

# DISCUSSION PAPER SERIES

No. 3890

**ARE HOUSEHOLD  
PORTFOLIOS EFFICIENT?  
AN ANALYSIS CONDITIONAL  
ON HOUSING**

Loriana Pelizzon and Guglielmo Weber

***FINANCIAL ECONOMICS and  
INTERNATIONAL MACROECONOMICS***



**Centre for Economic Policy Research**

[www.cepr.org](http://www.cepr.org)

Available online at:

[www.cepr.org/pubs/dps/DP3890.asp](http://www.cepr.org/pubs/dps/DP3890.asp)

# ARE HOUSEHOLD PORTFOLIOS EFFICIENT? AN ANALYSIS CONDITIONAL ON HOUSING

Loriana Pelizzon, Università di Padova  
Guglielmo Weber, Università di Padova and CEPR

Discussion Paper No. 3890  
May 2003

Centre for Economic Policy Research  
90–98 Goswell Rd, London EC1V 7RR, UK  
Tel: (44 20) 7878 2900, Fax: (44 20) 7878 2999  
Email: [cepr@cepr.org](mailto:cepr@cepr.org), Website: [www.cepr.org](http://www.cepr.org)

This Discussion Paper is issued under the auspices of the Centre's research programme in **FINANCIAL ECONOMICS** and **INTERNATIONAL MACROECONOMICS**. Any opinions expressed here are those of the author(s) and not those of the Centre for Economic Policy Research. Research disseminated by CEPR may include views on policy, but the Centre itself takes no institutional policy positions.

The Centre for Economic Policy Research was established in 1983 as a private educational charity, to promote independent analysis and public discussion of open economies and the relations among them. It is pluralist and non-partisan, bringing economic research to bear on the analysis of medium- and long-run policy questions. Institutional (core) finance for the Centre has been provided through major grants from the Economic and Social Research Council, under which an ESRC Resource Centre operates within CEPR; the Esmée Fairbairn Charitable Trust; and the Bank of England. These organizations do not give prior review to the Centre's publications, nor do they necessarily endorse the views expressed therein.

These Discussion Papers often represent preliminary or incomplete work, circulated to encourage discussion and comment. Citation and use of such a paper should take account of its provisional character.

Copyright: Loriana Pelizzon and Guglielmo Weber

## ABSTRACT

### Are Household Portfolios Efficient? An Analysis Conditional on Housing\*

In this Paper we argue that standard tests of portfolio efficiency are biased because they neglect the existence of illiquid wealth. In the case of household portfolios, the most important illiquid asset is housing: if housing stock adjustments are costly and therefore infrequent, we show how the dynamic optimization problem produces optimal portfolios in periods of no adjustment that are affected by housing price risk (through a hedge term). When the housing stock is not adjusted, we argue that tests for portfolio efficiency of financial assets must then be run conditionally upon housing wealth.

In our application, we use Italian household portfolio data from SHIW 1998 and time series data on financial asset and housing stock returns to assess whether actual portfolios are efficient. We first consider purely financial portfolios and portfolios that also treat the housing stock as another asset. We then consider the consequences of treating the housing stock as given and test for efficiency in this framework. Our empirical results support the view that the presence of illiquid wealth plays an important role in determining whether portfolios chosen by homeowners are efficient.

JEL Classification: D91 and G11

Keywords: efficiency, housing and portfolio choice

Loriana Pelizzon  
Dipartimento di Scienze Economiche  
Università di Padova  
Via del Santo 33  
35123 Padova  
ITALY  
Tel: (39 049) 827 4054  
Fax: (39 049) 827 4221  
Email: pelizzon@decon.unipd.it

Guglielmo Weber  
Dipartimento di Scienze Economiche  
Università di Padova  
Via del Santo 33  
I-35123 Padova  
ITALY  
Tel: (39 049) 827 4271  
Fax: (39 049) 827 4221  
Email: guglielmo.weber@unipd.it

For further Discussion Papers by this author see:  
[www.cepr.org/pubs/new-dps/dplist.asp?authorid=158993](http://www.cepr.org/pubs/new-dps/dplist.asp?authorid=158993)

For further Discussion Papers by this author see:  
[www.cepr.org/pubs/new-dps/dplist.asp?authorid=108952](http://www.cepr.org/pubs/new-dps/dplist.asp?authorid=108952)

\*We are grateful to Elena Parcianello and Viola Angelini for skilful research assistance and to Alessandro Buccioli, Marjorie Flavin, Raffaele Miniaci and Stephen Schaefer for helpful discussions. We are also grateful for comments by seminar audiences at CSEF (Salerno) and University of Padua. This research was partly financed by MURST and CNR.

Submitted 01 April 2003

## 1. Introduction

There has been an increased interest in recent years in household portfolio choice. A number of country studies have looked at the way households allocate their financial wealth across different financial instruments and found that a decreasing but still sizeable proportion of households fail to invest in the stock exchange (Guiso, Haliassos and Jappelli, 2002).

Households allocate their wealth into financial and real assets, but the portfolio allocation problem has typically been addressed empirically focusing solely on financial assets. A few studies have extended the analysis to cover other forms of household wealth, notably own business (Heaton and Lucas, 2000) and housing equity (Flavin, 2002). Both assets are illiquid, that is subject to non-negligible trading costs. These trading costs are likely to be particularly high for the housing stock for homeowners. When consumption and investment needs differ, and rental markets are imperfect (Henderson and Ioannides, 1983), short run adjustments can be all but impossible. Flavin and Yamashita (2002) stress that in this sense “demand for housing is over-determined”, and investment considerations may be of secondary importance.

In this paper we address the issue of efficiency of household portfolios when illiquid housing wealth is also considered. This issue has been investigated by Grossman and Laroque (1990) and more recently by Flavin (2002). Grossman and Laroque show that the standard CAPM holds in a dynamic setting when households derive utility from just one good that is durable and illiquid (and therefore infrequently adjusted). Flavin’s paper extends Grossman and Laroque’s model by allowing for the presence of two goods in the utility function, but considers the case of no correlation between housing returns and financial asset returns. Flavin proves that over those periods where the housing stock is not adjusted, all households hold a single optimal portfolio of risky assets (the standard Markowitz optimal risky portfolio), the standard CAPM holds and housing wealth affects portfolio allocations only through the relative risk aversion of individual investors. We extend Flavin’s analysis to cover the case where returns are correlated, and show how efficient portfolios should be over the periods of time when the housing stock is not adjusted. In these portfolios housing wealth affects the optimal shares in two distinct ways: indirectly, via risk aversion, and directly, via a hedge motive. In particular, we observe that all households will hold as single optimal

portfolio of risky assets (the standard Markowitz optimal portfolio) and a hedge term covering house price risk.

On the basis of our theoretical analysis, we expect optimally chosen financial portfolios not to be mean-variance efficient in the standard sense when asset and housing returns correlate. Also, if the housing stock is not frequently adjusted, we also expect the overall portfolios (that include financial assets and housing wealth) not to be mean-variance efficient. However, we show that optimal portfolios should be conditionally mean-variance efficient, that is mean-variance efficient when housing wealth is treated as given but stochastic. Our analysis provides the economic rationale for implementing the conditional test of mean-variance efficiency that treats housing wealth as predetermined, suggested by Gourieroux and Jouneau (1999).

In our application, we use Italian household portfolio data from the Bank of Italy Survey on Household Income and Wealth (SHIW) for 1998 and time series data on financial assets returns as well as housing stock returns to test the hypothesis that observed portfolios are efficient. Our application is of particular interest because in Italy home ownership is wide spread, whereas household stock market participation is low, but rapidly increasing. Pension wealth is still almost entirely provided by the public pay-as-you-go social security system and is therefore both out of individual investors' control and not directly related to the financial markets performance.

The paper is organized as follows: section 2 presents the theory, section 3 discusses the test statistic and econometric issues, section 4 describes the data used, sections 5-8 report estimation and test results and section 9 concludes.

## **2. Theory**

In this section we build upon Flavin's (2002) analysis of the dynamic optimization problem with housing, and use the same notation for comparison's sake. We then show that the dynamic optimization problem produces the same asset allocation rule as a static problem that treats housing wealth as given.

Flavin generalizes Grossman and Laroque's model by making current utility a function of both a durable good, a house (H), and a non-durable good (C). The non-durable good is infinitely divisible and costlessly adjustable. As in Grossman and Laroque, the durable good is instead subject to an adjustment cost proportional to its

value and is therefore adjusted infrequently. This generalization is of great relevance for the analysis of portfolio choice, because it allows to explicitly consider the relationship between the real rate of return on housing investment and the real rates of return on financial assets.

The household maximizes expected lifetime utility:

$$(1) \quad U = E_0 \int_0^{\infty} e^{-\delta t} u(H_t, C_t) dt$$

For analytical simplicity, the house is not subject to physical depreciation. Using the non-durable good as numeraire, define:

$$(2) \quad P_t = \text{house price (per square meter) in the household's market}^3.$$

Assume that wealth is held only in the form of financial assets and housing. The household can invest in a risk-less asset and in any of  $n$  risky financial assets. Holdings of the financial assets can be adjusted with zero transaction cost.

Thus wealth is given by:

$$(3) \quad W_t = P_t H_t + B_t + \underline{X}_t \underline{\ell}$$

where  $\underline{X}_t = (1 \times n)$  vector of amounts (expressed in terms of the non-durable good) held of the risky assets and  $\underline{\ell} = (n \times 1)$  vector of ones.  $B_t$  is the amount held in the form of the risk-less asset. All financial assets, including the risk-less asset, may be held in positive or negative amounts.<sup>4</sup>

Assuming that dividends or interest payments are reinvested so that all returns are received in the form of appreciation of the value of the asset, let  $b_{it}$  = the value (per share) of the  $i$ -th risky asset, and assume that asset prices follow an  $n$ -dimensional Brownian motion process:

$$(4) \quad db_{it} = b_{it}((\mu_i + r_f)dt + d\omega_{it})$$

The vector  $\underline{\omega}_{Ft} \equiv (\omega_{1t}, \omega_{2t}, \dots, \omega_{nt})$  follows an  $n$ -dimensional Brownian motion with zero drift and with instantaneous covariance matrix  $\Sigma$ , the corresponding vector ( $n \times 1$ ) of expected excess returns on risky financial assets is  $\underline{\mu} \equiv (\mu_1, \mu_2, \dots, \mu_n)$ , and  $r_f$  is the risk-less rate. The  $i$ -th element of  $\underline{X}_t$  in equation (3) is given by  $X_{it} \equiv N_{it} b_{it}$  where

<sup>3</sup> Unlike Flavin, we do not consider a separate price process for the next house to be bought

<sup>4</sup> This model does not deal with labor income or borrowing restrictions, that are instead considered in the single financial asset model developed by Cocco (2001).

$N_{it}$  is the number of shares held of asset  $i$ . Since asset prices,  $b_{it}$ , are taken as exogenous, the household determines  $X_{it}$  by its choice of  $N_{it}$ <sup>5</sup>.

House prices also follow a Brownian motion:

$$(5) \quad dP_t = P_t((\mu_H + r_f)dt + d\omega_{Ht})$$

where  $\omega_{Ht}$  is a Brownian motion with zero drift and instantaneous variance  $\sigma_P^2$ .

Stacking equations (4) and (5), define the  $((n+1) \times 1)$  vector  $d\underline{\omega}_t$ :

$$(6) \quad d\underline{\omega}_t = \begin{bmatrix} d\omega_{1t} \\ \vdots \\ d\omega_{nt} \\ d\omega_{Ht} \end{bmatrix}$$

which has instantaneous  $((n+1) \times (n+1))$  covariance matrix  $\Omega$ :

$$(7) \quad \Omega = \begin{bmatrix} \Sigma & \Gamma_{b_i,P} \\ \Gamma'_{b_i,P} & \sigma_P^2 \end{bmatrix}$$

where:

$$(8) \quad \Gamma_{bP} = \begin{bmatrix} \sigma_{b1P} \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \sigma_{bnP} \end{bmatrix}$$

Note that we depart from Flavin here, in that we do not assume the covariance matrix  $\Omega$  to be block diagonal. This is the substantial difference between our models, that generates qualitatively different results.

We show in the Appendix that, under the assumptions listed above, the optimal holding of risky financial assets, is given by:

---

<sup>5</sup> We follow Flavin, and use  $X_{it}$  rather than  $N_{it}$  as the choice variable representing the portfolio decision.



$$(9) \quad \underline{X}_0^T = \begin{bmatrix} -\frac{\partial \mathcal{V}}{\partial W} \\ \frac{\partial^2 \mathcal{V}}{\partial W^2} \end{bmatrix} \Sigma^{-1} \underline{\mu} - P_0 H_0 \Gamma_{bp} \Sigma^{-1}$$

and the amount held of the risk-less asset is:

$$(10) \quad B_0 = W_0 - P_0 H_0 - \underline{X}_0 \underline{\ell}$$

In equation (9), the expression in square brackets is the reciprocal of the coefficient of absolute risk aversion:

$$(11) \quad ARA \equiv - \frac{\frac{\partial^2 \mathcal{V}(W_t, H_t)}{\partial W_t^2}}{\frac{\partial \mathcal{V}(W_t, H_t)}{\partial W_t}} > 0$$

It is worth pointing out that risk aversion affects the first term on the RHS of equation (9) but not the second term, that bears the interpretation of a hedge portfolio<sup>6</sup>. In Flavin's analysis this second term disappears, because she assumes  $\Gamma_{bp} = 0$ , and therefore she can prove that the traditional CAPM holds.

We shall now prove that equation (9) can be derived in a static mean-variance analysis framework, if the existing housing stock is treated as an additional constraint to the optimization problem (see Anderson and Danthine, 1981, for the general case where an asset is constrained).

Let us consider a market with a risk-less asset,  $n$  unconstrained and one constrained risky assets. Denote the first two moments of asset returns as  $\underline{m} + r_f$  (where

$$\underline{m} = \begin{pmatrix} \underline{\mu} \\ \underline{\mu}_H \end{pmatrix} \text{ and } \underline{\mu} \text{ is the expected excess return) and } \Omega \text{ (see equation (7)).}$$

Consider an investor whose portfolio allocation in the risky assets is:

$$(12) \quad Z = \begin{pmatrix} x_0 \\ h_0 \end{pmatrix}$$

$$\text{where } \underline{x}_0 \equiv \frac{X_0}{W_0} \text{ and } h_0 \equiv \frac{H_0 P_0}{W_0}$$

---

<sup>6</sup> This term is different from the classical Merton hedge term that accounts for shifts in the investment opportunity set.

and  $(1-Z)^T \underline{1}$  in the risk-less asset ( $\underline{1}$  is an  $n+1$  vector of ones). Assume that this investor is constrained in his  $h_0$  (that is  $h_0$  is given and equal to  $\bar{h}_0$ ), but otherwise behaves according to the mean-variance model. The investor problem becomes:

$$(13) \quad \begin{cases} \min_Z Z^T \Omega Z \\ \text{s.t.} \begin{cases} Z^T \underline{m} + r_f = m^* \\ h_0 = \bar{h}_0 \end{cases} \end{cases}$$

where  $m^*$  is a given level of expected return.

The problem can be solved by defining the lagrangian:

$$(14) \quad \Lambda = \left( \underline{x}_0 \Sigma \underline{x}_0^T + h_0^2 \sigma_p^2 + 2h_0 \underline{x}_0 \Gamma_{bp} \right) - 2\gamma \left[ \underline{x}_0 \underline{\mu} + h_0 \underline{\mu}_H + r_f - m^* \right]$$

The first order conditions are:

$$(15) \quad \frac{\partial \Lambda}{\partial \underline{x}_0} = \left( 2\Sigma \underline{x}_0^T + 2h_0 \Gamma_{bp} \right) - 2\gamma \underline{\mu} = 0$$

$$(16) \quad \frac{\partial \Lambda}{\partial \gamma} = \underline{x}_0 \underline{\mu} + h_0 \underline{\mu}_H + r_f - m^* = 0$$

The solution is:

$$(17) \quad \underline{x}_0 = \gamma \Sigma^{-1} \underline{\mu} - h_0 \Gamma_{bp} \Sigma^{-1}$$

where  $\gamma$  is the Lagrange multiplier of the constraint on the expected return, that has the standard relative risk aversion interpretation (Samuelson, 1970). This is the same as equation (9), after multiplying through by  $W_0$ .

This result means that investors have to be efficient with respect to the risky financial assets and choose the efficient Markowitz portfolio according to their risk aversion (see Markowitz (1992)). However, they also use the risky financial assets to hedge their expositions on the constrained asset. If  $\Gamma_{bp}=0$  the problem is the same as in Flavin (2002) and portfolio choice can be separated between financial and real assets.

### 3. Econometric Issues

In section 2 we have seen that the notion of efficiency of household portfolios depends on the assumption we make on the nature of housing investment. If housing investment is costless, then the efficient frontier should be computed using all financial assets returns as well as the return on housing<sup>7</sup>. If transaction costs affect housing investment, then the analysis differs according to the correlation between housing and financial returns. If this correlation is zero, household portfolios will be mean-variance efficient in the usual sense (i.e. : with respect to the standard financial assets frontier). If this correlation is instead non-zero, household portfolios will be mean-variance efficient once we condition upon the value of the housing stock, as shown in equation (17).

In this section we show how we can test for the efficiency of the observed household portfolios in all cases discussed above. In order to do this, we use time series data on asset returns for a period prior to the survey to estimate the mean variance frontier, taking into account the theoretical assumptions of rational expectations and normal return distributions. In particular, we use weighted sample means and covariances in order to estimate expected excess returns and risk (i.e. the first two unconditional moments). The weights are a declining function of the time distance from the end of the sample period.<sup>8</sup>

In the vast literature on efficient portfolios, only a few papers incorporate real estate as an asset. Goetzmann and Ibbotson (1990) and Goetzmann (1993) used regression estimates of real estate price appreciation, and Ross and Zisler (1991) calculated returns from real estate investment trust funds, to characterize the risk and return to the real estate investment. Flavin and Yamashita (2002) use data from the 1968-1992 waves of the Panel Study of Income Dynamics that contain records on the owner's estimated value of the house and compute rates of return from regional real estate price data.

Mean-variance efficiency is usually assessed on the basis of a graphical comparison. However, Jobson and Korkie (1982,1989) and Gibbons, Ross and Shanken (1989) have

---

<sup>7</sup> Housing can be neglected if its return is spanned by financial assets.

<sup>8</sup> Another way is to consider the first two conditional moments from a time series model of the returns data that allows for time-varying conditional heteroskedasticity, as in Blake (1996). This modeling framework requires long time series.

proposed a test of the significance of the difference between the actual portfolio held by an investor and a corresponding efficient portfolio. This test is based on the difference between the slopes of arrays from the origin through the two portfolios in the expected return-standard deviation space. If the actual portfolio is an efficient portfolio, the two slopes will be the same; if the actual portfolio is inefficient, the slope of the efficient portfolio will be significantly greater.

Gourieroux and Jouneau (1999) derive efficiency tests for the conditional or constrained case, i.e. for the case where a subset of asset holdings is potentially constrained (housing in our case). They define the Sharpe ratio of the unconstrained risky financial assets portfolio as :

$$(18) \quad S_1 = \underline{\mu}^T \Sigma^{-1} \underline{\mu}$$

The Sharpe ratio for the observed (constrained) portfolio made of the first  $n$  (financial) assets is defined in this notation as:

$$(19) \quad S_1(Z) = \frac{[\underline{\mu}^T v_1]^2}{v_1^T \Sigma v_1}$$

where  $v_1^T = \underline{x}_0^T + h_0 \Gamma_{bP} \Sigma^{-1}$  (see equation 17), that is the actual risky financial asset portfolio after eliminating the hedge term.

When all asset returns are normally distributed, Gourieroux and Jouneau show that the Wald statistic

$$(20) \quad \xi_1 = T \frac{\hat{S}_1 - \hat{S}_1(Z)}{1 + \hat{S}_1(Z) \frac{Z^T \Omega Z}{v_1^T \Sigma v_1}}$$

is distributed as a  $\chi^2(n-1)$  under the null hypothesis that the risky financial assets portfolio (after eliminating the hedge term) lies on the financial efficient frontier<sup>9</sup>.

---

<sup>9</sup> For the sake of simplicity we do not stress in our notation that the test statistic is defined as a function of sample estimates of the first two moments of the rates of return distribution and takes observed portfolio shares as given.

Gourieroux and Jouneau also show that a test for the efficiency of the whole portfolio can be derived as a special case by setting  $v_1 = Z$ . The test statistic becomes

$$(21) \quad \xi_e = T \frac{\hat{S} - \hat{S}(Z)}{1 + \hat{S}(Z)}$$

where  $\hat{S} = \underline{m}^T \Omega^{-1} \underline{m}$  and  $\hat{S}(Z) = \frac{[\underline{m}^T Z]^2}{Z^T \Omega Z}$ .

$\xi_e$  is distributed as a  $\chi^2(n)$  under the null hypothesis that mean and standard deviation of the observed portfolio lie on the efficient frontier. In this special case, this test is asymptotically equivalent to the test derived by Jobson and Korkie (1982,1989) and Gibbons, Ross and Shanken (1989).

In our empirical analysis, we compute efficiency test statistics (either  $\xi_e$  or  $\xi_I$ ) for each household in our sample. In particular, we compute the standard test ( $\xi_e$ ) twice: once for the financial portfolio (as in standard practice), and once for the whole portfolio (inclusive of housing). In this latter case, we also compute the constrained test ( $\xi_I$ ).

We use the computed test statistics in two different ways. First, we show what proportions of household portfolios fail the efficiency tests for a range of possible test sizes (from .75 to .95). Second, we perform a Probit regression where the dependent variable takes value 1 if the observed portfolio is efficient and the regressors are household characteristics, as a way to investigate possible causes for inefficient portfolio allocations.

#### 4. Application.

To show the implications of our theoretical analysis we use data on Italian asset returns and household portfolios.

Italian households traditionally have held poorly diversified financial portfolios (Guiso and Jappelli, 2002). In the 1980s and even more in the 1990s, though, the stock exchange has grown considerably and mutual funds have become a commonly held financial instrument. Household financial accounts reveal that the aggregate financial

portfolio share in stocks and funds amounted to 16.15% in 1985, 20.69% in 1995 and rose to an unprecedented 46.95% in 1998. This growth in the equity market paralleled the sharp decrease in importance of bank accounts and short-term government debt in household portfolios. These aggregate statistics are uninformative on the participation issue, though. To this end, an analysis of survey data is required. The most widely used Italian survey data, the Bank of Italy-run Survey on Household Income and Wealth (SHIW), shows direct or indirect participation in equity markets (broadly defined to include life insurance and private pensions) to have increased from 26.43% in 1985 to 38.19% in 1995 and to 48.24% in 1998. For comparison, the percentage of homeowners in the same sample hovered around 63-65% over the period. Finally, the share of financial to total wealth in SHIW was 11.7% in 1991 and rose to 14.59% in 1998 – housing wealth accounted for a 68.91% of total wealth in 1991 and fell slightly to 65.81% in 1998 (50.11% to 48.84% if we consider the principal residence only).

These summary statistics clearly show that household financial portfolios have changed a lot over the years, but that a key role in total household wealth is played by real estate. It makes sense to consider the interaction of housing and financial wealth holdings when assessing the efficiency of household portfolios, as stressed by Flavin and Yamashita (2002). A financial portfolio may be away from the financial assets mean variance frontier simply because of its covariance properties with the return on housing equity. This is a relevant issue whether housing wealth is treated as liquid or instead as an illiquid asset.

In our application we use household portfolio data for 1998 and asset return data for the period 1989-1998.

The 1998 SHIW wave contains detailed information on asset holdings of 7115 households as of 31.12.1998, as well as self assessed value of their housing stock (both principal residence and other real estate) and actual or imputed rent for each dwelling. For each household we also know the region of residence and a number of demographic characteristics (that are used to characterize departures from efficiency). The survey does not over sample the very rich, and it therefore captures a fraction of about a third of total household financial wealth. It does cover a relatively large number of assets, including individual pension funds: these are still remarkably unimportant in Italy, though, partly because of inadequate tax incentives. Occupation pension schemes are

also relatively minor, even though recent reforms of the Italian Social Security system (particularly the Dini reform of 1995) imply that they should become wide-spread.<sup>10</sup>

Asset return data cover five major assets: short term government bonds (3-month BOT), medium term government bonds, long-term government bonds (BTP), corporate bonds and equity (the MSCI Italy stock index). We treat the short-term bond as risk free, and assume that this is the relevant return on bank deposits, once account is taken of non-pecuniary benefits. For medium term, long term and corporate bonds we derive the holding period returns by standard methods. In particular, for medium term we use the RENDISTAT index (the index of the medium term government bonds yields) and we determine the holding period return by assuming a duration of two years. For corporate bonds we use the RENDIOBB index (the index of Italian corporate bonds yields) and assume a duration of three years. For long term bonds we use the estimated term structure of interest rates and determine the holding period returns of an equally weighted portfolio based on two assets with a duration of three years and five years. We checked the quality of this estimation by regressing our monthly returns determined with this procedure on those of the MSCI Italian bond index (that are only available since December 1993) and found that the fit is almost perfect ( $R^2$  is equal to 99.62%).

We express all returns net of withholding tax, on the assumption that for most investors other tax distortions are relatively minor (financial asset income is currently subject to a 12.5% withholding tax. Housing is taxed on the basis of its ratable value, while dividends on stocks directly held and actual rental income is taxed at the marginal income tax rate).

To evaluate the efficiency of households portfolio we need to determine the expected return and the expected variance covariance matrix of the assets. Given long, stationary series we could simply compute the corresponding sample moments of the excess return of the different assets. However, this approach is unlikely to work in our case: in the decade we consider we observe a long convergence process of Italian interest rates to German interest rates that accelerated dramatically in the few years before the introduction of the Euro on January 1999. This would suggest estimating a VAR for the first moments and a multivariate GARCH for the second moments, but we

---

<sup>10</sup> Further information on the survey is provided in Guiso and Jappelli, (2002) and D'Alessio and Faiella (2000). Information on the Italian pension system and its recent reforms is presented in Brugiavini and Fornero (2001).

do not have enough data points to perform this estimation exercise. In fact, housing returns are available at a biannual frequency, and we are therefore forced to use at most twenty-one data points. For these reasons, we used a simple Weighted Least Squares procedure, where the raw return series data are weighted down more the farther away they are from December 1998. More precisely, we constructed the weights to be a geometrically declining function of the lag operator multiplied by  $\alpha$  (where  $\alpha$  is set to .8). The weights are then multiplied by a constant so that the expected returns on long term government bills are in line with the actual returns of the German Treasury bond in 1998-9.

In Table 1 we show the first and second moment of the excess returns data used. These are expressed as percentage bi-annual rates of return net of the time-varying risk-free rate: for the risk-free rate we report only the January 1999 six month Italian Treasury bill interest rate.

We see that stocks have higher expected return and higher variance than all other risky financial assets. Correlation coefficients between bonds are quite high (they range between .86 and .97) – correlation coefficients of stocks and bonds are much lower (between .33 and .55).

**Table 1: Sample first and second moments of asset excess returns (1989-98)**

	<b>BOT</b>	<b>BTP</b>	<b>MTG-Bonds</b>	<b>Corporate Bonds</b>	<b>Stocks</b>
<b>Expected return %</b>	1.3169	0.8021	0.427	0.4495	2.2932
<b>Standard Deviation %</b>		1.2223	0.6469	0.7809	7.4875

*Note: the risk-free return refers to the second half of 1998*

<b>CORRELATION</b>	<b>BTP</b>	<b>MTG-Bonds</b>	<b>Corporate Bonds</b>	<b>Stocks</b>
<b>BTP</b>	1	0.965024	0.855069	0.331997
<b>MTG-bonds</b>		1	0.901229	0.313592
<b>Corporate Bonds</b>			1	0.54595
<b>Stocks</b>				1

This picture is however largely incomplete. We know that two households out of three own real estate, and we argued that this type of investment is highly illiquid. It is therefore of great interest for us to compute first and second moments of the housing stock. To this end we use province-level biannual price data (source: Consolente



Immobiliare) covering the whole 1989-98 period. We compute the return on housing according to the formula:

$$(22) \quad r_{H,t} = \frac{P_t - P_{t-1}}{P_{t-1}} + \frac{D_t - COM_t}{P_{t-1}} = \frac{P_t - P_{t-1}}{P_{t-1}} + \kappa$$

where  $D$  denotes rent and  $COM$  maintenance costs. Given that we lack time series information on these, we set  $\kappa=.025$  (5% on an annual basis), as in Flavin and Yamashita (2002). The choice of  $\kappa$  is immaterial in the analysis of the constrained case, as long as  $\kappa$  is a fixed number (see equation (20)). It becomes important in the case where housing is treated as unconstrained, given that it affects its expected return directly. However, if rental income is time-varying, real estate indices based on observed house prices are flawed, as stressed by De Ronn, Eichholtz and Koedijk (2002).

Finally, we aggregate housing returns to the macro-region level (provincial resident population numbers were used to generate weights). This way we generate average return data for the North West, North East, Centre and South (inclusive of the Islands).<sup>11</sup> The first and second moment are then determined using the Weighted Least Squares procedure described above.

Table 2 reveals that expected excess returns on housing are highest in the North East and in the South and lowest in Central Italy (they range between 0.73% and 0.61% on a biannual basis). They are close to returns on bonds, but are much lower than returns on stocks. Housing excess return standard deviations range between 0.46% and 0.78%, and are therefore much lower than on stocks, but comparable to Government and Corporate Bonds. Of interest to us is the negative correlation between housing return and most financial asset returns.

---

<sup>11</sup> If we wanted to increase the number of observations, we could generate quarterly price data by using rent price index aggregate data and regress changes in the house price on changes in rent to fill in the gaps in the series. However, the fit of these regressions is relatively low, and this implies that the quarterly return data would be affected by a low signal to noise ratio. Given our interest in second moments this is potentially a serious problem, and we prefer to use biannual data throughout.

**Table 2: Expected excess returns and correlation matrix of housing (1989-98)**

	NW	NE	Centre	South
<b>Expected excess return %</b>	0.6143	0.7108	0.6517	0.7303
<b>Standard deviation %</b>	0.7816	0.4607	0.5439	0.4986

	NW	NE	Centre	South
<b>BTP</b>	-0.03977	-0.14022	-0.23704	-0.27408
<b>MTG-bonds</b>	-0.12893	-0.24622	-0.35476	-0.14243
<b>Corporate bonds</b>	-0.15509	-0.08582	-0.52375	-0.24496
<b>Stocks</b>	-0.51182	-0.2704	-0.67522	0.031486

The issue arises of whether these correlations are negligible. The simplest way to assess this, is to estimate the coefficients of the hedge term in equation (17), that is to estimate the beta hedge ratio  $\Gamma_{bp}\Sigma^{-1}$ . This can be done by running the regression of housing returns on financial asset returns, as suggested by de Roon, Eichholtz and Koedijk (2002). In our case we use WLS instead of OLS for internal consistency. Parameter estimates and their standard errors are summarized in Table 3.

**Table 3: Regression of excess return on housing on financial assets excess returns**

Variable	North West	North East	Centre	South
<b>Constant</b>	-0.00141 0.001073	-0.0003 0.000945	-0.00041 0.000952	-0.00037 0.001061
<b>R<sub>BTP</sub></b>	0.928275 0.282423	0.559817 0.248665	0.714788 0.250625	-0.82673 0.279198
<b>R<sub>MTG</sub></b>	-2.38735 0.609286	-1.88857 0.536458	-1.29275 0.540686	2.014983 0.602328
<b>r<sub>BONDS</sub></b>	1.080841 0.3165	0.848318 0.278669	-0.11126 0.280865	-0.73687 0.312885
<b>r<sub>STOCKS</sub></b>	-0.12004 0.017563	-0.04496 0.015464	-0.04314 0.015586	0.0354 0.017363
p-value	0.000034	0.014788	0.001414	0.02506
<b>R<sup>2</sup></b>	0.784422	0.518884	0.649422	0.482323

*Notes: Standard errors in parentheses. Number of observations = 21*

We see that in all regions there is at least one non-zero parameter at the 95% significance level and the slope coefficients are jointly significantly different from zero at the 95% level (the p-value of the F-test is reported at the bottom of the table, together with the  $R^2$ ). The region where this test is least significant is the South (with a p-value of 2.5%).

On the basis of this evidence, we conclude that housing returns present significant correlations with financial asset returns in Italy, and that this provides the basis for introducing a hedge term in household portfolios of house-owners. De Roon, Eichholtz and Koedijk (2002) find that a similar result is also true for some areas the U.S., but do not analyze the efficiency of U.S. household portfolios.

In Tables 4 and 5 we report the percentage participation for each asset and liability recorded in SHIW98 and the corresponding aggregate portfolio share. For instance, we see that almost 75% of the sampled households have a bank current (i.e. checking) account, and that the 27.24% of all financial wealth is held in such accounts.

We also show in the last column of Table 4 where each asset is classified, given that we use asset returns data at a much coarser aggregation level. So the first seven assets (cash, various deposits, and repos) are all classified as risk-free. Of particular interest is the relatively low direct stock market participation (7.42% hold listed shares; 1.58% shares in unlisted companies).

However, 10.86% of all households have mutual funds, and these holdings we classify partly as stocks and partly as bonds. Of great interest to us is the high proportion of households who own some housing stock (almost 70%) and the magnitude of this type of investment (that accounts for 85% of total wealth). Liabilities are relatively wide-spread (10.41% households report mortgage; 12.33% other forms of consumer debt), but their quantitative importance is relatively minor.

---

<sup>13</sup> However, the statistic proposed by Basak, Jagannathan and Sun tests for differences in portfolios standard deviation for a given expected return. This is conceptually different from the test used in this paper, that allows direct comparison of portfolios that have different expected returns.

**Table 4: Participation decision - individual financial and real assets**

<b>Asset</b>	<b>Participation</b>	<b>Broad Asset</b>
Cash	100%	Risk-free
Bank Current Account Deposits	74.94%	Risk-free
Bank Savings Deposits (Registered)	19.31%	Risk-free
Bank Savings Deposits (Bearer)	10.90%	Risk-free
Certificates of deposit	3.68%	Risk-free
Repos	0.94%	Risk-free
Post Office Current Accounts and Deposit Books	11.43%	Risk-free
Post Office Savings Certificates	6.55%	Long-Term
BOT (Italian T-bills)	9.67%	Risk-free
CCT (Italian T-certificates)	4.74%	Risk-free
BTP (Italian T-bonds)	2.70%	Long-Term
CTZ (Italian zero-coupon)	0.78%	Medium-Term
Other Italian Government Debt (CTEs, CTOs, etc.)	0.31%	Medium-Term
Corporate Bonds	5.55%	Bonds
Mutual Funds	10.86%	Bonds (1/2) Stocks (1/2)
Shares of listed companies	7.42%	Stocks
<i>of which:</i> of privatised companies	4.30%	Stocks
Shares of unlisted companies	1.58%	Stocks
Shares of limited liability companies	0.53%	Stocks
Shares of partnerships	0.15%	Stocks
Managed Savings (by banks)	2.03%	Bonds (1/2) Stocks (1/2)
Managed Savings (by other financial intermediaries)	0.5%	Bonds (1/2) Stocks (1/2)
Managed Savings by Trust Companies	0.06%	Bonds (1/2) Stocks (1/2)
Foreign bonds and government securities	0.52%	Bonds (1/2) Stocks (1/2)
Foreign Stocks and Shares	0.46%	Bonds (1/2) Stocks (1/2)
Other foreign assets	0.05%	Bonds (1/2) Stocks (1/2)
Loans to co-operatives	1.67%	Stocks
House	69.76%	House
Mortgage	10.41%	Long Term (neg. position)
Debt	12.33%	Bonds (neg. position)

**Table 5: Portfolio share - individual financial and real assets**

Asset	Portfolio share (financial assets)	Portfolio share (financial assets + House)
Cash	2.13%	0.31%
Bank Current Account Deposits	27.24%	2.86%
Bank Savings Deposits (Registered)	4.94%	1.00%
Bank Savings Deposits (Bearer)	2.75%	0.48%
Certificates of deposit	2.52%	0.50%
Repos	1.19%	0.25%
Post Office Current Accounts and Deposit Books	2.54%	0.38%
Post Office Savings Certificates	2.00%	0.31%
BOT (Italian T-bills)	7.64%	1.22%
CCT (Italian T-certificates)	3.92%	0.58%
BTP (Italian T-bonds)	2.14%	0.37%
CTZ (Italian zero-coupon)	0.31%	0.06%
Other Italian Government Debt (CTEs, CTOs, etc.)	0.34%	0.04%
Corporate Bonds	4.92%	0.74%
Mutual Funds	13.99%	2.25%
Shares of listed companies	5.90%	0.99%
<i>of which: of privatised companies</i>	1.86%	0.29%
Shares of unlisted companies	0.77%	0.12%
Shares of limited liability companies	2.19%	0.26%
Shares of partnerships	1.30%	0.16%
Managed Savings (by banks)	6.62%	1.23%
Managed Savings (by other financial intermediaries)	1.53%	0.31%
Managed Savings by Trust Companies	0.04%	0.01%
Foreign bonds and government securities	0.25%	0.08%
Foreign Stocks and Shares	0.14%	0.03%
Other foreign assets	0.00%	0.00%
Loans to co-operatives	0.80%	0.14%
House		85.05%
Mortgage		-2.07%
Debt		-0.54%

In Table 6, we treat mortgages as negative holdings of long-term bonds (the only long term bonds available are on government debt, BTP) and other debt as negative holdings of corporate bonds (other debt typically has medium term maturity like corporate bonds). On this basis we re-classify our households in 4 mutually exclusive groups. We then show how this classification changes according to the broad region: we follow standard practice and split the country in North West (that includes the three large industrial cities of Milan, Turin and Genoa), the North East (that includes many

middle-sizes cities and towns, such as Bologna, Venice, Verona, Trieste), the Centre (that includes the capital city, Rome, and many medium-sized town such as Florence, Perugia and Ancona) and the South (largely rural, but including Naples and Bari). The two large islands, Sicily and Sardinia, are also counted as South here.

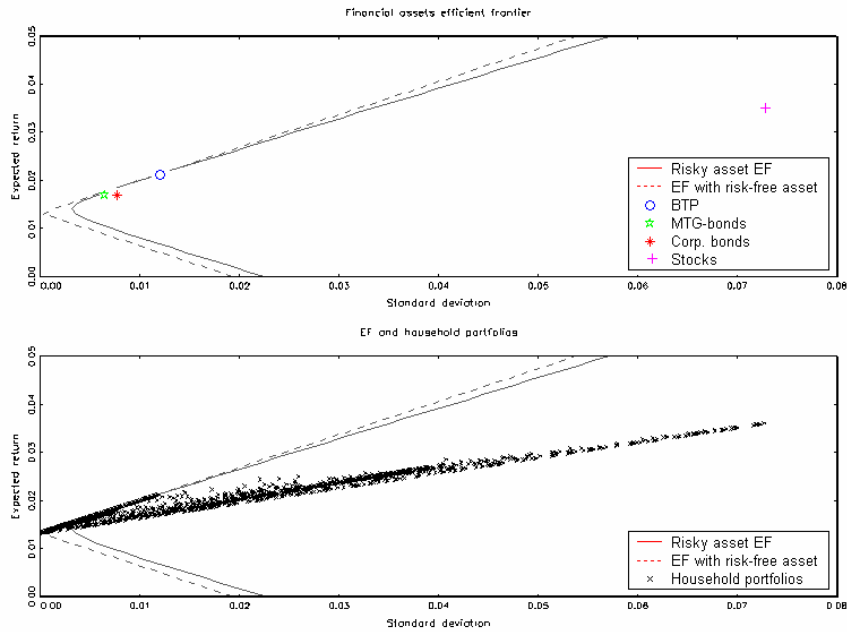
We see that the highest proportion of risk-free asset portfolios (30.1%) is found in the South, the lowest in the Centre (24.2%). The combination of risk-free and housing assets is highest in the South (49.4%), lowest in the North West (33.8%). The combination of risk-free and risky financial assets (included debts) is most common in the North East (5.6%), whereas the presence of all three assets is most common in the North East (36.2%) and least common in the South (only 18.4%).

**Table 6 – Classification by Region.**

	Total		NW		NE		Centre		South	
	n°	%	N°	%	N°	%	n°	%	n°	%
<b>Risk-free asset</b>	1567	26.47%	385	27.13%	217	20.43%	291	24.25%	674	30.10%
<b>Risk-free asset + housing</b>	2499	42.21%	479	33.76%	402	37.85%	511	42.58%	1107	49.44%
<b>Risk-free + risky assets</b>	223	3.77%	78	5.50%	59	5.56%	41	3.42%	45	2.01%
<b>Risk-free + risky assets + housing</b>	1631	27.55%	477	33.62%	384	36.16%	357	29.75%	413	18.45%
<b>Total assets</b>	<b>5920</b>	<b>100%</b>	<b>1419</b>	<b>100%</b>	<b>1062</b>	<b>100%</b>	<b>1200</b>	<b>100%</b>	<b>2239</b>	<b>100%</b>

## 5. Estimation and test results: financial assets portfolios.

First we show mean variance frontier for financial assets alone, using sample averages and variances. We follow the literature and neglect both housing wealth and mortgages and debts. Given that the latter are mostly incurred to purchase housing stock, this is the most natural course of action when analyzing purely financial decisions.



**Figure 1 – Risky financial assets efficient frontiers, efficient frontier with the risk-free asset and households portfolios.**

In the upper panel of Figure 1 we show the risky financial assets efficient frontier and the efficient frontier with the risk-free asset (this is a broken line). Individual assets are also displayed there: to the far right we have stocks (+ sign), to the extreme left of the risky frontier we find corporate bonds (denoted by a \*). In the lower panel we show where individual portfolios lie. Notice that households who have a financial portfolio are 5920 in total: 76.92% of these only have the risk-free asset while 23.08% also have risky assets.

**Table 7: MARKET PORTFOLIO WEIGHTS**

	Weights
BTP	0.2923
MTG Bonds	0.8932
Corporate bonds	-0.2030
Stocks	0.0175

The tangency of the upper portion of the broken line and the risky financial assets financial frontier defines the market portfolio. In Table 7 we report its weights:

the market portfolio is made of long positions in BTP (long-term government bonds) MTG bonds and stocks, and short position in Corporate bonds. In the literature, the short positions have attracted attention (see Jappelli, Julliard and Pagano, 2001), and the argument has been made that one should consider the constrained efficiency frontier, where negative holdings are not allowed. We, however, are interested in testing for efficiency, and for this reason consider here the unconstrained frontier. An extension to the case where there are constraints on some assets is discussed in Section 8.

In Table 8 we report the results of a formal efficiency test (described in Section 3) of observed household portfolios. The test statistic is computed for all valid observations (households whose wealth is not entirely in cash) and the percentages of non-rejections are computed at different values of the test size (from 75% to 95%).

**Table 8. Efficiency Test – Financial assets only**

1-test size	75%		90%		95%	
	N°	%	N°	%	N°	%
<b>Whole sample</b>	5016	84.73%	5166	87.26%	5920	100.00%
<b>Risk-free only</b>	4554	76,92%	4554	76,92%	4554	76,92%

**Note: The table reports the number and % of cases where efficiency is not rejected**

We see from Table 8 that all portfolios containing just the risky assets are (trivially) efficient. Of the remaining portfolios, in one case out of three the efficiency test rejects the null at 75%. Results for those households who hold at least some risky assets are summarised in Table 9: the Table highlights that at most 45% of all risky asset portfolios are efficient (when the size of the test is set at 10%)

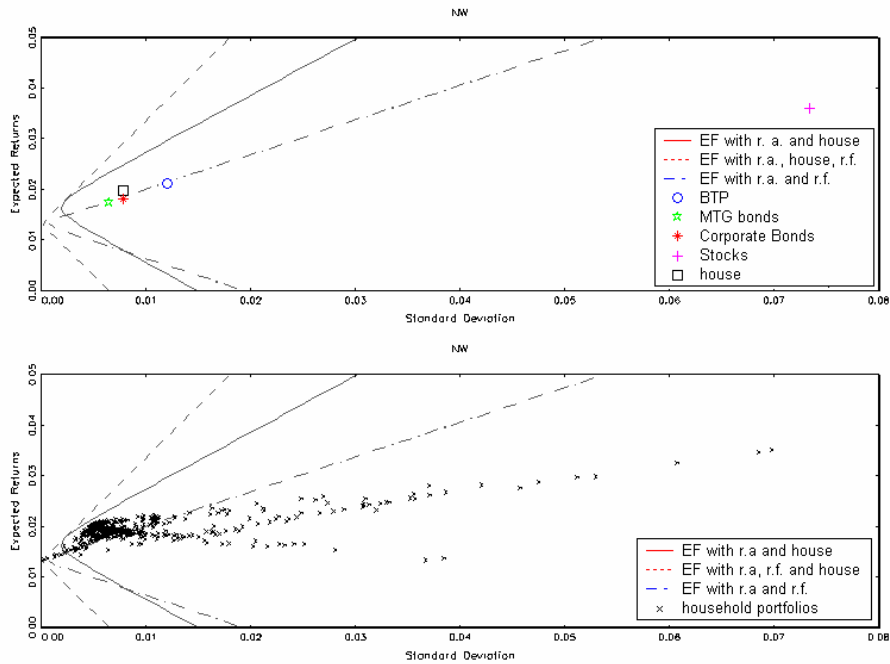
**Table 9. Efficiency Test. Diversified Portfolios only**

1- test size	75%		90%		95%	
	N°	%	N°	%	N°	%
<b>Risky financial assets</b>	462	38.82%	612	44.80%	1366	100.0%



## 6. Estimation and test results: housing and financial assets.

We show in Figures 2-5 the mean variance frontier for financial assets and housing: given that we know where the households live and house prices differ by region, we compute sample averages and variances for each broad region. We now treat outstanding mortgages as negative holdings of long-term bonds (BTP) and debts as negative holdings of medium term (corporate) bonds. In this Section we disregard transaction costs on housing and therefore treat housing as fully unconstrained.



**Figure 2 – North-West Efficient frontiers with housing**

In the upper panel of Figure 2 we show the risky assets efficient frontier and the efficient frontier with the risk-free asset (this is a broken line) for households living in North Western Italy. Individual assets are also displayed there: to the far right we still have stocks (+ sign), to the extreme left of the risky frontier we find MTG bond (denoted by a star) and corporate bonds (denoted by a \*). Just above corporate bonds is housing (denoted by a square). Even though corporate bond seems to be a dominated asset, we know from Tables 1 and 2 that its standard deviation is actually lower than the standard deviation on the house. Also, its highly positive correlation with MTG bonds, BTP and stocks gives its short position some insurance value. This is borne out by the market portfolio weights: as Table 10 shows, the optimal portfolio weight for housing in

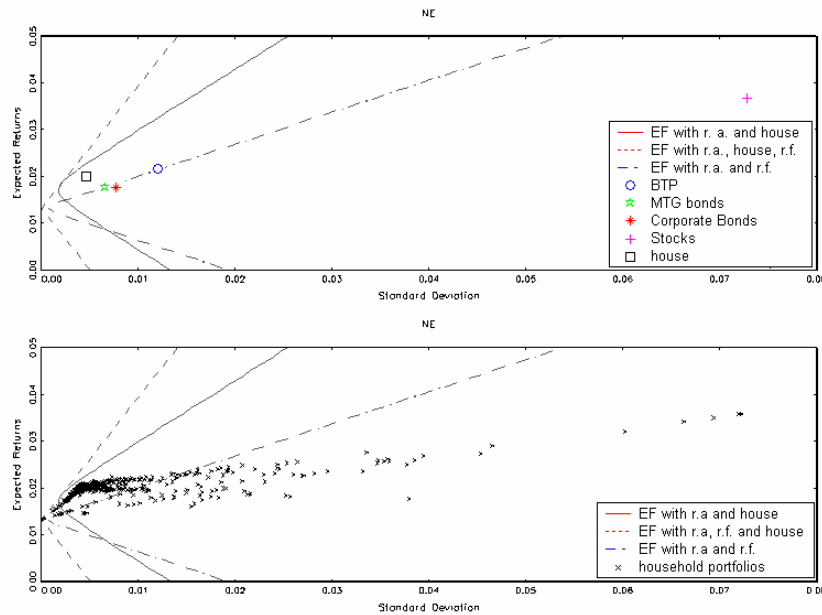
the NW region is 60% (and the BTP weight falls relatively to the purely financial portfolio shown in the first column). This high wealth percentage in housing is of course largely explained by our assumption that housing rental rate is as high as 5% in real terms.

In the lower panel we show where individual portfolios lie. A formal efficiency test is discussed later.

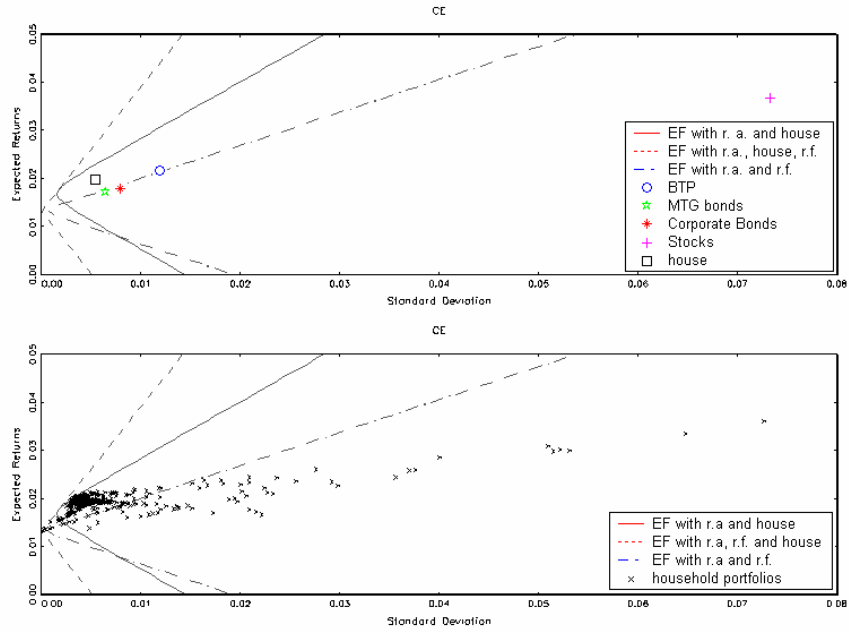
**Table 10: MARKET PORTFOLIO WEIGHTS WITH HOUSING**

	financial assets	NW	NE	Centre	South
<b>BTP</b>	0.2923	-0.5462	-0.3324	-0.3829	1.4222
<b>MTG-bonds</b>	0.8932	1.5346	1.21856	0.7724	-3.099
<b>Corporate bonds</b>	-0.2030	-0.6615	-0.5252	0.0476	1.1269
<b>Stocks</b>	0.0175	0.0735	0.02842	0.0243	-0.052
<b>House</b>		0.5995	0.61071	0.5386	1.6027

Figure 3 displays the efficient frontier for the North East. As we know from Table 2, the figure shows housing investment has higher expected return and lower standard deviation than in the North West. Its optimal portfolio weight is higher (almost 61%).

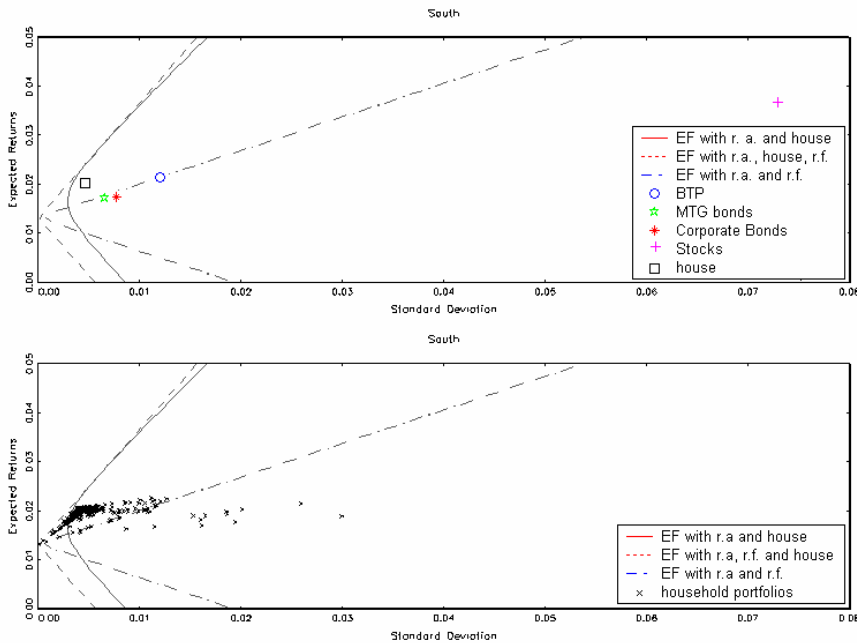


**Figure 3 – North-East Efficient frontiers with housing**



**Figure 4 – Centre Efficient frontiers with housing**

Figure 4 represents the efficient frontier for Central Italy. The house in this case is less attractive as an asset and its optimal portfolio weight is lower than in other macro-areas (54%). For the South the picture is quite different (see Figure 5): the housing expected return is quite large (similarly to North East). As a result the optimal portfolio has an extremely large weight on housing (160%).



**Figure 5 – South Efficient frontiers with housing**

We do not display formal efficiency tests in this case, because the test results are not particularly informative: at any size of the test, the only efficient portfolios are those made just of the risk-free asset (but for a handful of observations: 1 at 90% and 11 at the 95% level).

## **7. Estimation and test results: financial assets conditioning on housing.**

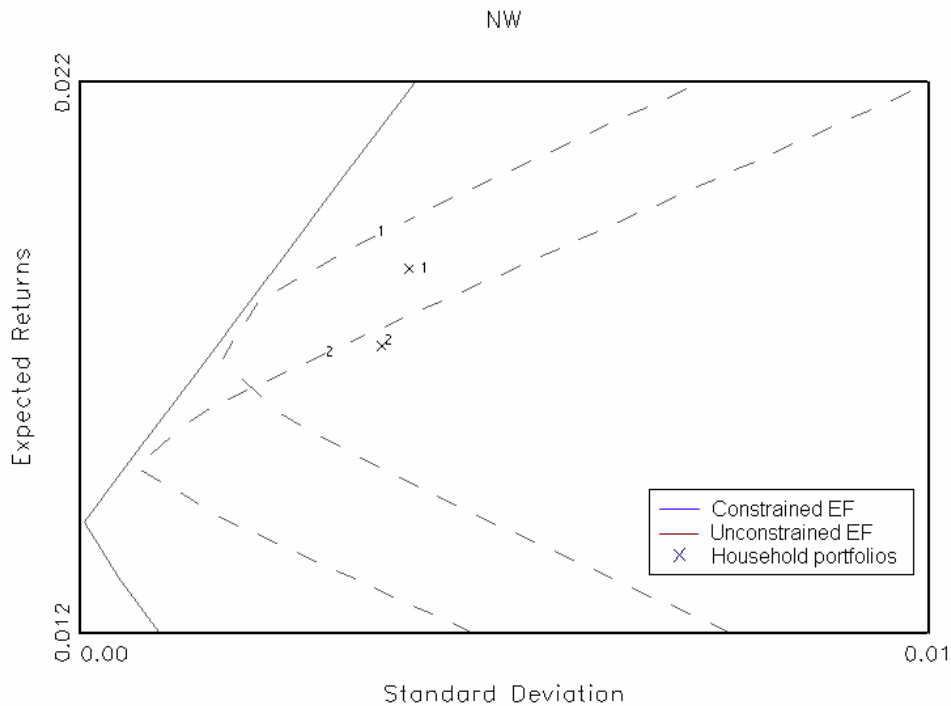
Our results so far can be summarized as follows:

- When we consider only financial assets, household portfolios are mostly trivially efficient (because 76% of the sample hold just the risk free asset). Of the diversified portfolios, at most 45% are mean-variance efficient at the 10% level.
- When we take a broader set of assets and liabilities (housing, mortgages and debt) into consideration, many more households hold diversified portfolios (a common combination is the risk-free asset and housing). However, only a few diversified household portfolios are now found to be efficient.

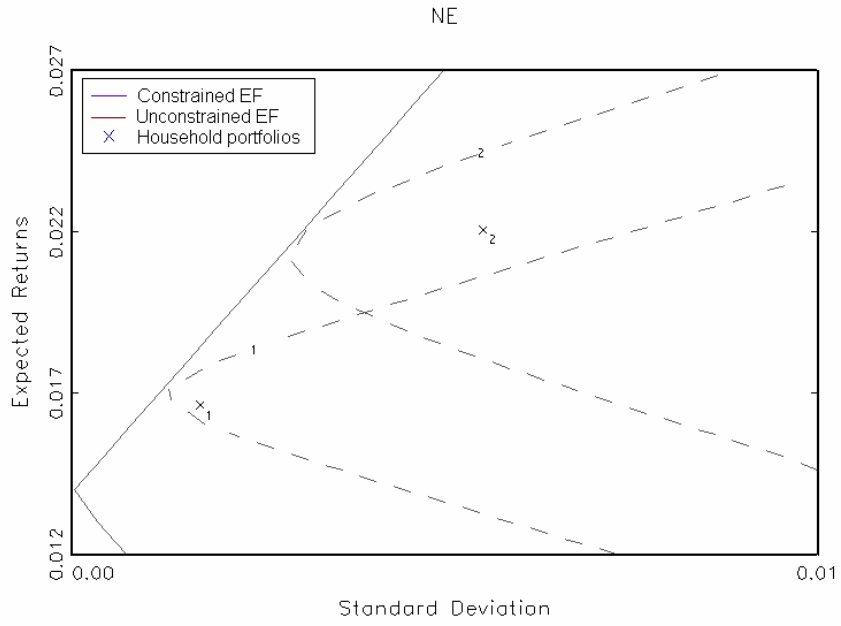
We have already argued (see Section 2) that the illiquid nature of housing should be taken into account. If consumers hold a large fraction of their wealth in housing for reasons other than investment (because rental markets are imperfect, due to information asymmetry, as argued by Henderson and Ioannides, 1983), and do not trade frequently because of high pecuniary and non-pecuniary costs (Flavin, 2002), then we should investigate their portfolio efficiency conditional on housing. It is in fact possible (and plausible) that their financial decisions are partly dictated by the need to hedge some of the risks connected with their illiquid housing investment.

For each household who has non-zero housing wealth we can compute a specific conditional efficiency frontier that treats housing as constrained (for those without housing the frontier displayed in Section 5 still applies). It's worth stressing that in the constrained case the risk-free portfolio cannot be attained, except trivially (zero housing). This explains why the efficient frontiers we display in Figures 6-9 are not broken lines, contrary to what we have in Sections 5 and 6. In each Figure we display the unconstrained and a few constrained frontiers, corresponding to a random sub-sample of house-owners whose actual portfolio is also shown (marked with a plus sign).

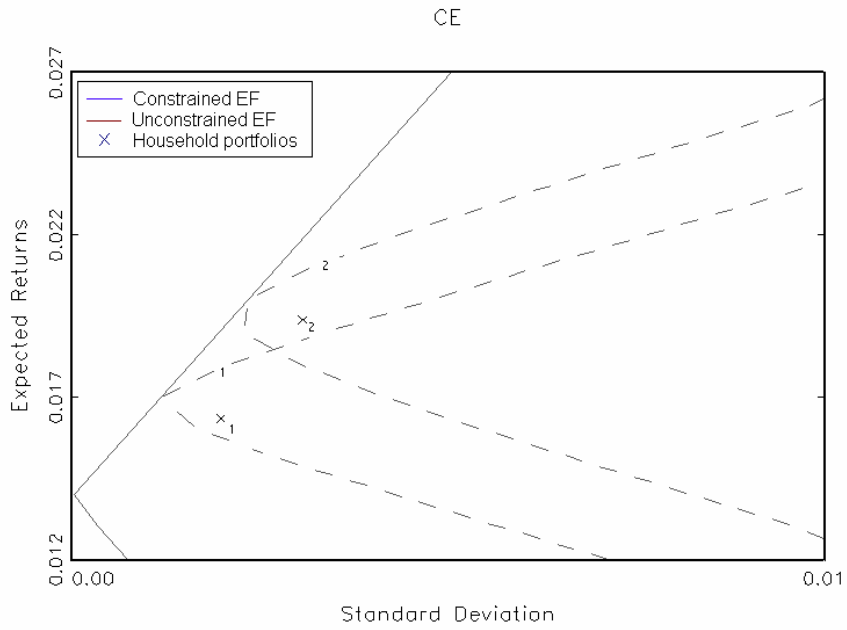
To illustrate, consider Figure 6. This depicts the unconditional frontier with housing for the North West: the presence of a risk-free asset makes it a broken line. In the Figure, we also show two constrained frontiers for the same region, corresponding to two different shares of housing to total wealth (the frontier marked 1 has 18% of wealth into housing; the frontier marked 2 has 47% of total wealth into housing). They correspond to two observed portfolios, displayed as points  $x_1$  and  $x_2$ ). These frontiers lie entirely to the right of the unconstrained frontier (in general, there could be a tangency point, corresponding to the case where the housing portfolio share is at its optimal value). They do not touch the vertical axis, because a risk free position cannot be achieved with positive housing wealth, given the correlations shown in Figure 2.



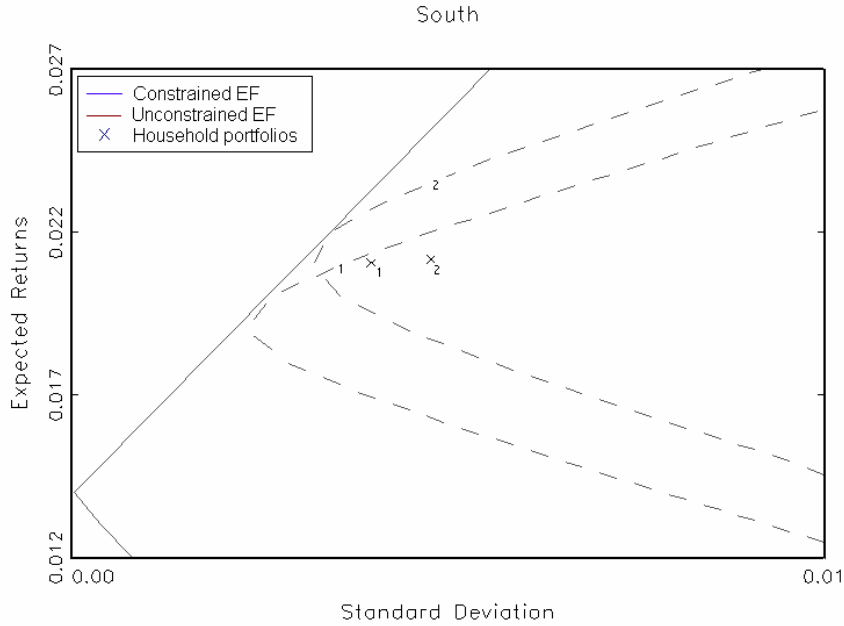
**Figure 6 – North West efficient frontiers conditional on housing**



**Figure 7 – North East efficient frontiers conditional on housing**



**Figure 8 – Centre efficient frontiers conditional on housing**



**Figure 9 – South efficient frontiers conditional on housing**

We can now compute the test statistic for the conditional portfolios,  $\xi_l$  (defined in equation (20), Section 3), and calculate for how many portfolios the test fails to reject the null hypothesis of mean variance efficiency. The test is trivial in the case of portfolios made entirely of risk-free assets (they are all efficient), and is identical to the standard test of Section 6 for portfolios consisting of just financial assets.

Looking at Table 11a, we do find a much smaller number of risk-free portfolios than in Table 8 (1567 instead of 4554). In fact, of those without risky financial assets, house-owners without a mortgage are now classified in the risk-free + house category, house-owners with a mortgage or debt appear in the last category (risk free+ house+ financial assets), because the mortgage is treated as a negative position on long term government bonds. We see from Table 11a that the test fails to reject efficiency in 1843 cases (31.1%) at the 75% significance level, and this number rises to 3551 (60.0%) at the 95% level. In contrast to what we found for financial portfolios (see Table 8), the number of trivially efficient portfolios is now small (1567 households in all, compared to 4554).

If we look at the group of households who have a well-diversified financial portfolio (but no housing), we find that 46.64% of these portfolios are efficient when the test is conducted at the 90% significance level. It's worth stressing that the households that fall in this category are just 223. A straight comparison with Table 9 is hard to make: there 1366 households are classified in the well diversified portfolios group (here we classify those with diversified portfolios and housing into a new category, and take debt into account). Note however that the 104 households that are now classified as efficient in this group were also classified as efficient in Tables 8 and 9.

Let us now consider those cases where there is at least some housing wealth (almost 70% of the sample, see table 6). In the case of risk free+ house portfolios, for a given significance level all portfolios in a region are either efficient or inefficient. We find that all NW and NE such portfolios are inefficient (efficient) at all chosen significance levels. The Centre and South portfolios are classified as inefficient at the 75% and 90% levels, as efficient at the 95% level. It is worth stressing that conditional portfolios made of just the risk-free asset and housing from a given region are either all efficient or all inefficient by construction.

In the more interesting case, where the household holds all asset categories, we find that 192 cases are efficient at the highly conservative 75% significance levels. When we run the test at the more conventional 90% significance level, we find that 27.2% of these households hold efficient portfolios (444 in all, see Table 11b). This number rises to 848 (52.0% of the group) at the 95% level.

**Table 11a. Efficient portfolios conditional on housing**

1- size of test	75%		90%		95%	
	N°	%	n°	%	n°	%
<b>Portfolios</b>						
<b>Risk-free</b>	1567	26.47%	1567	26.47%	1567	26.47%
<b>Risk-free + house</b>	0	0.00%	0	0.00%	913	15.42%
<b>Risk-free + Risky fin. ass.</b>	84	1.42%	104	1.76%	223	3.77%
<b>Risk-free + Risky fin. ass + house</b>	192	3.24%	444	7.50%	848	14.32%
<b>Total</b>	<b>1843</b>	<b>31.13%</b>	<b>2115</b>	<b>35.73%</b>	<b>3551</b>	<b>59.98%</b>



**Table 11b. Efficient portfolios conditional on housing: Diversified portfolios**

1- size of test	75%		90%		95%	
Portfolios	N°	%	n°	%	n°	%
<b>Risk-free + house</b>	0	0.00%	0	0.00%	913	36.53%
<b>Risk-free + Risky fin. Ass.</b>	84	37.67%	104	46.64%	223	100.00%
<b>Risk-free + Risky fin. ass + house</b>	192	11.77%	444	27.22%	848	51.99%
<b>Total</b>	<b>276</b>	<b>6.34%</b>	<b>548</b>	<b>12.59%</b>	<b>1984</b>	<b>45.58%</b>

The efficiency test results displayed in Table 11 suggest that a large proportion of house owners hold portfolios that are not far from their conditional (or constrained) mean variance frontier (this is particularly true for those who hold diversified portfolios).

It is worth stressing that the 548 efficient portfolios we observe, in most of the cases, are not the same as when we consider only financial assets (Table 9). In fact, classification differences arise because of debt and mortgages (that we neglected with financial portfolios). As many as 489 households who were classified as having only the risk-free asset, now hold diversified portfolios (risk free, mortgage or debt and housing). Of these, 235 are conditionally efficient and 254 conditionally inefficient.

Let us now consider the remaining 1142 fully diversified portfolios (risk free, risky financial assets and housing). In Table 12 we cross tabulate diversified financial portfolios and total conditional portfolios according to the efficiency criterion (at the 90% level):

**Table 12. How diversified portfolios are classified: a comparison**

	<b>Efficient (Financial)</b>	<b>Inefficient (Financial)</b>	<b>Total</b>
<b>Efficient (conditional)</b>	<b>95</b>	114	209
<b>Inefficient (conditional)</b>	413	<b>520</b>	933
<b>Total</b>	508	634	1142

We find that as many as 413 portfolios are classified as efficient when housing is neglected, but inefficient when it is considered. In the next section, we shall argue that this reveals that hedging opportunities are not fully exploited. This is partly

compensated by the presence of 114 portfolios for which the reverse holds. We shall argue that this is because housing has diversification properties (for house owners, financial risks are relatively small compared to total wealth). Given the high correlations found (see Table 3) and the very large weight attached to housing wealth, the failure to exploit hedging opportunities outweighs the benefits from diversification, and the number of conditionally efficient portfolios (209) is smaller than the number of efficient financial portfolios (508).

It's worth stressing that the estimated coefficients in Table 3 are the relevant indicators of the way hedging should be performed. For instance, in three regions out of four, more should be invested in MTG bonds compared to the market portfolio weights displayed in Table 7 (the exception is the South).

In Table 13 we display efficiency results by region: we see that a higher proportion of efficient portfolios obtains in the North West and in the South. However, in the North West this is due to the relatively large fraction of efficient diversified portfolios, whereas in the South this is achieved because the large proportion of portfolios made of the risk-free asset alone.

**Table 13. Efficient portfolios conditional on housing by region**

<b>1- size of test= 90%</b>	<b>NW</b>		<b>NE</b>		<b>Centre</b>		<b>South</b>	
<b>Portfolios</b>	<b>N°</b>	<b>%</b>	<b>n°</b>	<b>%</b>	<b>n°</b>	<b>%</b>	<b>N°</b>	<b>%</b>
<b>Risk-free</b>	385	27.13%	217	20.43%	291	24.25%	674	30.10%
<b>Risk-free + house</b>	0	0.00%	0	0.00%	0	0.00%	0	0.00%
<b>Risk-free + Risky fin. Ass.</b>	30	2.11%	23	2.17%	17	1.42%	34	1.52%
<b>Risk-free + Risky fin. Ass + house</b>	148	10.43%	104	9.79%	94	7.83%	98	4.38%
<b>Total</b>	<b>563</b>	<b>39.68%</b>	<b>344</b>	<b>32.39%</b>	<b>402</b>	<b>33.50%</b>	<b>806</b>	<b>36.00%</b>

The question more generally arises of what makes a household more likely to hold an efficient portfolio. To address this question we run a simple probit regression explaining the efficiency test result (at the 90% level) to observable household characteristics such as age, education (secondary junior school degree, high school

degree or graduate) and employment position (employee, entrepreneur, retired or unemployed) of the head, region, household income and wealth.

It is worth recalling that efficient portfolios are either made of the risk-free asset alone, or include housing wealth as well as financial assets (see Table 11). Clearly, these results are highly affected by the wide-spread presence of efficient portfolios characterized only by risk-free assets (indeed in the sample we observe that most of the unemployed hold only the risk-free asset), and by home-ownership.

In view of the consideration that for purely financial portfolios test results are mostly trivial (all risk-free portfolios are efficient), we focus attention on the more interesting group of house owners with risky financial assets. In Table 14 we report the results of the PROBIT regression: the sample size has been reduced from 5920 to 1617 (the dependent variable takes value 1 if the observed portfolio is classified as efficient – 14 observations were dropped because of missing values).

**Table 14. Results of PROBIT regression conditional on housing and risky financial assets (1 = efficient at 90%; 0 = inefficient )**

<b>Observations: 1617</b>				
Variable	Coefficient	Std. Error	z-Statistic	Prob.
<b>Constant</b>	27.16039	6.444539	4.214481	0.0000
<b>AGE-40</b>	-0.003856	0.005604	-0.688047	0.4914
<b>(AGE-40)^2</b>	-7.59E-05	0.000178	-0.427009	0.6694
<b>SEC.J.SCHOOL</b>	-0.274730	0.112769	-2.436210	0.0148
<b>HIGH SCHOOL</b>	-0.244461	0.106534	-2.294663	0.0218
<b>LOG(INCOME)</b>	1.691417	0.952572	1.775632	0.0758
<b>LOG(INCOME)^2</b>	-0.073613	0.044694	-1.647053	0.0995
<b>EMPLOYEE</b>	-0.013212	0.244321	-0.054077	0.9569
<b>ENTREPRENEUR</b>	-0.038727	0.253244	-0.152923	0.8785
<b>RETIRED</b>	-0.035138	0.252832	-0.138979	0.8895
<b>NW</b>	0.439936	0.099275	4.431499	0.0000
<b>NE</b>	0.339120	0.106745	3.176921	0.0015
<b>CENTRE</b>	0.324256	0.107115	3.027181	0.0025
<b>LOG(WEALTH)</b>	-5.540209	0.853464	-6.491437	0.0000
<b>LOG(WEALTH)^2</b>	0.203123	0.033919	5.988517	0.0000
Mean dependent var	0.270872	S.D. dependent var	0.444548	
S.E. of regression	0.422007	Akaike info criterion	1.084825	
LR statistic (12 df)	164.8954	McFadden R-squared	0.087290	

We see that not all the variables considered are statistically significant: the variables that are statistically significant are region of residence, wealth (and to a lesser extent income) and education. The efficiency index decreases if the head lives in the South, or has less than a college degree. The probability of being classified as efficient at first decreases with wealth, but then increases (the reverse holds true for income).

## 8. Discussion of empirical results and extensions

The empirical results presented in Section 7 must be interpreted with special care. In fact, the result that the test statistic

$$\xi_1 = T \frac{\hat{S}_1 - \hat{S}_1(Z)}{1 + \hat{S}_1(Z) \frac{Z^T \Omega Z}{v_1^T \Sigma v_1}} \xrightarrow{d} \chi_{n_1-1}^2$$

holds independently of the properties of the  $\Omega$  matrix.

An interesting special case derives when this matrix is block diagonal. In this case, we know from Flavin's analysis that the optimal portfolio of financial assets is the same as in the standard Markowitz case. However, the test statistic does not simplify to the Sharpe test applied to financial assets alone: the denominator involves the variance of the housing return multiplied by the square of its share in total wealth.

$$\xi_1^* = T \frac{(\hat{S}_1 - \hat{S}_1(Z))}{1 + \hat{S}_1(Z) \frac{1}{\underline{x}_0^T \Sigma \underline{x}_0} (\underline{x}_0^T \Sigma \underline{x}_0 + \underline{h}_0^T \sigma_H^2 \underline{h}_0)} \xrightarrow{d} \chi_{n_1-1}^2$$

By construction, this test statistic is lower than the standard Sharpe statistic for financial assets alone, unless the housing share,  $h_0$ , is zero, or housing is risk-less. This is because this statistic tests for the efficiency of the whole portfolio, conditional on one asset being given, and this is conceptually different from testing for efficiency of the allocation of unconstrained assets, even when the optimal financial portfolio does not include a hedge term.

The situation can arise where financial portfolios are inefficient according to a standard Sharpe test, and yet the corresponding overall portfolios (including housing) are found to be efficient even when housing is treated as given and there is zero covariance between housing and all financial assets. This is hardly surprising: if housing wealth is a large fraction of total wealth, the inefficiency of the financial wealth portfolio is relatively minor, compared to the diversification benefits that derive from the existence of this other form of wealth.

We know from Section 3 that for most Italian regions the  $\Omega$  matrix is not block-diagonal, but in others (notably, Southern Italy) the covariance between financial assets and housing is less marked. It is therefore possible that the diversification effect may be playing a prominent role in at least the South.

To assess the relative importance of the diversification effect we have tried a simple exercise: in all four regions we have set the covariance terms to be zero. Test results are reported in Tables 15a (all portfolios) and 15b (diversified portfolios). We see that now all portfolios that contain only the risk free asset and housing are classified as efficient at all significance levels: when the hedge motive disappears, conditional upon housing, risk-free portfolios are trivially efficient (even though the overall portfolio has positive variance). We recall that this does not happen with our estimated  $\Omega$  matrix (Table 11).

Of greater interest to us is the fact that more fully diversified portfolios are efficient at all significance levels (for instance: 1347 instead of 444 at the 90% level). This suggests that the hedge motive is not widely taken into consideration when households make their portfolio choice. Thus many household portfolios that would be classified as (constrained) efficient if the correlation between housing return and financial assets returns were zero, are instead inefficient.

**Table 15a. Efficient portfolios conditional on housing (block-diagonal  $\Omega$ )**

I- size of test	75%		90%		95%	
	N°	%	n°	%	n°	%
<b>Risk-free</b>	1567	26.47%	1567	26.47%	1567	26.47%
<b>Risk-free + house</b>	2499	42.21%	2499	42.21%	2499	42.21%
<b>Risk-free + Risky fin. Ass.</b>	84	1.42%	104	1.76%	223	3.77%
<b>Risk-free + Risky fin. ass + house</b>	1064	17.97%	1347	22.75%	1584	26.76%
<b>Total</b>	<b>5214</b>	<b>88.07%</b>	<b>5517</b>	<b>93.19%</b>	<b>5873</b>	<b>99.21%</b>

**Table 15b. Efficient portfolios conditional on housing (block-diagonal  $\Omega$ ): Diversified portfolios**

I- size of test	75%		90%		95%	
	N°	%	n°	%	n°	%
<b>Risk-free + house</b>	2499	100.00%	2499	100.00%	2499	100.00%
<b>Risk-free + Risky fin. Ass.</b>	84	37.67%	104	46.64%	223	100.00%
<b>Risk-free + Risky fin. ass + house</b>	1064	65.24%	1347	82.59%	1584	97.12%
<b>Total</b>	<b>3647</b>	<b>83.78%</b>	<b>3950</b>	<b>90.74%</b>	<b>4306</b>	<b>98.92%</b>

Another issue worth considering is the effect of differential underreporting. We know from D'Alessio and Faiella (2000) that SHIW98 underestimates financial wealth by a wide margin (it accounts only for a third of aggregate household financial wealth), whereas housing wealth is in line with aggregate statistics. The reasons why financial wealth falls short of aggregate statistics can be non-response among the rich and under-reporting among those who do respond. To assess whether the latter has an important impact on our test, we take the extreme case where differential non-response is not an issue, multiply all financial wealth holdings by a factor of three and re-run the test.

Tables 16a and 16b display efficiency test results in this hypothetical case, where all households report the same fraction of their financial wealth. If we compare these results with those in Tables 11a and 11b, we see that more fully diversified portfolios are counted as efficient (for instance: 718 instead of 444 at the 90% level). This increase is in line with expectations (the hedge motive is relatively less important if housing wealth has a lower portfolio weight) and is quite sizeable (now 44% of fully diversified portfolios are efficient, rather than 27%). It is worth stressing that its overall impact is limited, because it affects solely the group of fully diversified portfolios.

**Table 16a. Efficient portfolios conditional on housing – corrected for under-reporting**

I- size of test	75%		90%		95%	
	N°	%	n°	%	N°	%
<b>Risk-free</b>	1567	26.47%	1567	26.47%	1567	26.47%
<b>Risk-free + house</b>	0	0.00%	0	0.00%	913	15.42%
<b>Risk-free + Risky fin. ass.</b>	84	1.42%	104	1.76%	223	3.77%
<b>Risk-free + Risky fin. ass + house</b>	397	6.71%	718	12.13%	1174	19.83%
<b>Total</b>	<b>2048</b>	<b>34.59%</b>	<b>2389</b>	<b>40.35%</b>	<b>3877</b>	<b>65.49%</b>

**Table 16b. Efficient portfolios conditional on housing corrected for under-reporting : Diversified portfolios**

I- size of test	75%		90%		95%	
	N°	%	n°	%	N°	%
<b>Risk-free + house</b>	0	0.00%	0	0.00%	913	36.53%
<b>Risk-free + Risky fin. Ass.</b>	84	37.67%	104	46.64%	223	100.00%
<b>Risk-free + Risky fin. ass + house</b>	397	24.34%	718	44.02%	1174	71.98%
<b>Total</b>	<b>481</b>	<b>11.05%</b>	<b>822</b>	<b>18.88%</b>	<b>2310</b>	<b>53.07%</b>

Finally, in this paper we have ignored non-negativity constraints on asset holdings. In the case of long-term bonds and of corporate bonds this is not a problem for us: we assume negative holdings are allowed in the form of mortgages or other consumer debt. The same applies for the risk free asset, if we are willing to treat informal debt (from friends and relatives) or formal, variable interest debt (as in some mortgage contracts) as negative holdings. However, even in our framework it is hard to explain how consumers can take negative positions in medium term government bonds or stocks.

The efficiency of household portfolios cannot be assessed in this context using the Gouriéroux Jouneau test, but a new test has been proposed by Basak, Jagannathan and Sun (2002) that allows for non-negativity constraints (and can be adapted to treat housing as given)<sup>13</sup>. Preliminary results suggest that non-negativity constraints have a minor impact on the empirical analysis. This is not entirely surprising, because short positions on medium term government bonds or stocks do not appear in the Markowitz optimal portfolio (as shown in Table 7) and these assets have negative partial correlation with housing returns in all regions but the South (see Table 3).

## 9. Conclusions

In this paper we have argued that standard tests of portfolio efficiency are biased because they neglect the existence of illiquid wealth. In the case of household portfolios, the most important illiquid asset is housing: if housing stock adjustments are costly and therefore infrequent, optimal portfolios in periods of no adjustment are affected by housing price risk.

We have shown that, if financial assets' and housing returns are correlated, the intertemporal expected utility model subject to transaction costs in housing investment implies that financial decisions are affected by the need to hedge some of the risks connected with their illiquid housing investment. In particular, we have demonstrated that the investors' optimal strategy is to choose the standard Markowitz portfolio according to their risk aversion and use the risky financial assets to hedge their expositions on the constrained asset (this last decision is independent of their risk aversion). This hedging motive disappears in the case of zero correlation between housing return and financial returns, as pointed out by Flavin (2002), in which case housing price risk only affects the investor's degree of risk aversion.

We have also shown that the optimal investment in the risky financial asset is equal to the one derived in a static mean-variance analysis framework, if the existing housing stock is treated as an additional constraint to the optimization problem. Gouriéroux and Jouneau (1999) have proposed such a test for analyzing the performance of a portfolio of risky assets (in a mean-variance framework) when some constraints exist on a part of the portfolio. We are then able to claim that this test can be applied for a more general test of portfolio efficiency.

In our application, we have used household portfolio data from the 1998 SHIW and time series data on financial asset and housing stock returns to assess whether actual portfolios are efficient. We first consider purely financial portfolios and portfolios that also treat the housing stock as another asset. We then consider the consequences of treating the housing stock as given and test for efficiency in this framework.

Our empirical results support the view that the presence of illiquid wealth plays an important role in determining whether portfolios chosen by home-owners are efficient.

Our results can be summarized as follows:



- When we consider only financial assets, three portfolios out of four are made of just the risk free asset, and therefore trivially efficient. However, a large fraction (45%) of the diversified portfolios are mean-variance efficient;
- When we take a broader set of assets and liabilities (housing, mortgages and debt) into consideration, many more households hold diversified portfolios (a common combination is the risk-free asset and housing). But very few diversified household portfolios are found to be efficient.
- When we calculate the efficiency test conditional on housing we find that one in three of all fully diversified portfolios (that include the risk-free asset, housing and risky assets) are mean-variance efficient. However, these are largely not the same households whose financial portfolios were considered efficient. This mismatch is due to the interplay of a the hedging effect (financial assets should be used to hedge housing price risk; if they are not, this may lead to inefficiency) and the diversification effect ( that plays a less relevant role).
- When we consider how our results would change if the hedge motive disappeared or became less important, we find that more portfolios would appear to be efficient. This is due to the diversification effect: given that financial wealth is a small part of total wealth, its incorrect allocation needs not make the overall portfolio inefficient. This also suggests that most households do not currently use well the existing hedging opportunities for house price risks.

In summary, compared to the efficiency results relating to portfolios consisting solely of financial assets such as stocks, bonds and BOT, the introduction of housing and mortgage alters the risk and return trade-off in a direction which pushes fewer household portfolios to be efficient. This is no longer the case, once the illiquid nature of housing investment is taken into account, but there is evidence that hedging opportunities are not fully exploited even by those Italian households who hold well-diversified portfolios. This widespread failure to hedge house price risk has important implications for portfolio management.

## References

- Anderson, Ronald W., and Jean-Pierre Danthine, 1981, 'Cross Hedging', *Journal of Political Economy*, **89**(6), 1182-96
- Basak, Gopal, Ravi Jagannathan and Guoqiang Sun, 2002, 'A Direct Test for the Mean-Variance Efficiency of a Portfolio', *Journal of Economic Dynamics and Control*, **26**, 1195-1215.
- Blake, David, 1996, 'Efficiency, Risk Aversion and Portfolio Insurance: An Analysis of Financial Asset Portfolios Held By Investors in the United Kingdom', *The Economic Journal*, **106**, pp. 1175-1192
- Brugiavini, Agar and Elsa Fornero, 2001, "Pension Provision in Italy", in Richard Disney and Paul Johnson (eds.) Pension Systems and Retirement Income across OECD Countries, London: Edward Elgar, pp. 197-231
- Cocco, João F., 2001, 'Hedging House Price Risk with Incomplete Markets', LBS mimeo.
- D'Alessio, Giovanni and Ivan Faiella, 2000, "Italian Household Budgets in 1998", Supplements to Statistical Bulletin, Methodological Notes and Statistical Information, no. 22, Rome: Banca d'Italia (available at <http://www.bancaditalia.it>).
- De Roon, Frans, Piet Eichholtz and Kees Koedijk, 2002, "The Portfolio Implications of Home Ownership", CEPR Discussion Paper n. 3501
- Flavin, Marjorie, 2002, 'Owner-Occupied Housing in the Presence of Adjustment Costs: Implications for Asset Pricing and Non-Durable Consumption', UCSD mimeo
- Flavin, Marjorie and Takashi Yamashita, 2001, 'Owner-Occupied Housing and the Composition of the Household Portfolio over the Life Cycle', *American Economic Review*, forthcoming (also: NBER WP 6389)
- Gibbons, M., S. Ross and J. Shanken (1989) 'A test of Efficiency for a Given Portfolio', *Econometrica*, **57**, pp. 1121-1152
- Goetzmann, William N., and Roger Ibbotson (1990) "The Performance of Real Estate as an Asset Class." *Journal of Applied Corporate Finance*, 3(1), pp. 65-76.
- Goetzmann, William N. (1993) "The Single Family Home in the Investment Portfolio." *Journal of Real Estate Finance and Economics*, **6**(3), pp. 201-22.
- Gollier, Christian (2002) 'What Does the Classical Theory Have to Say about Household Portfolios?', chapter 2 in Guiso, Haliassos and Jappelli (eds.) Household Portfolios, Cambridge: MIT Press, pp. 27-54

- Grossman, Sanford J. and Guy Laroque (1990) 'Asset Pricing and Optimal Portfolio Choice in the Presence of Illiquid Durable Consumption Goods', *Econometrica*, **58(1)**, pp. 25-51
- Gourieroux, Christian and Francois Jouneau (1999) 'Econometrics of Efficient Fitted Portfolios', *Journal of Empirical Finance*, **6**, pp. 78-118
- Guiso, Luigi and Tullio Jappelli (2002) 'Household Portfolios in Italy', chapter 6 in Guiso, Haliassos and Jappelli (eds.) Household Portfolios, Cambridge: MIT Press, pp. 251-290
- Guiso, Luigi, Michael Haliassos and Tullio Jappelli (2002) Household Portfolios, Cambridge: MIT Press.
- Jobson J. D., and Robert Korkie (1982) 'Potential Performance and Tests of portfolio Efficiency', *Journal of Financial Economics*, **10**, 433-466
- Jobson J., and Robert Korkie (1989) 'A Performance Interpretation of Multivariate Tests of Assets Set Intersection, Spanning and Mean Variance Analysis', *Journal of Financial and Quantitative Analysis*, **24**, 183-204
- Haliassos, Michael and Carol C. Bertaut, 1995, Why so few hold stocks?, *Economic Journal*, **105**, pp. 1110-1129
- Heaton, John and Deborah Lucas (2000) 'Portfolio Choice and Asset Prices: The Importance of Entrepreneurial Risk', *Journal of Finance*, **55**, pp. 1163-1198
- Henderson, J. Vernon, and Yoannis Ioannides (1983) 'A Model of Housing Tenure Choice', *American Economic Review*, **73(1)**, pp. 98-113
- Jappelli, Tullio, Christian Julliard and Marco Pagano (2001) 'La diversificazione del portafoglio delle famiglie italiane', mimeo
- Markowitz, Harry (1992) Mean Variance Analysis in Portfolio Choice and Capital Markets, Oxford: Basil Blackwell.
- Ross, Stephen A. and Randall C. Zisler (1991) 'Risk and Return in Real Estate', *Journal of Real Estate Finance and Economics*, **4(2)**, pp. 175-90.
- Samuelson, Paul (1970) 'The Fundamental Approximation Theorem of Portfolio Analysis in Terms of Means, Variances and Higher Moments', *The Review of Economic Studies*, **37**, pp. 537-542

## APPENDIX – Derivation of equation (9)

We show here how the asset allocation rule of equation (9) can be derived by suitably extending Flavin's (2002) analysis. We take as given the result that there are finite periods of time when the household decides not to adjust the housing stock, because the benefit from adjusting is smaller than the transaction costs incurred. Our derivations hold for such periods of inaction.

Suppose that at time  $t=0$ , the household decides that it is not optimal to change the housing stock immediately. During a time interval  $(0,s)$  when the possibility of such change is negligible, wealth evolves according to:

$$(A1) \quad dW_t = \left[ P_t H_0 (\mu_H + r_f) + \underline{X}_t (\underline{\mu} + r_f) + r_f B_t - C_t \right] dt + \underline{X}_t d\omega_{Ft} + P_t H_0 d\omega_{Ht}$$

or, rewriting in order to eliminate the term representing risk-free bonds,

$$(A2) \quad dW_t = \left[ r_f W_t + P_t H_0 \mu_H + \underline{X}_t \underline{\mu} - C_t \right] dt + \underline{X}_t d\omega_{Ft} + P_t H_0 d\omega_{Ht}$$

Let  $V(H, W)$  denote the supremum of household expected utility be twice continuously differentiable, conditional on the current values of the state variables  $(H, W)$ , the Bellman's principle of optimality can be stated as:

$$(A3) \quad V(H_0, W_0) = \sup_{\{\underline{X}_t, \{C_t\}\}} E \left[ \int_0^s e^{-\delta t} u(H_0, C_t) dt + e^{-\delta s} V(H_0, W_s) \right]$$

subject to the budget constraint (10) and the process for house prices (5). The term inside the brackets intuitively represents the sum of the rewards on the interval  $(0,s)$  and the maximized expected value by proceeding optimally on the interval  $(s,\infty)$  with the system started at time  $s$  in state  $(H_0, W_s)$ <sup>14</sup>

Subtracting  $V(H_0, W_0)$ , dividing by  $s$  and taking the limit as  $s \rightarrow 0$  gives:

$$(A4) \quad 0 = \lim_{s \rightarrow 0} \sup_{\{\underline{X}_t, \{C_t\}\}} E \left[ \frac{1}{s} \int_0^s e^{-\delta t} u(H_0, C_t) dt + \frac{1}{s} (e^{-\delta s} V(H_0, W_s) - V(H_0, W_0)) \right]$$

Evaluating the integral and using Ito's lemma, equation (12) can be rewritten as:

$$(A5) \quad 0 = \sup_{\underline{X}_0, C_0} \left\{ u(H_0, C_0) - \delta V(H_0, W_0) + \frac{\partial V}{\partial W} (r_f W_0 + P_0 H_0 \mu_H + \underline{X}_0 \underline{\mu} - C_0) \right. \\ \left. + \frac{\partial V}{\partial P} P_0 \mu_H + \frac{1}{2} \frac{\partial^2 V}{\partial W^2} \left( \underline{X}_0 \Sigma \underline{X}_0^T + P_0^2 H_0^2 \sigma_P^2 + 2P_0 H_0 \underline{X}_0 \Gamma_{bP} \right) \right\}$$

<sup>14</sup> We assume that the transversality condition holds such that  $V(H_0, W_s)$  is bounded.

that is:

$$(A6) \quad 0 = \sup_{C_0} \left\{ u(H_0, C_0) - C_0 \frac{\partial V}{\partial W} \right\} - \delta V(H_0, W_0) + \frac{\partial V}{\partial W} (r_f W_0 + P_0 H_0 \mu_H) \\ + \frac{1}{2} \frac{\partial^2 V}{\partial W^2} P_0^2 H_0^2 \sigma_P^2 + \sup_{\underline{X}_0} \left\{ \frac{\partial V}{\partial W} \underline{X}_0 \underline{\mu} + \frac{1}{2} \frac{\partial^2 V}{\partial W^2} (\underline{X}_0 \Sigma \underline{X}_0^T + 2P_0 H_0 \underline{X}_0 \Gamma_{bp}) \right\}$$

Non-durable consumption satisfies the usual first order condition:

$$(A7) \quad \frac{\partial u}{\partial C} = \frac{\partial V}{\partial W}$$

and the necessary first order conditions for the investment in risky financial assets,

$\underline{X}_0$ , is:

$$(A8) \quad \frac{\partial V}{\partial W} \underline{\mu} + \frac{\partial^2 V}{\partial W^2} (\underline{X}_0 \Sigma + P_0 H_0 \Gamma_{bp}) = 0$$

Equation (9) follows from this last set of equations.