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ABSTRACT

Optimal Financial Market Integration and Security Design*

We study two-period pure-exchange Capital Asset Pricing Model (CAPM) economies, for given degrees of incompleteness of financial markets and given degrees of restricted participation of agents in the markets. We characterize the optimal financial market structure of this economy, as well as efficient financial innovations consisting of both the introduction of new assets and the integration of segmented markets. We show that the optimal financial market structures maximize a simple criterion that captures the dispersion of the 'betas' of the participating agents. Our results imply that, in contrast with the prescription of the Optimal Currency Area literature, maximal advantage from the integration of financial markets arises for those economic regions whose endowment processes are minimally or negatively correlated. Moreover, we show that some coordination of the innovation process, e.g., in the form of consolidation of exchanges, is likely to be desirable when the integrating economies have segmented asset markets.

JEL Classification: D52, D58 and G12
Keywords: CAPM, decentralizability, financial integration, incomplete markets, restricted participation and security design

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1 Introduction

Innovations in financial markets have taken different forms, from the introduction of new assets, to the integration of segmented markets. The list of new financial assets which have been introduced in the last few decades includes notably options, swaps, inflation-indexed bonds, and various other derivatives such as catastrophe bonds and credit protections (see Allen and Gale, 1994-a for a historical review). But many recent financial innovations represent instances of integration of segmented markets: the introduction of electronic trading has, for instance, reduced the transaction costs for small brokers and retail investors, thereby integrating their trades in many national financial exchanges (Naik and Yadav, 1999), and the practice of securitization has allowed liabilities such as mortgages, credit card debt, and local bank debt, originally held by specific classes of creditors, to be traded by national financial markets (Kendall and Fishman, 1998).[1] Also, the recent examples of integration of international commerce among the OECD countries (Wei, 1996) have led to the integration of previously segmented financial markets as evidenced by a decline in the "home bias" (see Mann and Meade, 2002).

The theoretical literature on financial innovation has successfully analyzed the effects of the introduction of new assets to span the uncertainty of agents' endowments and production plans in economies with incomplete financial markets (see the surveys in Allen and Gale, 1994-a, and Duffie and Rahi, 1995). For instance, the risk sharing rationale for the introduction of derivative securities and their effect on market volatility are well understood in finance.[2] The literature has, however, paid much less attention to the limited participation in financial markets and to the integration of segmented markets as a form of financial innovation.[3] This is true also for the study of innovations in international financial markets; for instance, Wincoop (1994), Athanasoulis and Shiller (1999), and Davis, Nalewaik and Willen (2001), have mostly dealt with the introduction of new assets rather than with the

[1]Indeed, the empirical evidence for the segmentation of financial markets is ample. Blume and Friend (1975), Blume and Zeldes (1993), and Mankiw and Zeldes (1991) document the fact that investors within an economy do not seem to hold the entire market portfolio, and, in fact, hold a very small number of individual securities. French and Poterba (1991), Kang and Stulz (1997), and Lewis (1999) document the "home bias puzzle," which is a form of limited participation by investors in foreign securities.

[2]Damodaran and Subrahmanyam (1992) provide a detailed summary of the theoretical and the empirical literature concerning the effect of derivative securities on markets for underlying assets.

[3]The effect of limited participation on financial markets has been examined in a few studies such as Allen and Gale (1994-b), who study volatility of asset prices in limited participation regimes vis-a-vis full participation regimes; Diamond (1997), who proposes restricted participation as an explanation for the existence of financial intermediaries; Basak and Cuoco (1998), who develop a continuous-time pure-exchange economy with restricted participation in the stock market; and Heaton and Lucas (1999), who show that an increase in stock market participation along with greater ability to diversify can at least partially explain observed price run-ups.

integration of segmented financial markets. Even most of the theoretical and empirical analysis of Optimal Currency Areas, including Helpman and Razin (1982), Neumeyer (1998), the recent survey by Ching and Devereux (2000), and Alesina, Barro and Tenreyro (2002), has concentrated on the effect of trade liberalizations and currency unions for economies already integrated in terms of their financial markets. Such limited attention to issues specifically of international financial integration is at odds with the reality of integration processes; in the recent European integration experience, for example, the formation of the currency union has led to cross-border operations by banks and financial institutions of member countries and to a movement towards the consolidation of national exchanges ultimately aimed at forming a pan-European stock exchange.

In this paper we study financial innovations consisting of both the introduction of new assets, and, notably, of the integration of segmented markets. This will allow us to study the interactions between these two forms of innovations. Our analysis addresses several interesting issues concerning the optimality of financial market structures, financial innovations, and the process of integration. We discuss and illustrate these issues below through a leading example economy.

Consider an economy composed of large investors such as firms and banks, and individual investors such as retail households. Large investors have unrestricted access to financial markets, while individual investors have access only to local markets. Local markets could be specialized financial markets, or geographically restricted markets; but they could also represent national markets in the context of international financial markets. Trade in local markets is organized through exchanges.

Suppose diseconomies of scale prevent local markets from fully integrating the trades of all investors in the economy. Diseconomies of scale can be due to monitoring costs, or to the costs of setting up an integrated clearing system. Also, in international financial markets diseconomies of scale are related to the difficulty of policy coordination in a currency union, or to the costs of social and economic policies in culturally heterogeneous societies. Suppose also that complete financial markets are not feasible, say because of transaction costs. Therefore, in this economy financial markets are incomplete and segmented, and exchanges have incentives to innovate by partially integrating their trades and by introducing new financial assets.

We first characterize the optimal structure of financial assets and the optimal level of integration of markets. What is the optimal structure of assets to be traded in the economy, or, in other words, which collection of assets provides the best risk sharing opportunities and the highest welfare in the economy? What is an optimal asset that maximizes the risk sharing opportunities and what should its payoff be correlated with? What do the welfare gains from integrating local financial markets depend upon? Which markets are best to integrate?

We show that the optimality of financial structures, in an economy in which lump-sum transfers compensate agents for negative relative price effects, is determined only by the 'betas' associated with the financial structures, i.e., by the lists of the covariances of each agent's endowment with each asset's payoff, normalized by the variance of the asset payoff. The aggregate welfare associated with a financial market structure increases with (an appropriate measure of) the dispersion of the betas. This characterization provides simple answers to the questions raised above.

An innovation consisting of the introduction of a new asset is optimal if it maximizes the sum of the distances between the betas of each pair of agents which are allowed to trade in the new security. Furthermore, the structure of financial assets is optimal if all assets are designed so as to maximize risk sharing. As a result, an optimal financial market structure is the one in which the payoffs of the assets traded are correlated with (that is, span) the most important factors (the principal components) which drive the dispersion of the endowments of agents in the economy.

Similarly, the integration of two groups of agents in the same market is optimal when it maximizes the distance of the mean betas of the agents in the two groups. For instance, two countries, whose financial markets are segmented because of the investors' home bias or because of the lack of harmonization of market regulations, gain maximally from financial integration if their aggregate outcome processes do not co-move, that is, they are negatively correlated. While this result is a natural consequence of the risk sharing motive for financial integration, it is a striking prescription because it is at odds with the prescriptions deriving instead from the theory of Optimal Currency Areas. This theory has suggested that the benefits of currency unions are the highest for those countries with the highest co-movements in output (see, e.g., Alesina, Barro, Tenreyro, 2002, page 9). Our results suggest a reconsideration of the estimated benefits of currency unions in many circumstances wherein the formation of the unions is also associated with the integration of financial markets, as in the case of the European financial integration.

The results discussed above concern the characterization of optimal innovations and financial market structures in our economy. Financial market structures are however the results of the innovations introduced by decentralized intermediaries. We also address in this paper the efficiency properties of a class of such decentralized innovation processes. Consider again for this purpose our leading example economy. The financial innovations of local exchanges will be uncoordinated (each exchange only considers its own local market given the other local financial markets) and possibly sequential (different exchanges innovate at different times). Does the financial market structure resulting from such a sequence of financial innovations necessarily coincide with the optimal financial market structure? Is the order of financial innovations immaterial for welfare considerations? If this is so, we say that the optimal financial asset structure is *decentralizable.* If decentralizability fails to hold, it is possible that the uncoordinated innovation process introduces a suboptimal set of innovations, and it may be economically advantageous to 'harmonize' by regulation the innovation process of decentralized exchanges.

We show that optimal financial structures do result from a decentralized innovation process whenever financial innovation consists of either the introduction of new assets into an economy *without* restricted participation constraints, or of the relaxation of restricted participation constraints for an existing asset. More importantly, we also show that optimal financial market structures may not result from a decentralized innovation process when the innovation consists of the introduction of new assets into economies *with* restricted participation: the introduction of new assets and the integration of segmented markets interact so that even the weak notion of optimality of financial intermediation that we are studying is not satisfied.[4] As a consequence, the order in which financial innovations are introduced has potential significance in terms of welfare considerations.

The intuition for this result is simple but subtle. Consider as before economies where large financial institutions participate in financial markets of all economies but each economy has a set of retail investors who only participate in local markets. The exchange in each economy introduces a new asset that maximizes the welfare of its 'participating' agents which comprise of the financial institutions and the retail investors of that economy. Trades in this new market, however, affect the trades of financial institutions in foreign markets which involve the foreign retail investors. This in turn affects endogenously the optimality of innovations by the foreign exchanges: each such exchange maximizes the welfare of its own 'participating' agents, taking as given the existing assets and in particular the assets already introduced by exchanges in other economies. It is exactly the possibility of such overlapping sets of 'participating' agents in different markets that potentially leads to the market failure of uncoordinated financial innovations, even though the objective of each exchange is aligned with the welfare criterion of the 'participating' agents.

To summarize, in the presence of restricted participation where the nature of restriction varies across financial assets, the optimality of financial market structures requires some form

[4]'Decentralizability' is, in fact, far from sufficient for financial intermediation to be optimal, because it requires that each innovation introduced into financial markets be optimal for the given existing financial market structure. In general, allowing for assets with correlated payoffs, and for innovations to maximize, e.g., trading volume (as in Duffie and Jackson, 1989), or intermediation profits (as in Pesendorfer, 1995), rather than an optimality criterion, might in fact introduce inefficiencies in the design of assets. By concentrating on 'decentralizability,' we abstract from such sources of inefficiency and concentrate rather on the possible inefficiency resulting from the uncoordinated introduction of innovations in the markets for financial intermediation.

of coordination amongst innovating intermediaries. Note however that such coordination of the innovation process might be achieved by the exchanges or the intermediaries themselves, without explicit regulation. In fact, this is currently being achieved by a consolidation of stock exchanges within Europe, as documented in McAndrews and Stefanadis (2002), and by alliances between exchanges across countries, e.g., the collaboration between the Chicago Board of Trade in the U.S. and the Euronext-Liffe in Europe on development of futures and options and five- and ten-year Treasury bond swap contracts (see the article "Capital Markets: CBOT, Euronext discuss link up," *Financial Times*, 28 January 2003).

We conclude this Introduction with a few words on the modelling and the methodology we adopt in the analysis. For the sake of tractability, we restrict our analysis of financial innovations to two-period pure exchange Capital Asset Pricing Model (CAPM) economies in which financial markets are incomplete, and traders' participation in financial markets is restricted. In particular, the class of economies we consider has normally distributed endowments and assets' payoff, and agents with exponential utility. We allow agents to consume in both periods, so that agents trade to smooth wealth across time as well as to diversify risk, a real risk-free interest rate is well defined, and financial innovations affect the equilibrium risk-free rate. In fact, financial innovations reduce the demand for precautionary savings and hence raise the interest rate in equilibrium. As a consequence, financial innovations have positive welfare consequences for agents who in equilibrium are net savers (lenders), and a negative effect on agents who are net borrowers. This allows for a much richer analysis of financial markets and innovations.[5]

We do not model explicitly the process by which innovations enter the financial markets. This would require an explicit modelling of the sources of market incompleteness and, in turn, of the direct or indirect costs associated to any specific financial innovation, and would also require a strategic analysis of financial institutions, intermediaries, and exchanges.[6] In other words, our simple analysis of the properties of optimal financial innovations and integration is silent on the associated costs, which might as well be greater than the advantages for

[5]The properties of competitive equilibria for the class of economies we examine have been extensively studied, notably by Willen (1997, 1999-a); see also the references therein. A characterization of optimal incomplete asset structures in one-period CAPM economies is derived in Demange and Laroque (1995). We extend their analysis (i) by considering two-period economies in which agents consume in both periods, thereby trading to smooth as well as to diversify their wealth process; (ii) by considering innovations which consist of the integration of markets as well as of the introduction of new assets; and (iii) by studying the 'decentralizability' of optimal financial structures and financial innovations.

[6]See for instance Bisin (1998), Cuny (1993), Duffie and Jackson (1989), Madan and Soubra (1991), Pesendorfer (1995), and Ross (1989), for economies with transaction costs; Boot and Thakor (1993), Demarzo (1999), Duffie and Demarzo (1999), Glaeser and Kallal (1997), and Nachman and Noe (1994), for economies with asymmetric information; and Athanasoulis and Shiller (2000) for economies with imprecise information about endowment processes of agents.

some specific innovation or integration process. Nevertheless our analysis sheds light on the benefits of different financial market structures and on their 'decentralizability' by means of financial innovations introduced sequentially by different intermediaries.

The rest of the paper is structured as follows. Section 2 sets up the two-period CAPM economy with incomplete markets and restricted participation. Section 3 characterizes the competitive equilibrium of such an economy and introduces the appropriate welfare measure to compare different financial market structures. Section 4 characterizes the optimality of financial market structures. Section 5 studies the welfare effects of financial innovations; and Section 6 their 'decentralizability' by means of a sequence of financial innovations. We have structured sections 4, 5, and 6 to be self-contained so that a reader mainly interested in our specific results of an applied nature and not in our general equilibrium analysis of innovation could safely skip Section 3. Section 7 concludes. All the proofs not in the main body of the paper are contained in the Appendix.

2 The Economy

We study the class of CAPM economies introduced by Willen (1997). An economy is populated by H agents, who live for two periods, 0 and 1.

Agent $h \in \mathcal{H} := \{1, \ldots, H\}$ has a safe endowment y_0^h in period 0, and a random endowment y_1^h in period 1, of the unique consumption good.

Assumption 1 *Endowments are normally distributed:*

- *Let v denote a N-dimensional vector of random variables, multivariate normals with mean 0 and variance-covariance matrix I, the identity matrix; endowments y_1^h, for any h, are*

$$y_1^h := Y^h v, \;\; Y^h \in \Re^N.$$

Agent h is also endowed with Von Neumann-Morgernstern preferences.

Assumption 2 *The utility function is:*

1. *time and state separable:*

$$u^h(c_0, c_1) := u^h(c_0) + u^h(c_1),$$

2. CARA with identical absolute risk aversion, $A > 0$, across agents:[7]

$$u^h(c) = -\frac{1}{A}e^{-A\,c}.$$

In financial markets, both a risk-free bond and J risky assets are traded. The bond, asset 0 in our notation, has a payoff $x_0 = 1$ (in units of the consumption good) with probability 1. Asset $j \in \mathcal{J} := \{1, \ldots, J\}$'s payoff, on the other hand, is random and is denoted x_j.

Assumption 3 *Assets' payoffs are normally distributed:*

- *Assets payoffs $x := [x_j]_{j \in \mathcal{J}}$ are multivariate normal random variables with mean 0 and variance-covariance matrix I, the identity matrix.*

Trading in the financial markets is possibly restricted. Let $\mathcal{J}^h$ denote the set of assets that agent h is allowed to trade. Let $\mathcal{H}_j$ denote the set of agents which are allowed to trade asset j.

In general, we allow for incomplete markets, $J < N$ and restricted participation, $\mathcal{H}_j \subset \mathcal{H}$, for some j. We only impose the following assumption on the form of restricted participation, an assumption which cannot be relaxed without losing the closed form characterization of the equilibrium.

Assumption 4 *All agents h are allowed to trade the risk-free bond:*

$$\mathcal{H}_0 = \mathcal{H}.$$

Fixing the endowments, $[y_0^h, y_1^h]$, and the set of agents in the economy, $\mathcal{H}$, we parameterize a financial market structure by the list of tradable assets, their payoffs and participation sets: $[x_j, \mathcal{H}_j;\ j \in \mathcal{J}]$. To introduce general financial innovations, we say that a financial structure $F' = [x'_j, \mathcal{H}'_j;\ j \in \mathcal{J}']$ *innovates* on $F = [x_j, \mathcal{H}_j;\ j \in \mathcal{J}]$, if F' either adds assets or relaxes some participation restriction (or both) with respect to F; i.e., when

$$\mathcal{J} \subseteq \mathcal{J}',\ \ x_j = x'_j \text{ if } j \in \mathcal{J},\ \text{ and } \mathcal{H}_j \subseteq \mathcal{H}'_j\ \forall j.$$

To any financial structure is associated a vector of 'betas,'

$$\beta_j^h := \frac{cov\left(y_1^h, x_j\right)}{var\left(x_j\right)};\quad \beta_j := \frac{cov\left(\frac{1}{H_j}\sum_{h \in H_j}\ y_1^h, x_j\right)}{var\left(x_j\right)}, \tag{1}$$

[7]Only notational complications are added by allowing heterogeneity in absolute risk aversion parameters.

where $h \in \mathcal{H},\ j \in \mathcal{J}$.

The converse is not true: assets' payoffs x_j and agents' endowments y^h are characterized by the variance of y^h, the variance of x_j, and their covariance (means are normalized to zero by Assumption 3), which cannot be recovered uniquely from knowledge of $\beta_j^h := \frac{cov(y_1^h, x_j)}{var(x_j)}$.

3 Competitive Equilibria and Welfare

We study and characterize competitive equilibria for the class of two-period economies just introduced.

Let $\pi_0 \in \Re_+$ denote the price of the risk-free bond, and $\pi_j \in \Re^J$ the price of the asset j. The problem of each agent h is to choose a consumption allocation, $[c_0^h, c_1^h]$, and portfolio position in the risk-free bond and in all tradable assets, $[\theta_0^h, \theta_j^h]_{j\in\mathcal{J}} \in \Re^{J+1}$, to maximize

$$u^h(c_0^h, c_1^h) := -\frac{1}{A}e^{-Ac_0^h} + E\left[-\frac{1}{A}e^{-Ac_1^h}\right] \tag{2}$$

subject to the budget constraints and the restricted participation constraints:

$$c_0^h = y_0^h - \pi_0\theta_0^h - \sum_{j\in\mathcal{J}} \pi_j\theta_j^h, \tag{3}$$

$$c_1^h = y_1^h + \theta_0^h + \sum_{j\in\mathcal{J}} \theta_j^h x_j, \text{ and} \tag{4}$$

$$\theta_j^h = 0, \quad j \notin \mathcal{J}^h. \tag{5}$$

Definition 1 *A competitive equilibrium is a consumption and portfolio allocation* $\left(c_0^h, c_1^h, \theta_0^h, \theta_j^h, \text{ for each } j\right)$, *for all agents* h, *and a price vector* $(\pi_0, \pi_j, \text{ for each } j)$, *such that the consumption and portfolio allocations maximize (2) subject to (3-5) for each agent* h, *and consumption and financial markets clear:*

$$\sum_h \left(c_0^h - y_0^h\right) \leq 0, \tag{6}$$

$$\sum_h \left(c_1^h - y_1^h\right) \leq 0, \text{ and} \tag{7}$$

$$\sum_h \theta^h = 0. \tag{8}$$

3.1 Characterization of Equilibria

Closed form solutions for equilibrium allocations and prices are easily derived (see Willen, 1997; we report the solution in the Appendix for completeness). It suffices here to note that at a competitive equilibrium,

- the price of any existing asset j, π_j, relative to the price of the bond, π_0, is independent of the set of assets traded in the economy; i.e., it only depends on the distribution of endowments and on the properties of the distribution of asset j's payoffs:

$$\frac{\pi_j}{\pi_0} = E\left(x_j\right) - A\, cov\left(\frac{1}{H_j}\sum_{h\in H_j} y_1^h, x_j\right), \quad j \in \mathcal{J}; \tag{9}$$

- agents' portfolios satisfy a three-fund separation property, as each agent holds the bond, the market portfolio and the un-hedgable component of his endowment:

$$c_1^h = y_1^h - \sum_{j\in\mathcal{J}^h} \beta_j^h x_j + \sum_{j\in\mathcal{J}^h} \beta_j x_j + \theta_0^h \tag{10}$$

where

$$\beta_j^h := \frac{cov\left(y_1^h, x_j\right)}{var\left(x_j\right)}; \quad \beta_j := \frac{cov\left(\frac{1}{H_j}\sum_{h\in H_j} y_1^h, x_j\right)}{var\left(x_j\right).} \tag{11}$$

3.2 Welfare

As a measure of the welfare associated to an arbitrary financial market structure F, we consider the average welfare gains associated to F with respect to the autarchic financial market structure, in which only the risk-free bond is traded. More precisely, the welfare of financial market structure F is measured by the *compensating aggregate transfer* of F, i.e., the reduction in the time 0 consumption allocation for the economy with financial structure F, which, when redistributed lump sum across all agents, makes them indifferent between the allocation associated with the economy with financial structure F and the allocation associated with the economy with autarchic financial structure. The compensating aggregate transfer is the appropriate measure of the welfare gains of a particular financial structure (with respect to autarchy) for an economy in which lump-sum transfers across agents to redistribute welfare gains and losses can be made.[8]

Using the closed-form competitive equilibrium solution, given in Appendix, it is straightforward to show that the compensating aggregate transfer of financial market structure F, μ_F, only depends on the equilibrium price of the risk-free bond in the economy with the financial structure F, π_0^F, and its counterpart in the economy with the autarchic structure, π_0^a; in particular (see Willen, 1997):

$$\mu_F = -\frac{1}{A}\, ln\, \frac{1+\pi_0^F}{1+\pi_0^a}. \tag{14}$$

The welfare associated to a financial market structure F decreases with π_0, the price of the risk-free asset in equilibrium, and thus increases with the return of the risk-free asset. The price of the risk-free asset is, in fact, low when the agents' precautionary component of savings is relatively low, and hence when a large fraction of the risk in the economy is hedged.

4 Optimal Financial Structures

We turn to the characterization of the optimality of financial market structures and of financial innovations.

Consider the set of economies in which, e.g., because of transaction costs, markets are not complete, $J < N$, and/or participation in financial markets is restricted, the cardinality of $\mathcal{H}_j$ (let it be denoted by H_j) is $< H$, for some j. The degree of market incompleteness as well as the degree of restricted participation is exogenous and determines the set of feasible

[8]Formally, let $[c_0, c_1] := [c_0^h, c_1^h]_{h\in H}$; let $U([c_0, c_1])$ denote the average welfare associated with the consumption allocation $[c_0, c_1]$:

$$U([c_0, c_1]) = \frac{1}{H}\sum_{h\in H}\left(-\frac{1}{A}e^{-Ac_0^h} + E\left[-\frac{1}{A}e^{-Ac_1^h}\right]\right). \tag{12}$$

Let $[c_0^F, c_1^F]$ be the equilibrium allocation of the economy with financial structure F; and let $[c_0^a, c_1^a]$ be the equilibrium allocation for the autarchic economy, in which no agent can trade any assets except the risk-free asset. Let π_0^F and π_0^a denote the price of the the risk-free asset, respectively, in the economy with financial structure F and in the autarchic economy.

The *compensating aggregate transfer* of F, μ_F, by definition, solves

$$U([c_0^a, c_1^a]) = U([c_0 - \mu_F, c_1]). \tag{13}$$

financial market structures for the economy. We study the optimality of financial market structures in such a restricted feasible set, i.e., we provide a characterization of the financial market structure $F = [x_j, \mathcal{H}_j;\ j \in \mathcal{J}]$ which maximizes μ_F, given J and H_j, for all $j \in J$.

The next Lemma characterizes such a financial structure. It shows that the optimality of a financial structure can be determined by looking only at its associated vector of betas, rather than at the whole variance-covariance matrix of endowments and assets' payoffs: the optimal financial structure maximizes an appropriate measure of the 'dispersion' of the betas in the population, specifically the sum of the squared distances between the betas of each pair of agents in each possible market.[9]

Lemma 1 (Beta Representation) *The compensating aggregate transfer μ_F is maximal for the financial structure F whose betas maximize*

$$\sum_{j \in \mathcal{J}} \sum_{h \in \mathcal{H}_j} \sum_{h' \in \mathcal{H}_j} \frac{1}{H_j} \left(\beta_j^h - \beta_j^{h'} \right)^2 . \tag{15}$$

Such characterization can be substantially sharpened if we consider economies in which each agent's participation in financial markets is unrestricted, i.e., $H_j = H$, for any j.

Suppose the economy's financial market structure is restricted to be composed of J assets which all agents can trade, for instance, because of transaction costs or other financial market frictions. Which are the J optimal assets for such an economy, i.e., the J assets whose betas maximize (15)? The next proposition answers this question.

Proposition 1 (Principal Components Characterization) *Suppose that market participation is not restricted in any asset:*

$$\mathcal{H}_j \equiv \mathcal{H}, \ \ \forall j \in \mathcal{J}.$$

Then a financial structure F maximizes the compensating aggregate transfer μ_F if the asset payoffs, $[x_j]$, are a linear combination of the agents' endowments:

$$x_j := R_j v, \ \ R_j \in \Re^N;$$

and the columns of $R' := [R_j]'$ are spanned by the J principal components associated with the J largest eigenvalues of the matrix

$$M = \sum_{h \in \mathcal{H}} (Y^h - Y)'(Y^h - Y),$$

[9] We assume in what follows that $H_j \geq 2$ for any j, without loss of generality (assets are in zero-net-supply, and hence an asset j such that $H_j = 1$ is not traded in equilibrium).

where $Y = \frac{1}{\#\mathcal{H}} \sum_{h \in \mathcal{H}} Y^h$.

The optimal financial structure of J assets is composed of the assets whose payoffs are those linear combinations of endowments that produce a maximum dispersion of betas across the agents (Lemma 1). If the participation in any asset is unrestricted, in particular, the optimal assets are the eigenvectors corresponding to the J-th largest eigenvalues of the matrix M, which captures the extent of risk-sharing provided by the assets in the financial structure $\mathcal{J}$.[10] The optimal assets are the principal 'factors' driving the dispersion in the individual agents' endowment processes, and hence capture as much of the risk sharing opportunities of the economy as possible. The optimal financial structure, therefore, is composed of as many such factors as possible, in the order of their importance for risk sharing opportunities, and, given the limitations to market completeness induced in our interpretation by transaction costs.[11]

The characterization of optimal financial market structures of Lemma 1 can also be specialized to study the optimal composition of the agents trading a given asset j, in terms of their betas. Consider an arbitrary asset j, with payoff x_j. Suppose that due to diseconomies of scale there is a maximal number H_j of agents who are allowed to participate in trading asset j. What is in this context the optimal composition of the set of agents who are allowed to trade, i.e., which set $\mathcal{H}_j$ maximizes aggregate welfare (15)? Diseconomies of scale can be due to the difficulty of policy coordination in a currency union, or due to the costs of social

[10]This characterization of optimal financial structures coincides with the one derived by Demange and Laroque (1995). This is true in spite of the fact that our economy differs in an important way from Demange-Laroque's in that we allow for consumption in period 0, and hence we allow for welfare effects through changes in the risk-free rate at the competitive equilibrium. The intuition of this result (and we thank an anonymous referee for pointing it out) is as follows. Risky assets are in zero-net supply in our set-up, and hence, prices of risky assets, *relative to the price of the risk free asset*, depend only on the covariances of the second period endowments with risky asset payoffs. In particular, this implies that prices of risky assets *relative to each other* are identical under the two set-ups. The risk free asset is traded by agents in our set-up as a precautionary motive against unhedged second-period risks. Thus, the price of the risk free asset captures precisely the average risk-sharing available to agents in the economy (as revealed by equation 14). In the one-period economy of Demange and Laroque, welfare is measured precisely in terms of this average risk-sharing. Since the welfare under the two set-ups is identical (up to constant terms), it follows that the optimal financial structures are identical as well.

[11]We implicitly assume that the intermediaries or the planner introducing financial assets know precisely the endowment processes. Athanasoulis and Shiller (2000) analyze innovation in a similar CAPM model where the innovator has imprecise knowledge about the endowment structure. They show that in this case, the optimal asset corresponds to a "market portfolio" that puts an appropriate weight on each of the principal components driving the dispersion in agents' endowments. While this result on optimal asset structure is interesting in its own right, our conjecture is that the results in this paper on the integration of financial markets and on the decentralizability of optimal financial structures are qualitatively robust to such an extension.

and economic policies in culturally heterogeneous societies. More specifically, the issue at hand is related to the issue of the optimal composition of a currency union or more generally of a financially integrated area with focus on the financial integration rather than on the policy coordination or on trade liberalization dimensions (as studied in Alesina, Spolaore, and Wacziarg, 2000).

The next proposition addresses this issue.

Proposition 2 (Traders' Composition) *Assume that agents' betas for asset j are distinct, and order agents so that*

$$\beta^N > \cdots > \beta^h > \cdots > \beta^1.$$

There exists an $r < H_j$ such that the optimal composition of agents trading asset j consists of the first r agents and the last $H_j - r$ agents; i.e., $\mathcal{H}_j = \{1, \ldots, r, H - (H_j - r) + 1, \ldots, H\}$.[12]

The optimal composition of agents allowed to trade an arbitrary asset j in an economy with restricted participation consists of two sets of agents, the sets being at the two extremes of the ranked betas of the agents with respect to the asset. The distribution of agents in the two extreme sets depends, in general, on the structure of the endowments. Once again, this characterization has the general flavor of our results that optimal financial market structures maximize a measure of the dispersion of betas across the agents affected by the innovations.

Consider for illustration the case of international financial integration of a collection of countries whose endowment processes are driven by a common series of factors. Each country's endowment has different loadings on such factors: each country's endowment is different in terms of the magnitude and the sign of its factor loadings. If only a subset of these countries can be allowed to participate in the process of financial market integration, maximal risk sharing and welfare of the union is achieved by selecting those countries whose dependence on the factors is most varied, for instance, countries whose endowments are maximally positively correlated with a dominant factor together with those whose endowments are maximally negatively correlated with the same factor.

To summarize, countries with segmented financial markets, e.g., because of the investors' home bias or because of the lack of harmonization of market regulations, mostly gain from financial integration if their endowment processes co-move minimally. While this is a natural consequence of the risk sharing motive for financial integration and of our general characterization of the optimality of financial market structures (Lemma 1), the result is striking in that it is at odds with the prescriptions derived by the literature on the theory of Optimal

[12] See Lemma A.1 in the Appendix, where the Proposition is proved.

Currency Areas, which notably abstracts from financial integration, and stresses the benefits of currency unions deriving from the co-movements in the member countries' outputs (see e.g., Alesina, Barro, Tenreyro, 2002, page 9.).

5 The Welfare Effects of Financial Innovations

We turn now to study the implications of the characterization of optimal financial structures derived in the previous section. Specifically, we study the welfare implications of financial innovations, in the form of the introduction of new assets and of the integration of existing but segmented financial markets. Note that, it follows from (14) that the measure of the welfare gains associated to a financial structure F' which innovates on F, with respect to F, is $\mu_{F'} - \mu_F$. The following is then a simple implication of Lemma 1.

Proposition 3 *If a financial structure F' innovates on another financial structure F, no welfare losses are possible, $\mu_{F'} \geq \mu_F$. Moreover, strictly positive welfare gains are realized, $\mu_{F'} > \mu_F$, if the innovation involves the integration of agents with different betas, or if the innovation involves the introduction of an asset tradable by agents with different betas, i.e., if for some $j \in \mathcal{J}'$, there exist $h, h' \in \mathcal{H}'_j$ such that either $j \notin \mathcal{J}$ or $h' \notin \mathcal{H}_j$, $\beta^h_j \neq \beta^{h'}_j$.*

Thus, financial innovations invariably have positive welfare effects, as measured by the aggregate compensation transfer. However, a subset of the agents might have to be compensated by lump-sum transfers after a financial innovation, as they might experience a loss due to the change in the risk-free asset's price which is a consequence of the innovation. This is a feature of the model with consumption in period 0;[13] the one-period CAPM economy studied by Demange and Laroque (1995) has no price effects due to financial innovations.

We study separately the welfare effects of two standard innovations: (i) the introduction of a new financial asset, and (ii) the integration of two distinct markets for the same financial asset. We consider first financial innovations consisting of the introduction of an asset, given a set of pre-existing assets in the economy.

Proposition 4 (Introduction of a new asset) *Suppose a financial innovation consists of introducing a new asset j', i.e., F' is as F except that $\mathcal{J}' = \mathcal{J} \bigcup \{j'\}$. The welfare gain of such innovation, $\mu_{F'} - \mu_F$, is increasing in*

$$\sum_{h \in \mathcal{H}_{j'}} \sum_{h' \in \mathcal{H}_{j'}} \frac{1}{H_{j'}} \left(\beta^h_{j'} - \beta^{h'}_{j'} \right)^2 . \tag{16}$$

[13]Willen (1999-b) addresses the difficult question of the distributional effects of financial innovations across heterogeneous agents.

The welfare gains due to the introduction of a new asset j' depend on the 'dispersion,' across the agents allowed to trade the asset, of the betas relative only to that same asset. In other words, the welfare gains associated to the introduction of asset j' are independent of the betas relative to all assets traded before the innovation. An innovation consisting of the introduction of a single asset with unlimited participation is optimal if its 'betas' are maximally dispersed across the agents, and hence, if the extent of risk-sharing introduced by the asset is maximal. In the context of a financial innovation of an asset market in which financial institutions from different countries will participate, the result above shows that the welfare gain associated with the asset's introduction is greater if the endowments (cash flows) of these participating financial institutions are dispersed in terms of their covariance with the new asset's cash flows.[14]

We can now turn to study the welfare effects of innovations which consist of the integration of two distinct markets for the same financial asset.

Proposition 5 (Integration of two distinct markets) *Suppose the financial structure F has the property that assets j and j' have the same payoff, $x_j = x_{j'}$, but are traded in distinct markets, $\mathcal{H}_j \bigcap \mathcal{H}_{j'} = \emptyset$. A financial innovation which integrates such markets, i.e., a financial structure F' which is as F except that $\mathcal{H}'_j = \mathcal{H}_j \bigcup \mathcal{H}_{j'}$, has a welfare gain $\mu_{F'} - \mu_F$ which is increasing in*

$$\frac{H_j \, H_{j'}}{H_j + H_{j'}} \, (\beta_j - \beta_{j'})^2. \tag{17}$$

The welfare gains of innovations which consist of the integration of markets increase in the number of agents integrated. Most importantly, keeping constant the number of agents in each group, the welfare gains of market integration only depend on the difference between the average betas of the two groups, β_j and $\beta_{j'}$, and do not depend on the individual betas of the agents in the groups. Keeping constant the size of the markets, the integration of two distinct markets is optimal when no other pair of markets exists whose difference in average betas is higher. For example, the welfare gain of allowing agent h' to trade asset j is increasing in $(\beta_j - \beta_j^{h'})^2$, the difference between the beta of agent h' with respect to asset j and the average of the betas of agents trading asset j before the integration; and the

[14] In a related set-up, but with no restricted participation, Duffie and Jackson (1989) show that the optimal innovation in the financial asset market maximizes the total transaction volume. It can be shown (Lemma 2, Appendix 2) that our optimality criterion is equivalent to maximizing the sum of squared transaction volume. This difference in the characterization of the optimal asset arises due to a difference in the notions of optimality employed, as Duffie and Jackson study the Pareto optimality of the innovation without allowing for lump-sum transfers.

integration in some market j of agents whose beta is right at the average of the betas of the traders in that market has no welfare effects.

In particular, the result above shows in a succinct manner that in order to estimate the benefits from financial integration of member countries in a currency union, it suffices to examine only the average endowment processes of agents or sectors of each economy that will participate in the integrated financial arena. Furthermore, the benefits from financial integration are minimal if these average endowment processes of member countries are close to each other. In contrast, if the average endowment processes of member countries are dispersed in terms of their loadings on traded financial assets, then the benefits arising from financial integration can be significant. This underscores once again the difference between our result and that of the Optimal Currency Areas literature which ignores the integration of financial markets and concludes that the benefits from harmonization are maximized if member countries are similar in their endowment processes.

6 Financial Innovations and Decentralizability

For an economy in which markets are not complete and/or participation in financial markets is restricted, we have provided in the previous section a characterization of the optimal financial market structure. But can such a financial market structure be decentralized as an equilibrium of economies in which financial intermediaries or exchanges introduce new securities and integrate segmented markets in an uncoordinated fashion?

We do not model in this paper the process by which innovations are determined in financial markets. The degree of market incompleteness as well as the degree of restricted participation are exogenous and only implicitly justified by transaction costs, and hence we cannot attempt to answer this question directly. Nevertheless our analysis potentially sheds light on the 'decentralizability' of optimal financial asset structure by means of financial innovations introduced sequentially by different intermediaries. More precisely, suppose that, given J, the number of asset markets, and H_j, the size of each market $j \in J$, each innovation is introduced into financial markets independently of the others, so as to satisfy the orthogonality of asset payoffs and so as to maximize the optimality criterion, (15), for the given existing financial market structure. Innovations might consist either of the introduction of a new asset or of the integration of two markets, or both. Does the financial market structure which results from such a sequence of financial innovations necessarily coincide with the optimal financial market structure? If this is so, we say that the optimal financial asset structure is *decentralizable*.[15]

[15]Hara (1997) asks a related question, that is: Does there exist a sequence of financial innovations such

6.1 Decentralizability

We turn now to the 'decentralizability' of the optimal financial market structure by means of independent financial innovations. A precise definition of 'decentralizability,' which applies generally to optimal asset structures as well as to optimal compositions of traders, is as follows.

Definition 2 *Let the optimal financial structure of an economy with J orthogonal financial assets and market participation structure $\mathcal{H}_j,\ j = 1, \ldots, J$ be denoted F. The betas associated with F maximize $\sum_{j \in \mathcal{J}} \sum_{h \in \mathcal{H}_j} \sum_{h' \in \mathcal{H}_j} \frac{1}{H_j} \left(\beta_j^h - \beta_j^{h'}\right)^2$ (by Lemma 1). Consider then another financial structure, $\hat{F}$, with J orthogonal financial assets and market participation structure $\mathcal{H}_j,\ j = 1, \ldots, J$. Suppose $\hat{F}$ satisfies the following:*

- *the betas associated with each asset j maximize $\sum_{h \in \mathcal{H}_j} \sum_{h' \in \mathcal{H}_j} \frac{1}{H_j} \left(\beta_j^h - \beta_j^{h'}\right)^2$, given $\mathcal{H}_j$;*
- *for any asset j, the components of $\mathcal{H}_j$ can be ordered so that, without loss of generality,*

$$h := \text{arg max}_{\ h' \geq h} \sum_{h''=1}^{h-1} \left(\beta^{h''} - \beta^{h'}\right)^2 .$$

We say that the optimal financial structure F is 'decentralizable' if F does not strictly dominate $\hat{F}$, in welfare terms, i.e. $\mu_F = \mu_{\hat{F}}$.[16]

The rationale behind studying 'decentralizability' is to see if a set of innovations that is introduced either sequentially or in an uncoordinated manner by a group of financial intermediaries leads to a financial structure that is overall optimal.[17] If 'decentralizability' fails to

that when introduced sequentially they are each Pareto-improving, and lead to the completion of financial markets? He answers the question in the affirmative. Our analysis of decentralizability is conceptually different from his analysis because we require each financial innovation in the sequence to satisfy an optimality requirement that is stricter than Pareto-improvement, and we are interested in the optimality of resulting financial market structures which are still incomplete, rather than in the possibility of completing the markets. Also, the sequence of financial innovations Hara constructs has the property that asset prices are not affected by the introduction of a new asset, while this is not the case in our model (as implied by equation (8)), because we only consider economies with a risk-free bond (Assumption 4). Finally, Hara does not consider innovations in the form of market integration.

[16] Since F is optimal, obviously, $\mu_F \geq \mu_{\hat{F}}$.

[17] The issue of the optimal order of financial innovation has been largely overlooked in the literature. A notable exception is Dow (1998) who considers the opening up of a new market where trading occurs only as a way of hedging by arbitrageurs exploiting private information in another existing market (that is

hold, it suggests that there are costs from having decentralized exchanges or intermediaries introducing innovations independently (in an uncoordinated fashion). The efficiency gains (from a financial optimality standpoint) that result from a harmonization of the innovation process may thus be significant for market structures that lack 'decentralizability.' Such gains are, in fact, a lower bound since 'decentralizability' is a relatively weak requirement for optimality of innovation process: it only requires that each innovation introduced into financial markets be optimal given the existing financial market structure. Allowing for assets with correlated payoffs and for innovations to maximize, e.g., trading volume or intermediaries' profits, might introduce other inefficiencies from such decentralized, sequential, and uncoordinated design of assets.[18]

In the case where financial innovations are restricted to the introduction of new assets, 'decentralizability' requires that the optimal financial structure can be obtained equivalently by introducing a first financial asset x_1 which maximizes

$$\sum_{h \in \mathcal{H}_1} \sum_{h' \in \mathcal{H}_1} \frac{1}{H_1} \left(\beta_1^h - \beta_1^{h'} \right)^2 ;$$

a second asset whose payoff is orthogonal to x_1 above, and maximizes

$$\sum_{h \in \mathcal{H}_2} \sum_{h' \in \mathcal{H}_2} \frac{1}{H_2} \left(\beta_2^h - \beta_2^{h'} \right)^2 ;$$

and so on sequentially until the J-th asset. Similarly, for the case of market integration in a given asset, 'decentralizability' requires that a sequential strategy of adding an optimal agent, given the existing participation in the asset, gives rise to the overall optimal composition of traders in that asset.

We examine the 'decentralizability' of the optimal financial market structures for three sets of economies: (i) first, economies in which the participation of agents in all financial markets is unrestricted, but the number of tradable assets is limited; (ii) second, economies in which the number and the payoffs of tradable assets are exogenously determined, but the

correlated with the new market). The new market may thus give rise to greater informed trading in the existing market reducing the existing market's liquidity and potentially reducing the average welfare of the economy. In contrast, he shows that in the reverse sequence of market introductions, each sequential market introduction improves welfare.

[18] Cuny (1993), for example, characterizes the financial asset structure arising from a sequential design by multiple exchanges in a setting where agents incur a fixed cost to participate in an asset market. Given the fixed switching cost assumption, Cuny shows that exchanges that move early choose contracts that ensure sufficiently high liquidity in order to deter potential competition from subsequent innovations. He does not, however, discuss the efficiency properties of these sequential innovations.

number of agents allowed to trade each particular asset is limited; and, (iii) finally, economies in which the agents' participation in financial markets is exogenously determined and the number of tradable assets is limited.

Consider first an economy in which the optimal financial structure consists of J assets with no restriction in participation. The following is a simple implication of the principal components characterization (Proposition 1).

Proposition 6 *The financial structure F, which is optimal in the class of financial structures with J assets, and such that participation is not restricted in any asset, $\mathcal{H}_j \equiv \mathcal{H}, \ \forall j \in \mathcal{J}$, is 'decentralizable.'*

The optimal financial asset structure defined by the principal component characterization has the property that the n-th asset is chosen so that the asset's payoff equals the eigenvector corresponding to the n-th largest eigenvalue of the matrix M defined in Proposition 3. The n-th asset in a sequence of financial innovations is chosen in exactly the same way.

We next examine the 'decentralizability' of the optimal financial asset structure of an economy in which the payoff of each asset available for trade is exogenously determined, and the number of agents allowed to trade each particular asset is limited.

Proposition 7 *The financial structure F, which is optimal in the class of financial structures with J assets with exogenously given payoff, and such that asset j is traded by no more than $n < H$ agents, is 'decentralizable.'*

The result can be extended to the case in which the payoffs of financial assets are *exogenous* and the restriction on the participation in financial market j depends on j itself, i.e., in which no more than H_j agents are allowed to trade asset j, with $H_j < H$ but H_j not necessarily equal to $H_{j'}$ if $j \neq j'$. This is not the case, however, when we consider financial structures with limited participation but with assets whose payoffs are *endogenous* or *optimally designed*.

The next proposition shows that, if the restricted market participation structure is different across different assets but is nevertheless overlapping for some of the assets, then the sequential introduction of financial assets can produce a financial market structure that is *not optimal* amongst all structures with the same number of financial assets with the given market participation structure. In other words, with restricted participation, the order of innovation of financial assets may affect aggregate welfare.

Proposition 8 *The financial structure F, which is optimal in the class of financial structures with J assets and given restricted participation structure $\mathcal{H}_j$, with $\mathcal{H}_j \subset \mathcal{H}$ for some j,* **may not be** *'decentralizable.'*

Proof of Proposition 8. We will prove the proposition by introducing an example economy in which the optimal financial structure can, in fact, *strictly dominate* the financial structure resulting from the sequential introduction of optimal innovations.

Consider an economy with $H = 3$, i.e., $\mathcal{H} = \{0, 1, 2\}$. Also the dimension of endowment space is $N = 3$. The agents' endowments are given by:

$$Y_0 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}; \; Y_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}; \; Y_2 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}.$$

The matrix M in the principal components characterization is

$$M = \tfrac{1}{3} \begin{bmatrix} 2 & -1 & -2 \\ -1 & 6 & 4 \\ -2 & 4 & 8 \end{bmatrix},$$

whose eigenvalues are $\lambda_1 = 11.62$, $\lambda_2 = 3.00$, and $\lambda_3 = 1.38$, with the corresponding eigenvectors:

$$x_1 = \begin{bmatrix} -0.27 \\ -0.80 \\ 0.53 \end{bmatrix}; \; x_2 = \begin{bmatrix} 0.22 \\ -0.59 \\ -0.78 \end{bmatrix}; \; x_3 = \begin{bmatrix} -0.94 \\ 0.09 \\ -0.34 \end{bmatrix}.$$

Thus, in absence of any restricted market participation, for $J = 1$, the optimal financial structure is (x_1), and for $J = 2$, the overall optimal financial structure is (x_1, x_2).

Next consider economies with $J = 2$ financial assets and with an exogenously given restricted market participation structure of $\mathcal{H}_1 = \{0, 1\}$, and $\mathcal{H}_2 = \{1, 2\}$. The solution to the optimization problem for the overall financial structure yields the optimal assets as:

$$x_1 = \begin{bmatrix} 0.10 \\ -0.28 \\ -0.96 \end{bmatrix}; \; x_2 = \begin{bmatrix} 0.48 \\ 0.86 \\ -0.20 \end{bmatrix}.$$

The corresponding betas of the participating agents are $\beta_1^0 = -3.32$, $\beta_1^1 = -1.13$, and $\beta_2^1 = 1.14$, $\beta_2^2 = 3.33$. The welfare measure for this structure is

$$\begin{aligned} ln(\mu_F) &= k_1 + k_2 \cdot \sum_{j=1}^{2} \sum_{h \in \mathcal{H}_j} \sum_{h' \in \mathcal{H}_j} \frac{1}{2} \left(\beta_j^h - \beta_j^{h'} \right)^2 \\ &= k_1 + k_2 \cdot \frac{1}{2}(4.791 + 4.791) = k_1 + k_2 \cdot 4.791, \end{aligned}$$

where k_1 and k_2 are positive constants irrelevant to the analysis.

On the other hand, for the case of sequentially optimal asset introduction, solutions to the optimization problems are given as:

$$x_1 = \begin{bmatrix} 0.00 \\ -0.45 \\ -0.89 \end{bmatrix}; \; x_2 = \begin{bmatrix} 0.49 \\ 0.78 \\ -0.39 \end{bmatrix}.$$

The corresponding betas of the participating agents are given as: $\beta_1^0 = -3.58$, $\beta_1^1 = -1.34$, and $\beta_2^1 = 0.88$, $\beta_2^2 = 2.93$. Thus, the welfare measure can be computed as

$$\begin{aligned} ln(\mu_F) &= k_1 + k_2 \cdot \sum_{j=1}^{2} \sum_{h \in \mathcal{H}_j} \sum_{h' \in \mathcal{H}_j} \frac{1}{2} \left(\beta_j^h - \beta_j^{h'} \right)^2 \\ &= k_1 + k_2 \cdot \frac{1}{2}(5.000 + 4.1999) = k_1 + k_2 \cdot 4.599. \end{aligned}$$

Note that the welfare measure is smaller than that with overall optimal structure (k_1 and k_2 are the same for a given economy). $\diamondsuit$

This example illustrates the fact that a sequentially optimal structure may not do as well in welfare terms as an overall optimal structure in the presence of restricted market participation; and this is because of lack of coordination in the innovation process.

As a motivation for the example, consider two exchanges in two economies. Each economy has its set of local retail agents who participate only in markets of local exchanges. These agents are numbered as 0 and 2, respectively, in the example. Across the two economies, there are financial institutions which participate in all exchanges. These agents are numbered as 1 in the example. The exchanges in the two economies innovate sequentially taking into account that the participation in the market introduced by an exchange consists only of the financial institutions (agent 1) and the respective retail investors (agent 0 or agent 2). The example thus maps into a natural market participation structure, where due to transaction costs or geographical and technological distance, retail investors display a home bias.

As an intuition of the result, notice that the first sequential asset produces a welfare change greater than each of the two assets under the overall optimal structure. However, the first sequential asset, i.e., the first innovating exchange, does not take into account the restricted market participation of the second asset (which is different from that of the first asset), and, as a result, the second sequential optimal asset introduced by the other innovating exchange is inferior in welfare terms to both the overall optimal assets. In fact, it is inferior enough that the sequential structure is *strictly dominated* by the overall structure.[19]

[19] It is important to note here that in our model, each sequential asset is restricted to be orthogonal to the set of existing assets. This is for the sake of tractability in our CARA-Normal set-up. The assumption is, however, not very far from the observed practice. Silber (1981) and Black (1986) document empirically that futures contract innovations from 1960 to 1980 have succeeded at exchanges (measured using the induced trading volumes) primarily when these have been new contracts, that is, when they provide risk sharing

We have already shown in Propositions 6 and 7 that optimal financial structures are 'decentralizable' whenever the financial innovation consists of either the introduction of new assets in an economy without restricted participation constraints, or the relaxation of restricted participation constraints for an existing asset. Proposition 8 shows that on the contrary, optimal financial structures are not 'decentralizable' when the innovation consists of the introduction of new assets into economies with given restricted participation: the introduction of new assets and the integration of segmented markets interact so that even the weak notion of optimality of financial intermediation, guaranteed by 'decentralizability,' is not satisfied. In presence of restricted participation, where the nature of restriction varies across financial assets and allows for partial integration (that is, markets are not completely distinct in term of participating agents), the optimality of financial market structures requires some form of coordination amongst innovating intermediaries. In this case the order in which financial innovations are introduced has potential significance in terms of welfare consideration.

Further, the intuition for the result discussed above suggests that the construction in the example above is, in fact, quite robust. A careful inspection of the endowments of different agents and the market participation structures reveals the following. If introducing an asset generates a large amount of risk-sharing between its participating agents (agent 0 and agent 1 for the first asset), such that it restricts the risk-sharing between some of the participating agents and the non-participating agents (agent 1 and agent 2, respectively), then the asset introductions that follow in the sequentially optimal design are likely to produce a less-than-optimal overall financial structure. The intuition being compelling enough, we do not delve into a more rigorous analysis of precise financial structures where 'decentralizability' fails to hold.[20]

This result on the lack of decentralizability has implications for recent examples of the integration of segmented markets, most notably for the financial integration in Europe. Financial institutions and large international banks participate in all markets in different economies of European Union. However, a large number of the retail household investors have access only to their domestic markets, giving rise to 'market segmentation' or 'restricted participation' in several financial markets. Since exchanges in different economies innovate

along dimensions of risk that are residual given the existing contracts. Attempts of exchanges to compete with existing contracts on other exchanges have usually failed. Tufano (1989) also documents such a first-mover advantage in the financial innovations undertaken by investment banks over the period from 1974 to 1986. Cuny (1993) demonstrates theoretically that in a sequential innovation process, each successive contract is, in fact, orthogonal to all the existing ones, whenever there is a fixed cost to exchanges (traders) for innovation (participation).

[20] We conjecture that a similar result, the lack of 'decentralizability,' holds for financial market structures which are optimal in the class of financial structures in which only the number of tradable assets and the number of the agents allowed to trade each asset are given.

and create new markets, our analysis suggests that some coordination of the innovation process is likely to be desirable.[21] The consolidation of exchanges in Europe to create a pan-European stock exchange and the collaboration between exchanges in the U.S. and the Europe, such as the one between CBOT and Euronext that is currently under way, may have desirable consequences in terms of the efficiency of induced innovations.

7 Conclusions

The theoretical results of this paper provide two important normative prescriptions: (i) financial innovations that generate a higher level of risk-sharing, as characterized by dispersion across agents of betas, i.e., the covariances of their endowments with traded financial assets, are more desirable than others from an overall welfare standpoint; and (ii) some form of harmonization or coordination of the innovation process of decentralized financial intermediaries is likely to be desirable when asset markets in the integrating economies are segmented with different but overlapping sets of participating agents.

While this paper only analyzes the optimality of financial structures for economies in which lump sum transfers across agents are possible, many interesting issues arise with regard to the individual welfare effects of financial innovations. These effects pertain to the welfare effects when lump sum transfers are not implementable. In this case, in fact, the welfare effect of financial innovations can in general be negative on a subset of the agents, as the relative price effects of innovation have welfare effects: an innovation leads to a lower demand for precautionary savings, and hence for the riskless asset. The riskless rate of interest in the economy as a consequence increases, and the agents, who need to borrow in equilibrium and do not benefit much from risk-sharing provided by financial innovation, are relatively hurt. Unfortunately, few implications of optimal financial innovations with respect to individual welfare can be derived analytically. Some results are contained though in Willen (1999-b).

Given the simple structure of competitive equilibrium allocations of CAPM economies, our analysis of optimal financial structure can in principle be extended to CAPM economies with infinite horizon, modelled, for example, in Willen (1999-a). However, several conceptual and technical issues that have not been addressed hitherto arise in this case. We leave these as a realistic agenda for future research.

[21] The result on the lack of decentralizability has also implications for the recent evolutionary approach to financial innovations (see, e.g., Bettzuge-Hens, 2000). In this literature financial innovations are introduced sequentially and their survival depends on market participation (or trade volume). Our results suggest that, when the innovation process stops before the completion of markets, because of any sort of trade frictions, then the evolutionary process might not in general reach an optimal financial market structure.

More generally though our analysis has extensively exploited the simple nature of CAPM economies and, in particular, the closed-form expressions for equilibrium allocations, prices, and aggregate welfare. Many relevant extensions of our analysis are not possible without losing the ability to derive closed-form solutions, and hence require computation and calibration methods. These extensions include, for instance, the analysis of other forms of market frictions and incompleteness of markets such as borrowing constraints, transaction costs, and asymmetric information.

We have repeatedly noticed that our analysis of financial market integration has different, in fact, opposite, welfare implications from those derived in the theory of Optimal Currency Areas, which stresses the integration of trades and international commerce, as well as various forms of policy integration. Our analysis, therefore, strongly suggests the importance of a joint analysis of welfare benefits of financial and trade integration. This analysis though requires studying economies with multiple consumption goods. Unfortunately, the study of economies with multiple consumption goods cannot be pursued without losing the ability to solve for equilibria in closed form. It is our future objective to develop a dynamic model of integration with multiple consumption goods which can be studied computationally. This would enable us to pursue the analysis of this paper empirically and to study the welfare benefits of Currency Unions and Optimal Currency Areas when the integration process involves the integration of financial markets as well as of international commerce.

APPENDIX 1: Competitive equilibrium (Willen, 1997)

The competitive equilibrium of the two-period CAPM economy, defined by equations (1)-(7), is characterized by prices of assets (π_j), portfolio choices (θ_j^h), and consumption allocations (c_t^h), given below. Note that $j = 0$ denotes the riskfree asset.

$$\pi_0 = exp\left\{A\left(y_0 - Ey_1\right) + \frac{A^2}{2H}\sum_{h\in H}\left((1-R_h^2)var(y_1^h) + \sum_{j\in J^h} var(\beta_j x_j)\right)\right\} \tag{18}$$

where

$$R_h^2 := \frac{\sum_{j\in J^h}\left(\beta_j^h\right)^2 var(x_j)}{var(y_1^h)} \tag{19}$$

$$\frac{\pi_j}{\pi_0} = E\left(x_j\right) - A\; cov\left(\frac{1}{H_j}\sum_{h\in H_j} y_1^h, x_j\right), \quad j\in\mathcal{J} \tag{20}$$

$$\theta_j^h = \beta_j - \beta_j^h, \;\; j\in\mathcal{J}^h, \quad \text{and } \theta_j^h = 0, \;\; j\in(\mathcal{J}^h)^c \tag{21}$$

$$\theta_0^h = \frac{1}{1+\pi_0}\left(y_0^h - E(y_1^h) - \sum_{j\in\mathcal{J}^h}\pi_j\theta_j^h + \frac{A}{2}\,var(c_1^h) - \frac{1}{A}\,ln(\pi_0)\right) \tag{22}$$

$$c_1^h = y_1^h - \sum_{j\in\mathcal{J}^h}\beta_j^h x_j + \sum_{j\in\mathcal{J}^h}\beta_j x_j + \theta_0^h \tag{23}$$

$$var(c_1^h) = var(y_1^h) - \sum_{j\in\mathcal{J}^h}(\beta_j^h)^2 + \sum_{j\in\mathcal{J}^h}\beta_j^2. \tag{24}$$

Consumption allocations c_0^h can be solved by using (18-23) and the budget constraint (3):

$$c_0^h = -\frac{1}{A}\,ln\,\frac{1}{\pi_0} + E(y_1^h) + \theta_0^h - \frac{A}{2}\,var(c_1^h). \tag{25}$$

APPENDIX 2: Proofs

Proof of Lemma 1. From Willen (1997),

$$\mu_F = -\frac{1}{A} ln \, \frac{1+\pi_0}{1+\pi_0^a} = -\frac{1}{A} ln \, \frac{\frac{1}{\pi_0^a} + \frac{\pi_0}{\pi_0^a}}{1+\frac{1}{\pi_0^a}}$$

where for simplicity, we denote π_0^F as simply π_0. Thus, aggregate welfare is maximized when the ratio, $\frac{\pi_0}{\pi_0^a}$, is minimized. But,

$$\pi_0^a = \ \exp\left\{A\left(y_0 - Ey_1\right) + \frac{A^2}{2H}\sum_{h\in H} \ \text{var}(y_1^h)\right\}, \ \text{and}$$

$$\pi_0 = \ \exp\left\{A\left(y_0 - Ey_1\right) + \frac{A^2}{2H}\sum_{h\in H} \left((1-R_h^2) \ \text{var}(y_1^h) + \sum_{j\in J^h} \ \text{var}(\beta_j x_j)\right)\right\}.$$

Using $R_h^2 \ \text{var}(y_1^h) = \sum_{j\in J^h} \ (\beta_j^h)^2 \ \text{var}(y_1^h)$,

$$\frac{\pi_0}{\pi_0^a} = \ \exp\left\{\frac{A^2}{2H}\sum_{h\in H}\left(-\sum_{j\in J^h} (\beta_j^h)^2 + \sum_{j\in J^h} (\beta_j)^2\right) \ \text{var}(x_j)\right\}$$

which, in turn, can be written as:

$$\frac{\pi_0}{\pi_0^a} = \ \exp\left\{\frac{A^2}{2H}\sum_{j\in J}\ \sum_{h\in H_j} \left(\beta_j^2 - (\beta_j^h)^2\right) \ \text{var}(x_j)\right\}. \tag{26}$$

Then, using $\beta_j = \frac{1}{H_j}\sum_{h\in H_j} \ \beta_j^h$, we get

$$\sum_{h\in H_j} \left(\beta_j^2 - (\beta_j^h)^2\right) = \frac{1}{H_j}\left(\sum_{h\in H_j} \beta_j^h\right)^2 - \sum_{h\in H_j} (\beta_j^h)^2 = -\frac{1}{H_j}\sum_{h\in H_j}\ \sum_{h'\in H_j} \left(\beta_j^h - \beta_j^{h'}\right)^2,$$

and hence, given $var(x_j) \equiv 1$, we have the result

$$\frac{\pi_0}{\pi_0^a} = \ \exp\left\{\frac{A^2}{2H}\sum_{j\in J}\ \sum_{h\in H_j}\ \sum_{h'\in H_j} -\frac{1}{H_j}\left(\beta_j^h - \beta_j^{h'}\right)^2\right\}. \quad \diamondsuit \tag{27}$$

Proof of Proposition 1. We first derive another characterization of the optimality of financial structures in terms of portfolio volumes.

Lemma 2 (Portfolio Representation) *The compensating aggregate transfer μ_F is maximal for the financial structure F whose equilibrium trading portfolios, $[\theta_j^h]_{j\in\mathcal{J}}^{h\in\mathcal{H}}$, maximize*

$$\sum_{j\in\mathcal{J}}\sum_{h\in\mathcal{H}_j}\sum_{h'\in\mathcal{H}_j}\frac{1}{H_j}\left(\theta_j^h-\theta_j^{h'}\right)^2. \tag{28}$$

This is equivalent to maximizing

$$\sum_{j\in\mathcal{J}}\sum_{h\in\mathcal{H}_j}\frac{1}{H_j}\left(\theta_j^h\right)^2. \tag{29}$$

Proof of Lemma 2. Follows from Lemma 1, equation (27), and $\theta_j^h=\beta_j-\beta_j^h$, for $j\in\mathcal{J}^h$. The second representation is a result of the fact that $\sum_{h\in\mathcal{H}_j}\ \theta_j^h=0$. $\diamondsuit$

Assume $H_j=H$, for any $j\in\mathcal{J}$. In an abuse of notation, we shall use μ_F to represent the measure of beta dispersion that maximizes it (See Beta Representation of Lemma 1). This is of course innocuous as far as the proof below is concerned. From Lemma 2, using $\theta_j^h=\beta_j-\beta_j^h$,

$$\mu_{F'}-\mu_F=\sum_{j\in J}\sum_{h\in H_j}\sum_{h'\in H_j}\frac{1}{H_j}(\beta_j^h-\beta_j^{h'})^2.$$

But, using the assumption $H_j=H$,

$$\mu_{F'}-\mu_F=\sum_{h\in H}\sum_{j\in J}(\beta_j^h-\beta_j^{h'})^2=\sum_{h\in H}\sum_{j\in J}\ \left[\ \mathrm{cov}(y_1^h,x_j)-cov(y_1,x_j)\right]^2.$$

Let $y_1^h=Y^hv$, and $Y=\frac{1}{H}\sum_{j\in J}\ Y^h$; i.e., $y_1=Yv$. Since assets' payoffs are normally distributed (Assumption 3), we can write $x=R'v+\epsilon$, with $cov(\epsilon,v)=0$. We will first derive the result for $\epsilon=0$, i.e., assets' payoffs are linear in the agents' endowments; and we will then show that such linearity actually must hold if the asset structure is optimal.

Let $R=\begin{bmatrix}R_1\\ \vdots\\ R_J\end{bmatrix}$, and let R^T denote the transpose of R. Then

$$\mu_{F'}-\mu_F=\sum_{h\in H}\sum_{j\in J}\ \mathrm{cov}(y_1^h-y_1,x_j)^2=\sum_{h\in H}\sum_{j\in J}\ \mathrm{cov}(Y^hv-Yv,R_jv)^2$$

$$= \sum_{h \in H} \sum_{j \in J} (Y^h - Y)\ R_j^T\ R_j\ (Y^h - Y)^T.$$

Since $R_j\ R_{j'}^T = 0$, for any $j \neq j'$, and $R_j\ R_j^T = 1, \forall j$, it follows that $R\ R^T$ is an identity matrix. Then

$$\begin{aligned} \mu_{F'} - \mu_F &= \sum_{h \in H} \sum_{j \in J} (Y^h - Y)\ R_j^T\ R_j\ R_j^T\ R_j\ (Y^h - Y)^T \\ &= \sum_{h \in H} (Y^h - Y) R^T\ R\ (Y^h - Y)^T. \end{aligned}$$

The rest of the proof for the case $x = R'v$ follows Proposition 2.3 of Demange and Laroque (1995), Page 226.

But in general, assets' payoff have the form $x = R'v + \epsilon$. It remains to be shown that $\epsilon = 0$ is required by optimality. If $x = R'v + \epsilon$, $\beta^h = \frac{cov(R'v+\epsilon, Y^{h'}v)}{var(x)} = \frac{R'Y^h}{R'R+\epsilon^2}$ (since $var(v) = 1$). Since the welfare criterion, (16), is homogeneous of degree 2 in betas, the optimal asset structure has $\epsilon = 0$. $\diamondsuit$

Proposition 2 is implied by Lemma A.1, stated and proved in Proposition 7 below.

Proof of Proposition 3. From $\mu_F = -\frac{1}{A} ln\ \frac{1+\pi_0}{1+\pi_0^a}$,

$$\mu_{F'} - \mu_F = -\frac{1}{A} ln\ \frac{1+\pi_0'}{1+\pi_0} = -\frac{1}{A} ln\ \frac{\frac{1}{\pi_0} + \frac{\pi_0'}{\pi_0}}{1 + \frac{1}{\pi_0}}.$$

Thus, as in Lemma 1, aggregate welfare is maximized when the ratio, $\frac{\pi_0'}{\pi_0}$, is minimized. Fix H_j, for any $j \in J$. Then from Lemma 1,

$$\frac{\pi_0'}{\pi_0} = \exp\left\{ \frac{A^2}{2H} \sum_{j \in J} \sum_{h \in H_j} \sum_{h' \in H_j} -\frac{1}{H_j} \left(\beta_j^h - \beta_j^{h'} \right)^2 \right\}.$$

It follows that $\mu_{F'} > \mu_F$ if the innovation consists of introducing an asset $j' \notin \mathcal{J}$, such that there exist $h, h' \in H_{j'}$ for which $\beta_{j'}^h \neq \beta_{j'}^{h'}$.

To study the case of market integration, suppose that there exist $j, j' \in J$ such that $x_j \equiv x_{j'}$ and $H_j \bigcap H_{j'} = \emptyset$, and consider a new economy in which $H_j' = H_{j'}' = H_j \bigcup H_{j'}$. Then

$$\frac{\pi_0'}{\pi_0} = \exp\left\{ \frac{A^2}{2H} \cdot \sum_{h \in H_j'} \sum_{h' \in H_j'} \left(\frac{(\beta_j^h - \beta_j^{h'})^2}{H_j + H_j'} - \frac{(\beta_j^h - \beta_j^{h'})^2}{H_j} - \frac{(\beta_j^h - \beta_j^{h'})^2}{H_j'} \right) \right\}.$$

Using (26):

$$\frac{\pi_0'}{\pi_0} = \exp\left\{\frac{A^2}{2H}\left(\sum_{h\in H_j'}(\beta_j')^2-(\beta_j^h)^2-\sum_{h\in H_j}(\beta_j)^2-(\beta_j^h)^2-\sum_{h\in H_{j'}}(\beta_{j'})^2-(\beta_{j'}^h)^2\right)\right\}$$

$$= \exp\left\{\frac{A^2}{2H}\left(\sum_{h\in H_j'}(\beta_j')^2-\sum_{h\in H_j}(\beta_j)^2-\sum_{h\in H_{j'}}(\beta_{j'})^2\right)\right\}.$$

But since, $\beta_j' = \frac{H_j\beta_j+H_{j'}\beta_{j'}}{H_j+H_{j'}}$, we have

$$\frac{\pi_0'}{\pi_0} = \exp\left\{\frac{A^2}{2H}\cdot\frac{-H_j\,H_{j'}}{H_j+H_{j'}}(\beta_j-\beta_{j'})^2\right\}. \tag{30}$$

It follows that, if the innovation consists of integrating assets j and j', then $\mu_{F'} > \mu_F$ if $\beta_j \neq \beta_{j'}$. The necessary condition for $\beta_j \neq \beta_{j'}$ is that there exist $h \in H_j$, and $h' \in H_{j'}$ such that $\beta_{j'}^h \neq \beta_{j'}^{h'}$ (but $\beta_{j'}^{h'} = \beta_j^{h'}$, since $x_j = x_{j'}$). $\diamondsuit$

Proposition 4 and Proposition 5 follow directly from Proposition 3. Proposition 6 follows immediately from Proposition 1.

Proof of Proposition 7. In the notation which follows the index referring to an arbitrary security j is dropped for simplicity.

Let H denote the number of agents in the population. Let n denote the maximum number of agents allowed to trade an arbitrary security. We study the optimal choice of the n agents. The only interesting case to consider is the case $2 \leq n < H$, obviously. Also, we restrict ourselves to the case in which agents' betas (for the arbitrary asset) are distinct. (The proof can be extended to take into account of identical betas with only notational complications.) This allows us to order agents, without loss of generality, so that

$$\beta^H > \cdots > \beta^h > \cdots > \beta^1.$$

Two definitions of optimality are compared, which we call 'optimality' and 'sequential optimality' in the following. Let $O^n \subset H$ denote the optimal set of agents, while $S^n \subset H$ denotes the sequential optimal set of agents (both sets have cardinality n by construction, of course).

In particular, if H^n denotes any arbitrary n-dimensional subset of H, and

$$\overline{\beta}^n := \frac{1}{n} \sum_{h \in H^n} \beta^h,$$

$$O^n := \ \text{argmax}_{H^n} \sum_{h \in H^n} \left(\beta^h\right)^2 - n \left(\overline{\beta}^n\right)^2.$$

Also, S^n is defined recursively as follows: $S^n = S^{n-1} \bigcup h'$, where

$$h' := \ \text{argmax}_{h' \notin S^{n-1}} \sum_{h \in S^{n-1}} \left(\beta^h\right)^2 + \left(\beta^{h'}\right)^2 - n \left(\frac{1}{n} \left(\sum_{h \in S^{n-1}} \beta^h + \beta^{h'} \right) \right)^2,$$

and $S^2 := O^2$. Note that, equivalently,

$$h' := \ \text{argmax}_{h' \notin S^{n-1}} \sum_{h \in S^{n-1}} \left(\beta^{h'} - \beta^h\right)^2. \tag{31}$$

Lemma A . 1 *There exists an $r_o^n < n$ such that O^n consists of the first r_o^n agents and the last $n - r_o^n$ agents; i.e., $O^n = \{1, \ldots, r_o^n, H - (n - r_o^n) + 1, \ldots, H\}$. Moreover, if we let $\overline{\beta}_o^n = \frac{1}{n} \sum_{h \in O^n} \beta^h$, then*

$$\beta^{r_o^n} < \overline{\beta}_o^n < \beta^{H-(n-r_o^n)+1}. \tag{32}$$

Proof of Lemma A . 1 By contradiction. Suppose the statement does not hold. Pick a couple $s_1, s_2 \in O^n$ such that $\beta^{s_1} \leq \overline{\beta}_o^n < \beta^{s_2}$. Then either

Case 1: $\left\{\beta^h \mid h \in O^n,\ h \leq s_1\right\}$ are not the r smallest betas, for some $r \leq s_1$; or
Case 2: $\left\{\beta^h \mid h \in O^n,\ h > s_2\right\}$ are not the $n - r$ largest betas, for some $r \leq s_1$ (i.e., $(n - r) > s_2$); or finally
Case 3: both the above cases hold.

We will now show that in either of these cases it is possible to pick an agent k and substitute him with another agent k' and improve welfare.

The change in welfare due to the substitution of k with k' can be calculated to be:

$$n \sum_{h \in O^n - \{k\} \bigcup \{k'\}} \left(\beta^h\right)^2 - \left(\sum_{h \in O^n - \{k\} \bigcup \{k'\}} \beta^h \right)^2 - n \sum_{h \in O^n} \left(\beta^h\right)^2 + \left(\sum_{h \in O^n} \beta^h \right)^2,$$

which, in turn, after some algebra, can be written as:

$$\left(\beta^{k'}-\beta^{k}\right)\left((n-1)\left(\beta^{k'}-\beta^{k}\right)+2n\left(\beta^{k}-\overline{\beta}_{O}^{n}\right)\right).$$

By construction, $\beta^{s_1} \leq \overline{\beta}_{O}^{n}$.

In Case 1 then, taking $k = s_1$ and k' such that $\beta^{k'} < \beta^{s_1}$, we can improve welfare. Similarly, in Case 2, taking $k = s_2$ and k' such that $\beta^{k'} > \beta^{s_2}$, we can improve welfare. In Case 3, welfare can be improved as in Case 1 (and also as in Case 2).

Moreover, (32) is implied by our construction. We have picked, in fact, s_1, s_2 such that $\beta^{s_1} \leq \overline{\beta}_{o}^{n} < \beta^{s_2}$, and we have just showed that $s_1 = r_o^n$ and $s_2 = H-(n-r_o^n)+1$. ♢

Lemma A . 2 *There exists an $r_s^n < n$ such that S^n consists of the first r_s^n agents and the last $n - r_s^n$ agents; i.e., $S^n = \{1,\ldots,r_s^n, H-(n-r_s^n)+1,\ldots,H\}$. Moreover, if we let $\overline{\beta}_{s}^{n} = \frac{1}{n}\sum_{h\in S^n} \beta^h$, then*

$$\beta^{r_s^n} < \overline{\beta}_{s}^{n} < \beta^{H-(n-r_s^n)+1}. \tag{33}$$

Proof of Lemma A . 2 By induction. The case $n = 2$ is trivial. Assume the statement holds for $n-1$. Let

$$\tilde{h} := \text{ argmax}_{h'\notin S^{n-1}} \sum_{h\in S^{n-1}} \left(\beta^{h'}-\beta^{h}\right)^2.$$

But note that $\sum_{h\in S^{n-1}} \left(\beta^{h'}-\beta^{h}\right)^2 = (n-1)\left(\beta^{h'}-\overline{\beta}_{s}^{n-1}\right)^2$; and hence $\tilde{h}$ is either $r_s^{n-1}+1$ or $H-(n-r_s^{n-1})$.

To prove (33), we first write

$$\overline{\beta}_{s}^{n} = \frac{1}{n}\left((n-1)\overline{\beta}_{s}^{n-1} + \beta^{\tilde{h}}\right), \tag{34}$$

and consider three cases.

Case 1: $\beta^{\tilde{h}} < \overline{\beta}_{s}^{n-1}$. Then, by the induction hypothesis, $\beta^{H-(n-r_s^{n-1})+1} > \overline{\beta}_{s}^{n-1}$, and hence, using (34), $\overline{\beta}_{s}^{n-1} > \overline{\beta}_{s}^{n} > \beta^{\tilde{h}}$, which implies (33).
Case 2: $\beta^{\tilde{h}} > \overline{\beta}_{s}^{n-1}$. Then, by the induction hypothesis, $\beta^{r_s^{n-1}} < \overline{\beta}_{s}^{n-1}$, and hence, using (34), $\overline{\beta}_{s}^{n-1} < \overline{\beta}_{s}^{n} < \beta^{\tilde{h}}$, which implies (33).
Case 3: $\beta^{\tilde{h}} = \overline{\beta}_{s}^{n-1}$. This case can only occur if $n = H$, which is excluded. ♢

Let $W_o^n := \sum_{h\in O^n} \left(\beta^h\right)^2 - n\left(\overline{\beta}_{o}^{n}\right)^2$, and $W_s^n := \sum_{h\in S^n} \left(\beta^h\right)^2 - n\left(\overline{\beta}_{s}^{n}\right)^2$.

Lemma A . 3 $W_o^n = W_s^n$.

Proof of Lemma A . 3 By induction. $O^2 = S^2$ by definition. Assume the statement holds for $n-1$. Let $O_{-1}^n \subset O^n$ denote a possible $(n-1)$-dimensional subset of O^n.
By the induction hypothesis,

$$\sum_{h,h' \in S^{n-1}} \left(\beta^h - \beta^{h'}\right)^2 \geq \sum_{h,h' \in O_{-1}^n} \left(\beta^h - \beta^{h'}\right)^2, \ \forall O_{-1}^n \subset O^n. \tag{35}$$

Also, by the definition of S^n, either $S^n = S^{n-1} \bigcup \{r_s^{n-1}+1\}$ or $S^n = S^{n-1} \bigcup \{H-(n-r_s^{n-1})\}$. We consider only the first case; the second is symmetric and is left to the reader: $r_s^n = r_s^{n-1}+1$. Then again by the definition of S^n, and (31),

$$\sum_{h \in S^{n-1}} \left(\beta^{r_s^n} - \beta^h\right)^2 \geq \sum_{h \in S^{n-1}} \left(\beta^{h'} - \beta^h\right)^2, \ \forall h' \notin S^{n-1}. \tag{36}$$

Let $O^n := O_{-1}^n \bigcup \{\hat{h}\}$. We now write

$$W_s^n = \sum_{h,h' \in S^{n-1}} \left(\beta^h - \beta^{h'}\right)^2 + \sum_{h \in S^{n-1}} \left(\beta^{r_s^n} - \beta^h\right)^2, \text{ and} \tag{37}$$

$$W_o^n = \sum_{h,h' \in O_{-1}^n} \left(\beta^h - \beta^{h'}\right)^2 + \sum_{h \in O_{-1}^n} \left(\beta^{\hat{h}} - \beta^h\right)^2. \tag{38}$$

But by (35), the first term of (37) is greater than the first term of (38). We now proceed by picking $\hat{h} \in O^n$ such that the second term of (37), also, is greater than the second term of (38).

We can first simplify

$$\sum_{h \in S^{n-1}} \left(\beta^{r_s^n} - \beta^h\right)^2 - \sum_{h \in O_{-1}^n} \left(\beta^{\hat{h}} - \beta^h\right)^2 \quad \text{to}$$

$$(n-1)\left(\left(\beta^{\hat{h}} - \overline{\beta}_s^{n-1}\right)^2 - \left(\beta^{r_s^n} - \frac{1}{n-1}\sum_{h \in O_{-1}^n} \beta^h\right)^2\right).$$

Then, using (36),

$$\sum_{h \in S^{n-1}} \left(\beta^{r_s^n} - \beta^h\right)^2 - \sum_{h \in O_{-1}^n} \left(\beta^{\hat{h}} - \beta^h\right)^2$$

$$\geq (n-1)\left(\left(\beta^{\hat{h}} - \overline{\beta}_s^{n-1}\right)^2 - \left(\beta^{\hat{h}} - \frac{1}{n-1}\sum_{h\in O_{-1}^n} \beta^h\right)^2\right), \quad \text{if } \hat{h} \notin S^{n-1}.$$

Finally,

$$(n-1)\left(\left(\beta^{\hat{h}} - \overline{\beta}_s^{n-1}\right)^2 - \left(\beta^{\hat{h}} - \frac{1}{n-1}\sum_{h\in O_{-1}^n} \beta^h\right)^2\right) \tag{39}$$

$$= (n-1)\left(\left(\frac{1}{n-1}\sum_{h\in O_{-1}^n} \beta^h - \overline{\beta}_s^{n-1}\right)\left(2\beta^{\hat{h}} - \frac{1}{n-1}\sum_{h\in O_{-1}^n} \beta^h\right)^2 - \overline{\beta}_s^{n-1}\right).$$

We are now ready to choose $\hat{h} \notin S^{n-1}$ so as to show that (39)> 0. There are several cases.

Case 1: $\beta^{r_o^n} = \beta^{r_s^n}$. In this case, trivially, $O^n = S^n$.
Case 2: $\beta^{r_s^n} < \beta^{r_o^n} \leq \overline{\beta}_s^{n-1}$. In this case, we pick $\hat{h} = r_o^n$ (note that $r_o^n \notin S^{n-1}$). Then,

$$\beta^{r_o^n} < \overline{\beta}_o^n < \frac{1}{n-1}\sum_{h\in O_{-1}^n} \beta^h \leq \overline{\beta}_s^{n-1},$$

which directly implies that (39)> 0.
Case 3: $\beta^{r_o^n} < \beta^{r_s^n}$. By the definition of S^n, then, $\beta^{r_o^n} < \beta^{r_s^{n-1}}$. In this case, we pick $\hat{h} = H - (n - r_o^n) + 1$.

We first show by contradiction that $H - (n - r_o^n) + 1 \notin S^{n-1}$. Suppose $H - (n - r_o^n) + 1 \in S^{n-1}$. Then $r_o^n < \hat{h} = H - (n - r_o^n) + 1 \leq r_s^{n-1}$. But then, (35) and (36) imply that $\overline{\beta}_o^n < \beta^{\hat{h}} < \beta^{r_s^n} < \overline{\beta}_s^n$; this is a contradiction, since in such construction O^n contains some elements in common with S^n and all other elements greater than the remaining elements in S^n.

The choice of $\hat{h} = H - (n - r_o^n) + 1$ implies that

$$\beta^{\hat{h}} > \overline{\beta}_o^n > \frac{1}{n-1}\sum_{h\in O_{-1}^n} \beta^h \geq \overline{\beta}_s^{n-1},$$

which in turn implies that (39)> 0.
Case 4: $\beta^{r_o^n} > \beta^{r_s^{n-1}}$. This case is not possible. Otherwise, by Lemma 1, $\overline{\beta}_o^n > \beta^{r_o^n}$. But $r_o^n \geq r_s^n$, and hence $\overline{\beta}_o^n > \overline{\beta}_s^n$, which is not possible since in such construction O^n contains

some elements in common with S^n and all other elements smaller than the remaining elements in S^n. $\diamondsuit$

Proposition 8 is proved in the text.

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