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ABSTRACT

Strategic Behaviour and Underpricing in Uniform Price Auctions: Evidence from Finnish Treasury Auctions*

We study uniform price auctions using a dataset that includes individual bidders' demand schedules in Finnish Treasury auctions during the period 1992-99. Average underpricing amounts to 0.041% of face value. Theory suggests that underpricing may result from monopsonistic market power. We develop and test robust implications from this theory and find that it has little support in the data. For example, bidders' individual demand functions do not respond to increased competition in the manner predicted by the theory. We also present evidence that the Finnish Treasury acts strategically, taking into account the fact that the auctions are part of a repeated game between the Treasury and the primary dealers. Empirically, the main driver behind bidder behaviour and underpricing is the volatility of bond returns. Since there is no evidence that bidders are risk averse, this suggests that private information and the winner's curse may play an important role in these auctions.

JEL Classification: D44 and G10
Keywords: demand functions, market power, multiunit auctions, seller behaviour, supply uncertainty, treasury auctions, underpricing and uniform price

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1 Introduction

We study individual bidders' actual demand schedules in 206 Finnish Treasury auctions conducted over the time period 1992-99. The auctions are carried out under the uniform price format, where all bidders pay the stop-out price for the units they are awarded. Since 1998, this has also been the standard format of U.S. Treasury auctions. Participation in the Finnish auctions is limited to the primary dealers, who vary in number between five and ten. The small number of bidders makes these auctions a suitable environment to study strategic behavior. We examine the main drivers behind bidding strategies and auction performance with particular attention devoted to whether bidders in these uniform price auctions have market power as in the imperfect competition models of Wilson (1979), Back and Zender (1993), and Kyle (1989). We also address strategic seller behavior in light of the fact that these auctions are part of a repeated game between the Treasury and the primary dealers.

The performance of treasury auctions is often gauged by the spread between the yield paid by winners in the auction and a comparable secondary market or when-issued yield (Cammack, 1991). In uniform auctions in the U.S., this spread tends to be a fraction of a basis point [Nyborg and Sundaresan (1996), Malvey and Archibald (1998)]. This is also what we find in Finland where, translated to price, the auctioned securities are underpriced by .041% of face value. While this is a small number per unit, the large volumes imply that the underpricing amounts to a significant profit for the bidders and a corresponding cost to the seller.

Previous work on *discriminatory* treasury auctions by Nyborg, Rydqvist, and Sundaresan (2002) and Bjønnes (2001) identifies the volatility of bond returns as the variable which has the most significant economic impact on bidder behavior and underpricing. These papers find that when volatility increases the typical bidder tends to reduce the average price at which he bids, reduce his total demand, and increase the dispersion of his bids. As a result, underpricing is found to be increasing in volatility. Since auction size has hardly any impact on behavior or underpricing, it appears that the importance of volatility has little to do with bidders being risk averse. Instead, the evidence suggests that the dominating element behind the observed behavior and underpricing is that bidders have private information and adjust rationally for the winner's curse/champion's plague.[1]

A crucial distinction between these papers and ours is that whereas they study discriminatory auctions, we study uniform auctions. This is important because the theory of uniform auctions is very different from that of discriminatory auctions. In particular, in the uniform auction model of Wilson (1979) and Back and Zender (1993), bidders disperse and shade their bids and underpricing arises even though bidders are risk neutral, have a common and constant marginal valuation of the auctioned asset, and do not possess private information.[2] This is a result of monopsonistic market power which arises endoge-

[1]In multiunit contexts, Ausubel (1997) has suggested the terminology "champion's plague" instead of the winner's curse, to reflect that the more units a bidder wins, the worse news it is.

[2] Back and Zender (1993) develop their basic argument under the assumption that bidders have private information which they do not use. The point is to show that underpricing equilibria from market power are possible also when bidders are privately informed. Wilson (1979) provides an example with private information where the stop-out price is perfectly revealing, but underpricing still occurs because

nously in equilibrium. What happens is that when all the other bidders submit downward sloping demand schedules, the remaining bidder faces an upward sloping residual supply curve over which he is a monopsonist. The bidder optimally exercises his monopsonistic market power by submitting a downward sloping demand schedule himself, thus cementing the equilibrium and pushing the stop-out price below the "true value" of the securities. This monopsonistic market power theory is sometimes cited as a serious disadvantage of the uniform format as compared with the discriminatory format, particularly since the underpricing can be arbitrarily large.

Our dataset affords us with the unique opportunity of testing this market power theory, and this is one of the main objectives of our paper. Furthermore, by comparing our results on bidder behavior and underpricing with those of Nyborg, Rydqvist, and Sundaresan (2002) we can say something about how uniform and discriminatory treasury auctions compare from an *empirical* perspective.

While the Wilson/Back and Zender model admits a plethora of equilibria, there is a unique class of equilibria which are robust to supply uncertainty (Back and Zender, 1993). In our empirical tests, we will focus on these, since supply uncertainty is an important feature of Finnish Treasury auctions. We will also test the implications of a related model by Kyle (1989), where risk averse bidders enjoy market power. We look at both the linear equilibrium presented by Kyle as well as the nonlinear equilibria found by Wang and Zender (2002).[3]

We develop a new methodology to test the theory. While the theory that we seek to test is developed in the context of "smooth" demand schedules, in practice bidders submit finite collections of price-quantity pairs as bids. This makes it difficult to compare the theoretical demand schedules with those that bidders actually submit. We handle this problem by computing summary statistics of the theoretical and empirical bid distributions. The validity of the theory is assessed by testing the predicted summary statistics against the observed statistics, including checking whether the empirical summary statistics react to exogenous variables as predicted by the theory.

Using this methodology, we examine the implication from the unique class of supply uncertainty robust demand schedules in the Wilson/Back and Zender model that the bid distribution exhibits negative skewness which decreases with the number of bidders. This is also a property of the nonlinear equilibria in Kyle's (1989) model found by Wang and Zender (2002), while Kyle's linear equilibrium predicts zero skewness. We find that the distribution of bids within the empirical demand schedules tends to be negatively skewed when there are few bidders (5-8), but that skewness becomes significantly positive when

of monopsonistic market power.

[3]Kyle (1989) is predominantly occupied with the question of information aggregation under imperfect competition in a noisy limit order market with privately informed risk averse players who can both buy and sell. Strictly speaking, his model is therefore not one of a multiunit auction, where bidders can only buy. Furthermore, private information is in the focus. However, if we strip the private information away and let the noisy supply have positive expected value, Kyle's model becomes one where risk averse bidders choose demand schedules as strategies in an analogous way to the Wilson/Back-Zender model where bidders are risk neutral. This version of his model (which Kyle also solves) is applicable to multiunit auctions. See Section 4. We wish to emphasize that when we refer to "testing Kyle (1989)" we refer to this version of his model where players do not have private information.

the number of bidders increases (9-10), i.e., bidders respond to competitive pressure by submitting a few bids which are *much higher* than the other bids. We interpret this behavior as evidence against the theory. Moreover, contrary to what the market power theory predicts, we find that bid shading and underpricing do not decrease with the number of bidders. However, since we also find that demand per bidder is increasing in the number of bidders, it is possible that bidders exercise some market power, albeit less than suggested by the theory.

In our sample, the variable that has the most significant economic impact on bidder behavior and underpricing across auctions is *volatility*, just as previous research has documented for discriminatory auctions. Moreover, we also find that an increase in volatility tends to lead to more bid shading and underpricing, reduced demand, and increased dispersion. We find no evidence that the importance of volatility is driven by risk aversion. It seems that primary dealers in the Treasury market have sufficient risk management tools to act as approximately risk neutral. The evidence thus points to private information and the winner's curse as being the key driver behind bidder behavior in uniform treasury auctions, just as it appears to be in discriminatory treasury auctions.

A special feature of the Finnish auctions is that the seller determines supply after observing the bids. There is no pre-announced reservation price. Thus the seller may also be strategic. We document that the Finnish Treasury *never* chooses supply to maximize revenue given the bids in an auction. Indeed, in 6 cases, the Treasury cancelled the auctions because bids were not deemed to be sufficiently high. This behavior suggests that the seller thinks of the auction as a repeated game, where the bids in subsequent auctions can be influenced by rejecting revenue increasing bids in the current auction. The way bidders respond to this may be an important contributing factor to why the monopsonistic market power theories do not perform so well.

Our paper extends the empirical literature on uniform price auctions. Umlauf (1993) studies Treasury auctions in Mexico and concludes that a change from the discriminatory-price format to the uniform price format had the effect of enhancing competition and reducing bidder profits. Nyborg and Sundaresan (1996) and Malvey and Archibald (1998) find that in U.S. Treasury auctions in the 1990's, uniform auctions had, if anything, less underpricing than discriminatory auctions. Feldman and Reinhardt (1996) show that the aggregate demand curve in IMF gold auctions in the 1970's shifted up under uniform auctions compared with discriminatory auctions. Scalia (1999) studies bidder behavior in uniform price auctions in Italy, but do not test any of the theories of uniform price auctions. Compared to previous studies our data set is unusually clean: *i)* The number of bidders is small and observable without error, *ii)* non-serious, outlier bids are virtually absent, and *iii)* transactions data from the secondary market allow us to estimate auction underpricing with higher precision than in studies which must rely on indicative bid quotes. For example, Cammack (1991) must rely on aggregate demand and supply statistics to estimate the number of bidders, and Scalia (1999) has to separate between large and small bidders among a total of 60 bidders per auction. The small number of bidders in our data set also distinguishes our paper from Kandel, Sarig, and Wohl (1999), who study IPO auctions in Israel.

The rest of the paper is organized as follows. The Finnish Treasury market is described in Section 2 and the data in Section 3. The theory of strategic bidding in uniform price auctions is surveyed in Section 4 with an emphasis on drawing out empirical predictions. Section 5 presents the empirical results. We describe the actual demand curves and show how their characteristics vary with volatility, the number of bidders, and expected auction size. Section 6 describes the strategic behavior of the seller and Section 7 offers some concluding remarks.

2 The Finnish Treasury Bond Market

The Finnish Treasury started issuing securities in 1991 and stopped in 1999. The selling activity during this time period reflects the financing needs of the Government budget deficit. The top panel in Figure 1 shows that the budget deficit was very large during the recession in the early 1990s, when GDP growth was negative, but turned into a surplus towards the end of the decade. As a result, the Finnish Treasury has been buying back securities since 2000. The middle panel shows the annual number of Treasury bond auctions (darker columns to the left) and the number of occasions when the Treasury offers additional securities for sale by fixed price tender (lighter columns to the right). The frequency of auctions is approximately evenly distributed over time except in the beginning and the end of the period. The total number of auctions is 232 and the number of fixed price tenders 48.[4] Finally, the bottom panel shows monthly average auction size. The plot shows that average auction size increases, and gets more volatile, towards the end of the period. Auction size is nominal, but inflation is only 10% over the entire period and averages to 1.2% per year.

The Treasury announces a preliminary auction schedule twice a year. Regular auctions are held every second Thursday, when one or two Treasury bonds are sold at the same time. Our data set contains 232 auctions which are spread out over 204 calendar days, 176 days with a single security for sale, and 28 days with two securities. The 232 auctions include 13 first issues of a new security and 219 reopenings of existing securities which are traded in the secondary market. All Treasury bonds are non-callable bonds with annual coupon payments and between 2 to 15 years to maturity.

One week before the auction, the Treasury announces which securities will be offered for sale, but the actual amount for sale is not announced. Instead, supply is determined after observing the bids. From 1998, the Treasury announces the maximum amount. The auction format is sealed, multiple bid, and uniform price. Bids are submitted by phone and confirmed by fax no later than 1 p.m. on the auction day. Individual bids are expressed in price per 100 markka face value. Any number of price-quantity bids is allowed, but individual bids must be separated by two decimals ending with 0 or 5 before May 1998 and an even number (0, 2, 4, 6, or 8) after this date. The coarse bid grid is to avoid reading errors from the fax prints. There is no official quantity multiple, but the smallest bid size observed in the data set is for 1 million of face value. Awarded bidders pay the stop-out

[4]In addition, the Treasury sells T-bills which are discount securities with up to one year to maturity. Sometimes, the Treasury retains T-bonds which the primary dealers can borrow if they have problems with meeting physical delivery. Finally, when old T-bonds mature, the Treasury redeems the old bonds for cash or new securities. The exchange for new securities is carried out by auction.

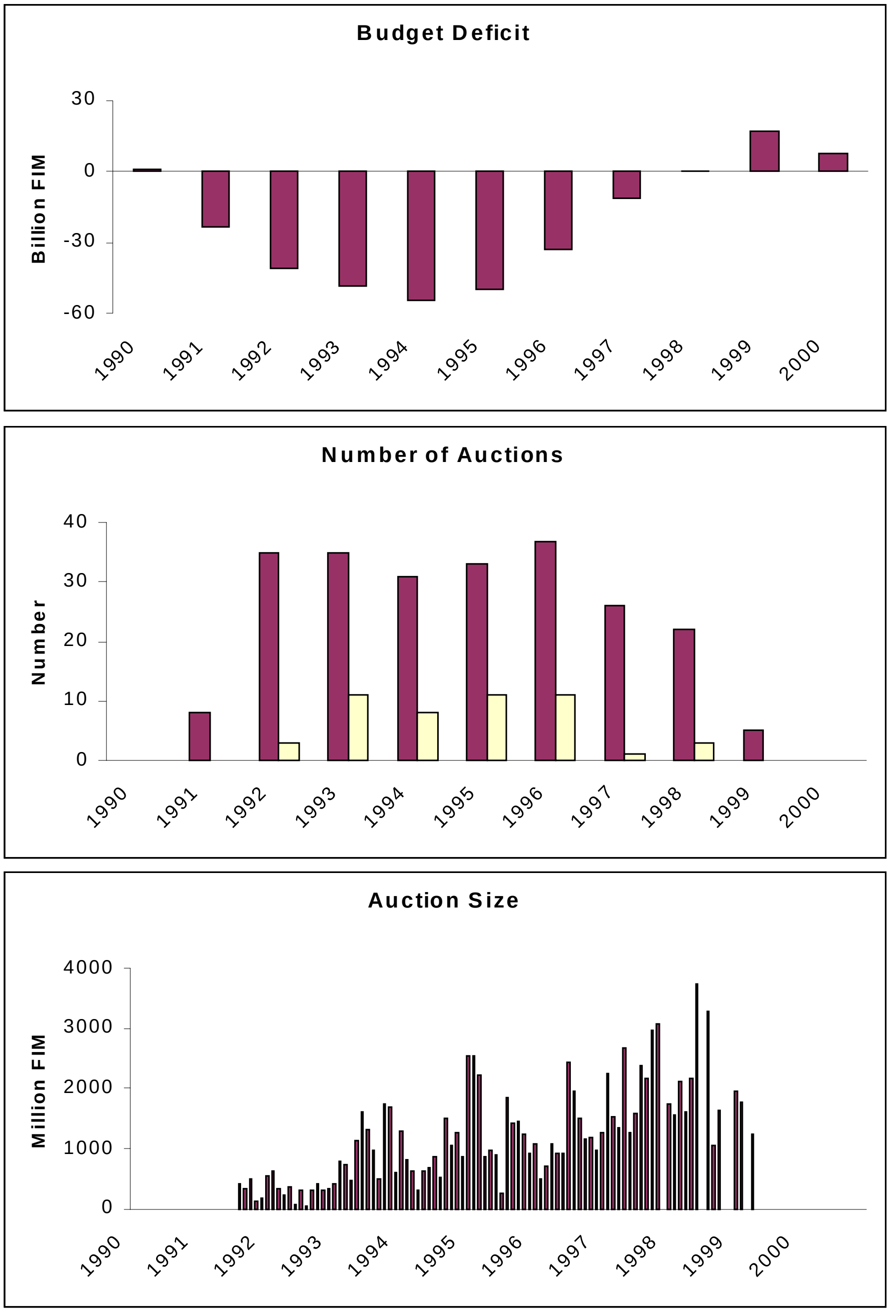

Figure 1: **Finnish Government Budget Deficit and Treasury Bond Auctions 1990-2000.** Top panel: Annual Net Balance of the Finnish government budget. Middle panel: Annual number of Treasury bond auctions along with the number of fixed price tender. Bottom panel: Monthly average auction size. An approximate exchange rate is 1 USD for 6 FIM.

price which is the price of the lowest awarded bid. The auction awards are announced half an hour later at 1:30 p.m. When additional securities are offered for sale by fixed price tender (see Figure 1), bidders who are awarded in the auction get the right to purchase the next day additional securities up to 30% of the auction awards at the auction stop-out price or higher.

Before August 1992, investors could bid directly in the auctions. During this period, the realization of the number of bidders per auction varies from 1 to 14. In August 1992, the Treasury designed a primary dealer system, where a few large banks were given the privilege, but also the obligation, to bid in the auctions. The primary dealers are six domestic banks, five foreign banks, and two domestic brokerage firms. In this system, investors who want to purchase securities in the auction must bid through one of the primary dealers. As can be seen in Figure 2, the number of primary dealers has increased over the sample period from five to ten.[5]

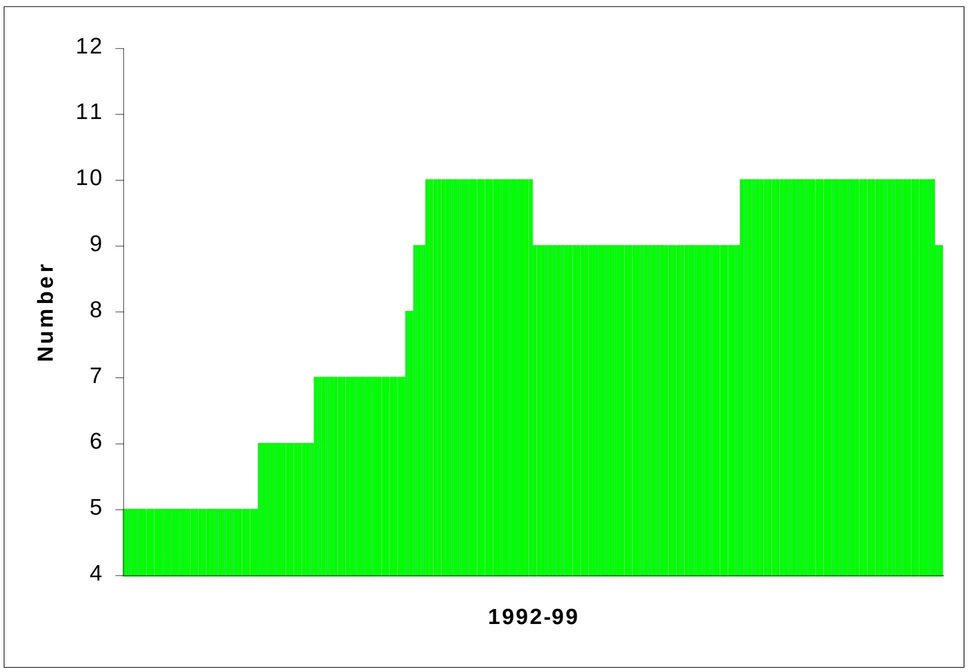

Figure 2: **Number of Primary Dealers**: Per auction from August 1992 to 1999.

In sum, there are two distinct characteristics of the Finnish Treasury bond auctions: *i)* The number of bidders is small, and *ii)* supply is determined after the seller has observed the bids. The Finnish auctions also differ from US Treasury auctions in three other ways: *iii)* There is no procedure for non-competitive bids at the stop-out price, *iv)* there is no limitation on how much can be awarded to a single primary dealer (no 35%-rule), and *v)* there is no when-issued market as the security is usually traded in the secondary market.

[5]For a more detailed description of the Finnish Treasury bond market, see Keloharju et al. (2002).

3 Data

3.1 Bid Distribution Data

For this study, the Finnish Treasury has produced a tape which contains all the bids in 231 of the 232 auctions. The last auction is missing. Each row of the tape displays the price per 100 markka face value, the yield to maturity, the face value demanded at that price, and a two-digit dealer code. The code remains constant within the auction and across auctions. The tape contains the demand schedules submitted by 28 different bidders of which 13 are primary dealers. The total number of demand schedules is 1,893, and the number of price-quantity pairs 5,163. We shall focus on the 206 auctions under the primary dealer system when the number of bidders is fixed prior to each auction. This reduces the number of demand schedules to 1,702 and the number of price-quantity pairs to 4,583.[6] The distribution of the number of bids per demand schedule can be seen in Figure 3. The average is 2.7, the median 2, and the mode 1. The maximum is 14 bids in one demand schedule.

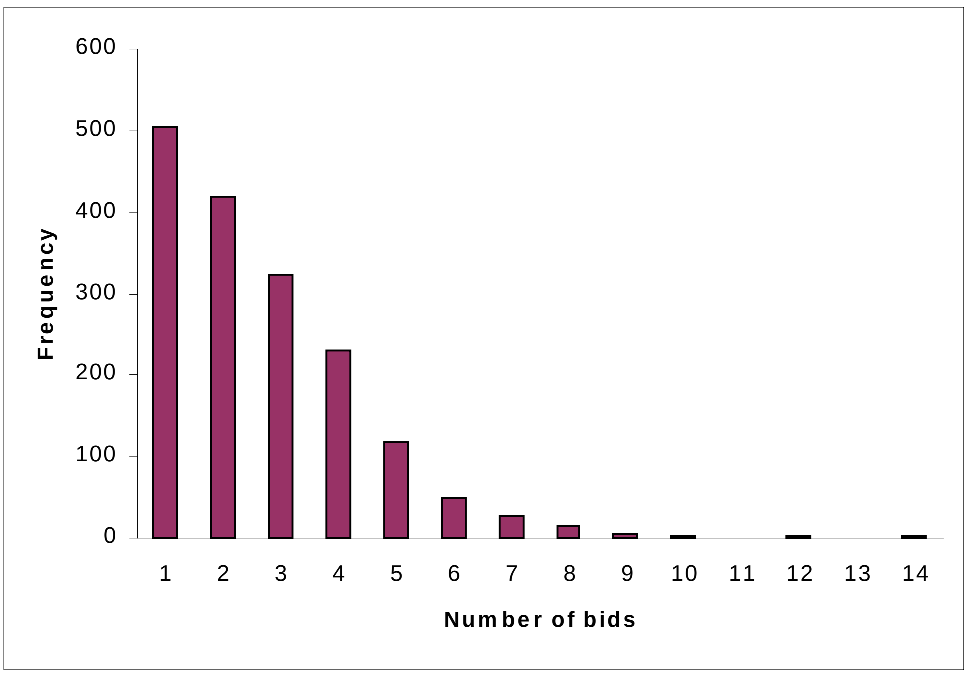

Figure 3: **Distribution of Number of Bids Per Demand Schedule**

[6]One outlying bid is excluded from our data set. This bid is the lower in a demand schedule of two bids, it is submitted at a price which is more than 6 percentage points below the higher bid, and for a quantity which exceeds the total auction awards. With so much weight on the lower of the two bids, the bid distribution exhibits *positive* skewness. See below in Section 4.3 for a definition of skewness.

3.2 Secondary Market Prices

This section describes the secondary market and the available price data. Secondary market prices are required for estimating bid shading and underpricing.

The secondary market for a new security opens immediately after the first auction. When the activity in the secondary market trading gets sufficiently high, a committee which consists of the Treasury and the primary dealers designates the security as a benchmark bond. The primary dealers are obliged to report all their transactions in benchmark bonds to the Bank of Finland, and they must also post bid and ask quotes. Usually, the dealers start posting quotes some time before the benchmark designation. The bond loses its benchmark status one year before maturity.

Time-series of daily bid quotes are provided by the Bank of Finland, which collects the average primary dealer quote at 1 p.m. The time-series cover 181 of the 206 auctions. The missing data are from the first few auctions of each security before dealers start posting quotes. Time-series of daily transactions data are also provided by the Bank of Finland. The transactions data are organized as *i)* purchases from customers, *ii)* sales to customers, and *iii)* purchases from other primary dealers. For each category, the Bank of Finland computes the equally-weighted average yield and aggregate trading volume. The next day the Bank of Finland releases to market participants the average yield across dealers and the aggregate trading volume. The time-series cover 153 of the 206 auctions. We shall construct secondary market prices from the bid quotes and use the transactions data to correct for the systematic deviation between posted quotes and transaction yields.

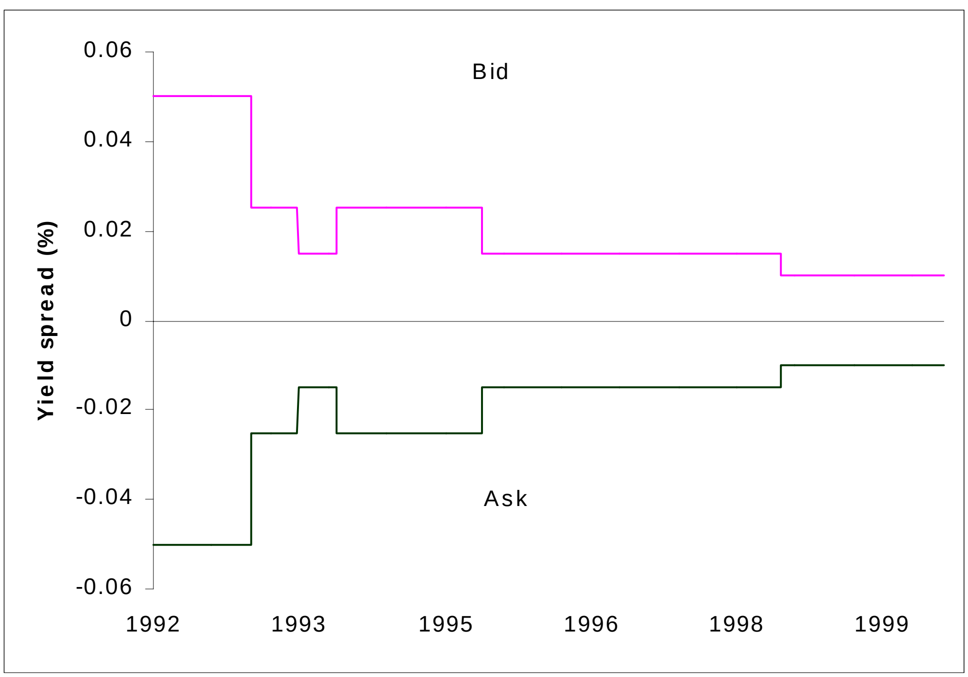

Figure 4: **Quoted Bid-Ask Spread**: Difference between the quoted bid and the ask yield by the largest primary dealer (Nordea).

The quoted bid-ask spread can be seen in Figure 4. The spread is constant over extended time periods and does not respond to daily changes in market conditions. Posted quotes are binding for 10 million, but this is a small amount compared to average daily trading volume which is about 450 million per bond, so the posted quotes are best interpreted as indicative.

On average, transaction yields are biased towards the bid quote. This can be seen by comparing the bid and ask quotes with the transaction yields. We pool the time-series and cross-section data and employ the 7,058 daily observations from August 1992 to April 1999 for which we have complete bid and ask quotes as well as transaction yields for purchases and sales to customers. Figure 5 reports the averages of three basic yield spreads: *i)* the bid quote minus the buy yield (1.01 bp); *ii)* the *effective spread*, i.e. the buy yield minus the sales yield (1.17 bp); and *iii)* the sales yield minus the ask quote (1.82 bp). The average quoted spread equals the sum of these spreads (4.00 bp). Clearly, the transaction yields are closer to the bid quote than the ask quote.

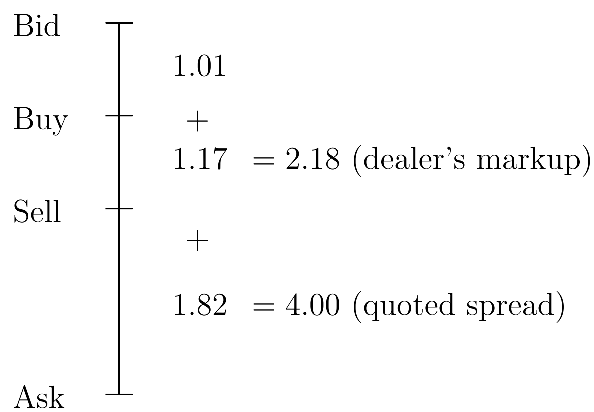

Figure 5: **Transaction Yields and Posted Quotes**: Bid and ask are the posted quotes at 1 p.m., and buy and sell are the average daily transaction yields for purchases and sales to customers. This is based on the pooled data set of 7,058 daily observations from August 1992 to April 1999. The numbers are expressed in basis points.

This bias suggests that a reasonable approximation of the market interest rate at the time of the auction would be the bid quote minus an adjustment for the general level of transactions yields relative to the bid quote itself. We therefore compute a fourth spread, namely the bid quote minus the transaction sell yield. We refer to this spread as the *dealer's markup* (2.18 bp), since it reflects a markup of the price dealers get from customers relative to the inter-dealer bid quote. The idea behind computing the markup based upon customers' sell yield is that dealers buy in the auction to sell in the secondary market. Table 1 shows that the dealers' markup varies with the size of the quoted spread.[7]

[7]We abstain from more sophisticated modelling of the markup, for example, using lags and volatility

Therefore, we estimate secondary market prices using the conditional markup. For example, if the posted bid quote is 5% and the quoted spread 2 bp, we infer the transaction yield to be $5 - .0094 = 4.9906\%$. While this means that we are measuring secondary market prices with error, we believe the error is reduced relative to relying on the bid quote or the midpoint of the spread. When bid quotes are missing, the observation is dropped from our data set. We do not attempt to extrapolate the missing secondary market yields from the sparse term structure data in Finland.

Quoted spread	2 bp	3 bp	5 bp	10 bp
Markup (bp)	0.94	1.74	3.14	3.84
Standard error	(0.06)	(0.05)	(0.10)	(0.22)

Table 1: **Quoted Spreads and Markups:** The markup is defined as the average difference between the bid quote at 1 p.m. and the average daily yield for sales to customers. The estimates are obtained from a dummy variable regression on the pooled data set of 7,058 daily observations from August 1992 to April 1999.

4 Theory of Bidder Behavior in Uniform Auctions

In uniform auctions, bidders compete by submitting collections of bids, or demand schedules, with awards allocated in the order of descending price until supply is exhausted. Moreover, winning bidders pay the stop-out price (the price of the lowest winning bid) for all units they are awarded, regardless of the price at which they submitted their bids. Hence, the uniform auction works much like a classical Walrasian market with the price and allocations being determined by the point where demand equals supply. The difference is that bidders in uniform auctions may submit their demand schedules strategically. This makes the study of uniform price auctions quite rich, even in the absence of private information. In particular, bidders can use their monopsonistic market power over the residual supply to submit demand schedules that generate an equilibrium price below the Walrasian price. Below we review this market power theory, with an emphasis on drawing out testable empirical implications. We consider in turn the cases that bidders are risk neutral and that they are risk averse.

4.1 Market Power when Bidders are Risk Neutral

The case of risk neutral bidders was first explored by Wilson (1979) and later by Back and Zender (1993) who introduce supply uncertainty. In their model, there are N identical bidders, each of whom can buy the entire auction. The auction size, Q, may be random

to forecast the markup on a daily basis. One reason is that the autocorrelation in the time-series is only .08, so there is little to gain from using lags. Another reason is that the transactions data are incomplete as a result of no trading and missing in about 11% of the trading days. The missing data would give rise to other estimation problems.

and is at most $Q_{\max}$. Bidders have identical valuations of $\bar{v}$ per unit. One can think of $\bar{v}$ as the expected secondary market price.[8] Wilson (1979) and Back and Zender (1993) show that there are numerous equilibria where bidders submit decreasing demand functions which result in underpricing, i.e. a stop-out price below $\bar{v}$.

The intuition behind the underpricing lies with the price-quantity tradeoff faced by each bidder when all the other $N-1$ bidders submit decreasing demand functions. In this case, a bidder can increase his share of the auction by submitting a higher demand function, but this comes at the expense of raising the stop-out price and thereby decreasing the profit per unit he buys. For a given stop-out price, the quantity a bidder receives is the residual supply – the quantity left over after other bidders' demand has been filled.[9] So each bidder is essentially maximizing his profit against an increasing residual supply curve, much like a monopsonist. The underpricing equilibria are cemented by the fact that each bidder can optimally exercise his monopsonistic market power by submitting a decreasing demand function.

When the auction size is known, the first order condition of a bidder's price-quantity tradeoff needs to be satisfied only at the stop-out price itself. As a result, there are numerous underpricing equilibria. However, when supply is uncertain and exogenous, the first order condition must be satisfied along the set of all possible stop-out prices. As a result, there is a unique class of supply uncertainty robust demand functions, as found by Back and Zender (1993). We shall focus on these equilibria, since bidders in the Finnish auctions do not know the supply when they submit their bids. The unique supply uncertainty robust equilibria are given by:

$$q(p) = a\left(1 - \frac{p}{\bar{v}}\right)^{\frac{1}{N-1}}, \tag{1}$$

where $a \geq Q_{\max}/N$ is the quantity demanded at a price of 0 and $Q_{\max}$ is the largest possible auction size. Given a, the inverse demand curve is:

$$p(q) = \left[1 - \left(\frac{q}{a}\right)^{N-1}\right]\bar{v}. \tag{2}$$

Under (1), demand at a price of zero is not boundless; it is a, while demand is zero at prices of $\bar{v}$ and higher. For $N \geq 3$, the demand schedule exhibits *strict concavity*, as illustrated in Figure 6. The intuition for this is also related to the price-quantity tradeoff faced by bidders: Given that the stop-out price is below $\bar{v}$, each bidder would appear to have an incentive to bid more aggressively to get a bigger share of the auction. So it must be that a large increase in quantity can only be achieved by a large increase in price. Furthermore, it must be that a small decrease in price will result in a large decrease in quantity; since otherwise bidders would have an incentive to be more passive. This is essentially a convexity condition on the residual supply and therefore a concavity

[8]In our exposition of the Wilson/Back and Zender model, the bidders are assumed to have no private information about the secondary market price. See footnote 2.

[9]In the underpricing equilibria, demand functions are strictly decreasing so rationing is not an issue. The importance of this is discussed by Nyborg (2002).

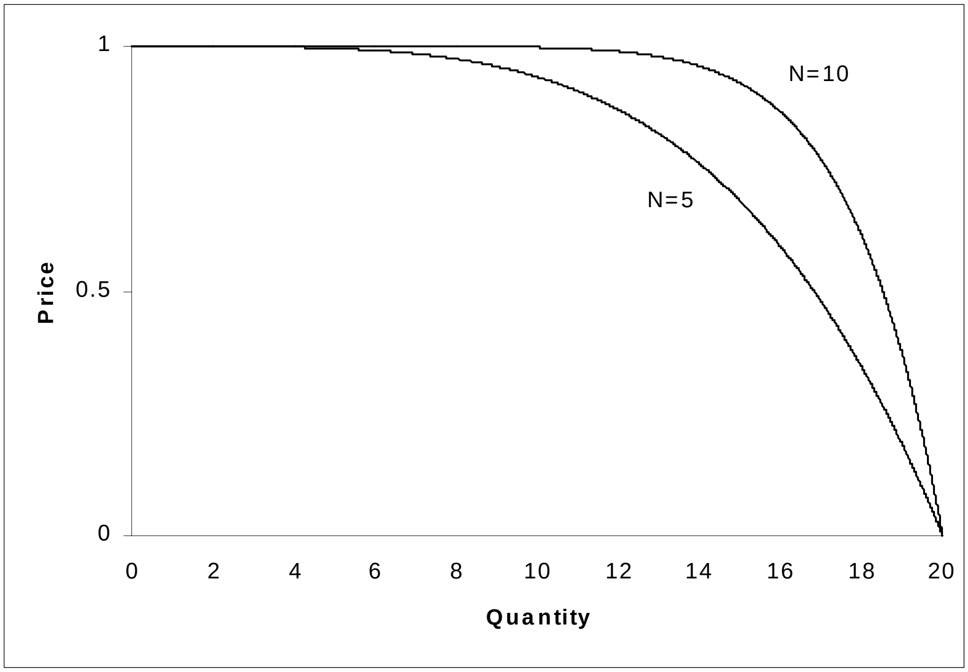

Figure 6: **Inverse Demand Curves Under Risk Neutrality**: Back and Zender (1993) inverse demand curves with five and ten bidders, respectively. The figure assumes that $\bar{v} = 1$, $Q_{\max} = 100$ and $a = 20$. The reservation price is zero.

condition on individual demand functions, especially since this must be satisfied along the continuum of possible stop-out prices [see Nyborg (2002) for further discussion]. Figure 6 also illustrates that concavity increases with N. This effect arises because as more bidders enter the auction, competition reduces the scope to exercise market power and so demand functions shift up. We show below how this crucial feature of the market power equilibria translates into a testable empirical prediction, even if a were to vary with N.

Under (1), the stop-out price, which equates demand and supply, is

$$p_0 = \left[1 - \left(\frac{Q}{aN}\right)^{N-1}\right] \bar{v}, \tag{3}$$

where Q is the realized auction size. Total revenue from the auction is thus $p_0 Q$ and depends upon $\bar{v}$, a, N, and Q.

The theory does not specify the parameter a. The seller's preferred equilibrium arises when $a = \infty$ which implies that demand curves are infinitely elastic and there is no underpricing ($p_0 = \bar{v}$). This contrasts with the bidders' preferred equilibrium where $a = Q_{\max}/N$. In this case, the seller would be giving away the securities for free if the auction size were $Q_{\max}$. The seller can reduce the extent of possible underpricing by imposing a reservation price $r > 0$. However, during the sample period, the Finnish Treasury never operated with pre-announced reservation prices.

Back and Zender (2001) show that the seller can reduce equilibrium underpricing by choosing the supply *ex post* to maximize revenue. If the seller behaves this way, in equilibrium bidders submit demand functions such that revenue is maximized at $Q_{\max}$ and the stop-out price is at least[10]

$$p_0 \geq \left(\frac{N-1}{N}\right)\bar{v}.$$

The demand schedule (1) is still equilibrium, but the lower bound on a increases to

$$a \geq \left(\frac{Q_{\max}}{N}\right) N^{\left(\frac{1}{N-1}\right)}.$$

Hence, with ten bidders, which is the maximum in our dataset, underpricing can be as large as 10%. This is much more than anything we observe.

4.2 Market Power when Bidders are Risk Averse

4.2.1 CARA Utility and Linear Equilibria

Kyle (1989) presents a model where bidders have CARA utility with risk aversion coefficient ρ. The post-auction value of the auctioned security, $\tilde{v}$, is normally distributed with expectation $\bar{v}$ and variance σ^2. We shall focus on the special case of his model where bidders do not have private information and thereby emphasize the implications of monopsonistic market power and risk bearing. Kyle (1989) demonstrates that there is a unique *linear* equilibrium which is robust to supply uncertainty, namely

$$q(p) = \left(\frac{N-2}{N-1}\right)\frac{\bar{v}-p}{\rho\sigma^2}. \tag{4}$$

We provide a straightforward derivation of this equilibrium in Appendix 1.[11] The inverse demand schedule is

$$p(q) = \bar{v} - \left(\frac{N-1}{N-2}\right)\rho\sigma^2 q. \tag{5}$$

To isolate the effect of market power from the effect of risk aversion, we can compare (4) to the corresponding Marshallian (or non-strategic) demand schedule under CARA utility. Standard arguments show that the Marshallian schedule is the linear function

$$q(p) = \frac{\bar{v}-p}{\rho\sigma^2}, \tag{6}$$

with inverse

$$p(q) = \bar{v} - \rho\sigma^2 q. \tag{7}$$

The negative slope is a result of risk aversion, and linearity is a result of CARA utility and normality. The strategic inverse demand schedule (5) is located below the Marshallian inverse (7), as illustrated in Figure 7 for $N = 5$, $N = 10$, and $\bar{v} = 1$. As N goes to infinity, the strategic equilibrium converges to the non-strategic one. As in the case of risk neutral bidders, this illustrates that a feature of supply uncertainty robust equilibria is that market power diminishes when N increases and eventually vanishes in the limit.

[10]If the seller is willing/able to sell an infinite amount, McAdams (1999) argues that underpricing from market power could be eliminated by the "maximize *ex post* revenue" rule.

[11]Note that Kyle (1989) considers the case that $\bar{v} = 0$, but it is straightforward to extend his analysis to $\bar{v} \neq 0$ (see the appendix). With respect to private information in Kyle's model, see footnote 2.

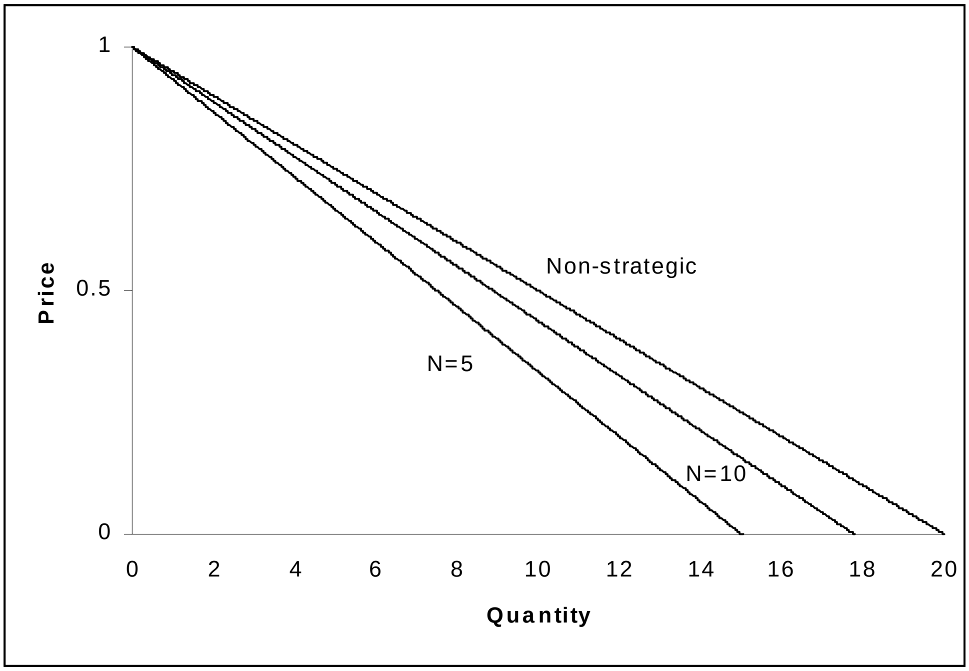

Figure 7: **Inverse, Linear Demand Curves under CARA Utility**: Graphs of Kyle's (1989) inverse demand curve with five bidders, ten bidders, and the corresponding inverse Marshallian demand curve. We let $\bar{v} = 1$, and $\rho\sigma^2 = .05$. The reservation price is zero.

Under the strategic demand schedule, (4), the stop-out price is:

$$p_0 = \bar{v} - \left(\frac{N-1}{N-2}\right)\frac{\rho\sigma^2 Q}{N}. \tag{8}$$

Under the non-strategic schedule, (6), it is

$$p_0 = \bar{v} - \frac{\rho\sigma^2 Q}{N}. \tag{9}$$

These formulas show that underpricing, $\bar{v} - p_0$, is larger when bidders are strategic. Furthermore, underpricing increases with the risk aversion coefficient and the amount of aggregate risk, $\sigma^2 Q$, that must be borne by a given number of bidders. An increase in N reduces underpricing primarily because more bidders share the aggregate risk, but also because market power is reduced.

4.2.2 CARA Utility and Nonlinear Equilibria

A surprising result is that Kyle's (1989) equilibrium does not converge to that of Back and Zender (1993) as the risk aversion coefficient goes to zero. The reason for this can be understood by looking at the general solution to Kyle's model, which has been shown by

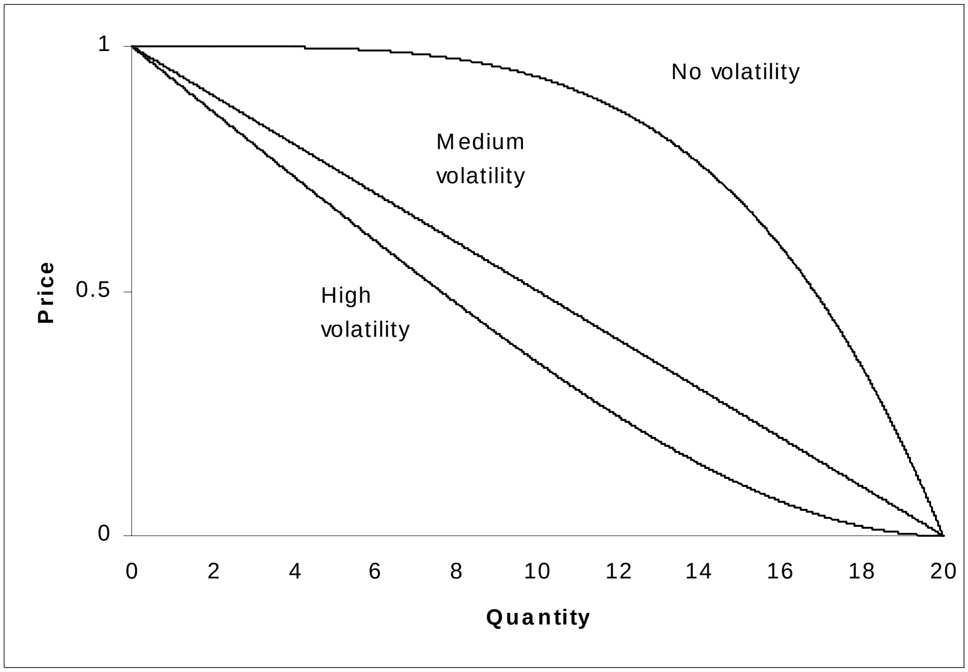

Figure 8: **Non-Linear Inverse Demand Curves under CARA Utility**: Graphs of Wang and Zender's (2002) general solution to Kyle's (1989) model (without private information). The graphs illustrate the role of changing volatility. We let $\bar{v} = 1$, $N = 5$, $r = 0$, $a = 20$, $\sigma = 0$ (no volatility), $\rho\sigma^2 = .015$ (medium volatility) and $\rho\sigma^2 = .05$ (high volatility). As we go from no to high volatility, the curves change shape from concave to convex. "Medium" volatility corresponds to the case that the demand function is linear.

Wang and Zender (2002) to be (in inverse form):[12]

$$p(q) = \left[1 - \left(\frac{q}{a}\right)^{N-1}\right]\bar{v} - \left[1 - \left(\frac{q}{a}\right)^{N-2}\right]\left(\frac{N-1}{N-2}\right)\rho\sigma^2 q, \tag{10}$$

where a is an arbitrary positive constant. These equilibria have the intuitive property that as ρ goes to zero, they converge to Back and Zender's equilibria (2). This also shows that the first term in (10) is a pure reflection of market power. The second term can be interpreted as a discount related to risk bearing.

The parameter a plays an important role. As long as $a \leq \frac{\bar{v}}{\rho\sigma^2}$, (10) is strictly decreasing and $p(a) = 0$; i.e., demand at a price of 0 equals a, as in Back and Zender's equilibrium. If $a = \frac{N-2}{N-1}\frac{\bar{v}}{\rho\sigma^2}$, (10) reduces to Kyle's linear equilibrium. So by imposing linearity, Kyle (1989) is essentially restricting the parameter a to move in a very specific way in relation to N and σ^2. A test of the empirical validity of (5) is therefore a test of this relation. The general solution of Wang and Zender admits a richer response of bidder behavior to changes in both N and σ^2.

Keeping a fixed, the effects of changing volatility in (10) are illustrated in Figure 8. The concave demand curve has $\sigma^2 = 0$ and is the same as Back and Zender's equilibrium,

[12] We provide a straightforward derivation of (10) in Appendix 1.

(2). As we increase volatility and risk bearing becomes more important, demand curves become less concave. Eventually, when σ^2 hits $\frac{N-2}{N-1}\frac{\bar{v}}{\rho a}$, the demand curve becomes linear. When volatility goes beyond this threshold, demand curves become convex, as shown in the figure. These volatility effects generate additional empirical predictions which are not present in Back and Zender (1993) and Kyle (1989).

4.3 Empirical Implications

In this section, we derive testable implications from the theories presented above. A contrast between theory and practice is that whereas the theory assumes that bidders submit "smooth" demand schedules, in practice, bidders submit collections of price-quantity pairs, implying that observed demand schedules are step-functions (see Figure 3). The theoretical demand schedules should therefore be viewed only as approximations. Furthermore, in our sample, there is variation in auction size and $\bar{v}$ from auction to auction and there is also variation in the number of bids submitted by individual bidders within an auction. Therefore, instead of trying to fit discrete empirical demand schedules to the smooth theoretical schedules, our approach is to compute a number of summary statistics of the predicted demand schedules and of auction performance which are straightforward to compare with what we see in the data.

We look at four measures of bidder behavior. The first is the *discount*, which measures the difference between the expected secondary market price and the quantity weighted average price of a bidder's demand schedule. Formally, we define the *discount* of demand schedule $q(p)$ to be

$$\bar{v} - \bar{p} = \bar{v} - \frac{1}{q(r)} \int_0^{q(r)} p(x)dx, \tag{11}$$

where $p(x)$ is the inverse demand schedule, $r \geq 0$ is the reservation price of the seller, and $\bar{p}$ is the quantity weighted average price along the inverse demand schedule for prices at or above the seller's reservation price. Note that $\bar{p}$ is defined by the last term in (11). This is the appropriate definition since $q(r)$, being the demand at the reservation price, is also the total demand of a bidder who uses $q(p)$. The discount is similar to, but not the same as underpricing, which is defined as the difference between the secondary market price and the auction stop-out price, $\bar{v} - p_0$.

The other three summary statistics are the standard deviation, skewness, and kurtosis of the inverse demand schedule. The standard deviation of bids along the schedule is:

$$st.dev \equiv \eta = \sqrt{\frac{1}{q(r)} \int_0^{q(r)} (p(x) - \bar{p})^2 dx}, \tag{12}$$

The formulas for skewness and kurtosis are, respectively,

$$skewness = \frac{1}{\eta^3 q(r)} \int_0^{q(r)} (p(x) - \bar{p})^3 dx, \tag{13}$$

and

$$kurtosis = \frac{1}{\eta^4 q(r)} \int_0^{q(r)} (p(x) - \bar{p})^4 dx, \tag{14}$$

Table 2 summarizes the predicted values of these four statistics along with quantity demanded, expected underpricing, and award concentration (see below) for *i)* Back and Zender's (1993) supply uncertainty robust equilibrium, *ii)* Kyle's (1989) linear equilibrium in his model under CARA utility, and *iii)* the corresponding Marshallian demand schedules. Statistics for Wang and Zender's (2002) general solution to Kyle's (1989) model are so long and complex that they would not fit in the table (see Appendix 2).[13]

	Back and Zender[a] (Market Power, Risk Neutral)	Kyle[b] (Market Power, Risk Aversion)	Non-Strategic[c] (Risk Aversion)
Discount	$\frac{\bar{v}-r}{N}$	$\frac{\bar{v}-r}{2}$	$\frac{\bar{v}-r}{2}$
St.Dev	$\frac{(N-1)(\bar{v}-r)}{N\sqrt{2N-1}}$	$\frac{\bar{v}-r}{2\sqrt{3}}$	$\frac{\bar{v}-r}{2\sqrt{3}}$
Skewness	$-\frac{2(N-2)\sqrt{2N-1}}{3N-2}$	0	0
Kurtosis	$\frac{3(2N-1)(6-5N+2N^2)}{(4N-3)(3N-2)}$	1.8	1.8
Quantity	$a\left(1-\frac{r}{\bar{v}}\right)^{\frac{1}{N-1}}$	$\left(\frac{N-2}{N-1}\right)\frac{\bar{v}-r}{\rho\sigma^2}$	$\frac{\bar{v}-r}{\rho\sigma^2}$
Underpricing[d]	$\bar{v}\left(\frac{Q}{aN}\right)^{N-1}$	$\left(\frac{N-1}{N-2}\right)\frac{\rho\sigma^2 Q}{N}$	$\frac{\rho\sigma^2 Q}{N}$
Award concentration[e]	1	1	1

Table 2: **Measures of Bidder Behavior and Auction Performance in Monopsonistic Market Power Models**

a. Back and Zender (1993) column uses (2). Model based on risk neutrality and strategic behavior.

b. Kyle (1989) column uses (5). Model based on CARA utility, normal distribution, strategic behavior.

c. Non-Strategic column uses (7). Model based on CARA utility and normal distribution.

d. Underpricing is measured as $\bar{v} - p_0$, where p_0 is the stop-out price and assumes that the reservation price is not binding. Otherwise, underpricing would equal $\bar{v} - r$.

e. Award concentration is the modified Herfindahl index, H^*, given by (15).

For the Back and Zender equilibrium, Table 2 reveals the striking result that the unknown parameter a does not figure in the expressions for the discounts or any of the higher order moments. Furthermore, skewness, kurtosis, and the ratio of the discount to the standard deviation, which equals $\sqrt{2N-1}/(N-1)$, depend only on N. *Hence these*

[13] Note that in the theory the smallest possible equilibrium stop-out price is $p_{\min} = p(Q_{\max}/N)$, where $p(q)$ is given by e.g. (2). So for $q > Q_{\max}/N$, the functional form of $p(q)$ is irrelevant and could be anything. However, there is no reason to expect that bidders submit bids which have no chance of getting awarded. Consequently, the formulas in Table 2 ignore such irrelevant demand. One can view r in the formulas as being the maximum of the actual reservation price and $p_{\min}$.

predictions are valid in a cross-section of auctions, even though a, $\bar{v}$, and r may vary from auction to auction. We think the intuition for the surprising result that the unknown parameter a drops out relates to the fact that the first order condition of a bidders' price-quantity tradeoff must be satisfied at every point along a supply uncertainty robust demand function. Specifically, Table 2 shows that the strict concavity of (2) implies that the bid distribution has *negative skewness*. Moreover, taking the derivative of the expression for skewness, we see that skewness gets more negative as the number of bidders increases, even if a were to vary systematically with N.

In some of the other comparative statics we can compute from Table 2 for the Back and Zender equilibrium, a and r do not drop out. For example, the discount and the standard deviation decrease with the number of bidders, keeping r fixed. The same holds for underpricing, if we also fix a and assume that the reservation price is not binding. Given a and r, quantity demanded increases with the number of bidders. In short, additional bidders induces more aggressive bidding, as a result of diminishing market power. This assumes that r and a do not vary with N in such a way as to offset this effect.

The Kyle equilibrium and its non-strategic counterpart offer the surprising result that the discount and the higher order moments do not depend on volatility, even though bidders are risk averse. There are three parameter free tests: Skewness and kurtosis are constants as a result of linearity, and the ratio of the discount to the standard deviation equals $\sqrt{3}$. All the action in these models appears to be in the quantity demanded. If bidders act non-strategically and risk bearing is their only concern (fourth column), quantity demanded decreases with volatility, but does not respond to changes in the number of bidders or expected auction size. If bidders act strategically (third column), quantity demanded increases with the number of bidders, which is the distinguishing qualitative feature between the two models. The comparative static results of the Kyle model depend critically on linearity, which, as shown in Section 4.2.2, implicitly imposes a constraint on the unknown parameter a.

While volatility has little impact on bidder behavior in Back and Zender's and Kyle's equilibria, volatility plays a significant role in Wang and Zender's equilibrium (10). When demand curves are strictly downward-sloping, i.e. $a \leq \bar{v}/\rho\sigma^2$, and the number of bidders varies between five and ten, one can show that bidders respond to an increase in volatility by lowering their bids (larger discounts) and increasing skewness, which turns from negative to positive as volatility gets very high (see Appendix 2 and also Figure 8). As a consequence of this behavior, underpricing increases with volatility. Bidders' response to an increase in the number of bidders is the same as in Back and Zender, (2).

Finally, we are interested in examining award concentration. With symmetric bidders and no private information, each bidder receives an equal share of the awards, which means that the Herfindahl index equals $1/N$. However, in our sample, the number of players varies over time. Hence, the Herfindahl index may give the wrong impression of award concentration. For example, if there are five bidders and one bidder gets all the awards, the Herfindahl index equals 1, which is also the case when one bidder gets all the awards in an auction with ten bidders. However, intuitively, the latter case involves more award concentration relative to the benchmark of equal awards to all bidders. To capture this, we employ a modified version of the Herfindahl index defined as

$$H^* = H \times N. \tag{15}$$

This measure equals 1 if bidders submit identical demand schedules, as in the models reviewed above. It is N if one bidder obtains all the awards.

5 Empirical Analysis: Bidder Behavior and Auction Performance

This section examines the extent to which the theories reviewed above are consistent with observed bidder behavior and auction performance. There are five main measures of bidder behavior: *discount*, *quantity demanded*, and *intra-bidder standard deviation, skewness* and *kurtosis*; and three measures of auction performance: *underpricing*, *realized auction size*, and *award concentration*. We examine how these endogenous variables vary with the following three exogenous variables: *volatility*, *the number of bidders*, and *expected auction size*. This allows us to test the qualitative performance of the theories. We also carry out some quantitative tests as well as a detailed examination of the non-linearities that are apparent in bidders' demand schedules.

5.1 Descriptive Statistics

Table 3 provides auction day summary statistics of the exogenous variables in Panel (a) and the endogenous variables in Panels (b)-(d). The 1,702 demand schedules are submitted in 206 auctions on 175 auction days. For each auction, we compute the equally-weighted average of all variables and, then, for each auction day, the equally-weighted average across the auctions (which are all held simultaneously). This procedure is a conservative way to eliminate correlations among the error terms. Hence, we treat each auction day average as an independent observation.

Panel (a) reports on volatility and the number of bidders. Volatility is measured as the daily standard deviation of bond returns imposing an ARCH(2) structure on volatility. The details of the estimation can be found in the Appendix 3. Daily volatility averages .346%, which is about one third of the volatility of S&P500. The number of bidders equals the number of primary dealers as shown in Figure 2.

Panel (b) reports on two measures of the location of bids relative to the secondary market price, discount and underpricing (or profit). To estimate the discount in auction j, we first compute the quantity-weighted average price p_{ij} for each bidder i, and then the equally-weighted average across bidders. The discount is defined as the difference between the secondary market price at the time of the auction (see Section 3.2) minus this average. Underpricing is simply defined as the difference between the secondary market price and the auction's stop-out price. Secondary market prices are available for 159 auction days, but underpricing can only be estimated for 156 auction days since three cancelled auctions without a stop-out price are lost. There are two important results: First, the average bid is submitted .081% below the secondary market price. Second, the auctions are underpriced

Variable	mean	std	s.e.	min	max	N
a) Exogenous						
Volatility	0.346	0.157	0.012	0.110	1.115	175
Number bidders	8	2	0.1	5	10	175
b) Location						
Discount	0.081	0.153	0.012	-0.397	0.920	159
Underpricing	0.041	0.144	0.012	-0.783	0.420	156
c) Dispersion						
St. deviation	0.065	0.049	0.004	0.003	0.279	175
Skewness	-0.009	0.428	0.032	-1.623	0.888	175
Kurtosis	2.907	1.547	0.117	1.000	11.184	175
d) Quantity						
Bid quantity	235	194	15	16	1,390	175
Tender volume	2040	1952	148	80	13,903	175
Auction size	1179	850	64	0	4,000	175
Award conc.	2.519	1.258	0.096	1.007	9.000	172

Table 3: **Descriptive Statistics:** Auction day averages. The symbol s.e. denotes the standard error of the mean, and N is the number of observations. Volatility is daily price standard deviation. Discount and underpricing are the difference between the secondary market price and the auction average price and the stop-out price, respectively. Intra-bidder dispersion, skewness, and kurtosis are quantity-weighted. Bid quantities, total tender volume, and realized auction size are expressed in million markka face value. Bid quantity is the quantity demanded by a single bidder, tender volume is the quantity demanded by all bidders in a given auction, and auction size is the quantity sold. Award concentration is measured by the modified Herfindahl index (15).

by .041%. The discount is significantly different from zero with a t-statistic of 6.7, and underpricing with a t-statistic of 3.4. The standard deviation of the discount is large, .153, so a significant fraction of the bids are submitted above the secondary market price. For example, on one auction day, the average bid is .414% above the secondary market price.

Panel (c) reports on the three measures of intra-bidder dispersion.[14] The average intra-bidder standard deviation is about one fifth of daily volatility. Average skewness is -.009, which is not statistically different from 0 and therefore consistent with Kyle (1989) or any other linear model. However, the table also shows that skewness varies widely across auctions. Furthermore, in the pooled sample of individual demand schedules intra-bidder skewness varies from -8.5 to 7.5, with a standard deviation of 1.17. This provides strong evidence against linearity at the individual bidder level. Further evidence against linearity is provided by the average kurtosis of 2.907, which exceeds 1.8 with a t-statistic of 9.5.

Finally, Panel (d) looks at four quantity measures. The first row shows that the average bid is for 235 million markka of face value, and the second row that aggregate demand averages to about 2 billion. The third row shows that the average quantity sold is about 1 billion per auction, but there is substantial variation across auctions. We also note that auction size is zero in three auctions when the Treasury rejected all bids. In the last row, we report the modified Herfindahl index according to (15). The average equals 2.5. This means that the bidders in our data set are not symmetric *ex post*.

5.2 Regression Analysis

In this section, we regress the bidding and auction performance variables on the exogenous variables. The regression results are reported in Table 4. One of the exogenous variables in these regressions is the expected auction size, since this is necessary to examine the hypothesis that bidders are risk averse. While it is clear that the expected auction sizes are linked to the Treasury's financing needs, a problem for us is that auction sizes were not pre-announced – the Treasury only announced maximum auction sizes after 1998 and never announced minimum auction sizes. Taking the point of view of a bidder, we therefore estimate expected auction size by the average of the realized sizes of the last three auctions. While this may be a fairly rough estimate, the major empirical results are robust to various alternative specifications, e.g., forecasting the auction size using the parameters from the size regression reported below. The regressions in Panels (a) and (b) are weighted with volatility. The three quantity regressions in Panel (c) are adjusted for first-order autocorrelation using the Cochrane-Orcutt transformation. The regression on award concentration is estimated with ordinary least squares.

Table 4 shows that volatility is statistically significant in all regressions, except for in the skewness and kurtosis regressions. In contrast, the number of bidders is significant only in the skewness and the quantity regressions. The expected auction size is significant only in the skewness, kurtosis, and quantity regressions. The overall impression from these

[14] For one-bid demand schedules, we set skewness equal to zero and kurtosis to one. The rationale is as follows: A single bid can be regarded as the limit as c goes to zero of two bids of identical sizes at prices $b+c$ and $b-c$. The standard deviation is c, the third moment is 0, and the fourth moment c^4. Hence, skewness is zero and kurtosis one. In the limit, as c goes to zero, skewness remains zero and kurtosis one.

Dependent variable	Constant	Volatility	Number bidders	Expected size	R^2	N
a) Location						
Discount	-0.037	0.318	0.003	-0.013	0.096	159
	(-0.7)	(3.8)[a]	(0.5)	(-1.0)		
Underpricing	-0.003	0.215	-0.003	-0.009	0.054	156
	(-0.0)	(2.4)[a]	(-0.4)	(-0.6)		
b) Dispersion						
St. deviation	0.038	0.161	-0.003	-0.019	0.222	175
	(2.2)[a]	(5.9)[a]	(-1.5)	(-0.4)		
Skewness	-0.678	-0.005	0.068	0.101	0.172	175
	(-3.6)[a]	(0.0)	(3.2)[a]	(2.0)[a]		
Kurtosis	2.374	0.436	-0.021	0.543	0.057	175
	(3.2)[a]	(0.4)	(-0.2)	(2.7)[a]		
c) Quantity						
Bid quantity	46	-227	21	0.702	0.296	175
	(0.6)	(-2.9)[a]	(2.5)[a]	(2.9)[a]		
Tender volume	-380	-2089	289	0.713	0.399	175
	(-0.5)	(-2.9)[a]	(2.9)[a]	(2.7)[a]		
Quantity sold	-61	-845	134	0.397	0.419	175
	(-0.2)	(-2.6)[a]	(3.7)[a]	(4.0)[a]		
Award conc.	1.580	0.388	0.109	-0.063	0.021	172
	(2.9)[a]	(0.6)	(1.6)	(-0.3)		

Table 4: **Determinants of Bidder Behavior:** The bid variables are regressed on volatility, the number of bidders, and the moving average of realized auction size from the past three auctions. Auction size is expressed per 1,000 million face value. t-statistics are in parentheses below, and index a denotes significance level 5% or better. Panel (a). Discount and underpricing are the difference between the secondary market price and the auction average price and the stop-out price, respectively. Panel (b). Intra-bidder dispersion, skewness, and kurtosis are quantity-weighted. Panel (c). Face value per bidder in million markka, total tender volume, and realized auction size. Award concentration is the Herfindahl index. The regressions in Panels (a) and (b) are estimated with weighted least squares using volatility as weight, the first three regressions in Panel (c) are corrected for autocorrelation using the Cochrane-Orcutt transformation, and the regression on award concentration is estimated with ordinary least squares.

regressions is that volatility is the primary driver behind bidder behavior and underpricing. Below we look more closely at the individual regressions and discuss where the equilibria presented in Section 4 succeed and where they fail.

Panel (a) reports that discounts and underpricing increase significantly with volatility but are unaffected by the number of bidders and expected auction size. Furthermore, volatility also has an economically significant impact. We see that a one standard deviation increase in volatility (.157%) raises the discount by .050% of face value, which is of the same order of magnitude as the average discount of .081% (Table 3). It also raises underpricing by .034% of face value, which is very close to the average sample underpricing of .041%. These magnitudes are close to what has been reported for discriminatory price auctions in Sweden [Nyborg, Rydqvist, and Sundaresan (2002)]. The fact that the number of bidders has no impact on the location of bids is hard to reconcile with the market power theory, since market power diminishes with the number of bidders. More precisely, the result on the discount is inconsistent with the equilibria of Back and Zender (1993), (2), and Wang and Zender (2002), (10), but consistent with that of Kyle (1989), (5). The finding on underpricing is inconsistent with all three models. This may be a consequence of how the Treasury sets the stop-out price, which we study in more detail in Section 6.

Panel (b) contains the regressions on the three intra-bidder dispersion measures. The skewness regression is of particular interest, since we saw in Section 4 that market power may manifest itself through skewness. Indeed, the skewness regression is the only non-quantity regression where the number of bidders has a significant impact. Skewness increases by .068 for each extra bidder in the auction and increases by .101 for each billion in expected auction size. Volatility has no effect. The systematic variation in skewness as N changes suggests that bidders employ non-linear bidding strategies in response to increased competition. What is really striking here, however, is the sign of the coefficient on N. It is the opposite of the negative effect predicted by Back and Zender's and Wang and Zender's equilibria. It is also inconsistent with Kyle's equilibrium, which predicts that there should be no effect.

Panel (b) also shows that intra-bidder standard deviation is increasing in volatility. Once again, this parallels the findings of Nyborg, Rydqvist, and Sundaresan (2002) for *discriminatory* treasury auctions in Sweden. In particular, the standard deviation increases by a significant .0161% of face value per .1 percentage point increase in volatility. This stands at odds with Kyle's equilibrium, where each risk averse bidder responds to uncertainty by reducing quantity demanded but not by increasing the dispersion of his bids. There is also no role for volatility in Back and Zender's equilibrium, since bidders are risk neutral and do not have private information. However, Wang and Zender's equilibrium could generate this result on standard deviation (see Appendix 2).

Panel (c) presents the results of the quantity regressions. In the regression on quantity bid by individual bidders, we have normalized the expected auction size regressor by dividing it by the number of bidders. There are four particularly interesting results. First, the quantity bid decreases with volatility, which is in line with Kyle's equilibrium. Second, individual bidders demand more when there are more bidders. For each new bidder who enters the auction, the typical bidder increases demand by a significant 21 million. This behavior is also consistent with Kyle's equilibrium. Third, bidders demand more when expected auction size increases. The striking thing here is that they do so *without lowering prices*, as can be seen in Panel (a). This is hard to reconcile with the hypothesis that bidders are risk averse. Fourth, award concentration is insensitive to changes in the exogenous variables. This contrasts with the findings on discriminatory auctions in Sweden (Nyborg, Rydqvist, and Sundaresan, 2002), where award concentration decreases with volatility. This is a potential advantage of uniform price auctions over discriminatory auctions, since sellers often have dispersed awards as an auction objective.

The empirical comparative statics from the above regression analysis are summarized in Table 5. A "+", "−", or "0" indicates that the regression coefficient is significantly positive, significantly negative, or not significant at the 5% level, respectively. The table also compares the empirical findings with the theoretical comparative statics from the Back and Zender and Kyle models. For each model, we mark with boldface if the predicted sign equals the empirically observed sign and, at the bottom, we report the number of correct and incorrect predictions.

Table 5 shows that Back-Zender's model delivers the right comparative statics in only 5 of 16 cases. Notably, the model fails with respect to the impact of the number of bidders. Most striking is that skewness varies with the number of bidders with the opposite sign in the data and the theory. Kyle's (1989) model does better and delivers the right comparative static result in 10 of 18 cases. Kyle predicts correctly that bid quantity decreases with volatility, but cannot explain the general importance of volatility. Kyle also predicts correctly that bid quantity increases with the number of bidders, which is suggestive of bidders having some market power. Overall, however, our findings suggest that market power is not a key factor.

5.3 Structural Tests of Standardized Discounts and Moments

Table 2 above shows that the ratios of the discount to the standard deviation and the higher order moments are functions of the number of bidders only (Back and Zender model) or constants (Kyle model). In this section, we test these quantitative implications by regressing observed values on predicted values:

$$\text{OBS} = \alpha + \beta \times \text{PRED} + \epsilon.$$

We test whether the intercept is zero and the slope coefficient one. The theory in Section 4 does not have a residual, but we can reject the quantitative implications even if we allow for one.[15] The test results are reported in Table 6.

We start with the Back and Zender (1993) model in Panel (a). The intercept is statistically significantly different from zero, and the slope coefficient is significantly different from one in all three regressions. Hence, the model is rejected. In particular, we notice that

[15]Some errors may arise from the fact that bidders must approximate the continuous demand functions with discrete price-quantity pairs. However, these errors should be relatively small. The price tick is .05 per cent of face value and the quantity multiple one bond with face value 100 markka. Hence, there are 2,000 price intervals between zero and one, and 200,000 quantity intervals between zero and 200 million which is the approximate average bid quantity. As a result, bidders should be able to approximate fairly closely the theoretical demand functions.

	Observed sign	Back-Zender[a] (Risk Neutral)	Kyle[b] (Risk Averse)
a) Discount			
Volatility	+	0	0
Number	0	−	**0**
Expected Size	0	**0**	**0**
b) Std deviation			
Volatility	+	0	0
Number	0	−	**0**
Expected Size	0	**0**	**0**
c) Skewness			
Volatility	0	**0**	**0**
Number	+	−	0
Expected Size	+	0	0
d) Kurtosis			
Volatility	0	**0**	**0**
Number	0	+	**0**
Expected Size	+	0	0
e) Bid Quantity			
Volatility	−	0	**−**
Number	+	**+**[c]	**+**
Expected Size	+	?	0
f) Underpricing			
Volatility	+	0	**+**
Number	0	−	−
Expected Size	0	?	+
Number correct	n.a.	**5**	**10**
Number incorrect	n.a.	11	8

Table 5: **Comparative Statics:** Summary of regression coefficients and comparative statics from two market power equilibria. Correct prediction is marked with boldface.

a. Back and Zender (1993) column uses (2). Model based on risk neutrality.

b. Kyle (1989) column uses (5). Model based on risk aversion. Linear equilibrium.

c. Assuming that $r > 0$. If $r = 0$, the comparative statics are 0.

? indicates ambiguity, since a may be increasing in Q as suggested by the regression results.

Dependent variable:	Constant α	Predicted β	F-value	R^2	N
	(a) Back and Zender (1993)				
Discount/st.dev.	3.065	-3.259	9.6	0.028	159
	(0.923)	(1.529)	(0.000)		
Skewness	-0.543	-0.256	2335.8	0.066	175
	(0.156)	(0.073)	(0.000)		
Kurtosis	1.964	0.140	640.6	0.029	175
	(0.428)	(0.061)	(0.000)		
	(b) Kyle (1989)				
Discount/st.dev.	-0.607	n.a.	n.a.	n.a.	159
	(0.158)				
Skewness	-0.009	n.a.	n.a.	n.a.	175
	(0.032)				
Kurtosis	1.107	n.a.	n.a.	n.a.	175
	(0.117)				

Table 6: **Structural Tests:** Panel a) Regressions of observed values on predicted values. The discount-to-standard deviation regression is weighted with the standard deviation. The other regressions are estimated with ordinary least squares. The F-statistic tests the joint hypothesis that the constant equals zero and the slope coefficient equals one. The numbers below the estimated coefficients are standard errors, and the numbers below the F-statistics are p-values. N is the number of observations. Panel b) The average difference between observed and predicted values.

the slope coefficients of the first two regressions are significantly negative, which means that standardized discounts and skewness *increase* with the number of bidders. Kurtosis increases with the number of bidders, but by less than the full amount according to theory.

The corresponding results for Kyle (1989) are reported in Panel (b). Since there is no cross-sectional variation in predicted values, we report the average difference between observed and predicted values and test whether this difference is zero. The difference is significantly different from zero for standardized discount and kurtosis, but not for skewness. The finding for the discount means that bidders in the Finnish auctions disperse their bids more than predicted by Kyle (1989). Excess kurtosis means that demand schedules are not linear which we return to in greater detail in the next section.

5.4 Non-Linearity: Skewness, Kurtosis, and Number of Bidders

In this section, we take a closer look at the nonlinearity of submitted demand functions and study how skewness and kurtosis vary with the number of bidders. Specifically, we show that bidders tend to submit one very low bid when there are few bidders, but that they drop the low bid and instead submit one very high bid when there are many bidders. Thus skewness tends to be negative for small N and positive for large N.

We first classify bidders' demand schedules by the number of bids they contain. A bidder's set of price-quantity pairs in a generic auction is given by the set $\{(p_k, q_k)\}_{k=1}^{m}$, where m is the number of bids and the bids are ordered by $p_1 > p_2 > ... > p_m$. We can think of a demand schedule with $m \geq 2$ as being "discrete-linear" if the bidder's marginal demand is the same at every price at which he submits a bid and these prices are spaced equally. To investigate whether bidders use discrete-linear strategies, we first compute the standardized difference between adjacent prices by

$$d_k^* = \frac{p_k - p_{k+1}}{p_1 - p_m} \bigg/ \frac{1}{m-1} .$$

There are $m - 1$ price differences. Under a discrete-linear strategy, $d_k^* = 1$. Furthermore, for any m, under a discrete-linear strategy, skewness is zero. Kurtosis increases in m and approaches 1.8 from below as m goes to infinity. We refer to the kurtosis predicted by a discrete-linear strategy as "linear kurtosis", to differentiate it from the realized kurtosis.

Table 7 contains our findings. Panel (a) covers the case with few bidders (5-8) and Panel (b) many bidders (9-10). Within each panel, the upper sub-panel provides the means of d_k^* across all demand schedules with $m = 1, \ldots, 8$ bids. The lower sub-panel contains the averages of the intra-bidder standard deviation, skewness, kurtosis, linear kurtosis, and the number of observations.

In Panel (a), we can see that, for all m, the lowest d_k^* exceeds one. This means that the last price difference is larger than the intermediate price differences. This explains why skewness is negative and kurtosis higher than predicted by a discrete-linear strategy. However, in Panel (b), we can see, for all m, that the highest d_k^* exceeds one, which means that the first price difference is larger than the intermediate price differences. Therefore, skewness turns from negative with few bidders to positive with many bidders. Moreover, this switching of sign is robust to the number of bids in a demand schedule; skewness is

(a) Few bidders (5-8)								
	$m=1$	$m=2$	$m=3$	$m=4$	$m=5$	$m=6$	$m=7$	$m=8$
d_1^*		1.000	0.935	0.989	1.116	0.829	0.947	1.468
d_2^*			1.065	0.853	0.756	0.786	0.914	0.863
d_3^*				1.156	0.882	0.890	0.820	1.059
d_4^*					1.246	1.292	0.825	0.775
d_5^*						1.207	0.797	0.541
d_6^*							1.700	1.195
d_7^*								1.097
F-test	n.a.	n.a.	7^a	8^a	8^a	4^a	4^a	0.3
Std deviation	0.000	0.055	0.078	0.106	0.157	0.138	0.145	0.277
Skewness	0.000	-0.104	-0.249	-0.174	-0.232	-0.521	-0.874	-0.155
Kurtosis	1.000	2.875	3.214	3.489	3.530	2.773	6.340	4.224
Linear Kurtosis	0	1	1.5	1.64	1.7	1.73	1.75	1.76
N	120	115	66	60	40	12	9	3

(b) Many bidders (9-10)								
	$m=1$	$m=2$	$m=3$	$m=4$	$m=5$	$m=6$	$m=7$	$m=8$
d_1^*		1.000	1.039	1.156	1.402	1.355	1.650	1.744
d_2^*			0.961	0.899	0.831	0.926	1.068	0.885
d_3^*				0.955	0.798	0.923	0.745	0.845
d_4^*					0.970	0.899	0.703	0.678
d_5^*						0.898	0.770	0.835
d_6^*							1.065	0.802
d_7^*								1.208
F-test	n.a.	n.a.	7^a	19^a	26^a	5^a	7^a	2^a
Std deviation	0.000	0.070	0.078	0.099	0.117	0.133	0.175	0.229
Skewness	0.000	0.038	0.105	0.180	0.285	0.347	0.238	0.323
Kurtosis	1.000	3.743	4.081	3.742	4.486	4.693	3.456	4.101
Linear Kurtosis	0	1	1.5	1.64	1.7	1.73	1.75	1.76
N	385	305	258	171	77	38	17	11

Table 7: **Intra-Bidder Dispersion, Skewness, and Kurtosis:** Demand schedules of up to eight individual bids. Upper sub-panels: Average standardized price differences. Lower sub-Panels: Average intra-bidder standard deviation, skewness, kurtosis. Linear kurtosis refers to what kurtosis would be if bidders employed discrete-linear strategies. N is the number of demand schedules. Super index a denotes significance level 5% or better.

consistently negative for 5-8 bidders and consistently positive for 9-10 bidders, regardless of m. In sum, Table 7 corroborates our earlier finding that while skewness is zero on the average, skewness is positively related to the number of bidders. Whatever generates this behavior, it is inconsistent with the three market power models discussed above.

6 Strategic Seller Behavior

Our findings thus far suggest that bidders in Finnish Treasury auctions act more competitively than predicted by the monopsonistic market power theory. In this section we explore the strategic behavior of the seller, with an adjunctive view to see how it might affect the behavior of bidders. Standard auction theory treats the seller as a non-strategic agent, who commits to sell a fixed quantity. However, in Treasury auctions in practice, the seller usually announces a target quantity while reserving the right to withdraw securities from the auction after observing the bids. For example, the U.S. Treasury "reserves the right to accept or refuse to recognize any or all bids".[16] This provides the seller with some protection against very low prices and also allows the seller to act strategically. The scope for strategic behavior by the seller is particularly large in our sample since the Finnish Treasury did not even announce maximum auction sizes before 1998. Recent theoretical advances have shown that underpricing can be reduced by, for example, choosing supply *ex post* so as to maximize revenue [Back and Zender (2001), McAdams (1999)]. While this specific strategy would not change the shape of equilibrium demand functions,[17] the general point is that a strategic choice of supply may help combat market power and underpricing.

We start by noting that the Finnish Treasury did not have an explicit policy regarding the choice of quantity and stop-out price. For example, they did not announce a reservation price or a supply curve. Conversations with one Treasury official revealed that, loosely speaking, their actual choices were influenced by *i)* the long-term revenue target, *ii)* market conditions, *iii)* the Treasury's own opinion about the true market price, and *iv)* unwillingness to spoil the market by accepting too low bids. This statement allows for almost any short-term behavior. Below we look at the Treasury's actual behavior. Our approach is motivated by the theoretical idea that the seller may wish to choose the stop-out price based on the revenue it will generate.

6.1 Stop-Out Price and Marginal Revenue Maximization

Figure 9 provides an example of the Treasury's typical behavior, using the auction held on 14 October 1993 for a bond maturing in 1996. In this auction, bids were submitted at 10 different price levels. This is close to the average number of price levels across all auctions, which is 9.4. For each price level, which are ordered from high price (level 1) to low price (level 10), we compute the total revenue the Treasury could obtain if that price level would have been chosen as the stop-out price. The figure depicts the normalized

[16] See ftp.publicdebt.treas.gov/gsr31cfr356.pdf, §356.33 Reservation of rights.

[17] See the discussion at the end of Section 4.1.

revenue curve, where the total revenue for each price level is expressed as a fraction of the maximum revenue which could be generated in the auction, given the submitted bids.

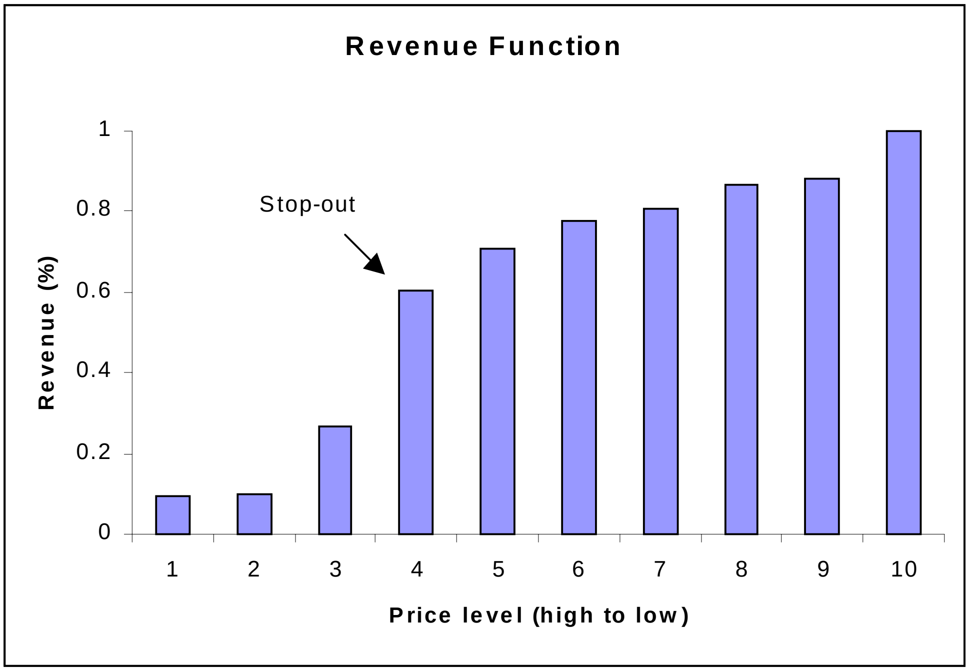

Figure 9: **Normalized Revenue Curve.** Auction held on 14 October 1993 of a treasury bond maturing in 1996. There are ten price levels in this auction, going from high price (level 1) to low price (level 10). Total revenue for each price level is expressed as a fraction of the maximum revenue that could be achieved in the auction. The stop-out price is chosen at the fourth highest price level. Total revenue is maximized at the 10th price level, and marginal revenue is maximized at the 4th price level.

Figure 9 illustrates four important and general facts. First, revenue is maximized at the lowest price level. Indeed, in 200 of the 206 auctions, this is the case.[18] Second, the revenue maximizing price level is not picked as the stop-out price, something which holds true in each and every auction in our sample. So the Treasury does not follow the strategy studied by Back and Zender (2001) and McAdams (1999). Third, *marginal revenue* is maximized at an internal price level (neither the highest nor the lowest price level). The marginal revenue at level l is defined as the difference in total revenue that could be generated at level l and level $l-1$. The maximum marginal revenue occurs at the highest price level in 14 auctions and at the lowest price level in 4 auctions, but is otherwise, in 188 auctions, located somewhere in the middle. Fourth, the chosen stop-out price coincides with the price at which marginal revenue is maximized.

To examine the generality of the fourth point, we have computed the normalized total and marginal revenues for each price level within each auction and then compared these

[18]In the remaining six auctions, the maximum would be attained at the second lowest bid (five cases) or the third lowest bid (one case). These six auctions have in common that the marginal demands at the lowest price are relatively small. Specifically, they are (in millions of markka) $1, 1, 5, 10, 10, 15$, and 60.

with the Treasury's choice of stop-out price. The results are in Table 8. In the table, p_0^* denotes the price level with the highest marginal revenue, p_{-1}^* denotes the price level immediately above, p_1^* denotes the price level immediately below, etc. The second column in the table shows the normalized marginal revenue as an average across all auctions in our sample, for five different price levels centered around p_0^*. We see that this average is 36% at the maximal marginal revenue price level, p_0^*. Given that the average number of price levels across auctions is 9.4, this illustrates that a typical auction has a price level where marginal revenue is considerably higher than at other price levels, like price level 4 in Figure 9. One can think of the demand function as exhibiting a kink, or a precipitous drop, at this price level. Alternatively, one can think of the inverse demand function as having a large flat at this price level.[19] The third column contains the key piece of information; namely how frequently the five price levels are chosen as the stop-out price. In particular, we see that p_0^* is chosen in 43.8% of the 203 completed auctions (recall that 3 auctions in our sample were cancelled). This illustrates the generality of the finding in Figure 9 that the Treasury tends to pick the stop-out price to coincide with the price level where marginal revenue is at its largest.

Price Level	Average normalized marginal revenue	Frequency stop-out	Frequency among rationed auctions	Ave marg rev if rationed
p_{-2}^*	.085	.049	.000	n.a.
p_{-1}^*	.108	.034	.190	.202
p_0^*	.360	.438	.571	.457
p_1^*	.132	.197	.048	.212
p_2^*	.094	.133	.000	n.a.
N	206	203	21	21

Table 8: **Marginal Revenue and Stop-Out Price:** For each auction, we identify the price level with the largest marginal revenue, p_0^*. The table reports the following statistics across all auctions for this price level and the two immediately above and below: Average normalized marginal revenue, frequency chosen as stop-out price, percentage of rationed auctions with the indicated price level as the stop-out, and the average normalized marginal revenue across the rationed auctions. There are 206 auctions in total, 203 auctions where the Treasury sold some bonds, and 21 auctions where the Treasury rationed bids placed at the stop-out price.

Another interesting feature of the Treasury's behavior is that it rationed marginal demand at the stop-out price in 21 auctions. The fourth column in Table 8 answers the question as to how many of these rationed auctions coincide with a stop-out price around p_0^*. We see that p_0^* is the stop-out price in 57.1%, or 12, of these auctions. The fifth column tabulates the average normalized marginal revenue at the five price levels for the rationed auctions. Comparing these numbers with those in the second column supports the view that rationing tends to happen when marginal demand at the stop-out price is

[19]As one might expect, p_0^* tends to be located reasonably close to the quantity weighted average price of the aggregate demand function. On average, p_0^* exceeds the auction mean by .032% of face value.

high. For p_0^*, marginal revenue increases from 36.0% in the sample as a whole (second column) to 45.7% in the sample of rationed auctions (fifth column). This increase is economically large, but not statistically significant due to the small number of observations. Similarly, at the adjacent price levels average marginal revenue approximately doubles from an unconditional average of around 10% to 20% when there is rationing.

The choice of the stop-out price as the price at which marginal revenue is maximized makes intuitive sense when one considers that the Treasury holds a sequence of auctions. However, what may be surprising is that the Treasury is able to raise the money it needs (to fund the budget deficit) without going below the maximum marginal revenue point more frequently. This could be a result of the Treasury exercising outside options to borrow elsewhere instead of borrowing expensively in the auction. But it could also be that the Treasury's marginal revenue maximization policy induces bidders to be more competitive than suggested by the market power theories. If bidders know that the seller will set the stop-out price where marginal revenue is at the highest, then a single bidder would have an incentive to concentrate demand on that price. However, if all bidders concentrate their demand on the same "consensus" price, rationing will occur. In this case, to avoid rationing, a bidder might find it preferable to concentrate his demand one tick above the others' "consensus" price. As a result, price competition would ensue and market power would break down. In our sample, the average quantity awarded to bids at the stop-out price is 495 million markka when the stop-out price is the marginal revenue maximizing price. In contrast, in the market power equilibria we tested above, the marginal quantity awarded to bids at the stop-out price has zero measure.[20] An important part of this argument is that the Treasury can credibly commit to the marginal revenue maximizing strategy. It may well be that the fact that the auctions in our sample essentially constitute a repeated game between the Treasury and the primary dealers plays an important role in communicating this policy and making it credible.

6.2 Underpricing and Auction Size

Within any given auction, the Finnish Treasury faces a price-quantity tradeoff. However, in this subsection we show that there is no evidence of such a tradeoff across auctions. There is no relation between underpricing and realized auction size. When demand is strong, the Treasury sells more securities, and when demand is weak, it holds back supply. In this subsection, to control for duration (and therefore indirectly for volatility) effects, we work with yields rather than prices. But we have also carried out the analysis below using prices – and reach the same conclusions.

Within each auction, bids are sorted by yield levels which are ordered from the lowest to the highest yield. For each level i, we compute the difference between the bid yield and the secondary market yield, ΔY_i. At the stop-out yield, this "markup" represents underpricing measured in yield space. For each yield level i, we also compute the aggregate quantity

[20] In the discrete version of these equilibria, the marginal quantity equals one quantity multiple, [Goswami, Noe, and Rebellow (1996), Nyborg (2002)]. The discrete theory is not fully developed when there is supply uncertainty.

	a	b	c	d	R^2	N
All yields	0.0300	0.0178	-0.0035	-0.0007	0.200	1,388
	$(15.7)^a$	$(9.6)^a$	$(-3.1)^a$	$(-3.7)^a$		
Stop-out only	0.0064	-0.0005	0.0024	-0.0001	0.014	175
	$(2.5)^a$	(-0.1)	(0.8)	(-0.1)		

Table 9: **Treasury Policy.** Regression of yield spread between each auction bid and the secondary market yield on the deviation between quantity bid at each level and the quantity expected for the auction. Estimation with ordinary least squares. t-statistics are below in parentheses with super-index a denoting significance level 5% or better.

bid up to this yield, Q_i. This is then standardized by the expected auction size:

$$X_i = \frac{Q_i - \bar{Q}}{\bar{Q}}.$$

For each auction, the locus of points $(\Delta Y_i, Q_i)$ essentially sketches out the aggregate (standardized) demand function.

We pool the data across all auctions and estimate a regression function as:

$$\Delta Y_i = a + bX_i + cX_i^2 + dX_i^3 + e.$$

This provides a characterization of the average aggregate demand schedule. A cubic functional form has been chosen because visual inspection shows that the aggregate demand curve within individual auctions tends to be S-shaped. The independent variable is highly skewed, so we adopt the transformation

$$X_i = log\left(1 + \frac{Q_i - \bar{Q}}{\bar{Q}}\right).$$

The regression coefficients evaluated at the stop-out yield characterize the seller's policy. The *tradeoff* policy says that $b > 0$. The *strong no-tradeoff* hypothesis says that $b = c = d = 0$.

The regression results are reported in Table 9. In the regression using all yields, the coefficient b is positive, which means that the aggregate inverse demand schedule (with yields on the y-axis) is upward sloping. In other words, within an auction, the Treasury faces a tradeoff between yield and quantity. The estimated values for c and d, both significantly negative, tell us that this tradeoff is nonlinear. This contrasts with the regression using only the observations at the stop-out yield. Here, only the constant is significantly different from zero.[21] This shows that while the auctions are underpriced on

[21] Lack of cross-section variation in the independent variable could explain why the regression coefficients in the smaller stop-out sample are insignificantly different from zero. However, the standard deviation of X_i is .749 in the stop-out sample compared with 1.505 in the full sample. Hence, there is substantial variation in the quantity at the stop-out yield, so lack of power does not explain the insignificant coefficients in the stop-out sample.

the average, across auctions the Treasury is not trading off underpricing, here measured in yield, and quantity. In other words, the outcome of the repeated game played between the Treasury and bidders is to keep the yield markup (underpricing) unaffected by quantity sold. There may be several reasons for this. First, bidders tend to respond to larger expected auction sizes by increasing quantity demanded without lowering discounts. This helps the Treasury to sell larger quantities without lowering prices. Second, since the Treasury tends to pick the price where the marginal demand is the largest as the stop-out price, it has scope for varying the quantity in an individual auction without changing the price. A further implication of these points is that when the expected auction size changes, the price level at which bidders place the largest marginal demand and where marginal revenue is at its largest, remains the same.

7 Conclusions

This paper has analyzed strategic behavior in uniform price treasury auctions with a small number of bidders. We have found that the bidders as well as the seller act strategically. Bidders increase quantity demanded and submit a few very high bids when competition increases, and the seller systematically rejects bids which would have raised revenue. Instead of choosing a stop-out price and quantity to maximize revenue, the seller systematically chooses as a stop-out price the price at which marginal revenue is maximized. We have also derived and tested robust implications from three non-informational models which emphasize bidders' monopsonistic market power and, to a smaller extent, risk sharing. The fact that individual bidders' demand increases when there are more bidders can be consistent with monopsonistic market power. However, the finding that discounts and underpricing are unaffected by the number of bidders (which is exogenous) is not. Moreover, the specific equilibria of Back and Zender (1993), Kyle (1989), and Wang and Zender (2002) cannot explain the observed non-linearities in bidders' demand schedules. Finally, risk sharing does not seem to influence bidder behavior. As auction size increases, bidders willingly purchase larger quantities without lowering the prices at which they bid.

There are several possible reasons why the extant models of monopsonistic market power are rejected by the data. We want to point out two. First, the models examined in this paper model the Treasury auction as a one-shot game while the Treasury auctions in our data are repeated. It seems implausible that the Treasury would be willing to tolerate very low prices in the auction without either disciplining primary dealers or taking its business elsewhere. Such implicit threats could serve to weaken primary dealers willingness and ability to coordinate on an underpricing equilibrium. However, repetition could also work in the opposite direction and enhance bidders' market power by facilitating coordination among the bidders, as emphasized in the experimental study by Goswami, Noe, and Rebello (1996) who find that subjects play Back-Zender type equilibria when they are allowed to communicate before the auction, but not otherwise. Weighing these views against each other, our evidence suggests that the Treasury's power to discipline dealers dominates the effect of dealers' enhanced ability to coordinate. Studying multiunit auctions as repeated games between the seller and the buyers seems to be an important direction for future research.

Second, we have documented that although the Treasury has no explicit policy with respect to its choice of stop-out price, it appears to have a policy which can best be described as a marginal revenue maximization policy. It may well be that this policy creates incentives for bidders to concentrate their demand around a "consensus" price. In turn, this may create competition for marginal units and thereby help break the noncompetitive market power equilibria, along similar lines as in Nyborg's (2002) analysis of a discretized uniform price auction. The importance of the repeated games idea in this context is that it may serve as a mechanism to give credibility to the Treasury's policy. This possible interaction between the auctions as repeated games and the effect of discreteness in bidders' strategy space could be interesting to explore in future research.

With respect to bidder behavior, our strongest empirical findings relate to the importance of volatility. Specifically, when volatility increases, bidders increase bid shading, reduce quantity demanded, and increase the dispersion of their bids. This is the same reaction as in Sweden's *discriminatory price* Treasury auctions (Nyborg, Rydqvist, and Sundaresan, 2002). This is noteworthy because monopsonistic market power should not be a concern in these auctions (Back and Zender, 1993) and there is little evidence that risk aversion is a significant driver of bidder behavior in Sweden either. Our findings on volatility appears to be consistent with the view that bidders have private information and are concerned with the winner's curse. However, this leaves us with a puzzle as to why discounts do not increase with the number of bidders. A possible explanation is that the winner's curse effect is offset by a market power effect.

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8 Appendix 1: Equilibria in Kyle's (1989) Model

This appendix shows the derivation of the equilibrium demand schedules in Kyle (1989) when bidders do not have private information. The approach follows Nyborg (2002). Suppose that supply Q is known. When all other bidders use $q(p)$, the "final" bidder's optimization problem can be written (because of CARA utility and normality):

$$\max_p (v-p)(Q-(N-1)q(p)) - \frac{1}{2}\sigma^2\rho(Q-(N-1)q(p))^2,$$

where $(Q-(N-1)q(p))$ is the residual supply. The first order condition is:

$$-(Q-(N-1)q(p)) - (N-1)(\bar{v}-p)q'(p) + \sigma^2\rho(Q-(N-1)q(p))(N-1)q'(p) = 0.$$

Using symmetry and market clearing, $Nq(p) = Q$, the first order condition is:

$$-q(p) - (N-1)(\bar{v}-p)q'(p) + \sigma^2\rho q(p)(N-1)q'(p) = 0. \tag{16}$$

This is an ordinary differential equation which is independent of Q. Therefore, the solution to the differential equation will work for any Q. There are many possible solutions. To get Kyle's solution, posit a linear equilibrium: $q(p) = \gamma - \gamma p$. Plug $q'(p) = -\gamma$ into (16). We get

$$q(p) = \frac{(N-1)(\bar{v}-p)\gamma}{\sigma^2\rho(N-1)\gamma+1}.$$

This implies that

$$\frac{(N-1)p\gamma}{\sigma^2\rho(N-1)\gamma+1} = \gamma.$$

Solving this for γ, we get

$$\gamma = \frac{N-2}{(N-1)\sigma^2\rho}.$$

Thus we get Kyle's solution (4).

The general solution to (16) is not known. However, we can obtain the general solution in inverse form by writing (16) in inverse form as follows:

$$p'(q)q - (N-1)[\bar{v}-p(q)] + (N-1)\sigma^2\rho q = 0. \tag{17}$$

The general solution to (17) is Wang and Zender's (2002) equilibrium (10), where $a > 0$. Note that the general solution is a polynomial function of order $N-1$ and therefore for $N > 5$, we are unable to find a general closed form solution for $q(p)$. (As is well known, Abel's classical theorem shows that there is no general formula for the root of a polynomial of degree 5 or higher).

9 Appendix 2: Summary Statistics Under Wang and Zender's (2002) Equilibrium

In this appendix, we report the summary statistics in Wang and Zender's (2002) equilibrium, (10) for $a \leq \bar{v}/\rho\sigma^2$. For simplicity, we report the statistics under the assumption that $\bar{v} = 1$, which is just a normalization (and will be reflected in σ). For reasons of tractability, it also assumed that $r = 0$.

$$discount = \frac{2 + a\,(-1+N)\,\rho\,\sigma^2}{2\,N}. \tag{18}$$

$$st.dev = \frac{\sqrt{(-1+N)^2\left(3\,(-2+a\,\rho\,\sigma^2)^2 + N\,(12 + a^2\,(-3+2\,N)\,\rho^2\,\sigma^4)\right)}}{2\,\sqrt{3}\,N\,\sqrt{-1+N+2\,N^2}}. \tag{19}$$

$$skewness = \frac{AB}{C}, \tag{20}$$

where

$$A = 12\,\sqrt{3}\,(-1+N)^3\,\sqrt{-1+N+2\,N^2}\left(2 - N + a\,(-1+N)\,\rho\,\sigma^2\right),$$

$$B = \left(4\,(1+N)\,(2+N) + a\,(-4+N)\,(2+N)\,\rho\,\sigma^2 + a^2\,(2+(-2+N)\,N)\,\rho^2\,\sigma^4\right),$$

$$C = (2+N)\,(-2+3\,N)\left((-1+N)^2\left(3\left(-2+a\,\rho\,\sigma^2\right)^2 + N\left(12 + a^2\,(-3+2\,N)\,\rho^2\,\sigma^4\right)\right)\right)^{\frac{3}{2}}.$$

$$kurtosis = \frac{D\,(E - F + G - H + I)}{JK}, \tag{21}$$

where

$$D = 9\,(1+N)\,(-1+2\,N),$$

$$E = 240\,(1+N)\,(2+N)\,(3+N)\,(1+2\,N)\,(-1+3\,N)\,(6+N\,(-5+2\,N)),$$

$$F = 480\,a\,(2+N)\,(3+N)\,(1+2\,N)\,(-6+N\,(23+2\,N\,(-7+2\,(-1+N)\,N)))\,\rho\,\sigma^2,$$

$$G = 40\,a^2\,(3+N)\,(-108+N\,(252+N\,(489+N\,(-862+3\,N\,(49+6\,N\,(5+4\,N))))))\,\rho^2\,\sigma^4,$$

$$H = 40\,a^3\,(-108+N\,(324+N\,(327+N\,(-1194+N\,(737+2\,N\,(-43+24\,N))))))\,\rho^3\,\sigma^6,$$

$$I = a^4\,\left(-540+N\,\left(2160+N\,\left(-75+N\,\left(-7335+N\,\left(10271+N\,\left(-5797+4\,N\,\left(368-39\,N+36\,N^2\right)\right)\right)\right)\right)\right)\right)\,\rho^4\,\sigma^8,$$

$$J = 5\,(2+N)\,(3+N)\,(1+2\,N)\,(-2+3\,N)\,(-1+3\,N)\,(-3+4\,N),$$

$$K = \left(3\left(-2+a\,\rho\,\sigma^2\right)^2 + N\left(12 + a^2\,(-3+2\,N)\,\rho^2\,\sigma^4\right)\right)^2.$$

$$underpricing = \frac{(-1+N)\,Q\,\rho\,\sigma^2}{(-2+N)\,N} - a^{1-N}\,\frac{Q}{N}^{-1+N}\left(-1 + \frac{a\,(-1+N)\,\rho\,\sigma^2}{-2+N}\right). \tag{22}$$

Note that this expression for underpricing holds true irrespective of r. Finally, note that when $a \leq \frac{v}{\rho\sigma^2}$, quantity demanded at a price of 0 is a. The modified Herfindahl index is 1.

Comparative statics

We have calculated comparative statics of the summary statistics above employing a combination of analytical and numerical methods, using $N \in [5, 10]$ (as in our data). No specific assumptions on ρ or σ^2 have been made. All comparative statics results are partial and, as seen in the formulas above, depend on a. We have not computed comparative statics with respect to Q, since it seems unreasonable that a would be unaffected by auction size.

In summary, the Wang and Zender (2002) model does very well with respect to volatility. Bidders are predicted to respond to an increase in volatility by increasing discounts, standard deviation,[22] and skewness, and by decreasing quantity demanded. The model fails only on the the relationship between skewness and volatility. However, like Back and Zender, it predicts that bidders respond to an increase in the number of bidders by decreasing discounts, standard deviation, and skewness, and by increasing bid quantity. Hence, it fails with respect to the number of bidders (except for quantity demanded), which is the variable at the heart of the imperfect competition story.

[22]Note the following qualifications: Under (10), standard deviation increases with σ^2 except when $a\rho\sigma^2$ is "small" relative to $\bar{v}$; decreases with N except when $N = 5$ and $a\rho\sigma^2$ are "close" to $\bar{v}$; and it increases with a except when $a\rho\sigma^2$ is "small" relative to $\bar{v}$.

10 Appendix 3: Volatility Estimation

We estimate conditional volatility as an ARCH(2) process of bond returns, which have been calculated from end-of-day bid quotes. The cross-section and time-series data are stacked. The level of the coefficients are about half of those from the Swedish data.

Let P_t be the bond price at time t and A is the one-day accrued interest for a coupon bond. We assume that bond returns follow a random walk with constant drift a:

$$\frac{P_t - P_{t-1} + A}{P_{t-1}} = a + e_t. \tag{23}$$

The cross-section and time-series data are pooled. The volatility of the error term is as

$$e_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \alpha_2 e_{t-2}^2 + \phi_1 DUR_t + \nu. \tag{24}$$

The estimated coefficients are:

α_0	α_1	α_2	ϕ_1
-0.0017	0.2959	0.2784	0.0179
(0.0013)	(0.0187)	(0.0182)	(0.0005)

When a new security is auctioned, there are no bond prices from the secondary market before the auction. In those cases, we use the prices of the traded T-bond with duration that most closely mimics the duration of the new T-bond. When a new T-bond is auctioned, we use the average winning auction yield to compute duration.