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## ABSTRACT

### Indirect Network Effects and Adoption Externalities\*

The conventional wisdom is that indirect network effects, unlike direct network effects, do not give rise to externalities. In this Paper we show that under very general conditions, indirect network effects lead to adoption externalities. In particular we show that in markets where consumption benefits arise from hardware/software systems, adoption externalities will occur when there are (i) increasing returns to scale in the production of software, (ii) free-entry in software, and (iii) consumers have a preference for software variety. The private benefit of the marginal hardware purchaser is less than the social benefit since the marginal hardware purchaser does not internalize the welfare-improving response of the software industry, particularly the increase in software variety, on inframarginal purchasers when the market for hardware expands.

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# 1 Introduction

A network effect exists if consumption benefits depend positively on the total number of consumers who purchase compatible products.<sup>1</sup> Katz and Shapiro (1985) distinguished between direct and indirect network effects. A direct effect arises when there is “a direct physical effect of the number of purchasers on the quality of the product” (p. 424). Katz and Shapiro (1994) term networks with a direct physical effect “communication networks”, and (not surprisingly) the canonical examples are communication technologies such as telephone networks and facsimile standards where it is intuitive that the value of joining a network depends on the number of other consumers who join by adopting the same, or a compatible, technology.

When the network effect is indirect, consumption benefits do not depend directly on the size of the network (the total number of consumers who purchase compatible products) per se. Rather individuals care about the decisions of others because of the effect that has on the incentive for the provision of complementary products. Users of Macintosh computers are better off the greater the number of consumers who purchase Macs because the larger the number of Mac users the greater the demand for compatible software, which if matched by an appropriate supply response—entry by software firms—will lead to lower prices and/or a greater variety of software which makes all Mac users better off. This hardware/software paradigm applies to many markets, including most consumer electronic technologies, including video-cassette recorders, televisions, and audio technologies. However, it is not restricted to consumer electronics: other examples include natural gas fueled automobiles (hardware) and natural gas filling stations (software); yellow pages (hardware) and listings (software); ATM/ABM cards (hardware) and compatible teller machines (software). Katz and Shapiro (1994) suggest that hardware/software systems can be viewed as “virtual networks

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<sup>1</sup>The seminal *modern* contributions on network effects are a series of papers by Farrell and Saloner (1985, 1986a, 1986b) and Katz and Shapiro (1985, 1986). See Liebowitz and Margolis (2002) for a discussion of historical antecedents.

which have similar properties as direct networks.<sup>2</sup>

In the case of direct network effects it is relatively uncontroversial that they can give rise to a network, or adoption, externality. Individuals when they join a network characterized by direct network effects do so on the basis of their private benefits and do not take into account that others on the network are also made better off by their decision to join. There has, however, been considerable disagreement in the literature over whether indirect network effects give rise to network externalities.

Liebowitz and Margolis in a series of contributions have argued that the term “externality” was used carelessly in reference to indirect network effects and that some researchers in using the term externality—with its connotation of market failure—have forgotten the importance of the distinction between technological and pecuniary externalities established in the 1920s.<sup>3</sup> Liebowitz and Margolis (1994, p.139) argue that indirect network effects “describe nothing more than welfare neutral interactions that occur in properly functioning markets” and thus do not give rise to an externality in the modern sense since they do not result in any welfare losses. The rationale for this dismissal is the assessment that “indirect network effects are ‘market mediated effects’ such as cases where complementary goods (e.g. toner cartridges) are more readily available or lower in price as the number of users of a good (laser printers) increases.” (1998, p. 671) As a result, these “market mediated (read price) effects are likely pecuniary” (1994, p.

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<sup>2</sup>See Katz and Shapiro (1985, 1994); Farrell and Saloner (1985); Chou and Shy (1990); and Church and Gandal (1992 1993).

<sup>3</sup>Liebowitz and Margolis (2002, p.79): “The early modern literature on network externality discusses direct and indirect network externalities, but does not acknowledge that the two have distinct economic properties. As a consequence, this literature repeats some mistakes of Marshall, Pigou, and others regarding pecuniary externalities.” Or Liebowitz and Margolis (1994, p. 137): “Today this distinction between technological and pecuniary externalities has largely been forgotten, perhaps because it is no longer needed to correct Pigou. But the trap that caught Pigou is still set, ready to spring on the unwary analyst. In fact, the pecuniary externalities that so perplexed Pigou walk and quack very much like the indirect network externalities that are waddling through the literature today.” Liebowitz and Margolis have made essentially the same argument regarding indirect network effects in a number of contributions. These include, at least, Liebowitz and Margolis (1994, 1995, 1998, and 2002).

138), not true externalities which impose welfare losses if left uninternalized.<sup>4</sup>

The objective of this paper is to establish the circumstances when network effects give rise to “real” externalities and Liebowitz and Margolis’ fundamental point that indirect network effects are most likely pecuniary externalities is incorrect and misleading. Circumstances, it appears to us, that characterize most hardware/software examples. In particular we demonstrate that the key indirect network effect—giving rise to a network externality—is a variety effect not a price effect. We demonstrate that the issue is not that toner cartridges become more readily available or lower in price as more people use laser printers. The issue is that marginal consumers do not account for the effect that extending the hardware network will have on the variety of software, and thus the benefit inframarginal consumers receive from being able to consume additional software varieties. It is the number of *different* software varieties that is important, not the quantity (or price) of a particular software variety. Therefore, contrary to Liebowitz and Margolis (1994, 1995, 1998, 2002), indirect network externalities are the result of *variety effects*, not price effects.

The critical requirements for indirect network effects to give rise to an adoption externality are three-fold: (i) increasing returns to scale in the production of software; (ii) free-entry into software; and (iii) consumer preferences for software variety. Under these circumstances we demonstrate using a simple model the existence of a positive adoption externality in a setting where consumption benefits arise from consuming systems composed of hardware and software, where the consumption benefits of hardware are increasing in the variety of compatible soft-

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<sup>4</sup>A pecuniary externality is a wealth transfer which imposes a private cost but not a social one. A technological or “modern” externality imposes a social cost, and a social cost reduces the wealth of society where as a private cost rearranges that wealth. Posner (1998, p.7) notes “competition is a rich source of pecuniary’ as distinct from ‘technological’ externalities – that is, of wealth transfers from, as distinct from cost impositions on, unconsenting parties. Suppose A opens a gas station opposite B’s gas station and as a result siphons revenues from B. Since B’s loss is A’s gain, there is no diminution in overall wealth and hence no social cost, even though B is harmed by A’s competition and thus incurs a private cost.” Therefore, while each firm acts in a way that harms the other firms in the industry, that harm is more than offset by transfers of surplus among firms and to consumers.

ware. The marginal adopter does not take into account the benefits that accrue to inframarginal adopters from the response of the software industry to an increase in hardware sales. When there are increasing returns to scale and free-entry into the production of differentiated software the key response to an increase in hardware sales is an increase in software variety which benefits inframarginal consumers. The manner in which inframarginal consumers benefit from indirect effects is identical to the manner in which they benefit when there are direct effects, the ability to create new systems of complementary products. Network externalities that arise in settings with indirect network effects have the same microfoundations as network externalities that rise in settings with direct network effects.

Section 2 explains why the microfoundations of direct and indirect network effects are identical, and in doing so justifies the assumption that consumers are likely to have a preference for variety. The assumptions of our formal model are set out in Section 3.<sup>5</sup> In Section 4 we find the market equilibrium and in Section 5 we derive the efficient allocation. In Section 6 we demonstrate the inefficiency of the market equilibrium by showing that the equilibrium market network size is smaller than the socially optimal size. Section 7 offers a brief conclusion.

## 2 Microfoundations of Network Benefits

Though it seems intuitively obvious that communications networks give rise to network benefits and an adoption externality, in the context of the debate over whether indirect network effects give rise to a similar adoption externality it is illuminating to ask why.<sup>6</sup> A more general perspective on both direct and indirect

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<sup>5</sup>The model presented here shares features with the work of Chou and Shy (1990) and Church and Gandal (1992, 1993). However, the existing literature is concerned with competition between different networks and a focus on whether the optimal technology was adopted in the market equilibrium, not whether adoption of a technology is sub-optimal and whether indirect network effects lead to network externalities.

<sup>6</sup>Our analysis here is based on, and extends, that of Economides and White (1998). Economides and White (1998) discuss both direct and indirect network effects, but they do not comment on whether they are pecuniary or technological, referring the reader to Liebowitz and Margolis (1994). Nor do they model indirect network effects.

network effects recognizes that in both cases—and what is perhaps the defining feature of a network—the products demanded are systems of components. Consumer demand is for a group of complementary products that when combined, or consumed, together provide value. It is often the case that the components by themselves have very little value.

A direct network consists of complements linked together to form a network. In this case horizontal compatibility allows for interconnection of the product purchased by a consumer with that of others. The classic example is a local telephone exchange. Consumers gain access to the network when they subscribe by purchasing a link from their location to the local switch. For a call to be made requires that both the caller and the person called have such a link and value is created by combining the two links through the switch. In this case the system that creates value is comprised of two complementary goods—the link of the caller and the link of the person called to the switch.

When an extra individual joins a network of  $n$  individuals, in addition to the  $n$  potential types of systems that are open to the new individual, the link of the new subscriber creates new systems for the  $n$  inframarginal adopters or existing subscribers. The addition of a new individual to an  $n$  individual network creates  $n$  new systems—combinations of complements that can be connected by existing subscribers to create a new good. It is this creation of new systems for existing subscribers/adopters that is the benefit to existing subscribers of network expansion and the source of the network externality.

As in the direct network externality case, when there are indirect network effects consumers benefit from the adoption by others of compatible hardware because it allows them to consume a wider variety of systems. In this case consumption benefits flow from creating systems consisting of one unit of hardware and one unit of software and the unit of hardware is typically compatible with many different varieties of software. If consumers value variety, then they will demand multiple systems, each involving one unit of hardware and a different variety of

software.<sup>7</sup> The advantage of more adopters of hardware to an existing subscriber arises if an increase in hardware adoption induces the production of more software varieties since existing adopters will then benefit from being able to create more two component systems. Adoption externalities that arise from both direct and indirect network effects have the same source—the creation of new goods for existing subscribers.<sup>8</sup>

### 3 The Model

#### 3.1 Consumer Preferences

We assume that the preferences of consumers are defined over hardware, software, and a competitively supplied outside good. The tastes of consumers for hardware are distributed uniformly along a line of unit length, the population is normalized to one, and all consumers have income of  $y$ . The consumption of a hardware technology different from the most preferred type imposes a utility cost on the consumer that is proportional to the distance separating the consumer and the hardware technology consumed.

The utility of a consumer located distance  $t$  from the hardware product consumed is

$$U = M^\beta + x - kt, \tag{1}$$

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<sup>7</sup>It is a rare individual indeed who listens to only one compact disc on their stereo, uses only one application program on their PC, or plays only one video game on their video game console.

<sup>8</sup>This systems creation benefit shares some similarity to that found in the “mix and match” literature (Matutes and Regibeau 1988; Economides 1989) in that consumers benefit from an increase in the variety of components of a system. Typically in that literature firms produce the two components of a system. Compatibility enables consumers to build a system that is closer to their ideal since compatibility increases the number of available systems in a duopoly from two to four. However, our framework differs from that of the mix and match literature in a couple of ways. Unlike the mix and match framework we assume that consumers have a preference for consuming a variety of systems—not just one—and the number of systems is determined endogenously by free entry in software. It is these two features, free-entry and a preference for variety, along with increasing returns to scale in software, that lead to adoption externalities when there are indirect network effects.

where  $M$  is the number of software varieties consumed,  $x$  is consumption of the competitively supplied numeraire good, and  $k$  measures the degree of hardware differentiation. The network benefit function is  $M^\beta$ . We assume that demand for both a unit of hardware and any given *variety* of software is completely inelastic.<sup>9</sup> We impose the restriction  $0 < \beta < 1$  so that, while the network benefit is increasing in the number of software varieties consumed, the marginal benefit of another variety is declining.

Every consumer who purchases a unit of hardware maximizes their utility by choosing software and consumption of the outside good subject to the following budget constraint

$$\sum_{j=1}^M \rho_j + x = y - p, \quad (2)$$

where  $\rho_j$  is the price of a unit of software variety  $j$ ,  $y$  is the income of the consumer,  $p$  is the price of the hardware technology, and  $M$  is the number of software varieties purchased.

Each consumer optimally selects the number and varieties of software to consume by ranking the software varieties in ascending order by price. The marginal benefit of another software variety is  $\beta M^{\beta-1}$ . Ignoring the integer problem, the optimal number of software varieties  $\widehat{N}(\rho_{\widehat{N}})$  for a consumer who purchases hardware is implicitly defined by

$$\rho_{\widehat{N}} = \beta \widehat{N}^{\beta-1}, \quad (3)$$

where  $\rho_{\widehat{N}}$  is the  $\widehat{N}^{th}$  most expensive software variety. In other words, the consumer purchases one unit of the  $\widehat{N}^{th}$  lowest-priced varieties, where  $\widehat{N}$  is such that the marginal benefit of the  $\widehat{N}^{th}$  software product equals  $\rho_{\widehat{N}}$ .<sup>10</sup>

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<sup>9</sup>Demand is not perfectly inelastic, since above a certain price consumers no longer purchase the product. However, up to that price demand is perfectly inelastic and henceforth demand will be referred to as being completely inelastic. Examples of different software varieties for personal computers are a spreadsheet, word processor, database, etc.

<sup>10</sup>We ignore the possibility that such an  $\widehat{N}$  might not exist. We show below that equilibrium pricing behaviour by software firms will ensure that it does.

Indirect utility as a function of software prices and the hardware price is

$$V = \left(\widehat{N}(\rho_{\widehat{N}})\right)^\beta + y - p - \sum_{j=1}^{\widehat{N}(\rho_{\widehat{N}})} \rho_j - kt. \quad (4)$$

The marginal consumer is indifferent between only consuming the numeraire good and purchasing hardware. Setting (4) equal to  $y$ , the utility from consuming only the numeraire good, the size of the network is

$$t = \frac{\widehat{N}^\beta - p - \sum_{j=1}^{\widehat{N}} \rho_j}{k}, \quad (5)$$

where  $1 \geq t \geq 0$ . Equivalently the size of the network is the demand function for hardware. The demand for hardware depends not only on the price of hardware, but also the number and prices of compatible software varieties.

## 3.2 Technology

We assume that the competitive suppliers of hardware all offer a product located at one end of the unit interval. We also assume that there are constant returns to scale in the production of hardware. Let the constant unit cost of hardware be equal to  $c$ . Our assumption of competition in the provision of hardware means that the price of hardware will be equal to  $c$ . Without loss of generality we set  $c = 0$ .

We assume that the production of software is characterized by increasing returns to scale: the marginal production cost of a unit of variety is zero, but associated with each software variety is a fixed cost equal to  $f$ .<sup>11</sup> Software firms are restricted to providing only a single software variety and there is free entry into the software industry.

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<sup>11</sup>Given that demand for a unit of hardware and each variety of software is completely inelastic for an individual consumer, the assumption of zero marginal cost in either hardware or software has no effect on the results. This assumption is made to simplify the presentation.

### 3.3 Timing

We assume a simple two-stage game and solve for the subgame perfect Nash equilibrium (SPNE). In the second stage the number of software varieties is fixed and there is price competition between software firms, and given the Nash equilibrium in software (and hardware) prices, consumers make their adoption decision. The assumption of competitive hardware means that the Nash equilibrium hardware price is simply its marginal cost. In the first stage there is free-entry into software. The effect of subgame perfection is that software firms anticipate correctly the dependence of second period equilibrium prices, hardware adoption, and profits on the number of software varieties.<sup>12</sup>

## 4 Market Equilibrium

### 4.1 Nash Equilibrium in Software Prices

The determination of the Nash equilibrium in software prices in the second stage when there are  $N$  software firms involves two steps: (i) finding equilibrium prices assuming that software firms believe that the size of the network, i.e. hardware sales, are invariant to software pricing; and (ii) showing that software firms will in fact price as if the size of the network is fixed, since a single software firm will not find it profitable to lower its price in order to induce more consumers to join the network by buying hardware.

#### 4.1.1 Step 1: Hardware Sales Invariant to Software Pricing

Suppose that there are  $N$  software firms which have each developed a single software variety, and contrary to the rules of the game, consumers have already purchased hardware so that the size of the network is fixed. Then the Nash equilib-

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<sup>12</sup>We adopt a two-stage game for ease of presentation. The Nash equilibrium to a simple static game where pricing and entry by software firms occur simultaneously is identical to that of the SPNE in the two-stage game considered here.

rium software price will be

$$\rho(N) = \beta N^{\beta-1}. \quad (6)$$

If the price of a variety of software exceeds  $\beta N^{\beta-1}$  (its marginal benefit), consumers will not purchase it. A price less than  $\beta N^{\beta-1}$  reduces profits since quantity is unchanged as the demand by a consumer for a variety of software is completely inelastic and the market size is assumed to be fixed.

A symmetric equilibrium software price given by (6) implies, from (5) and  $p = c = 0$ , an equilibrium network size of

$$t(N) = \frac{(1 - \beta)N^\beta}{k} \quad (7)$$

when there are  $N$  software firms.

#### 4.1.2 Step 2: An Individual Software Firm Does Not Have an Incentive to Price to Expand Network Size

However given the rules of the game, the size of the network is not *fixed*: from (5) it depends on the price of every software variety. While it is clear that no software firm will ever charge a price higher than (6)—since its sales would then be zero—we need to determine whether a software firm might find it profitable to unilaterally deviate from (6) and charge a lower price in order to induce more consumers to buy hardware and extend the network. The revenue for software firm  $j$  is

$$R_j = \rho_j t,$$

where  $t$  is given by equation (5). The change in the revenue of firm  $j$  given a change in the price of its software variety is

$$\frac{\partial R_j}{\partial \rho_j} = t + \rho_j \frac{\partial t}{\partial \rho_j}. \quad (8)$$

Decreasing the price of its software variety marginally will decrease its revenue by  $t$ , the loss in revenue on inframarginal units. A marginal reduction in  $\rho_j$  increases

the size of the network, and thus sales, by  $\partial t / \partial \rho_j = 1/k$ . An upper bound on the increase in revenue from marginal sales from a decrease in the software price is:

$$\rho_j(N) \frac{\partial t}{\partial \rho_j} = \frac{\beta N^{\beta-1}}{k}. \quad (9)$$

It is an upper bound since it values marginal sales by (6) and *not*  $\rho_j$ .

**Proposition 1** *When  $N > 1$  and  $\beta \leq 1/2$  the Nash equilibrium in software prices is given by (6), i.e.  $\rho(N) = \beta N^{\beta-1}$ .*

**Proof.** For  $t < 1$ , the loss on inframarginal units from a price decrease is  $t(N) = (1 - \beta)N^\beta/k$ , and the upper bound on the revenue from marginal sales from a price decrease is

$$\rho_j \frac{\partial t}{\partial \rho_j} = \beta N^{\beta-1}/k. \quad (10)$$

The decrease in revenue on the inframarginal units is greater than the increase in revenue from the marginal sales from a reduction in the software price  $\rho_j(N)$  if

$$\frac{(1 - \beta)N^\beta}{k} - \frac{\beta N^{\beta-1}}{k} > 0. \quad (11)$$

This is true when  $N > 1$  and  $\beta \leq 1/2$ . For  $t = 1$  a reduction in the price of software below (6) by firm  $j$  reduces revenue on inframarginal units but does not increase quantity. Prices greater than (6) result in sales of zero since price exceeds marginal benefit for all consumers. ■

In equilibrium each variety of software is priced such that consumers purchase one unit from each software firm and  $\widehat{N} = N$ . The price of software exactly equals its marginal benefit and all consumers who purchase hardware consume one unit of each software product.

## 4.2 Nash Equilibrium in Software Variety

In the first stage of the game there is free entry into software. The equilibrium number of software firms (and thus varieties) will be determined by the free-entry condition of zero profits given the equilibrium software price (6).

The equilibrium profits of a software firm when there are  $N$  software firms is

$$\pi(N) = \rho(N)t(N) - f. \quad (12)$$

Substituting (6) into (12) and setting the result equal to zero implicitly defines the free entry number of software firms:

$$\beta N^{\beta-1}t(N) = f. \quad (13)$$

The equilibrium network size,  $t(N)$ , is given by (7) provided  $t \leq 1$ . The free-entry equilibrium is defined by the simultaneous solution of (7) and (13).<sup>13</sup> We can distinguish between an interior equilibrium where  $1 > t > 0$ —in which only some consumers join the network—and universal adoption where  $t = 1$  and all consumers adopt the hardware. Whether the equilibrium involves universal adoption or not depends on the relationship between the extent of hardware differentiation and the value of software.

The effect of entry by another software firm (which supplies an additional variety of software) on equilibrium software profits is given by

$$\frac{\partial \pi(N)}{\partial N} = \frac{\partial \rho(N)}{\partial N}t(N) + \rho(N)\frac{\partial t(N)}{\partial N}. \quad (14)$$

The first term in (14) captures the competitive effect of additional entry: an additional competitor reduces equilibrium software prices. The second term captures the network effect: additional software makes purchasing hardware more attractive and hence extends the hardware market. If the second effect dominates, software firms will continue to enter the market until the market is covered, at which point the network effect is zero.<sup>14</sup> The existence of an interior equilibrium requires that entry from an additional firm will reduce profits.

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<sup>13</sup>The Nash equilibrium to an alternative game in which pricing and entry by software firms occurs simultaneously would be found by solving the same two equations. In this case  $N$  in (7) would be the expected number of software varieties, which if consumers have rational expectations, is given by (13), and  $t(N)$  in (13) is expected market size, which under rational expectations is given by (7).

<sup>14</sup>This terminology is consistent with the analysis and terminology used by Church and Gandal (1992) in their analysis of the incentives of software firms to supply complementary products when there are competing hardware products/networks.

However, this is not sufficient for an interior equilibrium. It must also be the case that the number of software firms which enter in the interior equilibrium is less than the number required to induce every consumer to purchase hardware or join the market. This implies a restriction on  $f$ .

**Proposition 2**

1. *An Interior Equilibrium exists if*

$$\beta(1 - \beta)/k > f > \beta \left( \frac{(1 - \beta)}{k} \right)^{\left( \frac{1-\beta}{\beta} \right)}$$

and

$$\beta < 1/2$$

then the equilibrium network size  $t_m$  and number of software varieties  $N_m$  are

$$t_m = \left[ \frac{(1 - \beta)}{k} \right]^{\frac{1-\beta}{1-2\beta}} \left[ \frac{\beta}{f} \right]^{\frac{\beta}{1-2\beta}} \quad (15)$$

and

$$N_m = \left[ \frac{(1 - \beta)}{k} \right]^{\frac{1}{1-2\beta}} \left[ \frac{\beta}{f} \right]^{\frac{1}{1-2\beta}} . \quad (16)$$

2. *Universal Adoption occurs when:*

$$\beta \left( \frac{(1 - \beta)}{k} \right)^{\left( \frac{1-\beta}{\beta} \right)} > f$$

or

$$\beta \geq 1/2$$

then the equilibrium network size and number of software varieties are

$$t_m = \bar{t} = 1$$

and

$$N_m = \bar{N} = \left(\frac{\beta}{f}\right)^{1/(1-\beta)}. \quad (17)$$

**Proof.** Substituting in the equilibrium network size given by (7) and the equilibrium software price given by (6) into the expression for profits of a software firm (12) and simplifying yields

$$\pi(N) = \beta N^{\beta-1} \left[ \frac{(1-\beta)N^\beta}{k} \right] - f. \quad (18)$$

The equilibrium number of software firms will exceed 1 (recall from Proposition 1 this condition is required for (6) to be the equilibrium software price) if  $\pi(N = 1) > 0$  and this will be true when  $\beta(1-\beta)/k > f$ .

For the interior equilibrium number of software varieties, set (18) equal to zero and solve for  $N$ . For the equilibrium network size, insert the result into (7).

An interior equilibrium exists if (i) the competitive effect dominates and (ii)  $t_m < 1$ . The competitive effect dominates if  $\partial\pi(N)/\partial N < 0$ . Differentiating (18) with respect to  $N$  yields

$$\frac{\partial\pi(N)}{\partial N} = -\beta(1-\beta)^2 \left[ \frac{N^{2\beta-2}}{k} \right] + \beta^2(1-\beta) \left[ \frac{N^{2\beta-2}}{k} \right] \quad (19)$$

which is negative when  $\beta < 1/2$ . If  $\beta \geq 1/2$ , then  $\partial\pi(N)/\partial N \geq 0$  and new software firms continue to enter the market until the market is covered and  $t = 1$ .

Setting (7) equal to one, universal adoption occurs when there are  $\bar{N}$  varieties of software

$$\bar{N} = \left[ \frac{k}{1-\beta} \right]^{\frac{1}{\beta}}. \quad (20)$$

The equilibrium number of software varieties will be less than the number required for full market coverage provided  $f$  is such that when  $N = \bar{N}$ , the profits of a software firm are negative

$$\beta \left[ \frac{k}{(1-\beta)} \right]^{\frac{\beta-1}{\beta}} - f < 0. \quad (21)$$

This is true when

$$f > \beta \left[ \frac{(1-\beta)}{k} \right]^{\frac{1-\beta}{\beta}}. \quad (22)$$

The free-entry number of software firms when there is universal adoption is found by setting  $t = 1$  in (13) and solving for  $N$ . ■

## 5 Social Optimum

In this section we derive the efficient network size. In the first-best allocation the social planner selects both network size ( $t_s$ ) and the number of software varieties ( $N_s$ ) to maximize welfare. In the second-best allocation the social planner can only mandate adoption of the hardware technology and cannot determine directly the number of software varieties. In the second-best allocation the social planner knows that for any  $t_s$ , the variety of software will be determined by the free-entry condition, (13).

### 5.1 First-Best Allocation

The objective function of the social planner is

$$W^* = \int_0^{t_s} [N_s^\beta + \bar{x} - kt_s] dt_s + \int_{t_s}^1 \hat{x} dt_s \quad (23)$$

The first term of (23) is the direct utility of those consumers on the network and the second term in (23) is the utility of those not on the network. Since profits in both hardware and software will be zero, only the surplus of consumers is relevant. In the first-best allocation the social planner maximizes (23) subject to the following resource constraint:

$$fN_s + \int_0^{t_s} \bar{x} dt_s + \int_{t_s}^1 \hat{x} dt_s = y \quad (24)$$

Integrating (23) and (24) and then substituting in the resource constraint, the first-best allocation is defined by maximizing

$$W^* = N_s^\beta t_s + y - \frac{kt_s^2}{2} - fN_s \quad (25)$$

with respect to  $t_s$  and  $N_s$ , i.e. the extent of the network (hardware quantity) and software varieties.

The two first-order necessary conditions are:

$$\frac{\partial W^*}{\partial N_s} = \beta N_s^{(\beta-1)} t_s - f = 0 \quad (26)$$

and

$$\frac{\partial W^*}{\partial t_s} = N_s^\beta - k t_s = 0. \quad (27)$$

The first term in (26) is the marginal social benefit of another software variety (the product of marginal utility and network size) and the second is the marginal social cost of another variety. The first term in (27) is the marginal social benefit to expanding the hardware network; the second term the marginal social cost.

## 5.2 Second-Best Allocation

In the second-best allocation the social planner can only determine the size of the hardware network and the number of software varieties is determined by free-entry. The constrained optimal market size maximizes

$$W = \int_0^{t_s} [(1 - \beta)N^\beta + y - k t_s] dt_s + \int_{t_s}^1 y dt_s \quad (28)$$

subject to

$$\beta N^{\beta-1} t_s = f. \quad (29)$$

The first term of (28) is the indirect utility of those consumers on the network and the second term in (28) is the indirect utility of those not on the network. Since profits in both hardware and software will be zero, only the surplus of consumers is relevant. The constraint is the free entry, zero-profit condition which determines the number of software firms and hence the software variety for any  $t_s$  chosen by the social planner.

Integrating (28) and substituting in the definition of  $N_s$  from the constraint the second-best allocation is defined by maximizing

$$W = (1 - \beta) \left( \frac{\beta t_s}{f} \right)^{\beta/(1-\beta)} t_s + y - \frac{kt_s^2}{2} \quad (30)$$

with respect to  $t_s$ .

In this simple model the first-best and second-best allocations are identical. To see this observe that the constraint in the second best problem (29) is identical to one of the first-order conditions in the first-best problem (26) and maximizing (30) with respect to  $t_s$  gives the same first order condition as (27):  $N_{t_s}^\beta - kt_s = 0$ . As the two allocations are identical, henceforth we will consider only the second-best problem and refer to it as the efficient outcome.

The efficient outcome can either involve partial or universal adoption. Partial adoption, or an interior solution, to the social optimum requires that the marginal benefit of increasing  $t_s$  be negative when  $t_s = 1$ . If it is positive then the efficient solution involves universal adoption, i.e.  $t_s = 1$ .

### Proposition 3

1. *Partial Adoption:*

*If  $1/2 > \beta$  and  $f > \beta(1/k)^{(1-\beta)/\beta}$  then the socially optimal network size and number of software varieties are*

$$t_s = \frac{1}{(k)}^{\frac{1-\beta}{1-2\beta}} \left[ \frac{\beta}{f} \right]^{\frac{\beta}{1-2\beta}}, \quad (31)$$

*and*

$$N_s = \frac{1}{(k)}^{\frac{1}{1-2\beta}} \left[ \frac{\beta}{f} \right]^{\frac{1}{1-2\beta}}. \quad (32)$$

2. *Universal Adoption:*

*If  $\beta < 1/2$  and  $\beta(1/k)^{(1-\beta)/\beta} \geq f$  then the socially optimal network size and number of software varieties are*

$$t_s = 1 \quad (33)$$

and

$$N_s = \left[ \frac{\beta}{f} \right]^{1/(1-\beta)}. \quad (34)$$

**Proof.** For the interior solution, integrate (28), substitute in the definition of  $N_s$  from the free entry constraint (29) and maximize with respect to  $t_s$ . An interior solution to the social optimum requires that the marginal benefit of increasing  $t_s$  be negative when  $t_s = 1$ . The marginal benefit of increasing  $t_s$  is

$$\frac{\partial W}{\partial t_s} = \left[ \frac{\beta t_s}{f} \right]^{\frac{\beta}{1-\beta}} - k t_s. \quad (35)$$

Given  $\beta < 1/2$ , this is negative at  $t_s = 1$  when  $f > \beta(1/k)^{(1-\beta)/\beta}$ .

If  $\beta < 1/2$  and  $f \leq \beta(1/k)^{(1-\beta)/\beta}$  then the marginal benefit of increasing  $t_s$  is positive at  $t_s = 1$  and the efficient solution is to maximize the size of the network. The optimal number of software varieties when there is universal adoption is found by substituting  $t_s = 1$  into (29) and solving for  $N_s$ . ■

## 6 Market Efficiency

**Proposition 4** *Market adoption is inefficient. This inefficiency manifests itself in two ways: (i) The market network is smaller than the socially optimal network when the market equilibrium does not involve universal adoption; and (ii) There are parameter values where the social optimum involves universal adoption but adoption in the market equilibrium is only partial.*

**Proof.** Follows directly from comparing Proposition 2 to Proposition 3. ■

Proposition 4 indicates that the “virtual networks” which exist in hardware/software industries are characterized by the same kind of inefficiency associated with physical networks. The inefficiency arises for the same reason: an adoption externality.

The social planner considers not only the welfare of the marginal consumer, but also the effect that extending the hardware network will have on the supply of software, and hence the welfare of inframarginal consumers. In the market equilibrium, marginal consumers consider only their private benefit when making their adoption decision.

To see this more clearly, observe that after integrating (28), the social planner's objective function is

$$W = (1 - \beta)N^\beta t_s - \frac{kt_s^2}{2} + y \quad (36)$$

The derivative of (36) with respect to  $t_s$  is

$$\frac{dW}{dt_s} = (1 - \beta) \left[ \beta N^{\beta-1} \frac{dN}{dt_s} t_s + N^\beta \right] - kt_s, \quad (37)$$

where from the constraint the social planner recognizes the dependence of  $N_s$  on  $t_s$ . If  $dN/dt_s = 0$ , then setting (37) equal to zero and solving for  $t_s$  would yield  $t_s(N) = t_m(N)$ , or equation (7). The reason that  $t_s(N) > t_m(N)$  is because the social planner recognizes that  $dN/dt_s > 0$ . Increasing the extent of hardware adoption will induce entry by additional software firms and a welfare increasing proliferation of software varieties. The marginal external benefit of adoption is

$$\beta(1 - \beta)N^{\beta-1} \frac{dN}{dt_s} t_s,$$

the increase in software varieties from increasing hardware adoption multiplied by the product of the marginal utility of another software variety and the size of the network. A social planner would be willing to subsidize network adoption in order to increase the number of software varieties, which benefits all adopters, not just the marginal adopter.

## 6.1 Subsidizing Hardware Adoption

That indirect network effects in these circumstances give rise to a positive adoption externality suggest that a tax and subsidy scheme can be adopted that by internalizing the externality results in an efficient market equilibrium. Suppose

that a subsidy of  $s$  is offered to hardware adopters—that is the price of hardware is reduced by  $s$ —and it is financed through a lump-sum tax. Then the demand for hardware (the network size) is no longer (7) but instead:

$$t(N, s) = \frac{(1 - \beta)N^\beta + s}{k} \quad (38)$$

reflecting that the price of hardware is zero and adopters receive a payment of  $s$ .

The free-entry condition, from (13), that determines the number of software varieties is

$$\beta N^{\beta-1} t(N, s) = f. \quad (39)$$

Solving (38) and (39) the subsidy required to achieve any market length  $t$  is

$$s(t) = kt - (1 - \beta) t^{\frac{\beta}{1-\beta}} \left( \frac{\beta}{f} \right)^{\frac{\beta}{1-\beta}}. \quad (40)$$

The relevant  $t$  for the social optimum is given by (31).

For instance, suppose that  $k = 2$ ,  $\beta = 0.25$ , and  $f = 0.05$ . Then  $t_m = 0.51$  and  $N_m = 3.52$ . The efficient outcome is  $t_s = 0.79$  and  $N_s = 6.25$ . The market equilibrium has a network size less than 65% of the efficient network size, with only just over 55% of the efficient software variety. An adoption subsidy of  $s = 0.40$  would result in  $t_m = 0.79$  and  $N_m = 6.25$ .

## 6.2 Understanding Indirect Network Externalities

Our analysis indicates that when market adoption leads to full coverage, there is no inefficiency. In this case, the market outcome and the social outcome are identical, and in particular, the number of software varieties are the same. This is similar to the findings of Spence (1976) and Dixit and Stiglitz (1977) on the optimality of monopolistic competition with an outside good. The market and social optimum would be the same if the market coverage of hardware in the market equilibrium was the same as the socially optimal coverage: the number of software products and their prices in the market equilibrium would be the same

as those in the constrained optimum. Our inefficiency arises because the market coverage of the hardware good is endogenous. Since the marginal consumer only considers their private benefit, there exists an externality and the extent of coverage in the hardware market is less than optimal. Therefore, contrary to the assertions of Liebowitz and Margolis (1994, 1995, 1998, 2002), indirect network effects give rise to network externalities that are technological, *not pecuniary*, and the equilibrium exhibits unexploited gains from trade regarding network participation.

Furthermore, the inefficiency of the market equilibrium in our model is not attributable to market power. The price of software is above marginal cost in order to ensure that software firms earn non-negative profits, but it equals average cost. Given the number of software varieties in the market equilibrium, the price of software is efficient. The inefficiency arises because there are too few software varieties in the market equilibrium since the network size—extent of hardware adoption—is too small.

The critical requirements for indirect network effects to give rise to an adoption externality are three-fold: (i) increasing returns to scale in the production of software; (ii) free-entry into software; and (iii) consumer preferences for software variety. To illustrate the role of (i) assume that there is constant returns to scale in the production of a software variety. Under these circumstances, the price of software would continue to be determined by the free entry, zero-profit condition. The equilibrium number of software varieties would be the number required to equate the marginal benefit of software consumption to average cost and since average cost does not vary with quantity because of constant returns to scale the equilibrium number of software varieties would be invariant to the number of consumers who purchase hardware. The effect of the marginal adopter is to simply increase sales of existing software varieties, not the number of varieties. If there is not free-entry into software—requirement (ii)—then clearly an increase in the number of adopters does not lead to an increase in the number of software varieties. Without a preference for software variety by consumers—requirement (iii)—the link

from increased supply of software varieties to increased utility for all consumers is broken. The additional demand for software from increased sales of hardware would result only in a price effect—a pecuniary externality—not a variety effect and a technological externality.

## **7 Conclusion**

In this paper we have presented a simple model that illustrates the circumstances under which indirect network effects give rise to an adoption externality similar to direct network effects. In doing so we challenge the conclusions of Liebowitz and Margolis that indirect network effects are pecuniary externalities that do not have welfare implications. The circumstances under which indirect network effects give rise to a network externality are (i) increasing returns to scale in the production of software; (ii) free-entry into software; and (iii) consumer preferences for software variety. In our view these circumstances apply to most examples suggested as illustrative of indirect network effects. By clarifying the circumstances under which indirect network effects give rise to adoption externalities, this paper provides the necessary foundation for the ongoing debate involving the broader concern of how public policy should change—if at all—given the increasing importance of industries characterized by indirect network effects in the economy.

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