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CUSTOMS UNION, TARIFF REFORM, AND TRADE IN DIFFERENTIATED PRODUCTS

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ABSTRACT

The paper analyses tax and tariff policy for trade between economies which each contain a monopolistically competitive industry producing differentiated products. The consequences of tax changes by a single country are examined, and the general desirability of taxing imports more heavily than domestically produced goods is established. Customs union theory is analysed by studying coordinated tax changes by a group of countries; circumstances under which customs union formation and enlargement will benefit members are found. Game theoretic techniques are used to investigate which countries will coalesce into a customs union.

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NON-TECHNICAL SUMMARY

This paper investigates tax and tariff policy for economies which contain a monopolistically competitive industry, and which trade internationally in the outputs of this industry. The monopolistically competitive industry contains firms each of which produces an output differentiated from the outputs of other firms, so giving each firm some degree of monopoly power. Firms' production takes place under conditions of increasing returns to scale, and the number of firms in each country is determined by the free entry and exit of firms in response to profits. The equilibrium of this model generally involves intra-industry trade in the differentiated products, as well as trade in other competitively produced homogeneous goods. The question addressed in the paper is, how do changes in the taxation of domestically produced and imported differentiated products affect trade and welfare?

The paper considers first the consequences of a single country independently changing taxes on domestic production, imports, and exports of differentiated products. It is established that, if retaliation does not occur, this country can raise its welfare by employing positive import tariffs, by subsidising its exports, or by subsidising domestic production, even beyond the point of marginal cost pricing. These policies work by increasing the number of firms in the country under consideration, and so matching the mix of product types produced more closely to preferences of consumers in this economy.

The paper then considers the effect of simultaneous and reciprocated tariff changes by a number of countries, i.e., the theory of customs union. If countries are sufficiently similar then customs union formation raises welfare in member countries, and reduces welfare of non-members. Continued enlargement of the union raises the welfare of the joining country, but may reduce the welfare of existing union members. The argument here is that the gains to firms in the joining country, and consequent increase in the number of firms in this country, may be large

enough to reduce the number of firms in existing member countries.

If countries are not sufficiently similar, then customs union formation may reduce welfare in some of the joining countries. For example, if countries differ in size then large countries will certainly gain from union formation, but small countries may lose. The reason for this is that trade liberalization will cause production to adjust to meet demand in the large, rather than the small market. A corollary of this is that customs unions may be expected to form between countries which are relatively similar, rather than between economies with very different characteristics.

1. Introduction. This paper investigates tax and tariff policy towards international trade in differentiated products. The economies under study each contain a perfectly competitive sector producing a composite commodity under conditions of constant returns to scale, and a monopolistically competitive industry producing differentiated products under increasing returns to scale. Both the composite commodity and the differentiated products are tradeable, and the international equilibrium generally involves both intra-industry trade in differentiated products, and trade in the composite commodity.[1] Tax and tariff policy may be used to change the international equilibrium, and the aim of this paper is to evaluate the welfare consequences of these changes.

The paper addresses three main issues. The first is to study the effects of tax changes by a single country acting in isolation. By changing taxes a country has the power to alter the profitability of firms in its own and other economies, and hence - with free entry and exit of firms - change the number of active firms in each economy. It is established that a country may raise its welfare by exploiting this power; both heavier taxation of imports than of domestically produced goods and the use of export subsidies are shown to raise welfare in the economy employing the taxes and subsidies.

Tax changes by a single country have welfare repercussions for the rest of the world. The second issue addressed is therefore to study the effects of coordinated tax changes by a group of countries acting together - that is, to develop a theory of customs union. The welfare consequences of customs union formation and enlargement are analysed. The gains from formation of a customs union are shown to depend on the characteristics of member countries, so formation of a union unambiguously raises the welfare of all members of the union only if all countries are symmetric. It is established that existing members of the union may suffer welfare reductions from union enlargement - unless side-payments are made by the entrant to existing members.

The sensitivity of the costs and benefits of customs union membership to the characteristics of members of the union raises the question of what unions may be expected to form when countries are asymmetric. The third issue addressed by the paper is therefore to use game theoretic techniques to study the question of coalition formation between countries. It is established that customs unions may be expected to form between countries which are, in an appropriate sense, relatively similar.

The model used throughout the paper is set out in the following section. Results are obtained by the use of calculus techniques to evaluate the consequences of small tax changes, and this is set out in general terms in section 3 of the paper. Section 4 studies tax and tariff changes by a single country acting in isolation, and sections 5, 6 and 7 evaluate the consequences of customs union formation and enlargement. Section 8 develops a three country example in which to explore the question of coalition formation.

2. The Model. There are ν countries, each of which is endowed with a fixed quantity of a single factor of production. Each economy may produce an identical composite commodity, which is tradeable,

will be assumed to be untaxed, and will be taken to be the numeraire, so ensuring a unit exchange rate between economies. The composite commodity is produced perfectly competitively, and under conditions of constant returns to scale. Factor efficiency in the production of this commodity may differ between countries.

In addition, each country may produce differentiated products. These are produced under conditions of increasing returns to scale, and by a monopolistically competitive industry. The number of these commodity types produced in country i is denoted n_i, and is endogenously determined. These commodities may be taxed, but assumptions on technology, preferences and tax rates will be employed to ensure that the differentiated products produced in any single economy are symmetric. The differentiated products are tradeable, and the quantity and consumer price of a single representative commodity type produced in country i and sold in country j will be denoted x_{ij} and p_{ij} respectively.

Demands in each country are derived from individualistic social welfare functions defined on the aggregate quantities of the commodities consumed in that country. The welfare functions will be assumed to be separable between the numeraire commodity and differentiated products. Each country then has a sub-utility function defined on the differentiated products, which will be assumed to have a constant elasticity of substitution form, as in Dixit and Stiglitz [1977]. All commodities from a particular country source will be assumed to enter country j's sub-utility function symmetrically, so if α_{ij} is a taste parameter describing the preferences of a consumer in j over products produced in i, and the sub-utility function is denoted y_j, it may be written as,

(1) $$y_j = \left[\sum_{i=1}^{\nu} n_i (\alpha_{ij} x_{ij})^{(\varepsilon_j - 1)/\varepsilon_j}\right]^{\varepsilon_j/(\varepsilon_j - 1)}, \qquad \varepsilon_j > 1.$$

Given preferences of this form consistent two stage maximisation is possible (see for example Green [1964]). y_j may be interpreted as a quantity index corresponding to which there is a price index, q_j, of the form,

(2) $$q_j = \left[\sum_{i=1}^{\nu} n_i (p_{ij}/\alpha_{ij})^{1-\varepsilon_j}\right]^{1/(1-\varepsilon_j)}$$

Consumers first allocate expenditure between the numeraire commodity and y_j, the index of differentiated products in aggregate, subject to their budget constraint, and guided only by knowledge of the price index q_j. It will be convenient to describe this first stage of consumer choice by an expenditure function, $e_j(q_j, u_j)$, where u_j is utility in country j. If m_j is economy j's endowment of its factor of production (measured in terms of the numeraire), and tax revenue in j is denoted r_j and distributed in a lump sum manner, then economy j's budget constraint is,

(3) $$m_j + r_j = e_j(q_j, u_j).$$

Demand for y_j is then given by the partial derivative,

(4) $$y_j = \partial e_j(q_j, u_j)/\partial q_j.$$

At the second stage the consumer allocates total expenditure on differentiated products, $q_j y_j$, between individual products, using individual consumer prices, p_{ij}. This gives demands,

(5) $$x_{ij} = p_{ij}^{-\varepsilon_j} \alpha_{ij}^{\varepsilon_j - 1} q_j^{\varepsilon_j} y_j.$$

Taxes are on sale rather than production, so tax revenue accrues to the country of sale. t_{ij} denotes the tax rate on sales in country j of differentiated products produced in country i, expressed as a proportion of the consumer price, (so $t_{ij} < 1$). If n_i products from i are imported, then revenue in j is

$$(6) \qquad r_j = \sum_{i=1}^{\nu} t_{ij} p_{ij} x_{ij} n_i .$$

Each type of differentiated product is supplied by a single firm, and each firm supplies only one type of differentiated product. Production of the differentiated products takes place under conditions of increasing returns to scale, which will be modelled by assuming that production incurs a fixed cost, then operates at constant marginal cost. All firms in a particular country are assumed to be symmetric, so a firm in country i has fixed costs denoted f_i, and, for sales in market j, marginal costs c_{ij}, both measured in terms of the numeraire. The profits of a representative firm in country i may now be written as,

$$(7) \qquad \pi_i = \sum_{j=1}^{\nu} x_{ij} [p_{ij}(1-t_{ij}) - c_{ij}] - f_i$$

The x_{ij} are chosen to maximise this, subject to the second stage demand function (equation (5)), but with the standard 'large group' assumption that q_j and y_j are assumed to be constant. First order conditions for this problem are,

$$(8) \qquad c_{ij} = p_{ij}(1 - t_{ij})(1 - 1/\varepsilon_j) \qquad \text{for all i, j,}$$

i.e., marginal revenue equals marginal cost, where ε_j is the elasticity of demand for a single differentiated product in country j.

Notice that firms' output decisions, as described by equations (7) determine equilibrium prices p_{ij} directly, so prices may then be regarded as parameters for the rest of the model. This may be exploited by defining two $\nu \times \nu$ dimension matrices, A and B as follows.

$$A = \{(p_{ij}/\alpha_{ij})^{1-\varepsilon_j}\}$$

(9)

$$B = \{(1-t_{ij})(p_{ij}/\alpha_{ij})^{1-\varepsilon_j}\}$$

Using the demand functions, equations (4) and (5), matrices with elements denoting the revenue of a single firm from i on sales in country j, at consumer and producer prices respectively, are

$$A\hat{e}_q\hat{q}^{\varepsilon} = \{p_{ij}x_{ij}\}$$

(10)

$$B\hat{e}_q\hat{q}^{\varepsilon} = \{(1-t_{ij})p_{ij}x_{ij}\}$$

In equations (10) and throughout the rest of the paper, upper case symbols are $\nu \times \nu$ matrices, and lower case symbols are ν dimensional column vectors, unless they are denoted $\hat{}$ in which case they are diagonal matrices formed from the corresponding vector. For example, $\hat{e}_q$ is a diagonal matrix with element $\partial e_j(q_j, u_j)/\partial q_j$ in the jth row and column.

The matrix A may also be used to obtain an expression for the number of active firms in each economy. Denoting the transpose of A by A^T, then from the price index, equations (2), the number of firms in each economy may be expressed as,

(11) $$n = (A^T)^{-1}q^{1-\varepsilon}.$$

The characterization of equilibrium may now be completed. There is free entry of firms in each country, and it will be assumed that each country produces a positive number of differentiated products, and that this number is large enough for n_i to be treated as a continuous variable. Free entry then ensures that at the equilibrium maximised profits are zero. Denoting the maximised profits of a single representative firm from each country by the vector π^*, and using (7), (8) and (10),

$$\pi^* = B\hat{\varepsilon}^{-1}\hat{e}_q q^{\varepsilon} - f = 0. \tag{12}$$

Equations (10), (11) and (6) may be used in the budget constraint, equation (3) to obtain,

$$m = e - [(A - B)\hat{e}_q \hat{q}^{\varepsilon}]^T (A^T)^{-1} q^{1-\varepsilon}, \text{ and rearranging,}$$

$$m = e - \hat{e}_q q + \hat{e}_q \hat{q}^{\varepsilon} B^T (A^T)^{-1} q^{1-\varepsilon}. \tag{13}$$

Equations (12) and (13) are 2ν equations in the 2ν unknowns q and u, so characterize the equilibrium of the model.

In order to analyse the effect of tax changes on the equilibrium the following assumption will be employed for the rest of the paper.

Assumption 1; All tax and tariff changes are evaluated around a point at which each country's import tariffs are equal to that country's tax on domestically produced and consumed output, i.e., $t_{ij} = t_j$, for all i.

This assumption does not of course preclude examination of situations where taxes and tariffs are not equal; these differences will be examined by taking small changes around a point where assumption 1 holds. Assumption 1 means that in evaluating the effects of differential changes in taxes or tariffs the following condition may be used (see equations (9)).

$$(14) \qquad B = A(I - \hat{t}).$$

Some results obtained below depend on the structure of firms' shares in different markets. The following assumption formalises the idea that firms have relatively larger shares of their home markets than they do of foreign markets, and will be used at various points in the rest of the paper.

Assumption 2; A is a Minkowski matrix, that is A^{-1} has positive elements on the leading diagonal, and negative elsewhere.

If A satisfies assumption 2 then so too does its transpose, and with assumption 1, so does B. Assumption 2 may be interpreted by considering two elements from the same column of A, say elements a_{ik}, a_{jk}. The ratio of these elements is the ratio of expenditures in country k on single products from the two row countries, that is, from (10),

$$(15) \qquad a_{ik}/a_{jk} = p_{ik}x_{ik}/p_{jk}x_{jk}.$$

Columns of A therefore contain the relative market shares of individual products from different countries in country k. If $\nu = 2$ assumption 2 is satisfied if A has a positive determinant, that is if each product type has a larger share of its domestic market than it does of its export market. If ε_j is the same in the two countries a necessary and sufficient condition for A to have a positive determinant is that $c_{11}c_{22}\alpha_{12}\alpha_{21} < c_{12}c_{21}\alpha_{11}\alpha_{22}$. This inequality may be secured either by assuming the existence of transport costs, so $c_{ii} < c_{ij}$, or by assuming that consumers in each country derive more utility from a unit of domestically produced differentiated good than from an import, so $\alpha_{ii} > \alpha_{ij}$, for all i, j, $i \neq j$. Notice that with $\alpha_{ii} > \alpha_{ij}$ and $c_{ii} = c_{ij}$, the price of a product produced in i is the same in all markets j. With this interpretation assumption 2 does not therefore require that firms can price discriminate between markets.

For $\nu = 3$ necessary and sufficient conditions for A to be Minkowski are set out in Kemp [1967], and are, (with (15)),

(16) $p_{ii}x_{ii}/p_{ji}x_{ji} > p_{ik}x_{ik}/p_{jk}x_{jk}$, for all i, j, k, $i \neq j$, $i \neq k$.

This states that comparing products produced in two countries i and j, a single product from i has a larger share of its domestic market, relative to a product from j, than it has of some other market, k. The underlying reason for the different relative market shares is again differences in costs or preferences. For $\nu > 3$ interpretation of assumption 2 is less straightforward. Technical properties of Minkowski matrices are discussed in Kemp [1967] and Kemp and Kimura [1971], and the implications of assumption 2 for the pattern of trade are analysed in Venables [1984].

3. Tax and tariff changes. Tax and tariff changes affect the equilibrium by changing elements of the matrices A and B, and hence changing profits of firms and government tax receipts. If dB denotes the matrix of changes in elements of B associated with some set of tax changes, and $d\pi$ denotes the direct effect of tax changes on the profits of a single representative firm in each

country, then, from (12),

$$(17) \qquad d\pi = dB\hat{\varepsilon}^{-1}\hat{e}_q q^{\varepsilon}.$$

dr denotes the direct effect of tax changes on tax revenue received in each country, so, from (13),

$$dr = -\hat{e}_q \hat{q}^{\varepsilon} d[B^T(A^T)^{-1}]q^{1-\varepsilon}$$

$$(18) \qquad = -\hat{e}_q \hat{q}^{\varepsilon}\hat{\varepsilon}^{-1} dB^T(B^T)^{-1}(I - \hat{t})q^{1-\varepsilon},$$

where differentiation of the matrices A^T and B^T and derivation of (18) is undertaken in appendix 1.

Given the direct effects of tax changes, the changes in q and u required to restore equilibrium may be found by totally differentiating the equilibrium conditions (12) and (13). So doing (with (14)) gives (19) and (20) respectively.

$$(19) \qquad -d\pi = B\hat{\varepsilon}^{-1}\hat{q}^{\varepsilon-1}[\hat{q}\hat{e}_{qu}du + (\hat{\varepsilon}\hat{e}_q + \hat{q}\hat{e}_{qq})dq]$$

$$(20) \qquad dr = [\hat{e}_u - \hat{t}\hat{e}_{qu}\hat{q}]du + [\hat{e}_q(I-\hat{t}) - \hat{t}\hat{q}\hat{e}_{qq}]dq.$$

Using (17) and (18) in (19) and (20), and eliminating dq, the welfare consequences of tax changes are,

$$(21) \qquad \hat{\sigma}du = [-\hat{\tau}Z + Z^T]1,$$

where 1 is the sum vector and,

$$(22) \qquad Z = -\hat{q}^{1-\varepsilon}(I-\hat{t})B^{-1}dB\hat{\varepsilon}^{-1}\hat{e}_q\hat{q}^{\varepsilon},$$

$$(23) \qquad \hat{\sigma} = \hat{e}_u + \hat{e}_{qu}\hat{q}\hat{e}_q\hat{\varepsilon}\{\hat{e}_q\hat{\varepsilon}+\hat{q}\hat{e}_{qq}\}^{-1}[(I-\hat{t})(I-\hat{\varepsilon}^{-1}) - I]$$

and

$$(24) \qquad \hat{\tau} - I = \hat{q}\hat{e}_{qq}\hat{\varepsilon}\{\hat{e}_q\hat{\varepsilon}+\hat{q}\hat{e}_{qq}\}^{-1}(I-\hat{t})^{-1}[(I-\hat{t})(I-\hat{\varepsilon}^{-1}) - I]$$

The second term on the right hand side of (21) gives the revenue effect of tax changes, and the first term the effect through perturbation of the industry equilibrium condition. To see this note that, using (18) and (22),

$$(25)\qquad Z^T 1 = -\hat{e}_q \hat{q}^{\varepsilon} \varepsilon^{-1} dB^T (B^T)^{-1} (I-\hat{t}) q^{1-\varepsilon} = dr.$$

σ_i therefore measures the change in lump sum income in country i required to raise country i's welfare 1 unit. We shall assume that $\hat{\sigma} > 0$. It will also be assumed that single firms' demand curves are more elastic than are the compensated demand curves for differentiated products in aggregate, so $\hat{e}_{qq}\varepsilon\{\hat{e}_q\hat{\varepsilon}+\hat{q}\hat{e}_{qq}\}^{-1} < 0$. We then have,

$$(26)\qquad \tau_i \gtrless 1 \quad \text{as} \quad 1 \gtrless (1-t_{ij})(1-1/\varepsilon_j) = c_{ij}/p_{ij} \text{ for all } j,$$

where the equality is from the price equations, (8). If $\tau_i < 1$ then firms selling in country i are subsidised such that the consumer price is less than the marginal cost of production, and if $\tau_i > 1$ then consumer price is greater than marginal cost.

Inspection of equation (21) with (26) yields one immediate result concerning world-wide efficiency.

Proposition 1. If differentiated products are subsidised to the point where price equals marginal cost, then there exists no set of small tax and tariff changes which can lead to a Pareto welfare improvement.

Proof; If $\hat{\tau} = I$ then $\hat{\sigma}du = [-Z + Z^T]1$. Since the sum of all elements of a matrix minus its transpose is zero, no row sum can be positive without some other row sum being negative. There is therefore no set of tax changes for which $du_i \geqslant 0$ for all i with strict inequality for some i.

In interpreting proposition 1, notice that world Pareto efficiency requires both that each firm supplies efficient quantities, x_{ij}, and that a socially efficient numbers of commodity types, n_i, are produced. Subsidising such that $\hat{\tau} = I$, achieves this by setting consumer prices equal to marginal cost, and simultaneously equating firms' revenues to the consumer benefit generated by each product type in the sub-utility functions. The ability of one set of instruments, the commodity taxes t_{ij}, to decentralise efficiency both in the output quantities and the number of commodity types is special to the iso-elastic form of the sub-utility functions. For examination of sub-utility functions with differing and variable elasticities see Venables [1982].

For the remainder of the paper we shall assume that all taxes which change, change by an equal amount, dt. dB may then be written as,

$$(27) \qquad dB = -\tilde{A}\hat{\varepsilon}\,\hat{dt},$$

where $\hat{dt}$ is a diagonal matrix with the common value dt in all diagonal elements, and $\tilde{A}$ is a matrix with element a_{ij} of A in any position where a tax has changed, and zeroes in all positions where taxes are unchanged, (see appendix 1) . Thus, if tax rates on domestically produced goods change in all countries, but all tariff rates remain constant, $\tilde{A}$ would be diagonal. With this assumption, and assumption 1, Z becomes (using equations (14) and (27) in (22)),

$$(28) \qquad Z = \hat{q}^{1-\varepsilon}A^{-1}\tilde{A}\hat{e}_q\hat{q}^{\varepsilon}\hat{dt},$$

Welfare consequences of different combinations of tax and tariff changes can now be found by analysing equations (21), that is,

(21) $\hat{\sigma} du = [-\hat{\tau} Z + Z^T] 1,$

together with (28) for different structures of the matrix $\tilde{A}$.

4. Tax and tariff changes by a single country: This section examines the welfare implications of tax and tariff changes by a single country acting in isolation, and thereby establishes a number of principles of tax and tariff policy for that country. The country under consideration will be taken to be country 1, so that $\tilde{A}$ contains non-zero elements only in its first column. The following proposition describes the optimal tax and tariff policy for country 1 if it is constrained to impose the same tax on all differentiated products, independently of source.

Proposition 2. If country 1 is constrained to maintain equality of taxes and tariffs, then (i) the optimal tax rate in 1 is to set t_1 such that $\tau_1 = 1$. (ii) Country 1's tax and tariff policy does not affect welfare in the rest of the world.

Proof; All taxes in country 1 change by an equal amount, and all taxes in the rest of the world are unchanged. $\tilde{A}$ therefore has first column with elements $\tilde{a}_{i1} = a_{i1}$ for all i, and all other columns zero, so $\tilde{a}_{ij} = 0$ for all $j \neq 1$. By the definition of an inverse $A^{-1}\tilde{A}$ has unity in row 1 column 1, and all other elements zero, so Z has element z_{11} taking sign dt, and all other elements zero (see (28)). From equation (21) we then have $du_i/dt_1 = 0$, for $i \neq 1$, and $du_1/dt_1 \gtrless 0$ as $\tau_1 \lessgtr 1$, so proving the proposition.

Part (ii) of proposition 2 establishes that countries other than country 1 need not care about tax and tariff policy in 1 providing that country 1 does not tax imports differently from domestically produced goods. Part (i) of proposition 2, together with proposition 1 establishes that if countries are not permitted to tax imports differently from domestically produced commodities, and can raise lump sum taxes to finance subsidies, then nationalistic choice of tax and tariff rates will support a world equilibrium which is Pareto efficient. This ceases to be the case if countries can tax imports differently from domestically produced goods, as is established in the next proposition.

Proposition 3. With assumption 2 (A is a Minkowski matrix) a small increase in country 1's tariffs on imports from any set of other countries, (i) raises welfare in country 1, and (ii) reduces welfare in all countries against which a tariff has been raised.

Proof; Let country 1 raise the tariff rate on its imports from countries $k = 2 \ldots \kappa \leq \nu$, so $\tilde{A}$ has all elements zero except in rows $2 \ldots \kappa$ of the first column, where it has elements a_{i1}, $i = 2 \ldots \kappa$. From equation (21) we may evaluate first $-\hat{\tau} Z1$. $A^{-1}\tilde{A}$ has zero elements except in the first column, where, with the Minkowski assumption, (assumption 2), the first element is negative and elements in rows $2 \ldots \kappa$ are positive. $-\hat{\tau} Z1$ is therefore a column vector with first element taking sign dt, and elements in rows $2 \ldots \kappa$ taking sign -dt. The revenue effect of the tariff change is, (using (11) in (28)),

$$Z^T 1 = dr = d\hat{t}\hat{e}_q \hat{q}^{\varepsilon} \tilde{A}^T n.$$

By non-negativity of n and the structure of $\tilde{A}^T$, $dr_1 > 0$ and $dr_i = 0$, for $i \neq 1$. Combining $-\hat{\tau}Z1$ and Z^T1 though equation (21) proves the proposition.

Proposition 3 establishes that asymmetric tax treatment of imported and domestically produced differentiated products is desirable, with import tariffs exceeding taxes on domestically produced and consumed goods. Notice that each economy is a price taker internationally, as the tariff does not change the unit cost to the economy of importing a particular type of differentiated product since $p_{ij}(1-t_{ij})$ is unchanged at $c_{ij}(1-1/\varepsilon_j)$, (equation (8)). The tariff does however change price indices, q, and numbers of commodity types produced domestically and imported. The mechanism here is the following. Suppose that country 1 imposes the tariff on imports from 2. The revenue of firms in country 2 is reduced by the tariff, so, if country 2's monopolistically competitive industry is to survive there must be an increase in some price indices, q. Since changes in q must be consistent with maintenance of zero profits in the rest of the world, q must increase in countries where country 2 has a large market share, and decrease in other countries. The Minkowski assumption ensures that this is an increase in q_2, and a reduction in the price indices q elsewhere. The changes in q are implemented through changes in the numbers of differentiated products produced in each country, and it can be shown that the tariff increase reduces n_2. These changes in n and the price indices q are by themselves sufficient to generate the welfare changes of the proposition. Additionally the country imposing the tariff benefits from receipt of tariff revenue.

Proposition 3 holds independently of the actual level of taxation in the country imposing the tariff. That is, even if domestic imperfections have been corrected by setting $(1-t_{ij})(1-1/\varepsilon_j) = 1$, asymmetric treatment of imports and domestically produced differentiated products is desirable. Heavier taxation of imports than of domestically produced goods is a first best policy instrument, and is employed to exploit power not over the prices of imports, but over the number of commodity types produced.

Propositions 2 and 3 may now be combined to obtain the welfare implications of a change in taxation of domestically produced goods, holding tariffs constant.

<u>Proposition 4</u>. With assumption 2, and $\tau_1 \geqslant 1$, a small reduction in taxation of domestically produced goods in country 1 raises welfare in 1, and reduces welfare in the rest of the world.

The proof is immediate, by regarding the increase in tax as an equal reduction in all taxes and tariffs, coupled with an increase in tariffs. The proposition establishes that a country has an incentive to subsidize production beyond the point at which price equals marginal cost. By so doing country 1 can alter the location and number of differentiated products produced, so matching world production of differentiated products more closely to country 1 preferences. This increases welfare in 1, but at the expense of causing welfare reductions elsewhere in the world.

The model has been formulated such that all taxes on commodities sold in a particular country accrue as revenue to that country, i.e., taxes are sales taxes rather than production taxes. Small taxes or subsidies on production for export may however be readily incorporated by regarding the tax on exports from, say, country k to country 1 as an import tariff in 1 coupled with a revenue transfer from 1 to k. If R is a matrix with unity in position row k column 1, and zero elsewhere then the welfare effects of a small tax imposed by country k on exports from k to 1 are obtained from the following modification of equation (21),

$$\hat{\sigma} du = [-\hat{\tau} Z + RZ^T]1. \quad (29)$$

Proposition 5. If country k employs a small subsidy on its exports to country 1, and if $\tau_k \geq 1$ and assumption 2 hold, then the subsidy raises welfare in country k and reduces welfare in country 1.

Proof; $\tilde{A}$ has all elements zero, except $\tilde{a}_{k1}$. $A^{-1}\tilde{A}$ has all elements zero except the first column in which, with assumption 2, the kth row has positive element and all other rows have negative element. If dt < 0 we therefore have (from equation (28)), all elements of Z zero except the first column in which $z_{k1} < 0$, and $z_{i1} > 0$, for all $i \neq k$. Rows 1 and k of equations (29) may be evaluated as,

$$\sigma_1 du_1 = -\tau_1 z_{11}$$

$$\sigma_k du_k = -\tau_k z_{k1} + \sum_{i=1}^{\nu} z_{i1}$$

$$= (1 - \tau_k) z_{k1} + \sum_{i \neq k}^{\nu} z_{i1} \quad (30)$$

Using the inequalities above on z_{il}, $du_l < 0$ and $du_k > 0$.

The intuition behind the desirability of export subsidies is as follows. Exports from k to l yield net revenue to firms in country k, since producer prices exceed marginal cost. Firms acting independently maximise this net revenue through choice of export prices and quantities, but in this maximization procedure take price indices q to be fixed. However, simultaneous action by firms in k can influence the price indices, and thereby increase net revenue. In particular, firms in country l can be forced out, so raising the price index q_l. The role of export subsidies is to induce firms in k to increase exports, and thereby exploit the fact that, including this increase in q_l, firms' export demand curves are more elastic than they are perceived to be in individual maximisation decisions. One qualification needs to be added to this argument. The export subsidy also involves a direct transfer of revenue from economy k's budget constraint to firms in country k. The social value of such a transfer depends on τ_k. If $\tau_k = 1$, then firms are subsidized such that the economy is at the social optimum, and the transfer has zero social value; if $\tau_k > 1$, the transfer to firms has positive social value since the differentiated products sector is smaller than it would be at the optimum, and there is a social premium on a revenue transfer which would induce expansion of the industry. This explains the first term on the right hand side of equation (30). Only if τ_k were considerably less than unity, so that the industry were subsidized significantly beyond marginal cost pricing, could the export subsidy reduce welfare in the country employing the subsidy.

Propositions 3 and 5 establish that import tariffs and export subsidies raise welfare in the country imposing the tariff or subsidy. These policies do however reduce welfare elsewhere. It is to multi-country policies that we now turn.

5. Customs unions: The welfare consequences of customs union formation may be modelled by looking at simultaneous and reciprocal tariff reductions by a set of countries. Assumption 1 will be retained so that tariff changes are evaluated around a point of equality of taxes and tariffs. To a first order approximation such changes are the same as reductions in tariffs to equality with domestic taxes from a situation in which tariffs were differentially higher.

Analysis is simplified if we restrict attention to circumstances in which all economies have, in an appropriate sense, symmetric technologies, tax rates and sub-utility functions for differentiated products. This may be formally stated as assumption 3.

Assumption 3. $f_i = f_j$, $t_i = t_j$, $\varepsilon_i = \varepsilon_j$,
$c_{ij}/\alpha_{ij} = c_{k\ell}/\alpha_{k\ell} > c_{ii}/\alpha_{ii} = c_{jj}/\alpha_{jj}$, for all i, j, k, ℓ,
$i \neq j$, $k \neq \ell$.

The assumptions on the ratios c_{ij}/α_{ij} do permit marginal costs and demand parameters to differ across countries and markets, but restricts differences such that the matrix A may, with an appropriate choice of units, be written as,

$$(31) \qquad A = \{\phi + \nu - 1\}^{-1} \begin{bmatrix} \phi & & \dots 1 \\ & \ddots & \\ 1 \dots & & \phi \end{bmatrix}$$

where $\phi = \{c_{ii}\alpha_{ii}/c_{ij}\alpha_{ij}\}^{1-\varepsilon} > 1.$

The inverse of A may be computed directly as,

$$(32) \qquad A^{-1} = (\phi - 1)^{-1} \begin{bmatrix} \phi+\nu-2 & & \ldots -1 \\ & \ddots & \\ -1 \ldots & & \phi+\nu-2 \end{bmatrix}$$

Notice that with $\phi > 1$, A is a Minkowski matrix (assumption 2). Assumption 3 therefore puts a symmetry on preferences and technology such that in each country the share of a single domestic firm in its home market is ϕ times the share of a single importing firm from any country. Assumption 3 does not require that economies have the same expenditure functions, nor the same factor endowments or factor productivity. Assumption 3 is therefore weaker than the symmetry assumptions employed elsewhere in the literature, see for example Krugman [1980], or Helpman [1981].

Assumption 3 also ensures that, in equilibrium, all elements of the vector $\hat{e}_q q^{\varepsilon}$ are equal. This may be established by noting that, from assumption 3, row sums of A^{-1} are unity, so that with assumption 1 (and equation (14)) the industry equilibrium conditions, equations (12), may be rearranged to give,

$$(33) \qquad \hat{e}_q q^{\varepsilon} = B^{-1}\hat{\varepsilon} f = \hat{\varepsilon}(I-\hat{t})^{-1} f.$$

Since $\hat{e}_q \hat{q}^{\varepsilon}$ is now a scalar times the identity matrix, the matrix Z (equation (28)) becomes,

$$(34) \qquad Z = \hat{e}_q \hat{q} A^{-1} \tilde{A} d\hat{t}.$$

We may now evaluate the effects of the formation of a customs union by a group of countries. If the first κ countries form the union, and change tariffs on intra-union trade leaving all other taxes and tariffs unchanged, then the matrix $\tilde{A}$ has $\tilde{a}_{ij} = a_{ij} = 1$, for $i, j \leqslant \kappa$, $i \neq j$, and $\tilde{a}_{ij} = 0$ elsewhere. Using equations (34) in (21) with (32) and (4), direct computation yields;

for $i \leqslant \kappa$:

$$\psi_i du_i = (\kappa-1)\Big[q_i y_i (1-\tau_i)(\phi-1+\nu-\kappa) - (\phi+\nu-\kappa)\Big\{q_i y_i - \sum_{\substack{j=1 \\ j\neq i}}^{\kappa} q_j y_j/(\kappa-1)\Big\} - \sum_{j=\kappa+1}^{\nu} q_j y_j\Big] dt \tag{35}$$

for $i > \kappa$:

$$\psi_i du_i = \tau_i \kappa(\kappa-1) q_i y_i dt, \tag{36}$$

where,

$$\psi_i = (\phi - 1)(\phi - 1 + \nu)\sigma_i > 0. \tag{37}$$

To interpret equations (35) and (36) it is easiest to look first at a case where expenditure on differentiated products, $q_i y_i$, is the same in all countries.

6. Equal expenditures on differentiated products: If $q_i y_i$ is the same for all countries i then equation (35), giving the effects of customs union formation on members of the union, reduces to,

$$\psi_i du_i = (\kappa-1) q_i y_i [(1 - \tau_i)(\phi - 1) - \tau_i(\nu - \kappa)] dt, \tag{38}$$

for $i \leqslant \kappa$.

The following proposition comes immediately from equations (36) and (38).

Proposition 6. With assumption 3 and equal expenditure on differentiated products in each economy, (i) a sufficient condition for customs union formation to raise the welfare of member countries is that $\tau_i \geqslant 1$. (ii) The welfare of countries outside the union is reduced by formation of the union.

Part (i) of this proposition is derived from equation (38). It may be noted that if the tariff reductions are worldwide, i.e., $\nu = \kappa$, then having $\tau_i > 1$ is a necessary as well as sufficient for the tariff reductions to increase welfare; this is as would be expected from proposition 1 describing world Pareto efficiency. If $\nu > \kappa$, then customs union formation increases the welfare of members, even if $\tau_i = 1$. The incentive to cut tariffs beyond marginal cost pricing may be explained as follows. Tariff cuts within the union raise the revenues of firms in the union, and thereby permit reductions in the price indices, q, consistent with zero profits in member countries. These reductions in price indices inside the union necessitate increases in price indices for countries outside the union, if their monopolistically competitive industries are to survive. These increases will further raise the export revenues of firms within the union, so permitting further falls in the price indices of union members. It is this ability to alter price indices which creates the incentive to subsidize intra-union trade beyond the point of marginal cost pricing. Changes in price indices are, as usual, implemented through changes in the number of active firms in each economy.

The dependence of the welfare effects of customs union formation on κ, the size of the union, may be investigated further by examining the welfare effects of customs union enlargement. If $du_i(\bar{\kappa})$ and $du_i(\bar{\kappa}+1)$ denote the welfare effects for country i of formation of customs unions of size $\bar{\kappa}$ and $\bar{\kappa} + 1$ respectively then evaluating equation (38) at $\kappa = \bar{\kappa}$ and $\kappa = \bar{\kappa} + 1$, gives, for existing members of the union, $i \leq \bar{\kappa}$, the welfare effects of customs union enlargement as,

$$(39) \quad \psi_i [du_i(\bar{\kappa}+1) - du_i(\bar{\kappa})] = q_i y_i [(1 - \tau_i)(\phi - 1) - \tau_i(\nu - 2\bar{\kappa})] dt$$

For the country joining the union we may evaluate equation (36) at $\bar{\kappa}$ and (38) at $\bar{\kappa} + 1$, to give, for the joining country, $j = \bar{\kappa} + 1$,

$$(40) \quad \psi_j [du_j(\bar{\kappa}+1) - du_j(\bar{\kappa})] = q_j y_j [(1 - \tau_j)(\phi - 1) - \tau_j(\nu - 2)] dt.$$

Proposition 7 follows from these two equations;

> Proposition 7. With assumption 3, and if all economies have the same income and preferences, then, (i) customs union enlargement raises the welfare of existing members of the union if $\bar{\kappa}$ is small relative to ν, but may reduce welfare for large $\bar{\kappa}$. (ii) If consumer prices are at least as great as marginal cost ($\tau_i \geq 1$) in the joining country its welfare is certainly increased by membership. (iii) If side payments are possible, the joining country can always compensate existing members such that customs union formation is a Pareto improvement for the $\bar{\kappa} + 1$ countries concerned.

Part (i) of proposition 7 comes from inspection of equation (39), so, if for example $\tau_i = 1$, then customs union enlargement benefits existing members only if $\bar{\kappa} < \nu/2$. The intuition here is the following. Firms in the joining country benefit by getting liberalized access to $\bar{\kappa}$ markets, and firms in countries already in the union benefit by getting liberalized access to one further market. The associated changes in profits cause changes in the values of the price indices consistent with zero profits, in particular reducing the price index in the joining country. If $\bar{\kappa}$ is large then, including these changes in the price indices, the net effect of enlargement could be to reduce profits of firms in countries already in the union, and thereby require an increase in q and reduction in welfare in these countries. The entrant certainly enjoys a welfare improvement, unless, from equation (40), $\tau_j < 1$, and very small relative to ϕ. For $\tau_j > 1$ it can also be shown (by multiplying (39) by $\bar{\kappa}$ and adding to (40)), that the welfare gain of the entrant is at least as large as the combined welfare loss of existing members. Under the conditions of the proposition the marginal utility of the numeraire is the same in all economies. Transfer payments are therefore possible such that the new entrant can bribe existing members to gain admission.

7. Expenditure differences. If expenditure on differentiated products is different between countries, then the welfare effects of customs union formation are given by equations (35) and (36). The size of an economy's expenditure on differentiated products is then an important determinant of the welfare effects of tariff reductions, as illustrated by the following proposition.

Proposition 8. If $\tau_i = 1$ worldwide tariff reductions ($\kappa = \nu$) increase welfare in countries which have larger than average expenditure on differentiated products, and reduce welfare in countries with below average expenditures.

This proposition is derived from equation (35), noting that with $\tau_i = 1$ the last term on the right hand side of the equation is zero, and in the second to last term $\sum_{j=1, j \neq i}^{\nu} q_j y_j/(\kappa-1)$ is the average expenditure on differentiated products in countries other than i. The differences in expenditure on differentiated products may arise from three sources; countries may have different expenditure functions, different size endowments, or differences in the productivity of their factor in producing the numeraire. Different expenditures derive their importance from the fact that, under the conditions of the proposition, economies with relatively small expenditures on differentiated products are net importers of differentiated products (see Venables [1984]). The direct revenue implications of worldwide tariff reductions are therefore damaging to small economies, and may outweigh the efficiency gains from moving towards marginal cost pricing.

For simplicity proposition 8 assumed that firms were subsidized to the point of marginal cost pricing ($\tau_i = 1$). The results of the proposition generalize however, so if $\tau_i > 1$ a country which has sufficiently small expenditure on differentiated products, relative to the average, will lose from world-wide tariff reductions. There is therefore a clear conflict of interest over multilateral tariff reductions between countries with large, and with small expenditures on differentiated products.

If tariff reductions occur between a subset of economies, $\kappa < \nu$, then the role of different expenditure levels is illustrated in proposition 9.

<u>Proposition 9.</u> If $\tau_i \geqslant 1$, a necessary condition for a country's welfare to be reduced by the formation of a customs union of

which it is a member is that its expenditure on differentiated products is below the average for the rest of the union.

This proposition is derived from equation (35). Clearly having smaller than average expenditure is not sufficient for welfare reduction; however, since the average expenditure on differentiated products by the rest of the union enters (35) negatively, if this is large enough it is certainly possible for a country with a small expenditure on differentiated products to suffer a welfare loss.

8. Coalition formation: an example. The results of the previous section suggest that, in the absence of side-payments, countries always want to be members of customs unions with countries with smaller expenditures on differentiated products than themselves. This raises the question of which customs unions will actually form when countries have different expenditures on differentiated products. The question may be analysed by thinking of the problem as one of coalition formation in a cooperative game without side-payments. Each country may form a union with a (possibly empty) set of other countries. The ensuing coalition structure will be termed 'stable' if there exists no set of countries, which, by forming a union, can guarantee themselves a higher level of welfare. (Notice that in this definition a coalition can be a single country). We may now investigate stable coalition structures for a three country example, in which assumption 3 holds. It will additionally be assumed that all countries have the same preferences, so differ only in endowment size, and therefore in expenditure on differentiated products.

Payoffs to countries when countries act singly (there are no customs unions) will be denoted $du_i\{i\}$, and since utility changes are measured from the situation where there are no customs unions,

(41) $$du_i\{i\} = 0.$$

If there is a coalition of two countries i, and j, payoffs to the three countries i, j, and k will be denoted $du_i\{i,j\}$, $du_j\{i,j\}$ and $du_k\{i,j\}$ respectively. From inspection of equation (35), for country i in the union,

(42) $$\psi_i du_i\{i,j\} = [q_i y_i(1-\tau_i)\phi - (q_i y_i - q_j y_j)(\phi+1) - q_k y_k]dt.$$

du {i,j} can be defined analogously, and for country k not in the union, from equation (36),

(43) $$\psi_k du_k\{i,j\} = 2\tau_k q_k y_k dt.$$

In the grand coalition the payoff to country i is denoted du {1,2,3}, and from equation (35) is,

(44) $$\psi_i du_i\{1,2,3\} = [2q_i y_i(1-\tau_i)(\phi-1) - \phi(2q_i y_i - q_j y_j - q_k y_k)]dt.$$

A stable coalition structure is found by looking at different possible coalitions and seeing if some alternative coalition can form which can guarantee its members higher utility. This process involves looking at series of inequalities between equations (41) - (44). Since these equations are functions of $q_i y_i$, a particular coalition will be stable for some (possibly empty) subset of values of $q_i y_i$, i = 1,2,3. Equations (41) - (44) are linear in $q_i y_i$, so we may normalize such that $\sum_{i=1}^{3} q_i y_i = 1$; a particular coalition may then be stable for some subset of values of $q_i y_i$

lying on the unit simplex. These various sets are defined precisely and for general parameter values in appendix 2. In the text, the sets are illustrated diagrammatically, and for the particular parameter values of $\tau = 2$ and $\phi = 2$. $\tau = 2$ implies that taxes and tariffs are such that consumer prices exceed marginal cost, although the magnitude of this difference depends on the parameters of the expenditure function (see equation (24)). $\phi = 2$ implies that costs and preferences are such that in each country, the sales of a single domestic firm are twice as large as the sales of a single foreign firm. If all countries were the same size and had the same number of firms half the market for differentiated products in each country would therefore be supplied by domestic firms, and the other half split equally between imports from the other two countries.

We may now consider the grand coalition, i.e., the customs union of all countries. This is stable unless it can be improved upon either by some single country being able to guarantee doing better outside the union, or by some coalition of two countries being able to do better outside the grand coalition. The sets of values of $q_i y_i$ for which these improvements occur are defined as B1 and B2 respectively in appendix 2. For $\tau = 2$ and $\phi = 2$ it can be established that a coalition of the two smallest economies can always improve upon the grand coalition. For these parameter values there is therefore no set of values of $q_i y_i$, $i = 1,2,3$ for which the grand coalition is stable.

A coalition of two countries may be improved upon in three possible ways. By either one of its members being able to guarantee itself higher welfare outside a customs union; by one of its members and the third country obtaining higher welfare by forming a coalition; or by all three countries achieving higher welfare in the grand coalition. The sets of values of $q_i y_i$ within which the coalition of countries 1 and 2 may be improved upon by these possibilities are sets B3 -B5 in appendix 2. The complement of the union of these sets contains values $q_i y_i$ for which the coalition of 1 and 2 is stable. For $\tau = 2$ and $\phi = 2$ this region is illustrated by the shaded area in figure 1.

Several remarks may be made about figure 1. First, regions within which the coalitions {2,3} and {1,3} are stable are apparent by the rotational symmetry of the diagram. For any configuration of country sizes there is therefore a unique two country customs union. Second, the coalition {1,2} is stable for values of $q_1 y_1$ and $q_2 y_2$ which are relatively close together. This is as would be expected from the analysis of the preceding section. If the expenditures were very different the smaller country would be able to do better either by acting independently, or by joining a union with the third country. Third, each of the two country coalitions are stable in two disjoint regions of the simplex. In the upper left section of the cross- hatched region, the coalition {1,2} is stable because $q_1 y_1$ and $q_2 y_2$ are much larger than $q_3 y_3$, so it is not in country 3's interest to join coalitions {1,3} or {2,3}. In the lower right region $q_1 y_1$ and $q_2 y_2$ are relatively small, so the coalitions {1,3} and {2,3} are prevented from forming by the fact that countries 1 or 2 would

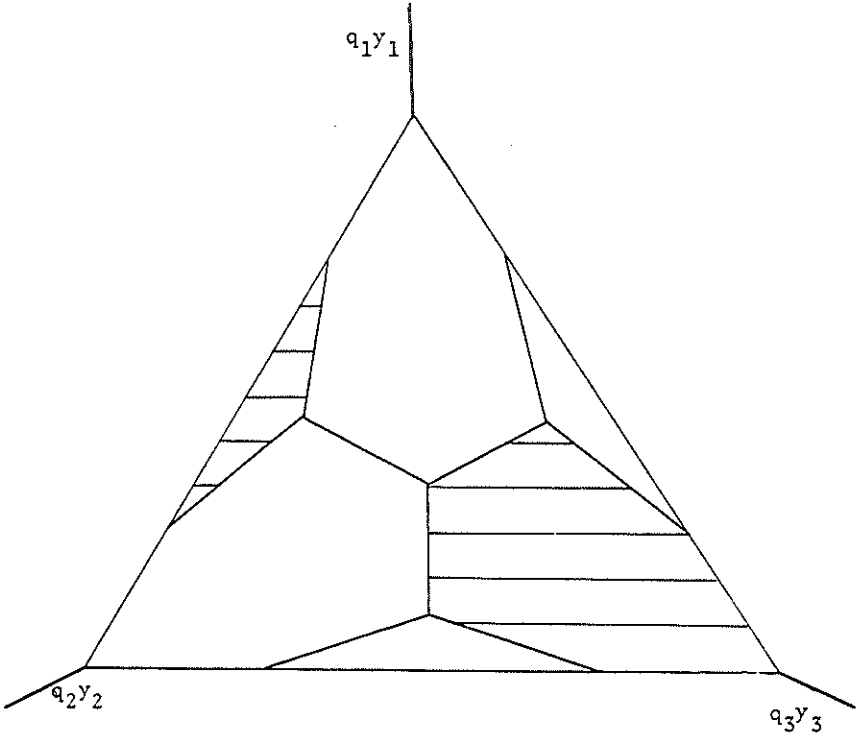

Figure 1

lose from joining a union with the larger country 3.

We may finally consider whether independent play by all three players is stable. Such a situation may be improved upon either by a two country coalition, or by the grand coalition. These improvements occur in the regions of the simplex defined by the sets B6 and B7 of appendix 2. A situation with no customs union is stable in the complement of the union of these sets; such regions are illustrated for $\tau = 2$, $\phi = 2$, in the cross-hatched regions of figure 2. As would be expected, these are regions where no two countries are of approximately equal size. A coalition is therefore prevented from forming by the fact that one of the countries concerned is necessarily much smaller than the other, so will lose from the union. In these regions the stable coalition structure is of course not unique; both the situation with no customs union, and a two country customs union are stable solutions to the customs union game.

As parameters τ and ϕ are varied the configurations of regions of figures 1 and 2 change. However, two general points seem to emerge. The first is that coalitions form only between countries with similar expenditures on differentiated products. The second is that, given the absence of side- payments, for some configurations of country size a situation in which countries all act independently is stable.

9. Conclusions. Three main conclusions may be drawn from the preceding analysis. The first is that a single country acting in isolation may increase its welfare by active tax and tariff policy. Each country is a price taker, but not a price index

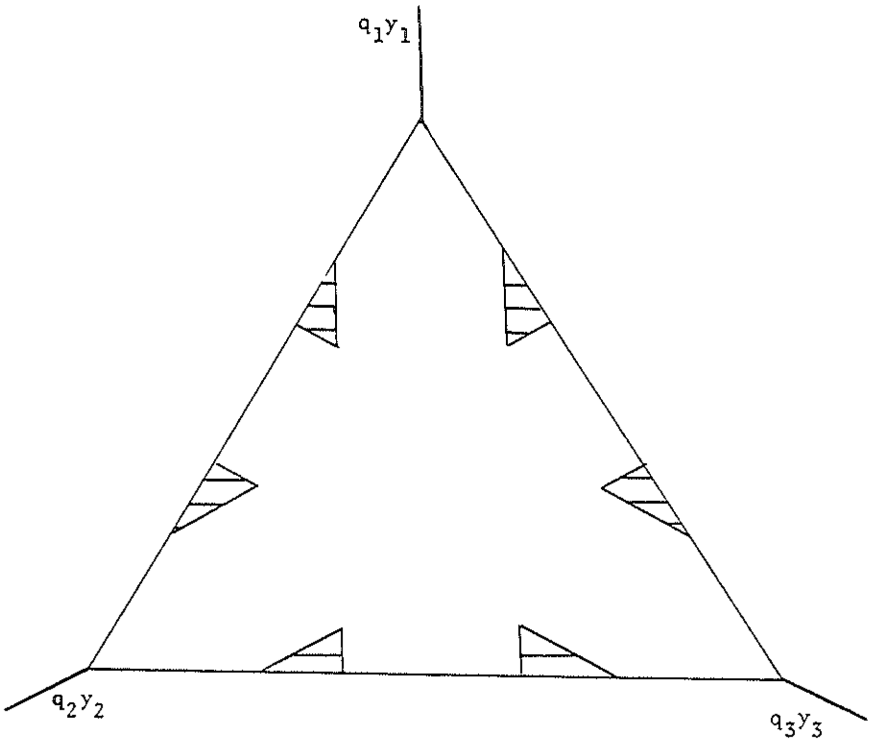

Figure 2

taker, as policy can influence the number of active firms producing in each economy. This generates an argument for the use of import tariffs and export subsidies.

Second, if all countries are symmetric then customs union formation raises the welfare of member countries and reduces the welfare of countries outside the union. Customs union enlargement raises welfare in the country joining the union, but may reduce welfare in existing member countries.

Third, if countries are not symmetric then reciprocated tariff reductions, either in a customs union or worldwide, may not benefit all countries. In particular, countries with relatively small monopolistically competitive industries are likely to suffer a welfare reduction from reciprocated trade liberalisation. This argument suggests that customs unions are likely to form between countries which are, in an appropriate sense, relatively similar.

Appendix 1

Totally differentiating equation (8)

(A1) $$dp_{ij}/p_{ij} = dt_{ij}/(1-t_{ij})$$

Differentiating equations (9) and using (A1), representative elements of dA and dB are,

(A2) $$dA = \{(1-\varepsilon_j)(1-t_{ij})^{-1}(p_{ij}/\alpha_{ij})^{1-\varepsilon_j}dt_{ij}\}$$

(A3) $$dB = \{-\varepsilon_j(p_{ij}/\alpha_{ij})^{1-\varepsilon_j}dt_{ij}\}$$

With assumption 1, (A2) and (A3) give,

(A4) $$dA = dB(I-\hat{t})^{-1}(I-\hat{\varepsilon}^{-1}).$$

The derivative of $B^T(A^T)^{-1}$ is,

$$d[B^T(A^T)^{-1}] = dB^T(A^T)^{-1} - B^T(A^T)^{-1}dA^T(A^T)^{-1}.$$

Assumption 1 states that $(A^T)^{-1} = (B^T)^{-1}(I-\hat{t})$, and using (A4),

$$d[B^T(A^T)^{-1}] = \hat{\varepsilon}^{-1}dB^T(B^T)^{-1}(I-\hat{t}).$$

If $\tilde{A}$ is the matrix A with elements deleted in all positions where the tax rate has not changed, then from equations (9) of the text and A2 above,

$$dA = \tilde{A}(I-\hat{\varepsilon})(I-\hat{t})^{-1}dt.$$

Using A4,

$$dB = -\tilde{A}\hat{\varepsilon}dt,$$

which is equation (27) of the text.

Appendix 2

(i) The grand coalition {1,2,3} may be improved upon by a single player in the set of values of qy defined by B1;

$$B1 = \bigcup_{i=1}^{3} \{qy \mid du_i\{j,k\} > du_i\{1,2,3\},\ i \neq j \neq k\}.$$

(ii) The grand coalition may be impoved upon by a two country coalition in B2;

$$B2 = \bigcup_{\substack{i,j=1 \\ i \neq j}}^{3} [\{qy \mid du_i\{i,j\} > du_j\{1,2,3\}\} \cap \{qy \mid du_j\{i,j\} > du_j\{1,2,3\}\}]$$

(iii) The grand coalition is stable in the complement, $\{B1 \cup B2\}^c$.

(iv) The two country coalition {1,2} may be improved upon by a single country in B3;

$$B3 = \bigcup_{i=1,2} \{qy \mid du_i\{j,3\} > du_i\{1,2\},\ i \neq j\}$$

(v) The two country coalition {1,2} may be improved upon by a coalition of two players in B4;

$$B4 = \bigcup_{i=1,2} [\{qy \mid du_i\{i,3\} > du_i\{1,2\}\} \cap \{qy \mid du_3\{i,3\} > du_3\{i,3\}\}]$$

(vi) The two country coalition {1,2} may be improved upon by the grand coalition in B5;

$$B5 = \bigcap_{i=1}^{3} \{qy \mid du_i\{1,2,3\} > du_i\{1,2\}\}.$$

(vii) The coalition is stable in $\{B3 \cup B4 \cup B5\}^c$.

(viii) A single player may be improved upon by a two country coalition in B6;

$$B6 = \bigcup_{\substack{i,j=1 \\ i \neq j}} [\{qy \mid du_i\{i,j\} > du_i\{i\}\} \cap \{qy \mid du_j\{i,j\} > du_j\{i\}\}].$$

(ix) A single player may be improved upon by the grand coalition in B7.

$$B7 = \bigcap_{i=1}^{3} \{qy \mid du_i\{1,2,3\} > du_i\{i\}\}.$$

(x) A situation with single players is stable in $\{B6 \cup B7\}^c$.

Footnotes.

1. For analysis of equilibrium in models of this type, see Krugman (1979, 1980, 1981).

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