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ABSTRACT

Monopoly Pricing of ‘Cyclical’ Goods*

Consumption of certain commodities produces transitory saturation, in the sense that potential instantaneous utility for an additional unit is very low immediately after a consumption episode, but increases over time. Such cyclical patterns of preferences have important implications for monopoly pricing: (i) In the absence of commitment, prices may be close to marginal cost. (ii) Prices may be non-monotonic with respect to the degree of commitment, reaching a maximum for intermediate degrees of commitment. (iii) Introduction of loyalty-rewarding schemes may benefit both buyers and sellers. (iv) Restrictions on the timing of purchases (purchase deadlines, sales, contracting both price and frequency) are likely to hurt consumers and increase efficiency.

JEL Classification: D42 and L14
Keywords: cyclical preferences, loyalty-rewarding schemes, monopoly pricing and repeat purchases

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1 Introduction

Repeat purchase patterns vary substantially across different classes of commodities. Some goods are consumed on a regular basis. For instance, individuals consume housing, transportation services, and certain types of food and clothing almost every day. In contrast, some goods are consumed less frequently or only occasionally: trips to exotic destinations, visits to amusement parks, a haircut, a rock concert, etc. Many goods that are consumed from time to time (in particular, most leisure goods and personal services) seem to share the same characteristic: Preferences for these goods follow a cyclical pattern. The day after visiting an amusement park, the utility derived from another visit is likely to be very low, but tends to increase over time. Most people also experience a similar change in preferences after dining at an ethnic restaurant or attending a concert by their favorite pianist. In other words, consumption of some commodities produces transitory saturation, at least for some consumers. One may even argue that some kind of saturation is associated with the consumption of most types of goods, at least at some level of disaggregation and on different time scales.

In this paper I propose a very simple characterization of such phenomena. I will refer to a 'cyclical' good as a non-durable good (purchase and consumption are simultaneous) for which consumers have cyclical preferences, and a new cycle starts every time individuals consume the good.[1] More specifically, consumers' potential instantaneous utility, R, measured in monetary units, increases monotonically with the time elapsed since the last consumption point, s, and falls discontinuously when consumption takes place. Thus, if a consumer pays a price p after s units of time since the last purchase, then she obtains an instantaneous net surplus of $R(s)-p$. Such a representation does not take account of two potentially important issues. First, various types of random shocks may play a role in many real world examples. In this paper I only consider a very simple specification of shocks. Second, while preferences for some 'cyclical' goods may exhibit long-run decreasing marginal utility, in the sense that the function $R(s)$ may shift downwards after every purchase, I assume stationary preferences throughout the paper. Thus, the current approach should be interpreted as a first step in modelling cyclical preferences.

[1] In the case of 'seasonal goods', the time pattern of preferences is exogenously given, and thus unrelated to the history of purchases.

The main goal of this paper is to study the monopoly pricing of 'cyclical' goods.[2] A crucial implication of the cyclical pattern of preferences is that expectations of future prices influence current demand. If the seller sets the price and buyers freely choose the timing of purchases, then the optimal timing balances the benefits of waiting: the increase in the instantaneous utility, and the costs of waiting: delaying the realization of the net surplus associated with the next purchase and with all other future purchases. If buyers expect higher future prices, then the costs of waiting are reduced and hence the next purchase is delayed. In general, higher prices are associated with a lower frequency of purchases. In static models we typically assume decreasing marginal utility of consumption and hence the monopolist faces a similar trade-off: a higher price implies lower demand. The current model emphasizes the time dimension and hence is concerned about the time consistency of pricing policies and the monopolist's commitment capacity. Similarly, in static models we analyze the optimality of non-linear pricing policies, while in this context we can study the role of intertemporal pricing schemes (such as loyalty-rewarding programs, sales, purchase deadlines, etc.).

The time dimension is also crucial in other contexts. In the absence of commitment capacity, a durable goods monopolist may end up setting prices very close to marginal cost (Coase conjecture).[3] Under asymmetric information and consumer learning, prices also follow various dynamic patterns.[4] In the current model the good is perishable and its quality is common knowledge; the time dimension matters only because the preferences of individuals vary systematically over time according to the endogenous timing of consumption. Analysis of this issue leads to new insights concerning the performance of a large set of markets.

I first analyze the case in which the seller can only post a single price and is always ready to transact at that price. It turns out that the seller's com-

[2]Other market structures are analyzed in a companion paper.

[3]Bulow (1982) formalized these ideas in a finite horizon model, and Stockey (1981) and Gul et al.(1986) in an infinite-horizon model.

[4]If consumers repeat purchases, monopolists set an introductory price to induce experimentation, and later increase the price as consumers become better informed (Milgrom and Roberts, 1986). When consumers purchase the good only occasionally, if the price is a positive signal of quality then a decreasing sequence of prices may be obtained in equilibrium (Bagwell and Riordan, 1991), although if consumers learn the quality of the good from market shares then the price sequence will, on average, increase (Caminal and Vives, 1996).

mitment capacity to keep the price unchanged is crucial. In one extreme, when the price can be changed almost immediately, then the Markov equilibrium price is arbitrarily close to marginal cost.[5] This result is analogous to the Coase conjecture, although the intuition is somewhat different. In the durable goods case, if the monopolist announces a constant price sufficiently above marginal cost and consumers believe it, then in the following period consumers with a high reservation price are out of the market and the monopolists finds it optimal to set a lower price. In the cyclical goods case, if the monopolist announces a constant price well above marginal cost and consumers believe it, then the seller has incentives to bring the purchase forward by lowering the price. Thus, in both cases monopolists have incentives to undercut expected prices, although the reasons are different.

The low level of profits associated with price flexibility may induce the seller to invest in various commitment devices.Two alternative degrees of price rigidity are considered. In the most extreme case the seller can commit to a constant price forever ('long-run commitment'). In the intermediate case the seller commits to maintaining the price unchanged until consumers make the next purchase but is free to change it right after ('medium-run commitment'). Other specifications of medium-run price rigidity are also discussed. It turns out that the Markov equilibrium price in the medium-run commitment game is higher than in the long-run commitment case, which in turn is higher than marginal cost. Thus, Markov equilibrium prices may be non-monotonic with respect to the degree of commitment (price rigidity), with a maximum reached at intermediate degrees. The intuition is the following. Under long-run commitment a lower price increases the net surplus of the next purchase as well as the net surplus of all future purchases. Thus, a price reduction has a large effect on the consumer's costs of waiting and hence on the timing of the next purchase. Alternatively, in the medium-run commitment case, consumers interpret a price below the long-run expected price as a transitory change, as one that increases the net surplus of the next purchase but does not affect the expected net surplus associated with future purchases. As a result the effect on current demand is small. Summarizing, in the medium-run commitment game, price elasticity is lower, and hence the optimal price is higher than in the long-run commitment case.

[5] Unfortunately, a pure strategy Markov equilibrium may or may not exist depending on parameter values.

If we relax the Markovian restriction on strategies and consider reputational effects in the medium-run commitment case, then a full interval of prices (constant over time) can be supported as subgame perfect equilibria. The existence of multiple equilibria is very common in dynamic games.[6] What is remarkable in the cyclical goods case is not the multiplicity of equilibria *per se*, but the fact that some of these equilibria can be Pareto ranked. In particular, there are equilibria for which both the seller and the buyer would prefer to switch to a different equilibrium involving lower prices.

In some circumstances the monopolist may be able to use more sophisticated pricing policies. In particular, it might be feasible to reward consumer loyalty. In my set up this is equivalent to allowing the seller to commit to a sequence of prices, with the nth price corresponding to the nth repeated purchase. In this case, the equilibrium policy includes a high price for the initial purchase and a price equal to marginal cost for the following ones.[7] The intuition goes as follows. The first price of the sequence only has an effect on the timing of the first purchase. However, successive prices affect not only the timing of the corresponding purchase but also the timing of the previous ones. Consider a sequence of prices that involves a positive margin in the nth purchase. The monopolist can make higher profits by raising the first price and lowering the nth price in such a way that the present value of prices (evaluated at the timing of purchases associated with the original price sequence) remains unchanged. The reason is that under the new price sequence, consumers do not alter the timing of the first purchase but they do bring forward the timing of the nth purchase. The introduction of such loyalty rewarding schemes obviously benefits the monopolist, in comparison commitment to a constant price, but its effect on consumer welfare is ambiguous.

This result is analogous to that obtained by Crémer (1984) for the case of experience goods. In a two-period model he showed that the monopolist finds it optimal to commit to a price sequence with similar characteristics: marginal cost pricing in the second period and a price equal to total gains from trade in the first. The crucial difference is that in Crémer's frame-

[6]For instance, in the durable goods case, Ausubel and Deneckere (1989) proved that as the discount rate approaches zero, all seller payoffs between zero and static monopoly profits are supported by subgame perfect equilibria.

[7]Limited commitment power and liquidity constrained consumers tend to smooth out the time profile of the equilibrium price sequence.

work the seller's commitment capacity hurts consumers, whereas in the case of cyclical goods, the first price must be relatively moderate in order to induce consumers to make the first purchase relatively soon. As a result, consumers may actually benefit from the introduction of such loyalty rewarding schemes. [8]

In some (properly defined) markets, products are available only occasionally. For instance, a performance by a particular artist in a specific city. More often, sellers may find the way of restricting the availability of the product, or, more generally, of affecting the timing of purchases. This is the aim of some common marketing techniques, like subscriptions, occasional sales and purchase deadlines. In my set up, the seller always finds it optimal to restrict the timing of purchases. Typically, consumers are worse off under restricted timing but social welfare is higher.[9]

The model is presented in the next section. Section 3 studies the Markov equilibrium of the price posting game under complete flexibility. Sections 4 and 5 are devoted to the cases of long and medium-run commitment. In Section 6 various robustness issues are discussed. Section 7 presents the case of commitment to a sequence of prices and Section 8 examines various policies that restrict the timing of purchases. Some concluding remarks follow.

2 The baseline model

Time is a continuous variable that runs from 0 to infinity. There is a single seller and a single consumer. We can think of a monopolist that sets customer-specific prices, or, alternatively, of an arbitrary number of identical consumers and perfectly synchronized purchases. The monopolist can instantaneously produce a homogeneous perishable good at zero cost. Consumer preferences can be in two different regimes: static and dynamic. Immediately after a purchase, consumer's potential instantaneous utility from an additional purchase is equal to $-L < 0$ and sticks to this

[8] In oligopolistic markets with random consumer preferences, loyalty-rewarding schemes create consumer switching costs. Consumers tend to lose when sellers use coupons to reward loyalty (Banerjee and Summers, 1987), but they may gain if sellers commit to prices for repeat purchases (Caminal and Matutes, 1990).

[9] Hahn (2001) shows that purchase deadlines can be an optimal commitment mechanism in a two-period model in which consumers purchase the good only once but are initially uncertain about their own valuation.

value unless there is regime switching, which occurs with probability $\mu e^{-\mu x}$, where x is the time elapsed since the last purchase and μ is a positive real number. After a regime switch, consumer's potential instantaneous utility evolves deterministically according to $R(s)$, where s is the time elapsed since regime switching, until a new purchase takes place. More specifically, $R(s)$ is a three times continuously differentiable function, from $\Re_{++}$ into $\Re$, satisfying (See Figure 1 for an illustration):

A.1. $R'(s) > 0, R''(s) < 0, R'''(s) > 0$.

A.2. $lim_{s\to 0}R(s) = -L < 0, lim_{s\to\infty}R(s) = M$.

Thus, the length of the time interval between purchases is $x + s$, the first component is a random variable and the second is a choice variable. Notice that the expected value of x is $\frac{1}{\mu}$. In the limit as μ goes to infinity the model becomes deterministic. Both seller and buyer discount the future at the rate $r > 0$. I assume that the realization of x is the consumer's private information while the rest of the model is common knowledge.

For simplicity, let us suppose that at time 0 the consumer has just purchased the good and hence is in the constant regime. If the buyer expects to pay a price p_n in the nth purchase, $n = 1, 2, ...$, and to spend s_n units of time in the dynamic regime between the $(n-1)th$ and the nth purchase, then the buyer's expected payoff at time 0 is given by:

$$U_0 = \sum_{n=1}^{\infty} \lambda^n \left[R\left(s_n\right) - p_n\right] e^{-r\sum_{j=1}^{n} s_j} \tag{1}$$

where $\lambda \equiv \frac{\mu}{r+\mu}$.

Similarly, the seller's payoff is given by:

$$\Pi_0 = \sum_{n=1}^{\infty} \lambda^n p_n e^{-r\sum_{j=1}^{n} s_j} \tag{2}$$

Finally, social welfare is the sum of the consumer's utility and the firm's profits:

$$W_0 = \sum_{n=1}^{\infty} \lambda^n R\left(s_n\right) e^{-r\sum_{j=1}^{n} s_j} \tag{3}$$

Let us turn to the characterization of the efficient allocation. The only variables that affect total surplus are the length of the time intervals between purchases. Thus, the efficient outcome is a sequence of time intervals with length $\{s_n\}_{n=1}^{\infty}$ that maximize 3. Analogously, we can set up the optimization problem as finding the optimal timing of the next purchase, s_1, that maximizes:

$$W_0 = \lambda e^{-rs_1}[R(s_1) + W_1^*]$$

where W_1^* is the maximum surplus that can be obtained after the first purchase (which is independent of s_1).

The solution is given by the first order condition:

$$R'(s_1) - r[R(s_1) + W_1^*] = 0$$

Thus, optimal timing is obtained by balancing the gains from waiting, i.e., the increase in instantaneous utility, and the costs of waiting, i.e., the interest on the capitalized gains from trade. The latter is the sum of the instantaneous utility plus the net present value of future gains from trade.

Since the optimization problem is stationary, the optimal time intervals are constant and the maximum surplus after a purchase is given by:

$$W^* = \frac{\lambda e^{-rs^o}}{1 - \lambda e^{-rs^o}} R(s^o)$$

where s^o is given by:

$$R'(s^o) - \frac{rR(s^o)}{1 - \lambda e^{-rs^o}} = 0. \tag{4}$$

Notice that s^o increases with r and is invariant to multiplicative transformations of $R(s)$.

3 Price flexibility

In this section I consider the discrete time version of the model. Trade can only take place at time $t = 0,\ \Delta, 2\Delta, ...$ The stochastic structure of preferences is adjusted as follows. If the consumer was in the static regime in the previous period (or she has purchased the good) then the probability of switching to the dynamic regime in the current period is $\mu \in (0, 1)$. Obviously, the time elapsed since the consumer switched to the dynamic regime, s, can only take discrete values, $s = \Delta, 2\Delta, ...$, and so does the potential instantaneous utility, $R(s)$.

I focus on Markov strategies; thus, actions only depend on payoff relevant variables and not on how the game was played in the past. A strategy for the buyer is a decision between buying or waiting as a function of the current price and the value of s (if the consumer is in the static regime, this is equivalent to $s = 0$). Since the consumer's payoff is monotone with respect to the price, then we can write the consumer's strategy as $\overline{p}(s)$, i.e., a reservation price as a function of her current potential instantaneous utility. In other words, the consumer purchases the good if and only if $p \leq \overline{p}(s)$. The only state variable for the seller is the probability distribution over s. Thus, a Markov strategy can be written as $p(H)$, where H denotes a probability distribution over s in that period. Along the equilibrium path the seller updates his beliefs about the distribution of s using Bayes' rule and on the basis of the characteristics of the random variable x, the buyer's strategy and the history of the game.

Let us first look at the optimal pricing policy for a given consumer behavior, $\overline{p}(s)$. As an auxiliary step consider the case that the seller is able to observe the realization of x. Then, the optimal price after a purchase would be the solution to the following optimization problem: choose s in order to maximize:

$$\Pi_0 = e^{-r(x+s)} \left[\overline{p}(s) + \Pi_1^*\right]$$

where Π_1^* denotes the seller's continuation value after the first purchase. Notice that the optimal solution is independent of the realization of x. Let us denote the solution to such optimization problem as s^*, and let $p^* = \overline{p}(s^*)$. For all $s < s^*$, $\overline{p}(s) < p^*$. Thus, for any $s < s^*$ it is optimal to set any price that discourages the consumer from purchasing the good ($p > \overline{p}(s)$).

Returning to the true specification of the model, provided that the time elapsed since the last purchase is less than s^*, and independently of the probability distribution over s, the optimal price is also any price that discourages a purchase (in particular, p^*). If the time elapsed since the last purchase is equal to s^*, then with probability μ the consumer's value of s is s^* (and with probability $1-\mu$, $s < s^*$), and hence the optimal price is p^*. If no purchase takes place at s^*, then Bayesian updating (using the seller's strategy) implies that the value of s in the next period is lower or equal to s^*. In fact, the probability distribution over s remains unchanged. Thus, the optimal price in period $s^* + \Delta$ is also p^*. This argument applies to all the following periods. Therefore, in equilibrium the seller's strategy can simply be characterized by a constant price.

Let us now turn to the consumer's optimal strategy for a constant price, $p^* < M$. The consumer's optimization problem after a purchase consists of choosing s in order to maximize:

$$U_0 = \lambda' e^{-rs} \left[R(s) - p^* + U_1^* \right]$$

where $\lambda' \equiv \frac{\mu}{1-(1-\mu)\beta}$ and U_1^* denotes the buyer's continuation value after a purchase. Given p^*, if the consumer finds it optimal to buy at s^* then:

$$R(s^*) - p^* + U_1^* \geq \beta \left\{ R(s^* + \Delta) - p^* + U_1^* \right\} \tag{5}$$

In equilibrium the seller sets the price that leaves the consumer indifferent (the consumer's reservation price). This implies that 5 holds with equality.

Let us denote the value of s as $\overline{s}$ such that the total surplus obtained if the transaction takes place at $s = \overline{s}$ is the same that at $s = \overline{s} - \Delta$ (See Figure 2), i.e.,

$$R(\overline{s} - \Delta) - \beta R(\overline{s}) + (1 - \beta) W_1^* = 0 \tag{6}$$

Notice that the efficient transaction point is the one that falls in the interval $[\overline{s} - \Delta, \overline{s}]$. From equation 5 (with equality), it follows that $p^* = 0$ if and only if $s^* = \overline{s} - \Delta$, and that p^* strictly increases with s^*.

Next, I characterize the set of prices that can be sustained in equilibrium. Let us first consider deviations that bring the purchase forward. At time $s^* - \Delta n$, $n \geq 1$, the seller could set a price that leaves the consumer with $x = 0$ indifferent. The seller would find it optimal to set such a price if total surplus at $s^* - \Delta n$, is greater than at s^*. Thus, it is not optimal to bring the purchase forward if and only if $s^* \leq \overline{s}$.

Next, I turn to deviations that delay the purchase. At $s^* + \Delta$, the seller's willingness to pay, $\overline{p}(s^* + \Delta)$, is given by:

$$R(s^* + \Delta) - \overline{p}(s^* + \Delta) + U_1^* = \beta \left[R(s^* + 2\Delta) - p^* + U_1^* \right]$$

The seller's incentives to deviate are given by:

$$D(s^* + \Delta) \equiv \beta \left[\overline{p}(s^* + \Delta) + \Pi_1^* \right] - (p^* + \Pi_1^*)$$

From the above two equations we obtain:

$$D(s^* + \Delta) = \beta \Psi(s^* + \Delta) - (1+\beta)\Psi(s^*) - (1-\beta) W_1^*$$

where

$$\Psi(s) \equiv R(s) - \beta R(s+\Delta)$$

with $\Psi'(s) > 0$ and $\Psi''(s) < 0$. Notice that $D(s^* + \Delta)$ decreases with s and hence the seller deviates if and only if $s^* \leq \underline{s}$, where $\underline{s}$ is implicitly given by $D(\underline{s} + \Delta) = 0$. Also, it can be checked that if the seller does not have incentives to deviate at $s^* + \Delta$ then neither will deviation occur at larger values of s. Thus, in equilibrium we must have $s^* \in [\underline{s}, \overline{s}]$. Since there is one and only one trading point in the time interval $[\overline{s} - \Delta, \overline{s}]$, an equilibrium may exist if $\underline{s} \leq \overline{s}$, although its existence can only be guaranteed if $\underline{s} \leq \overline{s} - \Delta$. The following lemma shows that the former inequality holds but the latter does not.

Lemma 1 $\overline{s} - \Delta < \underline{s} < \overline{s}$.

For proof of this lemma, see Appendix.

In other words, equilibrium exists if the trading point is close to $\overline{s}$, but it does not if it is close to $\overline{s} - \Delta$. If $s^* = \overline{s} - \Delta$ then the candidate to become an equilibrium price (equation 5) is $p^* = 0$. As s^* increases in the range $[\overline{s} - \Delta, \overline{s}]$ then the candidate price also increases. Only if such a price is sufficiently low does the seller have incentives to delay the purchase, which implies that a Markov equilibrium does not exist.

Finally, let us characterize the equilibrium whenever it exists. If we take limits, we obtain:

$$\lim_{\Delta \to 0} \overline{s} = \lim_{\Delta \to 0} \underline{s} = \lim_{\Delta \to 0} \overline{s} - \Delta = s^o$$

This implies that the equilibrium price goes to zero as Δ goes to zero. The following proposition summarizes this result.

Proposition 2 *If the distance between two trading points, Δ, is sufficiently small, and if a Markov equilibrium exists, the timing of purchases is efficient and the price is arbitrarily close to marginal cost.*

This result is driven by two main forces. First, for any Δ the timing of purchases is efficient. Alternatively, if s^* is higher than efficient, then the seller can deviate and offer a lower price at the efficient timing that leaves the consumer indifferent, which allows the seller to appropriate the extra surplus. Second, the price is constant over time (in all relevant periods). Because of the one-sided asymmetric information, if a buyer with $x = 0$ does not purchase at the prescribed price and date then the seller must interpret that $x > 0$, and the updated distribution of s remains constant. Under the expectation of a constant price the consumer finds it optimal to purchase at the efficient point only if the price is close to zero.

In other words, under price flexibility and for any expected price sufficiently higher than marginal cost, the consumer is very sensitive to any reduction in current price, since she presumes that such a bargain will not last. As a result, the seller prefers to set a price below expectations. Hence, the only consistent expectations involve a low margin.

Asymmetric information is a crucial assumption. In fact, if $\mu = 1$ then there exists a unique Markov Perfect Equilibrium such that the timing of purchases is efficient and the seller appropriates all the surplus (See Appendix). The equilibrium is sustained by a pricing policy that leaves the consumer indifferent at all points, $p = R(s)$. In contrast, we have shown that with an arbitrary amount of asymmetric information (μ could be arbitrarily close to 1) the seller cannot trace the consumer's instantaneous utility and as a result the equilibrium of the game changes drastically.[10]

Throughout this paper I assume that the seller sets buyer specific prices. This is a reasonable assumption if we are dealing with personal services but it does not fit very well with some of the examples discussed in the introduction. However, the Markov equilibrium characterized in this section could also be interpreted as an equilibrium with multiple buyers and no price discrimination. If consumers expect a constant price, then their reservation prices are the same $\overline{p}(s)$ as in the single buyer equilibrium. Also, given those reservation prices (and provided the mass of consumers with $s > s^*$ is zero) the profit maximizing price is $\overline{p}(s^*)$.

Thus, under price flexibility the seller's payoff is close to zero. This suggests that the seller could benefit from committing to future prices. In

[10]The result of the proposition also holds in the case that the probability of switching to the dynamic regime changes over time, as long as it is always strictly less than one. In any case we maintain the more restrictive assumption for expositional purposes.

the next section I consider the extreme case of long-run commitment and in Section 5 I deal with an intermediate case.

4 Commitment to a constant price

Let us consider the monopoly pricing problem under the assumption that the seller is able to commit to a constant price forever. I can now set the analysis in continuous time from the beginning.

At time 0 the seller sets a price p and the consumer chooses the timing of purchases. For a given price p, the consumer chooses , $s_1, s_2, \dots$ in order to maximize 1. Notice that the optimal plan is independent of the amount of time spent in the static regime, x. The first order condition is given by:

$$R'(s_1) - r[R(s_1) - p + U_1^*] = 0 \tag{7}$$

Given the stationarity of the problem, the buyer's continuation value can be written as:

$$U_1^* = \frac{\lambda e^{-r\widehat{s}}}{1 - \lambda e^{-r\widehat{s}}} \left[R\left(\widehat{s}\right) - p \right]$$

where $s_1 = \widehat{s}$. Thus, for a given price p equation 7 becomes:

$$R'(\widehat{s}) - \frac{r[R(\widehat{s}) - p]}{1 - \lambda e^{-r\widehat{s}}} = 0 \tag{8}$$

From the above expression we can compute the responsiveness of $\widehat{s}$ to changes in the (constant) price. In particular:

$$\frac{d\widehat{s}}{dp} = \frac{M(\widehat{s})}{1 - \lambda e^{-r\widehat{s}}}$$

where

$$M(\widehat{s}) \equiv \frac{r}{-R''(\widehat{s}) + rR'(\widehat{s})} > 0$$

Thus, a higher price increases the length of the time intervals between purchases (decreases frequency). Also notice that $M'(\widehat{s}) > 0$, and that $\widehat{s}\,(p = 0) = s^o$.

Next, let us turn to the monopolist's optimal pricing policy. The monopolist anticipates that consumers' behavior is given by equation 8 and chooses p in order to maximize 2. The first order condition characterizes the optimal price:

$$1-\frac{rp}{1-\lambda e^{-r\widehat{s}}}\frac{d\widehat{s}}{dp}=1-\frac{rpM(\widehat{s})}{(1-\lambda e^{-r\widehat{s}}\ )^{2}}=0 \tag{9}$$

Thus, equation 9 shows the trade-off faced by the monopolist: a higher price increases the margin but reduces the frequency of purchases. The size of the latter effect depends on $\frac{d\widehat{s}}{dp}$. Combining equations 8 and 9 we can characterize the equilibrium value of s, denoted by s^c:

$$R'(s^c)-\frac{rR(s^c)}{1-\lambda e^{-rs^c}}+\frac{1-e^{-rs^c}}{M(s^c)}=0 \tag{10}$$

Second order conditions imply that the left hand side of equation 10 decreases with s. Also, from equation 4, we know that the left hand side, evaluated at s^o, is positive. Hence, we obtain the following result.

Proposition 3 *Under commitment to a constant price, average interpurchase time periods are inefficiently long: $s^o < s^c$.*

The intuition is straightforward. A monopolist charges a price above marginal cost, which reduces the consumer's costs of waiting, and as a result the frequency of purchases decreases.

Finally, let us denote the equilibrium price under commitment to a constant price as p^c, i.e., the value of p such that $\widehat{s}\,(p^c)=s^c$. Notice that in this case there is no discontinuity in the limit as μ goes to infinity..

5 Medium-run commitment

Let us consider an intermediate situation between the case of complete price flexibility and commitment to a long-run price. Suppose now that at time 0 the monopolist sets a price, but can only commit to keeping that price unchanged until the next purchase. Immediately after the consumer purchases the good then the seller can set a different price. Below I argue that adding a deadline to the price commitment complicates specification of the equilibrium without altering the main effects.

In this case a Markov strategy for the seller is simply a price, since every time the seller sets a new price all payoff relevant variables take the same value. A Markov strategy for the seller can be expressed as a reservation price as a function of the current state of preferences, $\overline{p}(s)$ or, more conveniently, as a choice of the timing of next purchase as a function of the current price, $s(p)$. The consumer's optimization problem is similar to that of the previous section and thus $s(p)$ is also given by equation 8. The crucial difference is that now her continuation value, U_1^*, does not depend on the current price but only on expected future prices. In fact, the sensitivity of the timing of the first purchase, s_1, with respect to the current price, p, can be obtained by totally differentiating equation 8, whenever U_1^* is invariant to p:

$$\frac{ds_1}{dp} = M(s)$$

Hence, in this case, s_1 is less responsive to p than in the case of commitment to a constant price. The reason is that in the latter case a change in the price is expected to be permanent, while in the Markov Equilibrium of the current game any deviation from the equilibrium price is expected to be transitory (exclusively relevant for the next purchase).

Given consumer behavior, the seller's best response is the value of p which maximizes:

$$\Pi_0 = \lambda e^{-rs(p)}[p + \Pi_1^*]$$

where Π_1^* is independent of p. Thus, the first order condition can be written as:

$$1 - r(p + \Pi_1^*)\frac{ds_1}{dp} = 1 - r(p + \Pi_1^*)M(s) = 0 \tag{11}$$

Since the game is stationary, from equations 8 and 11 the equilibrium timing of purchases, s^d, is given by:

$$R'(s^d) - \frac{rR(s^d)}{1 - \lambda e^{-rs^d}} + \frac{1}{M(s^d)} = 0 \tag{12}$$

Also, from equation 8 we have that the equilibrium price, p^d, is such that $s^d = \hat{s}\left(p^d\right)$. Next, let us compare equations 10 and 12.

Proposition 4 *In the Markov Equilibrium of the medium-run commitment game the price is higher and the average interpurchase time interval longer than under commitment to a constant price, i.e.,* $p^d > p^c$*, and* $s^d > s^c$*.*

The intuition is the following. In the discretionary equilibrium any deviation from the equilibrium price is interpreted by consumers as a transitory cut and hence the impact on the timing of the next purchase is relatively small. In contrast, whenever the seller can commit to a constant price, a deviation is perceived as permanent, which has a greater effect on the frequency of purchases. As a result, in the Markov equilibrium of the medium-run commitment game the monopolist has incentives to charge a higher price than under long-run commitment.[11]

6 Discussion

6.1 Efficiency, distribution and the degree of price commitment

The last three sections have examined a standard price setting game under various assumptions about the seller's commitment capacity. In this game a single price is always posted and the seller is ready to transact at that price. The efficiency of the timing of purchases and the distribution of surplus vary substantially depending on the seller's commitment capacity. In particular, as we move from price flexibility to medium-run commitment, firm profits increase, but consumer surplus and total surplus decrease. However, if we move from medium to long-run commitment, then both buyer and seller payoffs increase. Hence, both consumer surplus and total surplus are non-monotonic with respect to the degree of price commitment.

This result suggests that since sellers benefit from any increase in commitment capacity they could be willing to invest in various commitment devices. In my model, if the seller can choose the degree of price commitment at the beginning of the game then he will choose to commit to a fixed price forever. However, in a richer model the benefits from price commitment analyzed in this paper would have to be traded-off against the costs of price rigidity. For instance, the seller could face random marginal costs. If the variance of these costs is sufficiently large then the costs of price rigidity

[11]There is no discontinuity in the limit as μ goes to infinity.

may overcome the benefits of commitment and the firm may choose some form of medium-run commitment. In this case, if we compare different markets with different cost volatility, then more flexible prices would be associated with lower efficiency.

6.2 Reputation

In the previous sections I have focused on Markov strategies. As usual if we allow for more general strategies then the set of equilibria will expand. In particular, in the case of price flexibility, it is straightforward to construct equilibria with trigger strategies that support prices significantly above marginal cost (at least when Markov Equilibria exist). In this case if we compare different equilibria, higher prices will be associated with lower consumer surplus (as well as lower total surplus) but higher firm profits. This is completely standard and I will not discuss it further.

The introduction of reputational arguments delivers more unusual results in the case of medium-run commitment. By using trigger strategies (see Appendix for details) we can support any price in the interval $\left[p_l, p^d\right]$, where $p_l < p^c < p^d$. If we compare the players' payoffs across these equilibria then a higher price in the interval $[p_l, p^c]$ is associated with lower consumer surplus and higher firm profits. However, a higher price in the interval $\left[p^c, p^d\right]$ is associated with lower payoffs for both the buyer and the seller. In other words, there are equilibria that are Pareto dominated: buyers and sellers could benefit from switching to a different equilibrium with lower prices.

6.3 Price commitment with a deadline

Under medium-run commitment I assumed that the seller has the capacity to commit to the price of the next purchase. Such a set up has been proved useful in illustrating the implications of less than full commitment. In particular, current demand becomes less sensitive to current price. However, an unpleasant feature of such a pricing policy is that the length of the commitment period depends on consumer behavior and it could potentially be arbitrarily large.

Alternatively, I could introduce price commitment with a deadline. In particular, suppose that at time 0 the seller announces a price and commits to maintaining it until a time interval of length Δ has elapsed with no purchase or until the consumer has made a purchase. Let us first discuss the case of complete information ($\mu = \infty$). In this case it is easy to show (see Appendix) that provided $\Delta > s^d$, the unique Markov Perfect Equilibrium also involves $\left(p^d, s^d\right)$. That is, the deadline does not matter. The argument is more complicated in the case of uncertainty ($\mu < \infty$). In this case, the seller will take into account that there is some probability that the deadline will matter. That is, there are some realizations of x for which a price below p^d would induce the consumer to purchase right before a new price is announced. Thus, I conjecture that in equilibrium the price would be lower than p^d, although such a difference is likely to shrink with μ (and with Δ) with no discontinuity in the limit as μ goes to infinity.

Finally, if we assume that every price announcement lasts for a period of length Δ (independently of whether a purchase has or has not occurred), then a Markov strategy for the consumer involves a reservation price not only as a function of the current price but also as a function of the time left for the next price announcement. Moreover, the players' continuation values also depend on the time left for the next price announcement. Those complexities suggest that exploration of this specification is not worthwhile.

7 Commitment to a sequence of prices

Let us now consider the possibility that the monopolist commits to a sequence of prices $\{p_n\}$ where n refers to the order of purchases, $n = 1, 2, ...$ Now the seller can reward or penalize consumer loyalty by setting a decreasing or an increasing price sequence. In order to simplify the presentation let us consider the deterministic case $(\mu = \infty)$. Given the sequence $\{p_n\}$, consumers choose the timing of purchases $\{s_n\}$ in order to maximize U_0 (equation 1). The optimality condition for the timing of the first purchase is well known by now and given by equation 7. Thus, as in the Markov Equilibrium of the medium-run commitment game, the effect of p_1 on s_1 is given by:

$$\frac{\partial s_1}{\partial p_1} = M\left(s_1\right)$$

However, the monopolist can influence s_1 not only through p_1 but also through $s_2, s_3, ...$ By the envelope theorem we have that for all n > 1:

$$\frac{\partial s_1}{\partial p_n} = M(s_1)\frac{\partial U_1^*}{\partial p_n} = M(s_1)\exp\left\{-r\sum_{j=2}^{n} s_j\right\}$$

Notice that a reduction in p_1 has the same effect on s_1 as an identical reduction in the present value of p_n. However, the reduction in p_n has the additional effect of reducing $(s_2, ..., s_n)$.

The monopolist chooses $\{p_n\}$ in order to maximize 2, anticipating how the timing of purchases is affected by the price sequence. The cumulative effect of future prices drives the following result:

Proposition 5 *The equilibrium price sequence includes a positive margin in the first purchase and zero margin in the following purchases, i.e.,* $p_1 > 0, p_n = 0$ *for all* $n > 1$. *As a result,* $s_1 > s^o$, $s_n = s^o$ *for all* $n > 1$.

Forr proof see Appendix.

Next, I characterize p_1 and s_1. Since, the consumer appropriates all the surplus after the first purchase the optimality condition for s_1 is an adaptation of equation 8:

$$R'(s_1) - r[R(s_1) - p_1 + \frac{e^{-rs^o}}{1-e^{-rs^o}}R(s^o)] = 0 \tag{13}$$

Since the monopolist makes zero profits after the first purchase, the optimality condition for p_1 is an adaptation of equation 11:

$$1 - rp_1\frac{ds_1}{dp_1} = 1 - rp_1 M(s_1) = 0 \tag{14}$$

Combining equations 13 and 14 we obtain:

$$R'(s_1) - r[R(s_1) + \frac{e^{-rs^o}}{1-e^{-rs^o}}R(s^o)] + \frac{1}{M(s_1)} = 0 \tag{15}$$

Thus, the optimal pricing policy rewards consumer loyalty. In fact, the result of marginal cost pricing after the first purchase is analogous to that of Crémer (1984) in a different context (consumer learning in a two-period model). The mechanism behind such a result is different although both models share the same principle. In both cases the first best can be implemented through a two-part tariff (and the monopolist can capture the entire surplus). In Crémer's two period model, the first period price is analogous to an upfront fee. In my model if the monopolist can charge a fee upfront (before the game starts and thus unrelated to any purchase) and a price per purchase then the profit maximizing policy includes a price equal to marginal cost in all purchases and a fee equal to the present value of total surplus. In most cases payment of an upfront fee is not feasible. Whenever the seller and the buyer are ready to sign a contract it is very likely that the buyer's potential utility changes over time.[12] Hence, the price of the first purchase is the instrument that the monopolist uses to collect rents, although the size of these rents is moderated by the incentives to induce consumers to make the first purchase relatively soon.

The equilibrium policy characterized in this section may look somewhat unrealistic. First, consumers could be liquidity constrained and unable to pay at the first purchase an amount equivalent to a significant fraction of the present value of all future gains from trade. Second, the monopolist's incentives to default on her promises are very powerful and therefore her commitment capacity must also be very strong. If we assume that consumers are unable to pay at a single purchase a price above a certain threshold, and/or that the monopolist is only subject to a finite (and relatively smalll) penalty if he defaults on the pricing policy announced at time 0, then the time profile slope of equilibrium prices is reduced, although the main qualitative features remain.

Do consumers benefit from such loyalty rewarding policies? Let us compare consumer payoffs in the equilibrium where the monopolist commits to a constant price (p^c, s^c) with the equilibrium where the monopolist can commit to a (decreasing) price sequence. In principle, there are two countervailing effects. On the one hand, the price charged after the first purchase is lower but, on the other hand, the price paid for the first purchase is higher than in the constant price equilibrium. The examples discussed in the Appendix suggest that consumers may actually lose or gain from loyalty rewarding policies. In particular, in Example 1 consumers lose if the monopolist can commit to a variable price policy, and in Example 2 consumers gain. The fact that in this context the introduction of loyalty rewarding schemes may be a Pareto improvement (and specially, the fact

[12]For instance, when a new variety is introduced consumers' potential utility is likely to be affected by the time period elapsed since the last purchase of a different variety. In this case, the buyer is willing to pay the upfront fee only at the moment of the first purchase.

that consumers may benefit from such practices) contrasts with the results of Crémer (1984), in spite of the analogies in the characteristics of the equilibrium price sequence.

8 Restricting the timing of purchases

Sellers may find a way of restricting the actual timing of purchases. For instance, they could announce a very high regular price with occasional and predetermined periods of 'sales'. Similarly, sellers could restrict the length of the time period for which new varieties of the same product are available (they may be able to commit to purchase deadlines). Finally, sellers could offer long-term contracts that include the price and the frequency of purchases. Real world examples of such practices are not hard to find. For instance, Disney video tapes are usually marketed under purchase deadlines, and subscriptions to magazines include a price and a frequency. Moreover, some products can only be offered occasionally at a particular location (for instance, live performances by a particular artist).

8.1 Occasional purchasing periods

As in the previous section let us concentrate on the deterministic version of the model. Suppose that the monopolist wishes to induce consumers to purchase every s units of time. In principle he could do that either by making the product available only at time $s, 2s, ...$, or by setting a very high price for purchases made at other points in time. Suppose that the monopolist cannot refuse to serve a consumer in ns who has not purchased in $(n-1)\,s$. In this case, the monopolist will be able to implement a price, p, and a time interval between purchases, s, provided:

$$R(s) - p + U_1^* \geq e^{-rs}\left[R(2s) - p + U^*\right] \tag{16}$$

In other words, the consumer purchases at time s only if it is not worthwhile to wait until the next trading period, $2s$. The gains from waiting have to do with the increase in the instantaneous utility, and the costs are due to the discounting. Since I only consider stationary policies, the continuation utility, U^*, is given by:

$$U^{*}=\frac{e^{-rs}}{1-e^{rs}}\left[R\left(s\right)-p\right] \tag{17}$$

Plugging equation 17 into condition 16 we obtain the highest price that the monopolist can charge for a given frequency:

$$p=\left(1+e^{-rs}\right)R\left(s\right)-e^{-rs}R\left(2s\right) \tag{18}$$

Thus, the optimal policy consists of choosing a pair (p, s) that maximizes 2 subject to constraint 18. By restricting the timing of purchases the monopolist faces a more favorable trade-off between prices and frequency. The optimal value of s, denoted by s^{r}, is given by:

$$rR\left(s^{r}\right)-\left(1-e^{-rs^{r}}\right)R'\left(s^{r}\right)=e^{-rs^{r}}\left(1-e^{-rs^{r}}\right)\left[2R'\left(2s^{r}\right)-R'\left(s^{r}\right)\right]+ \tag{19}$$

$$+re^{-rs^{r}}\left(2-e^{-rs^{r}}\right)\left[R\left(2s^{r}\right)-R\left(s^{r}\right)\right]$$

and the optimal price is given by condition 18 evaluated at s^{r}.

It would be interesting to compare the impact of restricting the timing of purchasing with the allocation resulting from the monopolist's commitment capacity to set a constant price and a sequence of prices, respectively. Unfortunately, no general result has been obtained. Instead let us turn to a particular example. Consider the following functional form:

$$R\left(s\right)=M\left(1-\frac{e^{-rs}}{1-z}\right)$$

In this case we can actually compute the payoffs under the various pricing policies (See Appendix for details). The following table reports the results for the limiting case of $z=0$, and $M=100$.[13]

	(1)	(2)	(3)
U_0	25	25	16.1
Π_0	25	50	38.3
W_0	50	75	54.4

[13] Payoffs turn out to be proportional to M, thus setting $M=100$ is only a normalization. However, the choice of z is not at all irrelevant.

Columns (1) and (2) contain the payoffs under a constant price (and $\mu = \infty$) and a sequence of prices, respectively. Column (3) presents the payoffs under the stationary policy with restricted timing analyzed in this section. Comparison between columns (1) and (2) illustrates the discussion of Section 7. The possibility of committing to a variable price sequence (introducing loyalty rewarding schemes) significantly increases the seller's payoff without necessarily hurting consumers. However, the seller's ability to restrict the timing of purchases increases firm profits in comparison with the case of commitment to a constant price (columns (1) and (3)), but it hurts consumers. Nevertheless, total surplus is higher. This is because by restricting the timing of purchases the seller is able to induce more frequent consumption at a higher price. That is, $s^o < s^r < s^c$. Finally, if we compare columns (2) and (3) we realize that restricting the timing of purchases is Pareto dominated by the commitment to a sequence of prices. In other words, the seller's ability to commit to trading exclusively at certain periods of time has only a modest impact on total surplus and the seller's ability to appropriate rents. This is because the seller cannot refuse consumers that did not purchase the good in the previous trading period. Therefore, the price cannot be too high otherwise consumers will find it optimal to wait until the next trading period.

8.2 Contracting price and frequency

Clearly, the monopolist could implement the first best (and appropriate all the surplus) if he has the ability to contract ex-ante both the price and the frequency of purchases. Subscription to magazines is one example of this type of contract. Other services such as house cleaning, maintenance of equipment, and so on, are sometimes marketed under contracts that include a price and a frequency (although usually the arrangement can be breached at no pecuniary cost). In particular, if the monopolist can commit to serving only those consumers that buy a contract that includes (p, s), then the optimal contract consists of $p = R(s^o)$ and $s = s^o$. Notice that if breaching the contract involves no cost, consumers will prefer to purchase at the price $R(s^0)$ at a lower frequency, and hence we are back to the case analyzed in Subsection 8.1.

9 Concluding remarks

This paper studies the monopoly pricing of 'cyclical' goods, that is of goods for which consumer preferences follow a cyclical pattern determined by the endogenous timing of purchases. It has been shown that such characteristics have important implications. First, in the absence of commitment, prices may be close to marginal cost (as in the durable goods case). Second, under an intermediate degree of price rigidity prices may be higher than under complete price rigidity. In fact, both buyers and sellers may benefit from a higher degree of commitment. Third, if we allow for reputational effects then under an intermediate degree of commitment we have multiple equilibrium prices; some of these equilibria are Pareto dominated by other equilibria featuring lower prices. Such a result points to the potential role of regulation, which may achieve more efficient outcomes with arbitrarily low punishments (regulation may be self-enforcing). Fourth, the introduction of loyalty rewarding schemes may also be a Pareto improvement, in contrast with most of the received literature. Fifth, marketing practices that restrict the timing of purchases (including purchase deadlines, sales, and contracts specifying frequency of trade) are likely to be detrimental to consumer welfare but have a positive impact on total welfare.

The model presented in this paper is extremely stylized. Product differentiation and interfirm competition are the subject of a companion paper. Considering more general preferences and heterogeneous consumers would appear to be profitable extensions.

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11 Appendix

11.1 Proof of lemma 1

If $s^* = \overline{s}$ then the incentives to deviate in $s^* + \Delta$ can be written as:

$$D(\overline{s}+\Delta) = (1+\beta)\left[\frac{1}{1+\beta}\Psi(\overline{s}-\Delta) - \Psi(\overline{s}) + \frac{\beta}{1+\beta}\Psi(\overline{s}+\Delta)\right]$$

Notice that $D(\overline{s}+\Delta) < 0$, since $\Psi(s)$ is an increasing and concave function and $\beta < 1$. Hence, $\underline{s} \leq \overline{s}$. If $s^* = \overline{s} - \Delta$:

$$D(\overline{s}) = \beta\left[\Psi(\overline{s}+\Delta) - \Psi(\overline{s})\right] > 0$$

Hence, $\underline{s} > \overline{s} - \Delta$.

11.2 Price flexibility with complete information

Consider the discrete time version of the model discussed in Section 3 and take $\mu = 1$. If $\mu < 1$ but x is observable, the result is analogous. Thus, the crucial change is not the stochastic component of preferences but the asymmetry of information.

A Markov strategy for the buyer is a sequence of reservation prices $\overline{p}(s)$, and a Markov strategy for the seller is a sequence of posted prices, $p(s)$.

Proposition 6 *The unique Markov Perfect Equilibrium (MPE) involves the efficient frequency of purchases and the seller appropriates all the surplus.*

Consider a candidate for MPE. Given the strategy of the seller, let us denote the timing of next purchase that maximizes consumer's utility after a purchase (at $s = 0$) as s_1 . Thus, along the equilibrium path, trade must take place after a period of length s_1. Similarly, I denote the timing of purchases that maximizes the consumer's utility subject to $s \geq s_1 + \Delta$ as s_2 That is, if for an exogenous reason trade does not take place at $s = s_1$, the MPE prescribes that players will trade at $s = s_2$. Analogously, we can define $s_3, s_4, ...$ The consumer's reservation price after a period of length s_j, $j = 1, 2, ...$, is given by:

$$R(s_j) - \overline{p}(s_j) + U_1^* = \beta^{s_{j+1}-s_j}\left[R(s_{j+1}) - p(s_{j+1}) + U_1^*\right] \tag{20}$$

Thus, $\overline{p}(s_j)$ is the price that leaves the consumer indifferent between purchasing after a period of length s_j or awaiting the most favorable timing.

Since the seller's strategy must be the best response to the buyer's strategy we must have that $p(s_j) = \overline{p}(s_j)$. Plugging this into 20, by iterative substitution we can write for any j:

$$R(s_1) - \overline{p}(s_1) + U^* = \beta^{s_j - s_1}\left[R(s_j) - \overline{p}(s_j) + U^*\right]$$

As j goes to infinity the right hand side goes to zero. Hence, in any MPE the consumer cannot obtain a strictly positive payoff, which implies that whenever trade occurs the consumer pays a price equal to her instantaneous utility.

Next, I argue that an MPE exists and involves an efficient timing. Let s^o be the length of the interpurchase period that maximizes total welfare, i.e., s^o is the solution to the following optimization problem: choose s in order to maximize:

$$W^* = \frac{e^{-rs^o}}{1 - e^{-rs^o}} R(s^o)$$

subject to $s \in \{0, \Delta, 2\Delta, ...\}$. Notice that as Δ goes to zero s^o satisfies 4. Consider the following strategy for the seller:

$$p(s) = \begin{cases} R(s^o) \ \ if \ s \leq s^o \\ R(s) \ \ otherwise \end{cases}$$

Given such a strategy the buyer is willing to purchase at s^o since a positive payoff cannot be obtained. Similarly, the seller obtains all the potential surplus, therefore he does not have any incentives to deviate either.

Can we have an equilibrium where the consumer makes zero surplus but timing is not efficient? Suppose in equilibrium purchases occur at $s^* < s^o$. Since $p(s) = R(s)$ for all $s \geq s^*$ then, the seller would find it optimal to deviate and set a price higher than the instantaneous utility for all $s < s^{o\cdot}$ Similarly, if in the equilibrium candidate purchases occur at $s^* > s^o$, then the seller finds it optimal to set a price $p(s^o) \leq R(s^o)$ and induce the buyer to make the purchase at the efficient timing.

11.3 Reputational equilibria

Let us consider the game in Section 5: the seller commits to the price of the next purchase. Suppose expectations about future prices are formed according to the following rule: If $p_1 = q$ then consumers expect future prices to be equal to q, otherwise they expect future prices to be equal to p^d. This is reminiscent of trigger strategies, in the sense that if the seller deviates from expected behavior then the punishment is that consumers will play the discretionary equilibrium from then onwards.

If the seller conforms to expected behavior then the optimal timing is given as usual by equation 8 in the text:

$$R'(\hat{s}) - \frac{r[R(\hat{s}) - q]}{1 - e^{-r\hat{s}}} = 0$$

and hence consumers purchase the good according to $\hat{s}(q)$. If the firm deviates and sets $p \neq q$, then s varies with p according to:

$$\frac{ds}{dp} = M(s)$$

Clearly, if the firm deviates then it faces the same incentives as in the discretionary equilibrium, and hence it will optimally set $p_1 = p^d$. As a result, a price q can be sustained in equilibrium if and only if:

$$\Pi(q) \equiv \frac{e^{-r\hat{s}(q)}}{1 - e^{-r\hat{s}(q)}} q \geq \frac{e^{-rs^d}}{1 - e^{-rs^d}} p^d.$$

As discussed in section 4, $\Pi(q)$ is a continuous function that increases if $q < p^c$ and decreases if $q > q^c$. Moreover, from Proposition 3 we know that

$p^c < p^d$. Hence, there exists a price, p^l, $0 < p^l < p^c$, such that any price q in the interval $[p^l, p^d]$ can be sustained as a stationary equilibrium.

11.4 Deadlines in the medium-term commitment game

Suppose that at time 0 the seller announces a price and commits to maintaining it until a time interval of length Δ has elapsed with no purchase or until the consumer has made a purchase. Let us assume that $\Delta > s^d$ and make the model deterministic ($\mu = \infty$). Also, suppose that at $s = \Delta$ the seller can choose the lowest price. Suppose that along the equilibrium path, transactions take place at $s' > \Delta$ at a price p'. Then I will show that there exists a price p'' that induces the buyer to purchase at $s \leq \Delta$ and that the seller finds it profitable to set immediately after a purchase. Let us introduce further notation: $w = s' - \Delta$. The highest price that the buyer is willing to pay at $s = \Delta$ (if at such a price the consumer buys at $s < \Delta$, that is even better for the seller) is denoted as p'' and given by:

$$R(\Delta) - p'' + U_1^* = e^{-rz}\left[R(\Delta + w) - p' + U_1^*\right]$$

Such a deviation is profitable if and only if $D > 0$, where D is given by:

$$D(w) = p'' + \Pi_1^* - e^{-rz}\left(p' + \Pi_1^*\right)$$

Plugging the definition of p'' on the above expression we can check that $D(0) = 0$ and $D'(w) > 0$ since $\Delta + w > s^o$. Therefore, in equilibrium transactions must take place at $s \leq \Delta$. Using a similar argument we can show that if we reach $s = \Delta$ with no purchase, then the equilibrium of the subgame involves a purchase in the interval $[\Delta, 2\Delta)$. In this interval the consumer's willingness to pay is given by the first order condition of the consumer's optimization problem:

$$R'(s) = r\left[R(s) - p + U_1^*\right]$$

This implies that at the moment of purchasing the good, the consumer gains a payoff of $\frac{R'(s)}{r}$. Thus, the seller's payoff at $s = \Delta$ is

$$\Pi_\Delta = e^{-r(s-\Delta)}\left[R(s) + W_1^* - \frac{R'(s)}{r}\right]$$

The optimal choice is $s = \Delta$. Finally, I consider the equilibrium price at $s = 0$. For any $s \in [0, \Delta]$ the consumer's willingness to pay is also given by the first order condition of the consumer's optimization problem. Hence, the seller's payoff at $s = 0$ is given by:

$$\Pi_0 = e^{-rs} \left[R(s) + W_1^* - \frac{R'(s)}{r} \right]$$

which implies that the seller optimally chooses s^d which is implemented by setting p^d.

11.5 Proof of Proposition 5

The monopolist chooses the price sequence $\{p_n\}$, $n = 1, 2, ...$ in order to maximize 2. The first order condition with respect to p_1 is given by:

$$\frac{\partial \Pi_0}{\partial p_1} = e^{rs_1} \left[1 - r(p_1 + \Pi_1^*) M(s_1) \right] = 0$$

where Π_1^* is the present value of profits after the first purchase, evaluated at the optimal solution. Similarly, the first order condition with respect to p_2 is given by:

$$\frac{\partial \Pi_0}{\partial p_2} = e^{r(s_1+s_2)} \left[1 - r(p_1 + \Pi_1^*) M(s_1) - r(p_2 + \Pi_2^*) M(s_2) \right] = 0$$

Combining the first order conditions with respect to p_1 and p_2 we get:

$$p_2 + \Pi_2^* = p_2 + e^{-rs_3} (p_3 + \Pi_3^*) = 0$$

From the first order condition with respect to p_3 we derive that $p_3 + \Pi_3^* = 0$, and hence $p_2 = 0$. If we repeat the procedure with the other first order conditions we can show that $p_n = 0$, for all $n > 1$.

11.6 Consumer welfare under various pricing policies

I wish to compare consumers' utility when the monopolist commits to a constant price and when the monopolist commits to a sequence of prices. As discussed above, the consumer's utility is a decreasing function of the optimal value of s_1:

$$U_0 = e^{-rs_1} \frac{R'(s_1)}{r}$$

Thus, consumers prefer the constant price policy over the variable price policy if and only if they find it optimal to make the first purchase sooner under the first pricing policy than under the second. Let us consider two examples. In the first, the constant price policy is strictly preferred to the variable price policy, while this result is reversed in the second example.

11.6.1 Example 1

Let us take $R(s)$ to be given by:

$$R(s) = M\left(1 - \frac{e^{-rs}}{1-z}\right)$$

where $1 > z > 0$. Notice that all the assumptions I made regarding $R(s)$ are satisfied (In particular, $L = \frac{zM}{1-z}$). Under the constant price policy, s_1 is given by equation 10. Thus, if we denote by $g^c \equiv e^{-rs^c}$, then we can write equation 10 as:

$$(1 - g^c)^2 (1 - 2g^c) = z \tag{21}$$

Similarly, under the variable price policy s_1 is given by equation 15. Let us denote by $z^o \equiv e^{-rs^o}$ and $z^v \equiv e^{-rs^v}$ where s^v represents the timing of the first purchase under the variable pricing policy. In this case we can write equation 15 as follows:

$$g^v = \frac{1}{4}\left[1 - z + \left(1 - \sqrt{z}\right)^2\right] \tag{22}$$

We can immediately see from equations 21 and 22 that as z goes to zero then both g^c and g^v go to one half, and as z goes to one then both g^c and g^v go to zero. Moreover, for any $z \in (0,1)$, $z^c > z^v$, i.e., consumers prefer the constant price over the variable price policy.

11.6.2 Example 2

Let us take $R(s)$ to be given by:

$$R(s) = M\left(1 - \frac{e^{-2rs}}{1-z}\right)$$

where $1 > z > 0$. Notice that once again all the assumptions I made regarding $R(s)$ are satisfied.

In this case, using the same notation as in Example 1, equation 10 can be written as:

$$\left[1 - (g^c)^2\right] - 2(g^c)^2(1-g^c)(4-3g^c) = z \tag{23}$$

Taking the limit as z goes to zero, then g^c is given by:

$$\Psi(g^c) = 0$$

where

$$\Psi(g) \equiv 1 + g - 8g^2 + 6g^3$$

Notice that $\Psi(1) = 0$, but $g^c = 1$ is not the limit of any meaningful solution to equation 23. It can quickly be confirmed that there is a single solution to $\Psi(g^c) = 0$ such that $0 < g^c < 1$, and that $\Psi(g) > 0$ for all $g < g^c$.

Similarly, from equation 15:

$$\lim_{z \to 0} g^v = \frac{1}{\sqrt{3}}$$

Finally, $\Psi\left(\frac{1}{\sqrt{3}}\right) > 0$. Thus, by continuity, if Δ is not too high $g^v > g^c$, i.e., consumers prefer the variable over the constant pricing policy.

Figure 1

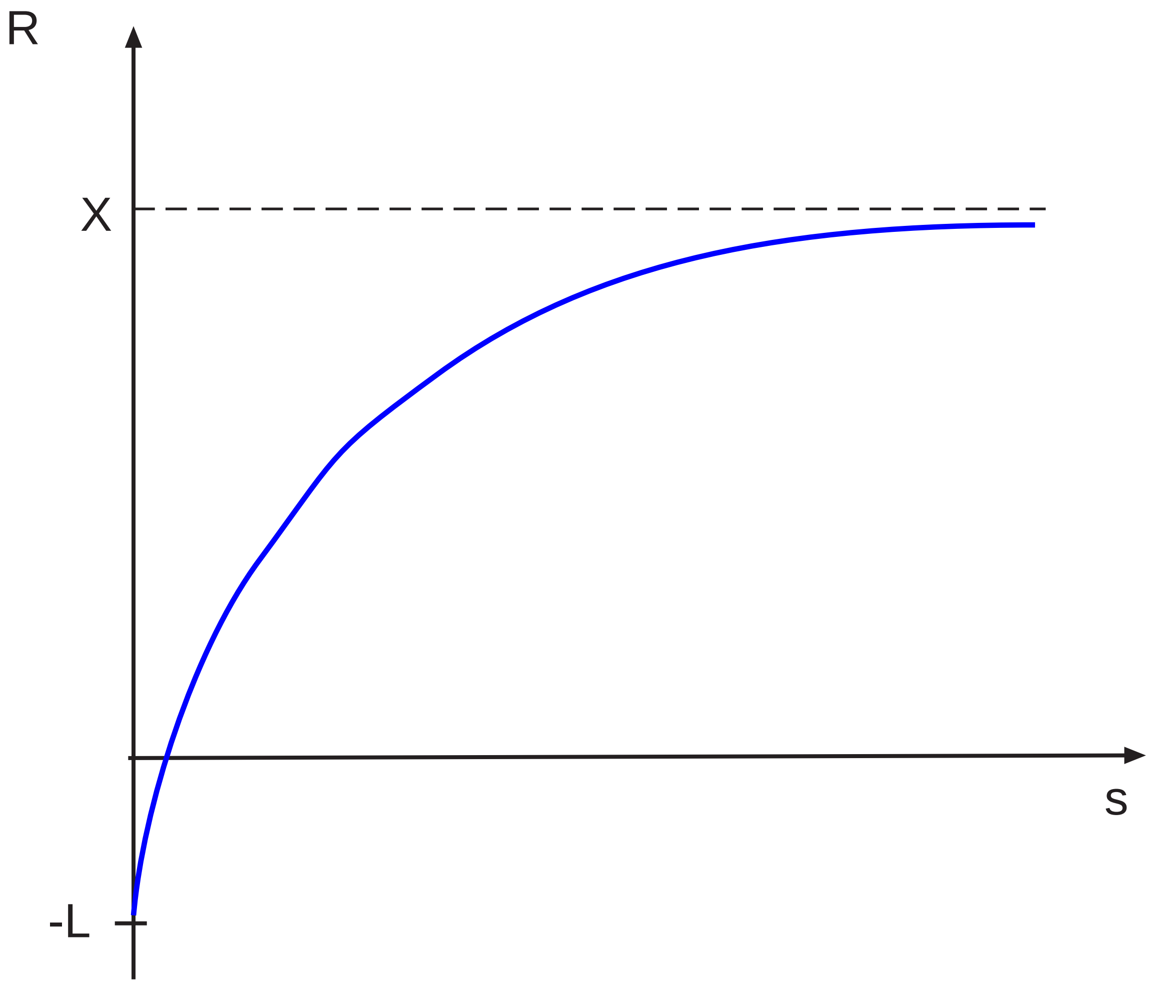

Figure 2