. Uniqueness requires a tough policy with respect to inflation which must be backed up by (relatively mild) responses to output. Moreover, the constant returns proposal ($\tau < 1$) always creates sunspot equilibria now, thus, the policy advice is different across technological regimes. For example, if the central bank sets $\tau = 1.5$, then sunspot cycles are ostracized for weak output-targeting $0.12 < \omega < 0.42$. However, at slightly smaller externalities at which indeterminacy does not arise from technology, say at $\gamma = 1.10$, that very policy creates indeterminacy.

In sum, simply applying very aggressive inflation-targeting does not bring about determinacy. This is the antithesis of the backward-looking policy's

[11] In the model, fiscal policy is passive in Leeper's (1991) sense. Therefore, it is possible for combinations of passive monetary and passive fiscal policies to lead to determinacy.

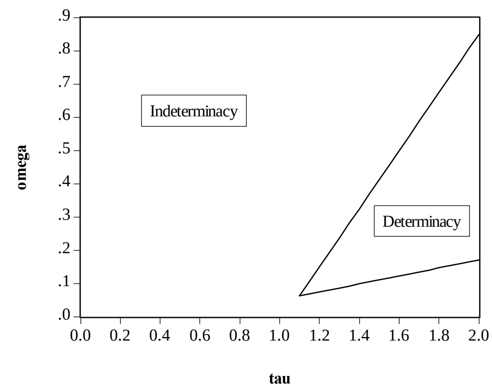

Figure 4: *Indeterminacy zones with increasing returns to scale* ($\gamma = 1.11$)*; forward-looking rules.*

proposal. The economics behind the result are parallel to those in Section 3.1.: an active interest rate policy can be used to counter the benefits of production bunching and it is therefore capable to automatically stabilizing the economy.

Proposition 2 *(Forward-looking rules) At constant returns to scale, passive policies rule out sunspot equilibria. In the presence of mild production externalities, output-targeting does not create determinacy alone and it must be supported by aggressive inflation-targeting.*

3.3 Indeterminacy with current-looking rules

Let us next consider current-looking rules of the form

$$\widehat{R}_t = \tau\widehat{\pi}_t + \omega\widehat{y}_t \qquad \tau \geq 0 \quad \omega \geq 0.$$

The dynamics of the complete model are given by

$$\begin{bmatrix} E_t\widehat{c}_{t+1} \\ E_t\widehat{R}_{t+1} \\ \widehat{k}_{t+1} \end{bmatrix} = \mathbf{W} \begin{bmatrix} \widehat{c}_t \\ \widehat{R}_t \\ \widehat{k}_t \end{bmatrix}.$$

3.3.1 Constant returns to scale and inflation-targeting

When $\omega = 0$ and when returns to scale are constant, the three eigenvalues of $\mathbf{W}$ are

$$\left\{0, \frac{\alpha}{1-\beta(1-\alpha)(1-\delta)}, \frac{1+\tau(1+(1-\alpha)(1-\delta)\beta)+\beta(1-\alpha)\delta}{\alpha\beta\tau}\right\}.$$

It is easy to show that the first two eigenvalues are always inside the unit circle. Since the underlying dynamical system involves two jump variables, the economy is always subject to indeterminacy.

3.3.2 Constant returns to scale and output-targeting

There is always indeterminacy even when output is targeted by the central bank. This can be seen by inspecting the eigenvalues of $\mathbf{W}$:

$$\left\{0, \frac{\alpha}{1-(1-\alpha)\beta(1-\delta)}, \frac{1+\tau-\omega-\beta(1-\omega-(1-\alpha)\delta(1+\tau-\omega)-\tau(1-\alpha)}{\alpha\beta\tau}\right\}.$$

The first two eigenvalues are always inside the unit circle. In fact, they do not even contain the parameter ω. Therefore, the upshot appears to be a clear case against using current-looking rules either with or without output-targeting. This is easy to understand from the simplified version of the model. The dynamics boil down to

$$0 = \frac{1+\omega-\tau}{1+\omega} E_t \widehat{\pi}_{t+1}$$

which implies that future rates of inflation are pinned down. However, the initial rate, $\widehat{\pi}_t$, is not pinned down. The same holds for period t's nominal interest rate. Real indeterminacy arises.

3.3.3 Increasing returns

The picture changes, however, when $\gamma > \gamma^{\min}$. Figure 5 plots determinacy regions while setting $\gamma = 1.11$. Now there exists an area in which applying a current-looking Taylor-rule can eliminate sunspot equilibria. In a nutshell, the central bank must react sufficiently to output and (to a lesser extent) to inflation.

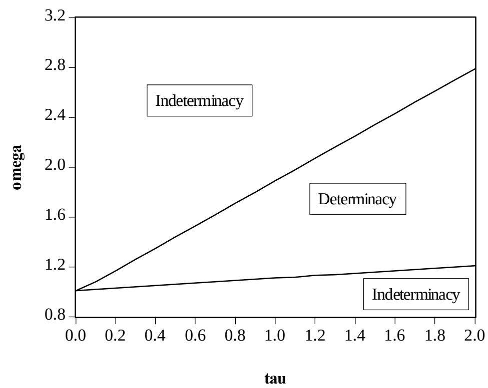

Figure 5: *Indeterminacy and determinacy regions when mild production externalities ($\gamma = 1.11$) are present; current-looking rules.*

Current-looking rules may therefore be a channel that eliminate sunspot fluctuations that arise from scale economies. In a sense, the result parallels Christiano (2000). He finds that – in the neighborhood of his economy's high level steady state – a policy that sets the nominal interest rate proportional to current employment can stabilize the economy. Here, in an economy with a more realistic departure from constant returns to scale a much more stringent requirement is placed on policy. In fact, output-targeting must sufficiently *lean-against-the-wind* and always be supported by inflation-targeting.

Proposition 3 *(Current-looking rules) Current-looking rules always imply indeterminacy at constant returns to scale. Indeterminacy arising from mild externalities can be eliminated by aggressive output-targeting (in combination with inflation-targeting).*

To conclude, Section 3 has shown that various formulations of the Taylor-rule generate very different economic dynamics depending on the economic environment. In particular, the presence of market imperfections (i.e. production externalities or monopolistic competition) fundamentally changes the policy recommendations that apply when these market imperfections are not present. This implies that pinning down the empirical degree of imperfections in actual economies may turn out to be an integral part of designing monetary policy. Table 2 summarizes the findings.

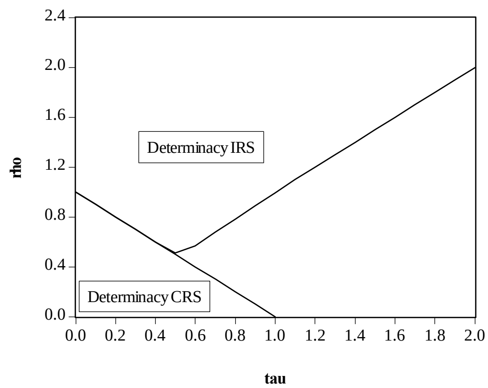

Figure 6: *The effect of interest smoothing: determinacy regions with constant returns to scale (CRS) and increasing returns to scale ($\gamma = 1.11$). The regions do not overlap; forward-looking rules.*

Table 2: Policies that lead to determinacy			
	Backward	Forward	Current
CRS	active π targeting	passive $\pi \& y$ targeting	n.a.
IRS	(active) y targeting	(active) $\pi \& y$ targeting	(active) $\pi \& y$ targeting

4 Nominal interest rate smoothing

Empirical studies on the Taylor-rule generally include the lagged interest rate. In what follows I will explore determinacy properties of interest smoothing in a forward-looking and a backward-looking Taylor-rule. The motivation arises from recent theoretical work that has suggested that indeterminacy can be relinquished by adding lagged values of the nominal interest rate to the standard Taylor-rule (see for example Rotemberg and Woodford, 1999, Giannoni and Woodford, 2002, and Benhabib et al., 2003).

4.1 Nominal interest rate smoothing and forward-looking inflation-targeting

Let us first consider hybrid Taylor-type rules such as

$$\widehat{R}_t = \rho \widehat{R}_{t-1} + \tau E_t \widehat{\pi}_{t+1} + \omega \widehat{y}_t \tag{10}$$

in which the parameter ρ stands for interest rate smoothing (the formulation applies Clarida, Gali and Gertler's, 1998, baseline case which they estimate for several central banks). Generally, estimates of (10) find a high degree of inertia and slightly greater than one-for-one increases in the nominal rate in response to inflation. Furthermore, the response to the output gap is mostly found to be small for the U.S. post-1980 period. Therefore, let us assume $\omega = 0$. Under constant returns to scale, the following four eigenvalues depict the dynamics

$$\left\{\frac{1-\beta(1-\alpha)(1-\delta)}{\alpha\beta}, \frac{\alpha}{1-\beta(1-\alpha)(1-\delta)}, \frac{1-\sqrt{1-4\rho\tau}}{2\tau}, \frac{1+\sqrt{1-4\rho\tau}}{2\tau}\right\}$$

(the dynamical system involves two jump variables). The first two eigenvalues split around the unit circle. If $\rho = 0$, the third eigenvalue is zero and the policy should be passive; of course, the argument simply repeats the finding for forward-looking rules (Section 3.2.1.). By making ρ positive, the last two eigenvalues become complex at

$$\rho' = \frac{1}{4\tau}$$

and cross the unit circle at

$$\rho'' = 1 - \tau.$$

We see from the ρ''-condition, that depending purely on a passive rule no longer guarantees determinacy since, *ceteris paribus*, large values of ρ push the economy out of the determinacy region. Empirical evidence points to central bank policies such as $\rho \approx 0.85$ and $\tau \approx 1.1$ – my analysis suggests that banks should avoid these policies to not create endogenous fluctuations. Figure 6 shows the determinacy regions for (i) constant returns to scale and (ii) mild increasing returns, $\gamma = 1.11$. The two areas do not overlap which underlines the central bank's dilemma. Taylor-rule prescriptions concerning the smoothing parameter differ across technological regimes: under constant returns the policy's response should be passive whereas strong interest rate smoothing will eliminate sunspot equilibria when increasing returns to scale are operating.

4.2 Nominal interest rate smoothing and backward-looking inflation-targeting

Rotemberg and Woodford (1999) and Giannoni and Woodford (2002) have suggested that backward-looking Taylor-rules' performances can be improved by adding lagged values of the nominal interest rate. In particular, they consider the rule

$$\widehat{R}_{t+1} = \rho \widehat{R}_t + \tau \widehat{\pi}_t$$

and find that a smoothing coefficient greater than one guarantees unique equilibria. Let us again begin by assuming constant returns to scale and endogenous capital accumulation (the above mentioned papers by Giannoni, Rotemberg and Woodford abstract from capital). The dynamics are characterized by the four eigenvalues

$$\left\{ \frac{1-\beta(1-\alpha)(1-\delta)}{\alpha\beta}, \frac{\alpha}{1-\beta(1-\alpha)(1-\delta)}, \tau+\rho, 0 \right\}.$$

The parameters τ and ρ enter in complementary fashion and determinacy simply requires that $\tau + \rho > 1$ (the underlying dynamical system has two jump variables). Thus, $\rho > 1$ is indeed a sufficient condition for ruling out indeterminacy and the result carries over to the current perfectly flexible price, cash-in-advance model.[12] However, the advice given is no longer valid once market imperfections are present. My numerical analysis suggests that there are no (τ, ρ)-constellations that deliver determinacy at $\gamma = 1.11$. For example, with $\tau = \rho = 1$, the four eigenvalues are

$$\{0, 0.8534 \pm 0.2941i, 2.0338\}.$$

Once again, the presence of market imperfections has nontrivial effects on the design of monetary policy: pushing up the smoothing parameter, ρ, no longer guarantees the elimination of indeterminacy.

Table 3: Policies that lead to determinacy

	Backward	Forward
CRS	strong smoothing	weak smoothing
IRS	n.a.	strong smoothing

[12] And, of course, it is the opposite of which is required for the forward-looking case.

Proposition 4 *(Interest rate smoothing) The exact dose of interest rate smoothing needed to rule out sunspot fluctuations is highly dependent on the specific technological environment. Moreover, the analysis suggests that no prescription can be found that secures a stable economy under both constant and increasing returns to scale.*

5 Concluding remarks

Recent literature on Taylor-rules has suggested that the monetary authority should adopt aggressive, backward-looking rules. For example, Carlstrom and Fuerst (2000) advise that

> "[t]o avoid doing harm, the central bank should place most weight on past movements in the inflation rate." [Carlstrom and Fuerst, 2000, p. 22]

The present paper has shown that we must be very careful with any generalized proposals. There are many dimensions, i.e. the modelling assumptions on the monetary side (Carlstrom and Fuerst, 2000, 2001) and the real side of the economy (the current paper), that have an impact on monetary policy's effects and that should accordingly be considered before spelling out a specific policy.

To demonstrate this, I have added a cash-in-advance superstructure to an otherwise well specified dynamic general equilibrium model that generates indeterminacy by externalities. Once increasing returns are present, most of the policy proposals contained in the existing literature – such as the ones prescribed by Carlstrom and Fuerst – are flipped on its head. For example, when formulating the Taylor-policy on past observations, inflation-targeting will not eliminate sunspot equilibria and aggressive output-targeting is required. Furthermore, rules which should be avoided (chosen) in perfect market environments often ensure (yield) unique (multiple) rational expectations solutions in alternative settings.

To summarize, essential information on how monetary design should be framed in practice must be inferred from empirical estimates of market imperfections. Unfortunately, the existing work on the issue does not offer a clear cut answer – the measurement of the degree of increasing returns is simply too imprecise – which given my results poses a dilemma for the central bank. Cole and Ohanian (1999) suggest a basic problem for the ambiguity:

insufficient variations in factor inputs. They conclude that currently available methods are not adequate to return estimates of scale economies such that we can eventually draw a conclusive diagnosis against or in favor of models with indeterminacy such as those summarized in footnote 1. I conclude that estimates on scale economies that are currently available are also not adequate to square conflicting Taylor-policy proposals.

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6 Appendix

The unique steady state is given by

$$
\begin{aligned}
\eta\frac{c}{y} &= \frac{\alpha}{lR} \\
u^{\theta} &= \alpha\frac{y}{k} \\
\theta\delta &= 1/\beta - 1 + \delta \\
\beta R &= \pi \\
1 &= \beta[\alpha\frac{y}{k} + 1 - \delta] \\
\delta &= \frac{y}{k} - \frac{c}{k}.
\end{aligned}
$$

Calibrating η, α, δ, β, l determines c/y, u, y/k, θ, R and π.

6.1 Backward-looking rules: analytical results

By employing the special case $\delta \to 1$ as in McCallum (1989), the basic insights for backward-looking policies can be derived from tractable analytical expressions. The four eigenvalues of $\mathbf{M}$ are

$$
\left\{\alpha, \frac{1+\alpha\beta \pm \sqrt{4\alpha\beta(\omega(1-\alpha\beta)-\tau)+(1+\alpha\beta\tau)^2}}{2\alpha\beta}, 0\right\}.
$$

It is quite instructive to consider $\tau = 1$, in which the central bank follows neither a passive nor an active policy. Starting at $\omega = 0$, the eigenvalues are

$$
\left\{\alpha, \frac{1}{\alpha\beta}, \tau, 0\right\}
$$

and the economy would drift into the indeterminacy zone by decreasing τ. It is easy to see that by increasing ω, the third eigenvalue

$$\frac{1+\alpha\beta-\sqrt{(1-\alpha\beta)(1-\alpha\beta(1-4\omega))}}{2\alpha\beta}$$

will be pushed inside the unit circle: indeterminacy arises. Analogously to the above simplified model, there is also an upper level of ω

$$\omega > \frac{2(1+\alpha\beta)}{1-\alpha\beta} > 0$$

at which determinacy enters again since two eigenvalues will have modulus larger than one. If on the other hand, $\tau \to 0$, determinacy is obtained for

$$\omega > \frac{1+\alpha\beta}{1-\alpha\beta} > 0.$$

Here the result from the capital-free version of the economy got repeated: output-targeting alone can produce determinacy. However, the parameter should not be chosen in a "medium range" otherwise sunspot cycles arise.

Next, I show how monetary policy must include output-targeting in order to eliminate indeterminacy that arises from externalities. Consider the case in which inflation-targeting is neither active nor passive, $\tau = 1$ (to simplify notation). Then the matrix $\mathbf{M}$'s eigenvalues at $\omega = 0$ are

$$\left\{0, 1, \frac{(-1+\beta(1+\alpha\gamma(\alpha(1+\beta)-1)) \pm \sqrt{\Gamma^2 - 4\alpha\beta\Delta}}{2\alpha\beta(\beta\gamma-1)}\right\}$$

$$\begin{aligned}\Gamma &\equiv 1-\beta(1-\alpha)(1+\alpha\gamma)-\alpha\beta^2(\alpha+\alpha\gamma)\\ \Delta &\equiv (1-\beta\gamma)(\alpha(1-\beta)+(1+\alpha-\beta(1+3\alpha+\alpha\beta))\gamma-1).\end{aligned}$$

Simple algebra delivers the minimum increasing returns

$$\gamma^* = \frac{(1+\alpha)(1-\beta)(1+\alpha\beta)}{1-\beta+\alpha^2\beta(1-\beta)+\alpha(4\beta+\beta-1)} - 1$$

at which indeterminacy kicks in. At this point, the four eigenvalues are

$$\left\{0, 1, -1, \frac{1+\alpha-\beta-2\alpha\beta-\alpha^3\beta^2(2-\beta)+\alpha^3\beta(3-\beta(2-\beta))}{\alpha\beta(1+\alpha(1-\beta(3-\alpha))}\right\}$$

and the third eigenvalue crosses into the unit circle for increasing returns higher than γ^*. While holding fixed the threshold level of increasing returns, I introduce output-targeting. In particular, by boosting $\omega \rightarrow \infty$, the eigenvalues become

$$\{0, \alpha, \infty, \infty\}.$$

That is, policies that are sufficiently offensive in countering output movements help to rule out indeterminacy.