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ABSTRACT

Weddings with Uncertain Prospects – Mergers under Asymmetric Information*

We provide a framework for analysing bilateral mergers when there is two-sided asymmetric information about firms' types. We introduce the concepts of essentially monotone decreasing (EMD) and increasing (EMI) functions, which generalize the respective mono-tonicity properties. If the profit differential between post-merger and pre-merger profits satisfies EMD, low-state firms gain more than high-state firms from mergers in expectation. Using this result, we characterize the equilibria of merger games with simultaneous and sequential moves. The application of our framework to specific oligopoly models illustrates that the introduction of two-sided asymmetric information may lead to considerable changes in the predicted merger pattern.

JEL Classification: D43, D82, L13 and L33
Keywords: asymmetric information, essentially monotone decreasing functions, merger, oligopoly and single crossing

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1 Introduction

When Dynegy was considering a takeover of Enron in autumn 2001, it was certainly aware that the potential acquiree was in trouble. Quite clearly, however, Dynegy initially underestimated the magnitude of the problem.[1] After the company had found out more about its target, it invoked a "material adverse change" clause to retreat from the deal. Yet, not all firms that recently considered mergers or acquisitions were lucky enough to find ways out of a transaction with partners that turned out to be less attractive than expected. For example, in the merger with the German Hypobank, it took Hypovereinsbank more than two years "to discover the full horror of its partner's balance sheet" (*The Economist*, July 20, 2000). More generally, many mergers are considered as failures with the benefits of hindsight.[2]

The asymmetric information surrounding mergers is typically two-sided: Each of the firms knows more about its "quality" than the potential partner does. Both parties thus face the risk of joining a "bad" partner who drives down profits of the merged entity.[3] In the present paper, we therefore consider mergers under two-sided asymmetric information about firms' types.[4] High types are defined as having high stand-alone profits—that is, high profits if no transaction occurs—and as contributing to high merger profits if the transaction occurs. We ask whether low types are more likely to engage in mergers than high types. Our setup resembles a standard lemon's problem in the sense that bad types will be worse off than good types if no transaction occurs, as bad types have lower stand-alone profits (*stand-alone profit effect*).

[1] Dynegy's former CEO C. Watson is cited in *The Economist* (November 15, 2001) to have "looked under the hood", finding that Enron "might need a new paint job and some new tyres, but its engine is sound."

[2] See, for instance, the studies of Ravenscraft and Scherer (1987; 1989), but note also the results of Healy et al. (1992).

[3] Only if the transaction is entirely financed with cash, the owners of the aquired firm have no stake in the merged entity and thus do not need to be concerned about the lack of knowledge about the potential partner.

[4] See Hviid and Prendergast (1993) for an analysis of one-sided asymmetric information.

In contrast to the standard lemon's problem, however, lower types will generally also have lower payoffs if a transaction occurs: Bad types will drive down profits of the merged entity, thereby adversely affecting their own share of the profit (*profit share effect*). This specific feature distinguishes mergers from transactions like the sale of used cars where the seller's profit is independent of the type of the car if the transaction occurs.[5] In the merger setting, it is thus not obvious whether bad types have more to gain from entering a merger than good types. Our goal is to determine under which conditions the stand-alone profit effect dominates over the profit share effect, so that, in equilibrium, only the relatively bad types are going to merge.

To this end, we analyze a merger game, in which two firms are matched whose types $z_i, i = 1, 2$, are drawn from distributions that are common knowledge.[6] After having observed their own type, both firms state whether they consent to a merger. If both firms consent, a merger takes place. If at least one firm declines, there is no merger. Following the merger game, an oligopoly game is played. If no merger occurs, both firms earn their stand-alone profits. If a merger does occur, the joint profit is shared according to some predetermined rule.

The critical aspect of our model is that a higher value of z_i has a positive effect both on stand-alone profits and on post-merger profits. Without additional assumptions, it is therefore ambiguous whether the profit differential $g_i(z_i, z_j)$ is increasing or decreasing in z_i and, thus, whether good or bad types are more likely to gain from merger deals.

To make progress on this issue, it is crucial to develop a better understanding of the profit differential function g_i. We search for a condition on g_i guaranteeing that the *expected* profit differential is positive for low and negative for high own states, that is, satisfies a single-crossing property for arbitrary distributions of competitors' types. We introduce the concept of "essentially monotone decreasing" (EMD) functions. The class of such func-

[5] We are obviously abstracting from warranty payments here.

[6] In specific applications, types may be interpreted as cost or demand parameters, with lower cost or higher demand corresponding to a better type.

tions has the desired property. Apart from monotone decreasing functions, it contains many single-peaked functions, for example. Whether EMD holds in specific applications, depends on both the oligopoly model under consideration (including the interpretation of the firms' types) and the rule that is applied to share profits in the merged entity. However, EMD will be seen to hold widely. Further, we provide a slightly weaker property that is necessary for single crossing to obtain. While this characterization of single crossing is of interest in its own right, we are mainly concerned with its application to our merger game.

We first show that the merger game always has a trivial Bayesian equilibrium where players never consent to a merger, no matter what their type is. More interestingly, if g_i satisfies EMD, every equilibrium must be of a cut-off form where bad types consent to a merger and good types do not. We also provide a condition for non-trivial cut-off equilibria to exist, that is, cut-off equilibria where some positive measure of players consents to a merger.

Our analysis has two implications that are potentially relevant to the "Merger Puzzle" (Scherer 2002), that, in spite of their ubiquity, mergers are often considered as failures. First, in the equilibrium of our game there is always a non-degenerate measure of types for which mergers are not profitable ex post: Types that are just below the cut-off value break even in expectation, but make losses if the partner is drawn from the lower end of the distribution. Second, the cut-off nature of the equilibrium suggests that bad types are more likely to merge than good types. While this does not explain why mergers often turn out to be non-profitable, it fits with the perception that merged entities do not do well relative to other firms. Our model supports the interpretation that merged entities do badly because they consist of bad firms.

We realize that our model is highly reduced. Obviously, the terms of a merger are usually the outcome of a complex bargaining process. However, the literature on bargaining with two-sided incomplete information (e.g. Ausubel et al. 2002) shows that such models often admit a multitude of equilibria, and the outcomes often depend delicately on the details of the bargaining protocol. Therefore, we believe that the reduced form approach adopted in this paper is useful for making progress with respect to the issues we are interested in.

We apply our results to specific oligopoly models to highlight the effects of two-sided asymmetric information on merger decisions. In a first example, we consider a Cournot model with linear demand and constant marginal costs where the merging firms earn a predetermined share of post-merger profits. With (the negative of) marginal cost as the state variable, a firm's profit differential g_i is typically monotone decreasing and thus EMD in its own type. Under certainty, bilateral mergers are unprofitable for homogeneous firms (except for mergers for monopoly), but profitable for sufficiently heterogeneous firms. In the case of asymmetric information with identical distributions of types, the only Bayesian equilibrium is the one where players never consent to a merger.

In a second example, we consider a chain of monopolies with linear demand where we suppose that firms differ in fixed costs. With (the negative of) fixed cost as the state variable, a firm's profit differential is monotone decreasing and thus EMD in own type. Under certainty, the standard double marginalization argument implies that vertical mergers are always profitable. Under uncertainty, in contrast, firms with low fixed costs might shy away from mergers, for fear of being stuck with a high cost firm. Thus, typically we obtain a cut-off equilibrium where only bad types merge.

These applications might suggest that, in spite of the different nature of transactions under consideration, the outcome is similar to the lemon's problem, with only bad firms entering agreements. While this is true in the two examples just mentioned, there are cases where it is not. For instance, in a Bertrand model with constant costs, linear demand and (the negative of) marginal costs as the state variable, g_i is typically single-peaked in z_i, but not EMD, so our result does not guarantee that only low-types consent. More importantly, there are cases where the outcome is actually the converse of the lemon's problem, i.e. only the good firms consent to an agreement: For instance, consider a joint project between two firms. Suppose that the relevant state variables are relation-specific in the sense that the firms' outside options are independent of states. Then, the project's value and thus the gains from an agreement will typically be increasing in firm states, so that the logic of the earlier examples is reversed.

This paper adds to a rather sparse literature on mergers under incomplete information. So far, this literature has largely focused either on one-sided incomplete information or on the competitive effects of horizontal mergers in the presence of two-sided incomplete information. To our knowledge, this is the first analysis of the impact of two-sided incomplete information about the potential partner's characteristics on the decision to carry out horizontal, vertical or conglomerate mergers.[7]

The paper is organized as follows. In section 2, we introduce the main assumptions of our model. Section 3 introduces the concept of essentially monotone decreasing functions and their properties. Section 4 contains the equilibrium classification for the merger game. Section 5 applies the general results to examples. Section 6 concludes.

[7] In an early contribution, Saloner (1987) investigates the rationality of price cutting to drive rivals out of the market (predatory pricing) and deter potential entrants (limit pricing) when there is one-sided incomplete information. He shows that rational price cutting can arise even if takeovers are also available as a means of achieving monopoly power. Gal-Or (1988) assumes a linear stochastic demand function of which each producer observes a signal. She shows that in the Cournot equilibrium, a merger may impose informational disadvantages on the merging firms, thereby further reducing the incentives to merge. In the Bertrand equilibrium, however, merging firms always benefit from a larger informational advantage than nonmerging firms. Building on this approach, Cabral (1999) compares mergers and joint ventures as information sharing vehicles. Finally, Stennek (2001) argues that when there is mutual uncertainty with respect to firms' marginal costs, mergers may generate efficiency gains due to the pooling of information.

2 Assumptions

We consider an oligopoly model with some exogenous number of firms. Two of these firms, denoted as $i = 1, 2$, are engaged in a merger game. The firms may be active in the same market (and thus contemplate a horizontal merger). Yet, they might as well be operating in a vertical relationship (vertical merger), or producing unrelated goods (conglomerate merger). Each firm is characterized by a type $z_i \in \mathbb{R}$, which influences its profitability. There is asymmetric information about the value of z_i, i.e. firms know their own z_i but not their competitor's. The ex-ante probability of z_i is described by a probability distribution F_i with density f_i and compact support $[\underline{z}_i, \overline{z}_i] \subset \mathbb{R}$.[8] F_i is common knowledge.[9]

There are two versions of the game. In the *simultaneous merger game*, both firms simultaneously announce whether they are willing to enter an agreement. The decision of firm i is summarized in a variable s_i such that $s_i = 1$ if it consents to an agreement and $s_i = 0$ if it rejects it. In the *sequential merger game*, firm 1 first decides whether to offer a merger to firm 2. If firm 2 receives an offer, it can either accept or reject it. If no merger occurs, each firm earns its stand-alone oligopoly profit $\pi_i(z_i, z_j)$. This function, which is defined on some set $\mathcal{Z} \supset [\underline{z}_1, \overline{z}_1] \times [\underline{z}_2, \overline{z}_2]$, reflects the nature of competition as well as the interpretation of the type. If a merger occurs, the owners of each of the formerly separate firms $i = 1, 2$ earn profits $\pi_i^M(z_i, z_j)$. In the simplest case, $\pi_i^M(z_i, z_j)$ is given by $\alpha_i \pi^M(z_i, z_j)$, where π^M denotes the profits of the merged entity and $\alpha_i \in [0, 1]$ is some predetermined profit share. It is natural to interpret α_i as the outcome of a bargaining process that

[8] It is possible to extend the analysis to the case where the support of F_i is not compact, though we do not pursue this issue here.

[9] Note that we allow for ex ante heterogeneity between firms, i.e. firms' types z_i may be drawn from different distributions. This is of particular importance for vertical or conglomerate mergers where firms are producing entirely different goods. Even the interpretation of the firms' types might differ. For vertical mergers, for instance, the types might correspond to the costs of input production for the upstream firm and marketing ability for the downstream firm.

precedes the merger decision summarized in s_i. In this bargaining process, firms typically both reveal information about their types, e.g. by way of signaling, and they are thus able to update their beliefs about the potential partner's type. That is, the distribution functions $F_i, i = 1, 2$, describing the probability of z_i should be regarded as reflecting the remaining uncertainty after the bargaining process has come to an end.[10] π_i^M generally depends both on the type of oligopoly competition and the profit sharing rule. We shall assume that both π_i and π_i^M are non-decreasing in z_i, so that a higher type corresponds to higher profits. Firm i's profit differential is denoted as

$$g_i(z_i, z_j) = \pi_i^M(z_i, z_j) - \pi_i(z_i, z_j)$$

throughout the paper.

As both π_i and π_i^M are non-decreasing in z_i, the effect of z_i on g_i is ambiguous. The most difficult aspect of this paper will be to develop an appropriate substitute for monotonicity of g_i in z_i to assure that good firms are less inclined to merge than bad firms. We do not make any monotonicity assumption on g_i with respect to z_j, since monotonicity in z_j is not necessary to establish most of our results.[11]

A broader interpretation of the game we are considering is that some pair of players is deciding on whether to enter any type of bilateral agreement in the presence of two-sided incomplete information, for instance a joint venture. The analysis in sections 3 and 4 carries over verbatim to such a setting, whereas the applications discussed in section 5 are largely restricted to the merger interpretation. Section 6 concludes.

[10] More general interpretations are compatible with this notation. For instance, if the individual states become verifiable after a merger agreement is in place, α_i can be conditioned on z_i and z_j. Thus π_i^M might also take the form $\pi_i^M(z_i, z_j) = \alpha_i(z_i, z_j)\,\pi^M(z_i, z_j)$.

[11] In many examples, however, g_i will be non-decreasing in z_j, since π_i is non-increasing in z_j for most forms of oligopoly competition, and profit sharing rules will often be such that each partner's profit is higher the higher joint profits, implying that π_i^M is non-decreasing in z_j.

3 Essentially Monotone Decreasing Functions

For the proof of our main result, it will be crucial that the expected value $\int_{\underline{z}_j}^{\overline{z}_j} g_i(z_i, z_j) f_j(z_j) dz_j$ of the profit differential g_i satisfies a suitable single crossing property in z_i. Roughly speaking, we require a condition on g_i guaranteeing that low types z_i have a positive *expected* profit differential, whereas high types have a negative *expected* profit differential. This condition will then be shown to guarantee that bad firms are more inclined to consent to mergers than good firms. Unsurprisingly, this condition will be satisfied if g_i is monotone decreasing in z_i. As argued above, monotonicity of g_i is a fairly strong requirement, even though it holds in some contexts. In this section, we shall therefore state the necessary and sufficient conditions on g_i guaranteeing that the expected value of g_i satisfies this single crossing property.

We first supply some notation and auxiliary results.

Definition 1 *Let $U \subset \mathbb{R}$, $f : U \longrightarrow \mathbb{R}$ be a function.*

(*i*) *f satisfies **strong single crossing (SSC)** if, for all $x_H, x_L \in U$ such that $x_H > x_L$, $f(x_L) \geq 0$ implies $f(x_H) \geq 0$ and $f(x_L) > 0$ implies $f(x_H) > 0$.*

(*ii*) *f satisfies **SSC$^-$** if $-f$ satisfies SSC.*

In the following, suppose $h : \mathcal{X} \times \mathcal{Y} \longrightarrow \mathbb{R}$ is an arbitrary function on $\mathcal{X} \times \mathcal{Y} \subset \mathbb{R}^2$. We introduce the following notation:[12]

Notation 1

- $A(x) \equiv A_h(x) \equiv \{y \in \mathcal{Y} \mid h(x, y) \geq 0\}$
- $D(x) \equiv D_h(x) \equiv \{y \in \mathcal{Y} \mid h(x, y) \leq 0\}$
- $C \equiv C_h \equiv \{x \in \mathcal{X} \mid \mu(A(x)) > 0 \wedge \mu(D(x)) > 0\}$.

[12] Throughout the paper we denote the standard Lebesgue measure on $\mathbb{R}$ as μ.

The special case outlined in section 2 (where $x = z_i, y = z_j$ and $h = g$) serves to motivate the notation. $A(z_i)$ corresponds to the set of types z_j for which firm i with state z_i accepts an agreement, and $D(z_i)$ corresponds to the set of types z_j for which it declines.[13] C is the "critical" set of types z_i who accept an agreement with some types z_j, but decline it with others. The following definition is useful.

Definition 2 *(a) h is **essentially monotone decreasing** (EMD) in x if the following properties are satisfied:*
(EMD1) $\forall x^2 \geq x^1 : \mu(A(x^1)) = 0 \Longrightarrow \mu(A(x^2)) = 0.$
(EMD2) $\forall x^1 \leq x^2 : \mu(D(x^2)) = 0 \Longrightarrow \mu(D(x^1)) = 0.$
(EMD3) $\forall x^1, x^2 \in C, x^1 < x^2 :$

$$h(x^1, y) > h(x^2, y) \text{ for } \mu\text{-almost all } y \in A(x^1) \text{ or } A(x^2), \qquad \text{or}$$
$$h(x^1, y) > h(x^2, y) \text{ for } \mu\text{-almost all } y \in D(x^1) \text{ or } D(x^2).$$

(EMD4) The restriction of h to the subset $C \subset \mathcal{X}$, $h|_C(x, y)$, is non-increasing in x for μ-almost all $y \in \mathcal{Y}$.
*h is **essentially monotone decreasing in x in the weak sense** (WEMD) if (EMD1)–(EMD3) hold.*

*(b) h is **essentially monotone increasing** (EMI) in x if $-h$ is EMD in x. h is **essentially monotone increasing in x in the weak sense** (WEMI) if $-h$ is WEMD in x.*

In the model of section 2, the interpretation of (EMD1) is that, when some low type will not enter an agreement with a positive measure of other types, then neither will any higher type. The interpretation of (EMD2) is similar: When some high type z_i will enter an agreement with almost every type z_j, then any lower type will do so, too. (EMD3) and (EMD4) are additional monotonicity requirements on the critical set of types who consent for some types, but not for others. The following statement is fairly obvious.

[13] In both cases, the types z_j for which firm i with state z_i is indifferent are also included.

Lemma 1 *If $h(x,y)$ is monotone decreasing in x for μ-almost all $y \in \mathcal{Y}$, then h is EMD in x.*

Proof. See Appendix. ■

The converse of Lemma 1 does not hold. For instance, suppose single-peaked functions $h(x,y)$ are given as in Figure 1, that is, $h\left(\underline{x},\underline{y}\right) > 0$ and h moves upward as y increases. Even though h is non-monotone in x, it is EMD.[14]

<Figure 1 here>

Lemma 2 *If h is EMD in x, then $h(x,y)$ satisfies SSC^{-} in x for μ-almost all $y \in \mathcal{Y}$.*

Proof. See Appendix. ■

Figure 2 illustrates that the converse of Lemma 2 does not hold, as EMD includes monotonicity properties for the critical set $C \equiv [\underline{c}, \overline{c}]$, whereas SSC^{-} does not. Even WEMD does not hold in this particular example.

<Figure 2 here>

Lemma 1 and Lemma 2 together immediately imply the following result.

Corollary 1 *monotone decreasing $\Longrightarrow$ EMD $\Longrightarrow$ SSC^{-}.*

Our next result demonstrates more generally that a large class of single-peaked functions satisfies EMD, thereby making the intuition given in Figure 1 more precise. As usual, we say that $h(x,y)$ is single-peaked in x for some $y \in \mathcal{Y}$ if there exists a unique (but not necessarily interior) $x^*(y)$ such that $h(x,y)$ is non-decreasing for $x \leq x^*(y)$ and non-increasing for $x \geq x^*(y)$. Define $\widehat{x} = \sup_{y \in \mathcal{Y}}(x^*(y))$ and $\widetilde{x} = \inf C$.

[14] This observation uses the fact that C is the interval $(\underline{c}, \overline{c})$. The x-values to the left of $\underline{c}$ satisfy $\mu(D(x)) = 0$; those to the right of $\overline{c}$ satisfy $\mu(A(x)) = 0$. For a proof of a more general result, consider Lemma 3 below.

Lemma 3 *Suppose* $\underline{x} \equiv \min \mathcal{X}$ *exists and* $\widehat{x} \leq \widetilde{x}$*. If* $\underline{h}(y) \equiv h(\underline{x}, y) > 0$ *and* $h(x,y)$ *is single-peaked in* x *for* μ*-almost all* $y \in \mathcal{Y}$*, then* h *is EMD in* x*.*

Proof. See Appendix. ∎

EMD and EMI may be satisfied simultaneously, as the following result shows.

Lemma 4 *If either* $h(x,y) > 0$ *for all* $x \in \mathcal{X}$ *and* μ*-almost all* $y \in \mathcal{Y}$ *or* $h(x,y) < 0$ *for all* $x \in \mathcal{X}$ *and* μ*-almost all* $y \in \mathcal{Y}$*, then* h *is EMD and EMI.*

Proof. See Appendix. ∎

In the setting of section 2, $g_i(z_i, z_j) > 0$ for all $z_i \in \mathcal{Z}_i$ and almost all $z_j \in \mathcal{Z}_j$ corresponds to the case where every type will consent to a merger with almost every other type, whereas $g_i(z_i, z_j) < 0$ for all $z_i \in \mathcal{Z}_i$ and almost all $z_j \in \mathcal{Z}_j$ corresponds to the case where no type will consent to a merger with almost every other type.

The next result follows immediately from applying Lemmas 1 and 3 to $-h$.

Corollary 2 (*i*) *If* $h(x,y)$ *is monotone increasing in* x *for* μ*-almost all* $y \in \mathcal{Y}$*, then* h *is EMI in* x*.*

(*ii*) *Suppose* $\underline{x} = \min \mathcal{X}$ *exists and* $-h$ *is single-peaked with peak* $x^*(y)$ *and* $\check{x} = \sup_{y \in \mathcal{Y}} (x^*(y)) \leq \widetilde{x}$*. Then, if* $\underline{h}(y) < 0$*,* h *is EMI in* x*.*

Our first main result states that (EMD) is sufficient to guarantee ESC$^-$.

Theorem 1 *If* h *satisfies (EMD), then it also satisfies* ***downward single-crossing in expectation*** *(ESC$^-$), i.e.* $\int_B h(x,y) f(y)\, dy$ *satisfies SSC$^-$ in* x *for every pair* (B, f) *where* B *is a subset of* $\mathbb{R}$ *and* $f : B \to \mathbb{R}_+$ *is a function with* $\int_B f(y)\, dy > 0$*.*

Proof. See Appendix. ∎

In the context of the merger model, if g_i is EMD in z_i, Theorem 1 will be shown to imply that better firms are less likely to consent to a merger for given expectations on the other firm's type. To understand why monotonicity in z_i is not necessary to obtain this result, consider the set of z_i such that $\mu(D(z_i)) = 0$ for all z_i. Thus all types z_i gain from a merger with almost every other type z_j, so they also gain in expectation, no matter from which distribution z_j is drawn. Thus, it is natural that monotonicity on $\{z_i \,|\mu(D(z_i)) = 0\}$ is not required.

An alternative sufficient condition for ESC$^-$ could have been obtained by applying the same logic as in Athey (2000, Theorem 5). To see this, consider any function h such that there exists $\lambda > 0$ for all $x_H \geq x_L$ such that $h(x_L, y) \geq \lambda h(x_H, y)$ for all $y \in Y$. Then

$$\int_B h(x_L, y) f(y) \, dy \geq \lambda \int_B h(x_H, y) f(y) \, dy \tag{1}$$

for arbitrary B and f, and $\int_B h(x, y) f(y) \, dy$ satisfies SSC$^-$, i.e., h satisfies ESC$^-$. However, we believe that conditions EMD1–EMD4 are more transparent than (1), and also more directly applicable.

An example of a function satisfying (1) that is not EMD is sketched in Figure 3. In this example $\mathcal{X} = \{x_L, x_H\}$ and $h\{x_L, y\} \geq 2h\{x_H, y\}$ for all $y \in \mathcal{Y}$. Clearly, the function does not satisfy (EMD4): Both x_L and x_H are in the critical set C, but $h(x_L, y) < h(x_H, y)$ for $y \in D(x_L) = D(x_H)$.

<Figure 3 here>

Thus EMD1–EMD3 are not necessary for ESC$^-$. Nevertheless, we can provide a partial converse of Theorem 1.

Proposition 1 *If ESC$^-$ holds, h satisfies WEMD.*

Proof. See Appendix ■

The next result follows immediately from Theorem 1 and Proposition 1.

Corollary 3 (i) *If h is EMI, then it also satisfies* ***upward single-crossing in expectation*** *(ESC), i.e. $\int_B h(x, y) f(y) \, dy$ satisfies SSC in x for every (B, f) where B is a subset of R and $f : B \to R_+$ is a function with $f(y) \geq 0 \; \forall \; y \in B$ and $\int_B f(y) \, dy > 0$.*

(*ii*) *If (ESC) holds, h satisfies (WEMI).*

Proof. Apply Theorem 1 with $-h$ for (i), Proposition 1 for (ii). ■

4 The Equilibrium of the Merger Game

This section explores the characteristics of the equilibria that can arise in the merger game. We first analyze the equilibrium structure of the simultaneous game and then show that the equilibria in the sequential game lead to the same outcome.

4.1 The simultaneous merger game

In Theorem 2, we first show that the simultaneous merger game has a trivial Bayesian equilibrium where no type ever agrees to a merger. We then demonstrate that, if g_i satisfies EMD, any equilibrium must satisfy a very simple "cut-off" rule: Low types consent to a merger whereas high types do not. We finally derive conditions for existence of "non-trivial" cut-off equilibria where a positive measure of types consents.

For $i = 1, 2$, if firm i plays a strategy $s_i(z_i)$, we define $B_i \equiv B_i(s_i) \equiv \{z_i \mid s_i(z_i) = 1\}$, i.e., B_i denotes the set of z_i for which firm i consents to a merger.

Theorem 2 (*i*) *Each pair (s_1, s_2) with $\mathbb{P}[B_i(s_i)] \equiv \int_{B_i} f_i(z_i)\,dz_i = 0,\ i = 1, 2,$ is a trivial Bayesian Equilibrium of the simultaneous merger game.*

(*ii*) *Suppose g_i is EMD in z_i for $i = 1, 2$. Then for every Bayesian Equilibrium (s_1^*, s_2^*) of the simultaneous merger game in pure strategies with $\mathbb{P}[B_i(s_i^*)] > 0$ there are cut-off values $z_i^* \in \mathcal{Z}_i$ such that*

$$s_i^*(z_i) = \begin{cases} 1, & \text{if } z_i \leq z_i^* \\ 0, & \text{if } z_i > z_i^* \end{cases}, \quad i = 1, 2.$$

(*iii*) *If further g_i is non-decreasing in z_j and if for all $i, j = 1, 2, i \neq j$, there exists $\widehat{z}_i > \underline{z}_i$ such that*

$$\int_{\underline{z}_j}^{\widehat{z}_j} g_i(\widehat{z}_i, z_j) f_j(z_j) dz_j \geq 0, \tag{2}$$

for all $i = 1, 2$, $j \neq i$, then there exists a Bayesian Equilibrium (s_1^*, s_2^*) of the simultaneous merger game in pure strategies with $\widehat{z}_i \leq z_i^* \leq \overline{z}_i$.[15]

(*iv*) *Suppose $\mathcal{Z}_i = \mathcal{Z}_j = \mathcal{Z}$. If, for $i = 1, 2$, g_i is increasing in z_j and no $\widehat{z}$ exists such that $g_i(\widehat{z}, \widehat{z}) > 0$, then there is no non-trivial Bayesian Equilibrium with $\mathbb{P}[B_i(s_i^*)] > 0$.*

Proof. See Appendix. ■

Result (i) is very intuitive: If both firms believe that the competitor will not consent to a merger—no matter what his type is—it is a (weakly) best response never to consent, and beliefs are correct in equilibrium. Thus, there is always a trivial equilibrium where firms merge with probability zero.

Result (ii) relies crucially on the profit differentials g_i being EMD. Suppose the profit differentials are EMD in the first argument. Then, using Theorem 1, if some type z_i consents to a merger, so will any lower type $z_i' < z_i$, no matter what the distribution of z_j is. The result in (ii) applies this idea to the distribution of z_j corresponding to the equilibrium behavior of the competitor.

Result (iii) demonstrates that, if the profit differential g_i is non-decreasing in the potential partner's state z_j (as in most oligopoly models) and the expected profit differential is positive for *some* state above the minimum level, there is an equilibrium where firms merge with positive probability and cut-off values are bounded away from the minimal states.[16] The result

[15]It is possible to derive a uniqueness condition, which makes sure that one reaction function is always steeper than the other. As this condition is not particularly illuminating, we refrained from stating it.

[16]Clearly, condition (2) is a necessary condition for mergers to occur. The proof shows that it is also a sufficient condition.

relies on the fact that, if a potential partner increases its cut-off value, one should reply with a higher cut-off value. Intuitively, the latter follows since a higher cut-off value z_j increases the chances of firm i to merge with a high-state firm j. Figure 4 illustrates the equilibrium for the case $\mathcal{Z}_i = \mathcal{Z}_j$, with $\tilde{R}_i(z_j)$ denoting the cut-off value that corresponds to the strategy which is best response to a strategy with cut-off value z_j.

Finally, result (iv) shows that, if a merger of symmetric firms never generates strictly positive profit differentials under certainty, the only possible equilibrium under uncertainty is the trivial one where firms merge with probability zero.

<Figure 4 here>

Theorem 2 has two implications that may be relevant to a better understanding of the "Merger Puzzle" (Scherer 2002). First, there is always a positive measure of firms that find mergers unprofitable ex post: Firms with types just below the cut-off value break even in expectation but suffer losses if the partner turns out to be from the lower end of the distribution. Second, the cut-off nature of the equilibrium suggests that bad types are more likely to merge if the relevant conditions hold. This conclusion, however, needs to be interpreted carefully. Suppose there is ex ante heterogeneity, i.e., firms' types are drawn from different distributions. For simplicity, assume that firm 2 is chosen from a distribution that is generated by a shift of firm 1's distribution to the right. Then a low-type firm 2 with state $\tilde{z}_2$ consenting to a merger might have a higher type than a high-type firm 1 with state $\tilde{z}_1$ that does not consent to a merger. Figure 5 illustrates this argument.

<Figure 5 here>

The next result is an immediate corollary of Theorem 2. If, contrary to the assumptions of Theorem 2, g_i is EMI, so that good firms gain more than bad firms from a merger in expectation, the equilibria will be of the opposite cut-off type, i.e. only good firms consent to mergers.

Corollary 4 *Suppose g_i is EMI in z_i for all $i = 1, 2$. Then for every Bayesian Equilibrium (s_1^*, s_2^*) of the simultaneous merger game in pure strategies with $\mathbb{P}[B_i(s_i^*)] > 0$ there are cut-off values $z_i^* \in Z_i$ such that*

$$s_i^*(z_i) = \begin{cases} 0, & \text{if } z_i < z_i^* \\ 1, & \text{if } z_i \geq z_i^* \end{cases}, \quad i = 1, 2.$$

The proof follows the same logic as part (ii) of Theorem 2.[17]

4.2 The sequential merger game

We now show that due to the simple structure of the merger game, the essential insight of Theorem 2 is robust to a change from simultaneous to sequential moves. Recall that in the sequential game, firm 1 first decides whether to offer a merger to firm 2. If firm 1 decides not to offer a merger, the merger game ends and both firms receive their stand-alone profits. If firm 1 offers a merger, however, firm 2 gets a chance to either accept or reject it. Thereafter, the merger game ends and firms receive their stand-alone or post-merger profits.

Theorem 3 *Suppose g_i is EMD in z_i for all $i = 1, 2$. Then the subgame-perfect equilibrium of the sequential merger game also has a cut-off structure. The cut-off values z_1^{**}, z_2^{**} of the sequential merger game equal the cut-off values z_1^*, z_2^* of the simultaneous merger game.*

Proof. See Appendix. ■

To grasp the intuition of this result, first note that, irrespective of the time structure, the outcome of the merger game is always the same as soon as at least one firm does not consent to a merger. In the *simultaneous* game, each firm knows that its own decision affects the game's outcome only if the competitor consents, and its decision is therefore based on the assumption that the competitor does consent. Similarly, in the *sequential*

[17] Analogous results to parts (iii) and (iv) of Theorem 2 are straightforward.

game, firm 2 gets to decide about the merger only if firm 1 has already consented to the merger. Since, in both cases, firm 2's choice is based on firm 1 consenting, it is unaffected by the change in the time structure of the game. Anticipating firm 2's reaction, firm 1's optimal choice is not changed relative to the simultaneous game, either. As a result, the cut-off values of the sequential game equal those of the simultaneous game.

5 Examples

The following examples serve to show that, with types defined appropriately, the profit differentials g_i associated with bilateral mergers satisfy EMD in many oligopoly models, so that cut-off equilibria of the type characterized in section 4 emerge in the corresponding games. In some of these examples, g_i is monotone decreasing in z_i, in others it is not. We also provide an example where g_i is EMI, so that only the good types merge. Finally, we give an example where g_i is neither EMD nor EMI.

Throughout this section, we compare the results for two-sided asymmetric information with the benchmark case of certainty. Under asymmetric information, firms will have to decide about the merger without knowing each other's type and thus post merger profits. As a result, mergers that turn out to reduce total profits of the merging firms may well occur. Under certainty, however, merger decisions are taken under common knowledge about types. Mergers will thus occur if and only if they increase total profits of the merging firms.

5.1 Horizontal Merger in the Linear Cournot Model

Consider a homogeneous Cournot market with inverse demand $P(Q) = a - bQ$ and at least three firms. Suppose firms $i = 1, 2$ play a merger game. The firms' types are given by $z_i = -c$, i.e., the negative of marginal costs. Suppose both values of z_i are distributed with compact support $\mathcal{Z} = \mathcal{Z}_1 = \mathcal{Z}_2 = [\underline{z}, \overline{z}]$ and identical distribution functions $F_1 = F_2 = F$.

It is well known that with homogeneous marginal costs c, bilateral mergers are not profitable (Salant et al. 1983). That is, for arbitrary sharing rules there must be at least one $i \in \{1, 2\}$ such that $g_i(z, z) < 0$ for any $z \in \mathcal{Z}$. However, Barros (1998) has shown that, with sufficient cost asymmetry, mergers may be mutually profitable if rationalization is possible.[18] Thus, under certainty, firms will merge only if they are sufficiently dissimilar.

Contrast this with the case of uncertainty where firms must commit to a merger and an associated profit sharing mechanism without knowing each other's state. Suppose that firms commit to profit shares α_i such that $0 \leq \alpha_i \leq \frac{27}{37}; \alpha_1 + \alpha_2 = 1$. Then it is easily shown that $\partial g_i / \partial z_i < 0, i = 1, 2$. Therefore, g_i is EMD in z_i (Lemma 1). Thus, by Theorem 2, there is a trivial Bayesian equilibrium in pure strategies where firms never merge. Further, it turns out that there cannot be a non-trivial equilibrium where firms merge with positive probability, since $\partial g_i / \partial z_j > 0$ and there is no $\widehat{z}$ such that $g_i(\widehat{z}, \widehat{z}) > 0$ in the linear Cournot model. Theorem 2 thus implies that firms will never merge under uncertainty, even if there are profitable mergers under certainty for some realization of types.

5.2 Vertical Merger in a Chain of Monopolies

In a vertical chain of monopolies, the benchmark case is opposite to the homogeneous Cournot case: Under certainty, a bilateral (vertical) merger always increases joint profits because it eliminates double marginalization. Thus, both firms will consent to a merger for suitable profit-sharing rules. More specifically, consider a case where inverse demand for the final product is given by $P(Q) = a - bQ$. Assume that the downstream monopolist's marginal cost is normalized to zero and its fixed cost is given by $\varphi_1 = -z_1$. In addition, suppose the upstream monopolist produces the intermediate good with constant marginal cost c and fixed cost $\varphi_2 = -z_2$. Standard calculations show that, in the absence of a merger, the profits of the respective firms are

[18] More specifically, it is assumed that the merged entity produces with the minimum of the two relevant marginal costs.

given by

$$\pi_1 = \frac{(a-c)^2}{16b} + z_1;\ \pi_2 = \frac{2(a-c)^2}{16b} + z_2. \tag{3}$$

For given profit shares α_i, the equilibrium post-merger profits are given by

$$\pi_i^M = \alpha_i \left(\frac{4(a-c)^2}{16b} + z_1 + z_2 \right), \quad i = 1, 2.$$

Thus, joint profits are higher after the merger.

Now consider the case of two-sided asymmetric information with respect to fixed cost. Suppose z_1 and z_2 are identically distributed with distribution functions F and compact support $\mathcal{Z} \equiv [\underline{z}, \overline{z}]$. Straightforward calculations show that $\partial g_i / \partial z_i < 0, i = 1, 2$. Therefore, g_i is EMD in z_i (Lemma 1). By Theorem 2, there is thus a trivial Bayesian equilibrium where firms merge with probability zero. Also, since g_i is EMD, every equilibrium is of the cut-off type. To characterize the equilibrium more precisely, suppose $a = 9, b = 1, c = 1, \underline{z} = -4, \overline{z} = 0$, and z_i is uniformly distributed.

Note that, in equilibrium, it is impossible that every type consents, unless $\frac{5}{7} \geq \alpha_2 \geq 2/3$: If every player 2 consents, player 1 with $z_1 = 0$ expects

$$\int_{-4}^{0} g_1(0, z_2) f(z_2)\, dz_2 = 56\alpha_1 - 16.$$

Thus, such a player will only consent if $\alpha_1 \geq 2/7$. Similarly, a player with $z_2 = 0$ needs to obtain at least $\alpha_2 > 2/3$ if every type of player 1 consents. Further, since the stand-alone profit of $z_i = -4$ is 0 for $i = 1, 2$, the cut-off value must be greater than -4 for every $\alpha_1 \in (0, 1)$. We thus find that even though vertical mergers are always profitable under certainty, some firms may not be willing to merge under uncertainty.

5.3 Horizontal Merger in the Linear Bertrand Model

Let us now provide an example where the profit differential associated with a bilateral merger does not satisfy EMD. Consider a homogeneous Bertrand duopoly with demand $D(p) = a - p$. Suppose that the state variable is $z_i = -c_i$, i.e. the negative of marginal cost. The z_i are distributed with compact support $\mathcal{Z} = [\underline{z}, \overline{z}]$ and identical distribution functions F_i. Standard calculations yield the following profits in the absence of a merger

$$\pi_i = \begin{cases} 0 & \text{if } z_i \leq z_j \\ (z_i - z_j)(a + z_j) & \text{if } z_j < z_i \leq 2z_j + a \\ [(a + z_i)/2]^2 & \text{if } z_i > 2z_j + a \end{cases}.$$

Now assume that the merged entity can produce the homogeneous good with the average $-\frac{z_1+z_2}{2}$ of the merging firms' marginal costs. This assumption contrasts with the common rationalization assumption that post-merger costs equal $\min\{c_1, c_2\}$, that is, the merged entity can apply the technology of the superior firm. Our assumption is a particularly simple way of capturing the idea that post-merger marginal costs are increasing in both c_1 and c_2.[19]

Fig. 6 illustrates the profit differential for the parameter values $a = 10$, $\alpha_1 = \alpha_2 = \frac{1}{2}$ and $z_i \in \mathcal{Z} = [-10, 0], i = 1, 2$. Straightforward calculations show that g_i is increasing in z_i for $z_i \leq z_j$ and decreasing for $z_i > z_j$; in particular g_i is single-peaked for any $z_j \in \mathcal{Z}$. Intuitively, a firm earns zero profits on its own as long as $z_i < z_j$. An increase of z_i within this region therefore only affects π_i^M. The higher z_i, the higher π_i^M and therefore the higher g_i. For $z_i > z_j$, a further increase in z_i affects both π_i and π_i^M positively, but the former effect dominates as $\alpha_i < 1$, since the effect on π_i^M is diluted by the fact that firm i only earns a share α_i of π^M, whereas it earns all of π_i.

<Fig. 6 here>

As pointed out above, g_i is not EMD in this particular case even though it is single-peaked. To see this, observe that $g_i(z_i, 0) > 0$ for all $z_i \leq 0$ and $g_i(z_i, -10) < 0$ for all $z_i > -10$; the critical set thus satisfies $C = \mathcal{Z} \backslash \{-10\}$.

[19]In our specific example, post-merger marginal costs are above $\min\{c_1, c_2\}$. This is not essential for the argument.

As $g_i(z_i, z_j)$ has an internal maximum at $z_i = z_j$ for every $z_i \in (-10, 0)$, (EMD) is violated.

Now consider a modified version of this example where both firms are also active as monopolists in additional markets, with marginal costs $-z_i$ and positive fixed costs. Suppose that, if a merger occurs, the merged entity continues to be active in these monopoly markets, with demand and cost conditions being unchanged. It can then be shown that the profit differentials g_i are typically first increasing, then decreasing, and, in the language of Lemma 3, $\widehat{x} < x$.[20] Application of Lemma 3 shows that g_i is thus EMD, and therefore a cut-off equilibrium exists.

5.4 Complementary Research Activities

We shall briefly sketch a final example that moves slightly beyond the scope of merger analysis. Suppose two parties can engage in a joint venture that requires specific inputs of both parties. Further, assume that the state variable relates to an ability that is valuable for the project, so that the project's value $\pi^M(z_i, z_j)$ is increasing in both z_i and z_j. Finally, assume that the states are relation-specific in the sense that they have no effect on the firms' profits if the project is not carried out, implying that π_i is independent of (z_i, z_j). Then, provided that each party receives a positive share of each increase in π^M, g_i is increasing in a firm's own state. Thus, g_i is also EMI and Corollary 4 shows that under these circumstances, only good firms consent in equilibrium. That is, the basic result that only bad firms consent is reversed in this particular case.

[20] Calculations for an example with monopoly demand $D(p) = 4 - 5p$ and fixed cost $F = 5$ are availabe on request.

6 Conclusions

This paper presents a theory of two-party agreements—such as bilateral mergers—, where the potential partners have to consent to an agreement in the presence of two-sided asymmetric information about the payoff-relevant types of the parties involved.

In this setting, we show that the Bayesian equilibrium can be characterized in a simple and intuitive way, provided that the payoff differential associated with entering the two-party agreement satisfies a weak monotonicity requirement in the own state variable: If the payoff differential is essentially monotone decreasing in the own state (EMD), equilibria are of a cut-off type where only low-state firms consent to a merger. Similarly, if the payoff differential is essentially monotone increasing (EMI), only high state firms consent.

Our findings bear relevance for the "Merger Puzzle" in two respects: First, in the presence of two-sided asymmetric information, there are always types for which the merger turns out to be non-profitable ex post. Second, since the equilibria are of a cut-off type, our model suggests that merged entities do badly because they are typically formed by bad firms.

Applications of the general approach to specific contexts require checking the shape of the profit differential function which depends both on the type of competition under consideration and on the way the potential partner share profits. In the linear Cournot case, our approach shows that there will be no mergers under asymmetric information, even if firms are sufficiently heterogenous such that they would merge under certainty. In the case of vertical chains of monopolies, where firms would always merge under certainty, mergers might not occur under two-sided asymmetric information: again, only low-state firms are likely to merge, while high-state firms shy away from mergers for fear of joining a low-state firm. In other situations, this result may be reversed—i.e. only high-state firms are willing to merge—, for instance, when the value of a joint project depends positively on both firms' states, and the outside options of both firms are independent of states.

Appendix

Proof of Lemma 1

To show (EMD1) let $x^1 \in \mathcal{X}$ such that $\mu(A(x^1)) = 0$. Then $h(x^1, y) < 0$ for μ-almost all $y \in \mathcal{Y}$. Since $h(x, y)$ is monotone decreasing in x for μ-almost all $y \in \mathcal{Y}$ it follows that $h(x^2, y) < h(x^1, y) < 0$ for all $x^2 > x^1$ and μ-almost all $y \in \mathcal{Y}$. Therefore $\mu(A(x^2)) = 0$ for all $x^2 \geq x^1$.
The proof of (EMD2) is analogous, (EMD3) and (EMD4) are obvious.

Proof of Lemma 2

Suppose $h(x, y)$ does not satisfy SSC$^-$ in x for μ-almost all $y \in \mathcal{Y}$. Then there exists $x^1 < x^2$ and $M \subset \mathcal{Y}$ with $\mu(M) > 0$ such that $h(x^1, y) \leq 0 \leq h(x^2, y)$ for all $y \in M$. This implies $\mu(A(x^2)) > 0$ as $M \subset A(x^2)$. Since $x^1 < x^2$ we get $\mu(A(x^1)) > 0$ by (EMD1). Thus $x^1 \in C$. An analogous argument shows that $x^2 \in C$. Now if $h(x^1, y) < h(x^2, y)$ for μ-almost all $y \in M$, we have a contradiction to (EMD4), and if $h(x^1, y) = 0 = h(x^2, y)$ for μ-almost all $y \in M$, we have a contradiction to (EMD3).

Proof of Lemma 3

(EMD1) Let $x^1 \in \mathcal{X}$ such that $\mu(A(x^1)) = 0$. Then $h(x^1, y) < 0$ for μ-almost all $y \in \mathcal{Y}$. Since $\underline{h}(y) > 0 > h(x^1, y)$ and $h(x, y)$ is single-peaked in x for μ-almost all $y \in \mathcal{Y}$, it follows that $h(x^2, y) < h(x^1, y) < 0$ for all $x^2 > x^1$ and μ-almost all $y \in \mathcal{Y}$. Therefore $\mu(A(x^2)) = 0$ for all $x^2 \geq x^1$.

(EMD2) Let $x^2 \in \mathcal{X}$ such that $\mu(D(x^2)) = 0$. Then $h(x^2, y) > 0$ for μ-almost all $y \in \mathcal{Y}$. Since $\underline{h}(y) > 0$ and $h(x, y)$ is single-peaked in x for μ-almost all $y \in \mathcal{Y}$, it follows that $h(x^1, y) > 0$ for all $x^1 \leq x^2$ and μ-almost all $y \in \mathcal{Y}$. Therefore $\mu(D(x^1)) = 0$ for all $x^1 \leq x^2$.

(EMD3) Let $x^1, x^2 \in C,\ x^1 < x^2$, and $y \in D(x^1)$. Since $\underline{h}(y) > 0 \geq h(x^1, y)$ and $h(x, y)$ is single-peaked in x for μ-almost all $y \in \mathcal{Y}$, it follows that $h(x^1, y) > h(x^2, y)$ for μ-almost all $y \in D(x^1)$.

(EMD4) As $h(x, y)$ is single-peaked in x, it is non-increasing in x for $x \geq \widehat{x}$ and μ-almost all $y \in \mathcal{Y}$. As $\widetilde{x} \geq \widehat{x}$, $h(x, y)$ is non-increasing in x for $x \in C$ and μ-almost all $y \in \mathcal{Y}$.

Proof of Lemma 4

If $h(x, y) > 0$ for all $x \in \mathcal{X}$ and μ-almost all $y \in \mathcal{Y}$, then $\mu(A(x)) > 0$, $\mu(D(x)) = 0$ for all $x \in \mathcal{X}$, and $C = \emptyset$. Therefore (EMD1)–(EMD4) are trivially satisfied. An analogous argument holds for $h(x, y) < 0$.

Proof of Theorem 1

(a) Let $x \in \mathcal{X}$ such that $\mu(D(x)) = 0$. Then $h(x, y) > 0$ for μ-almost all y. Therefore $\int_B h(x, y) f(y)\, dy > 0$ for all (B, f) with $\int_B f(y)\, dy > 0$.

(b) For $x \in \mathcal{X}$ with $\mu(A(x)) = 0$ an analogous argument shows that $\int_B h(x, y) f(y)\, dy < 0$ for all (B, f) with $\int_B f(y)\, dy > 0$.

(c) Let $x^1, x^2 \in C$ such that $x^1 < x^2$. From (EMD4) we know that $\int_B h(x^1, y) f(y)\, dy \geq \int_B h(x^2, y) f(y)\, dy$ for all (B, f) with $\int_B f(y)\, dy > 0$. Now suppose that there exists (B_0, f_0) with $\int_{B_0} f_0(y)\, dy > 0$ such that

$$\int_{B_0} h\left(x^1, y\right) f_0(y)\, dy = 0 = \int_{B_0} h\left(x^2, y\right) f_0(y)\, dy. \tag{4}$$

This implies $\mu(A(x^i) \cap B_0) > 0$ and $\mu(D(x^i) \cap B_0) > 0$ for $i = 1, 2$. Together with (EMD3) and (EMD4) we get $\int_{B_0} h(x^1, y) f_0(y)\, dy > \int_{B_0} h(x^2, y) f_0(y)\, dy$, which is a contradiction to (4). Thus, for h restricted to C, $\int_{B_0} h(x, y) f(y)\, dy$ satisfies single crossing.

(d) (EMD1) and (EMD2) guarantee that $\sup\{x \in \mathcal{X} \mid \mu(D(x)) = 0\} = \inf C$ and $\sup C = \inf\{x \in \mathcal{X} \mid \mu(A(x)) = 0\}$, and so (a)–(c) establish SSC$^-$ for $\int_B h(x, y) f(y)\, dy$ for all (B, f) with $\int_B f(y)\, dy > 0$.

Proof of Proposition 1

(a) Suppose that (EMD1) is not satisfied. Then there exists $x^1 < x^2$ such that $\mu(A(x^1)) = 0$ and $\mu(A(x^2)) > 0$. Therefore we can find $B \subset A(x^2)$ and f such that $\int_B f(y)\,dy > 0$. We thus get $\int_B h(x^1, y) f(y)\,dy < 0 \leq \int_B h(x^2, y) f(y)\,dy$, which is a contradiction to SSC$^-$.

(b) The necessity of (EMD2) is proven analogously.

(c) Suppose (EMD3) is not satisfied. Then there exists $x^1, x^2 \in C, x^1 < x^2$ and $M_1 \subset A(x^1), M_2 \subset D(x^1)$ with $\mu(M_i) > 0,\ i = 1, 2$, such that $h(x^1, y) \leq h(x^2, y)$ for all $y \in M_1 \cup M_2$. Therefore we can find $(B \subset M_1 \cup M_2, f)$ with $\int_B f(y)\,dy > 0$ and such that $0 = \int_B h(x^1, y) f(y)\,dy \leq \int_B h(x^2, y) f(y)\,dy$, which contradicts SSC$^-$. This completes the proof.

Proof of Theorem 2

(i) Suppose firm i plays a strategy $s_i(z_i)$ with $\mathbb{P}[B_i(s_i)] = 0$. Then the probability that a merger takes place is zero and therefore firm $j \neq i$ is indifferent between any strategies it can play; in particular, every strategy $s_j(z_j)$ with $\mathbb{P}[B_j(s_j)] = 0$ is a best response.

(ii) Firm i's expected net gain from consenting to a merger, facing firm j with strategy s_j, is

$$\begin{aligned} & \mathbb{P}[B_j]\,\mathbb{E}_{z_j}\left[\pi_i^M(z_i, z_j) \mid z_j \in B_j(s_j)\right] + \\ & (1 - \mathbb{P}[B_j])\,\mathbb{E}_{z_j}[\pi_i(z_i, z_j) \mid z_j \notin B_j(s_j)] - \mathbb{E}_{z_j}[\pi_i(z_i, z_j)] \\ = & \int_{B_j} g_i(z_i, z_j) f_j(z_j)\,dz_j. \end{aligned} \tag{5}$$

If (5) is positive, firm i will offer a merger, if (5) is negative it will stay alone. By Theorem 1 we know that $\int_{B_j} g_i(z_i, x) f_j(x)\,dx$ satisfies SSC$^-$ in z_i. Denote the single crossing points required by Definition 1 as $z_i^\circ(s_j)$. Now define

$$\tilde{R}_i(s_j) = \begin{cases} z_i^\circ(s_j), & \text{if } z_i^\circ(s_j) \le \overline{z_i} \\ \overline{z_i}, & \text{if } z_i^\circ(s_j) \ge \overline{z_i} \text{ or if } z_i^\circ(s_j) \text{ does not exist} \end{cases}.$$

Then firm i's optimal reaction is

$$R_i(z_i, s_j) = \begin{cases} 1, & \text{if } z_i \le \tilde{R}_i(s_j) \\ 0, & \text{if } z_i > \tilde{R}_i(s_j) \end{cases}.$$

In particular, for an equilibrium strategy s_j the best reply has the required cut-off structure.

(iii) By (ii), we can identify a best-response strategy with its cut-off value and write $\tilde{R}_i(z_j)$ to denote the cut-off value corresponding to the strategy that is best-response to a strategy with cut-off value z_j. We first show that $\tilde{R}_i(z_j)$ is increasing in z_j. To see this, suppose $z_j^H > z_j^L$. Then

$$\begin{aligned} & \int_{\underline{z}_j}^{z_j^H} g_i\left(\tilde{R}_i\left(z_j^L\right), z_j\right) f_j(z_j)\, dz_j \\ = & \int_{\underline{z}_j}^{z_j^L} g_i\left(\tilde{R}_i\left(z_j^L\right), z_j\right) f_j(z_j)\, dz_j + \int_{z_j^L}^{z_j^H} g_i\left(\tilde{R}_i\left(z_j^L\right), z_j\right) f_j(z_j)\, dz_j, \end{aligned}$$

where $\int_{\underline{z}_i}^{z_j^L} g_i\left(\tilde{R}_i\left(z_j^L\right), z_j\right) f_j(z_j)\, dz_j = 0$ by definition of $\tilde{R}_i\left(z_j^L\right)$. As g_i is non-decreasing in z_j, $g_i\left(\tilde{R}_i\left(z_j^L\right), z_j\right) \ge 0$ for all $z_j \in \left[z_j^L, z_j^H\right]$. Thus $\int_{\underline{z}_i}^{z_j^H} g_i\left(\tilde{R}_i\left(z_j^L\right), z_j\right) f_j(z_j)\, dz_j \ge 0$ and therefore $\tilde{R}_i\left(z_j^H\right) > \tilde{R}_i\left(z_j^L\right)$ by SSC^-. By (2) and SSC^- we have $\tilde{R}_i(\widehat{z}_j) \ge \widehat{z}_i$. Since $\tilde{R}_i(\cdot)$ is increasing we get $\tilde{R}_i([\widehat{z}_j, \overline{z_j}]) \subset [\widehat{z}_i, \overline{z_i}]$. Therefore, $(z_1, z_2) \longmapsto \left(\tilde{R}_1(z_2), \tilde{R}_2(z_1)\right)$ defines a non-decreasing mapping from $[\widehat{z}_1, \overline{z}_1] \times [\widehat{z}_2, \overline{z}_2]$ into itself. Therefore, Tarski's fixed point theorem guarantees the existence of equilibrium cut-off values $z_1^* \in [\widehat{z}_1, \overline{z}_1]$, $z_2^* \in [\widehat{z}_2, \overline{z}_2]$.

(iv) Suppose there is no $\widehat{z}$ such that $g_i(\widehat{z}, \widehat{z}) > 0, i = 1, 2$. Suppose w.l.o.g that there is a non-trivial cut-off equilibrium (z_1^*, z_2^*) with $z_1^* \ge z_2^*$.

Expected equilibrium payoffs for firm 1 for all $\underline{z} < z_1^* \leq z_2^* < \overline{z}$ would be $\int_{\underline{z}_2}^{z_2^*} g_1(z_1^*, z_2) f_2(z_2) dz_2 < 0$, because for all $z_2 \leq z_2^*, g_1(z_1^*, z_2) < g_1(z_1^*, z_2^*) \leq g_1(z_1^*, z_1^*) \leq 0$.

Proof of Theorem 3

The sequential merger game is solved by backward induction. Consider firm 2 first. Facing a strategy s_1 with $B_1 = B_1(s_1) = \{z_1 \in \mathcal{Z}_1 \mid s_1(z_1) = 1\}$, firm 2 will agree to a merger if and only if

$$\mathbb{E}_{z_1}\left[\pi_2^M(z_1, z_2) \mid z_1 \in B_1(s_1)\right] \geq \mathbb{E}_{z_1}\left[\pi_2(z_1, z_2) \mid z_1 \in B_1(s_1)\right],$$

which is equivalent to

$$\frac{1}{\mathbb{P}[B_1]} \int_{B_1} g_2(z_2, z_1) f_1(z_1) dz_1 \geq 0. \tag{6}$$

Denote the reaction function in the sequential game as $\tilde{R}_2^{se}$. Since the left-hand side of (6) is zero if and only if the integral in (5) is zero, $\tilde{R}_2(s_1) = \tilde{R}_2^{se}(s_1)$ for all strategies s_1.

Now we look at firm 1. Firm 1 will offer a merger if and only if

$$\begin{aligned} & \mathbb{P}[B_2]\,\mathbb{E}_{z_2}\left[\pi_1^M(z_1, z_2) \mid z_2 \in B_2(s_2)\right] + \\ & (1 - \mathbb{P}[B_2])\,\mathbb{E}_{z_2}\left[\pi_1(z_1, z_2) \mid z_2 \notin B_2(s_2)\right] - \mathbb{E}_{z_2}\left[\pi_1(z_1, z_2)\right] \\ = & \int_{B_2} g_1(z_1, z_2) f_2(z_2) dz_2 \geq 0. \end{aligned}$$

If firm 1 anticipates the reaction of firm 2, it expects $B_2 = \left[\underline{z}_i, \tilde{R}_2(s_1)\right]$. Therefore z_1^{**} satisfies

$$\int_{\underline{z}_i}^{\tilde{R}_2\left(z_1^{**}\right)} g_1(z_1^{**}, z_2) f_2(z_2) dz_2 = 0.$$

Thus z_1^{**} is a fixed point of $\tilde{R}_1 \circ \tilde{R}_2$. The same is true for z_1^*, and so $z_1^* = z_1^{**}$.

References

Athey, S. (2000): "Characterizing Properties of Stochastic Objective Functions", MIT Working Paper 96-1R.

Ausubel, L.M., Cramton, P., Deneckere, R.J. (2002): "Bargaining with Incomplete Information", in Aumann, R., Hart, S. (eds.): *Handbook of Game Theory*, 3, 1897-1945.

Barros, P.P. (1998): "Endogenous Mergers and Size Asymmetry of Merger Participants", *Economics Letters*, 60, 113-119.

Cabral, I. (1999): "Information Exchange: Mergers and Joint Ventures", European University Institute, Florence (mimeo).

Gal-Or, E. (1988): "The Informational Advantages or Disadvantages of Horizontal Mergers", *International Economic Review*, 29, 639-661.

Healy, P.M., Palepu, K.G., Ruback, R.S. (1992): "Does Corporate Performance Improve After Mergers?", *Journal of Financial Economics*, 31, 135-175.

Hviid, M., Prendergast, C. (1993): "Merger Failure and Merger Profitability", *Journal of Industrial Economics*, 41, 371-386.

Ravenscraft, D.J., Scherer, F.M. (1989): "The Profitability of Mergers", *International Journal of Industrial Organization*, 7, 101-116.

Ravenscraft, D.J., Scherer, F.M. (1987): *Mergers, Sell-Offs, and Economic Efficiency*, Washington.

Salant, S., Switzer, S., Reynolds, R.J. (1983): "Losses from Horizontal Merger: The Effects of an Exogenous Change in Industry Structure on Cournot-Nash Equilibrium", *Quarterly Journal of Economics*, 98, 185-199.

Saloner, G. (1987): "Predation, Mergers, and Incomplete Information", *Rand Journal of Economics*, 18, 165-186.

Scherer, F.M. (2002): "The Merger Puzzle", in Franz, W., Ramser, H.J., Stadler, M. (eds.): *Fusionen*, Wirtschaftswissenschaftliches Seminar Ottobeuren 31, 1-22.

Stennek, J. (2001): "Horizontal Mergers Without Synergies May Increase Consumer Welfare", University of Stockholm (mimeo).

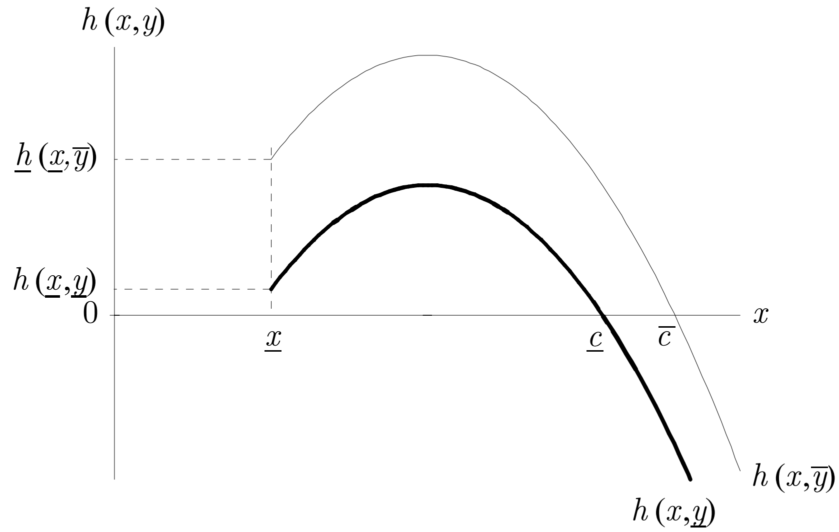

Figure 1: Single-peaked functions $h(x, y)$

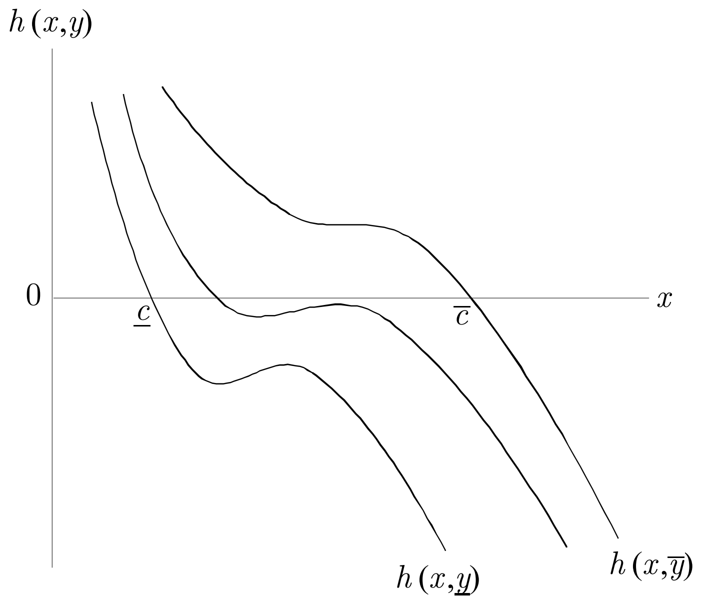

Figure 2: $h(x,y)$ is SSC$^-$ but not WEMD

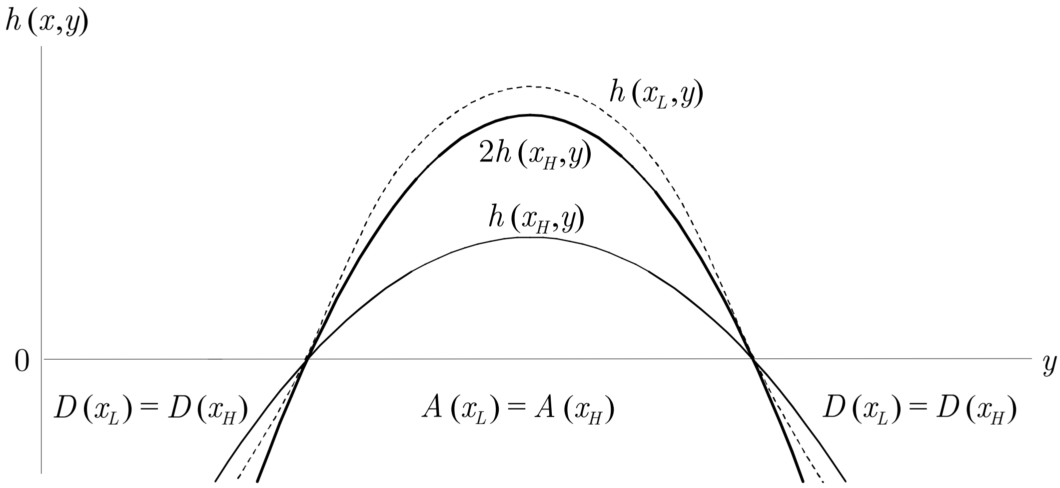

Figure 3: SSC$^-$ does not imply ESC$^-$

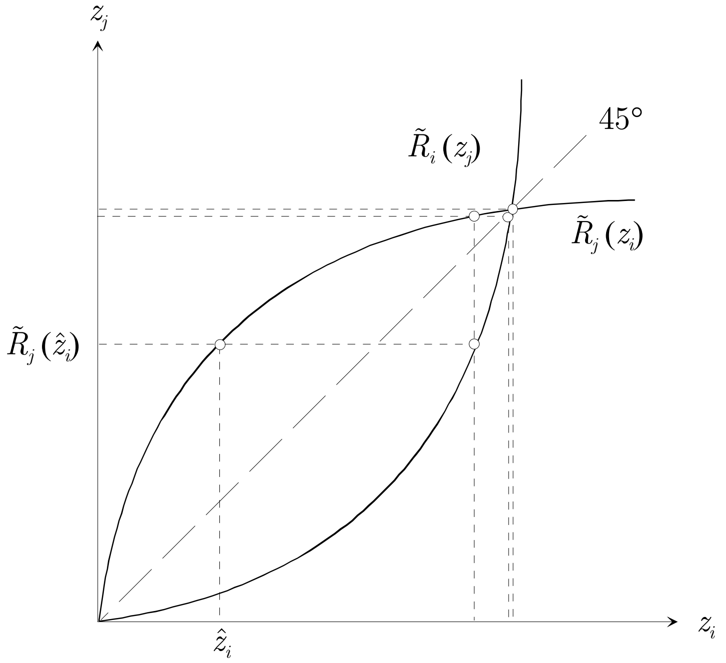

Figure 4: Symmetric equilibrium in cut-off strategies

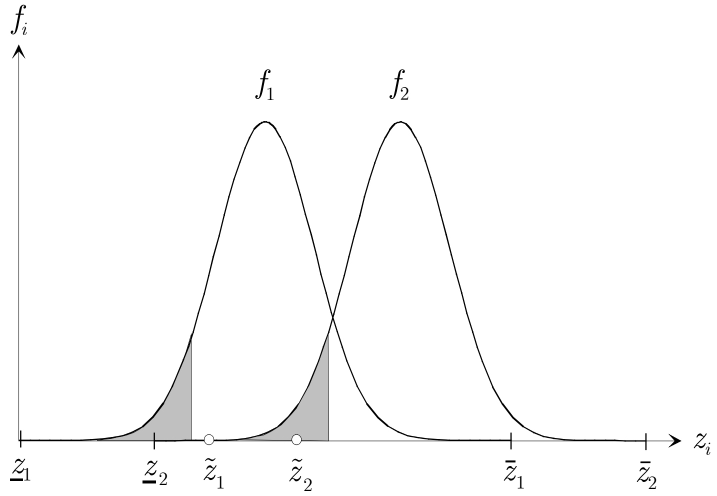

Figure 5: Ex ante heterogeneity

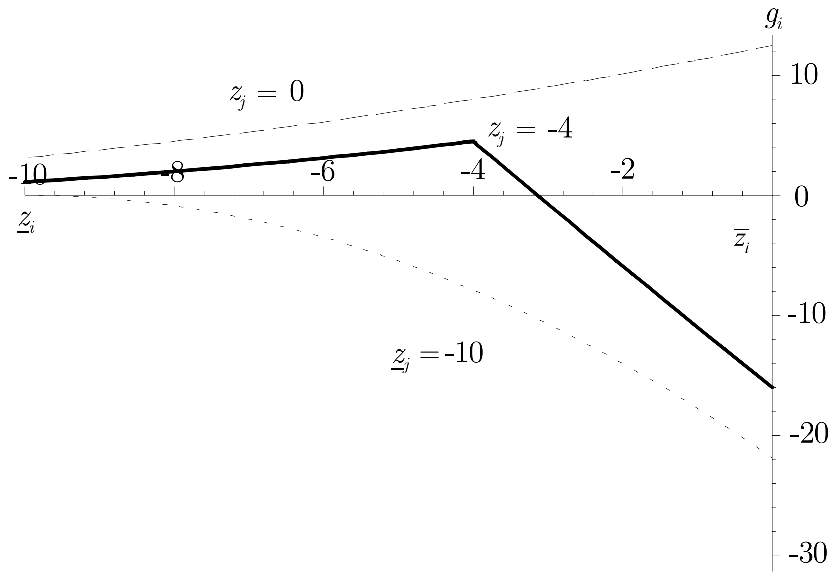

Figure 6: g_i in the homogenous Bertrand duopoly