DISCUSSION PAPER SERIES

No. 3752

INTEGRATION, REGIONAL AGGLOMERATION AND INTERNATIONAL TRADE

Philippe Monfort and Tanguy van Ypersele

INTERNATIONAL TRADE

Centre for Economic Policy Research

www.cepr.org

Available online at: **www.cepr.org/pubs/dps/DP3752.asp**

ISSN 0265-8003

INTEGRATION, REGIONAL AGGLOMERATION AND INTERNATIONAL TRADE

Philippe Monfort, Université Catholique de Louvain
Tanguy van Ypersele, Université de Namur and CORE and CEPR

Discussion Paper No. 3752
February 2003

Centre for Economic Policy Research
90–98 Goswell Rd, London EC1V 7RR, UK
Tel: (44 20) 7878 2900, Fax: (44 20) 7878 2999
Email: cepr@cepr.org, Website: www.cepr.org

This Discussion Paper is issued under the auspices of the Centre's research programme in **INTERNATIONAL TRADE**. Any opinions expressed here are those of the author(s) and not those of the Centre for Economic Policy Research. Research disseminated by CEPR may include views on policy, but the Centre itself takes no institutional policy positions.

The Centre for Economic Policy Research was established in 1983 as a private educational charity, to promote independent analysis and public discussion of open economies and the relations among them. It is pluralist and non-partisan, bringing economic research to bear on the analysis of medium- and long-run policy questions. Institutional (core) finance for the Centre has been provided through major grants from the Economic and Social Research Council, under which an ESRC Resource Centre operates within CEPR; the Esmée Fairbairn Charitable Trust; and the Bank of England. These organizations do not give prior review to the Centre's publications, nor do they necessarily endorse the views expressed therein.

These Discussion Papers often represent preliminary or incomplete work, circulated to encourage discussion and comment. Citation and use of such a paper should take account of its provisional character.

Copyright: Philippe Monfort and Tanguy van Ypersele

CEPR Discussion Paper No. 3752

February 2003

ABSTRACT

Integration, Regional Agglomeration and International Trade*

In this Paper, we develop a two-country four-region model allowing for a core periphery pattern inside countries. We then examine how both the integration and the agglomeration process inside a given country affects the pattern of specialization and international trade. We also analyse how agglomeration processes interact from one country to the other and in particular how agglomeration forces in one country are affected by the spatial distribution of activity in the partner country.

The main results of the analysis are the following: (i) agglomeration proves to be a source of comparative advantages in the industries featuring economies of scale on condition that this agglomeration process is driven by market forces; (ii) both integration and agglomeration in one country reduces the likelihood to observe agglomeration in the partner country.

JEL Classification: F12 and F15
Keywords: integration, international trade and regional agglomeration

Philippe Monfort
Departement de Sciences Economique
Université Catholique de Louvain
Place de Montesquieu 3
B-1348 Louvain-la-Neuve
BELGIUM
Tel: (32 10) 473 801
Fax: (32 10) 473 945
Email: monfort@ires.ucl.ac.be

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=146298

Tanguy van Ypersele
FUNDP Namur and Core
Université de Namur
Rempart de la Vierge, 8
B-5000 Namur
BELGIUM
Tel: (32 81) 72 4846
Fax: (32 81) 72 4840
Email: tanguy@fundp.ac.be

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=133931

*This Paper is produced as part of a CEPR Research Network on 'The Economic Geography of Europe: Measurement, Testing and Policy Simulations', funded by the European Commission under the Research Training Network Programme (Contract No: HPRN-CT-2000-00069).

Submitted 09 October 2002

Integration, Regional Agglomeration and International Trade

Philippe Monfort*and Tanguy van Ypersele†

19 March 2002 (Preliminary and incomplete)

Abstract

In this paper, we develop a two country four region model allowing for a core periphery pattern inside countries. We then examine how both the integration and the agglomeration process inside a given country affects the pattern of specialization and international trade. We also analyze how agglomeration process interact from one country to the other and in particular how agglomeration forces in one country is affected by the spatial distribution of activity in the partner country.

The main results of the analysis are the following: (i) agglomeration proves to be a source of comparative advantages in the industries featuring economies of scale on condition that this agglomeration process is driven by market forces; (ii) both integration and agglomeration in one country reduces the likelihood to observe agglomeration in the partner country.

1 Introduction

Economic geography proposes a formal and convenient framework to explore issues related to the spatial distribution of economic activities. In particular, it allows to isolate some important factors favoring the emergence of a core-periphery pattern in which activities agglomerate on a limited number of locations. This is indeed a well documented facts one can trace in various areas and circumstances. For example, two third of the European GDP is realized on around 4 % of its territory (EC, 1996). From a methodological point of view, this strand of literature allows to account for such phenomenon

*UCL-IRES Belgium.

†The University of Namur, CREW and CORE, Belgium.

by departing from the standard neo-classical model of trade as it relies on increasing returns and imperfect competition. Even though the pioneering works of Hotelling (1929), Lösch (1940) or Koopmans (1957) were already containing these basic ingredients one needs to explore such issues, it is the seminal contributions of the New economic geography that incorporate them in a coherent framework. These models therefore rigorously isolates several determinants of an agglomeration process. In particular, they emphasize the reduction of transport, or more generally, transaction costs as an important factor favoring the emergence of such a spatial distribution of economic activity.

However, few contributions examine the relation between the spatial distribution of economic activity observed in a country and its international trade performance. How would the volume and nature of trade be affected by the agglomeration of economic activities inside a given set of the trading countries? What consequences can we expect in terms of welfare for these countries that agglomerate their activities and for the others?

In order to relate these questions to the existing literature, it may be noted that, in many cases, agglomeration and market size are quite related issues. Amity (1998) examines how country size affects international trade pattern and shows how a market access effect attracts firm to the largest country to save on transport costs. Closer to our concern, Martin and Rogers (1995) show how the public infrastructure affects industrial location with the conclusion that infrastructure easing the contacts between consumers and producers can turn out to be a source of comparative advantages in sectors featuring economies of scale. Haaparanta (1999) examines how international trade can lead to the concentration of economic activities and shows how this can have some negative welfare effects. Finally, Monfort and Nicolini (2000) show how integration of international markets exacerbates the agglomeration forces at work inside the trading countries.

In this paper, we develope a two country four region model allowing for a core periphery pattern inside countries. We then examine how agglomeration inside a given country affects the pattern of specialization and international trade. We also analyze how agglomeration process interact from one country to the other and in particular how agglomeration forces in one country is affected by the spatial distribution of activity in the partner country.

The main results of the analysis are the following: (i) internal integration can act as a source of comparative advantages for industries characterized by increasing returns to scale in coherence with Martin and Rogers (1995); (ii) international trade fosters the agglomeration forces at work inside the trading countries; (iii) a necessary condition for agglomeration in one country to be a comparative advantage in the industry featuring economies of scale; (iv) both integration and agglomeration in one country reduces the likelihood to observe agglomeration in the partner country.

The paper is organized as follows. Section 2 develops a model based on Baldwin (1999) which allows to capture the different aspects relevant for this analysis. Section 3 examines the effects of integration and agglomeration both on international trade performance and on the nature of the spatial equilibrium. Finally, section 4 concludes.

2 The model

The model is a static variant of Baldwin (1999). Consider two countries (home and foreign), each of them divided in two regions (1 and 2). There are potentially, three sectors of production: services, food and manufacturing (S, F, M). Each region is endowed with one non tradable factor, labor (L). A fraction of the labor force has managerial abilities that can be used to create service firms. Whether they use this type of skill or not, managers also supply regular units of labor and earn a salary like the rest of the workforce.[1] Regions have similar endowment both in the total labor force and in managerial resources. Finally, consumers have identical preferences across countries.

Let us now detail the assumptions and functional forms adopted to describe the different agents' behavior.

2.1 Preferences

In each region, households consume the food and the differentiated manufacturing goods. Preferences are represented by the following utility function:

$$U = C_F^{1-\alpha} C_M^{\alpha}$$

$\alpha \in [0, 1]$, C_F is the consumption of food and C_M is the consumption of a composite of the differentiated manufacturing goods:

$$C_M = \left(\int_0^N c_k^{1-1/\sigma} dk \right)^{\frac{\sigma}{\sigma-1}}$$

[1]For instance, one can interpret this as a story in which each manager creates and owns a service firm in which he is also self-employed. Note that all our results hold if we assume that managers either work as managers or as workers. Adopting this assumption instead just introduces discontinuities that confuse the analysis.

N being the measure of the total number of variety of the manufacturing good, c_k the consumption of the variety k of the manufacturing good and $\sigma > 1$ the elasticity of substitution between varieties.

The Cobb-Douglas form adopted to represent preferences implies that a constant share $(1-\alpha)$ of the consumption spending, denoted by E, falls on food and the rest (α) on manufacturing. The demand in region x for a particular variety of manufacturing good is then:

$$c_k^x = \alpha E^x \frac{\left(p_k^x\right)^{-\sigma}}{\int_0^N \left(p_k^x\right)^{1-\sigma} dk} \tag{1}$$

where p_k^x is the price of the variety k prevailing in region $x, x = h1$, $h2$, $f1$, $f2$. Note that p_k^x are consumption prices and therefore include transaction costs. A typical regional consumption expenditure must correspond to the wage bill plus the managers remuneration.

$$E^x = w^x(L-R) + (w^x + r^x)R = w^x L + r^x R \tag{2}$$

where w^x and $r^x$are, respectively, the wage rate and the income associated to managerial activities in region x. R is the mass of managers (remember that we assume L and R to be identical among regions).

2.2 Production sectors

2.2.1 Food

Food is produced with a constant return to scale technology. One unit of labor is required to produce one unit of food:

$$F = L_F$$

This good is assumed to be traded at no cost and is used as the numéraire. Therefore, as long as it is produced by all regions, wages are equalized and $w^x = w = 1$ for all regions.

2.2.2 The service sector

The market for services is characterized by monopolistic competition. Services are differentiate good that are used as intermediary inputs in the local manufacturing sector. The production of varities exhibits increasing return to scale. Total cost in the production of a given variety of service is composed of a fixed cost, the manager input (one unit of R) and a constant marginal cost, each additional unit of service requiring a_s units of labor. The cost function for a service of type l in region x is:

$$C_s^x(r^x, s_l, w) = r^x + a_s w s_l$$

where s_l is the output level of variety l of services. Local services in region x are aggregated in a composite good s^x

$$s^x = \left[\int_{I^x} s_l^{1-1/\mu} dl\right]^{\mu/(\mu-1)}$$

with $\mu > 1$, the elasticity of substitution between varieties of services and I^x set of region's x services. The demand side on the market for service comes from the manufacturing industry. One unit of this composite service is required, as setup cost, for the production of each varity of the manufactured good. As usual in monopolistic competition model, firms are mill pricing and apply a fixed markup which is a decreasing function of the elasticity of demand. Choosing units such that $a_s = (\mu - 1)/\mu$, we have the following equilibrium prices :

$$p_{sl} = \frac{\mu}{\mu - 1} a_s w = 1$$

As managers are used only in the service sector, r^x is such that it exhausts the profits of service firms. All services are identical, therefore the demand for each varity is equal to the number of manufacturing varieties produced in that region,

$$r^x = n_x(1 - a_s) = \frac{n_x}{\mu} \tag{3}$$

where n_x is the number of manufacturing varieties produced in region x. For simplicity, we assumed that the mass of managers in each region is equal to unity. Given the definition of the service composite the appropriate form of the price index is:

$$P_s^x = \left[\int_{I^x} p_{sl}^{1-\mu} dl\right]^{1/(1-\mu)}$$

where p_{sl} is the price of service l.

This price index is then:

$$P_s^x = 1 \tag{4}$$

2.2.3 The manufacturing sector

The manufacturing sector is characterized by monopolistic competition. Production exhibits increasing return to scale. To start operations, the producer of a particular variety indeed supports a fixed cost as he has to invest in one unit of the locally produced composite service. The production of each additional unit requires a_M units of labor. The cost function of a manufacturing variety k in region x is therefore:

$$C^x_{Mk}(P_s, M_k, w) = P^x_s + a_M w M_k$$

where M_k is the production level of the variety k of the manufacturing good and P^x_s is the price of the service composite.

To spare notation, we choose units such that $a_M = (\sigma - 1)/\sigma$. As usual in this type of model, firms are mill pricing and apply a fixed markup which is a decreasing function of the elasticity of demand:

$$p_k = \frac{\sigma}{\sigma - 1} a_M w = 1 \tag{5}$$

where p_k is the mill price of variety k of the manufacturing good. It differs from p^x_k by the transaction cost, if any. We assume free entry in the manufacturing industry, in which case pure profits are exhausted. We then obtain that each variety supply is :

$$M_k = \frac{\sigma}{P^x_s} = \sigma. \tag{6}$$

2.2.4 Transaction costs

As already mentioned, food and manufacturing goods are traded among regions and among countries. Trade in manufacturing goods incurs some transaction costs (which are assumed of the Samuelsonian iceberg type) while food is traded at no cost. Specifically, we assume that international trade is more costly than inter-regional trade. Without loss of generality, we also assume that the home market is more integrated than the foreign one, i.e. that inter-regional trade is less costly at home than abroad. Formally, $\tau_I \geq \tau_f \geq \tau_h$ with τ_y (with $y = h$, f or I) the amount of good that has to be exported to get one unit at destination ($\tau_y > 1$).

2.3 Spatial equilibrium

We assume free entry on the manufacturing market. A spatial equilibrium is defined as the total (world) number of manufacturing firms and a distribution of those firms among countries and regions such that (i) pure profits are exhausted (zero in our case), (ii) the manufacturing markets clear and (iii) no existing firm has an incentive to move from one location to another. As we show below, the equilibrium can be stable or unstable. It is said to be stable if, following an infinitesimal perturbation, the system features a tendency to return to the original solution.

2.3.1 Equilibrium conditions

Let us first compute the equilibrium world number of manufacturing firms as well as the corresponding world expenditure level (respectively denoted by N^w and E^w). The Cobb-Douglas form of the utility function implies that a fraction α of world expenditure is spend on manufacturing goods, i.e.:

$$\sigma N^w = \alpha E^w$$

On the other hand, the aggregate budget constraint requires that the world expenditure corresponds to world revenue. By (3), we then have:

$$E^w = 4L + \frac{N^w}{\mu}$$

Using these two conditions, we have:

$$\begin{aligned} N^w &= 4L\frac{\alpha\mu}{\sigma\mu - \alpha} \\ E^w &= 4L\frac{\alpha\mu}{\sigma\mu - \alpha} \end{aligned} \tag{7}$$

In this model, the total number of firms is independent of any transaction cost. Only the spatial distribution of firms across and inside countries is function of transaction costs. This is a common feature to model with Dixit-Stiglitz preferences and monopolistic competition.

At equilibrium, demand equals supply for each variety in each region. A home firm in region 1 faces a demand of the type that is the sum of the goods that has to be produced in order to satisfy the demands for this variety from all regions. Using (1) and (5), it takes the following form

$$\begin{aligned} c_{h1} &= \frac{aE^{h1}}{n_{h1} + \phi_h n_{h2} + \phi_I(n_{f1} + n_{f2})} + \frac{aE^{h2}\phi_h}{\phi_h n_{h1} + n_{h2} + \phi_I(n_{f1} + n_{f2})} \\ &+ \frac{aE^{f1}\phi_I}{(n_{h1} + n_{h2})\phi_I + n_{f1} + \phi_f n_{f2}} + \frac{aE^{f2}\phi_f}{(n_{h1} + n_{h2})\phi_I + \phi_f n_{f1} + n_{f2}} \end{aligned}$$

where each fraction represents the gross demand from each region for a particular varity produced in region $h1$. We rewrite this demand as follows

$$c_{h1} = \frac{aE^w}{N^w}(S_{h1} + S_{h2}\phi_h + S_{f1}\phi_I + S_{f2}\phi_I)$$

where

$$\begin{aligned}
S_{h1} &= \frac{\theta e_{h1}\theta e_h}{\theta n_{h1}\theta n_h + (1-\theta n_{h1})\theta n_h \phi_h + (1-\theta n_h)\phi_I} \\
S_{h2} &= \frac{(1-\theta e_{h1})\theta e_h}{\theta n_{h1}\theta n_h \phi_h + (1-\theta n_{h1})\theta n_h + (1-\theta n_h)\phi_I} \\
S_{f1} &= \frac{\theta e_{f1}(1-\theta e_h)}{\theta n_h \phi_I + \theta n_{f1}(1-\theta n_h) + (1-\theta n_{f1})(1-\theta n_h)\phi_f} \\
S_{f2} &= \frac{(1-\theta e_{f1})(1-\theta e_h)}{\theta n_h \phi_I + \theta n_{f1}(1-\theta n_h)\phi_f + (1-\theta n_{f1})(1-\theta n_h)}
\end{aligned}$$

θe_{ij} and θn_{ij} are, respectively, the shares of region j in country i expenditure and the share of the country i manufacturing firms located in region j and, θe_i and θn_i denote the shares of country i in world expenditure and in the world number of firms.[2] Let $i = \{h, f\}$ and $j = \{1, 2\}$. We can check that S_{ij} denotes the share of sales of a firm located in region ij that is locally consumed.

At equilibrium, manufacturing markets have to clear. Using (6), the equilibrium condition for the home varities is given by

$$\sigma = \frac{aE^w}{N^w}(S_{h1} + S_{h2}\phi_h + S_{f1}\phi_I + S_{f2}\phi_I)$$

which is equivalent to

$$S_{h1} + S_{h2}\phi_h + S_{f1}\phi_I + S_{f2}\phi_I = 1$$

as $aE^w/N^w = \sigma$. Would the left hand side of the equation be larger (resp. smaller) than the right hand side, firms from the region 1 of the home countr would generate a positive (resp. negative) profit: at a price equal to one, they would sell more (less) than what is needed to cover the fixed cost i.e. σ.

To derive an equilibrium condition for each market, we define the following variables:

$$\begin{aligned}
B_{h1} &= S_{h1} + S_{h2}\phi_h + S_{f1}\phi_I + S_{f2}\phi_I \\
B_{h2} &= S_{h1}\phi_h + S_{h2} + S_{f1}\phi_I + S_{f2}\phi_I \\
B_{f1} &= S_{h1}\phi_I + S_{h2}\phi_I + S_{f1} + S_{f2}\phi_f \\
B_{f2} &= S_{h1}\phi_I + S_{h2}\phi_I + S_{f1}\phi_f + S_{f2}
\end{aligned}$$

[2] For instance, $E^{h1} = E^w\theta e_{h1}\theta e_h$ and $n_{h1} = \theta n_{h1}\theta n_h$.

Equilibrium conditions simplify to:

$$n_{ij}(B_{ij}-1)=0 \tag{8}$$

$$n_{ij}\geq 0 \text{ and } 1\geq B_{ij} \tag{9}$$

That are interpreted as follows, when a maket is active i.e. $n_{ij}>0$, demand has to equal supply ($B_{ij}=1$), when a market is not active profits have to be negative i.e. $1>B_{ij}$. Following Baldwin (1999), the B_{ij}'s represent the local markets biases, i.e. the extent to which sales of a local variety exceeds the average level of per variety sales (which is $\alpha E^w/N^w$).

To close the model, we need to specify the link between the shares of expenditure and the shares of firms. The expenditure of region ij is:

$$E_{ij}=L+\frac{n_{ij}}{\mu}$$

Using this definition and (7) , we have:

$$\begin{aligned}
\theta e_{1h}\theta e_h &= \frac{(1-b)}{4}+b\theta n_{1h}\theta n_h \\
(1-\theta e_{1h})\theta e_h &= \frac{(1-b)}{4}+b(1-\theta n_{1h})\theta n_h \\
\theta e_{1f}(1-\theta e_h) &= \frac{(1-b)}{4}+b\theta n_{1f}(1-\theta n_h) \\
(1-\theta e_{1f})(1-\theta e_h) &= \frac{(1-b)}{4}+b(1-\theta n_{1f})(1-\theta n_h)
\end{aligned}$$

where $b=\alpha/(\sigma\mu)\in[0,1]$.

The expenditure in one region has a fixed component that is the labor income and a variable one that is the income from the managers. The larger the number of firms in one region, the lager the manager income.

Therefore, the unknowns of system $(8-9)$ are the shares of the total number of firms located in each country (θn_h) and the share of these firms located in each region (θn_{1h} and θn_{1f}).

Note that, as we check below, the system allows for all types of agglomeration of the manufacturing industry, i.e. international and/or inter-regional. For simplicity however, this paper will only considers the cases for which countries do not entirely specialize in the production of agricultural good. The required restrictions are detailed below.

The subsequent part of this section is devoted to the analysis of the equilibrium distribution of firms and its stability. We first analyse how firms of a given country are distributed among regions. Later we concentrate on the equilibrium location of firms among countries. At the end of the section, we study the stability properties of this equilibrium.

2.3.2 Internal distribution of manufacturing firms

We first consider the equilibrium distribution of manufacturing firms inside countries for a given cross countries distribution. One easily checks that the homogenous distribution of manufacturing firms across the two regions of each country is always an equilibrium. Indeed, in this case, equilibrium conditions $(8-9)$ require $B_{h1} = B_{h2}$ and $B_{f1} = B_{f2}$ which is only true for $\theta n_{1h} = 1/2$ and $\theta n_{1f} = 1/2$.[3] This does not come as a surprise since, inside each country, regions are then perfectly symmetric.

For this to be a stable equilibrium, the transfer of an infinitesimal number of firms from one region to the other must decrease profits in the receiving regions relative to profits in the regions losing firms. In such case, the displaced firms would have an incentive to return to their original location which would leave the solution of the system unchanged. Formally, in our setting, the stability condition becomes :

$$\frac{\partial B_{h1}(\theta n_{1h} = 1/2)}{\partial \theta n_{1h}} = -\frac{\partial B_{h2}(\theta n_{1h} = 1/2)}{\partial \theta n_{1h}} < 0 \tag{10}$$

for the home economy and

$$\frac{\partial B_{f1}(\theta n_{1f} = 1/2)}{\partial \theta n_{1f}} = -\frac{\partial B_{f2}(\theta n_{1f} = 1/2)}{\partial \theta n_{1f}} < 0 \tag{11}$$

for the foreign economy.

The system also allows for corner solutions in which firms agglomerate in one of the two regions of the country. Without loss of generality, we will consider that agglomeration always occurs in regions 1, i.e. $\theta n_{1h} = 1$ and /or $\theta n_{1f} = 1$. The equilibrium condition is then:

$$B_{h1}(\theta n_{1h} = 1) > \ B_{h2}(\theta n_{1h} = 1)$$

[3] In order to have all equilibrium conditions fullfilled, θn has also to adapt so that B_{ij}'s are all equal to unity.

for the home economy and

$$B_{f1}(\theta n_{1f}=1) > \; B_{f2}(\theta n_{1f}=1)$$

for the foreign economy.

These stability conditions are derived in detail below and, in particular, expressed in terms of critical integration levels.

2.3.3 International distribution of manufacturing firms

¿From the former section, we conclude that θn_{ij}'s can take three different equilibrium values: $0, 1/2$ and 1. In the next section we consider the effects of an agglomeration process taking place in one of the country (say the home country) on the international repartition of the production. In other words, we compare the equilibrium θn when $\theta n_{1h} = \theta n_{1f} = 1/2$ to one with $\theta n_{1h} = 1$ and $\theta n_{1f} = 1/2$. We therefore solve the model for these two different configurations.

Dispersion in both countries We first consider a situation in which the manufacturing industry is dispersed in both countries, i.e. $\theta n_{1h} = \theta n_{1f} = 1/2$. As there are firms active in all regions, the interior equilibrium conditions are :

$$B_{h1} = B_{h2} = B_{f1} = B_{f2} = 1 \tag{12}$$

Solving (12) for θn, we have:

$$\theta n^d = \frac{1}{2} + \frac{\phi_I(\phi_h - \phi_f)}{(1-2\phi_I+\phi_h)(1-2\phi_I+\phi_f) + b(4\phi_I^2 - (1+\phi_h)(1+\phi_f))} \tag{13}$$

where the superscript d stands for dispersed equilibrium. This expression is relevant for parameters values such that $\theta n^d \in [0,1]$. When the right hand side of (13) is larger than 1, $\theta n^d = 1$ and when negative, $\theta n^d = 0$. As already mentioned, we only consider cases for which none of the country entirely specializes in the production of the agricultural good, i.e. we impose restrictions on the parameters such that $0 \leq \theta n^d \leq 1$.

In the appendix A1, we show that a necessary and sufficient condition for this to be satisfied is[4]:

$$b < b^d = \frac{\phi_h + \phi_f(1+\phi_h) + (1-2\phi_I)^2 - 4\phi_h\phi_I}{(1+\phi_h)(1+\phi_f) - 4\phi_I^2} \tag{14}$$

[4]This condition in fact guarantees that one country alone cannot meet world demand for agriculture. In other terms, α and hence b cannot be too large.

As far as the properties of θn^d are concerned, we should note that when $\phi_h = \phi_f$, $\theta n^d = 1/2$. In such case indeed, locations are symmetric which is reflected in the spatial distribution of firms. Also, when $\phi_I = 0$, $\theta n^d = 1/2$, for all ϕ_h, ϕ_f. In the absence of international trade, the two countries have manufacturing industries of similar size even if they differ in terms of their internal integration levels. This is easy to understand since in autarchy, the equilibrium number of firms in each country is independent of transaction costs as it was discussed previously at the world level.

International stability requires that transferring firms from one country to the other leads to a decline of firms' profits in the destination country and an increase in the profit of the firms remaining in the original country. Formally, for $\theta n_{1h} = \theta n_{1f} = 1/2$ and $\theta n = \theta n^d$, we must have

$$\frac{\partial B_{h1}}{\partial \theta n_h} = \frac{\partial B_{h2}}{\partial \theta n_h} = -\frac{\partial B_{f1}}{\partial \theta n_h} = -\frac{\partial B_{f2}}{\partial \theta n_h} < 0$$

In the appendix, we show that the no specialization requirement, i.e. imposing $b < b^d$, is sufficient to ensure this stability condition to be always verified.

Agglomeration in the home country In such case, $\theta n_{1h} = 1$ and $\theta n_{1f} = 1/2$. The interior equilibrium conditions are

$$B_{h1} = B_{f1} = B_{f2} = 1 \text{ and } B_{h2} < 1. \tag{15}$$

Solving (15) for θn, we obtain two roots. In the appendix A2, we show that one corresponds to a stable equilibrium and the other to an unstable one. One can check that for $b < b^d$, this last root is also negative. We therefore focus on the stable one which writes:

$$\theta n^a = \frac{-B - \sqrt{B^2 - 4AC}}{2A} \tag{16}$$

where the superscript a stands for agglomerated equilibrium and

$$\begin{aligned}
A &= 4(\phi_h - \phi_I)(b(1 + \phi_f - 2\phi_I^2) - (1 - \phi_I)(1 + \phi_f - 2\phi_I)) && (17)\\
B &= (\phi_I - \phi_h)((1 + \phi_f)(4\phi_I - (1 - b)) - 2\phi_I^2(1 + b)) && (18)\\
&\quad +2\phi_I\{2(\phi_I - (1 - b))(1 + \phi_f) + \phi_I(2 + (1 + b)(1 - \phi_I)) - 4\phi_I^2(1 + b)\} \\
&\quad +\phi_h\{(1 - b)(1 - \phi_I)(1 + \phi_f) + 4\phi_I^2\} \\
C &= \phi_I((1 + \phi_f)\{(1 - b)(1 + \phi_h) - 4\phi_I\} + 4\phi_I^2(1 + b)) && (19)
\end{aligned}$$

The expression of θn^a is tedious and makes its analysis cumbersome. It generally behaves like θn^d and in particular, we can show that $\theta n^a = 1/2$ when $\phi_I = 0$ (one indeed easily checks that in this case, $A = 2B$ and $C = 0$). Once again, this expression is valid for parameters value such that $\theta n^a \in [0, 1]$. Note that, when $\theta n^a \in [0, 1]$, it is an internationally stable equilibrium.

2.3.4 Stability conditions and openness to trade

We have completely solved the model for two particular configurations. Let us now detail the conditions required for these equilibrium to be stable. One important characteristic of new economic geography models is that a small decrease in transaction costs can lead to important discontinuity in the spatial distribution of firms. In particular, our model features catastrophic agglomeration in the sense that below a critical level of transaction costs, the only stable outcome is a core-periphery pattern in which the entire manufacturing industry agglomerates in one location. Since space is here composed of regions and countries, we could have two types of catastrophic agglomerations, an international one in which agglomeration occurs in a particular country and an inter-regional one in which the manufacturing industry of one country agglomerates in one of the two regions of this country. However, limiting ourselves to the analysis of cases for which countries do not entirely specialize in the production of agricultural good, we imposed $b < b^d$, which also guarantees that the international distribution of the manufacturing industry is stable (see the preceding sections). When agglomeration takes place in country 1, we saw in the former section that θn^a is an internationally stable equilibrium. This rules out the first type of catastrophic agglomeration. We therefore concentrate on the stability conditions relative to the internal (i.e., inside countries) distribution of the manufacturing industry.

We first focus on the conditions necessary for the equilibrium with $\theta n_{1h} = \theta n_{1f} = 1/2$ and $\theta n = \theta n^d$ to be stable.

Claim 1 *This equilibrium is stable in country h when $\phi_h < \phi_{hc}$ and in country f when $\phi_f < \phi_{fc}$, with:*

$$\phi_{hc} = \frac{1 + \phi_f - 2\phi_I - b(1 + \phi_f - 4\phi_I^2)}{(1 + b)(1 + \phi_f) - 2\phi_I} \leq 1 \tag{20}$$

$$\phi_{fc} = \frac{1 + \phi_h - 2\phi_I - b(1 + \phi_h - 4\phi_I^2)}{(1 + b)(1 + \phi_h) - 2\phi_I} \leq 1 \tag{21}$$

Proof. See the appendix A3. ■

Importantly, note that for all θn, the equation $B_{i1} = B_{i2}$ $(i = h,\ f)$ has only one root in θn_{i1} which is $\theta n_{i1} = 1/2$. Therefore, there are no partial agglomerations. Moreover, these conditions also delimit cases for which agglomeration is not an equilibrium.

Claim 2 *If the dispersed equilibrium is stable then agglomeration is not an equilibrium and vice versa*

Proof. See the appendix A3 ■

This means that we can only have one stable equilibrium. Either, there is only one equilibrium that is stable, i.e. the dispersed one or we have multiple equilibria but then only the agglomerated one is stable

3 Integration, agglomeration and trade

In this section, we investigate the effects both of regional/international integration and agglomeration on the principal properties of the model's equilibrium. To this end, we proceed to two different experiments. First, starting from a dispersed equilibrium in which both countries feature a homogeneous distribution of the manufacturing industry, we analyze the effects of different integration process. Second, comparing the dispersed and agglomerated equilibrium such as defined above, we emphasize the effect of an agglomeration process taking place inside a particular country, say in the home country. However, for this analysis to make sense, we need a stable dispersed equilibrium to exist in this country, i.e. ϕ_{hc} must belong to $[\phi_f, 1]$. Indeed, since we assumed that $\phi_h > \phi_f$, if $\phi_{hc} < \phi_f$ then agglomeration would be the only stable equilibrium in the home country for all $\phi_h > \phi_f$. Examining (21) and (20), one check that a sufficient condition for $\phi_{hc} \in [\phi_f, 1]$ is $b < b_0 = (1 - \phi_f)/(1 + \phi_f + 2\phi_I)$. Basic calculations also show that $b_0 < b^d$.

3.1 Integration

As already mentioned, the integration process is captured, in this paper, by a decrease in the extent of transaction costs. Considering a dispersed equilibrium, we examine the following: (i) the effect of internal integration and/or international on international trade and (ii) the effect of integration on the stability of the symmetric equilibrium. As far as the consequences of internal integration on international trade flows are concerned, our model reproduces , in a richer model, the results obtained by Martin and Rogers (1995). In addition and contrary to theirs, our framework contains the classic centripetal and centrifugal forces of an economic geography model and allows to study the determinants of the geographic distribution of economic activities. In the preceding section, we derived the conditions under which an agglomeration of the manufacturing sector in a particular country and/or region is an equilibrium. We can then examine how the likelihood of such an agglomeration, say in the home country, is affected by the extent of integration between home regional markets (as is usually considered in economic geography) but also between foreign regional markets as well as between home and foreign markets. Examining the properties of the dispersed equilibrium, we have the following:

Proposition 3 ***Domestic integration and trade.*** *In the presence of international trade, an increase in the level of internal integration in one country leads to an increase in the world share of manufacturing firms located in that country.*

Proof. *The proof is direct. Using the expression for θn^d, one first checks that $\theta n^d > 1/2$ for all $\phi_h > \phi_f$ and that, for $b < b_1 = (1 + \phi_f - 2\phi_I)/(1 + \phi_f + 2\phi_I)$, $\partial \theta n^d / \partial \phi_h > 0$ and $\partial \theta n^d / \partial \phi_f < 0$. Since $b^d < b_1$, this condition is always respected for the values of b that we consider.* ■

In other terms, an integrated internal market can be considered as a source of comparative advantage in the sense that it leads to a relative specialization of the most integrated country in the manufacturing industry. In such type of models featuring increasing returns to scale, this corresponds to the so-called "large market effect" (see, for instance, Krugman,1980) by which the advantage comes from the fact that economies of scale are better exploited on a location characterized by a large demand. This advantage is moreover monotonically growing with the extent of international integration.

Proposition 4 ***International integration and trade.*** *An increase in the level of international integration leads a larger share of the world manufacturing industry to locate in the country featuring the highest internal integration level.*

Proof. Using the expression for θn^d, as long as $b < b_1$, we have:

$$\frac{\partial \theta n_h}{\partial \phi_I} \gtreqless 0 \text{ when } \phi_h \gtreqless \phi_f$$

■

International integration benefits to the most integrated region in terms of the manufacturing industry world market shares. Here again, it is the comparative advantage created by the large market that induces this effect. The lower the international transaction costs, the more this comparative advantage can be exploited.

Finally, turning to the stability conditions of the dispersed equilibrium, we have the following:

Proposition 5 ***Internal integration and agglomeration.*** *An increase in the level of the internal integration level in one country decreases the likelihood of agglomeration inside the other country i.e.* $\partial\phi_{hc}/\partial\phi_f > 0$ *and* $\partial\phi_{fc}/\partial\phi_h > 0$.

Proof. See the appendix. ■

Proposition 6 ***International integration and agglomeration.*** *An increase in the level of international integration increases the likelihood of agglomeration inside both countries i.e.* $\partial\phi_{hc}/d\phi_I < 0$ *and* $\partial\phi_{fc}/d\phi_I < 0$.

Proof. See the appendix A4. ■

Interestingly, this result suggests that agglomeration forces working inside countries could be fostered not only by the internal integration of markets but also both by the integration of partner country's market and the international opening to trade. The intuition behind the effect of the integration in the partner country is the following. An increase in the size of the national manufacturing industry amplifies agglomeration forces. By proposition 2, an increase in the level of integration in one country reduces the manufacturing world market share of the other country which in turns reduces the strength of its agglomeration forces. On the other hand, integration of international markets has two effects. The first is a direct one and reflects the increase in competition stemming from the increased openness to international trade which exacerbates agglomeration forces inside countries. The second one is indirect and works through the change in the international distribution of the manufacturing firms following an increase in international integration. By proposition 3, an increase in international integration modifies the location of the manufacturing industry in favor of the most integrated country (home in our example). Consequently, the net effect on the catastrophic agglomeration critical value of integration is unambiguously negative in the home country i.e. $\partial\phi_{hc}/\partial\phi_I < 0$. For the foreign country, the two effects are playing in opposite direction but as shown in the appendix, the direct effect always dominates so that we also have $\partial\phi_{fc}/\partial\phi_I < 0$.

3.2 Agglomeration

Let us now focus on the case for which in one of the country (home), the manufacturing sector agglomerates in one of the region, for example $\theta n_{1h} = 1$ and $\theta n_{2h} = 0$. The objective is to examine the properties of the model in such case and compare them with the one obtained for the dispersed equilibrium.

Proposition 7 ***Agglomeration and trade.*** *The home world market share in the manufacturing industry increases with the agglomeration of the home manufacturing industry if and only if this agglomeration process is spontaneous i.e.* $\theta n^a > \theta n^d$ *iff* $\phi_h > \phi_{hc}$.

Proof. *See the appendix A4* ■

The agglomeration of the industrial sector characterized by increasing returns to scale can be a source of comparative advantage. This is because such an agglomeration creates a large market which, as in Krugman (1980), turns out to affect the international distribution of firms in favor of the largest economy. This is however not general. Indeed, as emphasized by the proposition above, this proves to be true on condition that $\phi_h > \phi_{hc}$ or in other words that the agglomeration process is spontaneous. As a consequence, if the agglomeration is spontaneous, policies aimed at restoring an homogeneous distribution of activities (for instance to prevent the emergence of regional disparities) can conflict with the international trade performance of the economy under consideration. In this case, such policies indeed prevent to reap the benefice tied to the creation of a large market. On the other hand, policies aimed at increasing competitiveness by favoring the concentration of activities (creation of industrial districts for instance), will only generate efficiency gains in this sense if they are supported by a natural tendency of activities to cluster.

Let us now turn to the analysis of how the spatial distribution of manufacturing firms in the home economy affects agglomeration forces in the foreign economy.

Proposition 8 ***Cross country interdependence of the spatial distribution of activities.*** *The agglomeration of the manufacturing industry in one country reduces the likelihood to observe its agglomeration in the other country.*

Proof. *The condition for agglomeration of economic activities in the foreign economy is derived above. In particular, the homogeneous spatial distribution of the manufacturing industry in the foreign economy is stable if and only if* $\phi_f < \phi_{fc}$. ϕ_{fc} *is the value of* ϕ_f *such that* $\frac{\partial Bf1}{\partial \theta n_{1h}} = 0$. *Its general* *expression is:*

$$\phi_{fc}(\theta n_h) = \frac{1 - b(1 + 4\phi_I \theta n_h)}{1 + b(3 - 4\theta n_h)}$$

(the expression given in claim 1 is obtained for $\theta n_h = \theta n_h^d$). One easily checks that $\phi_{fc}(\theta n_h)$ is an increasing function of θn_h when $b < (1-\phi_I)/(1+3\phi_I)$. Since, we restrict ourselves to the cases for which $\phi_{hc} > \phi_f$, i.e. $b < b_0$ and that $b_0 < (1-\phi_I)/(1+3\phi_I)$, we have the following: if ϕ_h is superior to ϕ_{hc}, then agglomeration takes place in the home country, θn_h increases (by proposition 5) and therefore ϕ_{fc} increases which implies that agglomeration is less likely in the foreign country. ■

This proposition emphasizes the fact that the spatial distribution of activities inside a given country can in part be explained by the one observed in its trading partners and suggests that, under certain conditions (namely that countries feature different internal integration levels), it is more likely to witness a world characterized by differentiated geographical patterns across countries.

4 Conclusion

We developed a two country four region model allowing for a core periphery pattern inside countries. We first analyzed the effect of international and inter-regional integration on the international distribution of the manufacturing industry as well as on the nature (dispersed or agglomerated) of the within countries distribution of activities. We also compared the properties of an equilibrium corresponding to a homogeneous distribution of activities inside both countries with the ones obtained in the case for which one of the country features an agglomeration of its manufacturing industry.

For the first part of the analysis, we showed that (i) internal integration can act as a source of comparative advantages for industries characterized by increasing returns to scale. Therefore, by decreasing its transaction costs, a country also increases its share in the manufacturing industry which is in coherence with previous results obtained by Martin and Rogers (1995); (ii) international integration increases the size of the manufacturing industry of the most integrated country at the expense of the other. Therefore, in terms of manufacturing world market shares, the international integration benefits to the country that already has the advantage in this matter; (iii) international integration fosters the agglomeration forces at work inside the trading countries making agglomeration more likely in both countries.

Concerning the effects of an agglomeration process taking place in one of the country, we showed that (i) the agglomerated country gains market shares in the manufacturing industries on condition that this agglomeration process is driven by market forces. Would this agglomeration be forced, that country would lose market shares; (ii) agglomeration, (like integration) in one country reduces the likelihood to observe agglomeration in the partner country.

To conclude let us illustrate our model with the Canadian Economic Nationalism example developed by Krugman (1991). In 1878, Canada introduced its National Policy consisting in increasing tariff and building a national rail road. This induced an international disintegration and a national integration as well as a rise in the agglomeration level of the Canadian manufacturing industry. The result has been an increase of the Canadian share of this type of industry which is conform to the prevision of our model.

References

[1] Amiti, M. (1998) Inter-industry trade in manufactures: Does country size matter? *Journal of International Economics* 44,231-255

[2] European Commission (1996) First report on Economic and social cohesion, *Office of the Official Publications of the European Commission*

[3] Haaparanta, P.(1998) Regional concentration, trade, and welfare, *Regional science & urban economics* 28, 445-463

[4] Hotelling H. (1929) Stability in competition, *Economic Journal* 39, 41-57.

[5] Koopmans T.C. (1957) *Three Essays on the State of Economic Science*, New York, McGraw-Hill.

[6] Lösch A. (1940), *Die Räumliche Ordnung der Wirtschaft*, Jena, Gustav Fischer. English translation: *The Economics of Location*, New Haven (Conn.), Yale University Press (1954)

[7] Krugman P. (1980), Scale Economies, Product Differentiation and the Pattern of Trade, *American Economic Review*, 70, 950-959

[8] Krugman P. (1991), *Geography and Trade*, MIT Press

[9] Martin, P. and C.A. Rogers (1995) Industrial location and public infrastructure, *Journal of International Economics* 39, 3-4, 335-351

[10] Monfort, Ph and R. Nicolini, (2000) Regional Convergence and International Integration, *Journal of Urban Economics* 48, 286-306

Appendix

A1: The study of θn^d

Claim 9 *θn^d is a stable interior equilibrium iff* $0 < b < b^d$

Proof:

To show this claim we first derive the conditions on b such that θn^d is interior and then show that the only range of b such that the interior equilibrum is stable is $0 < b < b^d$.

A) *The condition on b for θn^d to be an interior equilibrium*

By (13), we have the following:

1. θn^d has one root in $b = b_0^d$ with

$$b_0^d = \frac{\phi_h + \phi_f(1+\phi_h) + (1-2\phi_I)^2 - 4\phi_f\phi_I}{(1+\phi_h)(1+\phi_f) - 4\phi_I^2}$$

2. $\theta n^d = 1$ only for $b = b^d$ with

$$b^d = \frac{\phi_h + \phi_f(1+\phi_h) + (1-2\phi_I)^2 - 4\phi_h\phi_I}{(1+\phi_h)(1+\phi_f) - 4\phi_I^2}$$

3. θn^d has one asymptote in $b = b_a^d$ with

$$b_a^d = \frac{(1+\phi_h-2\phi_I)(1+\phi_f-2\phi_I)}{(1+\phi_h)(1+\phi_f) - 4\phi_I^2}$$

4. For $b = 0$,

$$\theta n^d = \frac{1}{2} + \phi_f \frac{\phi_h - \phi_f}{(1-\phi_f)(1+\phi_h-2\phi_f)} > 0 \text{ for } \phi_I < \phi_f < \phi_h$$

and

$$\frac{\partial \theta n^d}{\partial b} = \frac{(\phi_h - \phi_f)((1+\phi_h)(1+\phi_f) - 4\phi_I^2)}{(1+\phi_h-2\phi_I^2)(1+\phi_f-2\phi_I^2)} > 0$$

Summing up, we have one root, one asymptote and one value of $b = b^d$such that $\theta n^d = 1$. Noting that $b^d < b_a^d < b_0^d$, and using the point 4, we can describe θn^d as a function of b. It is a postive and increasing function from 0 to the asymptote; from the asymptote to b_0^d it is a negative and increasing function; and from b_0^d to 1, it is a positive function that take values smaller than one. Therefore we conclude that $\theta n^d \in [0, 1]$ if and only if $b \in [0, b^d] \cup [b_0^d, 1]$.

To complete the proof, we have.show that the interior equilibrium is stable only when $b \in [0, b^d]$.

B) *Stability of the interior equilibrium*

The stability condition for θn^d is

$$\frac{\partial B_{h1}(\theta n^d)}{\partial \theta n} < 0$$

This function has the following properties:

1. For $\theta n = \theta n^d$, it has two roots in b corresponding to b^d and b_a^d.

2. At $\theta n = \theta n^d$, it has two asymptotes in b

$$\begin{aligned} b_{a1}^B &= b_1 = \frac{1 + \phi_f - 2\phi_I}{1 + \phi_f + 2\phi_I} \\ b_{a2}^B &= \frac{1 + \phi_h - 2\phi_I}{1 + \phi_h + 2\phi_I} \end{aligned}$$

3. One easily checks that $b^d < b_{a1}^B < b_{a2}^B < b_0^d$.

4. One can check that at $b = b_0^d$,

$$\frac{\partial B_{h1}(\theta n^d)}{\partial \theta n} = (\phi_h - \phi_f)/(1 + \phi_f) > 0$$

5. Moreover, at $b = b^d$,

$$\frac{\partial^2 B_{h1}(\theta n^d)}{\partial \theta n \partial b} = \frac{(1 + \phi_h)(1 + \phi_f) - 4\phi_I^2}{4\phi_I(1 + \phi_h)} > 0$$

Focusing on interior equilibrium, we limit our investigation to $b \in [0, b^d] \cup [b_0^d, 1]$. Using point 3, we know that to the sign $\partial B_{h1}/\partial \theta n$ cannot change in these intervals. Therefore, by point 1 and 5, we have that $\partial B_{h1}/\partial \theta n < 0$ when $b \in [0, b^d]$. By point 4, we have that $\partial B_{h1}/\partial \theta n > 0$ for $b \in [b_0^d, 1]$.■

A2: The study of agglomerated equilibria

In this appendix, we show that there exists at most one stable agglomerated equilibrium. We can decompose the equilibrium condition (XX) in two parts: for $\theta n_{1h} = 1$ and $\theta n_{1f} = 1/2$, θn_h has to be such that i) $B_{h1} = B_{f1} = B_{f2} = 1$ and ii) $B_{h2} < 1$. We first, in *(a)*, find the θn_h's fufilling i), then, in *(b)* show one of the them is always negative in the parameter range we consider, in *(c)* we show that the other is stable when interior and finally, in *(d)* check under which condition it respects ii).

(a) $B_{h1} = B_{f1} = B_{f2} = 1$

First note that by the Walras law, as $B_{f1} = B_{f2}$, $B_{h1} = 1 => B_{f1} = 1$. We therefore have to solve $B_{h1} = 1$ in θn_h for $\theta n_{1h} = 1$ and $\theta n_{1f} = 1/2$. (b) show that only one of them could stable and interior and finally.

We can write

$$B_{h1} - 1 = \frac{(1-\theta n)}{4}\frac{N}{D} \tag{22}$$

with

$$N = A\theta n^2 + B\theta n + C \tag{23}$$

and

$$D = \left[(1+\phi_f)(1-\theta n) + 2\phi_I \theta n\right]\left\{\theta n(1-\phi_I) + \phi_I\right\}\left\langle \theta n(\phi_h - \phi_I) + \phi_I\right\rangle$$

and A, B and C defined by (17), (18) and (19). Note that when $\theta n \in [0,1]$, $D > 0$.

Solving (22) we get

$$\theta n^a = \frac{-B - \sqrt{B^2 - 4AC}}{2A},\ \theta n^{a1} = \frac{-B + \sqrt{B^2 - 4AC}}{2A} \text{ and } \theta n^{a2} = 1.$$

(b) θn^{a1} *is always negative in the parameter range we consider i.e* $b < b^d$

Consider the following properties of θn^{a1}

1. it has one root in $b = b_0^d$
2. it has no asymptote in b.
3. for $b = 1$, it is positive (the expression is nevertheless tedious and not reproduced here).

Recalling that $b^d < b_0^d$, these properties are sufficient to prove that for $b \in [0, b^d]$, $\theta n^{a1} < 0$.■

(c) Only θn^a is stable when interior

This amounts to show that when θn^a is an interior equilibrium, it is stable i.e. when $\theta n^a \in [0,1]$, $\left.\frac{d(B_{h1}-1)}{d\theta n}\right|_{\theta n=\theta n^a} < 0$.

Differentiating (22) with respect to the θn we get:

$$\frac{d(B_{h1}-1)}{d\theta n} = -\frac{N}{4D} + \frac{(1-\theta n)}{4}\frac{\frac{dN}{d\theta n}D - \frac{dD}{d\theta n}N}{D^2}$$

therefore, by definition of θn^α

$$\left.\frac{d(B_{h1}-1)}{d\theta n}\right|_{\theta n=\theta n^a} = \left.\frac{(1-\theta n)}{4}\frac{\frac{dN}{d\theta n}}{D}\right|_{\theta n=\theta n^a}$$

When $\theta n^a \in [0,1]$, $(1-\theta n) \geq 0$ and $D \geq 0$, therefore

$$sign\left.\frac{d(B_{h1}-1)}{d\theta n}\right|_{\theta n=\theta n^a} = \left.\frac{dN}{d\theta n}\right|_{\theta n=\theta n^a}$$

Differentiating (23) with respect to θn,

$$\frac{dN}{d\theta n} = 2A\theta n + B$$

therefore when $A > 0$,

$$\frac{dN}{d\theta n} < 0 \Leftrightarrow \theta n < \frac{-B}{2A}$$

and when $A < 0$

$$\frac{dN}{d\theta n} < 0 \Leftrightarrow \theta n > \frac{-B}{2A}$$

Therefore when $\theta n^a \in [0,1]$, $\left.\frac{d(B_{h1}-1)}{d\theta n}\right|_{\theta n=\theta n^a} < 0$ as $\theta n^a < \frac{-B}{2A}$ for $A > 0$ and $\theta n^a > \frac{-B}{2A}$ for $A < 0$.■

(c) Are profit in the home region 2 negative?

For θn^a to be an equilibrium, one need that not firm are willing to produce in that region. For $\theta n = \theta n^a$, $Bh_2 - 1$ can be positive or negative depending on the parmeters value. In particular, we show in claim xx of the appendix XXX that the conditions for $Bh_2 - 1$ to be positive or negative are the same as those related to the stability of the dispersed equilibrium.

A3: Existence of the dispersed equilibrium

In this appendix we first proof the claim 1 of the section 2.3.4. And then derive the conditions under which a dispersed equilibrium exists.

Claim 1: *A dispersed equilibrium exists when* $b < \min\{b_0, b_2, b^d\}$ with

$$\begin{aligned} b_0 &= \frac{1-\phi_f}{1+\phi_f+2\phi_I} \\ b_2 &= \frac{(1+\phi_h)(1-2\phi_I)+2\phi_I^2}{(1+\phi_h)(1+2\phi_I)-2\phi_I^2} \end{aligned}$$

and is stable in country h *when* $\phi_h < \phi_{hc}$ *and in country* f *when* $\phi_f < \phi_{fc}$*, with:*

$$\phi_{hc} = \frac{1+\phi_f-2\phi_I-b(1+\phi_f-4\phi_I^2)}{(1+b)(1+\phi_f)-2\phi_I} \leq 1 \tag{24}$$

$$\phi_{fc} = \frac{1+\phi_h-2\phi_I-b(1+\phi_h-4\phi_I^2)}{(1+b)(1+\phi_h)-2\phi_I} \leq 1 \tag{25}$$

Proof:

We first do the demonstration for the home country and then for the foreign one.

For the dispersed equilibrium to be stable, $\partial B_{h1}/\partial\theta nh1$ must be negative when evaluated at $\theta nh1 = \theta nf1 = 1/2$ and $\theta n = \theta n^d$. For these values, the equation $\partial B_{h1}/\partial\theta nh1 = 0$ has three roots in ϕ_h, 1, $\phi_{hc1}$and ϕ_{hc} with

$$\begin{aligned} \phi_{hc1} &= \frac{4\phi_I(1+\phi_f-b(1+\phi_I))}{(1-b)(1+\phi_f)} - 1 \\ \phi_{hc} &= \frac{1+\phi_f-2\phi_I-b(1+\phi_f-4\phi_I^2)}{(1+b)(1+\phi_f)-2\phi_I} \end{aligned}$$

One easily check that ϕ_{hc1} also corresponds to the value of ϕ_h such that $\theta n^d = 0$ with $\theta n^d < 0$ for $\phi_h \leq \phi_{hc1}$. Finally,$\partial B_{h1}/\partial\theta nh1$ has an asymptote in ϕ_h which we denote by ϕ_{ha} with

$$\phi_{ha} = 2\phi_I\frac{1+b}{1-b} - 1$$

Let us now define two critical values for b that are helpful:

$$b_0 = \frac{1-\phi_f}{1+\phi_f+2\phi_I}$$
$$b_1 = \frac{1+\phi_f-2\phi_I}{1+\phi_f+2\phi_I}$$

and one easily checks that $b_0 < b_1$ and $b^d < b_1$.

- For $b \leq b_0$, we have:

(i) $\phi_{hc} \geq \phi_f$, (ii) $\phi_{hc1} \leq \phi_f$, (iii) $\phi_{ha} \leq \phi_{hc1}$, (iv) $\frac{\partial B_{h1}}{\partial \theta n h 1} \leq 0$ for $\phi_h = \phi_f$

- For $b_0 \leq b \leq b_1$, we have:

(i) $\phi_{hc} \leq \phi_f$, (ii) $\phi_{hc1} \leq \phi_f$, (iii) $\phi_{ha} \leq \phi_{hc1}$, (iv) $\frac{\partial B_{h1}}{\partial \theta n h 1} \geq 0$ for $\phi_h = \phi_f$

- Finally, for $b_1 \leq b$, we have:

(i) $\phi_{hc} \leq \phi_f$, (ii) $\phi_{hc1} \geq \phi_f$, (iii) $\phi_{ha} \geq \phi_{hc1}$, (iv) $\frac{\partial B_{h1}}{\partial \theta n h 1} \geq 0$ for $\phi_h = \phi_f$

This is sufficient to conclude the following:

- for $b \leq b_0$, $\partial B_{h1}/\partial \theta n h 1 \leq 0$ for $\phi_h \leq \phi_{hc}$. We therefore have a range of $\phi_h \in [\phi_f, 1]$ such that for $\phi_h \leq \phi_{hc}$, the dispersed equilibrium is the only stable while for $\phi_h > \phi_{hc}$, the agglomerated equilibrium is the only one stable.
- for $b_0 \leq b \leq b_1$, $\frac{\partial B_{h1}}{\partial \theta n h 1} \geq 0$ for all $\phi_h \in [\phi_f, 1]$. Agglomeration is therefore the only stable outcome.
- we do not consider cases where $b_1 \leq b$ as we restricted our analysis to $b < b^d < b_1$.

The same analysis for the foreign country leads to similar results.

For the dispersed equilibrium to be stable abroad, $\partial B_{f1}/\partial \theta n f 1$ must be negative when evaluated at $\theta n h 1 = \theta n f 1 = 1/2$ and $\theta n = \theta n^d$. For these values, the equation $\partial B_{f1}/\partial \theta n f 1 = 0$ has three roots in ϕ_f, 1, $\phi_{fc1}$and ϕ_{fc} with

$$\phi_{fc1} = \frac{4\phi_I(1+\phi_h - b(1+\phi_I))}{(1-b)(1+\phi_h)} - 1$$
$$\phi_{fc} = \frac{1+\phi_h - 2\phi_I - b(1+\phi_h - 4\phi_I^2)}{(1+b)(1+\phi_h) - 2\phi_I}$$

One easily check that ϕ_{fc1} also corresponds to the value of ϕ_f such that $\theta n^d = 1$ with $\theta n^d > 1$ for $\phi_f \leq \phi_{fc1}$. We therefore restrict our analysis of $\partial B_{f1}/\partial\theta nf1$ on the range of $\phi f > \phi_{fc1}$

Finally,$\partial B_{f1}/\partial\theta nf1$ has an asymptote in ϕ_f which we denote by ϕ_{fa} with

$$\phi_{fa} = 2\phi_I \frac{1+b}{1-b} - 1$$

1) When $b < b_0$, one checks that $\phi_{fa} < \phi_I$ wich implies that the asymptot is not in the range of ϕ_f that we have to consider.

We define a critical values for b :

$$b_2 = \frac{(1+\phi_h)(1-2\phi_I) + 2\phi_I^2}{(1+\phi_h)(1+2\phi_I) - 2\phi_I^2}$$

We can show that for $b < b_2$, we have $\phi_{fc1} < \phi_{fc}$ and the $\partial B_{f1}/\partial\theta nf1$ is increasing in ϕ_f at $\phi_f = \phi_{fc}$. This implies that for all $\phi_f \in [\phi_{fc1}, \phi_{fc}]$, $\partial B_{f1}/\partial\theta nf1$ is negative and is positive for $\phi_f > \phi_{fc}$.

When $b > b_2$ we have $\phi_{fc1} > \phi_{fc}$ and the $\partial B_{f1}/\partial\theta nf1$ is increasing in ϕ_f at $\phi_f = \phi_{fc1}$. This implies that for all $\phi_f > \phi_{fc1}$,(remember we can restrict to this domain), $\partial B_{f1}/\partial\theta nf1$ is positive.

To sum up, this is sufficient to proof that when the interior dispersed equilibrium exists, it is stable for $\phi_f < \phi_{fc}$ as when $b < b_2$, $\partial B_{f1}/\partial\theta nf1$ is negative when $\phi_f \in [\phi_{fc1}, \phi_{fc}]$ and positive when $\phi_f > \phi_{fc}$ and, when $b < b2$, no stable dispersed equilibrium exists. Therefeore, the dispersed equilibrium exists iff $b < \min\{b_0, b_2, b^d\}$.■

A4: Proof of propositions 5 and 6

Proposition 5 : *Internal integration and agglomeration. An increase in the level of the internal integration in one country decreases the likelihood of agglomeration inside the other country i.e.* $\partial\phi_{hc}/\partial\phi_f > 0$ *and* $\partial\phi_{fc}/\partial\phi_h > 0$.

Proposition 6 : ***International integration and agglomeration.*** *An increase in the level of international integration increases the likelihood of agglomeration inside both countries i.e.* $\partial\phi_{hc}/d\phi_I < 0$ *and* $\partial\phi_{fc}/d\phi_I < 0$.

Proof:

We will prove both propositions in the same demonstration.

Using $(10-11)$, given θn, the critical levels of integration above which the dispersed equilibrium in the domestic (resp. foreign) economy is unstable are:

$$\begin{aligned}\phi_{hc}(\theta n) &= \frac{1-b-4b\phi_I(1-\theta n)}{1-b+4b\theta n} \\ \phi_{fc}(\theta n) &= \frac{1-b-4b\phi_I\theta n}{1-b+4b(1-\theta n)}\end{aligned} \tag{26}$$

Therefore, the expressions (21) and (20) are the solution of $\phi_{hc} = \phi_{hc}(\theta n^d)$ and $\phi_{fc} = \phi_{fc}(\theta n^d)$. We then have:

$$\frac{d\phi_{hc}}{d\phi_f} = \frac{d\phi_{hc}(\theta n)}{d\theta n}\frac{d\theta n^d}{d\phi_f} \tag{27}$$

$$\frac{d\phi_{fc}}{d\phi_h} = \frac{d\phi_{fc}(\theta n)}{d\theta n}\frac{d\theta n^d}{d\phi_h} \tag{28}$$

Moreover, by the implicit function theorem, we obtain:

$$\frac{d\phi_{hc}}{d\phi_I} = -\frac{\partial\phi_{hc}(\theta n)/\partial\phi_I + \partial\phi_{hc}(\theta n)/\partial\theta n\partial\theta n^d/\partial\phi_I}{-1+\partial\phi_{hc}(\theta n)/\partial\theta n\partial\theta n^d/\partial\phi_h} \tag{29}$$

$$\frac{d\phi_{fc}}{d\phi_I} = -\frac{\partial\phi_{fc}(\theta n)/\partial\phi_I + \partial\phi_{fc}(\theta n)/\partial\theta n\partial\theta n^d/\partial\phi_I}{-1+\partial\phi_{fc}(\theta n)/\partial\theta n\partial\theta n^d/\partial\phi_f} \tag{30}$$

- Examining (26), one checks that $\partial\phi_{hc}(\theta n)/\partial\phi_I < 0$ and $\partial\phi_{fc}(\theta n)/\partial\phi_I < 0$ when $\theta n \in [0,1]$
- One also checks that $\partial\phi_{hc}(\theta n)/\partial\theta n \lesseqgtr 0$ when $b \lesseqgtr (1-\phi_I)/(1+3\phi_I)$. Since we consider $b < b_0 < (1-\phi_I)/(1+3\phi_I)$, one have that $\partial\phi_{hc}(\theta n)/\partial\theta n < 0$
- It is also the case that $\partial\phi_{fc}(\theta n)/\partial\theta n \gtreqless 0$ when $b \lesseqgtr (1-\phi_I)/(1+3\phi_I)$. Since we consider $b < b_0 < (1-\phi_I)/(1+3\phi_I)$, one have that $\partial\phi_{fc}(\theta n)/\partial\theta n > 0$
- By proposition 3, we have that $\partial\theta n^d/\partial\phi_h > 0$ and $\partial\theta n^d/\partial\phi_f < 0$

Using the expressions above, it is then trivial to check that $d\phi_{hc}/d\phi_f > 0$ and $d\phi_{fc}/d\phi_h > 0$ which proves proposition 4. Finally, we also check that the denominators of (29) and (30) are both negative. The numerator of (29) is negative, which implies that $d\phi_{hc}/d\phi_I < 0$, but the one of (30) has an ambiguous sign. Indeed, an increase in ϕ_I has two opposite effects on ϕ_{fc}: on the one hand, the direct effect $(\partial\phi_{fc}(\theta n)/\partial\phi_I)$ is negative while the indirect one $(\partial\phi_{fc}(\theta n)/\partial\theta n\partial\theta n^d/\partial\phi_I)$ is positive. Nevertheless, one easily check that for $b < b_0$, the direct effect dominates so that $d\phi_{fc}/d\phi_I < 0$ which proves proposition 5.■

A5: Proof of proposition 6

Proposition 10 (6) *The home world market share in the manufacturing industry increases with the agglomeration of the home manufacturing industry if and only if this agglomeration process is spontaneous i.e.* $\theta n^a > \theta n^d$ *when* $\phi_h > \phi_{hc}$.

Proof: Let us define, $B_{h1}^a = B_{h1}|_{\theta nh1=1}$ and $B_{h1}^d = B_{h1}|_{\theta nh1=1/2}$. As, because of the stability of the agglomerated and dispersed equilibrium, we know that both are decreasing functions of the θ_n. Therefore, to proof our claim it is enough to show that $\forall\phi_h \in [\phi_f, 1]$, $\left(B_{h1}^a - B_{h1}^d\right)\big|_{\theta n=\theta n^d} > 0$ iff $\phi_h > \phi_{hc}$.

First note that, by definition of θn^d, $B_{h1}^d\big|_{\theta n=\theta n^d} = 1$, therefore we will show that $B_{h1}^a - 1 > 0$ at $\theta n = \theta n^d$.

By (22), one have that our claim is verified if N at $\theta n = \theta n^d$is positive. We denote by $N1(\phi_h, b, .)$ the value of $N|_{\theta n=\theta n^d}$.

- To analyze $N1(\phi_h, b, .)$, we have to recall that $b < b_1$ is a necessary condition for non international specialization. Indeed, the sufficient condition for non specialization is $b < b^d$ and, as shown above, $b^d < b_1$.

- One can easily check that $N1(\phi_h, b, .)$ has three roots in ϕ_h that are the ϕ_{hc}, ϕ_{hc1} and 1. These roots are the one discussed for the interregional stability. We know that $\phi_{hc} < 1$ and that $\phi_{hc1} < \phi_f$ when $b < b_1$.

- Let us show that $dN1(1, b, .)/d\phi h$ is negative for $b \in [0, b_1]$.

  - $dN1(1, b, .)/d\phi h$ has three roots in $b : 0, b_1$ and a third root, $((1 + \phi_f)(1 - 2\phi I) + 2\phi I^2)/(1 + \phi_f - 2\phi I^2)$ which is larger than b_1.
  - one easily check that $d^2N1(1, b_1, .)/(d\phi_h db) > 0$ therefore $\forall b \in [0, b_1], dN1(1, b, .)/d\phi h < 0$.

- As N is a continuous function of θn and as we assumed that parameters take values such that $\theta n \in [0, 1]$, $N1$ changes sign at its roots. Therefore, since
  - $dN1(1, b, .)/d\phi h < 0$ for $b < b_1$
  - $\phi_{hc1} < \phi_f$, for $b < b_1$
  - We have that $\forall \phi_h \in [\phi_f, 1]$, $N1 > 0$ iff $\phi_h > \phi_{hc}$.