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EFFICIENT UNEMPLOYMENT INSURANCE TIME PATH

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ABSTRACT

Efficient Unemployment Insurance Time Path*

The Paper examines the time sequencing of unemployment Insurance (UI) benefits in a general equilibrium framework, with random matching and endogenously determined wages. A key feature of the model is that policymakers exploit random matching to produce some assortative matching through UI policy.

The Paper considers a mechanism whereby a declining UI time profile makes unemployed workers relatively choosier at the beginning of their unemployment spell. Hence they tend to continue to search unless a sufficiently attractive offer is received. Later on, the reservation wage drops and agents are willing to take less attractive job offers. Firms respond by introducing endogenous market segmentation. In equilibrium more productive firms offer higher wages facing lower vacancy risk, whereas less productive firms offer lower wages and face higher vacancy risk.

The role of a declining profile UI regime is to ensure enough heterogeneity among workers, so as to obtain enhanced matching with heterogenous firms. The longer the duration, the higher the degree of induced heterogeneity. Such optimal UI policy is shown to crucially depend on the nature of technological dispersion.

JEL Classification: E24, J64 and J65
Keywords: benefit duration, matching, search, technology dispersion and UI

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EFFICIENT UI TIME PATH[1]

1 Introduction

The motivation for providing UI benefits is essentially twofold. First, it provides for better consumption smoothing between periods of employment and unemployment in the absence of private insurance markets. Second, even when agents are risk-neutral, UI can be used to induce agents to search more intensively, thereby improving the quality of matches between workers and jobs, when the market fails to do so due to externalities. UI programs entail, however, major work disincentives. One prevalent idea in this context is that limiting the duration of UI benefits may mitigate these disincentives. Examining the issue of UI duration is of key importance for researchers and policymakers alike and is the subject of this paper. In the current study we examine the time path of UI benefits, emphasizing the role of UI benefits in improving job-worker matching. A key feature of the model is that policymakers exploit random matching to produce some assortative matching through UI policy. Given the prevalent use of a declining UI path in the real world, one key idea is to study the welfare gains induced by such policy.

The paper examines the time sequencing of UI benefits within a general equilibrium framework, where wages are determined endogenously. We assume that agents are risk neutral and show that a declining profile is desirable under certain conditions, even in the absence of moral hazard. The model combines random matching with wage posting by the firms operating in the market. The intuition for the main result of the paper may be summarized as follows. Consider a frictional environment where assignment of unemployed workers to vacant jobs is entirely uncoordinated and governed by a random matching process. Firms, seeking to operate jobs with superior productivity, can do nothing to attract additional workers unless they can take advantage of either ex-ante heterogeneity (say, different imputed value of leisure as in Albrecht and Axell (1984)), or ex-post

[1]We thank seminar participants at Tel Aviv, Haifa, Ben Gurion, Tilburg and Zurich Universities for helpful comments. Any errors are our own.

heterogeneity which may be the result of on-the-job search (as in the Burdett and Mortensen (1998) model). All firms pay the reservation wage. Introducing a declining UI profile creates the missing heterogeneity. At the beginning of their unemployment spell agents are relatively more choosy and tend to proceed with search unless faced with a sufficiently attractive offer. Later on, the reservation wage drops and agents are willing to take less attractive job offers. Firms respond by introducing endogenous market segmentation.[2] In equilibrium, more productive firms offer higher wages, anticipating a tighter 'sub-market', namely lower vacancy risk. Less productive firms offer lower wages and face higher vacancy risk as only long-term unemployed workers will accept the offer. A declining UI profile thus results in voluntary unemployment by short term unemployed agents but features improved matching as it shifts the labor force towards more productive firms. The role of a declining profile UI regime is to insure enough heterogeneity among workers so as to obtain better matching. Hence policy induces discrimination to enhance efficiency. The longer the duration, the higher the degree of induced heterogeneity. Optimal UI policy is shown to crucially depend on the nature of technological dispersion.

The paper is organized as follows. Section 2 briefly discusses the relevant related literature. Section 3 presents the matching model with wage posting. Section 4 discusses the optimal UI time path. Section 5 concludes.

2 Background Literature

The main line of research on the optimal design of UI policy has focused on issues of moral hazard and consumption smoothing [see Karni (1999) for a recent survey]. This literature examines the impact of work disincentives on the design of optimal schemes. The seminal papers appeared in the late 1970s [Baily (1978), Flemming (1978) and Shavell and Weiss (1979)]. The main insight provided by these early partial equilibrium models was the desirability of a declining schedule, i.e.

[2] Contrary to the competitive search-equilibrium model [see Moen (1997)] where market segmentation is built in exogenously.

benefits should decline over the spell of unemployment so as to mitigate the moral hazard effect. The early models have been recently extended in several directions, some of them into general equilibrium frameworks. Hopenhayn and Nicolini (1997), as a notable example, enlarge the set of instruments by allowing for a wage tax after re-employment. This model preserves the sequencing structure of Shavel and Weiss (1979) but attains enhanced consumption smoothing.

One important feature of UI schemes, which has received far less attention, is the role UI benefits can play in attaining a better match between jobs and workers, an issue at the core of the current paper. Diamond (1981) discussed the role of UI in enhancing efficiency in the context of a steady state search model. In his model UI makes job-taking use more stringent standards, thereby raising the vacancy rate and improving the distribution of job offers. Albrecht and Axell (1984) have discussed the existence of a tradeoff between two effects of UI: an increase in the unemployment rate and a shift in the wage offer distribution, which brings about a reallocation of workers to more productive firms. A more recent contribution is Marimon and Zilibotti (1999). In their model UI improves matching between ex-ante heterogenous workers and heterogenous firms under random matching. The current paper differs from this literature by focusing on the time path of UI and on the role of UI policy in introducing missing heterogeneity for workers.

Another recent paper which bears a certain relation to this one is Acemoglu and Shimer (1999). The common point is that UI generates an increase in output, whereby more productive firms choose to offer higher wages and more workers are assigned to those firms. However the mechanism is very different: in Acemoglu and Shimer (1999) the key idea is that the labor market provides insurance for risk averse workers by reducing unemployment risk, charging the premium in the form of lower wages. By offering UI benefits, apart from the consumption smoothing argument, the policymaker induces risk-averse workers to take on a higher degree of unemployment risk, boosting investment by firms. The set-up is one with directed search, so externality issues do not arise. In the model presented below, the key point is UI policy turning random matching into assortative matching against the backdrop of heterogeneity in productivity. Thus the mechanism here does not relate to risk aversion (agents are risk-neutral) but rather explores the role of UI policy in internalizing externalities. It therefore builds upon essentially different features: search is random i.e. non-directed, and there is voluntary unemployment, i.e. non-acceptance of certain job offers.

3 The model

In what follows we describe the general set up (3.1) and then look at two alternative time paths for UI policy: constant (3.2) and declining (3.3).

3.1 The General Set-Up

There is a continuum of workers whose measure is given by $L > 0$, and a much larger continuum of firms (entrepreneurs) whose measure is given by $M >> 0$. Entrepreneurs differ in the technology they possess. Each entrepreneur can post a vacancy, incurring no costs, w.l.o.g. Once the vacancy is filled, the job produces x units of the single perishable consumption good, which price in normalized to unity. Production terminates with an exogenous Poisson parameter, $s > 0$. Otherwise the vacancy produces nothing.

The technological parameter, x, is assumed to be distributed according to the cumulative distribution function:

$$G(\cdot) \sim [\underline{x}, \overline{x}]$$

with strictly positive densities.

Workers are ex-ante identical in all respects and are assumed to be risk neutral. Let us note that this homogeneity assumption renders the model more tractable but is not crucial for the analysis. If workers were heterogenous, what would be needed is that their degree of heterogeneity be lower than the extent of technological dispersion. Firms are assumed to be expected-profit maximizers. To close the general equilibrium model we assume that workers possess an equal stake in each one of the firms.

In each period (time is assumed to be discrete) firms post wage rates. Then, unemployed workers are randomly assigned to vacancies. Without loss of generality we assume that each active entrepreneur posts a single vacancy. Due to lack of coordination there are cases where multiple applicants arrive at a job vacancy and others where either no one is assigned or where a single applicant arrives. Upon receiving a job offer, the worker decides whether to accept or reject the offer. Thus we allow for voluntary unemployment. If more than one applicant accepts the offer, the assignment choice is taken randomly. We assume that search intensity is fixed and normalized to one application per period. We assume that all UI benefits are financed via neutral lump-sum taxes.

We now turn to analyze the optimal behavior of the agents in steady-state equilibrium under different UI time paths.

3.2 A Constant UI Profile

Consider first a UI scheme with a constant time profile, i.e. infinite duration. Note that throughout we assume that agents search only when unemployed.

Following Diamond's (1971) seminal contribution, by the homogeneity of workers, and since on-the-job-search is ruled out and firms are committed to the wages posted prior to the arrival of the job applicants, the wage distribution collapses to a singleton. Denote the equilibrium wage rate, which coincides with the uniform reservation wage, by w. It therefore follows that:

$$w = a + h \tag{1}$$

where a and $h \leq \underline{x}$ denote the constant UI benefit and the imputed value of leisure, respectively.

The typical worker's optimal acceptance rule is trivial, namely accept the first offer received. Turning next to firms, it is easy to observe that firms, provided that they decide to operate, will choose to offer the uniform wage rate, w. This emerges from the fact that workers are assumed not to search on the job. Thus, applicants accept any job offer above the reservation instantaneously, and firms choose to offer precisely the reservation wage. Cutting this wage even slightly will increase vacancy risk to infinity, making profits drop to zero. Offering above it does not bring any gain, as it does not affect the vacancy risk because of the reservation strategy.

Firms differ in their productivity and correspondingly need to take a strategic decision, whether to enter the market or remain idle. Given w, the equilibrium wage rate, all firms possessing a technology $x \geq w$ will participate, for all expect the arrival of applicants with strictly positive probability.

Denote by U and V, the measure of unemployed workers and unfilled vacancies in steady state equilibrium, respectively. Let F denote the steady state measure of active firms, which is also the number of jobs, filled or vacant, in the economy. In equilibrium the following conditions hold, in addition to the wage determination condition given by equation (1) above:

$$L - U = F - V \tag{2}$$

$$M\left(1 - G(w)\right) = F \tag{3}$$

$$V\left(1 - e^{-\frac{U}{F}}\right) = s\left(F - V\right) \tag{4}$$

The interpretation of the equations above is straightforward. Equation (2) defines the equilibrium condition according to which the measure of matched workers (on the left-hand side) is equal to the measure of filled vacancies (on the right-hand side). Equation (3) defines the entry condition, given that the prevailing wage rate is w. Equation (4) states the familiar worker flow condition (the Beveridge curve). We assume Urn-Ball random matching so the distribution of the number of applicants that arrive to any posted vacancy is Poisson, where the expected number of applicants is given by the $\frac{U}{F}$ ratio.[3] On the left-hand side we have, therefore, the flow into the pool

[3]From the worker's point of view, the frictions are that the worker may approach a job already assigned and that

of filled vacancies (successful matches), given by the probability of having at least one applicant times the measure of unfilled vacancies. The right-hand side gives the flow out of the pool of filled vacancies.[4]

With equations (1)-(4) we can solve for the equilibrium recursively, for any level of the constant level of UI benefit a. First we substitute for w (from (1)) into (3) and obtain an explicit solution for F. We then obtain a system of two equations ((2) and (4)) solved for two unknowns, U and V.

It is easy to see that (2) is upward sloping and (4) is downward sloping in $U-V$ space as shown in Figure 1.

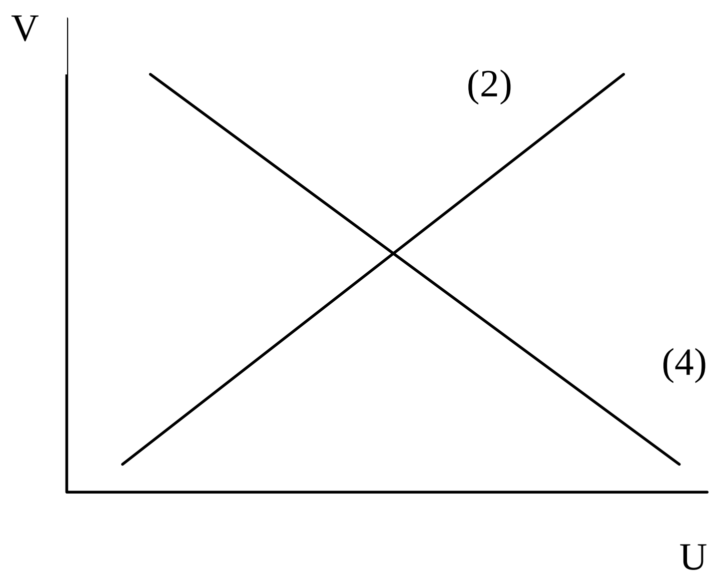

Figure 1: The Basic Equilibrium

there exists a possibility of multiple applicants at the same open vacancy.

[4]Observe that by the law of large numbers the product on the left-hand side of equation (4) gives the actual measure of successful matches, rather than the expected measure.

Thus, for quite general parametric assumptions a unique equilibrium exists. Furthermore, it easy to observe from Figure 1, that the measure of unfilled vacancies, V, rises unambiguously when the measure of posted vacancies, F, rises (both curves shift upwards). Fully differentiating both equations with respect to F, it can be shown that $\frac{dV}{dF} < 1$. Thus, the level of employment rises with respect to the measure of vacancies, as expected.

By relaxing the fixed arrival rate paradigm of conventional wage-posting search equilibrium models, we obtain the inherent trade-off between sorting and employment using an extremely simple framework. To see that, we follow Albrecht and Axell (1984) and let welfare be measured by per-capita utility, that is per-capita consumption plus the value imputed to per-capita leisure. By lowering a, the constant UI benefit, we lower the wage rate, w, thereby reducing unemployment. On the other hand, the fall in w, brings in less productive firms (the lower tail of the productivity distribution) that previously failed to break even. More formally, by the recursive nature of the equilibrium characterization, we can solve for the optimum by setting F optimally, then implement it via w, hence a. Given the welfare measure defined above, the maximization problem solved by the social planner is given by:

$$\max_a \{(L-U)E[x \mid x > w] + Uh\} \tag{5}$$

s.t. equations (2)-(4). Using these equations (5) may be re-written as a function of F :

$$\max_F \{(F-V)\left(E[x \mid x > G^{-1}(1-\frac{F}{M})] - h\right) + Lh\} \tag{6}$$

where, $E[\cdot]$ denotes the expectation operator, and the expression exists since by assumption densities are strictly positive.

When F is increased, the first term $(F-V)$ rises since $\frac{dV}{dF} < 1$. At the same time the second term $(E[x \mid x > G^{-1}(1-\frac{F}{M})])$ decreases, since entrepreneurs possessing technologies of lower quality enter the market, thereby reducing expected productivity. The optimal F balances the two opposing effects, namely employment versus enhanced matching. Once F is determined (by differentiating (6) with respect to F), we can set w and thus a, using (1) and (3). Note that without UI ($a = 0$) wages are determined by the imputed value of leisure (h) and are, therefore, independent of the productivity distribution. Hence a change in this distribution does not translate into a change in wages and in incentives.

3.3 A Declining UI Profile

In this sub-section we extend the model to allow for a declining UI profile. We confine attention to a two-tiered regime, in which agents are eligible for a short period of regular UI benefits, followed by an indefinite period of reduced compensation, which we refer to as income support. Let the level of income support be denoted by a, and let z denote the UI benefit ($z > a$). We assume that z is paid during the first two periods of unemployment and that all agents who exhaust their eligibility for UI benefits are henceforth indefinitely eligible for income support. UI eligibility is assumed to be independent of work history, for simplicity.

We first discuss a key feature of the model, according to which a declining profile implies a non-degenerate wage distribution. With a constant profile, there exists a unique wage rate in equilibrium. This derives from the wage-posting setting and the fact the agents search only when unemployed. While agents are ex-ante identical in all respects, the declining profile implies that short-term unemployed agents, faced with a non-degenerate distribution of wage offers, will have a higher reservation wage rate than long-term unemployed agents who have already exhausted their eligibility for UI benefits. The declining profile yields ex-post heterogeneity among ex-ante identical agents. For a sufficiently dispersed set of technologies, a two-wage equilibrium exists. Voluntary unemployment by short-term unemployed agents induces firms possessing more productive technologies to offer higher wages, thereby reducing their vacancy risk. Assuming two levels of UI benefits and two periods when the first level is in place, we can confine attention to a two-wage equilibrium.

We turn next to define the value functions for a typical agent in the economy. There are four states to consider: employment, long term unemployment (income support recipients) and two states of short-term unemployment (for each period of UI eligibility). We start with the three unemployment states. We denote by H_1, H_2, H, the continuation value functions for short-term unemployed agents (during the first and second period of the unemployment spell, respectively) and income support recipients. In steady-state equilibrium the following asset-value conditions hold:

$$H_1 = z + h + \beta[\overline{m}\max(\overline{J}, H_2) + \underline{m}\max(\underline{J}, H_2) + (1 - \overline{m} - \underline{m})H_2] \tag{7}$$

$$H_2 = z + h + \beta[\overline{m}\max(\overline{J}, H) + \underline{m}\max(\underline{J}, H) + (1 - \overline{m} - \underline{m})H] \tag{8}$$

$$H = a + h + \beta[\overline{m}\max(\overline{J}, H) + \underline{m}\max(\underline{J}, H) + (1 - \overline{m} - \underline{m})H] \tag{9}$$

where $\overline{J}$, $\underline{J}$ denote the high and low wage jobs continuation values, respectively, $\overline{m}$, $\underline{m}$ denote re-employment chances in firms offering high and low wage rates, respectively, and $\beta \in (0, 1)$ denotes the discount factor. Short-term unemployed get UI benefits z and long-term unemployed get income support a in addition to the value of leisure h. In the following period they move either to a job or to the next stage of unemployment.

he steady state value functions for the two types of jobs (those with high wage rate and low wage rate, respectively) are given by $\overline{J}$ and $\underline{J}$:

$$\overline{J} = \overline{w} + \beta[(1 - s)\overline{J} + sH_1] \tag{10}$$

$$\underline{J} = \underline{w} + \beta[(1 - s)\underline{J} + sH_1] \tag{11}$$

Workers get the relevant wage in each job and face the exogenous separation probability s.

By virtue of wage posting and the assumption that individuals search only while unemployed, in equilibrium firms offer such wages so as to satisfy the following reservation asset values:

$$\overline{J} = H_2 \tag{12}$$

$$\underline{J} = H \tag{13}$$

It is easy to verify that the reservation wage property is satisfied, by observing that $\overline{J} = H_2 > \underline{J} = H$. Thus, short-term unemployed agents will accept only high wage offers during their first period of UI eligibility. All other unemployed agents will accept any offer.

Manipulating equations (7)-(13) yields the following two conditions which determine the equilibrium wage offers as a function of the policy parameters (z and a), the re-employment probabilities ($\overline{m}, \underline{m}$), the discount factor ($\beta$) and the separation rate (s):[5]

$$z - a = \frac{\overline{w} - \underline{w}}{1 - \beta(1 - s)} \tag{14}$$

$$\overline{w} = (z - a)[(1 - \beta) + \beta\overline{m} - \beta^2 s(1 - \overline{m})] + h + a \tag{15}$$

We now explicitly formulate the re-employment probabilities. Using a Taylor's expansion, it is straightforward to verify that the urn-ball matching process (assuming a single job application per individual per period) implies a Poisson distribution of applicants, yielding the following conditions (for an explicit derivation see appendix B):

$$\overline{m} = \frac{\overline{V}}{U}(1 - e^{-\frac{U}{F}}) \tag{16}$$

$$\underline{m} = \frac{\underline{V}}{U - U_1}(1 - e^{-\frac{U - U_1}{F}}) \tag{17}$$

where $\overline{V}$ and $\underline{V}$ denote the measures of unfilled vacancies posted by firms offering the high wage rate and low wage rate, respectively; F denotes the measure of posted jobs, U denotes the measure

[5] See appendix A for the full derivation.

of aggregate unemployment and U_1 denotes the measure of short-term unemployed agents during the first period of UI eligibility.

The standard flow conditions are given by:

$$U_2 = (1 - \overline{m})U_1 \tag{18}$$

$$(U - U_1 - U_2)(\overline{m} + \underline{m}) = (1 - \overline{m} - \underline{m})U_2 \tag{19}$$

$$\overline{V}\left(1 - e^{-\frac{U}{F}}\right) = s\left(\overline{F} - \overline{V}\right) \tag{20}$$

$$\underline{V}\left(1 - e^{-\frac{U-U_1}{F}}\right) = s\left(F - \overline{F} - \underline{V}\right) \tag{21}$$

where , U_j, $j = 1, 2$ denotes the measure of unemployed agents during the jth period of UI eligibility and $\overline{F}$ denotes the measure of vacancies posted by firms offering the high wage rate in equilibrium.

To complete the characterization of steady state equilibrium we introduce the following conditions and interpret them:

$$L - U = F - \overline{V} - \underline{V} \tag{22}$$

$$F = M\left(1 - G(\underline{w})\right) \tag{23}$$

$$\left(1 - e^{-\frac{U}{F}}\right)(\widehat{x} - \overline{w}) = \left(1 - e^{-\frac{U-U_1}{F}}\right)(\widehat{x} - \underline{w}) \tag{24}$$

$$\overline{F} = M\left(1 - G(\widehat{x})\right) \tag{25}$$

Equation (22) is the condition according to which the measure of filled vacancies is equal to the measure of employed agents. Equation (23) is a consistency condition, which requires that the total measure of active firms (hence posted vacancies) should be equal to the fraction of the firms possessing a technology above the lower bound wage rate times the measure of potential firms. Equation (24) determines the wage distribution by defining a cutoff technology, $\widehat{x}$, above which all firms maximize expected profits by offering the high wage rate, and below which all firms maximize expected profits by offering the low wage rate. Equation (25) is a consistency condition similar to (23) with regard to the firms offering the high wage rate in equilibrium.

The optimality problem may be written as follows:

$$\begin{aligned} &\max_{a,z}\{(F-\overline{F}-\underline{V})E[x \mid \underline{w} \leq x \leq \widehat{x}] \\ &+(\overline{F}-\overline{V})E[x \mid x > \widehat{x}] \\ &+Uh\} \end{aligned} \tag{26}$$

subject to the above 12 equations [equations (14) – (25)]. Equilibrium is defined by this system of equations, where a and z are solved according to (26).

It can be shown that the above formulation, which focuses on choosing the levels of UI and income support, can be mapped into the following equivalent structure, focusing on allocation of production:

$$\max_{\underline{x},\widehat{x}} \left\{ M \int_{\underline{x}}^{\overline{x}} xdG(x) + Uh - \underline{V}\frac{\int_{\underline{x}}^{\widehat{x}} xdG(x)}{G(\widehat{x})-G(\underline{x})} - \overline{V}\frac{\int_{\widehat{x}}^{\overline{x}} xdG(x)}{1-G(\widehat{x})} \right\} \tag{27}$$

subject to equations (20), (21),(22), (23), (25) and :

$$U_1 = s\,(L-U)\,. \tag{28}$$

In other words the social planner chooses $\underline{x}$ and $\widehat{x}$ governing allocation. This can be implemented by choosing the relevant prices, i.e. the wage rates $\underline{w}$ and $\overline{w}$, through the choice of the two policy instruments a and z.

4 Optimal UI Time Path

The question we would like to address is a normative one, namely under what circumstances is a declining UI profile, hence a two-wage equilibrium, socially desirable. Intuitively it seems that the answer should relate to the extent to which the set of technologies is dispersed. We use a simple case of a discrete distribution of technologies, which comprises two elements in its support, so as to be able to infer some qualitative insights.[6] We start (4.1) by characterizing the types of worker sorting which emerge under the different UI paths analyzed in the previous section. We then study the relationship between the welfare levels induced by the different paths and the properties of the productivity distribution (4.2). Finally, we examine a special case whereby policy determines the duration of the first tier of the UI path (4.3).

4.1 Types of Sorting and UI Paths

Suppose there are two technologies, denoted by $\overline{x}$ and $\underline{x}$, where $\overline{x} > \underline{x} > h$. Denote by $0 < p < 1$ the fraction of firms (which measure is given by $M >> 0$) possessing $\overline{x}$. We henceforth restrict attention to pure-strategy equilibria.

There are three equilibrium configurations to consider. A benchmark case is the one in which the UI profile is constant and the benefit is set sufficiently low, so that all firms are active in equilibrium. We refer to this configuration as maximum employment or no-sorting, interchangeably. A second case, is the one in which the UI regime is constant, but the benefit is set high enough so as to crowd out the low-productivity firms. We refer to this configuration as high-sorting. The third configuration is when both technologies are active, but due to a declining UI profile, voluntary unemployment by short-term unemployed agents yields partial-sorting.

[6] The computations below are based on solutions of the allocation problem (27).

Denote by $E^{HS}, E^{PS}$and E^{NS}, the steady-sate measures of employed workers in the high-sorting, partial-sorting and no-sorting configurations, respectively (where $E = L - U$).

We already observed that $E^{NS} > E^{HS}$(see the characterization of equilibrium in the constant profile UI regime). Close inspection of (20)-(22) yields that $E^{NS} > E^{PS}$. This condition derives directly from the existence of voluntary unemployment in the partial-sorting configuration.[7]

Clearly, one can not relate E^{HS} and E^{PS} without making further assumptions. This ambiguity derives from the trade-off between involuntary unemployment in the high sorting case (when a lower measure of firms participates) and voluntary unemployment in the partial sorting case (due to workers' ex-post heterogeneity emerging from the declining UI profile).

Formulating the welfare measures for each one of the three configurations (maintaining our definition of welfare from the previous section), we obtain the following:

$$W^{NS} = E^{NS}[p\overline{x} + (1 - p)\underline{x} - h] + Lh \tag{29}$$

$$W^{PS} = E^{PS}[q\overline{x} + (1 - q)\underline{x} - h] + Lh \tag{30}$$

$$W^{HS} = E^{HS}[\overline{x} - h] + Lh \tag{31}$$

where $q = \frac{(pM - \overline{V})}{M - \overline{V} - \underline{V}}$ and it is easy to show, using (20) and (21), that $1 > q > p$.

Equation (29) is the "benchmark" case: a fraction p of workers go to the high technology firms and $1 - p$ to the low ones and E^{NS} is determined solely through random matching. At the other extreme there is (31) with a constant UI profile: here only the high technology is in operation. The two equations (29) and (31) express the trade-off between employment and sorting as $E^{NS} > E^{HS}$ and $[\overline{x} - h] > [p\overline{x} + (1 - p)\underline{x} - h]$. The intermediate case is that of a declining UI

[7] By aggregating (20) and (21) and comparing the expression to (4), it can be seen that for a given measure of active firms and for any level of unemployment, the market clears with a higher measure of aggregate unfilled vacancies relative to the constant UI regime.

profile − equation (30). This policy balances the two considerations, employment and productivity. In order to increase the degree of sorting in the market, namely shifting workers away from low-productivity firms to high-productivity ones, one inevitably sacrifices a rise in unemployment. The optimal policy depends on the properties of the productivity distribution as will be illustrated below.

4.2 UI Paths, Welfare and the Productivity Distribution

We now study the relationship between the UI path, welfare, and the productivity distribution. To do so we fix the average productivity in the economy and denote it by $x, x > h$, where $x = p\overline{x} + (1-p)\underline{x}$.

First, consider a collection of economies characterized by a spread on x, holding fixed the mean productivity (x) and the skewness (the parameter p, as the skewness is a function of p only). That is we examine the effect of a change in the variance of the productivity distribution on the degree of sorting, fixing the other moments of the distribution. That is, for some $\triangle \geq 0$, $\overline{x} = x + \triangle$ and $\underline{x} = x - \frac{p\triangle}{1-p}$, where $\underline{x} \geq h$. Reformulation of equations (29) -(31) yields the following:

$$W^{NS} = E^{NS}[x-h] + Lh \tag{32}$$

$$W^{PS} = E^{PS}[x-h] + E^{PS}\frac{q-p}{1-p}\triangle + Lh \tag{33}$$

$$W^{HS} = E^{HS}[x-h] + E^{HS}\triangle + Lh \tag{34}$$

In Figure 2 we depict each of the three configurations in welfare-productivity spread space ($W - \triangle$). Note that social optimum is given by the welfare frontier (the upper envelope of the figure).

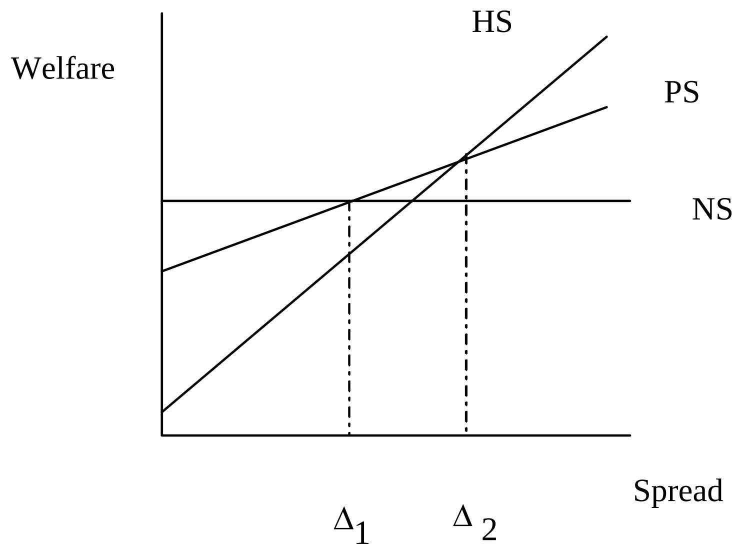

Figure 2: Welfare—variance (productivity spread) analysis

While Figure 2 is but an illustration of the optimum, its general properties are summarized by the following proposition.

Proposition 1 There exist thresholds $\triangle_1, \triangle_2$ where $0 < \triangle_1 \leq \triangle_2 \leq \frac{1-p}{p}[x-h]$, such that: for all $0 \leq \triangle \leq \triangle_1$, social welfare is maximized by the no sorting configuration, for all $\triangle_1 < \triangle \leq \triangle_2$ social welfare is maximized by the partial sorting configuration; and for all $\triangle_2 < \triangle \leq \frac{1-p}{p}[x-h]$social welfare is maximized by the high sorting configuration.

Proof. First observe that by virtue of linearity with respect to $\triangle$ of each one of the schedules (32)-(34), each configuration will appear at most once on the welfare frontier (single crossing property).

Next note that the finiteness of set of configurations ensures non-emptiness.

Note further that for $\triangle = 0$, the no-sorting configuration is welfare maximizing, since $x > h$ and $E^{NS} > \max[E^{PS}, E^{HS}]$. Then, by continuity, for sufficiently small $\triangle$, the no-sorting configuration is socially desirable.

Let NS denote the set of all $\triangle$, for which the no-sorting configuration is welfare maximizing. The set NS is non-empty (as just shown) and bounded from above (by construction of the spread $\triangle$). Thus it has a least upper bound. We denote it by $\triangle_1$

Next, note that if the high-sorting configuration is welfare maximizing for some $\triangle'$, then it remains the maximizing configuration for all $\triangle \geq \triangle'$. To see that, suppose, by way of contradiction, that the opposite holds true. Since the schedule of the no-sorting configuration is flat, whereas the high-sorting configuration is rising with respect to $\triangle$, the only case we need to examine is the possibility where partial-sorting attains a higher level of welfare than high sorting for some $\triangle$, $\triangle \geq \triangle'$. This necessarily implies that the slope of the partial sorting schedule with respect to $\triangle$ is steeper than the corresponding slope of the high sorting schedule. Formally:

$$E^{PS}\frac{q-p}{1-p} > E^{HS}$$

which implies

$$E^{PS} > E^{HS}$$

Thus, we obtain a contradiction, since it follows that for all $\triangle$, partial sorting is preferred to high sorting.

Let HS denote the set of all $\triangle$ for which the high-sorting configuration is welfare maximizing. The set is bounded from below (by construction of the spread $\triangle$). If it is non-empty, it has a highest lower bound. Let $\triangle_2$ denote the highest lower bound (if it exists) and set $\triangle_2 = (1-p)/p(x-h)$, otherwise.

This completes the proof. ■

The discrete two-technology case is highly stylized but it provides us with some clear insights regarding the forces at play. When the set of technologies is almost degenerate, i.e. the spread $\triangle$ converges to zero, there is little to gain from introducing voluntary unemployment and shifting the pool of workers away from low-productivity firms towards high-productivity ones. In this case UI is redundant. When, however, technologies are sufficiently dispersed, the partial sorting configuration is preferred to the no-sorting one.[8] When dispersion is very large, it is desirable to increase unemployment by eliminating employment at low-productivity firms altogether and obtaining high (full) sorting.[9] For intermediate values of the technological spread, social welfare is maximized by the partial sorting configuration, hence a declining UI profile is warranted.[10] In the two other cases, a constant profile (either no sorting or high-sorting) suffices.

Second, we examine the effects of the skewness of the productivity distribution. We do so through numerical analysis, as shown in Figure 3, holding the mean and the variance constant.[11]

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[8] In graphical terms, the partial sorting solution dominates in the interval $\triangle_1 - \triangle_2$. This interval is well-defined only when there is a substantial difference between the intercepts of the HS and PS schedules (on the vertical axis). This implies a sufficient degree of right skewness of the distribution.

[9] In a continuous case, this is equivalent to a situation in which there is a dense cluster of technologies at the top of the distribution.

[10] This discrete example extends to continuous cases with sufficient skewness, for example L-shaped distributions. In cases that are not skewed (or not sufficiently skewed), such as the uniform distribution, the constant profile dominates.

[11] The figure was produced using the following values: $M = 100$, $L = 70$, $s = 0.01, E(x) = 10$, $\sigma(x) = 8$.

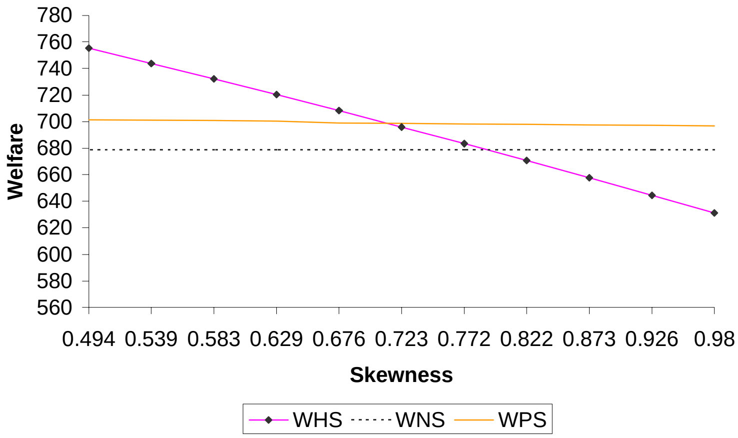

Figure 3: Welfare–skewness analysis

Note that for the skewness parameters chosen, UI is always desirable. For low enough skewness a flat scheme is dominating, i.e. the high sorting equilibrium regime . For skewness that is high enough, where there is a high mass of low productivity firms, the cost in terms of reduced employment is too high, thus a two-tiered scheme is called for, i.e. the partial sorting regime dominates.

It is easy to verify the existence of policy instruments that implement the social optimum. For the no sorting case set $a \leq \underline{x}$ and for the high sorting case set $\overline{x} \geq a \geq \underline{x}$. For the partial sorting configuration, see appendix B. Note that if taxation were distortionary,[12] then when $E^{PS} > E^{HS}$ the case for a declining UI profile is reinforced. This is so because expenditures on UI benefits are lower under the PS regime due to lower unemployment.

[12] Recall that we have assumed throughout that benefits are financed by lump-sum taxation.

4.3 Optimal Duration Policy

In this sub-section we turn to illustrate how duration can be set optimally so as to obtain a welfare gain. We retain the structure of a two-tiered UI regime – regular UI and income support. In this way we ensure that in equilibrium there would be at most two wage rates. While till now the first tier was assumed to last two periods, here we allow for more periods.

In order to maintain the simple property of a two wage equilibrium and in order to gain more flexibility with respect to duration policy, we confine attention to the case of myopic agents. Namely, we assume that the discount factor is given by $\beta = 0$. This is an extreme assumption but it helps us focus on the essential duration implications without complicating the analysis, which carries over to the more general case ($\beta > 0$). Myopia implies that along each of the two tiers the reservation wages, hence the equilibrium wage rates, will be fixed. Regardless of the length of the first tier of the UI regime, these will be given respectively by:

$$\underline{w} = a + h$$

$$\overline{w} = z + h$$

This result can be verified by substituting into the wage determination equations (14) and (15).

Under this set-up the social planner has one additional degree of freedom in choosing optimal UI policy, namely, setting the duration of the first tier of the regime, to be denoted $\bar{t}$. Let U_t denote the measure of unemployed agents during period t of UI (first-tier) eligibility. During the first $\bar{t}-1$ periods of UI eligibility agents will only accept high-wage offers. Maintaining our notation from previous sections, it follows that:

$$U_t = U_{t-1}(1 - \overline{m}) \quad 2 \leq t \leq \bar{t}$$

Denote by $\overline{U}$ the aggregate measure of unemployed agents whose reservation wage is high (given by $\overline{w}$). By construction:

$$\overline{U} = \sum_{t=1}^{\overline{t}-1} U_t = U_1 \frac{(1-(1-\overline{m})^{\overline{t}-1})}{\overline{m}} \tag{35}$$

Reformulating the allocation program (see equation (27)) yields:

$$\max_{\underline{x},\widehat{x}} \left\{ M \int_{\underline{x}}^{\overline{x}} x dG(x) + Uh - \underline{V} \frac{\int_{\underline{x}}^{\widehat{x}} x dG(x)}{G(\widehat{x}) - G(\underline{x})} - \overline{V} \frac{\int_{\widehat{x}}^{\overline{x}} x dG(x)}{1 - G(\widehat{x})} \right\} \tag{36}$$

subject to the same constraints as above (or re-formulated wherever relevant), reproduced here:

$$\overline{V}(1 - e^{-\frac{U}{F}}) = s(\overline{F} - \overline{V}) \tag{37}$$

$$\underline{V}\left[1 - e^{-\frac{U-\overline{U}}{F}}\right] = s\left[F - \overline{F} - \underline{V}\right] \tag{38}$$

$$L - U = F - \overline{V} - \underline{V} \tag{39}$$

$$F = M[1 - G(\underline{x})]] \tag{40}$$

$$\overline{F} = M[1 - G(\widehat{x})]] \tag{41}$$

$$\overline{U} \geq U_1 \tag{42}$$

$$\overline{U} \leq \frac{U_1}{\overline{m}} \tag{43}$$

$$U_1 = s[L - U] \tag{44}$$

$$\overline{m} = \frac{\overline{V}}{U}[1 - e^{-\frac{U}{F}}] \tag{45}$$

With (42) and (43) denoting constraints derived from (35) for the two limiting cases of $\bar{t} = 2$ and $\bar{t} \to \infty$.

Any change in duration policy, namely in $\bar{t}$, translates into a change in $\overline{U}$ (by virtue of (35)). To examine the implication of a change in duration policy, suppose that $\overline{U}$ is increased, fixing $\underline{x}$ and $\widehat{x}$. Fully differentiating equations (37)-(39) yields:

$$0 < \frac{dU}{d\overline{U}} < 1 \tag{46}$$

It follows that the matching probability for a firm posting a vacancy offering a high wage rate (given by the term in brackets on the left-hand-side of (37)) rises, whereas the corresponding matching probability for a low wage vacancy (given by the term in brackets on the left-hand-side of (38)) declines. Thus the rise in $\overline{U}$ implies a higher aggregate level of unemployment (U) accompanied by enhanced sorting – that is shift from low-wage vacancies towards high-wage vacancies, which in equilibrium results in a shift from low-productivity firms towards high-productivity ones.

Balancing between the two opposing forces will determine the optimum. To illustrate the point consider the following figure derived for a numerical simulation based on a two-point discrete example:[13]

[13] The following parameter values are used: $\overline{x} = 100, \underline{x} = 40, p = 0.5, M = 100, L = 80, s = 0.05, h = 20$ (with p denoting the fraction of high productivity firms).

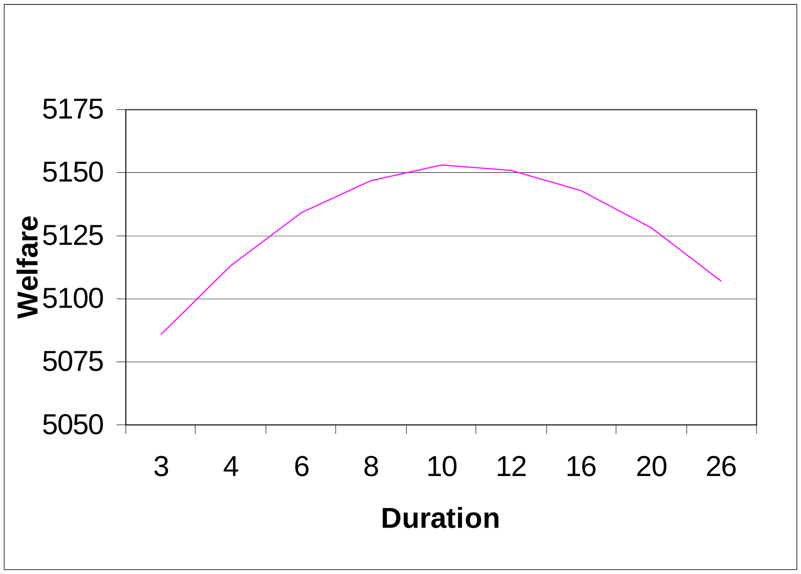

Figure 4: Optimal Duration

The figure illustrates that the optimal duration is given by $\bar{t} = 10$. It follows from the figure that for sufficiently short duration ($\bar{t} < 10$) the enhanced sorting effect dominates, whereas for sufficiently long duration ($\bar{t} > 10$) the increased unemployment effect dominates. The optimum at $\bar{t} = 10$ is obtained at the point of balance.

The general conclusion from the analysis above is that by extending the duration over a certain range, one can obtain sorting gains that outweigh the costs associated with a higher level of unemployment.

5 Conclusions

The model has examined the time path of UI from an assortative matching perspective. It has built on two key elements: heterogeneity in firm productivity and random matching.

The assumption of firm heterogeneity is well supported by empirical studies. Policy – by introducing a declining path of UI benefits – generates heterogeneity in reservation wages leading to worker sorting i.e. to some assortative matching.

The other building-block – random search – while much in use in the search literature, may appear more questionable empirically. However if we consider workers as being compensated not only by wages but also by other job attributes, such as job risk, work environment, promotion chances, amenities etc., then some imperfection in workers' information about jobs is quite realistic. Furthermore, the empirical evidence on job-worker mismatch makes the alternative concept of perfectly directed search an unrealistic assumption.

One can interpret the main result we obtain along traditional Pigouvian lines, according to which uniform taxation is dominated by differential taxation. In the current case, the latter implies giving higher subsidies to a fraction of the workers so as to induce them to search for more productive firms (that exert a larger positive externality). If the subsidy were uniform, firms would have no incentive to signal their higher productivity as workers would demand only the uniform reservation wage. However, with a declining UI path, short-term unemployed agents search for jobs in more productive firms, who are now willing to offer higher wages so as to attract workers with higher reservation wages. In equilibrium, on average, a larger fraction of workers is allocated to the more productive firms.

The role of a declining profile UI regime is to insure enough heterogeneity among workers, so as to obtain enhanced matching. The longer the duration of the first phase, the higher the degree of induced heterogeneity. A key lesson is that even in the absence of moral hazard arguments there is a role for such policy. By enhancing matching it operates to increase output and efficiency (in a constrained setting). In future work we hope to provide a mapping from empirically-relevant dispersion of firms' productivities to a larger set of UI policy instruments.

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6 Appendix A

Derivation of Wage Equations

We reproduce the relevant equations for convenience:

$$H_1 = z + h + \beta[\overline{m}\max(\overline{J}, H_2) + \underline{m}\max(\underline{J}, H_2) + (1 - \overline{m} - \underline{m})H_2] \tag{47}$$

$$H_2 = z + h + \beta[\overline{m}\max(\overline{J}, H) + \underline{m}\max(\underline{J}, H) + (1 - \overline{m} - \underline{m})H] \tag{48}$$

$$H = a + h + \beta[\overline{m}\max(\overline{J}, H) + \underline{m}\max(\underline{J}, H) + (1 - \overline{m} - \underline{m})H] \tag{49}$$

$$\overline{J} = \overline{w} + \beta[(1 - s)\overline{J} + sH_1] \tag{50}$$

$$\underline{J} = \underline{w} + \beta[(1 - s)\underline{J} + sH_1] \tag{51}$$

$$\overline{J} = H_2 \tag{52}$$

$$\underline{J} = H \tag{53}$$

Substracting (51) from (50) yields:

$$\overline{J} - \underline{J} = \frac{\overline{w} - \underline{w}}{1 - \beta(1 - s)} \tag{54}$$

Subtracting (49) from (48) yields:

$$H_2 - H = z - a \tag{55}$$

Substituting (52) and (53) into (54), and then substituting (54) into (55) yields:

$$z - a = \frac{\overline{w} - \underline{w}}{1 - \beta(1 - s)}$$

This is equation (14) in the main text.

Substituting (52) into (47) and noting that $\overline{J} = H_2 > \underline{J}$ yields:

$$H_1 = z + h + \beta H_2 \tag{56}$$

Substituting (52) into (50) yields:

$$[1 - \beta(1 - s)]H_2 = \overline{w} + \beta s H_1 \tag{57}$$

Solving (56) and (57) for H_2 and simplifying yields:

$$[1 - \beta(1 - s) - \beta^2 s]H_2 = \overline{w} + \beta s(z + h) \tag{58}$$

Substituting (52) and (53) into (48) and re-formulating yields:

$$(1 - \beta)H_2 = z + h - \beta(1 - \overline{m})[H_2 - H] \tag{59}$$

Substituting (55) into (59), then substituting from (58) into (59) for H_2 and simplifying, yields:

$$\overline{w} = (z - a)[(1 - \beta) + \beta\overline{m} - \beta^2 s(1 - \overline{m})] + h + a$$

This is equation (15) in the main text.

7 Appendix B

Derivation of Re-employment Probabilities

We show that:

$$\underline{m} = \frac{V}{U - U_1}(1 - e^{-\frac{U-U_1}{F}})$$

A symmetric argument proves the condition with respect to $\overline{m}$.

By definition:

$$\underline{m} = Prob[\text{being assigned to a vacancy offering } \underline{w}] \times Prob[\text{obtaining a job conditional on being assigned}].$$

Because of random matching across posted jobs, the probability of being assigned to a vacancy offering the lower wage rate is simply given by $\frac{V}{F}$.

The conditional probability takes into account the fact, that conditional on assignment, if another k applicants (who are willing to accept a low wage offer) arrive contemporaneously, a random draw from a pool of $k + 1$ applicants determines the winner.

Since agents during the first period of UI eligibility reject any low wage offer, it follows that the conditional probability is given by:

$$\begin{aligned} Prob[\text{obtaining a job conditional on being assigned}] &= \sum_{k=0}^{\infty} \frac{\frac{e^{-\lambda}\lambda^k}{k!}}{k+1} \\ &= \sum_{k=0}^{\infty} \frac{e^{-\lambda}\lambda^k}{(k+1)!} \end{aligned}$$

where $\lambda = \frac{U-U_1}{F}$

Using a Taylor expansion it follows that:

$$Prob[\text{obtaining a job conditional on being assigned}] = \frac{1 - e^{-\lambda}}{\lambda}$$

Substitution completes the derivation.

8 Appendix C

Implementation of the Partial Sorting Equilibrium

We set $F = M$ and $\overline{F} = pM$.

We substitute (16) and (17) in (18) and (19) correspondingly, and solve the system (18)-(22) for five unknowns: $\overline{V}, \underline{V}, U_1, U_2$, and U.

To insure existence of partial-sorting equilibrium, we need to verify that our solution satisfies (14), (15) and (24). Modified to the discrete case, eq. (24) turns into two inequality conditions:

$$\left(1 - e^{-\frac{U-U_1}{F}}\right)(\underline{x} - \underline{w}) \geq \left(1 - e^{-\frac{U}{F}}\right)(\underline{x} - \overline{w}) \tag{60}$$

$$\left(1 - e^{-\frac{U}{F}}\right)(\overline{x} - \overline{w}) \geq \left(1 - e^{-\frac{U-U_1}{F}}\right)(\overline{x} - \underline{w}) \tag{61}$$

This is easy to observe for we have two instruments at our disposal – a and z. Using (14) we fix some $\varepsilon > 0$ arbitrarily small, and set $z - a$ small enough such that $\overline{w} - \underline{w} = \varepsilon$. Using (15), we adjust a, such that $\overline{w} = \underline{x} + \frac{\varepsilon}{2}$. For $\varepsilon > 0$ sufficiently small, (24) is satisfied i.e. all high-productivity firms choose to offer $\overline{w}$, whereas all low-productivity firms choose to offer $\underline{w}$.