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Pascal Courty

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Pascal Courty, London Business School (LBS) and CEPR

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Centre for Economic Policy Research
90–98 Goswell Rd, London EC1V 7RR, UK
Tel: (44 20) 7878 2900, Fax: (44 20) 7878 2999
Email: cepr@cepr.org, Website: www.cepr.org

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ABSTRACT

Ticket Pricing Under Demand Uncertainty*

This Paper studies a monopolist selling tickets to consumers who learn new information about their demands over time. The monopolist can sell early to uninformed consumers, and/or close to the event date to informed ones, it can ration tickets and allow ticket holders to resell. I show that rationing and intertemporal sales are never optimal. More surprisingly, the monopolist cannot do strictly better by allowing resale despite the fact that consumers are willing to pay more when they can resell tickets. I discuss the implications of the model for the pricing practices observed in ticket markets.

JEL Classification: D42 and L82
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Pascal Courty
London Business School
Sussex Place
Regent's Park
LONDON
NW1 4SA
Tel: (44 20) 7262 5050 x3317
Fax: (44 20) 7402 0718
Email: pcourty@lbs.ac.uk

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1 Introduction

Incorporating demand uncertainty into monopoly price discrimination has helped to explain some pricing outcomes that were otherwise difficult to understand. One area where these contributions have increased our understanding is ticket markets. Demand uncertainty plays an important role in ticket markets because consumers typically do not know with certainty how much they value a ticket until just before they are about to use it. Their valuations may, for example, be contingent on their health or on some other variable influencing their enjoyment.

Consequently, ticket markets offer a rare opportunity to explore the role of demand uncertainty in the design of pricing policies. In that light, two contributions are particularly relevant. Lewis and Sappington (1994) have shown that a monopolist can strategically choose to withhold information from consumers. DeGraba (1995) has shown that a monopolist may deliberately choose to ration tickets to force consumers to make uninformed purchases in what he calls a 'buying frenzy'.

Although the existing literature sheds light on some of the questions that are specific to ticket pricing, it leaves many others unanswered. Consider for example the analysis of Lewis and Sappington. When translated to the context of ticket markets, their main result says that a monopolist should sell either long in advance, when consumers have uncertain demand, or close to the event date, when consumers are more sure about their demand, but not at some intermediate date. In ticket markets, however, the monopolist may want to sell tickets at different prices on different dates, a possibility that is not considered in their work. Similarly, DeGraba's finding about rationing rests on the assumption that the monopolist cannot commit to its future pricing decisions. It is not clear, however, whether rationing would still occur under commitment.

Despite the fact that these questions are relevant, perhaps the most important issue in ticket markets that begs for an explanation is that of resale. The evidence on resale shows that there is no prevailing industry practice. Some ticket agencies permit licensed brokers to resell tickets above face value. Other agencies try to control resale markets by lobbying law-makers to prohibit resale.[1] Why do some ticketing agencies support resale while others discourage it?

The primary goal of this work is to present a general framework that allows me to answer these questions. I present a model very similar to DeGraba and Lewis and Sappington. As in these two works, I focus on individual demand uncertainty and I assume that consumers' valuations are independent and identically distributed. As in Lewis and Sappington, the monopolist can commit to its pricing strategy. In contrast to these two works, I consider a monopolist who can sell both early to uninformed consumers and late to informed ones. Perhaps the most important new feature of my model is that the monopolist can strategically allow ticket holders to resell.

The main findings are as follows. First, I show that rationing is never optimal when the monopolist can credibly commit not to sell in the late market. Second, and somewhat more surprisingly, I show that it is never optimal to sell both far from and close to the event date. When consumers are identical it is never profitable to offer consumers the option to choose when they want to buy. Finally, and most importantly, I show that the monopolist will do as well by allowing resale and selling early than by selling late only. An implication of this last result is that the monopolist cannot do strictly better by allowing resale despite the fact that consumers are willing to pay more when they can resell tickets after they learn new information about their valuation.

This work contributes to the small but growing literature on ticket pricing. See Courty (2000) for an exhaustive review. Most notably, Rosen and Rosenfield (1997) extend second and third-degree price discrimination to questions specific to ticket markets, but consumers have deterministic preferences in their model. Dana (1999) and Courty and Li (2000) investigate ticket pricing under demand uncertainty but focus on different issues than the ones studied here. DeGraba and Mohammed (1999) and Courty (2001) extend DeGraba's work on buying frenzies and present some applications to ticket markets.

The paper is organized as follows. The next section introduces the model. Sections 3 and 4 present the main results on rationing, on the timing of ticket sales, and on

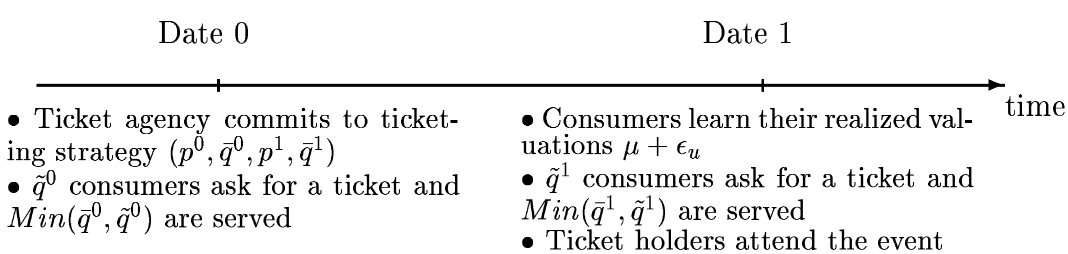
[1] For example, *Californians Against Ticket Scalping* is an association supported by event promoters who lobby lawmakers to prohibit resale (Philips, 1990).

the control of resale rights. Section 5 discusses the generality of the results. Section 6 summarizes the results and comments on the generality of the model.

2 The Model

There are two dates, the early date (date 0) and the late date (date 1). Figure 1 presents the timing of events. There is a unit continuum of consumers with unit demands. Consumer $u \in [0,1]$ values the event $\mu + \tilde{\epsilon}_u$ where μ is a constant and $\tilde{\epsilon}_u$ is a zero-mean random variable with cumulative distribution $F(.)$ and density $f(.)$. Consumer $u \in [0,1]$ privately observes the realization of $\tilde{\epsilon}_u$ at date 1. To focus on individual valuation uncertainty, I assume that the random variables $\tilde{\epsilon}_u$ are identically and independently distributed across consumers. This structure of preference follows when the population of consumers is homogenous (e.g. a large rock concert offering only general admission tickets).[2] The parameter μ will be interpreted as the popularity of the event.

Figure 1: Timing of Events



On the supply side, a monopoly ticket agency can sell early and/or late. I assume that the monopolist can commit to its actions in the late market. This assumption is reasonable, for example, when the event promoter is known and has an established

[2] A different branch of the price discrimination literature assumes that consumers differ ex-ante in the sense that they face different random demands (e.g. leisure and business demands). In these models the monopolist price discriminates in the early market by offering a menu of contracts. Clay, Sibley and Srinagesh (1992) and Miravete (1996) study optimal calling plans, Dana (1999) advance purchase discounts and Courty and Li (2000) refund contracts. The current paper focuses on a different set of issues and thereby deliberately ignores intertemporal contracts.

reputation. The importance of this assumption will become clear after Lemma 1. The cost of adding an additional seat is constant and equal to c. The monopolist announces early its pricing strategy, which consists in an early and a late price and a number of tickets available at these prices $(p, \bar{q}) = (p^0, \bar{q}^0, p^1, \bar{q}^1)$. For simplicity, the ticket agency and the consumers are risk-neutral and do not discount.

This section describes the firm-pricing problem under the assumption that consumers cannot resell. Section 3 solves that problem. Section 4 considers firm pricing problem where consumers can resell.

Given the monopolist's announcement of pricing strategy, consumers can choose to ask for a ticket early or wait. Let $\tilde{q}^0$ represent the mass of consumers who ask for a ticket early. If $\tilde{q}^0 > \bar{q}^0$ then consumers are rationed randomly. The actual number of tickets sold in the early market is $Min(\bar{q}^0, \tilde{q}^0)$. I assume without loss of generality that the firm does not carry over tickets from the early to the late market in the event there are unsold tickets in the early market.[3] Those consumers who decided to wait as well as those consumers who are rationed out in the early market can choose to buy late. Again, if $\tilde{q}^1$ is the mass of consumers who ask for a ticket late and $\tilde{q}^1 > \bar{q}^1$, then consumers are rationed randomly. The monopolist's profits are $p^0 Min(\bar{q}^0, \tilde{q}^0) + p^1 Min(\bar{q}^1, \tilde{q}^1) - c(\bar{q}^0 + \bar{q}^1)$.

Given announcement $(p, \bar{q})$, a coordination game between consumers follows. Consumers play a coordination game because the rationing probability is endogenously determined. I analyzed that game using backward induction and look for rational expectation equilibrium. Consider what happens at date 1. Given that $Min(\bar{q}^0, \tilde{q}^0)$ consumers bought in the early market, there are $1 - Min(\bar{q}^0, \tilde{q}^0)$ potential buyers in the late market. Out of these potential buyers, $(1 - Min(\bar{q}^0, \tilde{q}^0))(1 - F(p^1 - \mu))$ are willing to pay more than p^1 for a ticket. Therefore the late demand is

$$\tilde{q}^1 = (1 - Min(\bar{q}^0, \tilde{q}^0))(1 - F(p^1 - \mu)). \tag{1}$$

If $\tilde{q}^1 \leq \bar{q}^1$ then all consumers are served. Otherwise, consumers are rationed with probability $\frac{\bar{q}^1}{\tilde{q}^1}$.

[3] Carryover is not an issue when the monopolist can commit. This is not true, however, when the monopolist cannot commit. See discussion following Lemma 1.

In the early market, consumers have the option to buy at price p^0. Their utility conditional on getting a ticket is

$$U^0 = \mu - p^0. \tag{2}$$

Alternatively, they can wait for the late market and buy if their valuation is greater than the late price p^1. The expected utility from waiting is

$$U^1 = Min(1, \frac{\bar{q}^1}{\tilde{q}^1}) \int_{p^1-\mu}^{\infty} (\mu + x - p^1) f(x) dx. \tag{3}$$

Consumers ask for a ticket early if $U^0 > U^1$, wait if $U^0 < U^1$, and are indifferent if $U^0 = U^1$. The date 0 demand is

$$\tilde{q}^0 = \left| \begin{array}{l} 1 \; if \; U^0 > U^1 \\ 0 \; if \; U^0 < U^1 \\ x \in [0,1] \; if \; U^0 = U^1. \end{array} \right. \tag{4}$$

A rational expectation equilibrium conditional on firm strategy $(p, \bar{q})$ is a quadruplet $(\tilde{q}^i(p, \bar{q}), U^i(p, \bar{q}))_{i=0,1}$ that satisfies equations (1), (2), (3), and (4). A rational expectation equilibrium always exits.[4] To simplify the presentation, I denote the actual number of tickets sold in equilibrium $q^i = Min(\bar{q}^i, \tilde{q}^i)$. For a given firm announcement $(p, \bar{q})$, I do not distinguish the rational expectation equilibria that have identical equilibrium outcome q^i but different date 0 equilibrium demands $\tilde{q}^0$. The equilibrium prices, tickets sold, and profits are the same in these equilibria and the monopolist is indifferent among them.

The firm maximizes profits over all possible strategies $(p, \bar{q})$ subject to the constraint that consumer demands are part of a rational expectation equilibrium $((\tilde{q}^i)_{i=0,1}$ satisfy $(1-4))$. The maximization problem is not well-defined yet because I have not shown that there exists at most one rational expectation equilibrium for each firm strategies $(p, \bar{q})$. It will become clear after lemma 1 that the maximization problem is well defined.

The pricing problem differs from Lewis and Sappington (1994) in that I consider intertemporal sales while they assume that the monopolist can sell only at one point in time. The problem also differs from DeGraba (1995) because I assume intertemporal

[4] To show existence, define $\tilde{q}^1(\tilde{q}^0)$ from (1) and $U^1(\tilde{q}^1)$ from (3), then plug $U^1(\tilde{q}^1(\tilde{q}^0))$ in 4 and call the resulting equation $4'$. It is enough to show that there exists a $\tilde{q}^0$ that solves $4'$. If $U^1(\tilde{q}^1(\tilde{q}^0)) - U^0 > 0$ (< 0) for all $\tilde{q}^0 \geq 0$ then $\tilde{q}^0 = 0$ $(\tilde{q}^0 = 1)$. Otherwise, there exists a $\tilde{q}^0$ that solves $4'$ because $U^1(\tilde{q}^1(.))$ is a continuous function of $\tilde{q}^0$.

commitment while his monopolist cannot commit about its actions in the late market. To simplify the presentation, I deliberately keep the model as simple as possible. Section 5 discusses the role of the main assumptions on consumer preferences and on the rationing rule.

3 Ticketing Strategies

The first step is to show that the firm maximization problem is well defined. The maximization problem is not well defined if for a given firm strategy there is more than one rational expectation equilibrium. Multiple equilibria could emerge because of the implicit circularity in the equilibrium definition. It could happen, as in DeGraba for example, that for a given firm announcement, there exist one equilibrium where all consumers wait because they are better off waiting given that all consumers wait and another equilibrium where all consumers ask for a ticket early because they are better off doing so given that all consumers do so.

To deal with this multiple equilibrium issue, I define the concept of sellout rational expectation equilibrium. A quadruplet $(\tilde{q}^i(p,\bar{q}), U^i(p,\bar{q}))_{i=0,1}$ is a sellout rational expectation equilibrium if it satisfies $(1-4)$ and $\tilde{q}^i \geq \bar{q}^i$ for $i = 0, 1$. Lemma 1 shows that if a sellout equilibrium exists it is unique. This implies that the firm maximization problem is well defined after adding the constraints $\tilde{q}^i \geq \bar{q}^i$ for $i = 0, 1$. Adding these constraints, however, is not restrictive since it can be easily shown that the firm would never choose a strategy which is not followed by a sellout rational expectation equilibrium.[5]

Lemma 1 *For any firm strategy $(p, \bar{q})$, if a sellout rational expectation equilibrium exists then there is no other rational expectation equilibrium associated with $(p, \bar{q})$.*

[5]The proof follows by contradiction. Consider firm strategy $(p, \bar{q})$ and an associated rational expectation equilibrium such that the firm does not sellout. Lemma 1 implies that the firm does not sellout in any of the rational expectation equilibrium associated with $(p, \bar{q})$. Select out of all these equilibria the rational expectation equilibrium with highest profits and consider a new strategy in which the firm produces just enough to sell out in both market. There is a unique rational expectation equilibrium associated with this strategy and the firm's profits in this equilibrium dominate the firm's profits in any of the rational expectation equilibrium associated with $(p, \bar{q})$. This implies that the firm will never choose $(p, \bar{q})$, which is a contradiction.

Proof The proof follows by contradiction. First, consider the case where $\bar{q}^0 = 0$ or $\bar{q}^1 = 0$. In that case, there exists a unique rational expectation equilibrium. Next, consider the case $\bar{q} > 0$. Equation (4) implies that $U^0 \geq U^1$. Consider a firm strategy $(p, \bar{q})$ and a rational expectation equilibrium $(\tilde{q}^i(p, \bar{q}), U^i(p, \bar{q}))_{i=0,1}$ defined by $(1-4)$ and such that $\tilde{q}^i \geq \bar{q}^i$ for $i = 0, 1$. Assume that there is another rational expectation equilibrium that shares the same firm strategy but such that the equilibrium demand $\tilde{q}^{i'}$ and equilibrium sales $q^{i'}$ are different. Because the firm sells out in both markets in the original equilibrium, it has to be that $q^{i'} < q^i$ in at least one market. First, consider the case where $q^{0'} < q^0$. The utility from buying a ticket early is $U^0 = \mu - p^0$ in both equilibria. The utility from buying late is lower in the second equilibrium ($U^{1'} < U^1$) because $\tilde{q}^{1'} > \tilde{q}^1$ and consumers are more likely to be rationed. This implies that $U^{1'} < U^1 \leq U^0 = U^{0'}$. Consumers are strictly better off buying early in the second equilibrium. So $\tilde{q}^{0'} = 1$, which is a contradiction. Finally, consider the case where $q^{0'} = q^0$ and $q^{1'} < q^1$. $q^{0'} = q^0$ implies $\tilde{q}^{1'} = \tilde{q}^1$ and therefore $q^{1'} = q^1$, which is a contradiction. □

Adding the sellout constraints actually simplifies the pricing problem. Since $\bar{q}^i = q^i$, I will use the notation q^i to mean both the number of tickets offered for sale and the number of tickets actually sold. I denote (SO_i) the sell-out constraint in market i. Using equilibrium conditions $(1-4)$, the sellout constraints simplify to

$$\mu - p^0 \geq Min\left(1, \frac{q^1}{(1-q^0)(1-F(p^1-\mu))}\right)\int_{p^1-\mu}^{\infty}(\mu + x - p^1)f(x)dx \quad if \quad q^0 > 0 \qquad (SO_0)$$

$$q^1 \leq (1-q^0)\left(1 - F(p^1-\mu)\right) \qquad (SO_1)$$

The ticket pricing problem boils down to choosing strategy (p, q) to maximize $p^0q^0 + p^1q^1 - c(q^0 + q^1)$ subject to $(SO_i)_{i=0,1}$. Note that the sellout constraints do not have to bind. When inequality (SO_i) is strict, consumers will be rationed in market i.

Lemma 1 implies that the monopoly profits are unique. This finding should be contrasted with DeGraba's analysis. In DeGraba's model, the firm's pricing problem does not boil down to a maximization problem. The firm and consumers play a game, which can have multiple equilibria with different firm strategies and different level of profits. The difference between my model and DeGraba's is that the monopolist in DeGraba's model cannot commit; it sells all unsold tickets in the late market at market clearing price. Consumers know that if they strategically wait, the ticket agency will have to discount tickets in the late market. As a result, there can be multiple equilibria of the kind described in the introductory paragraph. In contrast, consumers cannot gain by waiting in my model. When consumers wait, they increase the demand in the date 1 market but the supply of tickets is constant.

The pricing problem dramatically simplifies when the ticket agency sells only early ($q^1 = 0$) or only late ($q^0 = 0$). After simplifications, one can show that in the former case the ticket agency maximizes profits by selling $q^0 = 1$ tickets early at $p^0 = \mu$ if $\mu > c$ and $q^0 = 0$ otherwise. In the latter case, the ticket agency chooses q^1 to maximize $q^1(\mu + F^{-1}(1 - q^1) - c)$. I will return to these benchmark cases after I have explored the general case of mixed sales.

I will solve the pricing problem in three steps. First, I show that the ticket agency never rations. Second, I show that the ticket agency never sells both early and late. Finally, I will revisit the two benchmark cases presented above since they are the only candidates left for the optimal pricing strategy. The next proposition shows that there is no rationing in equilibrium where rationing in market i is defined as a situation where more consumers strictly prefer to buy a ticket in market i than there are tickets available in that market.

Proposition 1 *It is never optimal to ration tickets.*

Proof The ticket agency never rations if it sells only early ($q^1 = 0$) or only late ($q^0 = 0$). The next step is to show that the ticket agency never rations when it sells both early and late. Consider rationing in the early market. Assume that it is optimal to ration in the early market ((SO_0) does not bind). If $p^0 - c < 0$, then a small decrease in q^0 increases profits and satisfies both sellout constraints; a contradiction. If $p^0 - c \geq 0$, a small increase in the early price increases profits and satisfies both sellout constraints; a contradiction again. Next, consider rationing in the late market. From constraint (SO_0) plug $p^0 = \mu - \frac{q^1}{(1-q^0)(1-F(p^1-\mu)} \int_{p^1-\mu}^{\infty} (\mu + x - p^1) f(x) dx$ in the objective function assuming that tickets are rationed in the late market ($min(1, \frac{q^1}{\tilde{q}^1}) = \frac{q^1}{\tilde{q}^1}$). Note that q^1 enters linearly both in the objective function and in (SO_1). Therefore, either (SO_1) binds or $q^1 = 0$ or $q^1 = 1$. In any of these three cases, there is no rationing in the late market. To conclude, both (SO_0) and (SO_1) bind. □

The result that rationing is never optimal in the early market should be contrasted with DeGraba's model of buying frenzy. In DeGraba's model, the monopolist cannot commit and there are multiple equilibria. The monopolist uses rationing in the early market to get consumers to buy early. In my model, the monopolist can commit. There is a single equilibrium and there is no need to ration tickets in the early market.

Although a monopolist who can commit does not need to ration tickets in the early market, commitment alone does not rule out the possibility that the monopolist may want to ration tickets in the late market. The monopolist may consider rationing tickets in the late market because doing so increases the early price as it decreases the consumers' waiting option in the early market. Interestingly, this rationale alone is not sufficient to warrant the use of rationing. Although rationing does increase the early price this increase is never sufficient to compensate for the opportunity revenue loss in the late market. The finding that the monopolist never rations either in the early or the late market demonstrates that lack of commitment is really what drives rationing in DeGraba's model.

Proposition 1 dramatically simplifies the pricing problem. The sell-out conditions can be rewritten as $p^0 = \mu - \int_{p^1-\mu}^{\infty} (\mu + x - p^1) f(x) dx$ and $p^1 = \mu + F^{-1}(1 - \frac{q^1}{1-q^0})$. The important thing to note is that the early and late prices depend on the early and late ticket supplies only through the fraction of late supply to late consumers $\frac{q^1}{1-q^0}$. The monopolist chooses q^0 and q^1 to maximize

$$q^0 \left(p^0 \left(\frac{q^1}{1-q^0} \right) - c \right) + q^1 \left(p^1 \left(\frac{q^1}{1-q^0} \right) - c \right),$$

where $p^0(.)$ and $p^1(.)$ are defined by (SO_0) and (SO_1). I show next that selling both early and late is always dominated by selling early only.

Proposition 2 *It is never optimal to sell both early and late.*

Proof Consider the change of variable $x = \frac{q^1}{1-q^0}$ and $q^1 = x(1-q^0)$. The profits are $q^0(p^0(x)-c) + (1-q^0)x(p^1(x)-c)$. The proof follows by contradiction. Assume that it is optimal to sell both early and late. Then the first-order condition with respect to q^0 holds:

$$p^0(x) - c = x(p^1(x) - c).$$

Plugging the first-order condition into the profits gives

$$q^0(p^0(x) - c) + (1 - q^0)(p^0(x) - c) = p^0(x) - c < \mu - c,$$

where the last inequality holds for any interior value. This implies that the profits earned at any interior value are lower than the profits from selling early, which is a contradiction. □

The intuition for this result is best understood if one starts by considering the general case where consumers are not assumed to be identical in the early market. In that case, there are two effects to switching from a regime where the ticket agency sells only on one date (early or late) to a regime where the agency sells both early and late. The first effect is a discrimination or sorting effect. Selling both early and late allows the agency to discriminate against those consumers who learn more information over time. These consumers will typically delay their purchasing decisions and pay a premium to purchase in the late market. The second effect is an arbitrage effect. Selling both early and late allows consumers buy in the market where they get the greatest surplus.

The decision to sell on both dates depends on the trade-off between these two effects. Little can be said about this trade-off for general populations of consumers, so it is difficult to predict in general whether the monopolist will sell on both dates. When consumers are identical, however, it is possible to conclude that selling both early and late is never optimal. In fact, there is no discrimination effect when the population of consumers is homogenous ex-ante. The arbitrage effect, however, is negative because $(S0_0)$ binds.

An implication of Proposition 2 is that the practice of advance purchase discounts is never profitable when consumers are identical in the early market. This is consistent with the observation that the literature on advance purchase discounts typically assumes some kind of consumer heterogeneity in the early market (e.g. Dana 1999 and Gales and Holmes 1993). Interestingly, ticketing agencies in the sports and entertainment industries rarely use advance purchase discounts. A candidate explanation suggested by the model is that these ticketing agencies face more homogeneous populations of consumers than do airlines or hotels, for example. According to that interpretation, the distinction between 'leisure traveller' and 'business traveller' would play a less important role in the sport and entertainment industries than in the airline and hotel industries.

Proposition 2 complements the analysis of Lewis and Sappington (1994). In the context of my model, their main result says that selling at an intermediate date (between early and late) is likely to be dominated by selling early or selling late.[6] Lewis and Sappington, however, do not consider the possibility of selling both early and late. Put together, my result and theirs suggest that the ticket agency should sell only once, either to fully uninformed consumers or to fully informed consumers.[7]

Early versus Late Sales

The analysis so far has shown that the optimal ticketing strategy entails the choice of

[6]Lewis and Sappington consider an information decision in the sense that they assume that the level of information varies along a continuum: Consumers can receive no information, full information, or partial information. They show that either full information or no information will typically dominate partial information. (The second part of their work extends these findings to multi-unit demands but this extension is not directly related to my work.) Although my analysis is a timing decision, there is a perfect mapping between the two models. Their information structure can be reinterpreted in my model by assuming that the monopolist can sell tickets at date $t \in [0,1]$ and F_t represents the distribution of consumer valuation at date t. Lewis and Sappington show that it may be optimal, although only in special cases, to provide partial information. Translated into the context of my model, this says that it may be optimal to sell tickets at, for example, date $t = 1/2$. To illustrate this point, consider a simple case with three dates $t = 0, 1/2, 1$. Assume $c = \mu = 0$. Consumers learn at $t = 1/2$ a random shock $\epsilon_{1/2}$ equal to $+/-\ 1/2$ with equal probability. They learn at date 1 a new shock ϵ_1 uniformly distributed over $[-1/2, 1/2]$. Given this information structure, consumers value a ticket 0 at date 0, $\epsilon_{1/2}$ at date 1/2, and $\epsilon_{1/2} + \epsilon_1$ at date 1. $\epsilon_{1/2} + \epsilon_1$ is uniformly distributed over $[-1, 1]$. Given this dynamic of consumer valuations, it is strictly optimal to sell at $t = 1/2$ to half of the consumers at price $p = 1/2$. Although intermediate sales are optimal in that specific example, Lewis and Sappington show that this will typically not be the case, at least under fairly general assumptions. In the context of my model, a sufficient condition for the conclusion that it is never optimal to sell at $0 < t < 1$ is that there exists ϵ^* such that $(\epsilon - \epsilon^*)(F_t(\epsilon) - F_{t'}(\epsilon)) \leq 0$ for $t > t'$ and for all ϵ in the support of F_t. (Using the inverse demand interpretation introduced in the next two paragraphs, this condition implies that the upper envelope of all the inverse demands (indexed by $t \in [0,1]$) coincides with the upper envelope of the early and late inverse demands.)

[7]One could consider the possibility to sell at more than two dates. The reasoning in Proposition 2 can be applied iteratively to show that this is never optimal.

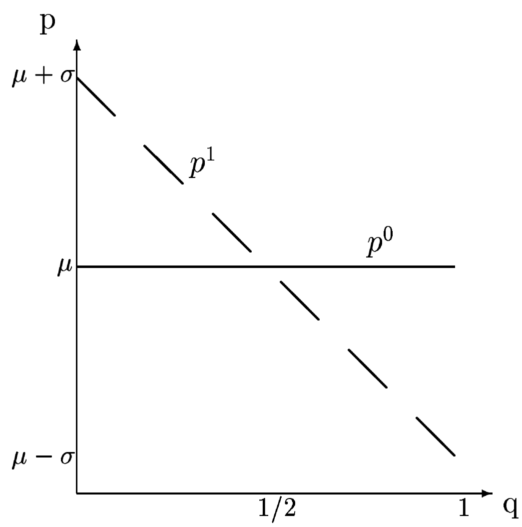
a selling date $t \in \{0, 1\}$, a price p and a quantity q. The optimal profits from selling early are $\Pi^0 = Max(\mu - c, 0)$. Let $q^{*1} = \text{Argmax}_q \; q(\mu + F^{-1}(1-q) - c)$ be the optimal ticket supply when the monopolist sells late and p^{*1} be the corresponding price. The optimal profits under late sales are $\Pi^1 = q^{*1}(p^{*1} - c)$. The analysis narrows down the agency's possible ticketing strategies to two generic ones. The ticket agency sells early when $\Pi^0 > \Pi^1$, late if the reverse inequality holds and is indifferent between selling early and late otherwise.

The trade-off between early and late sales is best understood by studying the shape of the early and late demands. The early inverse demand, p^0, equals the consumer's expected valuation independent of the ticket supply $p^0(q) = \mu$ if $q \in (0, 1)$, while the late demand is $p^1(q) = \mu + F^{-1}(1-q)$. Figure 2 graphs these two demands when the valuation shock is uniformly distributed over $[-\sigma, \sigma]$, so that $p^1(q) = \mu + \sigma(1-2q)$. This figure reveals some important insights. First, the early and late inverse demands are identical if consumers do not learn new information (in the uniform example, $\sigma = 0$) so that expected and realized valuations are the same. In general, however, the consumers' expected valuations are not always equal to their realized valuations; thus these two demands will differ and I will use the terminology *demand dynamic* to capture the idea that the demand for tickets changes over time.

Second, the late inverse demand lies above (below) the early inverse demand for small (large) ticket supplies. Actually, this property holds for any consumer preferences. To see that the late inverse demand exceeds the early inverse demand for small ticket supplies, notice that the early price for small ticket supplies is the highest expected valuation in the population while the late price is the average of the highest realized valuation. The claim follows from the observation that the average of the highest realized valuation is always greater than the highest expected valuation. For similar reasons, the early inverse demand exceeds the late inverse demand for large ticket supplies. Third and closely related to this last point, the late demand is steeper than the early one. The intuition for this result on steepness is that the late demand aggregates realized valuations which are more dispersed than expected valuations.

These simple points on the demand dynamic suggest some important implications on

Figure 2: Demand Dynamic



the ticket agency's choice of pricing policies. To investigate these implications, I start by assuming that the ticket agency cannot choose the ticket supply. I consider that case first to establish some basic points and once this is understood I will turn to more general comparative statics where the ticket agency also chooses the ticket supply.

The analysis of the demand dynamic demonstrates that early sales and large ticket supplies (and low prices) on the one hand and late sales and small ticket supplies (and high prices) on the other hand are complementary pricing instruments. In other words, delaying sales is profitable only when the ticket agency targets the happy few consumers who turn out to highly value the event. Selling early, on the other hand, is optimal when the ticket agency serves the entire market. Another way to look at the choice of selling date is as follow. For a given ticket supply, the ticket agency chooses the selling date that maximizes total surplus minus consumer surplus. Selling early tends to imply ex-post over-consumption (consumption inefficiency), while selling late tends to imply under-supply because of a steeper demand curve (production inefficiency). The ticket agency trades off these two effects. Selling early is likely to dominate selling late when the monopolist serves a large fraction of the market because the gains from re-allocating tickets to the most eager consumers are low.

It may appear surprising that selling early can dominate selling late despite the fact that selling late allocates tickets to those consumers who value them the most. It is obvious that selling late would be optimal if the monopolist could perfectly price discriminate in the late market, thus eliminating under-production. Under monopoly pricing, however, there is a dead-weight-loss and this explains why selling late can be sub-optimal. Although the finding that it may be optimal to strategically restrict consumers' information set may seem counter-intuitive, it is not new. Lewis and Sappington (1994) present this finding and offer the following intuition. They suggest considering the case where $c = 0$. In that case, the efficient outcome is to produce $q = 1$ and this is exactly what selling early accomplishes.[8]

From an empirical point of view, the above result is limited because it does not say when the ticket agency prefers selling late to selling early or vice-versa. A deeper understanding of how the trade-off between early and late sales depends on exogenous parameters defining the market environment will help to derive testable predictions about the timing of ticket sales. The relative advantage of selling late is

$$\Delta\Pi = \Pi^1 - \Pi^0 = \left(\mu - c + F^{-1}(1 - q^1)\right) q^1 - (\mu - c).$$

The choice of the timing of sales depends on the behavior of the function $\Delta\Pi$.

Proposition 3 *The ticket agency is more likely to sell early when the event is more popular (μ increases) and/or when the cost of capacity is lower (c decreases).*

The proof follows directly from the observation that $\frac{\partial \Delta\Pi}{\partial(\mu - c)} < 0$. Consider the case of an increase in event popularity. Under early sales, the early profits increase one-for-one with μ while under late sales the late profits increase by a lower amount than the increase in μ because the monopolist cannot capture all consumer surplus. An increase in μ increases the relative advantage of selling early. For events popular enough (or with low enough cost of capacity), the ticket agency supplies the whole market and sells early. This suggests the prediction that, everything else equal, tickets for more popular events are more likely

[8] I would like to thank an anonymous referee for pointing out this earlier finding by Lewis and Sappington.

to be sold longer in advance. In addition, more popular events will take place in larger venues not only because they are more popular but also because sales for these events are more likely to occur earlier.

Finally, consider the case of valuation uncertainty. To capture valuation uncertainty, I use Rothschild and Stiglitz's (1971) concept of mean preserving spread centered around zero. Distribution $\tilde{F}$ is more uncertain than distribution F in that sense if both distributions have zero mean and $\tilde{F}(\epsilon) > F(\epsilon)$ for $\epsilon < 0$ and, $\tilde{F}(\epsilon) < F(\epsilon)$ for $\epsilon > 0$.

Proposition 4 *The ticket agency is more likely to sell late when valuation uncertainty increases by a mean-preserving spread centered around zero.*

Proof I show that $\Pi^1 > \Pi^0$ implies $\tilde{\Pi}^1 > \tilde{\Pi}^0$. $\tilde{\Pi}^1 = (\mu + \tilde{F}^{-1}(1 - q^{\tilde{1}*}) - c)q^{\tilde{1}*} > (\mu + \tilde{F}^{-1}(1 - q^{1*}) - c)q^{1*} > \Pi^1 > \Pi^0 = \tilde{\Pi}^0$, where the first inequality holds because $q^{\tilde{1}*}$ is the optimal quantity under $\tilde{F}$ and the second inequality holds by the definition of the mean-preserving spread transformation. □

This result identifies a class of change in valuation uncertainty that unambiguously increases the comparative advantage of selling late. As valuation uncertainty increases, the early demand—and the early profits—remain unchanged. The late demand, on the other hand, becomes steeper because realized valuations become more dispersed and the late profits increase because the ticket agency increases ticket price to cash in on the happy few consumers who draw high valuations.

Propositions 3 and 4 show that the ticket agency will reschedule the selling date for large enough changes in the market environment. Figure 2 suggests another prediction about the qualitative change in price and ticket supply that should go along with a change in selling date. The upper envelope of the early and late inverse demands is locally convex where they intercept. It is never optimal for a monopolist to choose a ticket supply on a convex portion of an inverse demand.

Proposition 5 *Small changes in the market parameters (μ, c, or F) that cause the ticket agency to change the selling date will cause discontinuous changes in price and capacity.*

Proof The ticket agency is indifferent between selling early and late when $\Delta\Pi = 0$, that is, when $\left(\mu - c + F^{-1}(1 - q^{*1})\right) q^{1*} = \mu - c > 0$. When this is the case a small change in μ, c, or F will break the tie. Then the monopolist will strictly prefer one selling date to the other. When the ticket agency is indifferent between selling early and late, the optimal price is higher, and the optimal quantity lower, when it sells late than when it does so early ($q^{*0} > q^{*1}$ and $p^{*0} < p^{*1}$). But the optimal price and quantity under early sale (q^{*0}, p^{*0}) do not change with a change in the market parameters and the optimal price and quantity under late sale (q^{*1}, p^{*1}) change continuously with a change in market parameters. It follows that a small change in market parameters around the tie-break point will cause discontinuous changes in price and capacity. □

Consider for example a market environment where the ticket agency is indifferent between selling early and selling late. The convexity of the upper envelope of the early and late demands says that the optimal early ticket supply and price are quite distinct from the late ones. Thus, small changes in production cost, event popularity, or valuation uncertainty that trigger a change in the optimal timing of sales will trigger large changes in the ticket supply and price.[9] One may argue that this result is artificially created by the assumption that the selling date cannot vary continuously. This, however, is not the case since even if the ticket agency could vary the selling date continuously, it would typically prefer to sell at the starting or ending dates of the learning period (see Lewis and Sappington (1994) and Footnote 6). In the optimal ticketing strategy, the date of sales is itself a discontinuous variable.

4 Resale Right Allocation

In solving the ticket pricing problem, I restricted myself to the case where consumers could not resell tickets. In this section, I consider the same pricing game as the one outlined in Section 2 but I assume that resale is allowed in the late market. I show that allowing resale cannot increase profits.

[9] This prediction is exactly the opposite to the one found in non-collusive oligopoly theory (Sweezy, 1939). In these models, the demand is concave at its kink, implying rigidity in supply due to a gap in marginal revenue function.

Two resale situations have to be distinguished. In both cases, those consumers who buy early can resell in the late market. What distinguishes these two situations is whether those consumers who buy tickets from the monopolist at date 1 can or cannot resell. I will call the latter case resale/no-resale and the former case resale/resale. Although the resale/no-resale case may seem unrealistic, it is in fact not completely so. One way to implement that scheme, for example, would be to sell tickets to brokers early on and allow them to resell, but discourage resale by late buyers by, for example, limiting the number of tickets per buyer and controlling large purchases in the late market.

As before, I start by characterizing the rational expectation equilibria of the coordination game that take place after the monopolist has announced its pricing strategy. Using backward induction, consider what happens at date 1 after $Min(\bar{q}^0, \tilde{q}^0)$ consumers bought in the early market. I assume that consumers cannot collude in the late resale market. I denote $p^{1,r,n}$ ($p^{1,r,r}$) the competitive price in the resale market in the resale/no-resale (resale/resale) case. In the late market, the ticket agency offers $\bar{q}^1$ tickets at price p^1. Two situations may occur.

- If $1 - F(p^1 - \mu) \geq Min(\bar{q}^0, \tilde{q}^0) + \bar{q}^1$ then at least as many consumers want to attend the event at price p^1 as there are tickets available. The number of tickets sold in the late market is

$$q^1 = \bar{q}^1. \tag{5}$$

  Next, I solve for the price in the date 1 resale market. Consider first the resale/no-resale case. Consumers who value a ticket more than p^1 ask for a ticket first from the monopolist.[10] Consumers are rationed with probability $\frac{\bar{q}^1}{1-F(p^1-\mu)}$. Those consumers who are rationed out may buy from resellers. There are $1 - F(p^1 - \mu) - \bar{q}^1$ consumers left who value a ticket more than p^1 and these consumers value a ticket more than $p^{1,r,n}$ with probability $\frac{1-F(p^{1,r,n}-\mu)}{1-F(p^1-\mu)}$. Market clearing in the resale market requires that

$$(1 - F(p^1 - \mu) - \bar{q}^1)\frac{1 - F(p^{1,r,n} - \mu)}{1 - F(p^1 - \mu)} = Min(\bar{q}^0, \tilde{q}^0). \tag{6}$$

[10] Those consumers who hold a ticket from the early market ask for a ticket in the late market because they are allowed to resell the ticket they bought in the early market.

Consider next the resale/resale case. Consumers know that they can make a sure profit by buying at date 1 from the ticket agency. All consumers demand a ticket so they are rationed with probability $\bar{q}^1$.[11] The supply in the resale market is $Min(\bar{q}^0, \tilde{q}^0) + \bar{q}^1$ while the demand at price $p^{1,r,r}$ is $1 - F(p^{1,r,r} - \mu)$. Market clearing in the resale market requires that

$$1 - F(p^{1,r,r} - \mu) = Min(\bar{q}^0, \tilde{q}^0) + \bar{q}^1. \tag{7}$$

- If $1 - F(p^1 - \mu) \leq Min(\bar{q}^0, \tilde{q}^0) + \bar{q}^1$ then there are at least as many tickets as there are consumers who are willing to pay at least p^1. Ticket resellers can slightly undercut the monopolist. The price in the resale market is

$$p^{1,r,s} = Min(p^1, \mu + F^{-1}(1 - Min(\bar{q}^0, \tilde{q}^0))) \tag{8}$$

and the ticket agency sells q^1 tickets.

$$q^1 = Max(0, 1 - F(p^{1,r,s} - \mu) - Min(\bar{q}^0, \tilde{q}^0)). \tag{9}$$

A date 1 equilibrium given firm strategy $(p, \bar{q})$ and early demand $\tilde{q}^0$ is a pair $(q^1, p^{1,r,s})$ for $s = n, r$ which solves (5) and (6) if $s = n$ and $1 - F(p^1 - \mu) \geq Min(\bar{q}^0, \tilde{q}^0) + \bar{q}^1$; (5) and (7) if $s = r$ and $1 - F(p^1 - \mu) \geq Min(\bar{q}^0, \tilde{q}^0) + \bar{q}^1$; and (8) and (9) and if $1 - F(p^1 - \mu) \leq Min(\bar{q}^0, \tilde{q}^0) + \bar{q}^1$. Now that it is clear what the equilibrium is in the late market, we can move backward to the early market. In the early market, consumers have to make the decision to buy early at price p^0 or wait. Because resale is allowed, the decision to buy early is not a consumption decision as in the previous section. This decision is an investment decision. Given $p^{1,r,s}$, the date 0 demand is

$$\tilde{q}^0 = \left| \begin{array}{l} 1 \; if \; p^0 < p^{1,r,s} \\ 0 \; if \; p^0 > p^{1,r,s} \\ x \in [0,1] \; if \; p^0 = p^{1,r,s}. \end{array} \right. \tag{10}$$

A rational expectation equilibrium given firm strategy $(p, \bar{q})$ is a triplet $(\tilde{q}^0, q^1, p^{1,r,s})$ for $s = n, r$ such that $(q^1, p^{1,r,s})$ is an equilibrium given $((p, \bar{q}), \tilde{q}^0)$ and (10) holds. The firm

[11] Consumers could demand more than one ticket. This would not change the analysis as long as the resale market is competitive.

maximizes profits over all possible strategies $(p, \bar{q})$ subject to the constraint that consumer demands are part of a rational expectation equilibrium. The analysis of the firm pricing problem follows the analysis presented in section 3.

Proposition 6 *Under resale/no-resale and resale/resale, the firm profits are equal to the late profits Π^1. The firm produces q^{*1} tickets that are sold across the early and late market and the equilibrium prices are $p^0 = p^1 = p^{1,r,r} = p^{1,r,n} = p^{1*}$.*

Proof The proof follows the steps of Lemma 1 and Proposition 1. As in section 3, it is easy to show by contradiction that adding the condition that the firm sells out does not cost any loss of generality. Similarly, one can prove that if a sellout rational expectation equilibrium exists, then it is unique. The proof follows the proof in Lemma 1 and relies on the fact that $p^{1,r,s}$ in non-increasing in $\tilde{q}^0$. The firm maximization problem with the sellout constraints is well defined.

The next step is to show that $p^0 = p^{1,r,s}$ for $s = r, n$ in equilibrium. Assume that $p^0 > p^{1,r,s}$. Then consumers prefer to wait for the late resale market. Tickets do not sell out in the early market, which is a contradiction. Assume that $p^0 < p^{1,r,s}$. Then consumers can make a sure profit by buying early. The monopolist could raise the early price and consumers would still buy; again a contradiction.

The proof that there is no rationing in equilibrium follows the proof of Proposition 1. The fact that there is no rationing in date 1 market implies that $1 - F(p^1 - \mu) = \bar{q}^0 + \bar{q}^1$. Taken together with 6 and 7 this implies $p^0 = p^1 = p^{1,r,s} = \mu + F^{-1}(1 - \bar{q}^0 - \bar{q}^1)$. Define $q = \bar{q}^0 + \bar{q}^1$ and rewrite the pricing problem as maximizing $q(\mu + F^{-1}(1 - q) - c)$ subject to the constraints that $0 \leq q \leq 1$. This pricing problem is identical to selling tickets only in the late market. The optimal quantity is q^{*1} and the optimal price is p^{1*}. □

It does not matter whether capacity q^{*1} is sold early or late. Under perfect resale, consumers' early decisions to buy a ticket do not depend on their distributions of valuations. Consumers view the good as an asset whose value equals its expected resale payoff. All consumers make the same decision: if q tickets are sold across markets they are willing to pay $p^1(q)$ for a ticket.[12] Allowing resale does not increase profits because the monopolist cannot charge those consumers who do not buy early for the positive expected surplus they get by waiting for the resale market. These consumers get a free ride, and because consumers are indifferent between waiting and buying early, the early buyers get one, too.

One implication of the proposition is that allowing resale cannot increase profits above what the ticket agency can achieve with the best of early sales without resale and late sales. More generally, one can show that the ticket agency never wants to randomly choose whether consumers can resell. To see that, assume that the ticket agency sells early and allows resale with probability η. How much are consumers willing to pay when they can resell with probability η? If a consumer buys early, that consumer will not be able to resell with probability $1-\eta$. The consumer will consume and get expected utility μ. When the consumer can resell, the consumer values the ticket $p^1(q)$ and this outcome occurs with probability η. Consumers are willing to pay $(1-\eta)\mu+\eta p^1(q)$ in the early market. When the early price is set to that amount,

$$p^r(q,\eta)=(1-\eta)\mu+\eta p^1(q),$$

consumers are indifferent between buying early and waiting. This is therefore the equilibrium price and $p^r(q,\eta)$ is always lower than $Max(\mu,p^1(q))$. This shows that it is never optimal to artificially create some uncertainty about resale opportunities. Similarly, the ticket agency will never sell some tickets that can be resold and others that can't. Doing so is equivalent to selling both early and late, which again is sub-optimal.

The optimal ticketing strategy can now be restated taking into account resale as an additional pricing instrument. The ticket agency chooses among three ticketing strategies:

(1) Sell $q^0=1$ at $p^0=\mu$ early and prohibit resale.

(2a) Sell q^{*1} at p^{*1} late.

(2b) Offer q^{*1} tickets at price p^{*1}, allow resale, and let consumers choose when they want to buy.

The ticket agency is indifferent between strategies (2a) and (2b). Propositions 3 and

[12] Using the same reasoning, one can show that over-booking and under-booking have no effects on a firm's expected profit when resale is allowed, since the ticket agency can undo these decisions in the late competitive market. For a different analysis of overbooking, see Vickrey (1972).

4 say that strategy (1) is more likely to be optimal when the event is more popular, when the unit cost of production is lower, or when there is less valuation uncertainty. A new implication is that ticket agencies are more likely to have a negative attitude toward resale in these situations.

Although the model does not predict which ticketing strategies the monopolist will be inclined to select when (2a) and (2b) are optimal, there are at least two arguments (not formally modeled here) that push the balance in favor of allowing resale (2b). First, selling early does not prevent the monopolist from selling leftover tickets in the late market. Second, it may be more difficult for the ticket agency to reach those consumers with the highest valuations at the consumption date. The ticket agency may prefer to use professional brokers because they can more easily adjust the late price in a timely fashion to clear the late market. Decentralizing the late allocation of tickets may be a more efficient way to target the consumers with the highest valuations.

An additional prediction of the analysis is that when the ticket agency only imperfectly controls the resale parameter, it may still be optimal to sell early. Assume, for example, that consumers can resell with (exogeneously given) probability η. The price $p^r(q;\eta)$ computed above corresponds to the equilibrium price in the early market given that the monopolist sells q tickets. At that price, consumers are indifferent between buying early and waiting for the opportunity to buy in the resale market, and any price above that price is not an equilibrium because fewer than q consumers are willing to buy. The monopolist faces an implicit downward-sloping inverse demand curve in the early market even though consumers are identical. For low values of η, the optimal profits under early sales, which are equal to $Max_{q^0}\ q^0(p^r(q^0;\eta)-c)$, may still exceed the optimal profits under late sales. When this is the case, the optimal ticket supply under early sales might be lower than the unit mass of consumers ($q^0 < 1$). This does not imply, however, that there will be rationing in the early market. In equilibrium, there is no rationing in the early market because consumers are indifferent between buying early and waiting.

To summarize, the attitude of the ticket agency toward resale depends on whether it is optimal to sell late to informed consumers or early to uninformed ones. What is clear is that the ticket agency will typically prefer corner solutions where resale is either fully prohibited or completely legal. Evidence from the United States on the attitude of ticket agencies toward resale markets seems to be consistent with the model's prediction that ticket agencies will sometimes favor resale and other times discourage it. On the one hand, there is clear evidence that some sports and entertainment ticket agencies have tried to restrict resale. Promoters have tried to limit brokers' attempts to buy tickets at the issuing date and to resell these tickets at the consumption date by: (1) Limiting the number of tickets per buyer, (2) requiring that buyers pay with a credit card and checking that credit card number to control large purchases, and (3) lobbying state legislators to pass laws regulating resale prices and/or prohibiting resale. These practices are difficult to explain under standard price discrimination. Although ticket agencies sometimes price discriminate on the basis of quality (theaters 'scale the house' by varying prices inversely proportionally to their distance from the stage), this type of price discrimination does not require resale prohibition (Rosen and Rosenfield, 1997). Some ticket agencies also use third-degree price discrimination (e.g. targeted coupons to local market). This practice, however, is rare and can hardly explain the more widespread conflicts over resale.

On the other hand, there is some evidence that brokers are sometimes tolerated and even encouraged. Happel and Jennings (1995) survey anti-resale regulations in the United States both at the state and at the municipal level (local ordinances).[13] They find that recent resale laws acknowledge the role of elected brokers acting as agents on behalf of ticket agencies. These brokers can sell tickets above face value without restrictions on where they can operate. Overall, the authors conclude that this "second generation of scalping laws has brought the resale market closer to a purely competitive situation." The important point is that recent anti-resale regulations tend to support *discretionary* use of resale. This is consistent with the prediction of the model.

It is also interesting to look at the restrictions that are covered in some regulations. The restrictions prevent resale on the premises and prevent resale for profit, but allow resale at printed face value for sell-out and allow resale to close circles of friends and business acquaintances. This is consistent with the model because these resale activities

[13] See also Happel and Jennings (1989). For a rare study of the impact of resale laws on ticket prices see Williams (1994). His results, however, do not confirm or reject the theory presented here.

do increase the consumer's willingness to pay without directly competing inter-temporally with the ticket agency as brokers would.

5 Discussion

The model presented in Section 2 assumes random rationing and focuses on a restrictive class of consumer preferences where consumers have identically and independently distributed valuations.[14] This section explores the role of these assumptions and discusses how the analysis may change under different assumptions.

Rationing Rule

Throughout the analysis, I maintain the assumption that consumers are rationed randomly. Would the analysis still follow under more general rationing rules? To answer this question, I will distinguish the analysis without resale presented in Section 3 and the analysis of resale presented in Section 4.

I start by showing that the results presented in Section 3 follow under any rationing rule. I will show that Proposition 2, stating that it is never optimal to sell both early and late, holds under any rationing rule. This will imply that the firm maximization problem is well defined (Lemma 1) and that the firm will never ration (Proposition 1). The rest of the analysis presented in Section 3 follows from these results.[15]

To show that all results presented in Section 3 follow under any rationing rule, I will show that it cannot be optimal to sell both early and late (Proposition 2). The proof follows by contradiction. Consider a rational expectation equilibrium where the monopolist sells $q^0 > 0$ tickets early at price p^0 and $q^1 > 0$ tickets late at price p^1. The

[14]Another important assumption is that consumers are risk neutral. Considering risk aversion introduces some complex insurance issues that are not directly related to the main theme of this paper. Incidentally, these issues may not play an important role in ticket markets since the amounts at stake are very small.

[15]The proof presented below suggests that I could have skipped Lemma 1 and Proposition 1 in Section 3 and moved directly to the more general proof of Proposition 2 presented here. In fact, one can derive Proposition 2 without explicitly deriving the equilibrium price functions $p^1(.)$ and $p^2(.)$. This approach, however, leaves asides some important issues that emerge only when one solves the pricing problem step by step. In addition, the proof presented below does not hold for the analysis of resale presented in Section 4 (Proposition 6) so Lemma 1 and Proposition 1 had to be covered anyway. For clarity, I chose to present these results early on.

monopoly revenues are $q^0(p^0 - c) + q^1(p^1 - c)$. The sell-out condition in the late market requires that $q^1 \leq (1 - q^0)(1 - F(p^1 - \mu))$ and the early price in the early market is such that $\mu - p^0 \geq U^1$, where the expected utility from waiting, $U^1 > 0$, depends on the rationing rule and on what consumers do in equilibrium. I will show that the monopolist can always do better by selling either early or late. To do so, I distinguish two cases:

- Consider first the case $p^0 - c \geq \frac{q^1}{1-q^0}(p^1 - c)$. Then, the monopolist can do better by selling early only at price μ. Consumers buy because they have no choice. The early profits are such that $\mu - c > p^0 - c \geq \frac{q^1}{1-q^0}(p^1 - c)$. Profits increase since

$$\mu - c = q^0(\mu - c) + (1 - q^0)(\mu - c) > q^0(p^0 - c) + q^1(p^1 - c).$$

- Consider next the case $p^0 - c < \frac{q^1}{1-q^0}(p^1 - c)$. The monopolist can do better by selling late only to $\frac{q^1}{1-q^0} \leq 1$ consumers at price p^1. The monopolist sells out since the demand for tickets $1 - F(p^1 - \mu)$ is greater than $\frac{q^1}{1-q^0}$. The profits increase since

$$\frac{q^1}{1 - q^0}(p^1 - c) = q^0 \frac{q^1}{1 - q^0}(p^1 - c) + q^1(p^1 - c) > q^0(p^0 - c) + q^1(p^1 - c).$$

Therefore, it is never optimal to sell both early and late. This shows that the monopolist will sell either early or late. But the monopolist never rations when it sells only once. This concludes the proof that the analysis in Section 3 can be extended to any rationing rule.

Next, consider whether the analysis presented in Section 4 changes under more general rationing rules. To start, note that the rationing rule does not play any role when consumers can resell both the tickets they buy early and those they buy late (the resale/resale case). This is because the rationing rule in that analysis only determines how the rents from resale are allocated across consumers. The rationing rule does not determine the equilibrium price or the total number of tickets sold so the monopoly profits do not depend on the rationing rule. The resale/resale analysis follows under any rationing rule.

The case where consumers can resell only the tickets they buy in the early market is more interesting. Here, the rationing rule does play a role. Proposition 6 does not hold for any rationing rule in the resale/no-resale case. To show that, I present a simple example where the rationing rule may actually help the monopolist to increase profits. This example is insightful because it shows that the monopolist can leverage the rationing rule to segment the market more finely by price discriminating in the late market.

Consider the rationing rule that allocates tickets to those consumers who value them the least. This correspond to a reverse-parallel rule and would hold, for example, if low valuation consumers had lower cost of time and therefore were more willing to queue for tickets.[16] Consider the simple linear case presented in Section 3 and assume that $\mu = 1/2$, $\sigma = 1/2$, and $c = 1/2$. Then the optimal pricing scheme is to sell $q^0 = 1/6$ at date 0 at price $p^0 = 5/6$ and $q^1 = 1/6$ at date 1 at price $p^1 = 2/3$. In the late market, fraction 1/6 value a ticket more than 5/6 and fraction 1/6 value it between 5/6 and 2/3. There are only 1/6 tickets available in the late market and the rationing rule allocates these tickets to those consumers whose valuation is between 5/6 and 2/3. Those consumers who value a ticket more than 5/6 have to buy at price 5/6 from the early buyers. The profits are 1/12, which is greater than both the early profits (0) and the late profits (1/16).[17]

The monopolist uses the early market to target high-valuation consumers and the late market to target low-valuation consumers. A possible interpretation of this result is that the monopolist sells to brokers early on at a high price and rations tickets in the late market. Given the rationing rule, only low-valuation consumers will get a ticket from the monopolist in the late market. High-valuation consumers, who are excluded by the rationing rule, will have to buy at a higher price from brokers. This simple example shows that the monopolist can use the rationing rule to segment the late markets but this will work only if the monopolist can apply different resale rules to different ticket holders. In fact, Proposition 6 shows that if all ticket holders can resell then the monopolist cannot increase profits even under a reverse-parallel rationing rule.

Consumer Valuations

The model focuses on a specific class of consumer valuations. First, consumers are as-

[16]Consumers are still identical early on. They learn on date 1 their valuations as well as their willingness to wait and the two are inversely related.

[17]For that rationing rule, the analysis of Section 4 fails because pricing equation 6 does not hold. The logic presented in Proposition 1 does not extend to general rationing rule.

sumed to be identical early on. Second, the model considers only individual valuation uncertainty. It ignores aggregate uncertainty (e.g., public information about the event's attractiveness). I show that the analysis changes once one relaxes these assumptions.

To start, consider what happens to the analysis when consumers are heterogeneous in the early market (e.g., business versus leisure travellers as in the prototypical airline-pricing model). Under heterogeneous consumers, the proof of Proposition 2 no longer holds. Under heterogeneous consumer populations it may be optimal to sell both early and late. As a promising line of research, it would be interesting to establish when this proposition does not hold. This offers an alternative approach to Dana's work (1999) that assumes rigidities in the late market to explain some of the ticketing policies observed in the airline industry (as well as other industries) which use much more complex pricing schemes (e.g. yield management) than the ones observed in the event industry.

Another important assumption of the model is that it rules out aggregate uncertainty. The pricing game completely changes if one introduces aggregate shock. Consider for example the case where the aggregate shock does not become public information at date 1. In that case, the concept of late sale is no longer clearly defined. In fact, the profits under late sale depend on the mechanism the monopolist uses to distribute tickets in the late market. For example, the monopolist may be able to maximize profits by running an auction type game or some kind of priority pricing mechanism as in Harris and Raviv (1981).

6 Summary and Conclusions

This paper studies a monopoly ticket agency selling tickets to consumers who learn new information about their valuations over time. I consider a fairly general set of pricing strategies. The monopolist can sell early to uninformed consumers and/or late to informed ones, and it can choose to ration tickets and to strategically allow some ticket holders to resell. Somewhat surprisingly, I show that rationing and intertemporal sales are never optimal.

This narrows the set of candidate ticketing strategies to two. Under the first strategy, the agency sells to informed consumers and targets those consumers who have drawn high valuations. Under that scenario, the ticket agency will typically serve a small fraction of eager consumers. There are two ways to implement this strategy. The ticket agency can either sell close to the event date after consumers have learned their valuations or the ticket agency can sell early and allow resale. These two sub-strategies, or a mix of both, realize the ticket agency's ultimate goal of channeling tickets toward those consumers who value them the most. Somewhat surprisingly, the ticket agency does equally well under each of these scenarios. An implication of this result is that the monopolist cannot do strictly better by allowing resale despite the fact that consumers are willing to pay more when they can resell tickets.

Alternatively, the ticket agency may choose to sell to uninformed consumers in advance. The agency sells more tickets at lower prices. Consumers, however, are willing to buy early only if they expect that they will not be able to buy tickets later. This implies that the ticket agency must prohibit resale to prevent consumers from behaving strategically (i.e. wait for the resale market after they have learned new information about their valuations).

This characterization of generic ticketing strategies shows that the selling date, the ticket supply, the ticket price, and the decision to allow resale are complementary pricing instruments that should be chosen jointly as part of a coherent ticketing strategy. The second contribution of this work is to show that the choice between generic pricing strategies depends in a systematic way on the parameters describing the market environment. Selling early to uninformed consumers is more likely to occur when the event is more popular, when the cost of capacity is low, and when consumers learn less information over time. In these situations, the ticket agency sells to a large fraction of consumers at a relatively low price. This characterization of the ticket agency's choice of pricing strategy is a first step toward deriving empirical predictions.

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