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ABSTRACT

Time-to-Build and Strategic Investment Under Uncertainty*

We show that time-to-build, which creates a lag between the decision to invest and production, is an important element of industry structure. We study a multi-period investment game where there is demand uncertainty. Allowing for time-to-build alters, non-monotonically, the classic trade-off between making strategic commitments and exploiting the option to wait. At first, increases in time-to-build make commitment more likely as the committing firm has the market to itself for longer after the resolution of uncertainty. As time-to-build becomes more substantial, however, the likelihood of commitment declines as the follower firm increasingly makes its own *ex ante* investments. Furthermore, time-to-build gives rise to a novel incremental Cournot equilibrium where both firms make small *ex ante* investments, which they then scale up if uncertainty is positively resolved. This behaviour contrasts with most prior work on multi-period investment games where firms invest only once. We show how time-to-build affects firm heterogeneity, investment timing, the option value of waiting, the evolution of prices and social welfare.

JEL Classification: L13
Keywords: endogenous investment timing, real options and strategic commitment

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1. Introduction

To what extent do competing firms pursue strategies of flexibility in response to uncertainty? In the simplest case, this involves firms delaying some or all of an investment until after uncertainty is resolved. In opposition to this "real options" perspective on investment (Dixit and Pindyck, 1994), an extensive literature in IO including the work of Spence (1979) and Fudenberg and Tirole (1983) establishes that there may be commitment value to early, irreversible investments. The received wisdom in IO is that investment timing trades-off flexibility and commitment.[1] The conditions for the existence of strategic commitment are explored in an extensive literature on endogenous investment timing including Saloner (1987), Pal (1991), Mailath (1993), Daugety and Reinganum (1994), Maggi (1996) and Sadanand and Sadanand (1996). All of these papers, however, assume that investments are immediately productive, which means that firms can instantaneously increase their capacity in response to the resolution of uncertainty.

In reality, there are many factors that create a lag between the decision to invest and the start of production. Recruiting new employees, constructing physical facilities and gaining required governmental approvals are all time consuming activities. Despite this, **time-to-build** is not treated in the IO literature.[2] A strong motivation for doing so is that time-to-build is empirically observable (unlike, for example, private information) and highly variable. Koeva (2000) measures average time-to-build in 23 industries. She finds that it varies from 86 months for utilities, to 23 months for chemical plants, down to 13 months for rubber processing plants, with an average of 26 months. Table 1.1 reports more detailed data on the petrochemical industry (Pacheco-de-Almeida, 2002). It shows that time-to-build varies systematically across products and across countries. This brings us to our research questions. Given its empirical relevance, does time-to-build matter for the theory of strategic investment under uncertainty? And if it does, what is its impact on observables such as investment timing, firm heterogeneity in market shares and profits, and the evolution of prices?

We develop a theory of investment under uncertainty when there is time-to-build and imperfect competition. In our model, there are two firms that can invest in productive capacity. Initially, there is uncertainty about whether or not there will be demand for the output, with uncertainty resolved at a fixed date. Firms

[1] The received wisdom is well reflected in the following quotes from *Economics of Strategy* (Besanko et al., 2000: p. 282): "...the value of preserving flexibility, of keeping one's future options open, must be considered when evaluating the benefits of commitment," and "Of course, there are factors that limit option values. For instance, our example did not take into account the fact that by waiting the firm risks having the investment opportunity preempted by competitors." See also Vives (1999: pp. 277-9) and Ghemawat and del Sol (1998).

[2] Time-to-build does feature in the macroeconomic theory of business cycles (e.g., Kydland and Prescott, 1982; Gomme *et al.*, 2001). The closest we found in the IO literature is Hirokawa and Sasaki (2001) where there is a time lag required for the resolution of uncertainty that can be *reinterpreted* as time-to-build. However, they assume that firms can invest only once, which we show to be a restrictive assumption in a model with time-to-build (see discussion later in the introduction).

Product	US	Europe
Ethylene	33.8	36.7
Vinyl chloride monomer	27.0	38.0
Polystyrene	20.0	30.0
Polypropylene	19.8	23.0

Table 1.1: Average time-to-build of greenfield plants (in months) for selected products in the petrochemical industry (1978-1997). *Source: Oil and Gas Journal*

can make both *ex ante* and *ex post* investments (i.e. they have the option to wait). However, due to time-to-build, ex post investments only become productive with a lag.

The introduction of a lag between investment and production has substantial effects on the strategic interactions between the firms. As a benchmark, consider first what happens *without* time-to-build. In this case, there are two possible equilibria.[3] In the **Delay** equilibrium, both firms wait to invest until after the resolution of uncertainty, and the resulting simultaneous investment leads to a Cournot-like outcome. In the **Commit-delay** equilibrium, one firm acts as a leader and makes a large preemptive investment, while the other acts as a follower and waits to invest, a Stackelberg-like outcome. The magnitude of uncertainty determines which equilibrium exists. If uncertainty is sufficiently great, then the loss in flexibility required for commitment outweighs the gain and the unique equilibrium is Delay. Conversely, for low levels of uncertainty the unique equilibrium is Commit-delay. Thus, we start with a model that captures the classic trade-off between flexibility and commitment and, as is almost always the case in the received literature, has the property that firms do not make incremental investments (i.e., each firm only invests in one period).[4]

The first effect of introducing time-to-build to the model is to create an **initial price premium**. Immediately after the favorable resolution of uncertainty, capacity is limited because only ex ante investments are productive and hence price is high. Subsequently, ex post investments come on-line and the price declines. The extent to which time-to-build and the resulting price premium alter behavior depends on the magnitude of the length of the time-to-build.

For a sufficiently short time-to-build, the equilibrium still involves either both firms delaying or one firm committing. However, time-to-build makes it more likely that commitment occurs. There are two sources of this effect. First, delay is less attractive because profits come later. Second, the payoff from making a large preemptive investment is now greater due to the initial price premium. Thus, time-to-build

[3] We restrict attention to pure-strategy, subgame perfect equilibria.

[4] Usually, firms are restricted to a single investment period by assumption (eg, Mailath, 1993; Daugety and Reinganum, 1994; and Hirokawa and Sasaki, 2000, 2001). However, this can also emerge as an equilibrium property as in Pal (1991, 1996), who studies a duopoly model where firms can invest in either of two periods and where the investment cost varies across periods. Pal (1991) shows that all pure-strategy equilibria involve each firm investing only once. Pal (1996) shows that this result holds for mixed-strategy equilibria as well. An exception is Maggi (1996); see footnote 11.

alters the trade-off between commitment and flexibility. Moreover, for sufficiently low levels of uncertainty the commitment equilibrium takes the form of a **Commit-incremental** equilibrium where the follower (responding to the price premium) invests both ex ante and ex post.

Larger values of time-to-build have a more radical effect as an alternative to the Delay and two commitment equilibria arises. In this **Incremental Cournot** equilibrium both firms make small (i.e. non preemptive) ex ante investments. These investments are a fraction of the standard Cournot investments. Firms then scale up their investments to the full Cournot level if there is demand. We show that for a sufficiently long time-to-build, Incremental Cournot is the *unique* equilibrium. Although increases in time-to-build initially enhance the attractiveness of commitment by prolonging the initial price premium, they ultimately undermine the credibility of commitment. This occurs because the follower increasingly makes its own ex ante investments and hence investment decisions become simultaneous rather than sequential.[5]

In general, we find clear and consistent comparative statics *within* each of the equilibria. Social welfare is falling in time-to-build, largely because firms cannot respond as fast to the arrival of information about demand. The extent to which firms exploit the option to wait is also falling in time-to-build, which leads the value of the option to wait (to both firms and society) to fall. Finally, the price premium is falling in the time-to-build, which means there is a negative relationship between the duration of the initial price premium and its magnitude. However, these effects may reverse due to the (non-monotonic) effect of time-to-build on the existence of the commitment equilibria, which we show to have higher social welfare, less use of the option to wait and a smaller price premium. Finally, we find a clear non-monotonic relationship between time-to-build and firm heterogeneity: increases in a sufficiently short time-to-build increase heterogeneity while increases in a long time-to-build decrease heterogeneity.

The paper proceeds as follows. Section 2 describes the model. Section 3 characterizes how the set of equilibria vary with time-to-build and the level of uncertainty. Section 4 characterizes how firm heterogeneity, social welfare and the use and value of the option to wait change with time-to-build. Section 5 concludes. The proofs are in the Appendix of the paper.

2. Model

There are two firms indexed $k = 1, 2$ that have access to a potential market opportunity. There are three periods labeled I, II and III. Figure 2.1 shows the timing of the model, which is described below.

Investments can be made in both periods I and II. Firm k's investment in period I is denoted x_k, while its investment in period II is denoted y_k. We impose $x_k \geq 0$

[5] Thus, we show that firms want to make incremental investments if either the time-to-build is sufficiently long or uncertainty is sufficiently small. This calls into question the common assumption in the literature on endogenous timing that firms only invest at a single point in time.

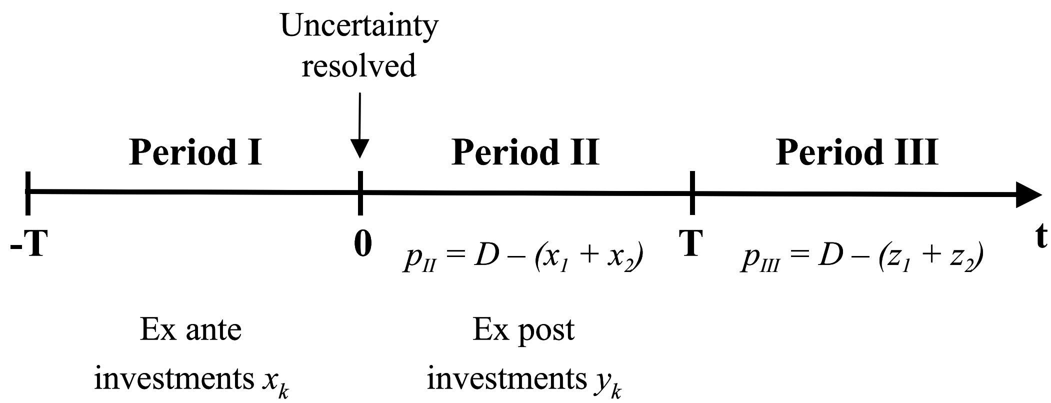

Figure 2.1: The timing of the model

and $y_k \geq 0$. The total investment of firm k is denoted $z_k = x_k + y_k$. Investment is irreversible.

A firm's output in a period is equal to the investments made in *prior* periods. The delay is due to time-to-build, which we parameterize by T.[6] Thus, there is no output in period I; in period II the output is equal to the period I investment $x_1 + x_2$; and the period III output is equal to total investment $z_1 + z_2$.

In period I, there is uncertainty about future demand. With probability $a < 1$, the per period demand is given by the inverse demand function $p(Q) = D - Q$. With probability $(1 - a)$ there is no demand and $p(Q) = 0$. We assume that $D = \bar{D}/a$ so that increasing a makes the investment less risky but does not change the expected demand intercept $\bar{D}$.[7] Uncertainty is resolved between periods I and II. When there is positive demand, period II prices are denoted $p_{II} = p(x_1 + x_2)$ and period III prices are denoted $p_{III} = p(z_1 + z_2)$. We refer to period I investment as *ex ante* and period II investment as *ex post*. Let Δ be the initial price premium relative to the final price:

$$\Delta = \frac{p_{II} - p_{III}}{p_{III}},$$

which is positive as long as there are some ex post investments.

Firms have a discount rate $\delta \in (0, 1)$. The cost of a unit of investment is denoted $\hat{c} > 0$, which is incurred at the time an investment is completed. Firms face no cost of production beyond the investment cost $\hat{c}$. We define $c = (1 - \delta)\hat{c}$.

We assume that firms seek to maximize expected, discounted profits given the discount rate δ. To specify the objective function it is necessary to know the relative length of periods II and III. Period II has length T.[8] We assume period III has infinite

[6] We assume that T is independent of the size of the investment, which is supported by the empiricial work of Koeva (2000).

[7] The expected demand intercept is $aD = \bar{D}$ and the variance is $\bar{D}^2(1-a)/a$, which is decreasing in a. In an earlier version of the paper (Pacheco-de-Almeida and Zemsky, 2001), we assume that D is independent of a without substantially changing the results, except for some of the comparative statics w.r.t. a.

[8] Recall that investments begun immediately after the resolution of uncertainty (i.e., at the

length. Formulating the problem in discreet time we have the following expression for discounted expected profits

$$\begin{aligned}\hat{\pi}(x_k, x_{-k}, z_k, z_{-k}) &= a\left(\sum_{t=0}^{T-1}\delta^t x_k p_{II} + \sum_{t=T}^{\infty}\delta^t z_k p_{III} - \delta^T \hat{c}(z_k - x_k)\right) - \hat{c}x_k \\ &= a\left(\frac{1-\delta^T}{1-\delta}x_k p_{II} + \frac{\delta^T}{1-\delta}z_k p_{III} - \delta^T \hat{c}(z_k - x_k)\right) - \hat{c}x_k\end{aligned}$$

where we have simplified the expression by using the fact that ex post investments $y_k = z_k - x_k$ are only made if there is demand. We multiply this $\hat{\pi}()$ by $(1-\delta)$ and substitute for prices to obtain the objective function we work with

$$\begin{aligned}\pi(x_k, x_{-k}, z_k, z_{-k}) &= a\left((1-\delta^T)x_k(D - x_k - x_{-k}) + \delta^T z_k(D - z_k - z_{-k})\right) \\ &\quad -a\delta^T c(z_k - x_k) - cx_k. \qquad (2.1)\end{aligned}$$

Thus, a firm maximizes a weighted average of its expected period II and III revenues less the expected cost of investment. The relative weighting depends on the time-to-build and the discount rate; specifically, the weight on period II is $(1-\delta^T)$ and the weight on period III is δ^T. The expected cost of period II investment are a fraction $a\delta^T$ of period I investment costs because they are only incurred when there is demand and because they are made later.[9]

Following Pal (1991) and Maggi (1996), we characterize the pure-strategy subgame perfect equilibria (SPE) of the model. We assume that costs are less than the expected demand intercept, $\bar{D} > c$, as otherwise ex ante investment is never profitable and the unique equilibrium is for both firms to invest ex post.

3. Equilibrium Analysis

We break the equilibrium analysis into four steps. In Section 3.1, we characterize the Nash equilibrium ex post investments for given ex ante investments. Then, in Section 3.2, we introduce an equilibrium typology. In Section 3.3, we characterize the set of equilibria when there is no time-to-build ($T = 0$). Finally, in Section 3.4 we characterize the effect of time-to-build on the set of possible equilibria.

3.1. Ex Post Investment

As usual, we work backwards to solve the model by first characterizing ex post investment. A useful preliminary is to consider the equilibrium if there is no ex ante

beginning of period II) are only productive at the beginning of period III due to the time-to-build T. Thus, the length of period II is precisely T.

[9] We do not derive comparative statics for δ. To do so, one needs to account for the dual impact on both the weight on each period through δ^T and on the effective cost of capacity $c = (1-\delta)\hat{c}$. An increase in δ is then equivalent to a decrease in both T and c. When comparative statics are the same for T and c (as in Proposition 4.4), then the reverse result holds for δ.

investment $(x_1, x_2) = (0, 0)$ and there is demand. The reaction functions are then $Z(z_{-k}) = \arg\max_z \delta^T z(p(z+z_{-k})-c)$, which yields $Z(z_{-k}) = (D-z_{-k}-c)/2$. There is a unique intersection of these reaction functions at the outcome $\bar{z} = (D-c)/3$, the standard Cournot outcome with linear demand. Now consider arbitrary ex ante investments $(x_1, x_2) \geq (0, 0)$. The best response of a firm to total investment z_{-k} is then $\max\{x_k, Z(z_{-k})\}$, which reflects the irreversibility of period I investments. When there is demand, the Nash equilibrium level of total investment is given by the intersection of these reaction functions, as first shown by Saloner (1987):[10]

Lemma 3.1. *Given (x_1, x_2) and positive demand, the equilibrium total investment of firm k is*

$$Z(x_k, x_{-k}) = \begin{cases} \bar{z} & \text{if } x_k \leq \bar{z} \text{ and } x_{-k} \leq \bar{z}, \\ Z(x_{-k}) & \text{if } x_{-k} > \bar{z} \text{ and } x_k < Z(x_{-k}), \\ x_k & \text{if } x_k > \bar{z} \text{ or } x_k \geq Z(x_{-k}), \end{cases}$$

If there is no demand then $Z(x_k, x_{-k}) = x_k$.

With this characterization of the period II equilibrium, we can write the payoffs from period I investment as $\pi(x_k, x_{-k}) = \pi(x_k, x_{-k}, Z(x_k, x_{-k}), Z(x_{-k}, x_k))$.

Figure 3.1 illustrates how Lemma 3.1 naturally divides the set of possible ex ante investments into regions. If ex ante investments are anywhere in region I (defined by $x_k \leq \bar{z}$ and $x_{-k} \leq \bar{z}$), then total equilibrium investment is $(\bar{z}, \bar{z})$. If ex ante investment is in one of the C regions (defined by $x_k > \bar{z}$ and $x_{-k} < Z(x_k)$ for some $k = 1, 2$), then one firm is beyond its reaction function and does not invest ex post ($z_k = x_k$) while the other firm invests ex post up to it's best response. In region $\oslash$, both firms are at or beyond their best response and there is no ex post investment $(z_1, z_2) = (x_1, x_2)$. However, ex ante investments in this region are not possible in equilibrium:

Lemma 3.2. *When there is demand, there is ex post investment in any pure-strategy SPE and hence the equilibrium price premium is positive $\Delta > 0$.*

Good news about demand, must stimulate additional investment by at least one firm. When this investment comes on-line, the price falls and hence there is necessarily a positive initial price premium.

3.2. Equilibrium Typology

We create a typology of possible equilibria based on the regions in Figure 3.1 and our interest in incremental investment. We first define three types of *ex ante* investment:

Definition 3.3. *We say that a firm k **delays** investment if $x_k = 0$, that it makes an **incremental** investment if $x_k \in (0, Z(x_{-k}))$ and that it **commits** if $x_k > \bar{z}$.*

[10] Although Saloner is looking at production spread over two periods, the structure of his model in the second production period is the same as in ours after the positive resolution of uncertainty.

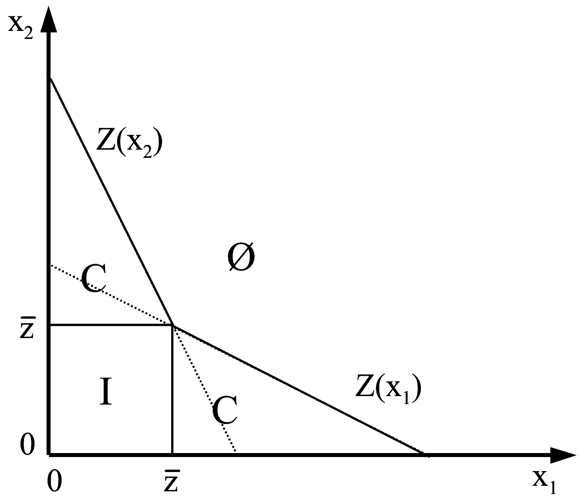

Figure 3.1: Ex ante investments classified into four regions

We distinguish between four types of equilibria based on the nature of ex ante investments.

Definition 3.4. *In a **Delay** equilibrium both firms wait to invest until after the resolution of uncertainty i.e.,* $(x_1, x_2) = (0, 0)$. *In an **Incremental Cournot** equilibrium both firms make the same incremental investment* $\bar{z} > x_1 = x_2 > 0$. *In a **Commitment** equilibrium one of the firms commits. We distinguish between two types of commitment equilibria. In a **Commit-delay** equilibrium one of the firms commits while the other delays investment, i.e.,* $x_k > \bar{z}$, $x_{-k} = 0$ *and in a **Commit-incremental** equilibrium one of the firms commits while the other makes an incremental investment, i.e.,* $x_k > \bar{z} > Z(x_k) > x_{-k} > 0$.

The Delay and Incremental Cournot equilibria lie in the I region of Figure 3.1. Delay is the point $(0, 0)$ and Incremental Cournot equilibria lie on the line between $(0, 0)$ and $(\bar{z}, \bar{z})$, as they must be symmetric. In both these equilibria, total investment when there is demand is the Cournot outcome $(\bar{z}, \bar{z})$. The region C shows possible Commitment equilibria. In a Commit-delay equilibrium, which are given by the intersection of the C regions and the axes, no firm makes an incremental investment: one commits with a large ex ante investment while the other firm delays. Other points in C represent Commit-incremental equilibria, where one firm commits and the other firm makes an incremental investment.

Lemma 3.5. *The only possible pure-strategy subgame perfect equilibria are Delay, Incremental Cournot, Commit-delay, and Commit-incremental.*

Given that Lemma 3.2 rules out equilibria with ex ante investment in the region Ø, the additional equilibria ruled out by Lemma 3.5 are equilibria in I where $x_1 \neq x_2$. Just as in a static model of quantity competition with linear demand and constant marginal costs, the first-order conditions in our model only allow a symmetric solution when firms are not making strategic commitments. We denote by x_I the ex ante investment of each firm in Incremental Cournot. For the Commitment equilibria, we refer to the firm that commits as the **leader** and denote its ex ante investment by x_L; we refer to the firm that does not commit as the **follower** and denote its ex ante investment by x_F.

Both Commit-incremental and Incremental Cournot are novel equilibria in the literature. The essential feature of these equilibria is that firms split investment across two periods, trading-off exposure to uncertainty and the lure of the initial price premium. It is the endogeneity of the price premium that leads to incremental investment: the premium falls with ex ante investment until the firm(s) making an incremental investment are just indifferent at the margin between ex ante and ex post investment.[11]

3.3. Instantaneous Investment

To understand the effect of time-to-build, it is useful to begin with the benchmark where investment is immediately productive (i.e., $T = 0$). Then, period II disappears and we are left with a simple two-period model where investment can occur in either of the two periods and all production and sales occur in the second. Hence, this case is already treated in the received literature on two-period Cournot models. In particular, Maggi (1996) considers a model similar to ours but with no time-to-build, while Saloner (1987) and Pal (1991, 1996) consider models without lags and without uncertainty (i.e., $a = 1$).

A key prior result is the characterization in Pal (1991) of equilibria when it is more costly to invest in the first period than in the second. He shows that pure-strategy equilibria are either Commit-delay or Delay, depending on the size of the cost differential. Firms have an incentive to delay investment as this is less costly. However, if one firm is delaying, then the other firm can choose between delaying as well, or investing early and committing. Which of the two holds, depends on whether the benefit from commitment outweighs the higher investment cost. This result is an important building block for our theory, as the main effect of uncertainty is to make period I investment more costly than period II investment because the former is made whether or not there is demand. This parallel between cost differentials and uncertainty was first pointed out by Maggi (1996). Formally, we have the following:

Proposition 3.6. *Suppose $T = 0$. i) There exists an $a_{DC} < 1$ such that there is a Delay equilibrium* iff *$a \leq a_{DC}$ and there is a Commitment equilibrium* iff *$a \geq a_{DC}$. ii) Firms never make incremental investments.*

[11] There can be other sources of incremental investment besides time-to-build. For example, Maggi (1996) studies a model with a continuously distributed uncertainty and finds that one firm may make an incremental investment. This occurs when a firm tries to commit but realized demand is so high that its ex ante investment is actually below the Cournot level. We would also expect incremental investment to occur if costs are sufficiently convex in the amount of investment initiated in a period.

We find that if uncertainty is not too great ($a > a_{DC}$), the equilibrium is Commit-delay. In this case, the cost of investing early is less than the benefit of committing. Otherwise the equilibrium is Delay.[12] Thus, Proposition 3.6 captures the classic trade-off between commitment and flexibility. Note that firms do not make incremental investments nor do firms make simultaneous ex ante investments.

3.4. The Effect of Time-to-Build

The effect of time-to-build depends on its magnitude. We start by considering the effect of lags less than a critical value $\bar{T}$ defined by $\delta^{\bar{T}} = 16/17 \approx 0.94$.

Proposition 3.7. *Suppose $T \in (0, \bar{T}]$. i) There exits a critical value $a_{DC} < 1$ such that there is a Delay equilibrium* iff *$a \leq a_{DC}$, there is a Commitment equilibrium* iff *$a \geq a_{DC}$. ii) The set of parameters for which there is a Commitment equilibrium is increasing in the time-to-build: $\partial a_{DC}/\partial T < 0$. iii) There exists an $a_{di} \in (a_{DC}, 1)$ such that there is a Commit-delay equilibrium* iff *$a \in [a_{DC}, a_{di}]$ and there is a Commit-incremental equilibrium* iff *$a > a_{di}$. iv) Incremental Cournot equilibria do not exist.*

As in the case of no time-to-build, the outcome is either Delay or Commitment.[13] However, time-to-build makes Delay less likely. First, it is less attractive to delay investment since profits are pushed into the future. At the same time, committing becomes more attractive as the leader benefits from the initial price premium. Hence, the longer the time-to-build, the greater the incentive to commit rather then to delay investment for a given level of uncertainty and the higher the level of uncertainty for which commitment equilibria are possible. In summary, we see that a small time-to-build alters the trade-off between flexibility and commitment.

For sufficiently small levels of uncertainty (i.e., $a > a_{di}$), the follower makes its own ex ante investment. The price premium is sufficient to offset the (small) cost of investing prior to the resolution of uncertainty. These are incremental investments and the follower scales up its capacity if uncertainty is favorably resolved.

We now turn to longer lags. Define $\hat{T}$ by $\delta^{\hat{T}} = (9 - \sqrt{17})/6 \approx 0.81$ and note that $\hat{T} > \bar{T}$.

Proposition 3.8. *Suppose $T > \bar{T}$. i) There exist critical values $a_D < a_C < a_I < 1$ such that there is a Delay equilibrium* iff *$a \leq a_D$, there is a Commitment equilibrium iff $a_C \leq a$ and there is an Incremental Cournot equilibrium* iff *$a_D < a \leq a_I$. ii) A Commit-delay equilibrium exists iff $T \leq \hat{T}$ and $a_C \leq a \leq a_{di}$ for some $a_{di} < 1$.*

For longer lags, an Incremental Cournot equilibrium is possible and for a range of parameters ($a_D < a < a_C$) it is unique. The intuition is as follows. There are two ways that Delay can break down as a increases. The first is that one firm finds

[12] The Delay equilibrium only exists for a sufficiently low if costs are not too small. Specifically, $a_{DC} > 0 \Leftrightarrow c > \bar{D}(3 - \sqrt{8})/6 \approx \bar{D}/35$.

[13] The Delay equilibrium exists for a sufficiently small as long as costs are not too small. Specifically, $a_{DC} > 0$ if $c > \bar{D}/17$ since $\lim_{T \to \bar{T}} a_{DC} = (17c - \bar{D})/(16c)$.

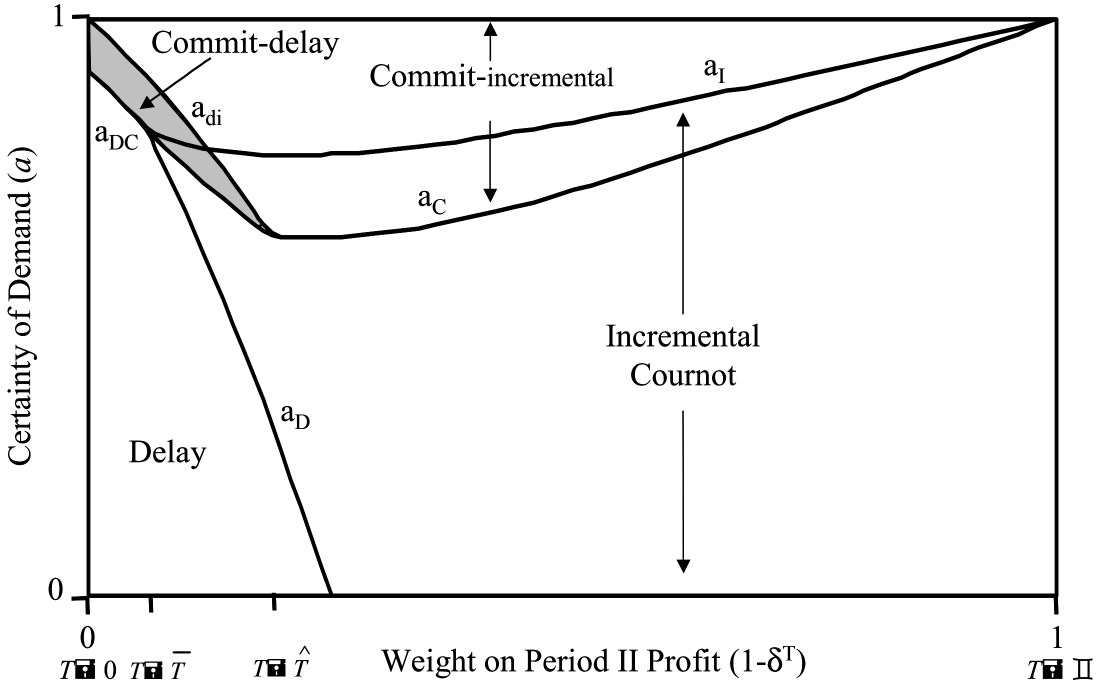

Figure 3.2: Equilibrium regions for $\bar{D}/c = 4$

it profitable to make a large ex ante investment and commit. The second is that both firms find it profitable to make small incremental investments, which leads to Incremental Cournot. Increases in T increase the incentive for both deviations because the initial price premium lasts longer. However, the incentive for an incremental deviation increases faster in T and hence, for T big enough, the Incremental Cournot equilibrium emerges.

We find that the range of parameters for which there is a Commit-delay equilibrium ($a_C \leq a \leq a_{di}$) is shrinking in T so that for $T > \hat{T}$ the only possible Commitment equilibrium is Commit-incremental.[14] In terms of limit behavior we have:

Proposition 3.9. *i) For all $a < 1$, the unique equilibrium for T sufficiently large is Incremental Cournot. ii) For all $T > 0$, the unique equilibrium for a sufficiently close to 1 is Commit-incremental.*

The range of parameters for which there is Delay or Commitment vanishes as the time-to-build becomes arbitrarily large, leaving Incremental Cournot as the unique outcome. Delay vanishes because profits from delayed investment are pushed into the infinite future. The breakdown of the Commitment equilibrium comes about

[14] Both a_C and a_{di} are falling in T: a longer lasting initial price premium increases both the willingness to commit and the willingness of the follower to invest ex ante. The effect of T on the incentive for incremental investment is stronger, as we saw with the emergence of the Incremental Cournot equilibrium.

because investment becomes increasingly simultaneous as the follower increases its ex ante investment and at some point commitment, although still profitable, is no longer credible. (The next section details how investment levels and profits change with T.)

Taking the limit as uncertainty becomes small ($a \to 1$), we find that the unique equilibrium is Commit-incremental. As the cost to investing before the resolution of uncertainty goes to zero, commitment becomes attractive and the follower becomes willing to make an incremental investment.

Figure 3.1 shows the equilibrium regions when $\bar{D}/c = 4$. In order to show all values of $T \in [0, \infty]$, we use the weight on period II profit, $(1 - \delta^T)$, for the x-axis. This quantity varies positively with T from 0 for $T = 0$ to 1 for $T = \infty$. We use the same axis for the graphs in the remainder of the paper.[15]

4. Comparative Statics

In this section, we characterize the effect of time-to-build on firm heterogeneity (Section 4.1), social welfare and first-best investment (Section 4.2), and the use and valuation of the option to wait (Section 4.3). As we saw in the last section, changes in T affect not only behavior within a given equilibrium, but also the set of possible equilibria. To account for the latter, we compare behavior across equilibria.

4.1. Firm Heterogeneity

Firms in our model are initially symmetric, but they may ultimately differ in terms of investments timing, market share and expected profit. Such heterogeneity only occurs in the Commitment equilibria, as Delay and Incremental Cournot are symmetric. Consequently, increasing a small T can create heterogeneity where there was none by making Commitment equilibria possible (Proposition 3.7 (ii)). Conversely, making T sufficiently large eliminates heterogeneity as only the Incremental Cournot equilibrium is possible (Proposition 3.9 (i)). We find the same non-monotonicity when we look at the effect of time-to-build *within* Commitment equilibria:

Proposition 4.1. *i) In a Commit-delay equilibrium the difference between the leader's market share and that of the follower is increasing in T; the difference in expected profits is increasing in T as well. ii) In a Commit-incremental equilibrium, the difference between the leader's market share and that of the follower (in both period II and III) is decreasing in T; the difference in expected profits is decreasing in T for a sufficiently close to 1. iii) As $\partial a_{di}/\partial T < 0$, there is Commit-delay for low values of T and Commit-incremental for high values of T.*

Heterogeneity is increasing in time-to-build for the Commit-delay equilibrium. The leader has a monopoly position for longer, which encourages more investment

[15] The equilibrium regions shift with $\bar{D}/c$. The set of parameters with a Delay equilibrium are decreasing in $\bar{D}/c$ (i.e., the expressions for a_D and a_{DC} in the Appendix are decreasing). Numerical analysis indicates that the set of parameters with Commitment are increasing (i.e., a_I and a_C are decreasing).

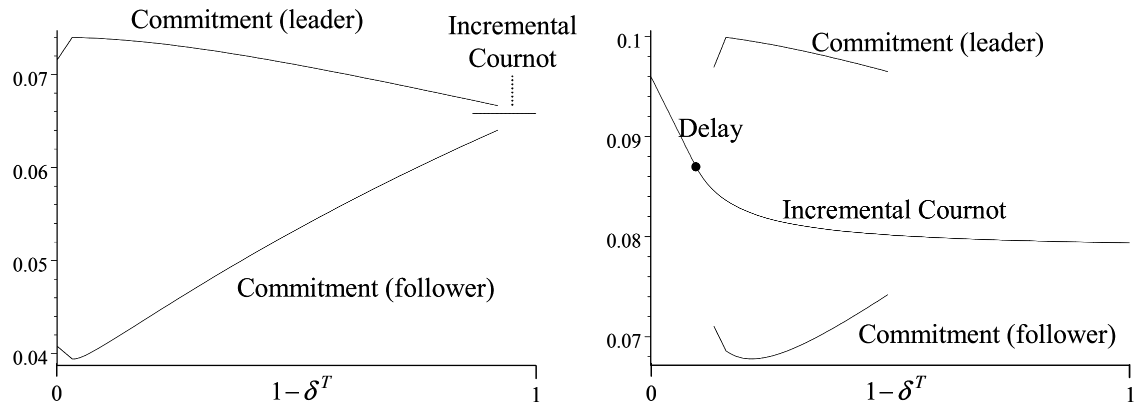

Figure 4.1: Firm profits when $\bar{D} = 1$, $c = .25$ and $a = .95$ (left graph) and $a = .7$ (right graph)

and increases profits. However, increasing T eventually shifts play to Commit-incremental. Then, the follower's ex ante investment is increasing in T, while the leader's investment falls. The falling difference in market share can also reduce profit differences.[16] Figure 4.1 shows how firm profit varies in T (or more precisely $(1-\delta^T)$) when $\bar{D} = 1$ and $c = .25$, parameters which satisfy the condition $\bar{D}/c = 4$ used for Figure 3.1. We look at both low uncertainty ($a = .95$) where Commitment is the unique equilibrium for all but high values of T and moderate levels of uncertainty ($a = .7$) where Commitment equilibrium only occur for intermediate levels of T. Profits are only plotted for an equilibrium when it exists.[17]

Note that our results on heterogeneity have their limitations. First, Commitment equilibria always come in pairs and we cannot explain which firm is the leader. Second, the heterogeneity we identify could be transitory in that growth in the market would be served by the follower because its lower market share gives the follower more incentive to expand output.

4.2. First Best Investment and Social Welfare

One source of time-to-build is government regulations and other institutional features of an economy. Thus, in Table 1.1, we see that time-to-build for chemical plants is on average lower in the US than in Europe. We now turn to the welfare implications of such differences. We proceed as follows. First, we define social wel-

[16] We do not get a stronger result on profit convergence because an increase in T has two effects within the Commit-incremental equilibrium. First, it erodes the leaders ability to commit, which reduces profit differences. Second, it reduces the present value of the follower's profits from ex post investment, which increases profit differences.

[17] When $a = .95$ a Delay equilibrium never exists, Commit-delay exists for $1 - \delta^T \in [0, .03]$, Commit-incremental exists for $1-\delta^T \in (.03, .92]$ and Incremental Cournot exists for $1-\delta^T \in (.87, 1]$. When $a = .7$, Delay, Incremental Cournot, Commit-delay and Commit-incremental exist for the following ranges, respectively, $[0, .09], (.09, 1], (.13, .16]$ and $(.16, .50]$. All values rounded to two decimal places.

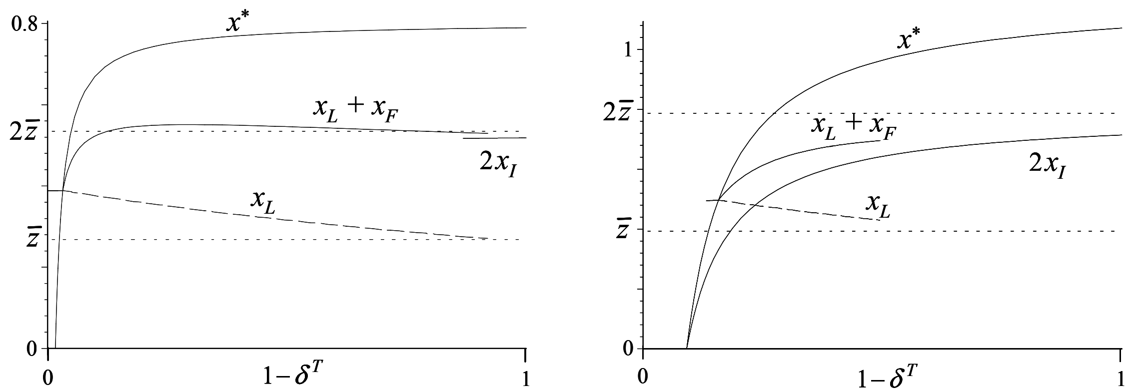

Figure 4.2: *Ex ante* investment when $\bar{D} = 1$, $c = .25$ and $a = .95$ (left graph) and $a = .7$ (right graph)

fare and characterize first-best investment levels. Then, we compare equilibrium outcomes to the first-best. Finally, we derive comparative statics on social welfare.

The social welfare from investments x and y is proportional to

$$W(x, z) = a\left((1 - \delta^T)u(x) + \delta^T u(z) - \delta^T(z - x)c\right) - xc$$

where $u(q) = \int_0^q p(q)dq = q(D - \frac{1}{2}q)$ is the total willingness-to-pay for some output q and where we have multiplied by $(1 - \delta)$ as we did with π. Let x^* and z^* be the investments that maximize the social welfare function. We have $z^* = D - c$, as this is the level of total investment which sets price equal to marginal cost (i.e., $p_{III} = c$). We have $x^* = 0$ iff $W_1(0, z^*) \leq 0$, which is equivalent to $a \leq (1-(1-\delta^T)\bar{D}/c)/\delta^T = a_D$. That is, there is a close relationship between the willingness of profit maximizing firms to delay investment and the willingness of a social planner. If $a > a_D$, the social planner wants to invest ex ante and the optimal level is $x^* = \frac{\bar{D}(1-\delta^T)-c(1-a\delta^T)}{a\left(1-\delta^T\right)}$. Comparing x^* to the equilibrium investment levels (given in the Appendix), we find:

Proposition 4.2. *i) In Delay and Incremental Cournot equilibria, ex ante and ex post investments are 2/3 of the corresponding first-best investment levels, i.e.* $2x_I = \frac{2}{3}x^*$ *and* $2\bar{z} = \frac{2}{3}z^*$. *ii) Ex ante investment is above the first-best level in a Commit-delay equilibrium* ($x_L \geq x^*$)*, and strictly below the first-best level in a Commit-incremental equilibrium,* ($x_L + x_F < x^*$)*.*

The Delay and Incremental Cournot equilibria just scale down the first-best investment levels and by the same fraction as in a standard Cournot model. Interestingly, the two Commitment equilibria can be distinguished based on whether or not the leader invests above or below the first-best level of ex ante investment, as this determines whether or not the follower can profitably invest ex ante. Figure 4.2 shows ex ante investments in each equilibria (when they exists) and the first-best ex ante investment; it uses the same parameters as Figure 4.1.

It follows from the proportionality result in Proposition 4.2 part (i) that

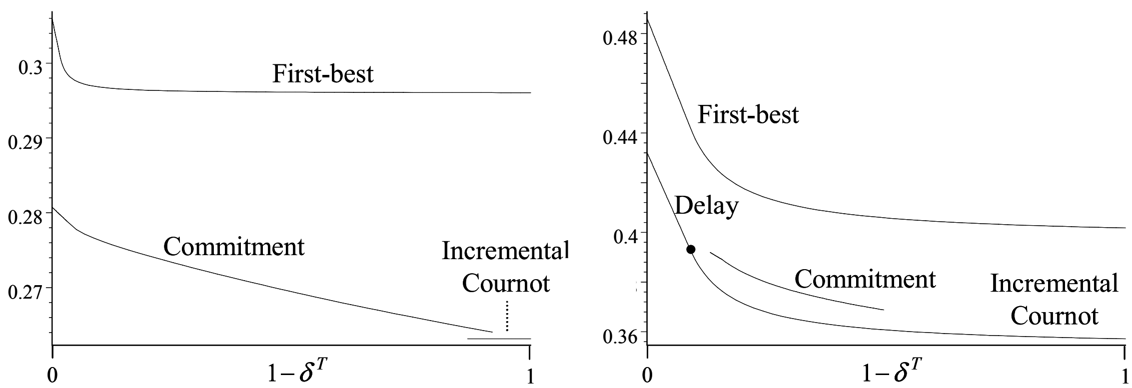

Figure 4.3: Social welfare when $\bar{D} = 1$, $c = .25$ and $a = .95$ (left graph) and $a = .7$ (right graph)

Lemma 4.3. *Social welfare in the Delay and Incremental Cournot equilibrium are 4/9 of the first best level of social welfare, which is given by*

$$W(x^*, z^*) = \begin{cases} \frac{1}{2a}\left(\left(\bar{D} - c\right)^2 + c^2 \frac{\delta^T}{1-\delta^T}(1-a)^2\right) & a > a_D, \\ \frac{1}{2a}\delta^T\left(\bar{D} - ac\right)^2 & a \le a_D. \end{cases}$$

Note that $a \le (>)\ a_D$ is a necessary condition for the existence of a Delay (Incremental Cournot) equilibrium. While the expressions for social welfare in the Commitment equilibria are more complex than $\frac{4}{9}W(x^*, z^*)$, we get general comparative statics results:

Proposition 4.4. *i) Social welfare is decreasing in T, a and c and increasing in $\bar{D}$ within all equilibria.*[18] *ii) Welfare is higher under Commitment than under Incremental Cournot (assuming both exist). The comparison of welfare under Commit-Delay and Delay is ambiguous.*

We find that welfare is declining in time-to-build within each of the equilibria. The main driver is that firms are not able to respond to the resolution of uncertainty as quickly. Interestingly, the result holds even in a Commit-delay equilibrium, where total investment is increasing in T, because the increase comes in the form of additional ex ante investment which is already above the first best level (Proposition 4.2, (ii)). However, the effect of time-to-build on welfare can reverse when one accounts for the effect on equilibrium existence. If an increase in T creates a Commitment equilibrium (as when $a = .7$ and $(1-\delta^T) = .13$), then welfare can increase according to part (ii) of Proposition 4.4. These results are illustrated in Figure 4.3.

Welfare is increasing in uncertainty (i.e., decreasing in a) because of the assumption that the demand intercept $D = \bar{D}/a$ is decreasing in a and because welfare is quadratic in D.

[18] Because profits in Delay and Incremental Cournot are $2/9W(x^*, y^*)$, the comparative statics in Propostion 4.4 part (i) also apply to firm profits in these two equilibria.

4.3. Real Options

A large literature in both financial economics (Dixit and Pindyck 1994; Trigeorgis 1996) and in strategic management (Kogut 1991; Luehrman 1998) has developed around the broad topic of real options. Research in this area looks at a variety of actions firms can take—building flexible manufacturing, forging alliances, developing a multi-country production base—to limit their downside exposure to uncertainty. In this section, we look at the effect of time-to-build on the most basic real option, the option to wait to invest until uncertainty is resolved. Two variables are of interest here: first, the extent to which competing firms exploit the option to wait; and second, the value of the option to wait in equilibrium. We define R as the fraction of total investment that exploits the option to wait:

$$R = \frac{a(y_1 + y_2)}{x_1 + x_2 + a(y_1 + y_2)}.$$

We now characterize the timing of investment.

Proposition 4.5. *i) R is decreasing in T and in $(\bar{D}/c)$ within all equilibria, except the Delay equilibrium where $R = 1$. ii) If there are multiple equilibria, then R is smaller in the Commitment equilibrium. iii) In an Incremental Cournot equilibrium with $a < 1/2$, R is increasing in a for T sufficiently large.*

Consistent with our finding that R falls in T, Bar-Ilan and Strange (1996) find that the use of the option to wait falls in T in a model of investment timing with with time-to-build, but without imperfect competition. However, in part (ii) we show that this result reverses when an increase in T shifts play out of the Commitment equilibria, as these equilibria naturally have less use of the option to wait. The effect of uncertainty on option use is ambiguous in our model.[19]

We now derive the value of the option to wait for both firms and society. That is, we derive the increase in profits (denoted V^F) and welfare (denoted V^S) created by the option to wait. As a benchmark, we also consider the increase in welfare created by the option to wait assuming first-best investment (denoted V^*).

Proposition 4.6. *i) For the Delay and Incremental Cournot equilibria, $V^F = \frac{2}{9}V^*$ and $V^S = \frac{4}{9}V^*$. ii) The option value under first-best investment,*

$$V^* = \begin{cases} \frac{(1-a)^2}{2a} \frac{\delta^T}{\left(1-\delta^T\right)} c^2 & a \geq a_D, \\ \frac{\delta^T}{2a} \left(\bar{D} - ac\right)^2 & a < a_D, \end{cases}$$

is decreasing in T and a and it is increasing (decreasing) in c for $a > (<)\ a_D$.

[19] Usually, the use of real options decreases in uncertainty. Simply decreasing the probality of demand does decrease the use of the option to wait (Pacheco-de-Almeida and Zemsky, 2001). However, decreases in a also increase the demand intercept $D = \bar{D}/a$, which increases the use of the option to wait. For low values of a, the second effect prevails.

Limiting the analysis to Delay, Incremental Cournot and first-best investment, we find that the value of the option to wait is decreasing in time-to-build and increasing in uncertainty. We find strong interaction between time-to-build, uncertainty and costs in the expression for the option value.

There is a close link between the use of the option to wait and the magnitude of the initial price premium. The more firms wait to see if there is demand for the product, the less capacity there is initially and the higher is the initial price premium. Thus, our theory elucidates the extent to which prices (e.g. for new products) fall over time.

Proposition 4.7. *i) The initial price premium Δ is decreasing in T and a within any given equilibrium, except Delay where Δ is independent of T and a. ii) When there are multiple equilibria, Δ is smaller in the Commitment equilibrium.*

Our theory offers an alternative to the standard explanation of price declines for new products based on scale economies and learning curves (Scherer, 1980). Unlike these explanations, we predict that the initial price premium varies with time-to-build and uncertainty.[20] We find an inverse relationship between the magnitude of the initial price premium and its duration.

Introducing time-to-build resolves a tension in traditional two-period Cournot models, which require that period I investment is somehow observable, as otherwise there is no commitment value to early investment. In our model, ex ante investment levels can be unobservable because firms can infer them from period II prices.[21] We find an assumption of observable prices to be more plausible than an assumption of observable capacity.

5. Conclusion

We argue that time-to-build is an important parameter for strategic investment under uncertainty. Introducing time-to-build i) shifts (non-monotonically) the classic trade-off between commitment and exploiting the option to wait, ii) gives rise to novel types of equilibria where firms make incremental investments, and iii) links models of investment timing to the evolution of product prices.

A natural setting to test theories of strategic investment is the petrochemical industry because it is a homogeneous-good oligopoly and data on capacity and prices are available. Prior work (e.g., Lieberman, 1987; Gilbert and Lieberman, 1987) does not incorporate time-to-build, despite the availability of data on project-level time-to-build in the trade press. An initial exploration of this data reveals that time-to-build is significant in this industry and varies across products and countries (Table 1.1). Hence, this seems to be a good setting in which to test for effects of time-to-build on strategic investment. There is anecdotal evidence (Henderson,

[20] As with prior results, the comparative statics on T can potentialy be reversed by its effect on the set of equilibria. The result for a does not reverse, but is reinforced.

[21] Given that investment is a strategic substitute, firms have an incentive to utilize all their capacity in period II so as to discourage rival investment.

1998) that firms in countries with relatively long time-to-build (like Japan) tend to make simultaneous, incremental investment, as our theory predicts.[22]

There are many ways to explore further the effects of time-to-build on investment games. How does time-to-build affect results in models with private information such as Daughety and Reinganum (1994)? If one incorporates flexible capacity (Vives, 1989) into the model, what is the relationship between time-to-build and the option to switch? Finally, what is the equilibrium time-to-build when firms face a trade-off between T and investment costs?

6. Appendix

6.1. Proofs for Sections 3.1 and 3.2

Proof of Lemma 3.2 Suppose $\Delta = 0 \Leftrightarrow x_k \geq Z(x_k, x_{-k})$ for $k = 1, 2$. Then $x_k \geq Z(x_{-k})$ which means that $\partial\pi/\partial x_k = 0$ is a necessary condition for equilibrium. Note that $\partial\pi/\partial z_k \leq 0$ is also a necessary condition for equilibrium. Without loss of generality, consider $x_1 \leq x_2$, which implies that $\partial Z(x_2, x_1)/\partial x_1 = 0$. Then, $\partial\pi/\partial x_1 = a(D - 2x_1 - x_2) - c$ and $\partial\pi/\partial z_1 = a\delta^T(D - 2x_1 - x_2 - c) = \delta^T(\partial\pi/\partial x_1 + c(1-a)) > 0$, which is a contradiction. ■

Proof of Lemma 3.5 We consider in turn each of the three possible alternative pure strategy equilibrium to Delay, Incremental Cournot, Commit-delay, and Commit-incremental. 1) Both firms invest $x_1 = x_2 = \bar{z}$. Then $x_k = Z(x_k)$ and there is no ex post investment (Lemma 3.1), which is not possible in a pure strategy SPE (Lemma 3.2). 2) Both firms commit, x_1, $x_2 > \bar{z}$, and hence $x_1 > Z(x_2)$. This cannot be a SPE as an investment $x_1 = Z(x_2)$ yields strictly higher profits whether or not there is demand. 3) Recall that in an Incremental Cournot equilibrium the firms make the same ex ante investment. Thus, the third alternative is that x_1, $x_2 < \bar{z}$ but $x_1 \neq x_2$. However, with linear demand and symmetric costs, there is only a symmetric solution to the first-order conditions $\pi_1(x_k, x_{-k}) = 0$ for $k = 1, 2$. ■

6.2. Proofs for Sections 3.3 and 3.4

We begin by deriving the *ex ante* investment levels for each of the four equilibria. By definition, in a Delay equilibrium $x_1 = x_2 = 0$. The first-order conditions in a Incremental Cournot equilibrium are

$$\pi_1(x_k, x_{-k}) = a(1 - \delta^T)(\bar{D}/a - 2x_k - x_{-k}) - c(1 - a\delta^T) = 0 \text{ for } k = 1, 2$$

and these have the unique solution $x_1 = x_2 = x_I$ where

$$x_I \equiv \frac{\bar{D}(1 - \delta^T) - c(1 - a\delta^T)}{3a\left(1 - \delta^T\right)}.$$

[22] It is straightforward to extend many, but not all, of the results in the paper to n firms. In particular, the Delay and Incremental Cournot equilbrium generalize to $x_k = \frac{1}{n+1}x^*$ and $z_k = \frac{1}{n+1}z^*$. Thus, for example, social welfare generalizes to $\frac{n(n+2)}{(n+1)^2}W(x^*, y^*)$ and social option values to $\frac{n(n+2)}{(n+1)^2}V^*$. Generalizing the Commitment equilibria is more complex as one must solve for the number of firms that commit (see Hirokawa and Sasaki, 2000).

The first-order conditions in a Commitment equilibrium depend on whether or not the follower delays investment. If the follower delays, then the first-order condition for firm k acting as the leader is $\pi_1(x_{Ld}, 0) = a(1-\delta^T)(\bar{D}/a - 2x_{Ld}) + a\delta^T(\bar{D}/a - 2x_{Ld} - Z(x_{Ld}) - x_{Ld}Z'(x_{Ld})) - c = 0$, which has the unique solution

$$x_{Ld} = \frac{\bar{D}(2-\delta^T) - c(2 - a\delta^T)}{2a\left(2-\delta^T\right)}.$$

If the follower makes an ex ante investment, then the first-order conditions when firm k is the leader are $\pi_1(x_{Li}, x_F) = a(1-\delta^T)(\bar{D}/a - 2x_{Li} - x_F) + a\delta^T(\bar{D}/a - 2x_{Li} - Z(x_{Li}) - x_k Z'(x_{Li})) - c = 0$ and $\pi_1(x_F, x_{Li}) = a(1-\delta^T)(\bar{D}/a - 2x_F - x_{Li}) - c(1 - a\delta^T) = 0$, which have the unique solution

$$x_{Li} = \frac{\bar{D} - c}{a\left(3-\delta^T\right)},$$

$$x_F = \frac{2(\bar{D}-c) - \delta^T\left(3-\delta^T\right)\left(\bar{D}-ac\right)}{2a\left(1-\delta^T\right)\left(3-\delta^T\right)},$$

where x_{Li} is the investment of the leader and x_F is the investment of the follower.

Propositions 3.6, 3.7 and 3.8 refer to the critical values a_{DC}, a_{di}, and a_D which are defined as follows:

$$a_{di} = \frac{2 - (2-\delta^T)(1-\delta^T)\bar{D}/c}{\delta^T\left(3-\delta^T\right)},$$

$$a_{DC} = \frac{18\delta^T - 17(2-\delta^T)\delta^T\bar{D}/c + 12\left(\bar{D}/c - 1\right)\sqrt{2\delta^T\left(2-\delta^T\right)}}{(17\delta^T - 16)\delta^T}$$

$$a_D = \frac{1 - (1-\delta^T)\bar{D}/c}{\delta^T}.$$

There are also closed form expressions for a_I and a_C, but these are more complex and we do not work with them directly. All five critical values depend on the ratio $\bar{D}/c$, rather than either $\bar{D}$ or c individually.

We prove most of Propositions 3.6, 3.7, 3.8 in following three lemmas.

Lemma 6.1. *For $T \le \bar{T}$, Delay is a SPE* iff $a \le a_{DC}$. *For $T > \bar{T}$, Delay is a SPE* iff $a \le a_D$.

Proof Delay is a SPE *iff* $\pi(0,0) \ge \max_x \pi(x,0)$. As $\pi(x,0)$ is a piecewise linear function of x with a kink at $x = \bar{z}$, we break the condition into two parts. First, $\pi(0,0) \ge \max_{x\in[0,\bar{z}]} \pi(x,0)$ *iff* $\pi_1(x_k, 0) \le 0$, which is equivalent to $a \le a_D$. The second part is $\pi(0,0) \ge \max_{x>\bar{z}} \pi(x,0)$. If $x_{Ld} < \bar{z}$ then $\max_{x>\bar{z}} \pi(x,0) < \pi(0,0)$. Suppose $x_{Ld} > \bar{z}$. Then $\pi(0,0) \ge \max_{x>\bar{z}} \pi(x,0) \Leftrightarrow \pi(0,0) \ge \pi(x_{Ld}, 0)$, which is equivalent to

$$\frac{\delta^T\left(\bar{D}-ac\right)^2}{9a} \ge \frac{\left(\bar{D}(2-\delta^T) - c(2-a\delta^T)\right)^2}{8a\left(2-\delta^T\right)}. \tag{6.1}$$

Condition (6.1) holds *iff* $a \leq a_{DC}$. One can show that $a > a_{DC} \Rightarrow x_{Ld} < \bar{z}$. Hence, Delay is a SPE *iff* $a \leq \min\{a_{DC}, a_D\}$.

Substituting $a = a_D$ into condition (6.1) when it holds with equality yields

$$\frac{c\,(D-c)^2\left(17\delta^T - 16\right)}{72\left(2-\delta^T\right)\left(D(1-\delta^T)-c\right)} = 0$$

and hence $a_{DC} = a_D$ when $T = \bar{T}$. Further, for $T = \bar{T}$, $\partial(a_D - a_{DC})/\partial T < 0$. Hence, for $T \leq \bar{T}$, $a_{DC} \leq a_D$ and Delay is a SPE *iff* $a \leq a_{DC}$. For $T > \bar{T}$, $a_{DC} > a_D$ and Delay is an SPE *iff* $a \leq a_D$. ■

Lemma 6.2. *For $T \leq \bar{T}$, an Incremental Cournot equilibrium does not exist. For $T > \bar{T}$, there exists an $a_I \in (a_D, 1)$ such that an Incremental Cournot equilibrium exists for $a \in (a_D, a_I]$.*

Proof Note that profits in an Incremental Cournot equilibrium are

$$\pi_I \equiv \pi(x_I, x_I) = \frac{1}{9a}\left(\delta^T\left(\bar{D} - ac\right)^2 + (1-\delta^T)\left(\bar{D} - c\frac{(1-a\delta^T)}{(1-\delta^T)}\right)^2\right).$$

A necessary condition for an Incremental Cournot equilibrium is $a > a_D$, as otherwise $x_I \leq 0$. In addition, for x_I to be globally optimal it must be that a firm does not want to commit, i.e. $\pi_I \geq \max_{x \geq \bar{z}} \pi(x, x_I)$. Let $\hat{x} = \arg\max_{x \geq \bar{z}} \pi(x, x_I)$. As $\pi_1(x, x_I) < 0$ for $x > Z^{-1}(x_I)$ we know that $\hat{x} \leq Z^{-1}(x_I)$. If $\hat{x} = \bar{z}$, then commitment is not attractive and $\pi_I > \hat{\pi}$. If $\hat{x} > \bar{z}$, we have $\pi_1(\hat{x}, x_I) = 0$ which yields

$$\begin{aligned}
\hat{x} &= \frac{\bar{D}(4-\delta^T) - c(4-a\delta^T)}{6a(2-\delta^T)}, \\
\hat{\pi} &= \pi(\hat{x}, x_I) = \frac{1}{9a}\frac{(\bar{D}(4-\delta^T) - c(4-a\delta^T))^2}{8(2-\delta^T)}.
\end{aligned}$$

Let $L(a) = (\pi_I - \hat{\pi})9a$. An Incremental Cournot equilibrium exists *iff* $a > a_D$ and $L(a) \geq 0$. We have $L'(a) < 0$ since one can show that $L'(1) < 0$ and $L''(a) > 0$. Further,

$$L(a_D) = \frac{\left(17\delta^T - 16\right)\left(\bar{D} - c\right)^2}{8\delta^T(2-\delta^T)}$$

For $T \leq \bar{T}$, we have $(17\delta^T - 16) \leq 0$ and hence $L(a) < 0$ for all $a > a_D$. Conversely, for $T > \bar{T}$, $L(a_D) > 0$ and an Incremental Cournot equilibrium exists for $a \in (a_D, a_I]$ where a_I is such that $\pi_I = \hat{\pi}$. We have $a_I < 1$ since $L(1) < 0$. ■

Lemma 6.3. *i) For $T \in [0, \bar{T}]$, there is a Commit-delay equilibrium iff $a_{DC} \leq a \leq a_{di}$ and a Commit-incremental equilibrium iff $a_{di} < a$. ii) For $T \in (\bar{T}, \hat{T}]$ there exists critical values $a_C \in (a_D, a_I)$ and $a_{di} \geq a_C$ such that a Commit-delay equilibrium exists iff $a_C \leq a \leq a_{di}$ and a Commit-incremental equilibrium exists iff $a_{di} < a$. iii) For $T > \hat{T}$, a Commit-delay equilibrium does not exist and there exists a critical value $a_C \in (a_D, a_I)$ such that a Commit-incremental equilibrium exists iff $a_C \leq a$.*

Proof A Commit-delay equilibrium exists iff $\pi_1(0, x_{Ld}) \leq 0$ and $\pi(x_{Ld}, 0) \geq \max_{x \leq \bar{z}} \pi(x, 0)$. A Commit-incremental equilibrium exists iff $x_F > 0$ and $\pi(x_L, x_F) \geq \max_{x \leq \bar{z}} \pi(x, x_F)$. We have $\pi_1(0, x_{Ld}) \leq 0 \Leftrightarrow x_F \leq 0 \Leftrightarrow a \leq a_{di}$.

i) Suppose $T \in [0, \bar{T}]$. Let $N(T) = a_{di} - a_{DC}$. Since $N(\bar{T}) > 0$ and $N'(T) < 0$, $a_{di} < a_{DC}$. We have $a \geq a_D \Leftrightarrow \pi_1(0,0) \leq 0 \Rightarrow \max_{x \leq \bar{z}} \pi(x, 0) = \pi(0,0)$. Further, $a \geq a_{DC} \Leftrightarrow \pi(x_{Ld}, 0) \geq \pi(0,0)$. Since $a_{DC} \geq a_D$ for $T \leq \bar{T}$(see proof of Lemma 6.1), a Commit-delay equilibrium exists iff $a_{DC} \leq a \leq a_{di}$. Let $\hat{x}_i = \arg\max_{x \leq \bar{z}} \pi(x, x_F)$ and let $M_i(a) = \pi(x_{Li}, x_F) - \pi(\hat{x}_i, x_F)$. Note that $a > a_{di} \Rightarrow \hat{x}_i > 0$. A Commit-incremental equilibrium exists iff $a > a_{di}$ and $M_i(a) \geq 0$. Since $M_i(a)$ is a quadratic function of a with $M_i(0) < 0$ and $M_i(1) > 1$ and $M_i(a_{di}) = \pi(x_{Ld}, 0) - \pi(0,0) \geq 0$, a Commit-incremental equilibrium exists iff $a > a_{di}$.

ii) and iii) Suppose $T > \bar{T}$. Let $\hat{x}_d = \arg\max_{x \leq \bar{z}} \pi(x, 0)$ and $M_d(a) = \pi(x_{Ld}, 0) - \pi(\hat{x}_d, 0)$. As with $M_i(a)$, we have $M_d(a)$ a quadratic function of a with $M_d(0) < 0$ and $M_d(1) > 1$. Since $M_i(a_{di}) = M_d(a_{di})$ there exists a unique $a_C \in (0,1)$ such that a Commitment equilibrium exists iff $a \geq a_C$. Moreover, $M_d(a_{di}) = 0$ at $T = \hat{T} > \bar{T}$ and $a_{di} < a_C$ for $T > \hat{T}$. Hence the Commit-delay equilibrium does not exist for $T > \hat{T}$. Since $x_I > x_F$ and ex ante investments are strategic substitutes, $M_i(a_I) > 0$ and $M_d(a_I) > 0$ and hence $a_I > a_C$. Finally, note that $a_C \geq a_{DC}$ and that for $T > \bar{T}$ we have $a_{DC} > a_D$ and hence $a_C > a_D$. ■

Proof of Proposition 3.6 i) Follows from Lemmas 6.1 and 6.3. ii) As $T < \bar{T}$, there is no Incremental Cournot equilibria by Lemma 6.2. We have $a_{di} = 1$ for $T = 0$ and hence the Commit-incremental equilibrium does not exist. By Lemma 3.5, this rules out the two possible equilibria with incremental investment. ■

Proof of Proposition 3.7 i) Follows from Lemmas 6.1 and 6.3. ii) For $\bar{D} = c$ we have $\partial a_{DC}/\partial T = 0$. Since one can show that $\frac{\partial^2 a_{DC}}{\partial T \partial \bar{D}} < 0$, we have $\partial a_{DC}/\partial T < 0$ since $\bar{D} > c$. iii) The existence of the critical value $a_{di} > a_{DC}$ follows from Lemma 6.3. Since $a_{di} = 1$ for $T = 0$ and since $\partial a_{di} \partial T < 0$, we have $a_{di} < 1$ for $T > \bar{T}$. iv) Follows from Lemma 6.2. ■

Proof of Proposition 3.8 The results follow from Lemmas 6.1, 6.2 and 6.3. ■

Proof of Proposition 3.9 i) For a given $a < 1$, we have that $\lim_{T \to \infty}(x_{Li} - x_I) = 0$ and hence in the limit the Commitment equilibrium breaks down. Thus, $\lim_{T \to \infty} a_C = 1$ and since $a_I \geq a_C$, this implies that $\lim_{T \to \infty} a_I = 1$. Finally, as $\lim_{T \to \infty} a_D < 0$ the unique equilibrium is Incremental Cournot. ii) Follows from a_D, a_I, a_C and a_{di} all less than 1 for $T > 0$. ■

6.3. Proofs for Section 4

Proof of Proposition 4.1 i) Consider a Commit-delay equilibrium. The result on market shares follows from $\partial x_{Ld}/\partial T = -(\ln \delta) \frac{(1-a)\delta^T c}{a\left(2-\delta^T\right)^2} > 0$ and the fact that the follower's ex post investment is falling in x_{Ld}. Given this, $\partial \pi(0, x_{Ld})/\partial T < 0$. We now show that $\partial \pi(x_{Ld}, 0)/\partial T > 0$, which is sufficient for the difference in expected profits to be increasing. Let $\lambda = \delta^T$ so that $\pi(x_{Ld}, 0) = \frac{\left(\bar{D}(2-\lambda) - c(2-a\lambda)\right)^2}{8a(2-\lambda)}$. Then $\partial \pi(x_{Ld}, 0)/\partial \lambda = -\frac{1}{8}\left(2\bar{D} - \bar{D}\lambda - 2c + c\lambda a\right)(2\bar{D} - \bar{D}\lambda - 4ac + c\lambda a + 2c)/(a(2-\lambda)^2) < 0$ since $x_{Ld} > 0 \Rightarrow 2Da - Da\lambda - 2c + c\lambda a > 0$. Since $\partial \lambda / T < 0$, this establishes that

$\partial\pi(x_{Ld},0)/\partial T > 0$.

ii) Consider a Commit-incremental equilibrium. We have $\partial x_{Li}/\partial T < 0$ and $\partial x_F/\partial T > 0$ (to see the latter note that x_F is a linear function of c and $\partial x_F/\partial T > 0$ for $c = 0$ and $c = \bar{D}$ and $c \in (0, \bar{D})$ by assumption). This established that the market shares in both periods II and III are getting more similar as T increases and hence we have $\partial\pi(x_{Li}, x_F)/\partial T < 0$ as well. We now show that $\partial\pi(x_F, x_{Li})/\partial T > 0$ for a sufficiently close to 1, which is sufficient for the difference in expected profits to be decreasing in T. Let $\lambda = \delta^T$ and let $f(a) = \partial\pi(x_F, x_{Li})/\partial\lambda$. Then $f(1) = -\frac{1}{2}(D-c)^2(2-\lambda)/(3-\lambda)^3 < 0$. Further $f'(1) = -\frac{1}{2}(D-c)(D+c)(2-\lambda)/(3-\lambda)^3 < 0$ and $f''(a) = c^2\frac{23+\lambda^3-17\lambda+\lambda^2}{2a^3(3-\lambda)^3(1-\lambda)^2} > 0$ and hence there exists some $\bar{a} < 1$ such that $\partial\pi(x_F, x_L)/\partial\lambda < 0 \Leftrightarrow \partial\pi(x_F, x_L)/\partial T > 0$ iff $a > \bar{a}$.

iii) We have $\partial a_{di}/\partial T = 2(\ln\delta)\left(3/\delta^T - 2\right)(\bar{D}/c - 1)/\left(3-\delta^T\right)^2 < 0$. ■

Proof of Proposition 4.2 i) In a Delay equilibrium $a \le a_D$ and hence $x_1 + x_2 = x^* = 0$. In an Incremental Cournot equilibrium $a > a_D$ and hence $x_1 + x_2 = 2x_I = \frac{2}{3}x^*$. In both equililbria $z_1 + z_2 = 2\bar{z} = \frac{2}{3}z^*$. ii) Let $O_d(a) = x_{Ld} - x^*$ and $O_i(a) = x_{Li} - x^*$. Note that both $O_d(a) = 0$ and $O_i(a) = 0$ have the unique solution $a = a_{di}$. As $O_d(1) = -\frac{1}{2}(\bar{D} - c) < 0$ and $O_i(1) = -\frac{1}{2}\left(\bar{D} - c\right)(2-\delta^T)/(3-\delta^T) < 0$, we have $x_L + x_F = x_{Ld} \ge x^*$ for $a \le a_{di}$, which is when a Commit-delay equilibrium exists, and conversely we have $x_L + x_F = x_{Li} + x_F < x^*$ for $a > a_{di}$, which is when a Commit-incremental equilibrium exists. ■

Proof of Proposition 4.4 i) Denote the social welfare in a Delay equilibrium by $W_D = W(0, 2\bar{z}) = \frac{2}{9a(1-\delta)}\delta^T\left(\bar{D} - ac\right)^2$. By inspection, $\partial W_D/\partial T < 0$, $\partial W_D/\partial a < 0$, $\partial W_D/\partial c < 0$, $\partial W_D/\partial\delta > 0$ and $\partial W_D/\partial\bar{D} > 0$. Suppose $a > a_D$, and denote the social welfare in an Incremental Cournot equilibrium by $W_I = W(2x_I, 2\bar{z}) = \frac{4}{9}W(x^*, z^*)$. By inspection, $\partial W_I/\partial T < 0$, $\partial W_I/\partial a < 0$, and $\partial W_I/\partial\bar{D} > 0$. We have $\partial W_I/\partial c = -\frac{\bar{D}}{a} + \frac{c}{a}\frac{1-a\delta^T(2-a)}{\left(1-\delta^T\right)}$, which is decreasing in a. As $\partial W_I/\partial c = -\bar{D} + c < 0$ at $a = a_D$ the limit value of a for which the equilibrium exists, $\partial W_I/\partial c < 0$.

Denote the social welfare in an Commit-incremental equilibrium by $W_{Ci} = W(x_{Li} + x_F, x_{Li} + Z(x_{Li})) = \frac{1}{8a(1-\delta)}\left(\left(\bar{D} - c\right)^2\frac{\left(8-3\delta^T\right)(4-\delta^T)}{\left(3-\delta^T\right)^2} + 3c^2\frac{\delta^T}{\left(1-\delta^T\right)}(1-a)^2\right)$. By inspection, $\partial W_{Ci}/\partial a < 0$ and $\partial W_{Ci}/\partial\bar{D} > 0$. We have $\partial W_{Ci}/\partial T < 0$ since for $f(\lambda) = \frac{(8-3\lambda)(4-\lambda)}{(3-\lambda)^2}$ we have $f'(\lambda) = 2(2-\lambda)/(3-\lambda)^3 > 0$. Finally, $\partial W_{Ci}/\partial c < 0$ because both ex ante investment and ex post investment are falling in c and both are less than the first best levels (Proposition 4.2, part (ii)).

Let $W = W(x_{Ld}, x_{Ld} + Z(x_{Ld}))$ denote welfare in a Commit-delay equilibrium. We will use the fact that W is a quadratic function of c. A Commit-delay equilibrium exists for $a \in [a_{DC}, a_{di}]$. Note that $a \le a_{di} \Leftrightarrow c \ge c_1 = \bar{D}\frac{2-3\delta^T+\delta^{2T}}{2-3a\delta^T+a\delta^{2T}}$. We have $\partial W/\partial c < 0$ since $\partial W/\partial c < 0$ for $c \in \{0, \bar{D}\}$ and $\partial W^2/\partial^2 c > 0$. We have $\partial W/\partial\bar{D} > 0$ since $\partial W/\partial\bar{D} > 0$ for $c \in \{0, \bar{D}\}$ and $\frac{\partial W^2}{\partial c\partial\bar{D}} > 0$ for $c = c_1$. We have $\partial W/\partial a < 0$ since $\partial W/\partial\bar{D} > 0$ for $c \in \{c_1, \bar{D}\}$ and $\frac{\partial W^2}{\partial c\partial a} < 0$ for $c = c_1$. Let $\lambda = \delta^T$. We have $\partial W/\partial\lambda > 0$ since $\partial W/\partial\lambda > 0$ for $c \in \{0, \bar{D}, c_2\}$ where c_2 is defined by $\frac{\partial W^2}{\partial c\partial a} = 0$. Hence $\partial W/\partial T < 0$.

ii) We first compare welfare under Incremental Cournot to Commit-incremental.

We have that $W_{Ci} > W_I$ because $2\bar{z} < x_{Li} + Z(x_{Li}) < 2z^*$ and $2x_I < x_{Li} + x_F < x^*$ with the last inequality following from Proposition 4.2 part (ii). We now compare welfare under Incremental Cournot to Commit-delay. Let $S(a) = (W_{Cd} - W_I)288a$. Note that $S(a)$ is quadratic in a and that a necessary for Commit-delay and Incremental Cournot to both exist is $a \in [a_D, a_{di}]$. $S(a_D) = \left(176 + 35\delta^{2T} - 140\delta^T\right) \left(\bar{D} - c\right)^2 / \left(\left(2 - \delta^T\right)^2 \delta^T\right) > 0$, $S(a_{di}) = 4\left(56\delta^T + 44 - 29\delta^{2T}\right)\left(\bar{D} - c\right)^2 / \left(\delta^T \left(3 - \delta^T\right)^2\right) > 0$ and $S'(a_{di}) = -4c\left(44 - 31\delta^T\right)(\bar{D} - c)/(3 - \delta^T) < 0$. Hence, $S(a) > 0$ for $a \in [a_D, a_{di}]$. Finally, we compare Delay and Commit-delay, which occurs when $a = a_{DC}$ and $T \in [0, \bar{T}]$. There is no global ranking as $W_{Cd} - W_D > 0$ for $T = 0$ and $W_{Cd} - W_D < 0$ for $T = \bar{T}$. ■

Proof of Proposition 4.5 i) In a Delay equilibrium we have $R = 1$ for all parameters. One can establish the comparative statics results from the following expressions. In Incremental Cournot,

$$R = \frac{a(1-a)}{(1-\delta^T)\bar{D}/c - \left(1 - a\delta^T - a + a^2\right)}$$

in Commit-incremental,

$$R = \left(1 + \frac{\bar{D}}{c}\frac{(4-g)(1-g)}{a(1-a)(3-g)} - \frac{4 + ag^2 - 3ag - 2g}{a(1-a)(3-g)}\right)^{-1}$$

and in Commit-delay

$$R = \frac{\left((2-\delta^T)\bar{D}/c + (2 + a\delta^T - 4a)\right)a}{\left(4 + 2a - \delta^T(2+a)\right)\bar{D}/c + a\left(2 + \delta^T(a+2)\right) - 4(1+a^2)}$$

ii) In an Incremental Cournot equilibrium $\lim_{T\to\infty} \frac{\partial R}{\partial a} = \frac{(1-2a)c\left(\bar{D}-c\right)}{\left(\bar{D}-c+ac(1-a)\right)^2}$ which is increasing in a for $a < 1/2$.

iii) For Delay $R = 1$ and for Commit-delay $R < 1$. R is increasing in $y_1 + y_2$ and decreasing in $x_1 + x_2$. Hence, to show that R is higher in an Incremental Cournot than in a Commitment equilibrium it is sufficient to show that $x_1 + x_2$ is lower and $y_1 + y_2$ is higher. The difference in period I investment is

$$2x_I - x_{Li} - x_F = -\delta^T \frac{\left(1-\delta^T\right)\left(\bar{D} - ac\right) + 2c(1-a)}{6a\left(3 - \delta^T\right)\left(1 - \delta^T\right)} < 0$$

and the difference in period II investment is

$$2(\bar{z} - x_I) - (Z(x_{Li}) - x_F) = \frac{c(1-a)}{6\left(1-\delta^T\right)a} > 0.$$

Now suppose that $a_C < a < \min\{a_I, a_{di}\}$ so that there is both an Incremental Cournot and a Commit-delay equilibria exist. Note that the difference in period I investment $2x_I - x_{Ld}$ is decreasing in a while the difference in period II investment

$2(\bar{z}-x_I)-Z(x_{Ld})$ is increasing in a. Hence, it is sufficient to show the result for $a=a_{di}$:

$$\begin{aligned}(2x_I - x_{PL})|_{a=a_{di}} &= -\frac{(\bar{D}-ac)\delta^T}{6a} < 0 \\ 2(\bar{z}-x_I) - Z(x_{PL})|_{a=a_{di}} &= \frac{\left(\bar{D}-ac\right)\left(2-\delta^T\right)}{12a} > 0.\end{aligned}$$

■

Proof of Proposition 4.6 i) If ex post investment is not possible, firm profits are given by $\dot{\pi}(x_k, x_{-k}) = ax_k(D - x_k - x_{-k}) - x_k c$ and social welfare is given by $\dot{W}(x) = au(x) - cx$. Let $\dot{x}^* = \arg\max_x \dot{W}(x)$. The unique Nash equilibrium investments are $x_1 = x_2 = \dot{x}^*/3$. If $a > a_D$ we have $\dot{x}^* = (\bar{D}-c)/a$. The value of the option to wait to society is then $V^S = W(0, 2\bar{z}) - \dot{W}(2\dot{x}^*/3)$ and the value to firms is $V^F = \pi(x_I, x_I) - \dot{\pi}(\dot{x}^*/3, \dot{x}^*/3)$. If $a \leq a_D$ we have $\dot{x}^* = 0$ and hence $\dot{W}(\dot{x}^*) = \dot{\pi}(\dot{x}^*/3, \dot{x}^*/3) = 0$. The option values are then $V^S = W(0, 2\bar{z})$ and $V^F = \pi(0,0)$. The results in the proposition follow. ■

Proof of Proposition 4.7 i) We have that in Incremental Cournot:

$$\Delta = \frac{2(1-a)}{(\bar{D}/c + 2a)(1-\delta^T)}$$

in Commit-incremental:

$$\Delta = \frac{\left(3-\delta^T\right)(1-a)}{\left(\left(2-\delta^T\right)\bar{D}/c + 1 + a(3-\delta^T)\right)\left(1-\delta^T\right)},$$

in Commit-delay:

$$\Delta = 1 - \frac{4a(2-\delta^T)}{(2-\delta^T)\bar{D}/c + 2 + 4a - 3a\delta^T}.$$

The comparative statics follow. ii) Like R, Δ is increasing in $y_1 + y_2$ and decreasing in $x_1 + x_2$. Hence, the proof is the same as in Proposition 4.5 part iii). ■

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