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Christian Keuschnigg

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Christian Keuschnigg, University of St Gallen and CEPR

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Centre for Economic Policy Research
90–98 Goswell Rd, London EC1V 7RR, UK
Tel: (44 20) 7878 2900, Fax: (44 20) 7878 2999
Email: cepr@cepr.org, Website: www.cepr.org

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ABSTRACT

Optimal Public Policy for Venture Capital Backed Innovation*

This Paper discusses the role of public policy towards the venture capital industry. The model emphasizes four margins: supply of entrepreneurs due to career choice, entry of venture capital funds and search for investment opportunities, simultaneous entrepreneurial effort and managerial advice subject to double moral hazard, and mark-up pricing when the successful firm introduces a new good. The Paper derives an optimal policy that succeeds in implementing a first best allocation in decentralized equilibrium. It also considers short- and long-run comparative static and welfare effects of piecemeal reform with regard to the capital gains tax, innovation subsidy, public R&D spending and other policy initiatives.

JEL Classification: D82, G24, H21, H25 and H32
Keywords: double moral hazard, innovation, public policy and venture capital

Christian Keuschnigg
University of St Gallen, IFF-HSG
Varnbüelstrasse 19
CH-9000 St Gallen
SWITZERLAND
Tel: (41 71) 224 3085
Fax: (41 71) 224 2670
Email: christian.keuschnigg@unisg.ch

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1 Introduction

Young technology firms are an important source of innovation and growth. For this reason, policy makers have increasingly focussed attention to the conditions for business creation in the high technology sectors. It seems that a large number of high quality entrepreneurs with promising ideas can emerge only in an active research environment. However, potential entrepreneurs must confront some formidable barriers if they intend to commercialize their ideas by starting a firm. Among the most often cited barriers to start new firms are lack of capital and commercial inexperience. Innovative young firms have high potential but are very risky as well. Developing a business idea into a new marketable product involves formidable technological and managerial risks. Given that these firms cannot show yet a past track record and have not yet accumulated sufficient collateral, it is difficult to raise external risk capital. A further difficulty for external financiers is that the main ideas are embodied in the entrepreneur's person. Start-up entrepreneurs have not yet proven their ability to manage a firm. They need not only money to finance start-up investment and research expenditure but also business contacts, strategic advice and other managerial support in building the firm. Experienced venture capitalists (VCs) can offer "informed capital" by giving both money and managerial advice.

There is a considerable theoretical and empirical literature in financial economics that explores how VCs screen, select, finance, monitor and advise their portfolio companies [see Aghion and Tirole (1994), Bascha and Walz (2001), Bergemann and Hege (1998), Casamatta (2002), Cumming and MacIntosh (2002), Hellmann (1998), Hellmann and Puri (2000,2002), Inderst and Müller (2003), Kanniainen and Keuschnigg (2002), Kaplan and Strömberg (2000,2001), Lülfesmann (2001), Repullo and Suarez (1999), Schmidt (2002), and the research summarized in Gompers and Lerner (1999)]. This literature is largely partial equilibrium in nature and studies in detail the dynamics of the relationship and optimal contractual arrangements between VCs and entrepreneurs. Gromb and Scharfstein (2002) and Michelacci and Suarez (2002) investigate models of equilibrium entrepreneurship. This paper, in contrast, is on the real side of VC finance which is less intensively studied in the literature [see the discussion of Gompers and Lerner (2001)]. An analysis of start-up entrepreneurship and innovation in industry equilibrium is all the more important since the main policy interest in a viable VC sector focuses on its role as an engine of innovation driven growth and job creation in new industries [European Commission (1994,1998), Botazzi and DaRin (2002)].

The analysis of a sample of firms located in Silicon valley by Hellmann and Puri (2000, 2002) shows that VC importantly enhances the professionalization and commercial orientation of young firms.[1] VC backed firms introduce more radical innovations and pursue more aggressive market strategies compared to other start-ups. For example, once a VC joins the firm and provides finance, the probability of introducing the new product jumps up by a factor of more than three! Rapid market introduction is strategically important because the first firm enjoys a first mover advantage. On a more aggregate level, Kortum and Lerner (2000) show that a Dollar of R&D spending in VC backed firms creates more patents and more radical innovations than the same expenditure in other firms. They calculate that VC financed R&D accounts for roughly 14 percent of U.S. industrial innovation in 1998 although it amounts to only about 3 percent of all R&D funds. This empirical evidence shows that VC significantly promotes innovation and business growth.

The real effects of venture capital, i.e. the ability to locate and select promising projects and to add value in terms of strategic business advice, depend not only on the VCs' own managerial qualifications and investment knowhow but also on their incentives to be actively engaged in portfolio companies. The supply of experienced financiers with useful business contacts and knowledge of the industry is a scarce resource that is not easily accumulated in short order and may become a considerable bottleneck in the development of a healthy VC sector. An equilibrium analysis of the VC industry should pay due attention to the slow entry of experienced financiers. The other precondition for the

[1] Keuschnigg and Nielsen (2002) model the coexistence of bank and VC financed firms in a common market where the strategic advice of VCs helps firms to earn a quality premium over other firms in the market. See Black and Gilson (1998) for an informal discussion of bank versus VC finance.

development of an active VC sector and a high rate of business creation is the supply of entrepreneurs with innovative ideas. In fact, representatives of the industry often complain about the lack of high quality entrepreneurs.[2] It seems that the creation of highly innovative firms requires an active research environment. It is no accident that the VC industry is geographically concentrated in the neighborhood of publicly and privately funded centers of basic research that host numerous researchers who might consider to turn their research ideas into a business start-up. The government might be able to concentrate research spending in areas with a potential for commercial applications and a correspondingly high probability of spin-offs. Basic research spending should thus raise the probability of researchers coming up with innovative business ideas.

To turn a promising idea into a new firm, researchers must also face the right incentives to give up safe employment for a risky entrepreneurial career. Given their lack of commercial experience, entrepreneurs need informed capital rather than passive bank lending. In VC finance, the entrepreneur and VC must each make a separate effort and contribute a unique input (research expertise of the entrepreneur and commercial experience of the VC) to turn the venture into a success. A suitable matching of entrepreneurs with financiers and, therefore, the frequency of VC financed business creation will depend on equilibrium market tightness, i.e. the relative supply of entrepreneurs looking for finance and of VCs in search of investment opportunities. In focusing on the effects of VC on the quality and quantity of innovative business creation, this paper necessarily adopts a somewhat simpler model of financial contracting that captures nevertheless the essence of the problem. Since both the entrepreneur and the VC must simultaneously contribute to the venture, they need both to participate in the upside potential of the firm. A straight equity contract is therefore optimal in this simple framework of double moral hazard.[3]

[2]Indeed, the case study by Becker and Hellmann (2002) identified the lack of skilled entrepreneurs as a prime reason for the slow development of VC in Germany.

[3]The finance literature has emphasized the importance of staging, syndication, allocation of control rights and use of convertible securities to overcome incentive problems [see Bergloef (1994), Gompers (1995), Hellmann (1998), Nöldeke and Schmidt (1998), Neher (1999), Hart (2001), Tirole (2001) and

The paper investigates the welfare properties of industry equilibrium and analyzes a range of public policies that are meant to promote innovation by facilitating start-up entrepreneurship in new industries [see Boadway and Tremblay (2002) for an informal policy discussion of welfare issues with respect to innovation]. A specific policy analysis of VC finance from a public finance perspective is largely missing. The financial economics literature has abstained from an explicit formal analysis of public policy [see Lerner (2002) for a verbal discussion of important policy issues], and the public finance literature has not considered specific models of VC backed entrepreneurship. Gompers and Lerner (1998) and Poterba (1989a,b) have empirically investigated the effects of the capital gains tax on VC financed entrepreneurship and have found that the impact of the tax is mainly on the entrepreneurs' career choice. Gordon (1998), Cullen and Gordon (2002), and Rosen (2002) demonstrate that taxes can impair entrepreneurship. None of these contributions in public finance consider the specific aspects of VC finance.

This paper is close to Keuschnigg (2002) and Keuschnigg and Nielsen (2003a,b) who analyze the effects of taxes on VC backed entrepreneurship. These authors, however, allocate all bargaining power to VCs, do not consider the matching of VC in a search market and also do not model the effect of entrepreneurship on innovation. Keuschnigg and Nielsen (2002) consider equilibrium entrepreneurship when a minority of entrepreneurs succeeds to raise VC finance and the rest must resort to bank finance. They discuss policy issues but exclude the role of VC in promoting innovation. They also do not focus on the entry of VCs in industry equilibrium which distinguishes the short- and long-run equilibrium in this paper. The paper is importantly related to Inderst and Müller (2003) and, in particular, shares their short- and long-run analysis of VC entry in a search market but importantly extends their work by endogenizing venture returns in an explicitly modeled output market for innovative goods and by an extended analysis of public policy.

Schmidt (2002), for example]. Taking account of these aspects of VC contracts requires a dynamic and more difficult modeling. Kaplan and Strömberg (2001, p.427) emphasize, however, that cash flow and control rights are more like complements rather than substitutes. Our simpler static model with straight equity finance should thus roughly capture the incentive problems between entrepreneurs and VCs.

The paper now proceeds with section 2 where a model of VC backed innovation is introduced. Out of a larger number of potential entrepreneurs, a fraction of them start a firm once they have met with a VC. The VC finances the start-up investment cost and advises the firm in exchange for an equity stake. Based on the entrepreneur's business plan, each start-up firm develops a new, specialized product or service and, if successful, introduces the brand on the market. Each VC fund finances and advises a portfolio of firms. In the short-run, the number of VCs is fixed. In the long-run equilibrium with free entry subject to a fixed start-up cost, not only the number of start-ups but also the number of VCs is endogenously explained such that the last fund raised just breaks even.

Section 3 considers the role of public policy in providing real inputs as well as strengthening private incentives. In principle, government should be active in four dimensions. First, it should spend on basic research and development and thereby provide a public good that helps potential entrepreneurs to come up with innovative ideas. Second, it should internalize the positive R&D spillovers of private start-up firms by means of an appropriate subsidy. Third, it should strengthen private incentives for joint effort which tend to be too low in a situation of double moral hazard in the relationship of VCs and entrepreneurs. And finally, if the bargaining power does not correspond to the effectiveness of financiers or entrepreneurs in generating a new profitable start-up firm, the search activity on one side of the market creates an externality on the other side which requires to appropriately encourage the search efforts of VCs or entrepreneurs [see Hosios (1990) on efficiency in search markets]. Section 3 analyzes the welfare properties of the market equilibrium and then derives an optimal policy that is able to decentralize a first best allocation in the VC industry. The policy involves basic research spending, output subsidies to successfully established firms, revenue subsidies to entrepreneurs and VCs, a tax on start-up investment spending, and specific taxes or subsidies on seed investments by entrepreneurs and acquisition activity of VCs. Starting from an untaxed market equilibrium, section 4 considers the comparative static and welfare effects of various piecemeal policy reforms. Section 5 concludes.

2 The Model

To take account of the small size of the venture capital (VC) industry, we distinguish a large traditional and a small innovative sector where start-up firms introduce new goods. Production and finance in the traditional sector poses no special problems. In the entrepreneurial sector, however, the market orientation and growth of new firms can benefit substantially from the close cooperation and joint effort of technology entrepreneurs and commercially experienced venture capitalists (VCs). VC is the preferred mode of finance in the early stage development of innovative start-up firms. Business creation involves a number of steps, starting with the entrepreneur's basic business idea and ending with the successful market introduction of a new good.

For the analytical investigation, it is useful to distinguish the following sequence of events ("time line"). First, the government specifies its policy towards the VC industry. Second, given a specific policy environment, researchers generate business ideas but have no own funds. If they decide to start their own firm, they search for VC finance. VCs, in turn, search for profitable investment opportunities. Third, after finding a suitable partner in a matching market, each pair of VC and entrepreneur bargain over an incentive compatible financial contract. Fourth, given the terms of the contract, and after having sunk the initial start-up investment cost, entrepreneurs and VCs expend joint effort to make the firm a success. At least in the early phase, the entrepreneur is relatively more preoccupied with further technological development while the VC supports the firm with managerial advice, helps in establishing contacts with key suppliers and clients, in hiring specialized personnel and in developing business strategies of the firm. The close cooperation of entrepreneurs and VCs is best seen as a joint effort in a team that is potentially fraught with incentive problems (double moral hazard). Fifth, risk is resolved, i.e. the firm matures to production stage or fails. The fraction of successful firms reflects the prior efforts of VCs and entrepreneurs. Finally, the profits of the firm are shared according to the terms of the contract, and agents spend on consumption of innovative and traditional goods. Given the sequential nature of private decisions, the model is solved by backward induction. We thus start with production of established firms and consumer spending, given the income derived in previous stages of the VC cycle.

2.1 Demand

Depending on the previous career, agents are endowed with disposable net income y^i which they spend on traditional and innovative goods,[4]

$$U^{i*} = \max_{D^i, Z^i} \left\{ u\left(D^i\right) + Z^i - l^i \quad s.t. \quad Z^i + V D^i \leq y^i \right\}. \tag{1}$$

Earning income may require a welfare reducing effort l^i by agent $i \in [0,1]$. The homogeneous traditional good is the numeraire and is consumed in quantity Z^i. The quantity D^i yields convave utility, $u\left(D^i\right) > 0 > u''\left(D^i\right)$, where D^i is a subutility over consumption x_j^i of differentiated goods as in Dixit and Stiglitz (1977). The upper index denotes the identity of the agent and the lower index indicates the specific brand or variety of the innovative good, $j \in [1, N]$, which is available at a price q_j. The basket of innovative goods is purchased at a price index V given by

$$V \cdot D^i = \min_{x_j^i} \left\{ \int_0^N q_j x_j^i dj \quad s.t. \quad \left[\int_0^N \left(x_j^i\right)^{1/\rho} dj \right]^\rho \geq D^i \right\}, \qquad \rho = \frac{\sigma}{\sigma - 1} > 1. \tag{2}$$

Different varieties are imperfectly substitutable where $\sigma = \frac{\rho}{\rho - 1} > 1$ is the constant elasticity of substitution.[5] Given a quantity D^i, standard optimization yields a closed form solution for derived demand and the price index,

$$x_j^i = \left(V/q_j\right)^\sigma \cdot D^i, \quad V = \left[\int_0^N q_j^{1/(1-\rho)} dj \right]^{1-\rho}. \tag{3}$$

Assuming further a convenient isoelastic specification of utility $u(D) = \phi^{1/\lambda} \cdot \frac{D^{1-1/\lambda}}{1-1/\lambda}$, demand for the final good (or basket of differentiated brands) is[6]

$$u'\left(D^i\right) = V, \qquad D = \phi V^{-\lambda}, \qquad \lambda < \sigma. \tag{4}$$

[4] One might also think of the 'new' and 'old' economy.

[5] When $\sigma \to \infty$, the innovative good is homogeneous with perfectly substitutable varieties.

[6] Firm entry expands supply but also stimulates demand. The restriction $\lambda < \sigma$ ensures that firm entry reduces excess demand in the product market.

The assumed structure of preferences implies several properties of demand. The separability and linearity of preferences in (1) eliminates income effects in effort choice and demand for innovative goods. All agents thus demand the same quantity D. Assuming a mass one of households, this is also aggregate demand. For the same reason, derived demand x_j is identical across different agents. With a large number N of varieties, individual agents take the price index and aggregate quantity, V and D, as given and independent of any specific variety price q_j. Therefore, the *perceived price elasticity of demand* for good j is $(dx_j/x_j) / (dq_j/q_j) = -\sigma$, see (3).

Since demand for innovative goods as in (2-4) is indepent of income, all changes in income are absorbed by demand for traditional goods, $Z^i = y^i - VD$. Substituting this into the utility function yields indirect utility equal to the sum of effort adjusted income and consumer surplus over innovative goods,

$$U^{i*} = y^i - l^i + C(V), \quad C(V) = u(D) - VD, \quad C'(V) = -D. \tag{5}$$

The derivative of consumer surplus C' uses the optimality condition (4).

2.2 Production and Effort

Production in the traditional sector is standard and poses no special incentive problems. One unit of labor yields one unit of output. Perfect competition implies a wage equal to the output price which is normalized to unity, $w = 1$. With L workers employed in manufacturing, supply of the traditional good is L.

In the innovative sector, each entrepreneurial firm is specialized in the production of one brand and supplies the entire market segment. Once a start-up firm has successfully matured to production stage, it produces x_j units of brand j, using x_j units of the traditional good as an input, i.e. the technology is linear with an input output coefficient of one. Government possibly offers an output subsidy z^X. Since firms are monopolists in their market segment and brands are imperfectly substitutable, they can exploit local market power to set a profit maximizing price,

$$\pi_j = \max_{q_j} \left\{ \left(1 + z^X\right) q_j x_j - x_j \quad s.t. \quad (dx_j/x_j) / (dq_j/q_j) = -\sigma \right\}. \tag{6}$$

Maximizing profits leads firms to set the price equal to a mark up ρ over unit costs (equal to one). Pricing thus results in a profit margin

$$\left(1 + z^X\right) q = \rho, \qquad \pi = (\rho - 1)\, x = \left(1 + z^X\right) qx/\sigma. \tag{7}$$

Sales are read from the demand curve in (3).

The profits of a venture as in (7) will materialize only if the firm has successfully matured to production stage. In the start-up phase, the original business idea of an entrepreneur must be developed to a marketable good which is a highly risky activity subject to a high failure rate. Only a fraction p of all start-up firms will eventually succeed while the rest fails before any profit is earned. In case of failure, output is zero by assumption and profits never materialize. It is assumed that the firm's success probability depends on the entrepreneur's more technologically oriented development effort e and the VC's managerial involvement and advice a. Efforts are assumed intangible, non-verifyable and non-contractible. Both types of inputs are continuous. To keep the analysis tractable, the success probability is specialized to

$$p = p\,(e, a) = p_0 e^{\epsilon} a^{\alpha}, \qquad \alpha + \epsilon < 1. \tag{8}$$

The parameter restriction implies decreasing returns to effort, i.e. the success rate is concave in a proportional increase in joint effort.

Apart from intangible effort, the firm must also invest a fixed quantity I of traditional goods during the start-up phase. The private cost amounts to $\left(1 + t^I\right) I$ since such investment is possibly subsidized ($t^I < 0$) or taxed ($t^I > 0$) by the state.

2.3 Making a Deal

Entrepreneurs are assumed to have no own wealth and must rely on an outside investor to fund the project. Their only capital is a business idea. Being inexperienced in commercial matters, they seek not only capital but also strategic business advice. Such informed capital is offered by VCs endowed with industry experience and managerial knowhow. After having located a suitable partner, the VC and the entrepreneur must strike a deal. When bargaining over a contract they anticipate how the financial incentives will later on determine the level of effort. Since both partners make a valuable contribution, they must both share in the upside potential of the firm to strengthen their incentives for effort at the margin. For this reason, a simple equity contract is optimal in the current framework. The deal is thus that the VC pays a price Q for a stake $1-s$ in the firm, leaving a share s to the entrepreneur. Since the entrepreneur has no own wealth, the VC's price $Q = B + \left(1+t^I\right) I$ must pay for the entire private start-up cost plus some upfront payment B. With this arrangement, expected income Y^E of the entrepreneur and Y^F of the VC (index F for financier) amounts to

$$Y^E = \left(1-t^E\right)\left[sp\pi + B\right], \quad Y^F = \left(1-t^F\right)\left[(1-s)\,p\pi - B - \left(1+t^I\right) I\right]. \tag{9}$$

If the firm were sold after the start-up phase, an investor would pay a price π equal to the level of profits at production stage. The value of the venture is π. The VC's expected capital gain on this deal is $(1-s)\,p\pi - Q$ and is subject to a capital gains tax at rate t^F if there is any. The entrepreneur makes an initial capital gain B upon concluding the deal, and $sp\pi$ when the firm is sold at the beginning of the production stage.[7]

The expected income or capital gain must be high enough to compensate for the entrepreneur's effort and opportunity cost which is the foregone outside wage in traditional manufacturing. Both agents incur intangible effort costs which are assumed linear in the level of efforts with β and γ being the relevant marginal costs. The entrepreneur's bargaining position is strengthened by the fact that she can leave the firm and earn a wage in the traditional sector if no agreement is found. By this participation condition,

[7] The distinction between capital gains and ordinary income is somewhat blurred in our framework. One might alternatively interprete t^E as the entrepreneur's personal income tax rate which applies to her dividends $s\pi$ received during production stage and her upfront payment B which could also be a non-performance related base salary.

her expected income net of effort costs must be at least as high as her outside wage, i.e. she must receive a non-negative surplus $S^E \geq 0$,

$$\begin{aligned} S^E &= Y^E - \beta e - w, \quad S^F = Y^F - \gamma a, \\ S &= Y^E + Y^F - \gamma a - \beta e - w. \end{aligned} \tag{10}$$

The VC must not only break even on its investments to pay back the funds raised but must also receive compensation for the managerial effort spent on advising the portfolio companies.[8] Adding up the surplus of each side, the joint surplus generated by a start-up is $S = S^E + S^F$.

When bargaining over a deal, the VC and entrepreneur must anticipate how the terms of the contract determine incentives for effort at later stages. At that time, the outside wage, the upfront payment, and capital spending are all sunk, leaving agents to choose simultaneously a level of effort that maximizes their remaining surplus. This gives rise to the two incentive compatibility constraints IC^E and IC^F which are stated as first order conditions below. The Nash bargaining problem is

$$\begin{aligned} \max_{s,B,e,a} \quad & \left(S^E\right)^{\xi} \cdot \left(S^F\right)^{1-\xi} \qquad s.t. \\ IC^E: \quad & \left(1 - t^E\right) s p_e\left(e,a\right) \pi = \beta, \\ IC^F: \quad & \left(1 - t^F\right) \left(1 - s\right) p_a\left(e,a\right) \pi = \gamma. \end{aligned} \tag{11}$$

Since the upfront payment B is already sunk at effort stage, it cannot have an impact on effort choice. Optimal efforts exclusively depend on the incentives created by the agreed profit share and are implicitely determined by the two incentive constraints, giving $e(s,\cdot)$ and $a(s,\cdot)$. Substituting into the definitions of surplus reduces the problem to finding profit maximizing values of the equity stake s and upfront payment B. These two components of the financial contract differ in an important way that allows for a recursive solution. The profit share influences incentives in subsequent effort stage and, thus, the size of the pie to be distributed, while the upfront payment B does not since it is not related to performance. In bargaining over the joint surplus, the entrepreneur and VC

[8] The VC's entry decision occurs at the beginning of the VC cycle. Her opportunity cost is distributed on a portfolio of firms, see section 2.5 below.

will therefore first agree on a Pareto efficient share s to advance their joint interest and then adjust B to find an appropriate distribution of the surplus.[9]

Proposition 1 ***(Profit Sharing and Joint Effort)*** *(a) Entrepreneurial effort e and managerial support a increase with venture returns π and decline with taxes t^E and t^F. A higher share s boosts the entrepreneur's effort but undermines VC advice.*
(b) The Pareto-optimal share s for the entrepreneur increases with a higher tax t^F on VC profits and falls with higher taxes on entrepreneurial income t^E. The profit share is independent of venture returns and also of a uniform tax $t^E = t^F$.

Proof. See appendix A and, in particular, equations (A.2) and (A.6) for part (a) and (A.6-8) for part (b). ■

Taxes discourage effort because they cut the returns to effort. More interestingly, effort depends not only on one's own tax rate but also on the tax rate applied to the other partner. When the VC gets taxed more heavily, she advises less intensively. Since efforts are complements, less VC support also reduces the returns to entrepreneurial effort. For this reason, a tax on VCs reduces the entrepreneur's effort. The same cross effect applies to the taxation of entrepreneurs, see (A.2). Furthermore, allocating a larger stake to the entrepreneur at the expense of the VC's share strengthens the returns to entrepreneurial effort while weakening the VC's incentives, with obvious consequences for effort choice.

Part (b) is also intuitive. Raising the entrepreneur's tax discourages her effort and makes her contribution to the joint surplus more expensive. Hence, the team relies more on VC effort by raising the VC's share and leaving a correspondingly smaller share to the entrepreneur. When the VC is taxed more, the team finds it optimal to raise the entrepreneur's share. If the same tax rate applies to both, the Pareto-optimal share is a constant as in (A.8), and exclusively depends on the relative effectiveness of the VC's and entrepreneur's effort in raising the survival rate. In the symmetric case of $\alpha = \epsilon$, the

[9] This corresponds to the notion of a "Pareto frontier" as in Inderst and Mueller (2003).

share is exactly half, $s = 1/2$. Note finally, that in all cases the agreed equity stake is independent of the upfront payment.

Proposition 2 ***(Sharing Joint Surplus)*** *Bargaining over the upfront payment B, the entrepreneur gets a share $\tilde{\xi}$ and the VC a share $1 - \tilde{\xi}$ of the joint surplus,*

$$S^E = \tilde{\xi} \cdot S, \quad S^F = \left(1 - \tilde{\xi}\right) \cdot S, \quad \tilde{\xi} \equiv \frac{\left(1 - t^E\right) \xi}{\left(1 - t^E\right) \xi + \left(1 - t^F\right) \left(1 - \xi\right)}. \tag{12}$$

Proof. Problem (11) yields a necessary condition $\left(1 - t^E\right) \xi S^F = \left(1 - t^F\right) \left(1 - \xi\right) S^E$. Noting $S = S^E + S^F$ gives (12). The implicitely determined upfront payment B allocates to each partner a share of the joint private surplus as in (12). ■

The joint surplus is divided among the two partners according to their modified bargaining power $\tilde{\xi}$. Only if the capital gains tax is uniform, the entrepreneur's share in the joint surplus becomes independent of tax rates, $\xi = \tilde{\xi}$. Note finally that any previous investments or search costs are sunk at this stage and therefore cannot affect the bargaining over the distribution of the joint surplus by means of B.

2.4 Matching of Venture Capital

In the VC market, F VCs search for profitable investment opportunities and R researchers, or potential technology entrepreneurs, consider to start a firm and seek informed capital. VCs are specialized in rather different industries and therefore have accumulated specific contacts with clients, suppliers, and specialized personnel that are valuable only in the same industry. The same holds for the specific knowledge of the market opportunities of certain technologies. The VCs' commercial and managerial knowhow is therefore not equally useful for all entrepreneurial ventures. A productive matching of VCs and entrepreneurs is therefore subject to frictions. It takes time to locate a suitable partner especially since a pair of VC and entrepreneur enter a close relationship in the start-up firm. In a static model, this time consuming search activity is reflected in the fact that only part of the potential entrepreneurs participating in the search market locate a financier, and only part of the financing offers are successfully invested.[10]

Formally, we assume that each of the R researchers who consider entrepreneurship prepares a business plan and searches for informed capital.[11] On the other side of the VC market, each VC chases for v deals, giving vF financing offers in total, but only a smaller number E of them is successfully matched and results in a deal. We refer to E as the number of entrepreneurs, or matched researchers that succeeded to locate a VC. A linear homogeneous technology $E = E(vF, R)$ determines the matching rates[12]

$$E/vF = f(\theta), \qquad E/R = f(\theta)/\theta, \qquad \theta \equiv R/vF. \tag{13}$$

Using $f(\theta) \equiv E(1, \theta)$, the matching rates satisfy $f'(\theta) > 0$ and $d[f(\theta)/\theta]/d\theta < 0$ as well as $0 < f(\theta) < 1$ and $0 < f(\theta)/\theta < 1$. Out of all vF financing offers, only a fraction $f(\theta)$ is actually financed. Therefore, a VC finances and advises an average number of portfolio firms equal to $E/F = f(\theta)v$. The ratio θ of potential entrepreneurs R to the number of financing offers vF is a measure of market tightness. In taking market tightness as given, a VC can generate a larger number of deals and end up with a larger portfolio of firms if she raises her search intensity v.

On the other market side, only a fraction $f(\theta)/\theta$ of potential entrepreneurs locate a VC. Matching splits potential entrepreneurs into $E = [f(\theta)/\theta]R = f(\theta)vF$ active VC financed entrepreneurs while the others, $R - E$, turn to their outside opportunity and remain workers. The higher is θ, the tighter is the VC market, and the less likely will

[10] In reality, the funds raised are usually considerably higher than the funds invested. The excess funds presumably wait for other promising investments.

[11] Before a VC joins the firm, these seed investments are often financed by family or other sources. The model treats this as intangible research and search effort rather than real expenditure.

[12] In the labor market, $E = m \cdot (vF)^{1-\eta} R^{\eta}$, $0 < \eta < 1$, is an empirically valid form, giving $f(\theta) = m \cdot \theta^{\eta}$ and $f(\theta)/\theta = m/\theta^{1-\eta}$. See Petrongolo and Pissarides (2001) on microeconomic underpinnings of the matching function. Diamond (1990) has introduced a search model of the credit market. Michelacci and Suarez (2002) and Inderst and Müller (2003) similarly apply a matching function to study the allocation of informed capital to start-up firms. They do not endogenize the VC's search intensity.

potential entrepreneurs locate VC finance to start a firm, $d\left[f\left(\theta\right)/\theta\right]/d\theta<0$. In contrast, VCs will identify profitable investments more easily, $f'\left(\theta\right)>0$, so that a larger fraction of their offers translates into a successful deal. The change in market tightness reflects the increase in R and vF on each side of the market. The sensitivity of successful matches, i.e. the number of VC backed start-ups, with respect to aggregate search activity of VCs and potential entrepreneurs is measured by the matching elasticities

$$\eta=\frac{R}{E}\frac{dE}{dR}=\frac{\theta f'}{f},\qquad 1-\eta=\frac{vF}{E}\frac{dE}{dvF}=\frac{f-\theta f'}{f}. \tag{14}$$

2.5 Searching for a Deal

Research and managerial skills are fundamentally different. It is thus assumed that an exogenously given part M of the population of mass one is endowed with managerial skills and the rest $1-M$ with variable research abilities that might lead with some effort to new technological ideas. To allow for a finitely elastic supply of potential entrepreneurs, we assume that agents differ by their inventive ability, i.e. the required research effort for a new business idea.[13] When an agent decides to enter the search competition for VC, she incurs a cost k^i and becomes a potential technology entrepreneur. This cost is interpreted as an effort cost or time input that is necessary to produce an idea for a new product or service and prepare a business plan. We assume, for simplicity, that the population with research skills is distributed uniformly in k^i, i.e. all types are represented with uniform density. Without loss of generality, research effort costs are specialized to

$$k^i=k\left(i,G\right)=\frac{k_0}{G}\cdot i^{\kappa},\quad k_i>0>k_G,\quad \kappa\equiv ik_i/k^i,\qquad i\in\left[0,1-M\right]. \tag{15}$$

Private research effort directed towards start-up activity can be reduced by public spending on basic research and development (R&D) in areas that are particularly amenable to

[13] The literature, in contrast, assumes either inelastic or perfectly elastic supply. Another key assumption of the paper is that this heterogeneity in research cost does not carry over to the quality of the business idea. Once the firm is started, they are all homogeneous. This helps to concentrate on the moral hazard problems during the start-up phase but cuts out adverse selection.

commercial applications. Basic research is a public good G that reduces the research cost of potential entrepreneurs and thereby raises the likelihood of a spinoff.

Preparing for an entrepreneurial career requires to invest in research effort and prepare a business plan that can be presented to a VC for financing the venture. The government may wish to offer a research grant or entry subsidy z^R to encourage potential entrepreneurs in the seed phase to experiment with new ideas. An agent considers entrepreneurship and participates in the search market for VC only if the expected return pays at least for this (re-)search effort, net of the entry subsidy. The expected return is the share of the joint surplus of the start-up firm that she expects to bargain with the VC as in Proposition 2. The number of entrepreneurs in search of VC is now defined by the critical agent R who just breaks even on her research effort cost net of the entry subsidy,

$$\frac{f(\theta)}{\theta} \cdot \tilde{\xi} S = k(R, G) - z^R. \tag{16}$$

Figure 1 illustrates how the indifference condition for the marginal researcher splits the population between entrepreneurs who do research on a potential business idea and, based on this seed investment, prepare a business plan to be presented to VCs. The figure also shows how increased public R&D funding as well as research grants for seed investments can raise the supply of entrepreneurs.

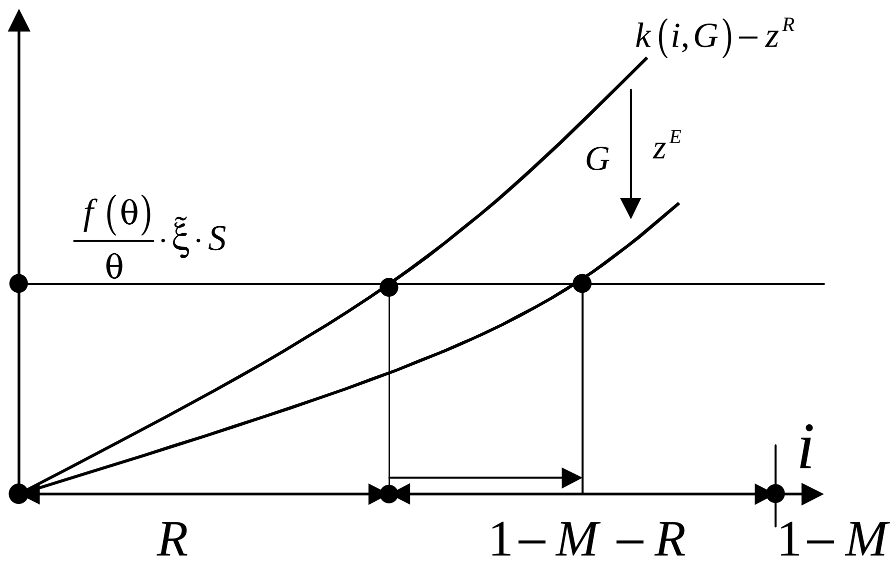

Figure 1: Seed Investments

In chasing for investment opportunities, a VC endogenously chooses search intensity v which corresponds to the number of financing offers posted. She thereby incurs progressively increasing search effort costs (time input)

$$\delta(v) = \delta_0 \frac{v^{1+\omega}}{1+\omega}, \qquad \delta'(v) > 0, \qquad \delta''(v) > 0. \tag{17}$$

The VC's search activity results in a deal only with some probability. The matching probabilities derive from aggregate VC search effort equal to $vF = \int_0^F v^i di$ under symmetry. In making v^i offers, an individual VC takes market tightness θ as given and expects to realize $f(\theta) v^i$ deals where each one yields a surplus S^F as in (12). A VC's optimal search intensity equates marginal expected benefit and cost from an extra offer,

$$\Omega = \max_{v^i} \left\{ v^i f(\theta) S^F - \delta\left(v^i\right) - w + z^F \right\} \quad \Rightarrow \quad f(\theta) \cdot \left(1 - \tilde{\xi}\right) S = \delta'\left(v^i\right). \tag{18}$$

This condition determines individual portfolio size $f(\theta) v$ which depends on individual search intensity, the VC's surplus, and aggregate market tightness.

A VC fund is viable only if expected surplus (profits less advisory effort cost) on the entire portfolio of firms suffices to compensate for total search cost $\delta(v)$ plus the foregone wage income w of its manager. It is assumed that VCs give up a managerial job in the old economy when choosing to set up a VC fund. The government may wish to stimulate entry of VCs with an entry subsidy z^F which will be relevant to determine the number of VC funds in the long-run equilibrium. In the short-run with a fixed number of VCs, the subsidy affects profits and, thereby, incentives for entry or exit of VCs.

2.6 Equilibrium

Only a fraction of R potential entrepreneurs with a business plan succeed to locate a VC, leading to E start-up firms as in (13). Depending on the effort of both the entrepreneur and VC, a new firm succeeds to develop and introduce the product on the market with probability $p(e, a)$ only. In the aggregate, only a fraction p of all start-ups matures to production stage, leaving N firms to supply differentiated goods,

$$N = p(e,a) \cdot E, \qquad E = f(\theta)\, v \cdot F. \tag{19}$$

In restricting attention to symmetric equilibria, the price and quantity indices in (2-3) are

$$V = N^{1-\rho} q, \qquad D = N^{\rho} x, \qquad VD = Nqx. \tag{20}$$

Since the component price is a fixed mark-up over the exogenously given unit cost as in (7), it cannot adjust to clear the product market. Instead, market clearing depends on sales x and thereby profits π per firm. Since profits are the ultimate reward of business formation, they determine the supply of new goods as in (19). On the demand side, when market size is finite, higher sales per brand must reduce demand for the number of differentiated products. The price index V increases with the price of components q but declines with product variety N. The latter effect reflects the gains from increasing specialization. More variety stimulates demand. Substituting demand for the composite good in (4) into (3) and replacing the price index by its symmetric form in (20) yields the consumers' trade-off between scale and variety. This trade-off yields the demand for variety as a function of quantity per brand,

$$N = \left[\frac{\phi}{xq^{\lambda}}\right]^{\mu}, \qquad \lambda < \sigma \quad \Rightarrow \quad \mu = \frac{1}{\rho - (\rho - 1)\lambda} > 0. \tag{21}$$

While the supply of variety in (19) increases in profits $\pi = (\rho - 1)\, x$ and thereby in sales per firm, the demand for variety in (21) falls when each single brand is consumed in larger quantity. The product market clearing level of sales per firm is achieved when the number of products supplied equals the number of products demanded.

Equilibrium must satisfy the resource constraints. The population is of mass one and is composed of different skills and professions. First, we have assumed in section 2.5 that there is an exogenously given mass M of agents with unique managerial skills who may choose among a management career in manufacturing or set up a VC fund to coach new firms. With F denoting the number of financiers, or VCs, the managerial skills are endogenously split according to $M = F + (M - F)$. The rest of the population

$1 - M$ is endowed with research skills enabling them to become workers or researchers, giving a split $1 - M = R + (1 - M - R)$ as in Figure 1. Finally, among all researchers with a business idea only a part E succeeds to obtain finance and are able to start a firm. After having sunk an effort cost in the initial attempt at entrepreneurship, the rest $R - E$ turns to employment in the traditional sector. The matching allocation in (13) implies a decomposition $R = E + (R - E)$. Total employment L in the traditional sector thus consists of $1 - M - R$ 'born workers' who avoided entrepreneurship right from the beginning, plus $R - E$ unmatched researchers, plus $M - F$ born managers who chose not to enter the VC business:

$$L = (1 - M - R) + (R - E) + (M - F) = 1 - E - F. \tag{22}$$

In other words, who is not a start-up entrepreneur or a VC ends up being a worker in the traditional sector. For simplicity, all types of agents earn the same wage in the old economy, irrespective of whether they are endowed with managerial or technological skills.

Several features of the model simplify the equilibrium solution. First, mark-up pricing in (7) fixes the compenent price q solely in terms of the output subsidy. Second, profit is proportional to output, $\pi = (\rho - 1)\, x$, so that the scale of production x also stands for profits or venture returns. Third, proposition 1 states that the entrepreneur's profit share is independent of venture returns and other endogenous variables, see also (A.5). Profit sharing may be affected only by differential tax rates. Therefore, joint effort and the success probability depend on profits, and thus x, as in (A.2), $p\,(x)$. Fourth, the bargaining solution of Proposition 2 together with (9-10) implicitely determines an upfront payment $B\,(x)$ which, in turn, fixes the joint surplus as a function of x, $S\,(x)$. Search investments depend on joint surplus and market tightness. With these results, the supply of variety in (19) depends on x and must match with variety demand in (21) which gives the product market condition PM below. Next, the entry condition (16) shows how many agents decide for a seed investment and participate in the search market, and is repeated as SR.

Condition SF gives the search effort of VCs as in (18):

$$\begin{aligned} PM: \quad \left[\phi/\left(xq^{\lambda}\right)\right]^{\mu} &= p(x)\cdot f(\theta)\, v\cdot F, \\ SR: \quad k(R,G)-z^{R} &= S(x)\cdot\tilde{\xi}\cdot f(\theta)/\theta, \qquad R=\theta vF, \\ SF: \quad \delta'(v) &= S(x)\cdot\left(1-\tilde{\xi}\right)\cdot f(\theta), \\ LR: \quad \Omega &= S(x)\cdot\left(1-\tilde{\xi}\right)\cdot f(\theta)\, v-\delta(v)-w+z^{F}. \end{aligned} \tag{23}$$

The supply of experienced VCs cannot expand in short order. The number of VC funds F is assumed fixed in the short-run. These VCs may, however, attract a variable number of start-ups if they raise their search intensity. In the *short-run equilibrium* with fixed F, the product market and investment conditions PM, SR and SF determine three unknowns x, θ and v. Note that SR and SF solve for $\theta(x)$ and $v(x)$ when $R=\theta vF$ is inserted. Substituting into PM then yields the equilibrium production scale, or venture returns.

In the short-run, VC funds derive rents on their scarce managerial resource. Such rents should succeed to attract more VCs after a sufficiently long period of skill formation. Rents tend to dissipate when the industry is crowded with more VCs. In the *competitive long-run equilibrium* with free entry of VCs, a zero profit condition $\Omega=0$ endogenizes F, the equilibrium number of VCs. Technically, SF and LR combine to $v\delta'(v)=\delta(v)+z^{F}-w$ and thereby fix the search intensity v. Given v, SR and SF solve for $\theta(F)$ and $x(F)$ which are substituted in PM to obtain the equilibrium number F of VCs.

3 Optimal Public Policy

This section identifies potential market distortions by comparing a socially optimal, first best allocation with the market equilibrium. The social optimum maximizes aggregate welfare as given in (A.13). Substituting consumer surplus by (5) and using $VD=Nqx$ and $N=pE$ yields the social objective function U^{*}. For convenience, we also repeat the other technological relationship that constrain the optimization problem:

$$\begin{aligned} &U^{*}=1+u(D)-xp(e,a)\,E-(\beta e+\gamma a+I+1)\,E-(\delta(v)+1)\,F-K-G, \\ &D=N^{\rho}x, \quad N=p(e,a)\,E, \quad E=f(\theta)\,vF, \quad \theta=\tfrac{R}{vF}, \quad K=\textstyle\int_{0}^{R}k(i,G)\,di. \end{aligned} \tag{24}$$

Now we find the efficient allocation x, e, a, v; R, F, G that maximizes social welfare. Denote the shadow price of the composite good by $u'(D) = V$, and that of a single component by q, i.e. $u'(D) \cdot \partial D / \partial x_j = q_j$. With symmetry,[14] $q = q_j = u'(D) \cdot N^{\rho-1}$. With these shortcuts, the necessary conditions for a socially optimal allocation are

$$
\begin{array}{llcl}
(a) & \frac{dU^*}{dx} & = & (q-1)\,N = 0, \\
(b) & \frac{dU^*}{de} & = & \left[(q\rho-1)\,xp_e - \beta\right] E = 0, \\
(c) & \frac{dU^*}{da} & = & \left[(q\rho-1)\,xp_a - \gamma\right] E = 0, \\
(d) & \frac{dU^*}{dv} & = & S^* \cdot \frac{dE}{dv} - \delta'(v)\,F = 0, \\
(e) & \frac{dU^*}{dR} & = & S^* \cdot \frac{dE}{dR} - k(R,G) = 0, \\
(f) & \frac{dU^*}{dF} & = & S^* \cdot \frac{dE}{dF} - \delta(v) - 1 = 0, \\
(g) & \frac{dU^*}{dG} & = & K_G - 1 = 0, \quad K_G \equiv -\int_0^R k_G(i,G)\,di, \\
(h) & S^* & = & p\pi - \beta e - \gamma a - I - 1, \quad \pi^* = (q\rho - 1)\,x.
\end{array}
\tag{25}
$$

An optimal public policy chooses taxes, subsidies and government expenditure such that the efficient allocation characterized by (25) is replicated as a decentralized market equilibrium. Consider first the optimal production scale at the plant level. According to (25a), the scale is optimal if the marginal valuation q of an extra unit of output equals the social resource cost given by the input output coefficient, $q = 1$. In the untaxed private equilibrium, mark-up pricing as in (7) violates this condition. In exploiting local market power, producers charge a price in excess of marginal cost and thereby restrict demand in order to boost profits. These profits are necessary to reward entrepreneurs and financiers for the effort and other start-up cost of a new venture. The optimal policy is to give an output subsidy that induces producers to charge the efficient demand price $q = 1$ but nevertheless allows them the same profit margin. The effect will be to spur demand and raise output which is too low in private equilibrium. Comparing (25a) and (7), the optimal output subsidy is

$$z^{*X} = \rho - 1 \quad \Rightarrow \quad q = 1. \tag{26}$$

[14] From (2), we have $\frac{\partial D}{\partial x_j} = \left[\int_0^N (x_j)^{1/\rho}\, dj\right]^{\rho-1} \cdot (x_j)^{1/\rho - 1}$. Imposing symmetry gives $\frac{\partial D}{\partial x_j} = N^{\rho-1}$.

To find the socially optimal effort levels of the entrepreneur and VC, substitute $q = 1$ and $\pi = (\rho - 1) x$ into (25b,c) and get the conditions $\pi p_e(e, a) = \beta$ and $\pi p_a(e, a) = \gamma$. Accordingly, each individual should be full residual claimant on her investment of extra effort so that marginal effort costs β and γ are equated with the full social returns πp_a and πp_e. In private equilibrium with double moral hazard and simultaneous effort choice, both agents must simultaneously participate in the upside potential of the firm and must therefore agree on a profit sharing. Therefore, the incentive constraints in (11) result in a natural underinvestment of entrepreneurial effort and managerial support since agents must bear the full private cost of effort but share the marginal return. The government can, in principle, remedy the problem by giving a revenue subsidy (negative taxes) as in (27) which are able to decentralize the conditions for socially optimal effort levels. The revenue subsidy both to entrepreneurs and VCs must compensate for the beneficial spillovers of individual effort on the other side of the team,[15]

$$\left(1 - t^{*E}\right) s = 1, \qquad \left(1 - t^{*F}\right)(1 - s) = 1. \tag{27}$$

To determine optimal policy with respect to search effort, we need to compare the social surplus S^* from an extra match with the private surplus S. Note that optimal (negative) tax rates imply $1 = -t^E s - t^F(1 - s)$. With this optimal policy in place, and S^* as defined in (25h), we can write the private surplus stated in (9-10) as

$$S - S^* = p\pi + \left(t^{*F} - t^{*E}\right) B - \left[\left(1 - t^{*F}\right)\left(1 + t^I\right) - 1\right] I, \quad S \gtreqless S^* \Leftrightarrow t^I \lesseqgtr \bar{t}^I. \tag{28}$$

With the optimal policy in place, there exists an investment tax $\bar{t}^I$ such that the right hand side is zero, implying $S = S^*$. Since $t^{*F}, t^{*E} < 0$, and with sufficient symmetry (i.e. t^{*F}, t^{*E} very much the same), the right hand side is zero only with a positive tax

[15] Before drawing any immediate policy conclusions, one should emphasize that the industry has partly developed its own solutions to alleviate the problem. Schmidt (2002) among others has shown that the use of convertible securities can go a long way to allocate the right incentives at the right time to each partner. Stage financing, i.e. the tying of further financing rounds to the achievement of predefined milestones, also addresses the problem. However, as long as both inputs must be supplied simultaneously at least in some stages of the firm's development, the problem never fully disappears.

$\bar{t}^I > 0$.[16] For any given desired gap $S - S^*$, one can find a value of the investment tax t^I that supports this equation. For a tax rate smaller than $\bar{t}^I$, the private joint surplus exceeds the social one, and vice versa.

The relation in (28) is now exploited to find an optimal policy for an efficient search allocation of VC. Writing $\frac{dE}{dv} = (f - \theta f') F$, $\frac{dE}{dR} = f'$ and $\frac{dE}{dF} = (f - \theta f') v$ on account of the matching constraint (24), we can compare the conditions for socially optimal search in (25d-f) with the relevant private ones in (16,18) where $\Omega = 0$ is the free entry condition determining F in long-run private equilibrium. Dividing the search conditions in (25) with the private ones and noting the matching elasticities in (14) yields

$$v^* : \frac{S^*}{S} \cdot \frac{1-\eta}{1-\tilde{\xi}} = \frac{\delta'(v)}{\delta'(v)} = 1, \qquad F^* : \frac{S^*}{S} \cdot \frac{1-\eta}{1-\tilde{\xi}} = \frac{1+\delta(v)}{1+\delta(v) - z^F}, \tag{29}$$

for search intensity and long-run entry of VCs. Comparing the decision of entrepreneurs to participate in the search market gives

$$R^* : \frac{S^*}{S} \cdot \frac{\eta}{\tilde{\xi}} = \frac{k(R,G)}{k(R,G) - z^R}. \tag{30}$$

The general principle is that optimal search intensity and entry all require that the property rights are appropriately allocated (via bargaining power) to correspond to each party's effectiveness in generating a match [see Boone and Bovenberg (2002) in a labor market context]. In standard search models, private equilibrium provides the right incentives if the so-called Hosios (1990) condition holds. In the present context, the Hosios condition must be generalized since the effective bargaining power also depends on relative taxation. With symmetric efforts, however, $s = 1/2$ which calls for a uniform revenue subsidy $t^{*E} = t^{*F} < 0$, implying $\tilde{\xi} = \xi$ so that $\xi = \eta$ is the relevant Hosios condition for search efficiency. In this case, optimal search intensity in (29) results only with $S = S^*$ which requires $t^{*I} = \bar{t}^I$ as defined in (28). The start-up tax is meant to undo the stimulating

[16] With exact symmetry, i.e. $\alpha = \epsilon$, $s = .5$ by (A.8), giving a uniform subsidy $t^* = -1$ according to (27). Thus, $S - S^* = p\pi - \left[(1-t^*)\left(1+t^I\right) - 1\right] I$ can be zero only if t^I is positive. If t^I were zero but $t^* = -1$, then $S - S^* = p\pi - I > 0$ since the joint surplus must be positive in equilibrium.

effects of the revenue subsidy on entry. According to (29-30), the other subsidies related to search efforts on both sides of the market must then be set to zero: $z^{*F} = z^{*R} = 0$.

Suppose now that entrepreneurs' bargaining power exceeds the matching elasticity, $\tilde{\xi} > \eta$. Entrepreneurs are therefore able to negotiate an overly large share of the joint private surplus, leaving a too small share to VCs which impairs their search incentives. In the short-run, given that search effort v is not observable, the VC's search incentives can be strengthened only by creating a larger joint surplus to be shared privately which also raises the VCs' reward for search. With capital gains taxes already used to address moral hazard, the surplus is increased only by charging a small, or even negative, start-up tax: $t^{*I} < \bar{t}^I$ so that $S > S^*$ until the condition for v^* is fulfilled in (29). The same policy also provides the right incentives for VC entry F^* in the long-run. Therefore, no further tax or subsidy on VC entry is desired, $z^{*F} = 0$. Note, however, that entrepreneurs now face excessive incentives to start firms. Not only do they get an overly large share $\tilde{\xi} > \eta$ of any given joint surplus but the private surplus is also increased beyond the social value to attract more VC activity, $S > S^*$. To prevent excessive start-up activity of entrepreneurs, the government must now tax it, $z^{*R} < 0$. We summarize:

$$\tilde{\xi} \gtreqless \eta \quad \Rightarrow \quad S \gtreqless S^* \Leftrightarrow t^{*I} \lesseqgtr \bar{t}^I \quad \Rightarrow \quad z^{*R} \lesseqgtr 0, \quad z^{*F} = 0. \tag{31}$$

The opposite case $\tilde{\xi} < \eta$ is also intuitive. If VCs are endowed with overly large bargaining power, the government's only possibility to suppress the VCs' excessive search activity is to reduce the joint surplus, $S < S^*$, by charging a larger start-up tax $t^{*I} > \bar{t}^I$ until the condition for v^* in (29) is fulfilled. Note that the large start-up tax is partly returned by the revenue subsidy once the firm matures to production stage. Since the VC now shares in a lower joint surplus, her search incentives are reduced accordingly. According to (29), VC entry or exit needs no further stimulus since the conditions for search intensity and entry are identical for $z^{*F} = 0$. The large investment tax t^{*I}, however, now creates a double reason to strengthen entrepreneurial start-up activity. First, entrepreneurs are endowed with too weak bargaining power relative to their effectiveness in generating matches. Second, the joint surplus is reduced by the investment tax which further weakens the entrepreneurs' incentives to enter. Therefore, the government should offer research grants $z^{*R} > 0$ to stimulate experimentation with entrepreneurship.

Finally, the efficiency condition (25g) identifies a productive role of the government for which there is no private counterpart. The government should allocate public R&D spending to activities that are more amenable for commercial applications. The optimal amount of the public good is attained when the marginal reduction of the entrepreneurs' entry costs in the aggregate just balances with the marginal budget cost, $\int_0^R -k_G(i,G)\,di = 1$. Publicly funded R&D leads to a larger frequency of start-up entrepreneurship by reducing private (re-)search efforts of potential entrepreneurs.

As with all other policy instruments, we have assumed that any net budgetary requirement is financed with a lump-sum tax that involves no excess burden. Note, however, that the budget cost of the optimal policy may not be overly large. The policy is at least partly self-financing on account of the start-up tax t^I which is required to offset the effects of the revenue subsidies on entry. The insights of this section are summarized by:

Proposition 3 ***(Optimal Policy)*** *The first best allocation is decentralized as follows:*
(a) The output subsidy (26) offsets mark-up pricing of innovative goods.
(b) The revenue subsidies (27) eliminate the spillovers due to double moral hazard and induce efficient entrepreneurial effort and managerial support by VCs.
(c) If the distribution of bargaining power induces efficient search activity ($\tilde{\xi} = \eta$), an investment tax $t^{*I} = \bar{t}^I > 0$ is needed to offset the effects of the revenue subsidy on entry.
(d) If bargaining power deviates from the matching elasticity ($\tilde{\xi} \neq \eta$), the investment tax t^{*I} and an entry tax/subsidy z^{*R} for entrepreneurs are chosen as in (31) and must favor the market side with the matching elasticity exceeds bargaining power. The optimal entry subsidy for VCs is always zero, $z^{*F} = 0$.
(e) Basic R&D as a public good G should be expanded until the marginal budget cost is balanced by the reduction in aggregate private research costs of potential entrepreneurs.

In our model framework, the main policy challenge is not only to create more start-ups to internalize the gains from product innovation but also start-up firms of higher quality where quality relates to the survival prospects. The main logic of the proposed optimal policy is thus to reward success which strengthens incentives for joint effort and thereby raises the success rate. Output and revenue subsidies may, however, boost entry by more than what is desirable. For this reason, a start-up investment tax is introduced to prevent excessive entry. In the end, the net cost to public revenues might not be that large. One could discuss though the government's ability to implement the optimal policy relating to search externalities even if it had sufficient information about the precise nature of the distortions. In particular, it might be difficult to enforce a tax $z^R < 0$ on an entrepreneur's seed investment. This tax would be useful to prevent excessive entry into the search market when entrepreneurs' bargaining power exceeds their matching effectiveness, $\tilde{\xi} > \eta$. Agents who produced a business plan, would be liable for the tax even if they were unable ot obtain VC finance and start a firm. If unsuccessful, they face an incentive to deny the search activity so that the government might find it difficult to implement a tax on entrepreneurial research effort in the seed phase. By way of contrast, when the entrepreneurs' bargaining power is overly small, the government would find it easy to administer an entry subsidy $z^R > 0$ on experimental research in the seed phase. Agents would apply for a subsidy and the government could check whether a serious business plan was produced even if the venture did not get funded by a VC after all. Finally, if search were efficient, policy should abstain from targeting entry of either entrepreneurs or VCs ($z^{*R} = z^{*F} = 0$).

4 Piecemeal Reform

Public policy towards the VC industry includes a number of actual and potential strategies to encourage start-up entrepreneurship, VC finance and innovation. A theoretical rationalization based on an explicit structural model of the VC industry is largely missing, however. To begin with, policy makers and representatives of the industry often consider the capital gains tax as the most important tax barrier to VC investments. One scenario thus deals with the effects of a uniform capital gains tax, $t = t^E = t^F$. Second, actual policy initiatives in many countries often subsidize start-up capital cost by means of interest subsidies and direct investment tax credits to young firms which corresponds to choosing $t^I < 0$ in the present model framework. Third, there are many programs which encourage entrepreneurial experimentation such as research grants for seed investments, z^R in our model. Other initiatives might consider to encourage the creation of additional VC funds, for example by awarding an entry subsidy z^F. Fourth, one of the most important elements of public innovation policy is the funding of basic R&D in applied areas with promising commercial applications, G. Such spending raises the probability of spin-offs and is conducive to innovative business start-ups by high quality entrepreneurs. Obviously, the VC industry can flourish only in an active research environment and is, therefore, geographically concentrated around major centers of basic and applied research. Fifth, innovation is rewarded by local market power and monopolistic profits for the specialized brands that successful start-ups introduce in the market. On the negative side, mark-up pricing restricts demand and consumer surplus which can be remedied in terms of an output subsidy z^X.

This section computes the comparative static effects and welfare consequences of such policy initiatives. We consider both a short-run effect where the number of VCs with specialized investment knowhow is fixed, and a long-run response with free entry and zero profits. To this end, we log-linearize the equilibrium conditions in (23) and investigate how the initial equilibrium is displaced after a small policy change. The hat notation considers logarithmic changes relative to the initial position, $\hat{x} \equiv d\ln(x) = dx/x$, for example. To avoid division by zero, one defines $\hat{z} = dz/(1+z)$ for subsidies and $\hat{t} = dt/(1-t)$ for tax rates. To further simplify the analysis, we assume $z^R = z^F = 0$ in the initial state. Detailed calculations are found in appendix C.

4.1 Output Subsidy on Innovative Goods

The invention and introduction of new goods allows new firms to enjoy local market power on account of product differentiation. In our model of horizontal product innovation, mark-up pricing over marginal cost unduly restricts demand and reduces consumer surplus. Further, if a specialized producer introduces a new good, the gains from variety reduce the price index, thereby stimulate demand and boost sales and profits of other producers as well. Mark-up pricing and demand externalities can be addressed by an optimal output subsidy. Starting from a laissez-faire equilibrium with other taxes set to zero, this section considers the effects of a small output subsidy given to VC backed firms.

The immediate effect of the subsidy is to reduce the demand price of innovative goods while the producer price is still chosen as a fixed mark-up over unit cost. Profits of a specialized firm are not immediately affected but increase only if the subsidy stimulates demand and sales in equilibrium. By (7),

$$\hat{q} = -\hat{z}^X, \qquad \hat{\pi} = \hat{x}. \tag{32}$$

Consider now the effects on market equilibrium when the number of VCs is fixed in the short-run. The reduction in demand prices creates excess demand in the product market and, thus, stimulates sales and profits of mature firms. The profit per firm is a measure of venture returns which affects incentives in earlier stages of the VC cycle when the product is not yet introduced in the market. Entrepreneurs supply more effort during the start-up phase when the stakes are larger on account of higher venture returns. VCs advise more intensively as well to make the start-up firm a success, see the incentive constraints in (11). Since efforts are complements, they are mutually reinforcing, leading to an overall increase in the firms' success probability as in (A.14). One stage earlier, the entrepreneur must bargain with a VC to finance the start-up investment cost of the venture. The VC buys an equity stake for a price that pays for the start-up cost and in addition includes a non-performance related upfront payment to the entrepreneur. The contract results in the entrepreneur and VC sharing the joint surplus of the start-up firm as in Proposition 2. An increase in venture returns $\hat{\pi} = \hat{x}$ obviously boosts the joint surplus of a start-up firm as in (A.16).[17]

[17]Since this scenario considers only an isolated increase of the output subsidy, all other policy instruments are set to zero, i.e. $\hat{t} = \hat{t}^I = 0$ in (A.18).

In the seed phase, researchers anticipate their share in the joint surplus when they consider to start their own firm and conduct experimental research leading to a business plan. The marginal investment condition is stated in (16) and illustrated in Figure 1. The expected return on experimental research or seed investment prior to VC finance is $\frac{f(\theta)}{\theta} \cdot \xi S$ and consists of the probability to locate a VC times the entrepreneur's share in the expected joint surplus of the firm. The expected return just matches the research cost $k(R, G)$ of the marginal entrepreneur R. The higher is market tightness θ, the lower is the probability of obtaining finance and the expected return to seed investment which shifts down the horizontal line in Figure 1. Consequently, fewer researchers will attempt entrepreneurship so that fewer financing offers v^D are needed to satisfy their demand for VC finance. The investment condition thus defines a demand curve for VC in terms of market tightness θ which is downward sloping as in Figure 2.[18]

The supply curve follows from the search investment of VCs who expend effort to locate profitable investment opportunities. The expected return on search is the probability $f(\theta)$ of locating a profitable investment opportunity times the VC's share in joint surplus, and must match with marginal cost of search effort. The marginal cost increases with search intensity or the number of financing offers, and the matching probability increases with market tightness. For any given S, the investment condition (18) thus defines an upward sloping supply curve v^S for VC as illustrated in Figure 2. In reducing demand prices, the output subsidy creates excess demand for innovative goods and thereby boosts the joint surplus S of a start-up firm that is shared among entrepreneur and VC. The subsidy thus shifts up both the supply and demand curves for VC and thus expands VC finance as captured by search intensity v. If entry of entrepreneurs is more elastic relative to the search activity of VCs, $\omega > \kappa$, market tightness will increase as well, see (A.20) for the formal analysis. We emphasize this condition which would always hold if the supply of

[18] Formally, replacing R by $\theta v F$, and taking the expexted surplus S as given, condition (16) can be solved for demand v in terms of θ. Solving equation SR in (A.19) for $\hat{v}$ gives the log-linearized demand curve while SF is the supply curve for VC. Note that demand and supply is defined *per* VC, i.e. v, whereas the aggregate quantity is vF.

entrepreneurs were perfectly elastic ($\kappa = 0$) as is often assumed in the literature.

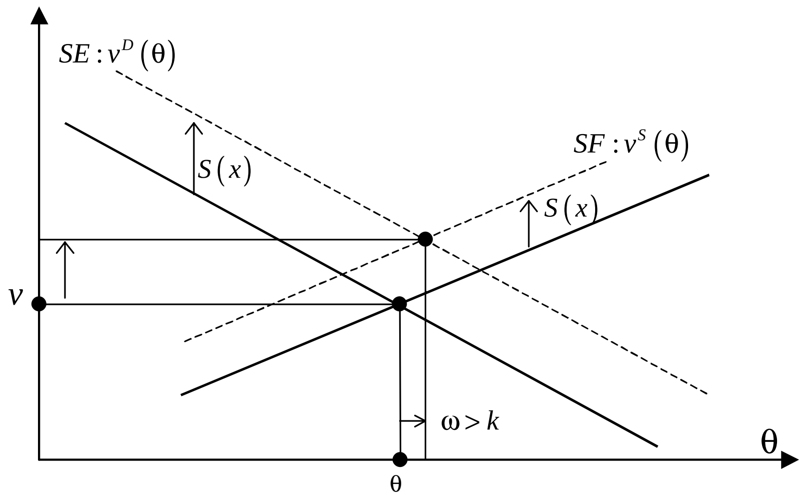

Figure 2: Supply and Demand for VC

Note that $\hat{S}$ and $\hat{x}$ are positively linked as in (A.16). Therefore, improved venture returns boost joint surplus which in turn raises market tightness and search intensity. Each VC thus posts not only a larger number of financing offers but also succeeds to strike a deal on a larger share of these offers. On account of higher joint effort, a larger fraction of these deals eventually result in market introduction of new goods. Putting these effects together shows how an increase in venture returns raises the number of mature firms N so that a larger range of differentiated products are on offer. This upwards sloping supply curve is illustrated in Figure 3 and is formally derived in condition PM in (A.17) or (A.22) in its final form. For any given component price q, the demand side (3-4) defines a trade-off between scale and variety in consumption. In other words, more quantity x per brand reduces the demand for the number of goods N. This trade-off thus defines a downward sloping demand curve for new goods in terms of sales x where the component price q is a shift parameter. Figure 3 illustrates what is shown formally in PM of (A.17), i.e. $\hat{N} = -\mu\hat{x} + \mu\lambda\hat{z}^X$. The intersection yields the market clearing level of venture returns x. Raising the output subsidy cuts the consumer price of all brands and stimulates demand. Holding sales x and thereby the scale of consumption constant, the demand curve for new goods shifts to the right in Figure 3. The subsidy leads to a new product market equilibrium with increased sales per product and a larger number of specialized firms. Formally, (A.22-23) state the relative changes compared to the initial equilibrium, $\hat{x} = \zeta_x \hat{z}^X > 0$ and $\hat{N} = (\Delta_X - \mu) \frac{\mu\lambda}{\Delta_X} \hat{z}^X > 0$.

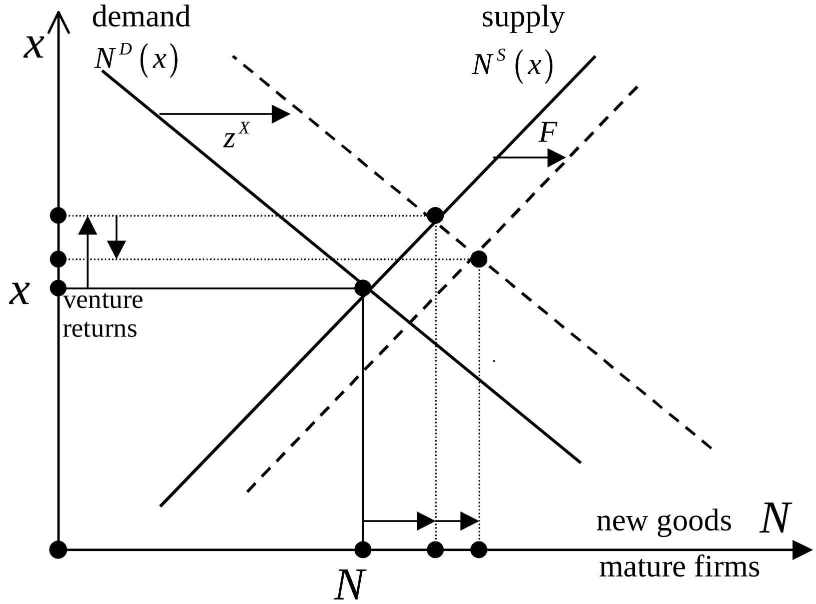

Figure 3: Product Market Equilibrium

Knowing the equilibrium adjustment of venture returns, one can calculate all other repercussions of the output subsidy. Of particular interest are the welfare consequences which are formally derived in (A.19). Keeping other policy instruments at zero, the output subsidy yields welfare gains on several margins:

$$dU^* = (q-1)\, xN \cdot [\hat{x} + \rho\zeta_{PX} \cdot \hat{\pi}] + \frac{(1-s)\,\epsilon + s\alpha}{1-\epsilon-\alpha} \pi N \cdot \hat{\pi} + (\eta - \xi)\, SE \cdot \hat{\theta}. \qquad (33)$$

The first two terms relate to the output market distortion with respect to scale and variety resulting from with monopolistic competition.[19] The subsidy squeezes demand prices and thereby offsets markup pricing of innovative firms. Since demand is repressed on account of prices in excess of marginal costs, this demand stimulation yields welfare gains proportional to the price wedge, $(q-1)\, xN\hat{x}$. Second, the increase in sales of individual firms boosts profits, $\hat{\pi} = \hat{x}$. Improved venture returns sharpen incentives for joint efforts

[19]Note that the same wedge $q-1$ measures two distortions with respect to scale and variety. In standard models of monopolistic competition, the scale effect is often absent, leaving only the variety effect.

and thereby raises the success probability, $\hat{p} = \zeta_{PX}\hat{x}$, and the number of mature firms in the market. The introduction of a larger variety of innovative goods raises consumer surplus due to gains from variety, $(q-1)\, xN \cdot \rho\hat{p}$. The third term reflects the beneficial effects of the output subsidy on joint effort. At the market equilibrium, the wedge between the social and private returns to effort is $1-s$ for the entrepreneur and $1-(1-s)$ for the VC, leading to inefficiently low effort levels in private equilibrium. In raising venture returns, the output subsidy boosts both entrepreneurial effort and managerial support and thereby raises expected joint surplus of start-up firms by the third term.

Finally, the subsidy boosts the expected surplus of new start-up firms that is shared among entrepreneurs and VCs as a result of bargaining. It thereby raises the prospects of initial research efforts of potential entrepreneurs and increases the number of business plans proposed while VCs search more intensively for profitable investment opportunities. If entry of potential entrepreneurs is more elastic than the supply of informed capital offered by a fixed number of VCs in the short-run, $\omega > \kappa$, the policy raises market tightness in the market for VC as in Figure 2. The effectiveness of entrepreneurial seed investments in generating matches and raising the number of VC backed start-ups is measured by the elasticity η. If the entrepreneurs' bargaining power determining their share in the joint surplus falls short of their matching effectiveness, then entrepreneurial entry is too hesitant in laissez faire equilibrium relative to the willingness of VCs to finance new ventures. Consequently, an increase in market tightness boosts welfare in case of $\eta > \xi$. If the property rights on initial seed investments and VC search effort are efficiently distributed and exactly correspond to the effectiveness of entrepreneurs and VCs in launching new start-up firms ($\eta = \xi$), the search market for VC is efficient so that a small change in market tightness is devoid of welfare consequences. Obviously, $\eta < \xi$ implies that a tighter market for VC is detrimental to welfare. If entrepreneurial entry is more elastic than VC search activity, $\omega < \kappa$, the subsidy reduces market tightness which turns around the welfare results. The preceeding discussion is summarized in

Proposition 4 ***(Output Subsidy in the Short-Run)*** *(a) Starting from an untaxed*

position, the output subsidy increases venture returns x which boosts entrepreneurial effort e and managerial VC support a and results in a higher success rate p of start-up firms.
(b) It raises market tightness θ (if $\omega > \kappa$), VC search intensity v and rents and Ω, VC portfolio size $vf(\theta)$, and the number N of mature firms.
(c) Starting from laissez-faire, it yields welfare gains by raising consumer surplus due to (i) higher demand for each variety and (ii) more product variety, (iii) by encouraging joint effort and (iv), if $\eta > \xi$, by encouraging entrepreneurial entry relatively more than VC search activity (higher market tightness). If $\eta \leqslant \xi$, the welfare gain from higher market tightness is zero or negative. The welfare results in (c.iv) are reversed if entrepreneurial entry is relatively less elastic, $\omega < \kappa$, and market tightness declines.

Proof. (a) See (A.22) for x and (A.14) for e, a, and p. (b) (A.20) reports $\hat{\theta}$, $\hat{v}$ and the effect on portfolio size, $\hat{v} + \eta\hat{\theta}$, (A.28) shows that rents increase in line with search intensity, and (A.21-22) give $\hat{N}$. (c) See the discussion of (33). ■

The number of VCs is assumed fixed in the short-run. It takes time to acquire the necessary market knowledge, financial expertise, entrepreneurial experience and reputation of a successful VC. However, when an output subsidy or any other structural change boosts profits, these rents should eventually succeed to attract more VCs to the industry. When more projects get funded by a larger number of VCs, the supply curve for new products shifts to the right in Figure 3 which reduces equilibrium venture returns by $\hat{x} = -\zeta_F \hat{F}$ as noted in (A.22). When the joint surplus of VC backed firms erodes and more VCs must share the market, each one will search less intensively and finance a smaller portfolio as in (A.20), and end up with smaller rent as in (A.28). The process continues until rents are exhausted and entry comes to a halt. Following an increase in the short-run, VC search intensity is reduced as a consequence of continued entry until it is back to the original value in the long-run, $\hat{v} = 0$ in (A.28). To accommodate the extra demand created by the output subsidy, the number of VCs increases to an extent given in (A.31), $\hat{F} = (\mu\lambda/\nabla_F)\,\hat{z}^X > 0$.

Since entry $\hat{F} > 0$ erodes venture returns, it works to dampen the short-run increase in (A.22), $\hat{x} = \zeta_x \hat{z}^X - \zeta_F \hat{F}$, and by implication joint efforts, the success rate and the joint surplus. The number of mature firms, in contrast, increases beyond the short-run effect simply because there are more VCs to finance projects. The final, long-run effects of the output subsidy are computed in (A.32). Interestingly, the long-run change in market tightness is in the opposite direction of the short-run effect. This reversal is intuitive, however. Since entrepreneurial talent is distributed unevenly, the supply of entrepreneurs remains finitely elastic even in the long-run while free entry makes the supply of VCs perfectly elastic. Obviously, market tightness should fall in the long-run as is verfied by (A.32). The same basic formula (33) applies to judge the long-run welfare consequences.

Proposition 5 ***(Output Subsidy in the Long-Run)*** *(a) Free entry of VCs dampens the short-run increase in scale x and profits, efforts e and a, and the success rate p.*
(b) The long-run effect on VC search is zero while the subsidy reduces market tightness θ and portfolio size $vf(\theta)$. The number N of mature firms expands beyond the short-run.
(c) A small output subsidy raises consumer surplus by raising (i) scale, (ii) variety, and (iii) joint efforts. The gains are positive but smaller than in the short-run. Since market tightness declines in the long-run, the short-run welfare effects from entry and search (iv) are reversed and negative if $\eta > \xi$, but positive (zero) if $\eta < (=)\xi$.

Proof. (a) $\hat{e}$, $\hat{a}$ and $\hat{p}$ are proportional to $\hat{x}$ but the short-run increase in venture returns is dampened by $\hat{F} > 0$, see $\hat{x} = \zeta_x \hat{z}^X - \zeta_F \hat{F}$ in (A.22). (b) Read (A.32). Given constant v, portfolio size $vf(\theta)$ declines in line with θ. The effect on $\hat{N}$ follows from (A.22) combined with the result that the increase in $\hat{x}$ is larger in the short-run than in the long-run. (c) Since $\hat{\pi} = \hat{x} > 0$, welfare increases by the first three terms in (33), verifying (i-iii). Part (c.iv) relates to the last term which changes sign along with θ. ■

4.2 Entry Subsidy to Venture Capitalists

How does the structure of the VC sector change if policy succeeds to attract more managers to finance and advise entrepreneurial firms? One possibility is to encourage the creation of new VC funds by means of an entry subsidy z^F. The analysis starts from a laissez-faire position with zero VC profits and supposes lump-sum finance of the subsidy. Since the entry subsidy is sunk once a fund is established, it cannot affect later stages of the VC cycle. Consequently, there are no immediate effects on the short-run equilibrium since it does not enter the short-run conditions PM, SR and SF in (23) or in the log-linearized version (A.17). The subsidy simply raises VC profits as in (A.28).

If the policy persists, it will eventually attract new managers to the industry to take advantage of these rents. With given market tightness and the same aggregate demand for VC, each individual VC fund receives less demand when more of them compete for investments. Therefore, the demand curve for the financial offers *per* VC shifts down in Figure 2, see SR in (A.17), while the supply curve reflecting the investment condition SF of an individual VC is not affected. Drawing the demand shift shows that equilibrium market tightness relaxes. Despite of the fact that search intensity and portfolio size also decline as in (A.20), the larger number of funds succeeds to finance more start-up firms.[20] A fraction p of them succeeds to introduce their product to the market. With a larger number of mature firms, $\hat{N} > 0$, the product supply curve shifts to the right in Figure 3. The excess supply of new goods erodes equilibrium venture returns by $\hat{x} = -\zeta_F \hat{F}$ as computed in (A.22). The joint surplus thus falls and thereby discourages efforts and the success rate. The lower surplus partly destroys the returns to search investments and impairs somewhat the incentives of entrepreneurs to pursue seed investments and of VCs to search for investment opportunities. The net effect of VC entry is stated in

Proposition 6 ***(VC entry subsidy)*** *(a) VC entry $\hat{F}$ reduces equilibrium venture returns, joint surplus of VC backed firms, and search intensity of VCs. Market tightness and portfolio size decline. Entry stops when the rents due to the subsidy are exhaused.*
(b) Welfare declines on account of negative effects on variety, scale, joint effort, and market tightness (in case of $\eta > \xi$).

[20] The number of VC backed start-up firms is $E = vf(\theta) F$ and changes by $\hat{E} = \hat{v} + \eta\hat{\theta} + \hat{F}$. Using (A.20), we obtain $\hat{E} = \frac{(1-\eta)(1+\kappa)\omega}{\Delta_S} \hat{F} > 0$.

Proof. (a) $\hat{x} < 0$ by (A.22), $\hat{S} < 0$ by (A.25), and $\hat{v} < 0$ by (A.20). A sufficient condition for $\hat{\theta} < 0$ and $\hat{v} + \eta\hat{\theta} < 0$, i.e. for market tightness and portfolio size to decline, is $\omega > \kappa$, see (A.20). Entry stops when search intensity is reduced to the extent given in (A.28) which implies a larger number of VC funds as in (A.31). Some long-run effects are given in (A.33). (b) The welfare effects are read form (33). ■

4.3 Basic Research and Research Grants

The supply of high quality entrepreneurs with new innovative ideas is a precondition for the development of a healthy VC sector.[21] It is no coincidence that VC is geographically concentrated in active areas of basic and applied research. Public research spending G helps more people to invent new commercial applications and thereby raises the probability of spinoffs.[22] When more researchers attempt entrepreneurship with some initial seed investments, the demand for VC rises. The government might also consider encouraging experimentation with new business ideas by handing out small research grants z^R. Such grants might be given subject to the requirement to produce a business plan, irrespective of whether the entrepreneur succeeds to attract VC finance or not.

In our model, both initiatives reduce private research cost. In shifting down the cost schedule in Figure 1, they raise the mass of agents who try entrepreneurship. The demand for VC grows. In Figure 2, the demand schedule shifts up (not drawn). Since this policy does not directly affect the investment criterion of financiers, the supply curve for VC remains invariant. Market tigthness increases when more entrepreneurs ask for finance. Accordingly, the VC's probability to locate profitable investments rises which strengthens her incentives for search, leading to an increase in the supply of VC in terms of financing offers. With each VC tending a larger portfolio $vf(\theta)$ of firms, more seed investments get

[21] Becker and Hellmann (2002) emphasize this argument.

[22] Zucker, Darby and Brewer (1998) show empirically that the human capital created by frontier research around great universities was a principal determinant of the growth and location of the US biotechnology industry. The conclude "This industry is a testament to the value of basic scientific research."

started as a VC backed firm, $E = vf(\theta) F$, of which a fraction p matures to production stage, $N = pE$. Consequently, market supply in Figure 3 shifts to the right. Equilibrium venture returns decline as in (A.22) when more firms crowd the market.

The research subsidy does not directly affect the joint surplus of a start-up firm. However, since increased start-up activity spoils sales and profits of individual firms, it thereby retards entrepreneurial effort and managerial support and erodes the joint surplus in the start-up phase. This negative feedback from the market weakens the incentives for search effort that were initially created by the research policy and dampens other reactions as well. However, if the elasticity for variety demand μ is sufficiently large, the erosion of venture returns will not overturn the incentives for start-up activity created by the research policy. With a large elasticity μ, the product demand curve in Figure 3 is flat so that even a small reduction in sales x per brand creates a large demand for new products. Consequently, a minor reduction of sales and profits suffices to create new markets for the extra start-ups resulting from a more active research policy. With this profit destruction effect being small, it cannot overturn the incentives of VCs to search more intensively for new investments when the research policy raises market tightness by stimulating seed investments. Although lower venture returns erode the incentives for effort and thereby reduce the success rate and joint surplus from start-up firms, this negative effect remains small as well when the product demand curve is flat.

In stimulating more seed investments by potential entrepreneurs, the policy contributes to increased market tightness and thereby raises the VCs' probability to attract investments. Larger company portfolios create VC rents which attract new managers to the industry. As more start-ups get financed, more firms will eventually mature to production stage and crowd the market with new products. The reduction in venture returns is thus magnified in the long-run by the entry of VCs, see (A.22). Consequently, joint efforts, the success rate and joint surplus all shrink beyond the short-run effect which impairs search incentives. Entry continues until the short-run increase in VC search intensity is fully reversed and VC rents are exhausted. When there is free entry of VCs in the long-run, the number of VC backed firms in the market is larger but VCs are less engaged to support the portfolio companies with active managerial advice. In this sense, there is more quantity but less quality of VC.

Proposition 7 ***(Research Policy)*** *(a) In the short-run, research policy boosts the supply of new products N but erodes venture returns x. Joint efforts, the success rate and surplus, e, a, p and S, fall. Market tightness θ, VC search intensity v, portfolio size $vf(\theta)$ and VC rents Ω increase if the demand elasticity μ for variety is sufficiently large.*
(b) In the long-run, VC search intensity is back to its initial value ($\hat{v} = 0$) but portfolio size increases with market tightness θ. VC entry is positive, $\hat{F} > 0$, venture returns x and thereby e, a, p and S fall while the number of mature firms N increases.
(c) Basic R&D spending G and small research grants z^R both diminish consumer surplus by reducing (i) scale, (ii) variety, and (iii) joint efforts. (iv) The increased market tightness yields a positive welfare effect if $\eta > \xi$, and a negative (zero) one if $\eta < (=)\xi$.
(v) Starting with small levels, public R&D spending G reduces private research costs by more than its resource costs and thereby raises welfare until the Samuelson condition for optimal public goods supply is satisfied.

Proof. (a) In the short-run, $\hat{x} < 0 < \hat{N}$ by (A.22). By (A.14), $\hat{e}$, $\hat{a}$ and $\hat{p}$ fall in line with $\hat{x}$ while $\hat{S} < 0$ is calculated in (A.25). By (A.26), search intensity increases despite of the negative influence of $\hat{S} < 0$ if μ is sufficiently large. Since a large μ dampens the effect of a smaller surplus, market tightness and portfolio size similarly fall on account of (A.20). VC rents increase with search intensity by (A.28). (b) Long-run effects follow from (A.28) and (A.34). (c) Noting $\hat{\pi}, \hat{x} < 0$, the welfare effects follow from (A.29) where the Samuelson condition is $K_G = 1$, see (25g). ■

4.4 Tax Policy

Industry representatives often cite the capital gains tax as the most important tax barrier against VC investments. Policy makers have often called for tax breaks or even an entire elimination of the capital gains tax to expand the VC industry and thereby stimulate growth and innovation. In general, all business taxes on the profits of start-up firms and VC funds are relevant. To mimick the consequences of a tax break but nevertheless avoid tedious tax base effects, we start from an untaxed equilibrium and introduce a small negative tax (i.e. a subsidy), $\hat{t} < 0$. To avoid any cost to the tax payer, this tax break is financed by a start-up tax $\hat{t}^I > 0$. Since entrepreneurs are wealth constrained, the start-up tax is in fact shouldered by financiers who must pay a larger price for their desired equity stake, see (9). This tax policy initiative is, in fact, a step in the direction of the optimal policy proposed in section 3. Will it succeed to expand the industry? What are the implications for the quantity and quality of VC finance, and for welfare?

To keep the proposal revenue neutral, the two instruments must be adjusted simultaneously to satisfy the government budget constraint. With a uniform tax, (A.9) implies $t \cdot \left[p\pi - \left(1 + t^I\right) I\right] = -t^I I$. Starting from an untaxed position, reducing the capital gains tax requires an increase in the start-up tax equal to

$$I \cdot \hat{t}^I = -\left(p\pi - I\right) \cdot \hat{t}. \tag{34}$$

The immediate effect of the tax cut, prior to the adjustment of venture returns, market tightness and search intensity, is to boost joint efforts and the success rate as in (A.14). The start-up tax, in contrast, has no impact at effort stage since it is sunk at that time. Both instruments, however, affect the joint surplus of a start-up firm. Imposing the government budget constraint (34) together with $t = t^I = 0$ an $I^N = I$ in (A.16) yields

$$\hat{S} = -\left(\zeta_{SX} - 1\right) \frac{p\pi}{S} \cdot \hat{t} > 0. \tag{35}$$

Since $\zeta_{SX} > 1$, the immediate effect of the tax cut $\hat{t} < 0$ at given venture returns is to boost the joint surplus of start-up firms. Since this higher surplus is shared among VCs and entrepreneurs, it raises the returns to search. Accordingly, VCs look more intensively for investments and more researchers decide for an attempt at entrepreneurship. The demand and supply curves for VC in Figure 2 shift up, implying a higher search intensity and market tightness (when entrepreneurial entry is more elastic than VC search, $\omega > \kappa$).

When VCs expand their company portfolios and finance a larger number of start-ups, as $\hat{S} > 0$ implies in (A.20), and when the success rate among these start-up firms is higher on account of increased effort, more firms mature to production stage and succeed to introduce new goods. The product supply curve in Figure 3 shifts to the right, indicating $\hat{x} < 0 < \hat{N}$. The policy initiative thus erodes equilibrium venture returns. The formal solution in (A.22) takes account of the fact that an erosion of venture returns negatively feeds back on the incentives to expend effort in earlier stages of the VC cycle. Substituting (34) into (A.22), yields after some manipulations, the equilibrium impact of the tax cut:

$$\begin{aligned} \hat{x} &= \left[\zeta_{PX} + (\zeta_{SX} - 1)\, \tfrac{1+\kappa+(\omega-\kappa)\eta}{\Delta_S} \tfrac{p\pi}{S}\right] \tfrac{1}{\Delta_X} \hat{t} < 0, \quad \hat{N} = -\mu \hat{x} > 0, \\ \hat{x} - \hat{t} &= -\left[\mu + \tfrac{1+\kappa+(\omega-\kappa)\eta}{\Delta_S} \tfrac{p\pi}{S}\right] \tfrac{1}{\Delta_X} \hat{t} > 0. \end{aligned} \tag{36}$$

When firms in their mature stage are less effective in generating sales and profits, $\hat{x} < 0$, entrepreneurs and financiers must expect a smaller joint surplus in the start-up phase. After taking account of this negative feedback, the net effect is calculated from (A.25). Imposing the policy restriction (34) yields

$$\hat{S} = -\frac{(\zeta_{SX} - 1)\, \mu - \zeta_{PX}}{\Delta_X} \frac{p\pi}{S} \cdot \hat{t} > 0. \tag{37}$$

This profits destruction effect is small when the demand schedule in Figure 3 is flat, i.e. when the demand elasticity μ is large. Equilibrium venture returns then decline by a relatively minor amount so that the direct effect of the tax cut on joint surplus remains dominant. VCs share in this larger surplus and, in addition, will search more intensively to expand their portfolios of companies. VC rents accordingly increase in the short-run which eventually attracts more managers to set up new VC funds. This relaxes the scarce managerial resource in the long-run. We find the following results for tax policy:

Proposition 8 ***(Self-financed Tax Cut)*** *(a) In the short-run, the self-financed tax cut given in (34) boosts the supply of new products N but erodes venture returns x. Efforts and the success rate, e, a, and p, all increase. Joint surplus increases if μ is large. Market tightness θ (if $\omega > \kappa$ and μ large), VC search intensity v, portfolio size $vf(\theta)$ and VC*

rents Ω increase in proportion to the joint surplus.
(b) In the long-run, VC search intensity is back to its initial value ($\hat{v} = 0$). VC entry is positive, $\hat{F} > 0$. The tax cut raises joint surplus but shrinks portfolio size together with market tightness θ. Venture returns x decline but the tax cut boosts efforts and the success rate, e, a, p. The number of mature firms N increases.
(c) The tax cut (i) diminishes welfare on account of a negative scale effect but (ii) raises welfare due to a positive variety effect and (iii) by stimulating joint efforts. These effects are qualitatively the same in the short- and long-run. (iv) The short-run increase in market tightness (if $\omega > \kappa$ and μ large) yields a welfare gain if $\eta > \xi$, and a loss if $\eta < \xi$. In the long-run, market tightness relaxes if μ is relatively large, yielding a welfare loss if $\eta > \xi$, and a gain if $\eta < \xi$.

Proof. (a) In the short-run, $\hat{x} < 0 < \hat{N}$ and $\hat{x} - \hat{t} > 0$ by (36). This raises efforts and the success rate as in (A.14) while $\hat{S} > 0$ is calculated in (37) for μ sufficiently large. The changes in θ, v, $vf(\theta)$ and Ω follow from (A.20) and (A.28). (b) Free entry in the long-run results in $\hat{v} = 0$ and $\hat{F} > 0$ by (A.28) and (A.35) where the latter effect hinges on the demand elasticity being large. Although venture returns decline, see (A.36), the tax cut raises joint surplus relative to the initial equilibrium, consistent with positive entry, $\hat{F} > 0 \Rightarrow \hat{S} > 0$ and $\hat{\theta} < 0$, see (A.29). The tax cut enhances efforts and boosts the success rate as in (A.14) sind $\hat{x} - \hat{t} > 0$ by (A.36). Given smaller sales per firm, $\hat{x} < 0$, market demand supports a larger number of mature firms, see PM in (A.17). (c) The welfare effects follow from (A.19) after substituting $\hat{x} < 0$, $\hat{x} - \hat{t} > 0$ and $\hat{\theta}$. ■

The tax cut fails to achieve unambiguous welfare gains, even if one excludes search distortions ($\eta = \xi$). Since the tax cut expands the supply of successful start-up firms, it reduces sales per firm and thereby produces a negative scale effect. Smaller demand per variety is detrimental since demand was already repressed due to mark-up pricing. The scale effect subtracts from the otherwise positive welfare effects of the tax cut that result from stimulating private effort and joint surplus per project, and from the extra product variety offered by new firms. An output subsidy, in contrast, subsidizes consumer prices and thereby creates extra demand. Consequently, sales per firm *and* product variety can expand at the same time, allowing for gains from variety and scale. Higher sales mean higher venture returns which strengthen the incentives for private effort. Except possibly for the search distortions, an output subsidy yields welfare gains on all fronts. We conclude that a targeted output subsidy for innovative firms is superior to a tax cut.

5 Conclusions

This paper proposed a rich model of the venture capital industry. It emphasizes the need for an active research environment as an important precondition for the supply of high quality entrepreneurs and the contribution of new firms to innovation. The supply of entrepreneurs depends on the willingness of inventive persons to give up alternative career opportunities. Start-up entrepreneurs are often commercially inexperienced and thus need informed capital that not only provides finance but also managerial support. This model of venture capital backed start-up firms is embedded in a model of industry equilibrium where a downward sloping demand curve for innovative goods determines the overall size of the product market. The inelastic entry and search activities of entrepreneurs and venture capitalists lead to an endogenously determined market tightness in venture capital finance. The first activity of entrepreneurs is a seed investment in experimental research leading to a business plan that can be presented to a venture capitalist who could provide the required funds and help with strategic business advice. The paper is unique in discussing not only inelastic supply of entrepreneurs but also inelastic supply of venture capital which consists of an optimally determined portfolio size of a fixed number of venture capitalists in the short-run and an endogenously explained number of venture capital funds in the long-run.

Based on this structural model of the venture capital industry, we have considered a rich menu of policy instruments that are able to influence the evolution of the industry. We have derived an optimal policy that consists of the following elements. First, basic

R&D as a public good optimally reduces the private R&D entry costs of potential entrepreneurs. Second, an output subsidy addresses the distortions in variety and scale that result from the monopolistic market power of innovative firms. Third, a revenue subsidy (negative capital gains or profits tax on entrepreneurs and venture capitalists) coupled with a start-up tax on initial investment outlays addresses the underinvestment in effort resulting from a double moral hazard in the relationship between entrepreneurs and venture capitalists. Finally, if the distribution of bargaining power among entrepreneurs and venture capitalists is not well aligned with the effectiveness of their search efforts in launching start-up firms, market tightness is distorted. This can, in principle, be addressed by an entry subsidy either to entrepreneurs (e.g. in terms of an initial grant for experimental research investments in the seed phase) or to venture capitalists.

The optimal policy derived in this paper involves several simultaneous interventions to correct for market distortions and is able to decentralize a first best equilibrium. Some of the popular policy measures often proposed by governments receive no support in this model. For example, research grants to encourage fresh entrepreneurs, or capital subsidies to reduce the cost of start-up investment, succeed to expand the VC sector. However, they do not help with mark-up pricing and they discourage private effort rather than promote it as is required in a situation of double moral hazard. Similarly, spending on basic research alone is not successful in addressing output market distortions or the problem of double moral hazard, although it will yield welfare gains by providing a public good to private researchers. The upshot is that policy should be active on several fronts at the same time. An output subsidy is seen to yield multiple dividends, however, if only a single measure is pursued in isolation. It not only shifts the scale and variety of new firms in the right direction but also sharpens incentives for joint effort.

Appendix

A Profit Sharing and Joint Effort

Effort Stage: The incentive constraints in (11) implicitely solve for efforts $a(s,\cdot)$ and $e(s,\cdot)$. To find the optimal profit share, one must determine, by linearization, how efforts respond to a variation of s. Since the incentive constraints are mutually dependent, this boils down to solving two reaction functions with s being a shift parameter. The hat notation indicates relative changes, for example, $\hat{a} = da/a$. The exceptions are variables such as tax rates which can be zero in the initial state. In this case, we define the change relative to the net tax price, $\hat{t}^E = dt^E/\left(1-t^E\right)$, or $\hat{z}^X = dz^X/\left(1+z^X\right)$ etc. The linearized reaction functions are[23]

$$
\begin{aligned}
IC^E: &\quad (1-\epsilon)\,\hat{e} = \alpha\hat{a} + \hat{s} + \hat{\pi} - \hat{t}^E, \\
IC^F: &\quad (1-\alpha)\,\hat{a} = \epsilon\hat{e} - \tfrac{s}{1-s}\hat{s} + \hat{\pi} - \hat{t}^F.
\end{aligned} \tag{A.1}
$$

The reaction functions are solved to find the equilibrium response of efforts to a variation in the profit share and other parameters,

$$
\begin{aligned}
\hat{a} &= \tfrac{1}{1-\epsilon-\alpha}\left[\hat{\pi} - \tfrac{s-\epsilon}{1-s}\hat{s} - \epsilon\hat{t}^E - (1-\epsilon)\,\hat{t}^F\right], \\
\hat{e} &= \tfrac{1}{1-\epsilon-\alpha}\left[\hat{\pi} + \tfrac{1-s-\alpha}{1-s}\hat{s} - (1-\alpha)\,\hat{t}^E - \alpha\hat{t}^F\right], \\
\hat{p} &= \tfrac{1}{1-\epsilon-\alpha}\left[(\epsilon+\alpha)\,\hat{\pi} + \tfrac{(1-s)\epsilon - s\alpha}{1-s}\hat{s} - \epsilon\hat{t}^E - \alpha\hat{t}^F\right].
\end{aligned} \tag{A.2}
$$

Contract Stage: Anticipating the effort response, bargaining as in (11) finds the optimal contract s and B. Only the profit share influences incentives for effort and, thus, the size of the pie to be distributed while B does not. This implies a recursive solution. The two parties first agree on a share s to maximize joint surplus and then bargain over B to find an appropriate distribution. The Pareto efficient share thus follows from

$$
S = \max_s \left\{Y^E + Y^F - \gamma a - \beta e - w \quad s.t. \quad (9), (A.2)\right\}. \tag{A.3}
$$

[23] By (8), we have $\hat{p} = \varepsilon\hat{e} + \alpha\hat{a}$, $\hat{p}_e = -(1-\varepsilon)\,\hat{e} + \alpha\hat{a}$, and $\hat{p}_a = \varepsilon\hat{e} - (1-\alpha)\,\hat{a}$.

Using the IC's in (11), the optimality condition is

$$\frac{dS}{ds} = \left[\left(t^F - t^E\right) + \left(1 - t^F\right)(1-s)\frac{p_e}{p}\frac{de}{ds} + \left(1 - t^E\right) s\frac{p_a}{p}\frac{da}{ds}\right] p\pi = 0. \tag{A.4}$$

Expand the square bracket, $[\cdot] = \left(t^F - t^E\right) + \left(1 - t^F\right) \frac{1-s}{s}\frac{ep_e}{p} \cdot \hat{e}/\hat{s} + \left(1 - t^E\right) \frac{ap_a}{p} \cdot \hat{a}/\hat{s}$. Using $ep_e = \epsilon p$ and $ap_a = \alpha p$ from (8), noting the elasticities in (A.2), and canceling all proportional terms, the first order condition for s is equivalent to

$$S_1 = \frac{\left(1 - t^E\right)(1-\epsilon)(1-s-\alpha)}{1-s} - \frac{\left(1 - t^F\right)(1-\alpha)(s-\epsilon)}{s} = 0. \tag{A.5}$$

The second order condition is also satisfied, $S_2 = -\frac{dS_1}{ds} > 0$. (A.5) and (8) imply

$$1 - s - \alpha > 0, \qquad s - \epsilon > 0. \tag{A.6}$$

Both terms must be of the same sign to satisfy (A.5). Adding up yields $1 > \alpha + \epsilon$ in line with (8). If both terms were negative, (8) would be violated. Condition (A.6) ensures that a higher s raises the entrepreneur's effort but reduces managerial effort in (A.2).

The differential of (A.5) shows how taxes affect optimal profit sharing,

$$\hat{s} = -\frac{\left(1 - t^E\right)(1-\epsilon)(1-s-\alpha)}{(1-s)\, s \cdot S_2} \cdot \hat{t}^E + \frac{\left(1 - t^F\right)(1-\alpha)(s-\epsilon)}{s^2 \cdot S_2} \cdot \hat{t}^F. \tag{A.7}$$

A uniform tax has no effect on optimal profit sharing, $\hat{s} = -S_1/(sS_2) \cdot \hat{t} = 0$ on account of (A.5). The tax cancels from both sides of the optimality condition. In this case, we can explicitely solve for the optimal profit share. From $S_1 = 0$, we obtain $(1-\epsilon)\, s\alpha = (1-s)\left[s\alpha + (1-s-\alpha)\epsilon\right]$. Exand by $+s\alpha\epsilon - s\alpha\epsilon$ and write the square bracket as $\left[(1-\epsilon)\, s\alpha + (1-s)(1-\alpha)\epsilon\right]$. We get $(1-\epsilon)\,\alpha s^2 = (1-s)^2 (1-\alpha)\,\epsilon$ or

$$\frac{s^2}{(1-s)^2} = \frac{(1-\alpha)\cdot\epsilon}{\alpha\cdot(1-\epsilon)}. \tag{A.8}$$

B Income and Welfare

This appendix derives income and welfare of the entire population. Given an exogenous division of people with technological and managerial skills, the matching allocation noted in (22) determines R potential entrepreneurs in the seed phase, F VCs searching for investments and E start-up entrepreneurs receiving VC finance. Out of E start-up firms, only $N = pE$ will successfully mature to production stage. Taking account of different average incomes of these groups, and introducing a per capita lump-sum tax T, the government budget constraint is

$$\begin{aligned} \$ &= T + t^E [sp\pi + B] E + t^F \left[(1-s) p\pi - B - \left(1 + t^I\right) I\right] E + t^I IE \\ &= G + z^X qxN + z^F F + z^R R. \end{aligned} \tag{A.9}$$

Now consider income in different groups of the population. Of all $1-M$ technologically talented people, a part $1 - M - R$ never even tries entrepreneurship and earn a net wage income $w-T$. All R researchers aspiring for entrepreneurship receive a lump-sum research subsidy or grant z^R to encourage a seed investment. Only E or them obtain finance, start a firm and derive an expected income Y^E as in (9). The $R-E$ unmatched entrepreneurs revert to a job in manufacturing at a wage $w = 1$, after having received a research subsidy z^R and having sunk a (non-monetary) effort k. Since research effort is interpreted as a time input rather than real expenditure, incomes of workers and unmatched entrepreneurs are the same, except for the research grant z^R. Of all M people with managerial skills, $M - F$ remain with a managerial career in the traditional sector, earning them a wage income. The other part F prefers to help young entrepreneurial firms with managerial advice. Each of them raises a VC fund, potentially receives an entry subsidy, and earns an expected income from their equity stakes in a portfolio of companies. All agents are subject to a lump-sum tax T,

$$\begin{aligned} 1-M-R: &\quad w - T, & M - F: &\quad w - T, \\ E: &\quad Y^E + z^R - T, & F: &\quad vf(\theta) Y^F + z^F - T, \\ R - E: &\quad w + z^R - T. \end{aligned} \tag{A.10}$$

To compute aggregate income, multiply the income terms in (A.10) with the size of the population groups and add up. Using (9), (A.9), $\pi = \left(1 + z^X\right) qx_j - x_j$ in (6) and $N = pE$ in (19) yields, after some manipulations,

$$Y = w + [(q-1) px_j - I - w] E - wF - G. \tag{A.11}$$

Equate spending with aggregate income in (A.12), $Y = VD + Z$. Spending on innovative goods is $VD = x_j^D qN$, where the superscript indicates the demand side as in (2). The wage rate is $w = 1$, manufacturing supply $L = 1 - E - F$ by (22), and $N = pE$ by (19). The aggregate income spending identity thus implies Walras' Law which says that valued excess demands sum up to zero,

$$\left(x_j - x_j^D\right) N \cdot q + (L - IE - x_j N - G - Z) = 0. \tag{A.12}$$

Demand for traditional goods stems from consumer demand Z, government R&D spending G, and demand of start-up firms for intermediates and capital goods. Only successful start-ups mature to production stage and, thus, require intermediate inputs $x_j N$ for the production of innovative varieties while IE is capital demand of *all* start-ups. By Walras' Law, market clearing for innovative goods, $x_j - x_j^D$, also implies market clearing for traditional goods where supply is L.

Indirect utility in (5), $U^{i*} = y^i - l^i + C$, depends on income, effort costs, and consumer surplus. Aggregate income is given in (A.11). Effort costs consist of productive efforts e and a and search efforts δ and $k(i, G)$ by VCs and entrepreneurs, giving a total cost of $\gamma a E + \delta F$ by VCs and $\beta e E + K$, $K = \int_0^R k^i di$, by entrepreneurs. Using (A.11) together with $w = 1$ and (13), we have

$$\begin{aligned} U^* &= C(V) + 1 + S^* E - (\delta(v) + 1) F - K - G, \\ S^* &= (q - 1) xp - \beta e - \gamma a - I - 1. \end{aligned} \tag{A.13}$$

Note that S^* is the social surplus from a new start-up, summing up the surplus derived by the VC, the entrepreneur and the government. This social surplus is different from the private one as in (10).[24]

[24] The social surplus results by adding net expected tax revenue per project in (10),

$$S^* = S + t^E [sp\pi + B] + t^F \left[(1 - s) p\pi - B - \left(1 + t^I\right) I\right] - z^X qxp + t^I I.$$

C Comparative Statics

C.1 Preliminaries

This appendix computes the log-linearized form of the system in (23) and proves the results of section 4. The solution strategy follows backward induction. Beginning with the product market, component prices and profits change by $\hat{q} = -\hat{z}^X$ and $\hat{\pi} = \hat{x}$, see (32). Looking one stage earlier, the linearized incentive constraints (11) yield the effects on efforts as given in (A.2). We restrict attention to a uniform capital gains tax $t = t^E = t^F$. The profit share is thus remains fixed by (A.8), giving $\hat{s} = 0$. Efforts change proportionately and affect the firm's success rate by

$$\hat{e} = \hat{a} = \frac{1}{1-\epsilon-\alpha}\left(\hat{x}-\hat{t}\right), \qquad \hat{p} = \zeta_{PX}\cdot\left(\hat{x}-\hat{t}\right), \qquad \zeta_{PX} \equiv \frac{\epsilon+\alpha}{1-\epsilon-\alpha}. \tag{A.14}$$

The joint surplus (10) is $S = (1-t)\left[p\pi - \left(1+t^I\right)I\right] - \gamma a - \beta e - 1$ and changes as

$$\begin{aligned} dS &= (1-t)\left[p\pi\cdot\hat{x} - \left(1+t^I\right)I\cdot\hat{t}^I\right] - (1-t)\left[p\pi - \left(1+t^I\right)I\right]\cdot\hat{t} \\ &\quad + \left[(1-t)\,\pi p_a - \gamma\right]da + \left[(1-t)\,\pi p_e - \beta\right]de. \end{aligned} \tag{A.15}$$

Replace marginal effort costs β and γ by (11) and note $ep_e/p = \epsilon$ and $ap_a/p = \alpha$ by (8). The second line emerges as $(1-t)\,p\pi\left[s\alpha\hat{a} + (1-s)\,\epsilon\hat{e}\right]$. Substituting the elasticities in (A.15) yields after some rearrangements

$$\begin{aligned} \hat{S} &= \tfrac{(1-t)p\pi\cdot\zeta_{SX}}{S}\cdot\hat{x} - \tfrac{I^N}{S}\cdot\hat{t}^I - \tfrac{(1-t)p\pi\cdot\zeta_{SX}-I^N}{S}\cdot\hat{t}, \\ \zeta_{SX} &\equiv \tfrac{1-s\epsilon-(1-s)\alpha}{1-\epsilon-\alpha} > 1, \qquad I^N \equiv (1-t)\left(1+t^I\right)I. \end{aligned} \tag{A.16}$$

Knowing how joint surplus relates to venture returns (as determined by the level of output x) and tax rates, one derives the log-linearized form of (23):

$$\begin{aligned} PM:&\quad -\mu\hat{x} + \mu\lambda\hat{z}^X &&= \hat{p} + \eta\hat{\theta} + \hat{v} + \hat{F}, \\ SR:&\quad \hat{S} - (1-\eta)\,\hat{\theta} &&= \kappa\cdot\left(\hat{\theta} + \hat{v} + \hat{F}\right) - \left(\hat{G} + k^{-1}\hat{z}^R\right), \\ SF:&\quad \hat{S} + \eta\hat{\theta} &&= \omega\hat{v}, \\ LR:&\quad \hat{\Omega} &&= \hat{S} + \eta\hat{\theta} + \hat{v} - \tfrac{(1+\omega)\delta}{1+\delta}\hat{v} + \tfrac{1}{1+\delta}\hat{z}^F = 0. \end{aligned} \tag{A.17}$$

In the entrepreneur's research condition SR, we assume $z^R = 0$ prior to the shock and note that $\tilde{\xi} = \xi$ remains constant when the capital gains tax is uniform. Linearization of SR thus yields $\hat{k} - (1/k)\,\hat{z}^R = \hat{S} + \hat{f} - \hat{\theta}$. Using (15), $\hat{k} = \kappa\hat{R} - \hat{G}$, and noting $\hat{R} = \hat{\theta} + \hat{v} + \hat{F}$ gives the result for SR. Next, the condition SF results upon noting that the constant elasticity form in (17) yields $\hat{\delta'} = \omega\hat{v}$. Finally, the last condition LR is evaluated at $z^F = 0$ and $\Omega = 0$, implying $S(x)(1-\xi) f(\theta) v = \delta + 1$. Defining $\hat{\Omega} \equiv d\Omega/(\delta+1)$ and noting $v\delta'/\delta = 1 + \omega$ by (17) gives the last result in (A.17).

The welfare change derives from the differential of (24). The derivatives in (25) yield

$$\begin{aligned} dU^* &= (K_G - 1)\, G\hat{G} + (q-1)\, xN\hat{x} + (q-1)\,\rho x N\hat{p} \\ &+ [1-(1-t)\,s]\,\pi N\epsilon\hat{e} + [1-(1-t)(1-s)]\,\pi N\alpha\hat{a} \\ &+ \left[\frac{\eta}{\xi}\frac{S^*}{S} - 1\right] kR\hat{R} + \left[\frac{1-\eta}{1-\xi}\frac{S^*}{S} - 1\right] F\left[v\delta'\hat{v} + (\delta+1)\,\hat{F}\right]. \end{aligned} \tag{A.18}$$

The third to fifth terms follow after expanding $q\rho - 1 = (q-1)\,\rho + \rho - 1$ in (25b,c), using $\pi = (\rho - 1)\,x$, replacing effort costs β, γ from (11) and noting $ep_e = \epsilon p$ and $ap_a = \alpha p$ as well as $\hat{p} = \epsilon\hat{e} + \alpha\hat{a}$ and $N = pE$. In the third line, the first term follows from (25e) upon substituting $k = S\xi f/\theta$ from (16) and $dE/dR = f'$ and using (14). It compares with equation (30). The coefficient of $\hat{v}$ similarly derives from (25d) upon substituting δ' from (18). Finally, the effect of $\hat{F}$ relies on the assumption that the initial equilibrium is one with zero profits and a zero entry subsidy, $\Omega = 0$ and $z^F = 0$, so that $(1-\xi)\, S \cdot f(\theta)\, v = \delta + 1$. The wedges in the third line relate to matching externalities that arise whenever the conditions in (29) and (30) are not fulfilled.

Starting from an untaxed position and considering a uniform tax increase, one can use (A.14) to simplify the second line. For the third line, compare (9-10) with (25h) to see that $S = S^*$ in the absence of taxes. Using the investment and zero profit conditions in (16) and (18), one obtains $kR = \xi S \cdot Rf/\theta$, $Fv\delta' = (1-\xi)\, S \cdot vfF$ and $(\delta+1)\, F = (1-\xi)\, S \cdot vfF$. Since $Rf/\theta = E = vfF$, the welfare effects from search are propotional to the change in market tigthness, $\hat{\theta} = \hat{R} - \hat{v} - \hat{F}$,

$$\begin{aligned} dU^* = &\ (K_G - 1)\, G\hat{G} + (q-1)\, xN\hat{x} + (q-1)\,\rho xN \cdot \zeta_{PX}\left(\hat{x} - \hat{t}\right) \\ &+ \frac{(1-s)\epsilon + s\alpha}{1-\epsilon-\alpha}\pi N\left(\hat{x} - \hat{t}\right) + (\eta - \xi)\, SE \cdot \left(\hat{R} - \hat{v} - \hat{F}\right). \end{aligned} \tag{A.19}$$

C.2 Short-Run Equilibrium

The comparative statics of the short-run equilibrium is defined by PM, SR and SF in (A.17). The number of VCs is fixed but they may finance and advise a variable number of portfolio companies. To prepare for long-run results, it is insightful to consider also the impact of an exogenous entry $\hat{F}$ of VCs. The solution first solves for the intersection $\theta(x)$ and $v(x)$ of the supply and demand curves for VC defined by the search conditions SR and SF, taking venture returns x as given. Figure 2 illustrates. The next step solves the product market condition PM to obtain equilibrium returns x. With joint surplus uniquely related to venture returns by (A.16), VC market equilibrium yields

$$\begin{aligned} \hat{\theta} &= \tfrac{1}{\Delta_S}\left[(\omega-\kappa)\hat{S}+\omega\left(\hat{G}+k^{-1}\hat{z}^R\right)-\omega\kappa\hat{F}\right], \\ \hat{v} &= \tfrac{1}{\Delta_S}\left[(1+\kappa)\hat{S}+\eta\left(\hat{G}+k^{-1}\hat{z}^R\right)-\eta\kappa\hat{F}\right], \\ \eta\hat{\theta}+\hat{v} &= \tfrac{1}{\Delta_S}\left[(1+\kappa+(\omega-\kappa)\eta)\hat{S}+(1+\omega)\eta\left(\hat{G}+k^{-1}\hat{z}^R-\kappa\hat{F}\right)\right], \\ \Delta_S &\equiv (1-\eta+\kappa)\omega+\kappa\eta>0. \end{aligned} \tag{A.20}$$

We largely focus on the case $\omega>\kappa$ where the elasticity ω of marginal search cost of VCs exceeds the elasticity κ of entrepreneurial entry cost k. This is justified by the limit case of perfectly elastic supply of entrepreneurs where $\kappa=0$ and entry cost is the same for the entire population such that $k=k_0/G$ in (15). With entrepreneurial entry being more elastic than VC supply, an increase in the joint surplus of start-up firms attracts relatively more potential entrepreneurs with demand for VC compared to financing offers v. The market for VC thus becomes increasingly tight and $\theta=R/vF$ increases as in (A.20).

At this stage, we know how venture returns or profits, as implied by sales x, affect efforts and the success rate, joint surplus, market tightness and VC search intensity. Using (A.14,16,20) in condition PM in (A.17), $\hat{N}=\hat{p}+\eta\hat{\theta}+\hat{v}+\hat{F}$, one finds how venture returns determine the number of mature firms. Collecting terms and using Δ_S gives

$$\begin{aligned} \hat{N} \;=\; & \left[\zeta_{PX}+\tfrac{1+\kappa+(\omega-\kappa)\eta}{\Delta_S}\tfrac{(1-t)p\pi\cdot\zeta_{SX}}{S}\right]\hat{x}-\left[\zeta_{PX}+\tfrac{1+\kappa+(\omega-\kappa)\eta}{\Delta_S}\tfrac{(1-t)p\pi\cdot\zeta_{SX}-I^N}{S}\right]\hat{t} \\ & -\tfrac{1+\kappa+(\omega-\kappa)\eta}{\Delta_S}\tfrac{I^N}{S}\hat{t}^I+\tfrac{(1+\omega)\eta}{\Delta_S}\left(\hat{G}+k^{-1}\hat{z}^R\right)+\tfrac{(1-\eta)(1+\kappa)\omega}{\Delta_S}\hat{F}. \end{aligned} \tag{A.21}$$

This equation defines, in log-linearized form, an upward sloping supply curve of new goods. The left hand side of PM in (A.17) states the demand curve for product variety,

$\hat{N} = -\mu\hat{x} + \mu\lambda\hat{z}^X$. Figure 3 illustrates. The intersection yields scale and variety of innovative goods or, equivalently, venture returns and the number of mature firms in product market equilibrium. Formally,

$$\hat{x} = \zeta_t\hat{t} + \zeta_I\hat{t}^I + \zeta_x\hat{z}^X - \zeta_G\left(\hat{G} + k^{-1}\hat{z}^R\right) - \zeta_F\hat{F}, \qquad \hat{N} = -\mu\hat{x} + \mu\lambda\hat{z}^X. \tag{A.22}$$

The term Δ_X stands for the elasticity of excess demand for variety with respect to venture returns. All ζ-coefficients are defined positive,

$$\begin{array}{rlrl}
\Delta_X \equiv & \mu + \zeta_{PX} + \frac{1+\kappa+(\omega-\kappa)\eta}{\Delta_S}\frac{(1-t)p\pi\cdot\zeta_{SX}}{S}, & \mu \equiv & \left[(1-\lambda)\rho + \lambda\right]^{-1} > 0, \\
\zeta_t \equiv & \frac{\zeta_{PX}}{\Delta_X} + \frac{1+\kappa+(\omega-\kappa)\eta}{\Delta_X\Delta_S}\frac{(1-t)p\pi\cdot\zeta_{SX}-I^N}{S}, & \zeta_I \equiv & \frac{1+\kappa+(\omega-\kappa)\eta}{\Delta_X\Delta_S}\frac{I^N}{S}, \\
\zeta_G \equiv & \frac{(1+\omega)\eta}{\Delta_X\Delta_S}, \qquad \zeta_x \equiv \frac{\mu\lambda}{\Delta_X}, & \zeta_F \equiv & \frac{(1-\eta)(1+\kappa)\omega}{\Delta_X\Delta_S}.
\end{array} \tag{A.23}$$

To see the effect of the tax on efforts in (A.14), one requires $\hat{x} - \hat{t} = (\zeta_t - 1)\,\hat{t}$, where

$$1 - \zeta_t = \frac{\mu}{\Delta_X} + \frac{1+\kappa+(\omega-\kappa)\eta}{\Delta_X\Delta_S}\cdot\frac{I^N}{S} > 0. \tag{A.24}$$

A key result is the policy impact on VC search intensity which importantly depends on joint surplus. Replacing $\hat{x}$ in (A.16) by (A.22) yields

$$\begin{array}{rl}
\hat{S} = & -\frac{(\mu+\zeta_{PX})I^N}{S\Delta_X}\hat{t}^I - \frac{\mu\left[(1-t)p\pi\cdot\zeta_{SX}-I^N\right]-\zeta_{PX}I^N}{S\Delta_X}\hat{t} \\
& +\frac{(1-t)p\pi\cdot\zeta_{SX}}{S}\left[\zeta_x\hat{z}^X - \zeta_G\left(\hat{G} + k^{-1}\hat{z}^R\right) - \zeta_F\hat{F}\right].
\end{array} \tag{A.25}$$

The coefficient of $\hat{t}^I$ is $\frac{(1-t)p\pi\cdot\zeta_{SX}}{S}\zeta_I - \frac{I^N}{S} = -\frac{(\mu+\zeta_{PX})}{\Delta_X}\frac{I^N}{S}$ which follows from some manipulations after substituting ζ_I and using Δ_X. Further, the coefficient for $\hat{t}$ is $\frac{(\zeta_t-1)(1-t)p\pi\cdot\zeta_{SX}+I^N}{S}$ and emerges as in (A.25) upon substitution of $\zeta_t - 1$ and further manipulations using Δ_X. The tax rate reduces joint surplus if the elasticity μ of variety demand is large.

Using $\hat{S}$ in (A.20), we find that the only ambiguity in the effect on search intensity is in the impact of $\hat{G} + k^{-1}\hat{z}^R$. In this case, substitute ζ_G and use Δ_X and Δ_S and get after some manipulations

$$\begin{array}{rcl}
\hat{v} & = & \frac{1}{\Delta_S}\left[(1+\kappa)\,\hat{S} + \eta\left(\hat{G} + k^{-1}\hat{z}^R\right)\right] \\
& = & \left[\mu + \zeta_{PX} - \frac{(1-t)p\pi\cdot\zeta_{SX}}{S}\right]\frac{\eta}{\Delta_S\Delta_X}\left(\hat{G} + k^{-1}\hat{z}^R\right) \gtrless 0.
\end{array} \tag{A.26}$$

Entrepreneurship changes by $\hat{R} = \hat{\theta} + \hat{v} + \hat{F}$. Substituting from (A.20),

$$\hat{R} = \frac{1}{\Delta_S}\left[(1+\omega)\,\hat{S} + (\omega+\eta)\left(\hat{G} + k^{-1}\hat{z}^R\right) + (1-\eta)\,\omega\hat{F}\right]. \tag{A.27}$$

C.3 Long-Run Equilibrium

To link the short-run and long-run effects, one must show first how policy shocks create rents Ω on VC investing for a given number of VCs F and, second, how entry $\hat{F}$ eliminates rents to determine a long-run number of VCs (free entry). Substituting SF in LR in (A.17), one finds that policy creates rents whenever it boosts VC search intensity $\hat{v}$,

$$\hat{\Omega} = \frac{1}{1+\delta}\left[(1+\omega)\,\hat{v} + \hat{z}^F\right] = 0 \quad \Rightarrow \quad \hat{v} = -\frac{1}{1+\omega}\hat{z}^F. \tag{A.28}$$

If a policy raises search intensity it creates rents and thereby eventually attracts more VCs, $\hat{F} > 0$. As entry continues, rents get eroded until no further entry is profitable. We now show that an exogenous increase in F reduces individual search intensity and short-run rents to VC investing. Now consider (A.17). For any given market tightness and search intensity, the supply curve for VC shifts to the right since more VCs are able to finance more firms. As Figure 2 indicates, market tightness and equilibrium search intensity both decline to equilibrate the market for VC, see (A.20). The implied reduction in portfolio size retards the initial supply expansion and stops the decline in venture returns. On net, VC entry boosts the number of mature firms and thereby erodes venture returns by $\hat{x} = -\zeta_F \hat{F}$ as in (A.22). Figure 3 illustrates. Since $\hat{F} > 0 > \hat{S}$ both reduces search intensity, we have shown that VC entry reduces rents in (A.28). If a policy boosts search intensity and VC rents in the short-run, it triggers entry of VCs. Increasing competition reduces the returns to search and discourages new investments until rents are finally exhausted and the long-run search intensity attains the value in (A.28). Quite intuitively, a higher entry subsidy reduces the net opportunity cost $1 - z^F$ of a VC such that a smaller portfolio is required to break even. Search intensity is reduced in the long-run.

Next, the investment conditions determine market tightness and joint surplus of start-ups although the number of VCs is not yet known and must be inferred from the product market condition. Solving SR and SF in (A.17) in terms of $\hat{v}$ yields the solution for the required surplus and market tightness,

$$\begin{aligned}
\hat{\theta} &= \tfrac{1}{1+\kappa}\left[\left(\hat{G}+k^{-1}\hat{z}^R\right)-\kappa\hat{F}+(\omega-\kappa)\,\hat{v}\right], \\
\hat{S} &= \tfrac{\Delta_S}{1+\kappa}\hat{v}-\tfrac{\eta}{1+\kappa}\left(\hat{G}+k^{-1}\hat{z}^R\right)+\tfrac{\kappa\eta}{1+\kappa}\hat{F}.
\end{aligned} \tag{A.29}$$

By (A.16), to sustain this surplus, the necessary venture returns must amount to

$$\begin{aligned}
\hat{x} = & \tfrac{S}{(1-t)p\pi\cdot\zeta_{SX}}\left[\tfrac{I^N}{S}\hat{t}^I+\tfrac{(1-t)p\pi\cdot\zeta_{SX}-I^N}{S}\hat{t}\right] \\
+ & \tfrac{S}{(1-t)p\pi\cdot\zeta_{SX}}\left[\tfrac{\Delta_S}{1+\kappa}\hat{v}-\tfrac{\eta}{1+\kappa}\left(\hat{G}+k^{-1}\hat{z}^R\right)+\tfrac{\kappa\eta}{1+\kappa}\hat{F}\right].
\end{aligned} \tag{A.30}$$

Now get $\hat{F}$ from product market condition, $\mu\lambda\hat{z}^X=(\mu+\zeta_{PX})\,\hat{x}+\eta\hat{\theta}+\hat{v}+\hat{F}-\zeta_{PX}\hat{t}$, after substituting the results for $\hat{\theta}$ and $\hat{x}$:

$$\begin{aligned}
\nabla_F\cdot\hat{F} &= \mu\lambda\hat{z}^X-\tfrac{(\mu+\zeta_{PX})I^N}{(1-t)p\pi\cdot\zeta_{SX}}\hat{t}^I \\
&- \tfrac{\mu\left[(1-t)p\pi\cdot\zeta_{SX}-I^N\right]-\zeta_{PX}I^N}{(1-t)p\pi\cdot\zeta_{SX}}\hat{t} \\
&+ \tfrac{\mu+\zeta_{PX}-(1-t)p\pi\cdot\zeta_{SX}/S}{(1-t)p\pi\cdot\zeta_{SX}/S}\tfrac{\eta}{1+\kappa}\left(\hat{G}+k^{-1}\hat{z}^R\right) \\
&- \left[\tfrac{1+\kappa+(\omega-\kappa)\eta}{1+\kappa}+\tfrac{(\mu+\zeta_{PX})S}{(1-t)p\pi\cdot\zeta_{SX}}\tfrac{\Delta_S}{1+\kappa}\right]\hat{v}, \\
\nabla_F &\equiv \tfrac{1+(1-\eta)\kappa}{1+\kappa}+\tfrac{(\mu+\zeta_{PX})S}{(1-t)p\pi\cdot\zeta_{SX}}\tfrac{\eta\kappa}{1+\kappa}>0.
\end{aligned} \tag{A.31}$$

At first sight, the coefficients of $\hat{t}$ and $\hat{G}+k^{-1}\hat{z}^R$ are ambiguous which is a mirror image of the short-run ambiguity in search intensity and VC rents, see (A.25-26). The short-run effect of the tax rate on search intensity and VC rents and, thereby, on long-run VC entry is negative if the demand elasticity μ is large. This condition is met if the demand elasticity of the final good $\lambda\leq\sigma$ is large.[25] If μ is sufficiently large, the coefficient of $\hat{G}$ is positive as well so that the tax reduces VC entry while more R&D spending and research grants for seed investments raise it in the long-run.

Output Subsidy: Setting $\hat{v}=0$ as in (A.28) and taking the product market clearing number of VCs from (A.31), $\hat{F}=\frac{\mu\lambda}{\nabla_F}\hat{z}^X>0$, one derives from (A.29-30)

$$\begin{aligned}
&\hat{S}=\tfrac{\kappa\eta}{1+\kappa}\hat{F}>0, \quad \hat{x}=\tfrac{S}{(1-t)p\pi\cdot\zeta_{SX}}\tfrac{\kappa\eta}{1+\kappa}\hat{F}>0, \\
&\hat{\theta}=-\tfrac{\kappa}{1+\kappa}\hat{F}<0, \quad \hat{E}=\hat{\theta}+\hat{v}+\hat{F}=\tfrac{1}{1+\kappa}\hat{F}>0.
\end{aligned} \tag{A.32}$$

[25] Note that $\mu>0$ requires $\lambda\leq\sigma$ where the upper bound $\sigma>1$. If $\lambda\to\sigma$ from below, $\mu\to\infty$.

Subsidizing VC Entry: The first line results from (A.28) and (A.31). The second is shown after some manipulations by substituting (A.31) into (A.29). The effect on $\hat{x}$ follows from inverting (A.16) and noting the impact on $\hat{S}$. Lastly, the impact on $\hat{\theta}$ follows from (A.20) and the signs of $\hat{S}$ and $\hat{F}$:

$$\begin{aligned} &\hat{v} = -\tfrac{1}{1+\omega}\hat{z}^F < 0, \quad \hat{F} = -\tfrac{1}{\nabla_F}\left[\tfrac{1+\kappa+(\omega-\kappa)\eta}{1+\kappa} + \tfrac{(\mu+\zeta_{PX})S}{(1-t)p\pi\cdot\zeta_{SX}}\tfrac{\Delta_S}{1+\kappa}\right]\hat{v} > 0, \\ &\hat{S} = \tfrac{1}{1+\kappa}\left[\Delta_S\hat{v} + \eta\kappa\hat{F}\right] = \tfrac{(1-\eta)\omega}{\nabla_F}\hat{v} < 0, \\ &\hat{x} = \tfrac{S}{(1-t)p\pi\cdot\zeta_{SX}}\hat{S} < 0, \quad \hat{N} = -\mu\hat{x} > 0, \quad \hat{\theta} = \tfrac{1}{\Delta_S}\left[(\omega-\kappa)\hat{S} - \omega\kappa\hat{F}\right] > 0. \end{aligned} \tag{A.33}$$

Research Policy: The first line results from (A.28) and (A.31), with $\hat{F} > 0$ for μ large. The square brackets in (A.26) and (A.31) are identical so that the sign of the short-run effect on $\hat{v}$ is the same as the sign of the long-run effect on VC entry $\hat{F}$. Substituting $\hat{F}$ from the first line of (A.34) and using the definition of ∇_F, one obtains after some manipulations $\hat{G} + k^{-1}\hat{z}^R - \kappa\hat{F} = \left(\hat{G} + k^{-1}\hat{z}^R\right)/\nabla_F$. Using this result in (A.29-30) yields the second and third lines where $\hat{N}$ simply reflects the product demand curve:

$$\begin{aligned} &\hat{v} = 0, \qquad \hat{F} = \tfrac{[\mu+\zeta_{PX}-(1-t)p\pi\zeta_{SX}/S]\eta}{\nabla_F(1+\kappa)(1-t)p\pi\zeta_{SX}/S}\left(\hat{G} + k^{-1}\hat{z}^R\right) \gtrless 0, \\ &\hat{x} = -\tfrac{S}{(1-t)p\pi\zeta_{SX}}\tfrac{\eta}{1+\kappa}\tfrac{1}{\nabla_F}\left(\hat{G} + k^{-1}\hat{z}^R\right) < 0, \qquad \hat{N} = -\mu\hat{x} > 0, \\ &\hat{S} = -\tfrac{\eta}{1+\kappa}\tfrac{1}{\nabla_F}\left(\hat{G} + k^{-1}\hat{z}^R\right) < 0, \qquad \hat{\theta} = \tfrac{1}{1+\kappa}\tfrac{1}{\nabla_F}\left(\hat{G} + k^{-1}\hat{z}^R\right) > 0. \end{aligned} \tag{A.34}$$

Self-financed Tax Cut: Free entry implies $\hat{v} = 0$. Imposing the budget restriction on (A.31), one gets

$$\hat{F} = -\frac{[(\zeta_{SX}-1)\mu - \zeta_{PX}]}{\zeta_{SX}\nabla_F}\cdot\hat{t} > 0. \tag{A.35}$$

The square bracket is positive under the same condition that also makes the short-run effect on search intensity positive. The number of VCs increases if the demand elasticity μ is large enough. Since $\hat{v} = 0$, entry of VCs reduces market tightness and raises joint surplus in the long-run, see (A.29). Venture returns emerge after imposing the budget restriction on (A.30) and some tedious manipulations:

$$\begin{aligned} \hat{x} = &\ \tfrac{\zeta_{SX}-1}{\zeta_{SX}}\cdot\hat{t} + \tfrac{S}{p\pi\zeta_{SX}}\tfrac{\kappa\eta}{1+\kappa}\hat{F} = \tfrac{(\zeta_{SX}-1)[1+(1-\eta)\kappa]+\zeta_{PX}\kappa\eta S/(p\pi)}{(1+\kappa)\zeta_{SX}\nabla_F}\hat{t} < 0, \\ \hat{x} - \hat{t} = &\ \tfrac{-1}{\zeta_{SX}}\cdot\hat{t} + \tfrac{S}{p\pi\zeta_{SX}}\tfrac{\kappa\eta}{1+\kappa}\cdot\hat{F} > 0. \end{aligned} \tag{A.36}$$

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