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Patrick Schmitz

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Patrick Schmitz, University of Bonn and CEPR

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Centre for Economic Policy Research
90–98 Goswell Rd, London EC1V 7RR, UK
Tel: (44 20) 7878 2900, Fax: (44 20) 7878 2999
Email: cepr@cepr.org, Website: www.cepr.org

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ABSTRACT

On Second Price Auctions and Imperfect Competition*

Consider two sellers, each of whom has one unit of an indivisible good, and two buyers, each of whom is interested in buying one unit. The sellers simultaneously set reserve prices and use second price auctions as rationing device. An equilibrium in pure strategies where each seller has a regular customer is characterized. The result is applied in order to demonstrate that not allowing sellers to use second price auctions may enhance total surplus.

JEL Classification: D43, D44 and D82
Keywords: auctions, duopoly, private information and selling mechanisms

Patrick Schmitz
Wirtschaftspolitische Abteilung
Universität Bonn
Adenauerallee 24-42
D-53113 Bonn
GERMANY
Tel: 737937
Fax: 739221
Email: schmitz@wipol.uni-bonn.de

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1 Introduction

While there exists a vast literature on auctions and optimal selling mechanisms for the case of privately informed buyers,[1] most papers in this literature are focused on a monopolistic seller. In contrast, in this note I consider two capacity constrained sellers who compete by setting prices and who may use second price auctions as rationing devices if more buyers show up than there are goods. I analyze this situation in the simplest possible model where the buyers's valuations are independently distributed.

The virtues of a second price auction in the monopolistic framework are by now well understood. In particular, in accordance with the textbook analysis of monopolistic price discrimination, the expected total welfare would clearly be reduced if a monopolist were not allowed to use a second price auction and could instead only set a fixed price (and select the winning buyers randomly if there are more buyers than goods).

At first glance, one might guess that allowing the use of second price auctions as rationing device must also be welfare enhancing in the case of competing sellers. After all, with an auction a seller can extract more additional surplus when he attracts more buyers by decreasing the price, so that we might expect lower prices and thus more trade. However, the willingness of a buyer to go to a cheaper seller can be smaller if the buyer knows that the seller will extract more of the surplus by using an auction. Thus, a price reduction can be less profitable, so that in fact higher prices could be sustainable if sellers are allowed to use second price auctions. As a consequence, second price auctions might be anti-competitive, so that total welfare could be increased if sellers were not allowed to price discriminate.

I am aware of only a few other papers that analyze auctions in a competitive framework, in particular McAfee (1993), Peters and Severinov (1997),

[1]For surveys, see e.g. McAfee and McMillan (1987), Matthews (1995), or the recent book by Krishna (2002).

and Burguet and Sákovics (1999).[2] These papers are mainly focused on the case of large markets with many buyers and/or sellers, where buyers randomize among sellers. In contrast, I analyze a duopolistic situation and focus on pure strategies. This note is also related to Wang (1993), who compares auctions and posted prices in the monopolistic framework (his results are driven by the assumption that auctioning is costly, which is not made here).

2 The model

There are two risk-neutral sellers, A and B, each of whom possesses one unit of an indivisible good. For simplicity, the sellers' valuations are assumed to be zero. Moreover, there are two potential buyers, $i \in \{1, 2\}$. Buyer i's willingness-to-pay for one unit of the good is denoted by $v_i \in [0, 1]$. The buyers' valuations are private information; yet, it is commonly known that they are independently and symmetrically distributed according to the distribution function F. The density function is denoted by f. For simplicity, let us assume that F is sufficiently well behaved, such that the first-order approach will be applicable.[3] In particular, the well-known monotone hazard rate property $\frac{d}{dv}\frac{1-F(v)}{f(v)} < 0$ is supposed to hold; i.e., we are in Myerson's (1981) 'regular case'. Following the mechanism design approach, it is assumed that the sellers can make take-it-or-leave it offers to the buyers. Specifically, a seller can commit not to sell her good if no buyer is willing to pay the price that she has posted.

Consider the following game. At date 1, the sellers A and B simultaneously announce reserve prices p_A and p_B, respectively. At date 2, the buyers

[2] See also the related work on common agency and the revelation principle [c.f. Page and Monteiro (2001), Martimort and Stole (2002)] and on decentralized trading mechanisms [c.f. Coles (1998), De Fraja and Sákovics (2001), and the literature discussed there].

[3] Specifically, it is assumed that the maximizer of expression (2) below is characterized by the first-order condition. It can be checked that this is e.g. satisfied in the case of the uniform distribution.

simultaneously decide whether to go to seller A, to seller B, or to no seller at all. If only one buyer shows up at seller j's store, he receives the good and pays the price p_j, $j \in \{A, B\}$. If two buyers show up, seller j determines the winner of the good by a second-price auction with p_j as minimum bid.

In a second-price auction (which is also often referred to as Vickrey auction), the buyer submitting the highest bid wins and pays a price equal to the second-highest bid.[4] As is well known, it is a weakly dominant strategy to bid one's true valuation in such an auction.[5] Hence, the following analysis will be simplified by considering a reduced-form game in which the buyer with the higher valuation gets the good and pays a price equal to the lower valuation whenever two buyers go to one seller.

In the remaining game to be analyzed, a strategy of seller j is simply a price announcement $p_j \in [0, 1]$. A strategy of buyer i is a decision of whether to go to seller A, to seller B, or to no seller, depending upon buyer i's type. Formally, buyer i's strategy can thus be described by two disjoint subsets of the unit interval, Θ_i^A and Θ_i^B. If $v_i \in \Theta_i^j$, then buyer i goes to seller $j \in \{A, B\}$. In what follows, only pure strategies are considered.

In a perfect Bayesian equilibrium each buyer makes his date-2 decision in order to maximize his expected payoff given his own type, taking the other buyer's type-contingent strategy as given, for any pair of prices that the sellers might have set. In the first stage, each seller chooses a price in order to maximize her expected payoff, taking the buyers' strategies as given.[6]

[4] Note that the commonly used English auction is equivalent to the second price auction in the private values framework studied here. See e.g. Krishna (2002) for a recent exposition.

[5] In order to see this intuitively, note that my bid has no impact on the price I pay given that I win. Thus, bidding more than my true valuation could only change my payoff if this made me the winner, which would mean that I had to pay more than my valuation. Similarly, bidding less than my true valuation could only imply that I do not win even though I would have made a positive profit. See e.g. Matthews (1995) for a detailed formal proof.

[6] Note that the (uninformed) sellers move first and then the buyers move simultaneously;

In order to analyze the second stage, let the announced prices p_A and p_B be given. Assume that if $p_A = p_B$, then buyer 1 always goes to seller A, and buyer 2 always goes to seller B, provided that the respective buyer's valuation exceeds the price (formally, $\Theta_1^A = [p_A, 1]$, $\Theta_1^B = \emptyset$, $\Theta_2^A = \emptyset, \Theta_2^B = [p_B, 1]$). In order to justify the selection of an equilibrium out of the infinitely many possibilities, one might assume that buyer 1 lives closer to seller A's store and incurs costs ε when he goes to seller B (with $\varepsilon \to 0$). In this case, it is a focal point for the buyers to coordinate on the equilibrium in which buyer 1 is seller A's regular customer and buyer 2 is seller B's regular customer.

Consider now the case $p_A < p_B$. If buyer 1 still never goes to seller B, buyer 2 will now only go to seller B if his valuation is sufficiently large. It is obvious that buyer 2 will go to A if $v_2 \in [p_A, p_B]$. But even if v_2 is slightly larger than p_B, buyer 2 may prefer to go to A, because the smaller probability of getting a good there can be overcompensated by the potentially lower price. It is demonstrated in the following result that there exists a cut-off level $x \in [p_B, 1]$ such that buyer 2 goes to B whenever $v_2 \geq x$.

Lemma 1 *If $p_A \leq p_B$, there is a second stage Bayesian Nash equilibrium characterized by $\Theta_1^A = [p_A, 1]$, $\Theta_1^B = \emptyset$, $\Theta_2^A = [p_A, x], \Theta_2^B = [x, 1]$, where x is implicitly defined by*

$$G(x) := p_B - x + \int_{p_A}^{x} F(v)dv = 0$$

if $G(1) \leq 0$, and $x = 1$ otherwise. Analogously, if $p_A > p_B$, an equilibrium is characterized by $\Theta_1^A = [y, 1]$, $\Theta_1^B = [p_B, y]$, $\Theta_2^A = \emptyset$, $\Theta_2^B = [p_B, 1]$, with

$$H(y) := p_A - y + \int_{p_B}^{y} F(v)dv = 0$$

if $H(1) \leq 0$, and $y = 1$ otherwise.

Proof. Note that buyer i will never go to seller j if $v_i < p_j$. Consider the case $p_A \leq p_B$. Assume that buyer 1 follows the strategy characterized in

i.e., even though this is a dynamic game with incomplete information, there is no need to worry about making inferences from observed actions of informed players.

the lemma. If buyer 2 goes to B, his payoff is $v_2 - p_B$. If he goes to A, he receives the good for the price p_A if buyer 1's valuation is smaller than p_A, and otherwise (since truthful bidding is a weakly dominant strategy in a second price auction) for the price v_1 if buyer 1's valuation is smaller than v_2, so that his expected profits are

$$(v_2 - p_A)F(p_A) + \int_{p_A}^{v_2} (v_2 - v_1)dF(v_1) = \int_{p_A}^{v_2} F(v)dv.$$

Thus, buyer 2 goes to seller A if and only if

$$G(v_2) = \int_{p_A}^{v_2} F(v)dv - v_2 + p_B > 0.$$

Notice that $\frac{dG(v_2)}{dv_2} = F(v_2) - 1$, so that $G(v_2)$ is strictly decreasing for all $v_2 < 1$. Moreover, $G(p_B) \geq 0$. Thus, if $G(1) \leq 0$, there exists a unique cut-off level $x \in [p_B, 1]$ such that $G(x) = 0$. Otherwise, buyer 2 never goes to seller B.

Assume now that buyer 2 follows the strategy given by the lemma. If buyer 1 goes to seller A, his expected payoff is

$$(v_1 - p_A)(F(p_A) + 1 - F(x)) + \int_{p_A}^{\min\{v_1, x\}} (v_1 - v_2)dF(v_2),$$

since buyer 2 only goes to A if $v_2 \in [p_A, x]$, in which case buyer 1 wins if $v_2 < v_1$. Consider first the case $v_1 \leq x$. Buyer 1 prefers to go to A whenever

$$(v_1 - p_A)(1 - F(x)) + \int_{p_A}^{v_1} F(v)\ dv \geq (v_1 - p_B) F(x).$$

It is sufficient to show that $\int_{p_A}^{v_1} F(v)\ dv \geq v_1 - p_B$ holds, which is the case since $v_1 < x$ implies $G(v_1) \geq 0$. Consider next the case $v_1 \geq x$. If buyer 1 goes to seller B, his expected profits are

$$(v_1 - p_B) F(x) + \int_{x}^{v_1} (v_1 - v_2)dF(v_2).$$

Thus, buyer 1 goes to A if

$$v_1 - xF(x) - p_A(1 - F(x)) + \int_{p_A}^{x} F(v)\ dv - (x - p_B) F(x) - \int_{x}^{v_1} F(v)\ dv \geq 0.$$

Notice that the derivative of the LHS with respect to v_1 is given by $1-F(v_1) \geq 0$. Thus, it is sufficient to check that the condition holds for $v_1 = x$, which has already been shown. ■

We are now in the position to analyze the first stage. Given the symmetry of the game, it is natural to look for an equilibrium in which the sellers set equal prices, so that according to the regular customer behavior outlined above, no rationing will actually occur.

Proposition 1 *Anticipating the buyers' behavior characterized in Lemma 1, seller A and seller B maximize their expected profits by setting $p_A = p$ and $p_B = p$, respectively, where p is given by*

$$\frac{1}{1-F(p)}p = \frac{1-F(p)}{f(p)}. \quad (1)$$

Proof. First note that the monotone hazard rate implies that the price $p \in (0,1)$ implicitly defined in (1) is unique.[7] Now take p_B as given. If seller A chooses $p_A \geq p_B$, her expected profits are $p_A[1-F(y)]$, which is concave in p_A due to the monotone hazard rate. Given p_B, the first order condition for a maximum of $p_A[1-F(y)]$ is $1-F(y)-p_A f(y)\frac{dy}{dp_A} = 0$. Implicit differentiation of the definition of y shows that $\frac{dy}{dp_A} = \frac{1}{1-F(y)}$ for $y < 1$. Thus, if $p_A = p_B = p$, upward deviations cannot be profitable.

If seller A chooses $p_A \leq p_B$, her expected payoff is

$$\begin{aligned} &p_A\left[(F(p_A)+1-F(x))\,(1-F(p_A)) + (F(x)-F(p_A))\,F(p_A)\right] \\ &+ \int_{p_A}^{x}\int_{v_1}^{x} v_1 dF(v_2)dF(v_1) + \int_{p_A}^{x}\int_{p_A}^{v_1} v_2 dF(v_2)dF(v_1) \\ &+ \int_{x}^{1}\int_{p_A}^{x} v_2 dF(v_2)dF(v_1). \end{aligned}$$

This expression immediately follows from inspection of Figure 1, which displays seller A's profit for every state of the world $(v_1, v_2) \in [0,1]^2$, and can

[7] When p moves from 0 to 1, then the left hand side of (1) monotonically increases from 0 to ∞, while the right hand side monotonically decreases from $1/f(0)$ to 0.

be simplified to

$$p_A\,(1 - F\,(x) + 2F(p_A)(F(x) - F(p_A))$$
$$+ \int_{p_A}^{x} (1 + F\,(x) - 2F(v)\,)vdF(v). \quad (2)$$

The first order condition of this expression, using $\frac{dx}{dp_A} = \frac{-F(p_A)}{1-F(x)}$, and setting $p_A = p_B$, yields again (1). Hence, if $p_A = p_B = p$, there are no profitable downward deviations, which completes the proof. ■

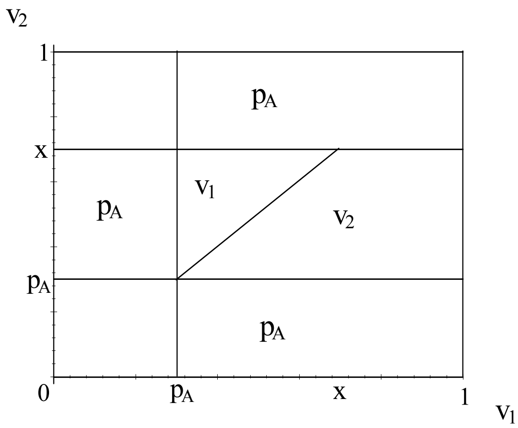

Figure 1. Seller A's profit if $p_A < p_B$.

Note that in the equilibrium derived in Proposition 1, the price is smaller than the price that a monopolist with one potential buyer would set (she would maximize $p(1 - F(p))$ and thus set $p = \frac{1-F(p)}{f(p)}$). Accordingly, competition reduces the expected profit, as one would expect.[8] Note also that the fact that reserve prices are not driven to zero is not really surprising, because the sellers are capacity-constrained.[9]

[8]Bulow and Klemperer (1996) have shown that the expected profit that a monopolist facing one buyer would make is in turn smaller than the expected profit of a monopolist with two potential buyers who conducts a second price auction without reserve price. This latter profit equals the one a seller would expect in the present model under symmetric information, namely the expected value of the lower valuation. Hence, asymmetric information reduces the expected profits of the sellers in the analysis of the present model.

[9]Cf. the traditional IO literature on duopolistic price competition under capacity constraints, e.g. Kreps and Scheinkman (1983).

3 An application: Second price auctions can be anti-competitive

An interesting application of the main result can now be demonstrated. Recall that in the textbook model of a monopoly, the possibility of the monopolist to price discriminate may reduce the consumers' surplus, but it increases total welfare. A similar conclusion is also true in the case of a monopolistic seller who faces privately informed buyers and who can use a second price auction.[10] As a simple illustration, consider a seller with zero costs who has one unit of a good and two potential buyers, whose privately known valuations are independently drawn from the uniform distribution on the unit interval. It is well known that the optimal reserve price in a second price auction is given by $p = \frac{1-F(p)}{f(p)}$; i.e., the monopolist sets $p = \frac{1}{2}$.[11] If the seller is only allowed to post a price, and select the winning buyer randomly (say, with equal probabilities) if two buyers show up, then she maximizes $p(1 - F(p)^2)$, so that $\frac{2F(p)}{1+F(p)}p = \frac{1-F(p)}{f(p)}$; i.e., the price is $p = \sqrt{\frac{1}{3}} \approx 0.577$. Hence, more gains from trade are realized if the seller is allowed to use a second price auction as rationing device.

However, as has been argued in the introduction, not allowing sellers to price discriminate by using second price auctions can be welfare enhancing if there are competing sellers. From Proposition 1, the price in the case of competition with second price auctions is given by $p = \frac{3}{2} - \frac{1}{2}\sqrt{5} \approx 0.38$. If the sellers are not allowed to price discriminate and have to choose a buyer randomly if two buyers show up, it is shown in the appendix that one can find symmetric equilibria in analogy to Lemma 1 and Proposition 1, where the sellers set $p_A = p_B = p$ with $p \in [2 - \sqrt{3}, \frac{1}{3}]$. Thus, if the sellers are allowed to price-discriminate, then they may use second price auctions in

[10] For more on the interesting analogies between auctions and standard monopoly models, see Bulow and Klemperer (1996).

[11] See e.g. the seminal paper by Myerson (1981).

order to sustain higher prices.[12] Due to this anti-competitive effect, it may be welfare-enhancing if competing sellers are not allowed to use second-price auctions as rationing devices. This is in stark contrast to the conclusions drawn from the monopolistic model.

4 Conclusion

In this note a pure strategy equilibrium of the simplest possible model depicting two price-setting sellers who are capacity constrained and use second price auctions as rationing devices has been characterized. This characterization has been applied in order to demonstrate that despite the well-known virtues that second price auctions have in the usually analyzed monopolistic setting, they may in fact be anti-competitive and more gains from trade may be realized if the sellers are only allowed to post prices.

There are straightforward variants of the model that might be interesting to analyze in future research. For example, one might consider situations in which buyers can simultaneously participate in both auctions. One could also analyze a prior stage in which the sellers choose their capacities. Of course, if the number of potential buyers exceeds the number of available goods (so that rationing does occur in equilibrium), the potentially anti-competitive effect of second price auctions may be overcompensated again by the usual efficiency gains due to the fact that the goods are awarded to the buyers with the largest valuations. Since we are still just beginning to understand how auctions work in competitive environments, these seem to be difficult but interesting avenues for future research.

[12] Notice that in equilibrium no rationing occurs, so a higher price unambiguously means that less surplus is realized.

Appendix

Consider the case in which the sellers can only post prices and serve each buyer with probability $\frac{1}{2}$ if two buyers show up. In analogy to Lemma 1, the buyers' equilibrium strategies are such that, in the case $p_A \leq p_B$, buyer 1 never goes to seller B, and buyer 2 goes to B whenever $v_2 \geq \tilde{x} := \min\left\{\frac{2p_B - p_A(1+p_A)}{1-p_A}, 1\right\}$. Buyer 2 does not deviate since

$$\begin{aligned} v_2 - p_B &> (v_2 - p_A)\left(\Pr\{v_1 < p_A\} + \frac{1}{2}\Pr\{v_1 > p_A\}\right) \\ &= \frac{1}{2}(v_2 - p_A)(1 + p_A) \end{aligned}$$

if $v_2 \geq \tilde{x}$. Buyer 1 does not deviate, because one can show in a straightforward way that

$$\begin{aligned} &(v_1 - p_A)\left(\Pr\{v_2 \notin [p_A, \tilde{x}]\} + \frac{1}{2}\Pr\{v_2 \in [p_A, \tilde{x}]\}\right) \\ = \; &(v_1 - p_A)\left(1 - \frac{1}{2}(\tilde{x} - p_A)\right) \end{aligned}$$

is always larger than

$$(v_1 - p_B)\left(\Pr\{v_2 < \tilde{x}\} + \frac{1}{2}\Pr\{v_2 > \tilde{x}\}\right) = \frac{1}{2}(v_1 - p_B)(1 + \tilde{x})$$

for $v_1 \geq p_B$. The case $p_A > p_B$ can be handled analogously; the relevant cut-off level is then given by $\tilde{y} := \min\left\{\frac{2p_A - p_B(1+p_B)}{1-p_B}, 1\right\}$.

Taking the buyers' behavior as given, seller A's profit if $p_A \geq p_B$ is given by $p_A(1 - \tilde{y})$, which is concave in p_A. The first order condition for the maximization of this expression with regard to p_A is $4p_A - p_B^2 - 1 = 0$. Thus, seller A would want to set $p_A > p_B$ whenever $p_B < 2 - \sqrt{3} \approx 0.267$. Seller A's profit if $p_A \leq p_B$ is given by $p_A(1 - p_A(p_A + 1 - \tilde{x}))$, which is also concave, and the first order condition is $2p_A^3 - (1 + 2p_B)\,p_A^2 - (4 - 4p_B)\,p_A + 1 = 0$. Hence, seller A would want to set $p_A < p_B$ whenever $p_B > \frac{1}{3}$.[13] Taking these results together, it follows that the sellers' symmetric equilibrium strategies are characterized by $p_A = p_B = p$ with $p \in [2 - \sqrt{3}, \frac{1}{3}]$.

[13] This can easily be seen by drawing the reaction curve and the 45 degree line in a diagram with p_A on the x-axis.

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