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ABSTRACT

Choice-Based Elicitation and Decomposition of Decision Weights for Gains and Losses Under Uncertainty*

This Paper reports the results of an experimental parameter-free elicitation and decomposition of decision weights under uncertainty. Assuming cumulative prospect theory, utility functions were elicited for gains and losses at an individual level using the trade-off method. Then decision weights were elicited using certainty equivalents of uncertain two-outcome prospects. Furthermore, decision weights were decomposed using observable choice instead of invoking other empirical primitives as in the previous experimental studies. The choice-based elicitation of decision weights allows for a quantitative study of their characteristics, and also allows, among other things, to confront the sign-dependence hypothesis with observed choice under uncertainty. Our results confirm concavity of the utility function in the gain domain and bounded sub-additivity of decision weights as well as choice-based subjective probabilities. We also find evidence of sign-dependence of decision weights.

JEL Classification: D81
Keywords: Choquet expected utility, cumulative prospect theory, decision under uncertainty (ambiguity), decision weights, probability weighting and subjective probabilities

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1. Introduction

For decision situations under uncertainty subjective expected utility theory (SEU; Savage 1954) is regarded as the standard normative theory. The famous contributions by Allais (1953) and Ellsberg (1961) and numerous experimental studies thereafter (see e.g. Camerer and Weber 1992), however, questioned the descriptive validity of Savage's model. As a response, new theories of choice were proposed that could account for the observed behavior. Among the most influential of these are Choquet expected utility (CEU) theory (Gilboa 1987, Schmeidler 1989) and Cumulative Prospect Theory (CPT) (Tversky and Kahneman 1992, Wakker and Tversky 1993). For decision under risk, i.e., when objective probabilities are available, similar generalizations of expected utility are rank-dependent utility (RDU; Quiggin 1981) and CPT.

While SEU evaluates an uncertain alternative as the sum of utilities from outcomes weighted by the corresponding subjective probabilities (i.e., measure of belief), CEU and CPT replace the additive probabilities by non-additive decision weights depending on the rank-ordering of consequences. Decision weights are obtained through a weighting function (i.e., a non-additive set function). CPT is more general than CEU because it allows decision weights to be sign-dependent (i.e., depend on whether the associated consequence is a gain or a loss) through the use of two event weighting functions. For decision under risk, RDU and CPT use a probability weighting function to transform objective probabilities into decision weights.

Under uncertainty, decision weights reflect additional considerations, called decision attitude, over and above a measure of belief (Wakker 2001b). Consequently, researchers suggested to decompose decision weights into a component of belief (i.e., a subjective probability), independent of the particular decision context, and a component reflecting decision attitude. Such a decomposition was initially proposed by Fellner (1961). Subsequently, Tversky and Fox (1995), Fox et al. (1996), Fox and Tversky (1998), Wu and Gonzalez (1999), and Kilka and Weber (2001) used subjective probabilities resulting from direct judgment, i.e. judged probabilities, to operationalize the decomposition in experimental investigations. The theoretical problem of a completely choice-based decomposition of decision weights was however recently solved in Wakker (2001b).

Decision weights can be investigated through two research strategies. The first strategy consists in testing simple preference conditions to obtain information about the shape of the weighting function (Wu and Gonzalez 1999). The second strategy consists in eliciting decision weights from individual preferences. Tversky and Fox (1995) and Fox et al. (1996)

used a certainty equivalent method and two-outcome prospects to elicit individual decision weights in the gain domain. It should be noted, however, that utility functions were not elicited at the individual level. Indeed, Fox and Tversky (1995) used a unique power utility function for all subjects, while Fox et al. (1996) used a similar methodology with a linear utility function (inferred from a preceding experimental study). Subsequently, Wakker and Deneffe (1996) proposed the trade-off method as a utility function elicitation technique which is applicable also in case of distorted or unknown probabilities. With this method at hand, it is possible to elicit utilities without the interference of decision weights, and then elicit decision weights in a second step. Abdellaoui (2000) and Bleichrodt and Pinto (2000) used this methodology in the domain of risk.

This paper reports the results of an experimental study in which CPT decision weights are elicited, for gains and losses, at an individual level through a parameter-free elicitation method based on choice questions. Decision weights are then decomposed using choice-based subjective probabilities as in the recent theoretical decomposition in Wakker (2001b). Using this decomposition, we obtain probability weighting functions for gains and losses. Their characteristics are then compared to those obtained through other approaches.

The elicitation of CPT at an individual level aims to take into account individual differences in preferences (see e.g. Tversky and Kahneman 1992, p. 306) and, thereby, go one step further than elicitations where between subjects utility differences were not taken into account. In our choice-based approach, the trade-off method is used to elicit utilities first, without using given probabilities as in Fennema and van Assen (1999), Abdellaoui (2000) and Bleichrodt and Pinto (2000). Then, decision weights are elicited using utility values as inputs. Our parameter-free elicitation of decision weights allows for a ‘fully quantitative’ study of their characteristics, and also allows, among other things, to confront the sign-dependence hypothesis with observed choice under uncertainty.

The present paper is structured as follows. Section 2 briefly reviews CPT and introduces the two-stage approach linking decision making under uncertainty to decision making under risk. Section 3 describes the procedure to successively elicit the utility function and decision weights. Section 4 completes the description of our experimental set-up. The results of our study are presented in Section 5. The paper concludes with a summary and discussion of the main findings.

2. Theoretical Framework

In decision making under uncertainty, uncertainty is modeled through a set S, called the *state space*. The decision maker knows that exactly one of these states will occur, but she does not know which one. Subsets of S are called *events*. The objects of choice are called prospects, which are functions from S to X, the set of *outcomes*. A *prospect P* can be represented as $(x_1, A_1; x_2, A_2; ...; x_n, A_n)$, where $(A_1, A_2, ..., A_n)$ is a partition of S and $x_i \in X$ is the outcome associated with the states contained in A_i. A two-outcome prospect $(x_1, A_1; x_2, A_2)$ is denoted $(x_1, A_1; x_2)$. In decision making under risk, events are replaced by their (exogenously given) objective probabilities in the corresponding prospects. The objects of choice in decision making under risk are therefore probability distributions over outcomes. Under SEU, the state space is endowed with a *subjective probability* measure resulting from preferences over acts (Savage 1954).

This paper adopts the theoretical framework of cumulative prospect theory (Tversky and Kahneman 1992, Tversky and Wakker 1995). For both risky and uncertain prospects, outcomes are evaluated by a strictly increasing utility function defined from $\Re$ to $\Re$ satisfying $u(0) = 0$. For decision making under uncertainty (risk), probabilities are replaced by *decision weights* generated by means of two weighting (probability weighting) functions. A weighting (probability weighting) function for gains is denoted by $W^+(.)$ $(w^+(.))$, a weighting (probability weighting) function for losses by $W^-(.)$ $(w^-(.))$. A *weighting function* $W(.)$ $(w(.))$ is a function from the set of all subsets of S ([0, 1]) to [0, 1] that satisfies $W(\emptyset) = 0$ $(w(0) = 0)$, $W(S) = 1$ $(w(1) = 1)$ and monotonicity with respect to set inclusion, i.e. $W(A) \leq W(B)$ for all events A, B with $A \subset B$ (that is strictly increasing over [0, 1]). W is *convex* if $W(A) + W(B) \leq W(A \cup B) + W(A \cap B)$ for all events A, B. W is *concave* if the reversed inequality holds.

Under CPT, the value of a prospect $P = (x_1, A_1; x_2, A_2; ...; x_n, A_n)$ in which $x_1 \leq ... \leq x_k \leq 0 \leq x_{k+1} \leq ... \leq x_n$, denoted $CPT(P)$, is given by the following formula

$$(1) \qquad CPT(P) = \sum_{i=1}^{k} \pi_i^- \cdot u(x_i) + \sum_{i=k+1}^{n} \pi_i^+ \cdot u(x_i)$$

where the *decision weights* π_i^-, π_i^+ are defined by $\pi_i^- = W^-(A_1 \cup \ldots \cup A_i) - W^-(A_1 \cup \ldots \cup A_{i-1})$ and $\pi_i^+ = W^+(A_i \cup \ldots \cup A_n) - W^+(A_{i+1} \cup \ldots \cup A_n)$.[1]

[1] We follow the usual convention that $A_1 \cup ... \cup A_{i-1} = \emptyset$ for i = 1, and that $A_{i+1} \cup ... \cup A_n = \emptyset$ for i = n.

Cumulative Prospect Theory for decision under uncertainty is a generalization of CEU introduced by Gilboa (1987) and Schmeidler (1989). When the utility function satisfies the condition $u(0) = 0$, CPT encompasses CEU as a special case where the *duality* condition $W^{-}(A) = 1 - W^{+}(S - A)$ holds for all events A. In the special case that the weighting function W satisfies additivity, i.e. W is a probability measure, CEU coincides with SEU.

Convexity of the weighting function was usually used to formalize the idea of ambiguity aversion in the literature (Schmeidler 1989). Subsequently, convexity (concavity) was shown to correspond to a pessimistic (optimistic) preference relation under uncertainty (Wakker 2001a). Empirical findings show, however, that the weighting function is inverse-S shaped, i.e., overweights unlikely events and underweights likely events (e.g., Tversky and Fox 1995, Wu and Gonzalez 1999, Kilka and Weber 2001). These findings were formalized by means of *bounded subadditivity* (Tversky and Wakker 1995), which comprises *lower subadditivity* (LSA) and *upper subadditivity* (USA). LSA (USA) corresponds to concavity (convexity) on unlikely (likely) events. LSA and USA of $W(.)$ are defined as follows:

(2) LSA: $W(A) \geq W(A \cup B) - W(B)$, provided that $W(A \cup B) \leq W(S - E)$;

(3) USA: $1 - W(S - A) \geq W(A \cup B) - W(B)$, provided that $W(B) \geq W(E')$;

where E and E' are boundary events. LSA (USA) implies that the impact of an event on $W(.)$ is greater when it is added to the null event (subtracted from the universal event) than when it is added to some non-null event (subtracted from some "non-universal" event). LSA (USA) therefore captures what is known as the "possibility effect" ("certainty effect").

Under CPT, decision weights are no longer pure measures of belief. Following Fellner (1961), a decomposition of decision weights into a belief component and a decision attitude component was introduced by Tversky and Fox (1995) and Fox and Tversky (1998). Judged probabilities were used to capture the belief component, which is related to decision weights according to the equation:

(4) $W(A_i) = w(q(A_i))$,

with $q(A_i)$ being the judged probability of event A_i and $w(.)$ the (CPT-) probability weighting function under risk.

A different approach by Wakker (2001b) starts out from the (empirically founded, see e.g. Tversky and Fox 1995) assumption that decision makers are less sensitive to uncertainty than to risk. He shows that this condition is equivalent to the existence of a subadditive and choice-based function $\hat{q}(.)$ such that

$$(5) \qquad W(A_i) = w(\hat{q}(A_i))$$

for all events A_i, where $w(.)$ denotes the probability weighting function under risk as in Equation (4). Wakker (2001b) also discusses the interpretation of the function $\hat{q}(.)$ and its relation to judged probabilities of support theory.

3. Elicitation of Utility Function and Decision Weights

3.1. Elicitation of the Utility Function

Our approach for eliciting decision weights amounts to a two-step procedure. The first step, which is based on the trade-off method, elicits the utility function by determining a standard sequence of outcomes, i.e. a sequence of outcomes equally spaced in utility terms. In the second step, decision weights can be computed using the utility values obtained in the first step as inputs.

In our experimental investigation, standard sequences are elicited for gains and losses, using monetary outcomes. The elicitation of the standard sequence for gains is performed as follows. Let $R^+ < r^+ < x_0 = 0$ denote three fixed outcomes and A a specified event. As a first step, the outcome x_1^+ is determined such that the decision maker is indifferent between the prospects $(x_0, A; r^+)$ and $(x_1^+, A; R^+)$. As a second step, the decision maker is called to state the outcome x_2^+ such that indifference between the prospects $(x_1^+, A; r^+)$ and $(x_2^+, A; R^+)$ holds. Assuming that CPT is the adequate descriptive theory of choice, the combination of the equations resulting from the above two indifference statements implies the equality of $u(x_2^+) - u(x_1^+)$ and $u(x_1^+) - u(x_0)$.

The next steps follow the general principle that once outcome x_i^+ has been elicited, outcome x_{i+1}^+ leading to indifference between $(x_i^+, A; r^+)$ and $(x_{i+1}^+, A; R^+)$ has to be determined. The elicitation procedure results in an increasing sequence of outcomes $x_0, x_1^+, ..., x_n^+$ such that

$$(6) \qquad u(x_{i+1}^+) - u(x_i^+) = u(x_i^+) - u(x_{i-1}^+), \quad i = 1, ..., n-1 .$$

Likewise, with $R^- > r^- > x_0 = 0$ and the same event A, a decreasing standard sequence of losses $x_0, x_1^-, ..., x_n^-$ is elicited.

3.2. Elicitation of Decision Weights

Let Γ denote a family of events based on the relevant source of uncertainty and A_j a typical element thereof. The determination of the decision weights for the gain domain, which builds upon the standard sequence of outcomes for gains $x_0 = 0, x_1^+, \ldots, x_n^+$, proceeds as follows. For each event $A_j \in \Gamma$, the certain amount of money, denoted CE_j^+, that the decision maker finds equally desirable to the prospect $(x_n^+, A_j; 0)$ is assessed. Under CPT, and with the normalization conditions $u(0) = 0$ and $u(x_n^+) = 1$, this indifference statement translates into:

(7) $$W^+(A_j) = u(CE_j^+)\,.$$

Using the normalization conditions $u(0) = 0$ and $u(x_n^-) = -1$, decision weights for the loss domain are obtained by means of $W^-(A_j) = -u(CE_j^-)$.

Except by coincidence, $CE_j^\bullet$ itself is not an element of the standard sequence of outcomes which means that $u(CE_j^\bullet)$ is not immediately available.[2] Because utility is approximately linear over small intervals of outcomes, we determine $u(CE_j^\bullet)$ by linear interpolation between the two adjacent elements of the standard sequence (e.g., Bleichrodt and Pinto 2000). Furthermore, we additionally fit a number of parametric families to our data and use the estimated functions to determine utilities in order to check the robustness of our results. Details of our approach will be set out in Subsection 5.2.

3.3. Elicitation of Subjective Probabilities

The central idea underlying the two-stage approach is that decision weights comprise a component reflecting belief and a component reflecting decision attitude. In order to make the belief component explicit, a subjective probability $\hat{q}(A_j)$ is assessed such that the decision maker is indifferent between the risky prospect $(x_n^+, \hat{q}(A_j); 0)$ and the uncertain prospect $(x_n^+, A_j; 0)$, for each event $A_j \in \Gamma$. Assuming CPT, this indifference statement implies

(8) $$w^+(\hat{q}(A_j)) = W^+(A_j)\,.$$

It should be noted that the latter equality is derived without invoking the original two-stage approach purely by matching a risky simple prospect to an uncertain simple prospect involving the same non-null outcome. The belief component is thus elicited in a "choice-based" manner, as opposed to the approach of asking subjects to state judged probabilities utilized by

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[2] Here and henceforth, the superscript $^\bullet$ stands for both $^+$ (gains) and $^-$ (losses).

e.g. Tversky and Fox (1995) or Fox et al. (1996). Equation (8) corresponds to the relation obtained in Theorem 1 of Wakker (2001b).

Subjective probabilities are determined through the use of gain prospects. They can be applied to decompose decision weights for losses according to the equation:

(9) $w^{-}(\hat{q}(A_j)) = W^{-}(A_j)$.

4. Experiment

4.1. Subjects and Procedure

Forty-one subjects (thirty-three male, eight female) participated in our study. All of them were graduate students of business administration at the University of Mannheim. They were enrolled in a decision analysis course and hence familiar with probability and expected utility basics. Each subject received a fixed payment of DM 50 (≈ 25 $) for participation.

The experiment was conducted in the form of computer-based individual interview sessions. A special software had been developed for purposes of the present experiment. The subject and the experimentalist were seated in front of a personal computer. The experimentalist presented the subject with various choice situations associated with the different experimental tasks and entered the subject's statements into the computer. Subjects were encouraged to take as much time for reflection as they considered necessary. The mean time needed to complete the experiment as a whole amounted to approximately two hours. Subjects were given the opportunity of a break of few minutes after the first half of the experiment.

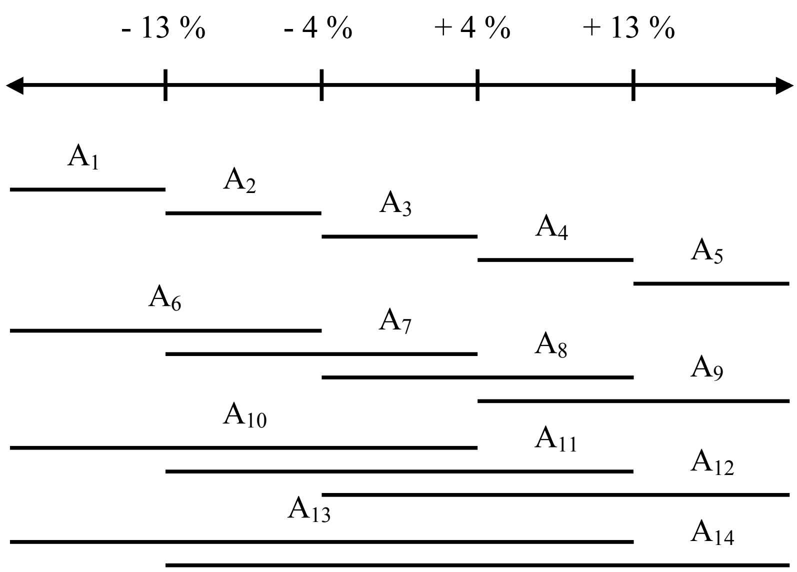
Participants were not directly asked for the specific (outcome or probability) value leading to indifference, i.e. we did not employ a matching procedure. Instead, every value was assessed by means of a series of binary choice questions. The screenshot in Figure 1 in the Appendix displays the typical choice situation that participants faced. If the participant expressed preference for the left-hand (right-hand) prospect, the right-hand prospect was modified to become more (less) attractive. This process continued until the participant regarded the two prospects appearing on the screen as equally desirable.

4.2. Method and Stimuli

The utility function was elicited from six indifferences. For gains (losses) this amounts to the construction of a standard sequence $x_0 = 0, x_1^+, \ldots, x_6^+$ ($x_0 = 0, x_1^-, \ldots, x_6^-$). We chose R^+ = -400 DM (R^- = 400 DM) and r^+ = -100 DM (r^- = 100 DM). The event *A* was defined as "CDU wins the German general election in 2002". The stimuli were chosen to guarantee that the utility curvature over the interval $[0, x_6^+]$ ($[x_6^-, 0]$) would not be negligible (see Wakker and Deneffe 1996, p. 1139).

The source of uncertainty used to elicit and decompose decision weights was the stock index DAX, which is computed as a capitalization-weighted average of the stock prices of the thirty largest companies listed in Germany. We defined the relevant source of uncertainty as "percentage change of the DAX index over the next six months", measured from the respective day on which the experiment took place.

Figure 2: Event space



By the construction of a stock index, the state space is bounded below by -100%, whereas there is no logical upper bound. We partitioned the space of feasible percentage changes to create five elementary events. Abbreviating "percentage change of the DAX index over the next six months" by ΔDAX, these are $A_1 = \{\Delta\text{DAX} < -13\%\}$, $A_2 = \{-13\% \leq \Delta\text{DAX} < -4\%\}$, $A_3 = \{-4\% \leq \Delta\text{DAX} < +4\%\}$, $A_4 = \{+4\% \leq \Delta\text{DAX} < +13\%\}$ and $A_5 = \{+13\% \leq$

ΔDAX}.[3] Our event space comprises the five elementary events plus all unions formed from the elementary events that result in contiguous intervals. The event space is depicted in Figure 2.

The order in which the fourteen events appeared during the experiment was determined randomly for each subject. It was fixed for all tasks that were based on this set of events (i.e. the decision weights tasks (gains and losses) and the subjective probabilities task).

In order to be able to assess the reliability of subjects' answers, we presented them twice with several choice situations, or more specifically, with those leading to the determination of x_1^+, x_1^-, CE_2^+, CE_2^-, CE_{12}^+, CE_{12}^-, $\hat{q}_2$, and $\hat{q}_{12}$. These choice situations reappeared at the end of the respective task they belong to except for the outcomes x_1^+ and x_1^- which were elicited once again at the end of the whole experiment.

5. Results

5.1. Reliability

For purposes of the present study, reliability denotes the proximity of the values elicited from a subject when an identical choice task is presented twice. Table 1 displays the results of the paired *t* tests performed to test reliability.

Table 1: Tests of reliability (paired *t* tests, two-tailed)

	$x_1^{\bullet}$	$CE_2^{\bullet}$	$CE_{12}^{\bullet}$	$\hat{q}_2$	$\hat{q}_{12}$
Gains	$t_{29} = -1.46^{ns}$	$t_{40} = 0.40^{ns}$	$t_{40} = -1.73^{ns}$	$t_{40} = -0.08^{ns}$	$t_{40} = -2.05^{*}$
Losses	$t_{29} = -1.42^{ns}$	$t_{40} = 0.71^{ns}$	$t_{40} = 1.60^{ns}$	---	---

ns : non-significant for α = 0.05; *: $p < 0.05$

The overall picture confirms the consistency of subjects' responses. Only for one of the subjective probabilities tasks ($\hat{q}_{12}$), a slightly significant difference is detected by the paired *t* test ($t_{40} = -2.05$, $p = 0.05$; two-tailed).

[3] The rationale behind the chosen partition is as follows: Assuming that the index change roughly follows a normal distribution, we estimated its standard deviation based on current financial data (adjusting for the time horizon in our study) and set the expected change equal to zero (because of the relatively short time horizon in our study). We then fixed interval boundaries such that the cumulative density of each interval amounts to 0.2.

5.2. Utility Function

To classify the curvature of participants' utility functions, we define $d_i^\bullet = x_i^\bullet - x_{i-1}^\bullet$, i = 1, ..., 6, and $\Delta_j^\bullet = \text{sign}(d_{j+1}^\bullet - d_j^\bullet)$, j = 1, ..., 5, for both domains separately. A utility function is labeled concave / linear / convex in a particular domain if at least three out of five $\Delta_j^\bullet$'s equal 1 / 0 / -1, and mixed otherwise. The idea behind this classification scheme is that even if the utility function of the subject's "true" preference functional (which we assume is CPT) exhibits a unique curvature, the stochastic component of her preference is likely to prevent all Δ_j's from being equal. An equivalent classification scheme has been applied in the studies of Abdellaoui (2000) and Bleichrodt and Pinto (2000). As opposed to the present study, however, both Abdellaoui (2000) and Bleichrodt and Pinto (2000) employed risky prospects for the elicitation of the utility function.

Table 2: Classification of utility functions (Trade-off method)

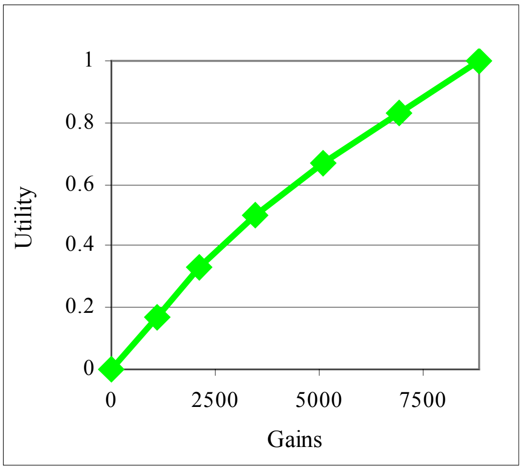
	Gains				Losses			
	concave	linear	convex	mixed	concave	linear	convex	mixed
Present study[1]	46.3 %	7.3 %	19.5 %	26.8 %	22.0 %	22.0 %	24.4 %	31.7 %
Abdellaoui (2000)[2]	52.5 %	17.5 %	20 %	10 %	20 %	25 %	42.5 %	12.5 %
Bleichrodt and Pinto (2000)[2]	59.2 %	26.5 %	2.0 %	12.2 %	no data			

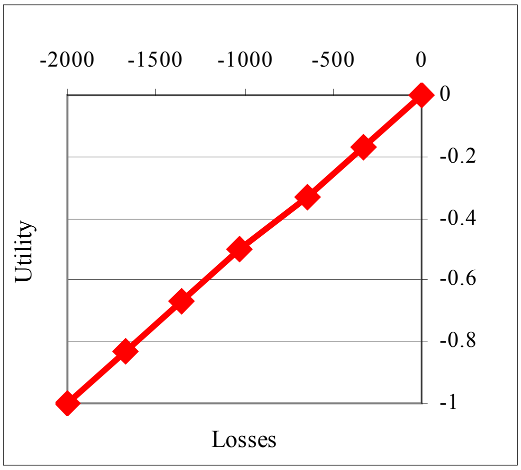
[1]: Utility elicited with unknown probabilities; [2]: Utility elicited with known probabilities

The results of the present study are displayed in the first row of Table 2. With respect to the gain domain, 19 subjects (46.3 %) are classified as having a concave utility function. Besides, there are 3 classifications as linear (7.3 %), 8 classifications as convex (19.5 %), and 11 classifications as mixed (26.8 %). The modal curvature of the utility function in the gain domain is thus concavity, in accordance with both the contention in Tversky and Kahneman (1992) and the respective results of the two related studies. What distinguishes our findings most remarkably from the two related studies is the comparatively high percentage of mixed classifications. The relative frequency of concave shapes conditional upon a classification as non-mixed amounts to 63.3 %, which is quite in line with the corresponding figures of 58.3 % in Abdellaoui (2000) and 67.4 % in Bleichrodt and Pinto (2000).

With respect to the loss domain, only 10 subjects (24.4 %) exhibit the hypothesized convex shape of the utility function. Besides, there are 9 classifications as linear (22.0 %), 9 classifications as concave (22.0 %), and 13 classifications as mixed (31.7 %). Whereas the preponderance of mixed shapes parallels the earlier observation of the relatively high number of mixed classifications in the gain domain, the lack of a distinct pattern among the non-mixed classifications comes as a surprise. It is not entirely unprecedented, though, as both in Abdellaoui (2000) and in Fennema and van Assen (1999) the evidence in favor of convexity in the loss domain is somewhat weaker than the evidence in favor of concavity in the gain domain.

Figure 3: Utility functions (for mean values)





When only concave and convex shapes are taken into account, a binomial test reveals that there are significantly more concave utility functions in the gain domain than convex ones ($p = 0.03$; one-tailed), whereas the reverse statement for the loss domain cannot be established ($p = 0.5$; one-tailed). These findings are also mirrored in the aggregate data. Figure 3 displays the utility functions, separately for each domain, that result when the elements of the respective standard sequences are averaged over participants.

In addition to the non-parametric classification of utility functions presented above, parametric estimations were conducted. Their purpose is twofold: First, they provide another way of looking at the utility function data and allow for comparisons with studies that adopt parametric approaches. Second, the estimated utility functions serve as input in the determination of decision weights as explained in Subsection 3. Two parametric forms were used: power and exponential (Table 3).

Table 3: Parametric specifications

	Power	Exponential
Gains	$z^{\alpha+}$	$(1-exp(-\lambda^{+}z))/(1-exp(-\lambda^{+}z))$
Losses	$-z^{\alpha-}$	$-(1-exp(-\lambda^{-}z))/(1-exp(-\lambda^{-}z))$

The power family is frequently employed in experimental studies on utility measurement. Its widespread use possibly has roots in evidence that specifically demonstrates its suitability to approximate subjects' utility function (e.g. Tversky 1967). Exhibiting the simple property of constant absolute risk aversion, the exponential specification is also extensively used in utility assessment from laboratory experiments as well as decision analysis interviews. The domain $[0, x_6^{+}]$ for gains ($[x_6^{-}, 0]$ for losses) is mapped into the positive (negative) unit interval through the rescaling $z = x/x_6^{+}$.

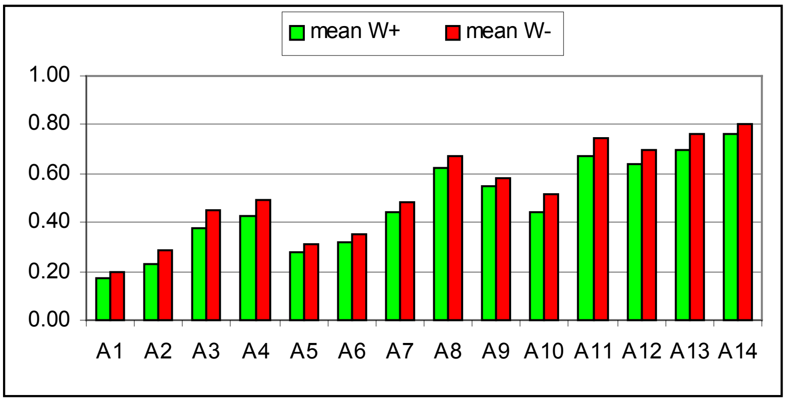
Table 4: Summary statistics for parameters of the utility function

	Gains			Losses		
	median	mean	st.dev.	median	mean	st.dev.
Power: $\alpha^{\bullet}$	0.91	0.91	0.32	0.96	1.08	0.47
Exponential: $\lambda^{\bullet}$	0.28	0.57	1.21	0.09	0.03	1.19

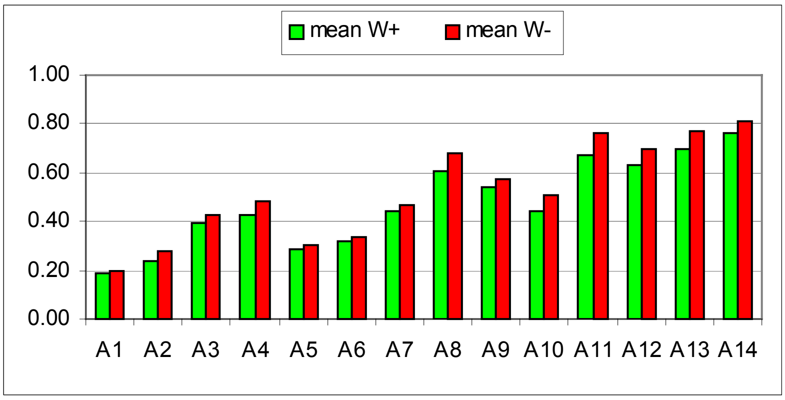
For gains, both the power utility function and the exponential utility function exhibit a concave shape for the mean and median parameter estimates, so that the non-parametric findings are affirmed. The estimate for the power utility function almost coincides with the median estimate of 0.89 in Abdellaoui (2000) and the median estimate of 0.88 in Tversky and Kahneman (1992). With regard to the loss domain, the parametric fitting also supports the non-parametric results at variance with the hypothesized convex shape. For the power utility function and the exponential utility function, the mean and median parameter estimates are near to or even on either side of the respective borderline separating concave and convex shapes (i.e. 1 for the power utility function and 0 for the exponential utility function). The parameter estimate for the power utility function therefore deviates from the median estimate of 0.92 reported in Abdellaoui (2000) and the median estimate of 0.88 obtained by Tversky and Kahneman (1992).

Figure 4: Mean decision weights

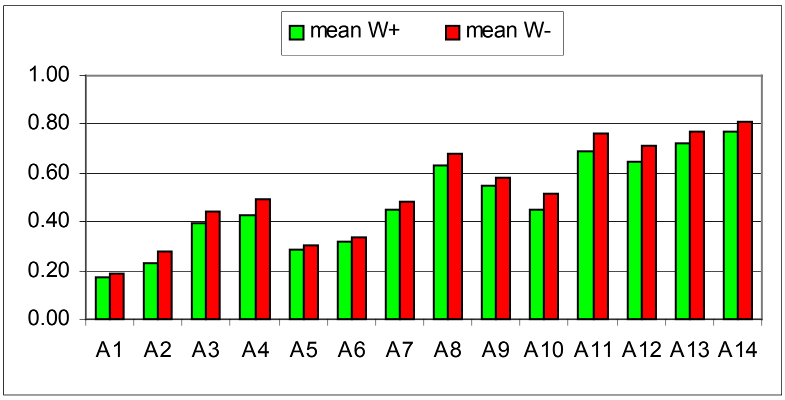
I: Linear interpolation



II: Power approximation



III: Exponential approximation



5.3. Decision Weights

As explained in Subsection 3.2, the decision weight of event A_j, $W^+(A_j)$ ($W^-(A_j)$), equals the (absolute value of the) utility of the certainty equivalent of the prospect (x_n^+, A_j; 0) ((x_n^-, A_j; 0)). The utility function is obtained by both linear interpolation between the outcomes of the standard sequence and parametric fitting of the families listed in Table 3.

Figure 4 displays mean decision weights for gains and losses. It can be seen that mean decision weights satisfy all monotonicity conditions imposed by the structure of the event space as depicted in Figure 2, which underscores the high quality of subjects' responses. Overall, Figure 4 suggests that, for each outcome domain, mean decision weights are not sensitive to the parametric specification used for the utility function. One factor ANOVA tests with repeated measures detect significant differences between the three specifications in 11 (5) out of 28 cases for α fixed at 0.05 (0.01). It seems however that the "traditional" power family produces decision weights that are closer to those computed by means of linear interpolation. This is confirmed by (two-tailed) paired t tests which detect significant differences between linear interpolation and the power specification in only 4 (2) out of 28 cases if α is fixed at 0.05 (0.01).

Figure 4 also shows that mean decision weights referring to the loss domain exceed their gain domain counterparts for all events. In the terminology used for the probability weighting function under risk (Gonzalez and Wu 1999), decision weights for the loss domain seem to exhibit more elevation. This statement is statistically supported more for likely events than for unlikely ones. As can be seen from Table 5, paired t tests, conducted separately for each event, lead to a rejection of the null hypothesis $W^+(A_j) = W^-(A_j)$ in favor of $W^+(A_j) < W^-(A_j)$ in about one half of the 14 cases ($\alpha = 0.05$, one-tailed).

It should be observed, however, that the event-wise comparison of decision weights is not necessarily a very effective technique to detect potential differences between the two domains. This limitation can be overcome using the two-stage approach that permits a characterization of the elevation of a participant's decision weights by means of a single overall parameter. We will therefore return to the aspect of elevation in Subsection 5.4.

A second important feature of decision weights that has to be distinguished from elevation is sensitivity. Sensitivity is a global measure that characterizes the increase in weight induced by "adding" some (disjoint) event to an underlying (non-null) event. The notion of sensitivity is thus related to the bounded subadditivity property of decision weights presented in Section 2 which says that the impact of an event is particularly strong if it is added to the null event or subtracted from the universal event.

Table 5: Elevation of decision weights (paired t tests, one-tailed)

	Linear interpolation	Power approximation	Expo. approximation
	t_{40}	t_{40}	t_{40}
A_1	0.72 ^{ns}	0.26 ^{ns}	0.48 ^{ns}
A_2	2.26 *	1.75 *	1.80 *
A_3	2.13 *	1.39 ^{ns}	1.51 ^{ns}
A_4	1.67 ^{ns}	1.36 ^{ns}	1.39 ^{ns}
A_5	0.65 ^{ns}	0.05 ^{ns}	0.15 ^{ns}
A_6	1.02 ^{ns}	0.64 ^{ns}	0.52 ^{ns}
A_7	1.25 ^{ns}	0.96 ^{ns}	0.87 ^{ns}
A_8	1.81 *	2.25 *	1.83 *
A_9	1.00 ^{ns}	1.04 ^{ns}	0.87 ^{ns}
A_{10}	2.23 *	2.20 *	1.99 *
A_{11}	2.13 *	2.42 *	2.00 *
A_{12}	1.62 ^{ns}	2.17 *	1.83 *
A_{13}	1.83 *	2.21 *	1.74 *
A_{14}	1.29 ^{ns}	1.70 *	1.14 ^{ns}

ns: non-significant for $\alpha = 0.05$; *: $p < 0.05$

The structure of the event space in the present study leads to 16 conditions to test for LSA[4] and to 6 conditions to test for USA[5], separately for each domain. For each participant, the respective number of conditions satisfied is computed. To obtain conservative results, a condition counts as satisfied only if strict inequality holds. Figure 5 displays the empirical distribution functions of conditions satisfied.

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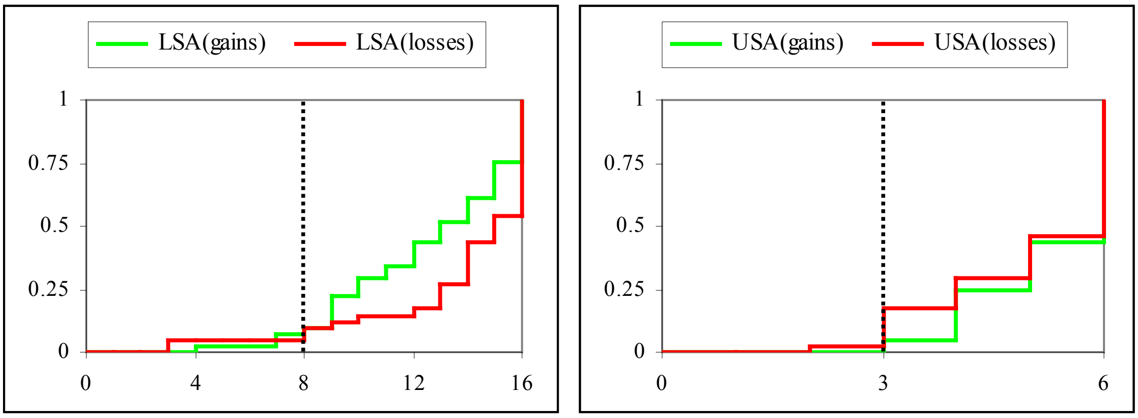
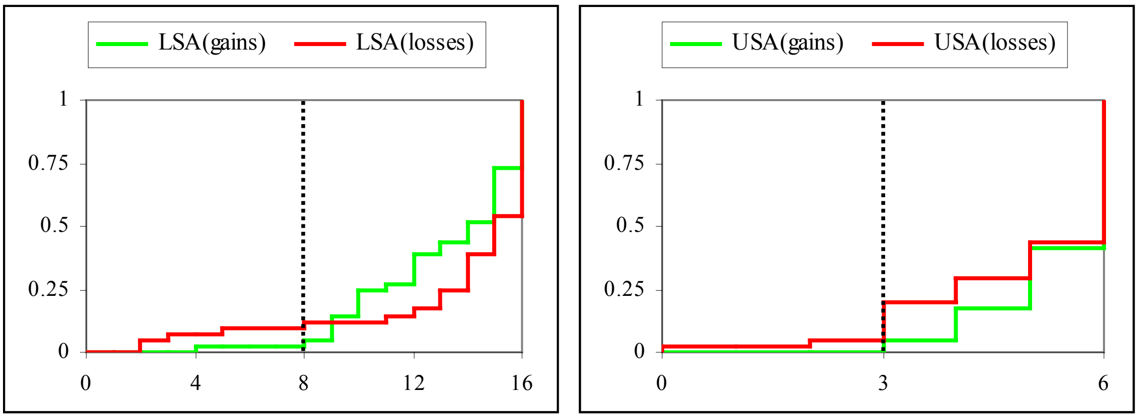
[4] LSA implies e.g. that $W^\bullet(A_1) \geq W^\bullet(A_6) - W^\bullet(A_2)$, where $A_6 = A_1 \cup A_2$. The fifteen other triples of events (A_i; $A_i \cup A_j$, A_j) allowing a test of LSA are (A_2; A_7, A_3), (A_3; A_8, A_4), (A_4; A_9, A_5), (A_1; A_{10}, A_7), (A_2; A_{11}, A_8), (A_3; A_{10}, A_6), (A_3; A_{12}, A_9), (A_4; A_{11}, A_7), (A_5; A_{12}, A_8), (A_1; A_{13}, A_{11}), (A_2; A_{14}, A_{12}), (A_4; A_{13}, A_{10}), (A_5; A_{14}, A_{11}), (A_6; A_{13}, A_8), and (A_7; A_{14}, A_9). All the triples satisfy the boundary condition $W^\bullet(A_i \cup A_j) \leq W^\bullet(S - E)$ for some non-null event E.

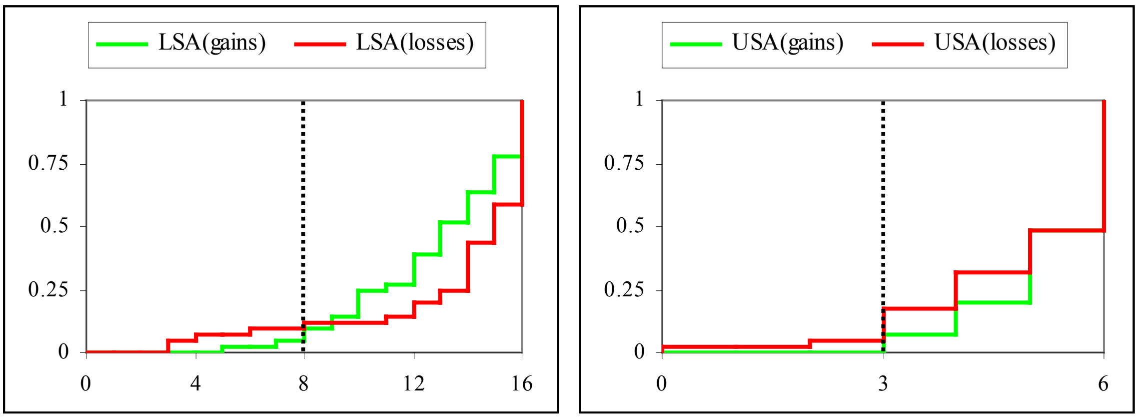
I: Linear interpolation



II: Power approximation



III: Exponential approximation



If decision weights were perfectly additive, the number of conditions satisfied would equal zero in all cases. If decision weights were fundamentally additive but with a nonsystem-

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[5] USA implies e.g. that $1 - W^{\bullet}(A_{14}) \geq W^{\bullet}(A_6) - W^{\bullet}(A_2)$, where $S - A_{14} = A_6 - A_2 = A_1$. The five other triples of events ($S - A_i$; $A_i \cup A_j$, A_j) allowing a test of USA are (A_{14}; A_{10}, A_7), (A_{14}; A_{13}, A_{11}), (A_{13}; A_9, A_4), (A_{13}; A_{12}, A_8), and (A_{12}; A_{10}, A_3). All the triples satisfy the boundary condition $W^{\bullet}(A_j) \geq W^{\bullet}(E')$ for some non-null event E'.

atic error component, the number of conditions satisfied would be stochastic and its distribution should be centered around one half of the number of available conditions.

Figure 5 provides strong evidence against "noisy additivity" of decision weights and therefore against the descriptive validity of SEU. With respect to LSA (USA), more than 8 (3) conditions are satisfied by 90.2 % (95.1 %) of the subjects in the gain domain and by 90.2 % (82.9 %) of the subjects in the loss domain. Aggregating over all subjects, the proportion of LSA (USA)-conditions satisfied amounts to 78.7 % (87.8 %) in the gain domain and to 86.4 % (84.1 %) in the loss domain. These figures closely resemble the results reported in Tversky and Fox (1995), table 4 (p. 276). The median number of LSA (USA) conditions satisfied is 13 (6) in the gain domain and 15 (6) in the loss domain.

To also obtain a quantification of the degree of subadditivity of decision weights, the measure initially applied by Tversky and Fox (1995) and thereafter used by e.g. Kilka and Weber (2001) is computed. An equivalent statement of the condition defining LSA (USA) is $W^{\bullet}(A_i) + W^{\bullet}(A_j) - W^{\bullet}(A_i \cup A_j) \geq 0$ ($1 - W^{\bullet}(S - A_i) - (W^{\bullet}(A_i \cup A_j) - W^{\bullet}(A_j)) \geq 0$). The index of LSA (USA), henceforth denoted by μ_W(LSA) (μ_W(USA)), is computed as the mean value of the left-hand side of the above inequalities, taken over the 16 (6) triples of events allowing a test of the respective condition. It is computed separately for each participant and each domain. The median values (where medians are taken over participants) of the indices amount to μ_W^+(LSA) = 0.18 (μ_W^+(USA) = 0.20) in the gain domain and to μ_W^-(LSA) = 0.25 (μ_W^-(USA) = 0.18) in the loss domain. The degree of subadditivity in our data is consistent with the findings of Tversky and Fox (1995), table 5 (p. 277), and to the findings concerning the source of uncertainty with higher perceived competence in Kilka and Weber (2001), table 1 (p. 1719). The pervasiveness of subadditivity is also mirrored by the fact that the LSA (USA)-index is strictly positive for 90.2 % (100 %) of the subjects in the gain domain and for 92.7 % (92.7 %) of the subjects in the loss domain. Both t tests of the hypothesis that the mean index value is zero and binomial tests of the hypothesis that positive and negative index values are equally likely result in $p < 0.001$ for each of the four domain-property-combinations.

It can also be asked whether the degree of subadditivity of decision weights differs across the two domains. The null hypothesis of equal mean (respectively median) LSA / USA-index values cannot be rejected by either paired t tests or sign tests at conventional levels of significance (each $p > 0.05$, one-tailed). This result suggests that the sensitivity of decision weights does not exhibit domain-dependence. An alternative characterization of decision weights in terms of elevation and sensitivity will be presented in the following subsection.

Table 6: Tests of duality (t tests, two-tailed)

	Linear interpolation		Power approximation		Exponential approximation	
	mean	t_{40}	mean	t_{40}	mean	t_{40}
$W^{-}(A_1) + W^{+}(S - A_1)$	0.955	-1.42 ^{ns}	0.954	-1.41 ^{ns}	0.965	-1.09 ^{ns}
$W^{-}(A_5) + W^{+}(S - A_5)$	1.012	0.29 ^{ns}	0.997	-0.08 ^{ns}	1.012	0.30 ^{ns}
$W^{-}(A_6) + W^{+}(S - A_6)$	0.989	-0.28 ^{ns}	0.971	-0.74 ^{ns}	0.985	-0.39 ^{ns}
$W^{-}(A_9) + W^{+}(S - A_9)$	1.020	0.48 ^{ns}	1.013	0.33 ^{ns}	1.032	0.74 ^{ns}
$W^{-}(A_{10}) + W^{+}(S - A_{10})$	1.067	1.62 ^{ns}	1.054	1.25 ^{ns}	1.073	1.63 ^{ns}
$W^{-}(A_{12}) + W^{+}(S - A_{12})$	1.012	0.32 ^{ns}	1.023	0.69 ^{ns}	1.032	0.88 ^{ns}
$W^{-}(A_{13}) + W^{+}(S - A_{13})$	1.044	1.30 ^{ns}	1.065	1.90 ^{ns}	1.064	1.77 ^{ns}
$W^{-}(A_{14}) + W^{+}(S - A_{14})$	0.973	-0.80 ^{ns}	0.998	-0.05 ^{ns}	0.985	-0.42 ^{ns}

ns: non-significant for $\alpha = 0.05$

If subjects' decision weights systematically violate duality ($W^{-}(A) = 1 - W^{+}(S - A)$ holds for all events A), one of the additional degrees of freedom introduced by CPT (as compared to CEU) is empirically warranted. Owing to the structure of the event space in the present study, there are eight pairs of event and complementary event that provide a test of the duality condition. Table 6 displays mean values of the respective sums of decision weights. Additionally, it shows t values for tests of the hypothesis that mean sums equal 1.

The overall picture is remarkably clear. The respective duality conditions are not significantly violated ($\alpha = 0.05$, two-tailed).

Table 7: Summary statistics for subjective probabilities

	$\hat{q}_1$	$\hat{q}_2$	$\hat{q}_3$	$\hat{q}_4$	$\hat{q}_5$	$\hat{q}_6$	$\hat{q}_7$	$\hat{q}_8$	$\hat{q}_9$	$\hat{q}_{10}$	$\hat{q}_{11}$	$\hat{q}_{12}$	$\hat{q}_{13}$	$\hat{q}_{14}$
median	0.10	0.17	0.35	0.42	0.20	0.25	0.40	0.66	0.55	0.50	0.75	0.72	0.75	0.85
mean	0.13	0.20	0.36	0.43	0.23	0.27	0.42	0.62	0.54	0.48	0.73	0.68	0.75	0.82
st.dev.	0.09	0.13	0.19	0.21	0.15	0.15	0.19	0.20	0.21	0.17	0.16	0.17	0.13	0.13

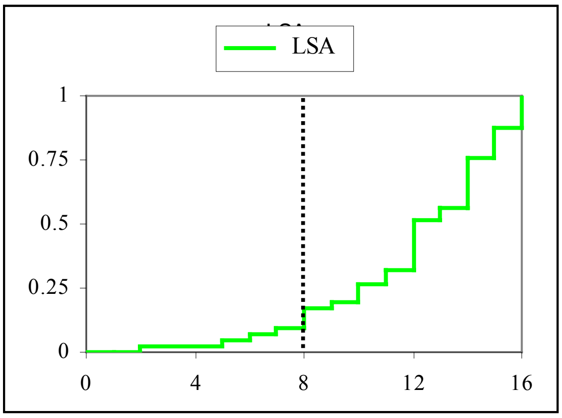
5.4. Subjective Probabilities and Decision Weights

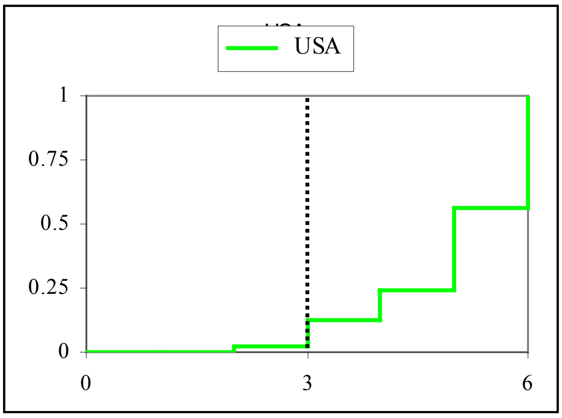
In this subsection, decision weights are no longer treated as an entity, but are related to subjective probabilities via the two-stage approach. The theoretical foundation of the two-stage approach was set out in Section 2.

Table 7 contains summary statistics (median, mean, and standard deviation) for the subjective probabilities. Parallel to the finding for decision weights, it can be verified that median and mean subjective probabilities satisfy all monotonicity conditions (for subjective probabilities) imposed by the structure of the event space.[6] We again view this as evidence supporting the high quality of the subjects' responses.

As pointed out in detail in Sections 2 and Subsection 3.3, subjective probabilities in the present study are determined through choices and in this respect differ from judged probabilities elicited in studies relying on the original two-stage model. This aspect naturally leads to the question of whether the properties of the measure of belief are affected by the elicitation mode. Subadditivity is one of the key properties of judged probabilities in support theory (Tversky and Koehler 1994) and also crucial in the derivation of the decomposition in Wakker (2001b). Parallel to the analysis of the subadditivity property for decision weights, Figure 6 displays the empirical distribution functions of LSA / USA-conditions satisfied.

Figure 6: Subadditivity of subjective probabilities





The graphs in Figure 6 closely resemble their decision weights counterparts in Figure 5. With respect to LSA (USA) of subjective probabilities, more than 8 (3) conditions are satisfied by 82.9 % (87.8 %) of the subjects. We also compute indices of subadditivity for subjective probabilities, denoted by μ_q(LSA) and μ_q(USA), which are defined analogously to the respective measures for decision weights, μ_W(LSA) and μ_W(USA), presented in Subsection 5.3. The LSA (USA)-index is strictly positive for 90.2 % (97.6 %) of the subjects. The median values (where medians are taken over participants) of these indices amount to μ_q(LSA) =

[6] Assuming the two-stage decomposition $W^{\bullet}(A_i) = w^{\bullet}(\hat{q}(A_i))$, the monotonicity condition for subjective probabilities, i.e. $\hat{q}(A) \leq \hat{q}(B)$ for all events A, B with $A \subset B$, follows from the respective condition for decision weights and the fact that the transformation function $w^{\bullet}(.)$ is non-decreasing.

0.12 and μ_q(USA) = 0.13. Two conclusions can be drawn from these figures. First, they provide strong empirical support for subadditivity of subjective probabilities, which thereby share an important property with judged probabilities (Tversky and Koehler 1994, Tversky and Fox 1995, Fox et al. 1996, Wu and Gonzalez 1999, Kilka and Weber 2001). Second, since the indices of subadditivity are smaller in value for subjective probabilities than for decision weights, our data are consistent with an interpretation of decision weights as a subadditive measure of belief transformed by a subadditive function (Fox and Tversky 1998, Wakker 2001b).

Table 8: Decision weights vs. subjective probabilities (paired *t* tests, one-tailed)

	Linear interpolation		Power approximation		Expo. approximation	
	Gains	Losses	Gains	Losses	Gains	Losses
	t_{40}	t_{40}	t_{40}	t_{40}	t_{40}	t_{40}
$W^\bullet(A_1) - \hat{q}_1$	1.51 ^{ns}	3.92 ***	2.06 *	3.63 ***	1.56 ^{ns}	3.63 ***
$W^\bullet(A_2) - \hat{q}_2$	1.56 ^{ns}	3.63 ***	1.40 ^{ns}	4.17 ***	1.06 ^{ns}	4.09 ***
$W^\bullet(A_3) - \hat{q}_3$	1.06 $^{ns.}$	4.09 ***	0.87 ^{ns}	3.06 **	0.92 ^{ns}	3.46 ***
$W^\bullet(A_4) - \hat{q}_4$	0.92 ^{ns}	3.46 ***	-0.06 ^{ns}	1.64 $^{n.s.}$	0.05 ^{ns}	1.79 *
$W^\bullet(A_5) - \hat{q}_5$	0.05 ^{ns}	1.79 *	2.20 *	3.18 **	1.99 *	3.20 **
$W^\bullet(A_6) - \hat{q}_6$	1.99 *	3.20 **	2.06 *	2.89 **	2.05 *	2.98 **
$W^\bullet(A_7) - \hat{q}_7$	2.05 *	2.98 **	0.76 ^{ns}	2.08 *	1.00 ^{ns}	2.29 *
$W^\bullet(A_8) - \hat{q}_8$	1.00 ^{ns}	2.29 *	-0.20 ^{ns}	2.87 **	0.27 ^{ns}	2.89 **
$W^\bullet(A_9) - \hat{q}_9$	0.27 ^{ns}	2.89 **	-0.20 ^{ns}	0.97 ^{ns}	0.25 ^{ns}	1.20 ^{ns}
$W^\bullet(A_{10}) - \hat{q}_{10}$	0.25 ^{ns}	1.20 ^{ns}	-1.54 ^{ns}	0.97 ^{ns}	-1.14 ^{ns}	1.21 ^{ns}
$W^\bullet(A_{11}) - \hat{q}_{11}$	-1.14 ^{ns}	1.21 ^{ns}	-1.81 *	1.11 ^{ns}	-1.33 ^{ns}	1.17 ^{ns}
$W^\bullet(A_{12}) - \hat{q}_{12}$	-1.33 ^{ns}	1.17 ^{ns}	-1.44 ^{ns}	1.31 ^{ns}	-0.98 ^{ns}	1.43 $^{ns.}$
$W^\bullet(A_{13}) - \hat{q}_{13}$	-0.98 ^{ns}	1.43 ^{ns}	-1.82 *	1.28 ^{ns}	-1.18 ^{ns}	1.41 $^{ns.}$
$W^\bullet(A_{14}) - \hat{q}_{14}$	-1.18 ^{ns}	1.41 ^{ns}	-2.12 *	-0.71 ^{ns}	-1.57 ^{ns}	-0.68 ^{ns}

ns: non-significant for $\alpha = 0.05$; *: $p < 0.05$; **: $p < 0.01$; ***: $p < 0.001$

It was already mentioned that CEU coincides with SEU for the special case that decision weights are subjective probabilities. Contrasting decision weights and subjective probabilities, it can be asked whether systematic deviations occur. Table 8 displays the results of paired t tests conducted to check whether mean decision weights equal mean subjective probabilities, separately for each event and each domain.

The pattern in the gain domain reflects an overweighting of less likely events and an underweighting of rather likely events (relative to the respective subjective probabilities). This finding is broadly consistent with evidence from studies of decision making under risk, particularly an inverse S-shaped transformation function (see e.g. Tversky and Kahneman 1992, Wu and Gonzalez 1996, Bleichrodt and Pinto 2000). The statistical significance of the observed differences is not high, however. The pattern in the loss domain, which is foreshadowed by the pattern in the gain domain and the comparison of decision weights across domains as presented in Figure 4, indicates a marked overweighting of less likely events. An underweighting of rather likely events cannot be found in the data, yet the decrease of the difference between decision weights and subjective probabilities as one moves towards more likely events is common to both domains. From an overall perspective, the results contained in Table 8 suggest that CPT is descriptively more adequate than SEU.

Fitting the two-stage model to our data amounts to estimating the parameter(s) of the probability weighting function via nonlinear regression with decision weights as dependent variable and subjective probabilities as explanatory variable. The estimations are conducted separately for each participant. We employ several parametric forms originally proposed for the probability weighting function under risk. One parametric specification that proves to be particularly useful is the linear-in-log-odds form applied by e.g. Goldstein and Einhorn (1987), Lattimore et al. (1992) and Gonzalez and Wu (1999):

(10) $$w(p) = \frac{\delta \cdot p^{\gamma}}{\delta \cdot p^{\gamma} + (1-p)^{\gamma}} .$$ [7]

Its usefulness derives from the fact that it permits a distinction between two essential features of the probability weighting function: elevation and curvature (i.e. sensitivity in the context of probability weighting). For the linear-in-log-odds specification, the parameter δ mainly controls elevation, whereas the parameter γ mainly controls curvature. The issues of elevation and curvature of decision weights addressed in Subsection 5.3 can therefore be reanalyzed in the

[7] The linear-in-log-odds property is demonstrated in Gonzalez and Wu (1999), p. 139.

light of the respective parameter estimates of the probability weighting function in the two-stage model.[8]

Table 9: Summary statistics for parameters of the probability weighting function

		Linear-in-log-odds				Linear weighting			
		elevation (δ)		curvature (γ)		elevation		curvature (β)	
		median	mean (s.e.)	median	mean (s.e.)	median	mean (s.e.)	median	mean (s.e.)
Linear inter-polation	Gains	0.911	1.081 (0.106)	0.779	0.783 (0.057)	0.471	0.476 (0.018)	0.826	0.820 (0.064)
	Losses	1.195	1.520 (0.184)	0.786	0.783 (0.036)	0.537	0.535 (0.017)	0.820	0.860 (0.047)
Power ap-proximation	Gains	0.894	1.067 (0.098)	0.689	0.758 (0.056)	0.481	0.477 (0.018)	0.801	0.815 (0.068)
	Losses	1.209	1.445 (0.147)	0.809	0.846 (0.052)	0.532	0.536 (0.017)	0.834	0.882 (0.064)
Exponential approximation	Gains	0.924	1.231 (0.180)	0.826	0.795 (0.054)	0.484	0.483 (0.020)	0.877	0.846 (0.059)
	Losses	1.182	1.585 (0.210)	0.848	0.869 (0.051)	0.531	0.538 (0.018)	0.853	0.909 (0.059)

A parsimonious parametric form for the probability weighting function which nevertheless incorporates a clear separation between elevation and curvature is the linear approximation

(11) $w(p) = \alpha + \beta \cdot p \quad for \ p \in (0,1), w(0) = 0, w(1) = 1$,

applied in e.g. Wu and Gonzalez (1996) and Kilka and Weber (2001). Curvature is captured by the parameter β, whereas elevation can be characterized through $\int w(p)\mathrm{d}p$.[9]

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[8] The same kind of decomposition is feasible for the (two-parameter) probability weighting function derived axiomatically by Prelec (1998): $w(p) = exp(-\beta \cdot (-log(p))^{\alpha})$, where the parameter β mainly controls elevation and the parameter α mainly controls curvature. We also applied this probability weighting function in the analyses to be presented subsequently. For a small number of subjects, we obtained parameter estimates – especially for the parameter β – that were quite high, which would have unduly affected summary statistics and statistical inference. We therefore focus on the linear-in-log-odds function, complementing it with a parsimonious and particularly robust specification of the probability weighting function to avoid dependence upon a single parametric form.

By construction, probability weighting functions with a single free parameter like the one used by Karmarkar (1978), Tversky and Kahneman (1992) or the one-parameter variant in Prelec (1998) do not permit an independent variation of elevation and curvature. For this reason, the following presentation of results is restricted to the two-parameter specifications.

Table 9 conveys summary statistics (median, mean, and standard error) for the individually estimated parameters of the probability weighting function, separately for each domain. It can be seen that the mean value of the curvature parameter of the linear-in-log-odds specification is quite similar across the two domains ($\gamma^+ = 0.78$, $\gamma^- = 0.78$ for the linear interpolation; $\gamma^+ = 0.76$, $\gamma^- = 0.85$ for the power approximation; $\gamma^+ = 0.80$, $\gamma^- = 0.87$ for the exponential approximation). In contrast, the mean value of the corresponding elevation parameter varies markedly across domains ($\delta^+ = 1.08$, $\delta^- = 1.52$ for the linear interpolation; $\delta^+ = 1.07$, $\delta^- = 1.45$ for the power approximation; $\delta^+ = 1.23$, $\delta^- = 1.59$ for the exponential approximation), signifying more pronounced elevation in the loss domain.

Paired *t* tests are conducted to investigate the hypothesis of equal parameter values in the gains and loss domains. Whereas the curvature parameter (γ) is statistically indistinguishable across domains at conventional levels of significance ($t_{40} = 0.00$, $p = 1.00$ for the linear interpolation; $t_{40} = 1.26$, $p = 0.21$ for the power approximation; $t_{40} = 1.18$, $p = 0.25$ for the exponential approximation; two-tailed), the domain-dependence of the elevation parameter (δ) is confirmed ($t_{40} = 2.04$, $p = 0.02$ for the linear interpolation; $t_{40} = 2.08$, $p = 0.02$ for the power approximation; $t_{40} = 1.23$, $p = 0.11$ for the exponential approximation; one-tailed).

The conclusions derived for the linear-in-log-odds specification turn out to be robust when the simple linear probability weighting function is considered instead. Paired *t* tests show no significant difference between the curvature parameter (β) of the gain and loss domains ($t_{40} = 0.56$, $p = 0.58$ for the linear interpolation; $t_{40} = 0.78$, $p = 0.44$ for the power approximation; $t_{40} = 0.85$, $p = 0.40$ for the exponential approximation; two-tailed). The measure of elevation in the loss domain exceeds its gain domain counterpart significantly ($t_{40} = 2.34$, $p = 0.01$ for the linear interpolation; $t_{40} = 2.33$, $p = 0.01$ for the power approximation; $t_{40} = 2.04$, $p = 0.02$ for the exponential approximation; one-tailed).

It must be mentioned that the picture is somewhat less clear at the level of individual subjects with 26 out of 41 participants satisfying $\delta^- > \delta^+$ (for linear interpolation, power ap-

[9] Bounded subadditivity of the probability weighting function corresponds to $\alpha > 0$ and $\alpha + \beta < 1$, in which case the measure of elevation equals $\alpha + \beta/2$ (see also Kilka and Weber 2001, p. 1717).

proximation, and exponential approximation), p = 0.06 for a binomial test (one-tailed). The general conclusion is strengthened again by the results of the linear probability weighting function where the measure of elevation is higher in the loss domain for 28 subjects(for linear interpolation, power approximation, and exponential approximation), p = 0.01 for a binomial test (one-tailed).

6. Discussion and Conclusion

This paper provides a parameter-free and a fully choice-based elicitation and decomposition of decision weights under CPT. We found that SEU is violated in a systematic fashion in both gains and loss domains. The elicited weighting functions as well as choice-based subjective probabilities seem to be consistent with the psychological principle of diminishing sensitivity stipulating a decrease in marginal effect as a distance from a reference point increases. The reference points seem to be 0 and 1 for $w^+(.)$ and $w^-(.)$, and $\varnothing$ and S for $W^+(.)$, $W^-(.)$ and $\hat{q}(.)$. This suggests that the subjective treatment of uncertainty, in the presence of exogenously given probabilities as well as in their absence, is mainly governed by a simple psychological principle. On this point our paper extends the previous experimental findings by Tversky and Fox (1995), Fox and Tversky (1998), Wu and Gonzalez (1999) and Kilka and Weber (2001).

The other findings regard the shape of the utility function in both gain and loss domains and the usefulness of the introduction by CPT of a specific weighting function for losses. The paper reports the results of an experimental elicitation of utility using the trade-off method with unknown probabilities. For gains, concavity is the dominating shape of the utility function. Our results are particularly similar to those obtained under risk and without taking into account the null monetary outcome as a reference point (Wakker and Deneffe 1996, Bleichrodt and Pinto 2000, Abdellaoui 2000). For losses, no clear evidence in favor of convexity was observed, and this result is also consistent with previous findings under risk (Fennema and van Assen 1998).

Decision weights for the loss domain exhibit more elevation, particularly for more likely events. When the two-stage model is assumed, the resulting probability weighting functions exhibit more elevation for losses. Furthermore, the hypothesis of equal curvature across domains is not rejected. This is consistent with similar findings under risk (Abdellaoui 2000). The duality condition is not contradicted by our data, suggesting that CEU might approximate CPT in particular choice situations.

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Appendix

Figure 1: Screenshot of a typical choice task