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ABSTRACT

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1 Introduction

In this paper we determine the optimal combination of taxes on money, consumption and income in transactions technology models where exogenous government expenditures must be financed with distortionary taxes. We show that the optimal policy does not tax money, regardless of whether the government can use an income tax, a consumption tax, or the two taxes jointly, as alternative fiscal instruments to the tax on money. This result contradicts the claims raised in the optimal inflation tax literature, that the conditions for the optimality of a zero tax on money depend on the choice of the alternative tax instrument. We argue that the reason for this divergence is an inappropriate specification of the transactions technology adopted in the literature.

The debate on the optimal tax on money dates back to Friedman (1969) and to his policy rule of a zero nominal interest rate. This rule was justified with the first best argument that the price charged for the use of real balances should be set equal to the production cost, which is approximately zero. Phelps (1973) challenged the relevance of Friedman's (1969) result by arguing that, in a second best world where government expenditures must be financed with distortionary taxes, liquidity should be taxed as any other good. Recent literature has revised the argument of Phelps (1973). In particular, Kimbrough (1986) criticizes Phelps's (1973) conclusion on the basis that liquidity should not be modelled as a final good but rather as an intermediate good in the production of transactions. Kimbrough (1986) assumes a transactions technology where transactions time is a function of consumption expenditures gross of taxes and real money holdings. For a technology that is homogeneous of degree one, Kimbrough (1986) shows that the Friedman rule is optimal in a second best environment. This result appears to be consistent with Diamond and Mirrlees (1971) optimal taxation rules of intermediate goods. According to these rules, if the technology is constant returns to scale and if taxes on final consumption goods are available, intermediate goods should not be taxed.

Correia and Teles (1996) generalize the optimality of the Friedman rule to homogeneous transactions technologies of any degree and provide a different explanation for the result. The optimality of the Friedman rule is a general result because money is costless to produce. If money were modelled as a costly intermediate good, the optimal tax on money would be zero only if the transactions technology were homogeneous of degree one. For homogeneous technologies of other degrees, such as the one considered by Baumol (1952) and Tobin (1956), the tax would be different from zero. However, as the cost of producing money is assumed to be arbitrarily close to zero, even if the optimal ad valorem tax or subsidy is bounded away from zero, the optimal price charged for the use of money also approaches zero.

Correia and Teles (1996) solve the optimal inflation tax problem by comparing the inflation tax to an income tax. The policy exercise performed by Kimbrough (1986), instead, was to compare the inflation tax to a consumption tax. This is an important distinction because, while income taxes do not affect monetary transactions directly, consumption taxes need to be paid by the households each time a consumption good is purchased. When consumption taxes are considered, the issue arises of how exactly these taxes affect households' transactions. Kimbrough (1986) proposes a specification for the transactions technology in which money is not unit elastic with respect to the price level gross of consumption taxes. This transactions technology was later adopted by Végh (1989), Dixit (1991), Guidotti and Végh (1993), and Mulligan and Sala-i-Martin (1997). Guidotti and Végh (1993) and Mulligan and Sala-i-Martin (1997) show that, when the alternative to inflation is a consumption tax, the optimality of the Friedman rule does not generalize to homogeneous transactions technologies of any degree. Additional restrictions, that Kimbrough (1986) assumes, are necessary. According to Mulligan and Sala-i-Martin (1997), these restrictions turn the optimality of the Friedman rule into a fragile result.

Under the specification of the transactions technology proposed by Kimbrough (1986), since the optimal allocations may depend on the choice of the alternative tax instrument to the inflation tax, income and consumption taxes are not equivalent fiscal instruments. When two taxes are not equivalent and there is no reason to exclude one of them, they should be jointly considered. We allow for both income and consumption taxes as alternatives to the inflation tax, and find that the Friedman rule is again optimal for all homogenous transactions technologies. However, we obtain the disturbing result that, under certain conditions, the optimal policy mix requires to fully tax income and subsidize consumption. The conditions under which this extreme policy mix is optimal are also conditions under which the Friedman rule is not optimal when only consumption taxes are considered.

These results arise because, under Kimbrough's (1986) specification of the transactions technology, money is not always unit elastic with respect to the price level gross of consumption taxes. When the technology is homogeneous of degree greater than zero, the elasticity is higher than one. We show that, in this case, there may be an incentive for the government to reduce consumption taxes, in order to reduce the volume and costs of households' transactions. If both consumption and income taxes are available, it may be optimal to fully tax income and subsidize consumption. Under this policy, gross consumption expenditures are zero, and the resources used for transactions are minimized. The government performs all transactions on behalf of the households, at zero cost.

We argue that the transactions technology should be restricted to ensure a unitary elasticity. In the absence of consumption taxes, a natural property of the transactions technology is that money be unit elastic with respect to the price of the consumption goods, because only real money matters for the provision of liquidity services. In an environment with consumption taxes, money should be unit elastic with respect to the price level gross of consumption taxes, because both the goods and the consumption taxes on those goods need to be paid with money. If the price level gross of consumption taxes is increased and money is increased in the same proportion, transactions should not be affected.

We propose a specification of the transactions technology such that money is always unit elastic with respect to the price level gross of consumption taxes. Under this technology, the Friedman rule is optimal when the alternative tax instruments are an income tax, a consumption tax or both taxes. Consumption and income taxes are equivalent fiscal instruments.

The paper proceeds as follows: In Section 2, we describe the model under the standard specification of the transactions technology used in the literature, and under a specification where money is unit elastic with respect to the price level gross of consumption taxes. In Section 3, we consider the standard transactions technology and compute the optimal mix of inflation, consumption and income taxes. We show that the Friedman rule is optimal for all homogenous technologies. However, under certain conditions, we find that the optimal policy mix requires full taxation of income and subsidization of consumption. We relate the conditions under which this prescription is obtained to the conditions under which the Friedman rule fails to be optimal, when only consumption taxes can be used as an alternative to inflation. In Section 4, we propose a specification of the transactions technology, where money is always unit elastic with respect to the price level gross of taxes. We show that, under this specification, consumption and income taxes are equivalent fiscal instruments. The Friedman rule is optimal for all homogenous transactions technologies. In Section 5, we interpret the results by relating them to the optimal taxations rules of costly intermediate goods. Section 6 contains the conclusions.

2 The Transactions Technology Model

We consider a monetary model with a transactions technology, as in Kimbrough (1986), Guidotti and Végh (1993), Chari, Christiano and Kehoe

(1996), Correia and Teles (1996, 1999) and Mulligan and Sala-i-Martin (1997).[1] We allow for two different specifications of the transactions technology, the standard formulation as in Kimbrough (1986), and an alternative formulation that exhibits unitary elasticity of money with respect to the price level gross of consumption taxes.

The economy is populated by a large number of identical households and by a government that must finance an exogenous sequence of public expenditures $\{g_t\}$ with inflation, consumption and income taxes.

The households have preferences defined by

$$\sum_{t=0}^{\infty} \beta^t U(c_t, h_t), \tag{1}$$

where c_t and h_t denote the amount of consumption and leisure enjoyed at time t. The instantaneous utility U is assumed to be an increasing and concave function.

Each household is endowed with one unit of time that can be allocated to labor, n_t, leisure, h_t, or transactions, s_t. Labor is used to produce one good that can be used for private or public consumption, $c_t + g_t = y_t$, according to the linear technology, $y_t = n_t$. In each period t, the households choose money holdings, M_t, to be used for transactions in that same period, and nominal bond holdings, B_t. These bonds entitle the households to $(1 + i_t)B_t$ units of money in period $t + 1$. i_t is the net nominal interest rate. We denote the consumption tax and the income tax by θ_t and τ_t respectively. The budget constraints for $t \geq 0$ are given by conditions

$$M_0 + B_0 \leq W_0, \tag{2}$$

$$P_t(1+\theta_t)c_t + M_{t+1} + B_{t+1} \leq P_t\left(1 - \tau_t\right)\left(1 - h_t - s_t\right) + M_t + (1+i_t)B_t, \ t \geq 0, \tag{3}$$

together with a no-Ponzi games condition. P_t is the price of the consumption good in units of money, before taxes. For simplicity, throughout the paper we assume that $W_0 = 0$.

The households need to use transactions time and money in order to acquire the consumption good, according to a transactions technology. We consider two specifications of this technology.

The standard specification of the transactions technology, first proposed by Kimbrough (1986) and then widely used in the literature, is represented

[1] The set up is a slight variation of the model in Kimbrough (1986), Guidotti and Végh (1993) and Correia and Teles (1996) in that the timing of the transactions is different. This distinction, however, is not relevant for the results.

by

$$s_t \geq l\left((1+\theta_t)c_t, \frac{M_t}{P_t}\right), \tag{4}$$

with $\theta_t \geq -1$. Let $m_t \equiv \frac{M_t}{P_t(1+\theta_t)}$. We can then state the following assumptions on the transactions function l, in line with previous literature:

*A*1. The function l is homogeneous of degree $k \geq 0$.

*A*2. The function l is such that $l_c(c_t, m_t) \geq 0$, $l_m(c_t, m_t) \leq 0$, $l_{mm}(c_t, m_t) \geq 0$.

From homogeneity, the transactions technology (4) can also be written as $s_t = (1+\theta_t)^k l(c_t, m_t) = [(1+\theta_t)c_t]^k\, l(1, \frac{m_t}{c_t})$. This allows us to characterize the point of *full liquidity*, $\overline{\frac{m}{c}}$, as the minimum value of the ratio $\frac{m}{c}$ such that $l_{\frac{m}{c}}(1, \overline{\frac{m}{c}}) = 0$. Since $l_m(c,m) = c^{k-1} l_{\frac{m}{c}}(1, \frac{m}{c})$, at full liquidity $l_m(c,m) = 0$. At that point, an additional unit of real balances does not reduce transactions time, for any θ_t, so that the households are satiated with liquidity. Notice that satiation in real balances can also be achieved if the government can fully subsidize consumption and set $\theta_t = -1$.

The condition on the minimum degree of homogeneity of the transactions technology, $k \geq 0$, has to be imposed because otherwise $l_c(c_t, m_t)$ could be negative at full liquidity, for a subset of the class of homogeneous transactions technologies.[2] It may be necessary to impose further restrictions on the minimum value of the degree of homogeneity of the transactions technology, k, depending on the curvature of the utility function, in order to ensure that the private problem is concave.

According to the standard specification of the transactions function, as described by (4), the households purchase $(1+\theta_t)c_t$ units of goods using time, s_t, and real money before taxes, $\frac{M_t}{P_t}$. This transactions technology can be interpreted as if the consumption tax was paid in units of consumption goods, which in turn need to be purchased with money. Of the real goods that are purchased, $(1+\theta_t)c_t$, the households keep c_t, giving the rest to the government when the tax is positive. When the tax is negative, the households only need to use money and time to purchase a part of the goods, receiving the rest as a real subsidy from the government. Under this specification, by subsidizing consumption the government is performing transactions on behalf of the households.

[2] To see this, notice that from the homogeneity assumption, $kl = cl_c + ml_m$. At full liquidity, $l_m = 0$. It follows that, when l is strictly positive at full liquidity, k must be non negative for l_c not to be negative at that point.

From homogeneity, the function (4) can also be written as

$$s_t \geq (1+\theta_t)^k l\left(c_t, \frac{M_t}{P_t(1+\theta_t)}\right). \tag{5}$$

When $k=0$, M_t is unit elastic with respect to $P_t\,(1+\theta_t)$, so that when the tax is increased, and M_t is increased in the same proportion, transactions are not affected. It is possible to keep constant transactions time, s_t, and the real quantity of goods that the household takes home, c_t. When $k>0$, this is no longer the case. If the tax were to be increased, money would have to increase more than proportionately, in order to keep s_t and c_t constant. Alternatively, we can consider a reduction of the consumption tax θ_t, and thus of the price level gross of taxes, $P_t\,(1+\theta_t)$, together with a reduction of M_t in the same proportion. If consumption were kept constant, time used for transactions would be reduced. In particular, by setting $\theta_t=-1$ time used for transactions can be made equal to zero. In this case, the amount of real money necessary to achieve the desired level of consumption can also be zero. The households are still able to consume c_t, because they receive it from the government as a full subsidy.

We propose an alternative specification of the transactions technology, where money, M_t, is unit elastic with respect to the price level gross of consumption taxes, $P_t\,(1+\theta_t)$,

$$s_t \geq l\left(c_t, \frac{M_t}{P_t(1+\theta_t)}\right). \tag{6}$$

The function l satisfies the same two assumptions stated above. According to (6), the households purchase c_t units of goods using time, s_t, and real money after taxes, $\frac{M_t}{P_t(1+\theta_t)}$. This is the natural specification when the consumption tax is paid in units of money. It turns out to be a convenient formulation to analyze other Ramsey problems in monetary models with transactions technologies, such as in Correia, Nicolini and Teles (2001).

When $k=0$, it does not matter whether taxes are paid in units of goods or of money. Money is unit elastic with respect to the price level gross of consumption taxes and the specifications (4) and (6) are equivalent.

3 The Optimal Policy Mix under the Standard Transactions Technology

In this section, we solve the Ramsey problem using the specification of the transactions technology proposed by Kimbrough (1986) and used by Guidotti and Végh (1993) and Mulligan and Sala-i-Martin (1997), among others. In that literature the conditions for the optimality of the Friedman rule depend on whether the alternative tax instrument is the consumption or the income tax. The two taxes are, thus, not equivalent instruments, and therefore they should be jointly considered.

In what follows, we show that once the government is allowed to use both taxes, the Friedman rule is optimal for all homogeneous transactions technologies, as in the case when the only alternative to the inflation tax is an income tax. However, under certain additional conditions on the transactions technology, the unique Ramsey solution requires the government to tax away all income and give the revenue back to the households as a consumption subsidy. Under those conditions, if the income tax is restricted to be zero, the Friedman rule is not optimal. The lack of equivalence of the consumption and the income taxes, the extreme policy prescription for the two taxes, and the non-optimality of the Friedman rule when only the consumption tax is considered are explained by the assumption that the transactions technology does not exhibit a unitary elasticity of money with respect to the price level gross of consumption taxes.

We consider an economy where transactions are performed according to the standard transactions costs technology, (4). The budget constraints of the household for $t \geq 0$ are (2) and (3), where s_t is given by (4).

The maximization problem of the households implies the following marginal conditions:

$$\frac{U_c(t) - U_h(t)(1+\theta_t)^k l_c(c_t, m_t)}{U_h(t)} = \frac{1+\theta_t}{1-\tau_t}, \tag{7}$$

$$-(1+\theta_t)^k l_m(c_t, m_t) = \frac{(1+\theta_t)}{(1-\tau_t)} i_t, \tag{8}$$

$$\frac{U_h(t)}{\beta U_h(t+1)} = \frac{(1+i_{t+1}) P_t}{P_{t+1}} \left(\frac{1-\tau_t}{1-\tau_{t+1}}\right). \tag{9}$$

Condition (7) sets the marginal rate of substitution between consumption and leisure, adjusted for the need to use time for transactions, equal to the relative price. Condition (8) equates the marginal gain of using real money to its marginal cost. Condition (9) is an intertemporal marginal condition.

Let $Q_t = \frac{1}{(1+i_0)\ldots(1+i_t)}$. The intertemporal budget constraint for the households can be written as

$$\sum_{t=0}^{\infty} \frac{Q_t P_t}{P_0}(1+\theta_t)c_t + \sum_{t=0}^{\infty} \frac{Q_t P_t}{P_0}(1+\theta_t) i_t m_t \tag{10}$$

$$= \sum_{t=0}^{\infty} \frac{Q_t P_t}{P_0} (1-\tau_t) \left[1 - h_t - (1+\theta_t)^k l(c_t, m_t)\right].$$

The resources constraints are given by

$$c_t + g_t \leq 1 - h_t - (1+\theta_t)^k l(c_t, m_t), \, t \geq 0. \tag{11}$$

In order to minimize the excess burden of taxation, the government solves a Ramsey problem. This amounts to choosing, within the set of competitive equilibria determined by conditions (7)-(11), the path for the quantities, prices and taxes that maximizes welfare.

3.1 The Ramsey Solution

When the government has access to taxes on money, consumption and income, the restrictions of the Ramsey problem are the resources constraints, (11), the condition $\theta_t \geq -1$, and an implementability constraint. This latter can be obtained by using equations (7)-(9) to substitute $\left\{\tau_t, \; i_t, \frac{(1+i_{t+1})P_t}{P_{t+1}}\right\}_{t=0}^{\infty}$ into the budget constraint (10).

In order to build the implementability constraint, we use the first order condition (9) to express the discount factor as

$$\frac{Q_t P_t}{P_0} = \frac{1}{(1+i_0)} \frac{\beta^t U_h(t) (1-\tau_0)}{U_h(0) (1-\tau_t)}.$$

Plugging this into the intertemporal budget constraint (10), we get

$$\sum_{t=0}^{\infty} \beta^t U_h(t) \left[\left(\frac{1+\theta_t}{1-\tau_t} \right) c_t + \frac{(1+\theta_t)}{(1-\tau_t)} i_t m_t - \left[1 - h_t - (1+\theta_t)^k l(c_t, m_t)\right] \right] = 0. \tag{12}$$

Equations (7)-(8) can then be used to substitute i_t and τ_t into (12). Notice that one of the three taxes cannot be substituted in the constraints of the Ramsey problem, since conditions (7)-(8) can only determine two unknowns. For convenience, we choose not to substitute θ_t. Using the fact that the function $l(c_t, m_t)$ is homogeneous of degree k to write $kl(c_t, m_t) = l_c(c_t, m_t) c_t + l_m(c_t, m_t) m_t$, we obtain the implementability condition

$$\sum_{t=0}^{\infty} \beta^t \left\{ U_c(t) c_t - U_h(t) \left[(1-h_t) - (1-k)(1+\theta_t)^k l(c_t, m_t) \right] \right\} = 0. \tag{13}$$

The Ramsey problem consists of the choice of the path of quantities and of the tax on consumption, $\{c_t, \, h_t, \, m_t, \, \theta_t\}_{t=0}^{\infty}$, that maximizes welfare, subject to the restrictions (11), (13), and $\theta_t \geq -1$.

Let $\beta^t \lambda_t$ and ψ be the multipliers of the resources constraints, (11), and the implementability condition, (13). Since they measure, respectively, the excess burden of taxation and the shadow price of resources, they must be positive at the optimum. The relevant first order conditions of the Ramsey problem are those for real balances and the consumption tax. They are given respectively by

$$-\left[\psi U_h(t)(k-1)+\lambda_t\right](1+\theta_t)^k l_m(c_t,m_t)=0 \tag{14}$$

$$-\left[\psi U_h(t)(k-1)+\lambda_t\right] kl(c_t,m_t)(1+\theta_t)^{k-1}\leq 0. \tag{15}$$

The following two propositions state the main results of this section.

Proposition 1 *Let the government finance expenditures through an income tax, a consumption tax or an inflation tax, and let the transactions technology take the form in (4). Then the Friedman rule, $i_t = 0$, for all t, is optimal for all homogeneous transactions technologies.*

Proof. As shown in the Appendix, the term in square brackets in condition (14) is strictly positive. The optimal solution is thus characterized by $(1+\theta_t)^k l_m(c_t,m_t)=0$. From (7) and (8), it must be that $i_t = 0$. ■

The Proposition states that the Friedman rule is always optimal for homogeneous transactions technologies, as when the only alternative to inflation is the income tax. If the optimal solution has $\theta_t > -1$, then it must be that $l_m(c_t,m_t)=0$, which defines the point of full liquidity, $\overline{\frac{m}{c}}$. When $\theta_t = -1$, there is a large set of ratios $\frac{m}{c}$ that is consistent with the optimal solution. This is not surprising because, with full subsidization of consumption, the household is able to achieve satiation in real balances for any ratio $\frac{m}{c}$. The transactions are carried out by the government so that the amount of real balances held by the household is irrelevant. Increasing real balances does not help in reducing transactions time.

Proposition 2 *Let the government finance expenditures through an income tax, a consumption tax or an inflation tax, and let the transactions technology take the form in (4). Then, it is optimal to set $\theta_t = -1$ and $\tau_t = 1$. Whenever $k > 0$ and $l(c_t,m_t) > 0$, at full liquidity, then $\theta_t = -1$ and $\tau_t = 1$ is the unique solution. Otherwise, a continuum of combinations of θ_t and τ_t are solutions of the Ramsey problem.*

Proof. From condition (15), it follows that, whenever $k > 0$ and $l(c_t,m_t) > 0$ at full liquidity, it is optimal to increase the subsidy on consumption to its maximum level, $\theta_t = -1$. From the household's optimality condition (7),

$\theta_t = -1$ requires that $\tau_t = 1$. When the transactions function is homogeneous of degree zero, or when $l(c_t, m_t) = 0$ at full liquidity, the Ramsey condition (15) is satisfied as an equality. Therefore, any combination of θ_t and τ_t that satisfies (7) is a solution. ■

Once we allow for both consumption and income taxes, the result of optimality of the Friedman rule for all homogeneous transactions technologies is recovered. However, under certain conditions, the unique Ramsey solution recommends full taxation of income and subsidization of consumption. Those conditions are that the degree of homogeneity is strictly positive $k > 0$, and that time at full liquidity is also strictly positive, $l\left(1, \frac{\overline{m}}{c}\right) > 0$.

Whenever $kl\left(1, \frac{\overline{m}}{c}\right) = 0$, it is not possible to determine both the consumption and the income tax. This means that the two taxes are equivalent fiscal instruments. The interpretation of the conditions under which this equivalence holds is straightforward, in light of the discussion of the properties of the transactions technologies in the previous section. When $k = 0$, the standard transactions technology has the property that money is unit elastic with respect to the price level gross of consumption taxes. When the elasticity is unitary it is not possible to save on resources by reducing the consumption tax. The other condition for equivalence is that time at full liquidity is zero. In this case, even if the elasticity is not unitary, it is not possible to save on transactions time by reducing the consumption tax.

3.2 Restrictions on Tax Instruments

In this section, we review the results in the literature when only one alternative tax instrument is allowed for. We relate the conditions for equivalence of the two tax instruments, shown above, to the conditions for the Friedman rule to be optimal when only consumption taxes are considered.

When the tax on consumption is restricted to be zero, $\theta_t = 0$, the economy collapses to the one described in Correia and Teles (1996). In that case, the first order conditions of the private problem, equations (7)-(9), can determine the three unknowns, $\left\{\tau_t,\ i_t,\ \frac{(1+i_{t+1})P_t}{P_{t+1}}\right\}_{t=0}^{\infty}$. It follows that the two restrictions of the Ramsey problem, the resources constraints (11) and the implementability condition (13), are stated in terms of quantities only. The first order conditions of the Ramsey problem are the same, with $\theta_t = 0$, except that condition (15) disappears. From (14), we have

$$[\psi U_h(t)(1-k) - \lambda_t]\, l_m(c_t, m_t) = 0. \qquad (16)$$

The result obtained by Correia and Teles (1996) follows immediately. The

Friedman rule is optimal for any degree of homogeneity of the transactions technology.

We consider now the case when the income tax is restricted to be zero, $\tau_t = 0$. The first order conditions of the private problem, equations (7)-(9), can be used to determine the three unknowns, $\left\{\theta_t,\ i_t,\ \frac{(1+i_{t+1})P_t}{P_{t+1}}\right\}_{t=0}^{\infty}$, in terms of the quantities. Here, however, it is not possible to write θ_t, and thus i_t, explicitly as a function of the quantities. Using condition (7), we express θ_t implicitly by the function $\theta(t) \equiv \theta(c_t, h_t, m_t)$.

The restrictions of the Ramsey problem are given by (11), (13) and the restriction that $\theta_t = \theta(t)$, which is defined implicitly by (7). The government has to choose the path $\{c_t, h_t, m_t\}_{t=0}^{\infty}$ that maximizes (1), subject to (11) and (13) , where $\theta_t = \theta(t)$.

The marginal condition of the Ramsey problem with respect to m_t is given by

$$-\left[\psi U_h(t)(k-1) + \lambda_t\right]\left\{(1+\theta_t)^k l_m(c_t, m_t) + k(1+\theta_t)^{k-1} l(c_t, m_t)\,\theta_m(t)\right\} = 0, \tag{17}$$

where

$$\theta_m(t) = \frac{-(1+\theta_t)^k l_{cm}(c_t, m_t)}{1 + k(1+\theta_t)^{k-1} l_c(c_t, m_t)}, \tag{18}$$

which is obtained from the implicit function $\theta(t)$.

Notice that, from $s_t = (1+\theta(t))^k l(c_t, m_t)$, the term in curly brackets in (17) is $\frac{\partial s_t}{\partial m_t}$, i.e. the total marginal effect of changing m_t on the time used for transactions, keeping consumption and leisure constant. The first term is the direct effect on transactions time and thus on resources, while the second term is the indirect effect through the change in the consumption tax. When this second term is zero, this means that it is not possible to reduce time used for transactions by affecting the consumption tax. Then the optimal solution requires $i_t = 0$, for all t. In fact, as the term in square brackets is strictly positive, $l_m(c_t, m_t) = 0$ is the solution of (17).

The indirect effect will be zero, at the point of full liquidity, when either $kl(c_t, m_t)$ or $l_{cm}(c_t, m_t)$ is zero. $kl(c_t, m_t) = 0$, at full liquidity, is the condition derived in section 3.1 for consumption and income taxes to be equivalent. Since the Friedman rule is optimal for all homogeneous technologies when only income taxes are considered, if $kl(c_t, m_t) = 0$ the Friedman rule must also be optimal for all homogeneous technologies when only consumption taxes are considered.

Even if consumption and income taxes are not equivalent instruments and the optimal solution is to fully subsidize consumption and to fully tax income when both taxes are available, it can still be the case that the Friedman rule is optimal when only consumption taxes are used. That is the case when $l_{cm}(c_t, m_t) = 0$, at full liquidity.[3] From (18), it follows that in this case it is not possible to affect the consumption tax by marginally changing real money.

Kimbrough (1986) analyzes essentially the same model with the consumption tax as the only alternative tax instrument. In his model, it is assumed that the transactions technology is homogeneous of degree one and that time for transactions $l(t)$ is zero at full liquidity, so that the Friedman rule is optimal. In Guidotti and Végh (1993) and Mulligan and Sala-i-Martin (1997), time spent transacting at full liquidity can take a positive value and the transactions technology is homogeneous of any degree. In those models, the Friedman rule is optimal if it is assumed that $l_{(1+\theta)c}((1+\theta)c, (1+\theta)m) = 0$ at full liquidity. This assumption is equivalent to $k(1+\theta_t)^k l(c_t, m_t) = 0$ at full liquidity.[4] Condition (17) shows that the Friedman rule may still be optimal when $k(1+\theta_t)^k l(c_t, m_t) \neq 0$, provided that $l_{cm}(t) = 0$ at full liquidity. Mulligan and Sala-i-Martin (1997) rule out this case by assuming that $l_{cm}(t) < 0$.[5]

4 The Optimal Policy Mix with a Unit Elastic Transactions Technology

In this section, we compute the Ramsey solution when the transactions technology is described by the alternative specification (6), which exhibits unit elasticity of money with respect to the price level gross of consumption taxes. We show that the consumption and income taxes are equivalent and the conditions for the optimality of the Friedman rule are independent of the choice of the alternative tax instrument.

The private problem is defined by the maximization of (1), subject to (2), (3) and (6). As before, we assume that $W_0 = 0$. At the optimum, the following marginal conditions must be satisfied:

$$\frac{U_c(t) - U_h(t)\, l_c(c_t, m_t)}{U_h(t)} = \frac{1+\theta_t}{1-\tau_t}, \tag{19}$$

[3] That would be the case, for instance, with the transactions technology $l(c_t, m_t) = \left(\zeta + \eta \frac{c_t}{m_t}\right) c_t$.

[4] In fact, using Euler's theorem we have $k(1+\theta_t)^k l(c_t, m_t) = kl((1+\theta_t)c_t, (1+\theta_t)m_t) = l_{(1+\theta_t)c_t}(t)(1+\theta_t)c_t + l_{(1+\theta_t)m_t}(t)(1+\theta_t)m_t$. Since, at full liquidity, $l_{(1+\theta_t)m_t}(t)(1+\theta_t)m_t = 0$, the two assumptions are equivalent.

[5] This excludes the Baumol-Tobin transactions function, whose specification is such that $l_{cm}(t) = 0$ at full liquidity.

$$-l_m\left(c_t,m_t\right)=\left(\frac{1+\theta_t}{1-\tau_t}\right)i_t, \tag{20}$$

$$\frac{U_h(t)}{\beta U_h(t+1)}=\frac{\left(1+i_{t+1}\right)P_t}{P_{t+1}}\left(\frac{1-\tau_t}{1-\tau_{t+1}}\right). \tag{21}$$

The intertemporal budget constraint for the consumers can be written as

$$\sum_{t=0}^{\infty}\frac{Q_tP_t}{P_0}(1+\theta_t)c_t+\sum_{t=0}^{\infty}\frac{Q_tP_t}{P_0}(1+\theta_t)i_tm_t=\sum_{t=0}^{\infty}\frac{Q_tP_t}{P_0}\left(1-\tau_t\right)\left(1-h_t-l\left(c_t,m_t\right)\right), \tag{22}$$

The resources constraints are given by

$$c_t+g_t\leq 1-h_t-l\left(c_t,m_t\right),\ t\geq 0. \tag{23}$$

Conditions (19)-(23) define the set of feasible and implementable allocations, $\{c_t,\ h_t,\ m_t\}_{t=0}^{\infty}$ and prices and taxes $\left\{\theta_t,\ \tau_t,\ i_t,\ \frac{(1+i_{t+1})P_t}{P_{t+1}}\right\}_{t=0}^{\infty}$. In order to construct a single implementability condition, we use the first order conditions (21) and obtain $\frac{Q_tP_t}{P_0}=\frac{1}{(1+i_0)}\frac{\beta^tU_h(t)(1-\tau_0)}{U_h(0)(1-\tau_t)}$. Substituting in (22), we obtain

$$\sum_{t=0}^{\infty}\beta^tU_h(t)\left[\left(\frac{1+\theta_t}{1-\tau_t}\right)c_t+\left(\frac{1+\theta_t}{1-\tau_t}\right)i_tm_t-\left(1-h_t-l\left(c_t,m_t\right)\right)\right]=0. \tag{24}$$

Using conditions (19), (20) to substitute for $\frac{1+\theta_t}{1-\tau_t}$ and i_t into the intertemporal budget constraint (24), and using the fact that $l\left(t\right)$ is homogeneous of degree k, we obtain the implementability condition

$$\sum_{t=0}^{\infty}\beta^t\left\{U_c(t)c_t-U_h(t)\left[(1-h_t)-(1-k)l\left(c_t,m_t\right)\right]\right\}=0. \tag{25}$$

Notice that it is not possible to determine both the consumption tax, θ_t, and the income tax, τ_t. Only the ratio $\frac{1+\theta_t}{1-\tau_t}$ can be determined. This means that the two taxes are equivalent fiscal instruments.

The Ramsey problem is the choice of $\{c_t,\ h_t,\ m_t\}_{t=0}^{\infty}$ that maximizes welfare in the set that satisfies conditions (23) and (25). Let $\beta^t\lambda_t$ and ψ be the new multipliers of the resources conditions, (23), and of the implementability condition, (25). The marginal condition of the Ramsey problem for m_t is given by

$$-\left[\psi U_h(t)\left(k-1\right)+\lambda_t\right]l_m\left(c_t,m_t\right)=0. \tag{26}$$

We can now state the following proposition.

Proposition 3 *Let the government finance expenditures through an income tax, a consumption tax or an inflation tax, and let the transactions technology take the form in (6). The Ramsey solution is decentralized by $i_t = 0$, for all t, and by any combination of θ_t and τ_t that satisfies condition (19). The Friedman rule $i_t = 0$, for all t, also decentralizes the optimal solution when the alternative instrument to the inflation tax is either an income tax or a consumption tax.*

Proof. The restrictions of the Ramsey problem, i.e., the implementability and feasibility conditions, are the same, whether the alternative tax instrument is a consumption tax, an income tax, or both. It can be shown, as done in the Appendix for the model under the standard transactions technology, that the term in square brackets in (26) is strictly positive. Then, from (26), an interior solution must satisfy $l_m(t) = 0$. This can be decentralized by $i_t = 0$, for all t. When both consumption and income taxes are available, the optimal allocation is decentralized by any combination of θ_t and τ_t that satisfies condition (19). ■

The Friedman rule is optimal for any homogenous transactions technologies. The assumption of homogeneity is justified by both theoretical and empirical work. In Baumol (1952), Tobin (1956), Barro (1976), Guidotti (1989) and Jovanovic (1982), the transactions technologies are homogeneous of degree zero. Marshall (1992) proposes and estimates a transactions technology that is homogeneous of degree one and Braun (1994) estimates the degree of homogeneity to be .98. Homogeneous functions have the property that the scale elasticity of the money demand is one,[6] at full liquidity. Indeed, at full liquidity $l_{\frac{m}{c}}(1, \overline{\frac{m}{c}}) = 0$, so that $m = vc$, where v is a constant. While there is empirical evidence, often contradictory, on the scale elasticity, there is virtually no evidence on the elasticity at full liquidity. The closest evidence is the result in Mulligan and Sala-i-Martin (1996), which suggests that the scale elasticity of the household demand for money approaches one for interest rates that are close to zero.[7]

5 Why is the Friedman Rule Optimal?

In this section we interpret the results obtained above on the optimality of the Friedman rule. We show that the Friedman rule is generally optimal because

[6] The scale elasticity is defined as the elasticity of money demand with respect to consumption, which is given by $-\frac{c}{m}\frac{l_{cm}}{l_{mm}}$.

[7] Using a calibrated model with US data on the money demand, Correia and Teles (1999) show that, if the transactions technology is not homogeneous, the optimal inflation tax remains very close to the Friedman rule.

money is costless to produce. Furthermore, we show that Kimbrough's (1986) intuition, that money should not be taxed because it is an intermediate good, is not correct under his specification of the transactions technology.

We follow the strategy used by Correia and Teles (1996, 1999) and slightly modify the economies by modelling money as a good that is costly to produce. We solve the corresponding Ramsey problems for a government that can use a tax on consumption as the alternative instrument to the tax on money. The Ramsey solution can thus be related to the optimal taxation rules for costly intermediate goods derived by Diamond and Mirrlees (1971). According to these rules, if the technology is constant returns to scale and consumption taxes are available, it is not optimal to tax intermediate goods.

We start by considering the model with a unit elastic transactions technology, as in (6). In this economy money requires labor to be produced, according to the technology $n_{2,t} = \alpha m_t$. The households supply labor for the production of the consumption good in the amount of $n_{1,t} = 1 - h_t - s_t - n_{2,t}$ units of time. The only modification in the Ramsey problem solved in Section 4 is that the resources constraint will have to include the real resources used in the supply of real money,

$$c_t + g_t \leq 1 - h_t - l\left(c_t, m_t\right) - \alpha m_t,\ t \geq 0. \tag{27}$$

The problem of the planner is thus to choose the path $\{c_t, h_t, m_t\}_{t=0}^{\infty}$ that maximizes (1), subject to the implementability condition, (25), and the resources constraints, (27). The marginal condition of the Ramsey problem with respect to real balances is given by

$$-\left[\psi U_h\left(k-1\right) + \lambda_t\right] l_m\left(c_t, m_t\right) - \alpha\lambda_t = 0, \tag{28}$$

which collapses to the Ramsey first-order condition (26) when money is costless, $\alpha \rightarrow 0$. Condition (28) can be rewritten as

$$-l_m\left(c_t, m_t\right) = \frac{\alpha}{\left[1 + \frac{\psi U_h(k-1)}{\lambda_t}\right]}. \tag{29}$$

In this environment, the first best is characterized by $-l_m\left(c_t, m_t\right) = \alpha$. The marginal rate of technical substitution in the production of transactions, $-l_m\left(c_t, m_t\right)$, has to be equal to the social cost, α. When k is equal to one, it is optimal not to distort production by setting a zero proportional tax on money, in line with Diamond and Mirrlees' (1971) taxation rules. Otherwise, depending on whether k is greater or lower than 1, it is optimal to set either a subsidy or a tax, respectively, on the use of money in the production of transactions. As α converges to zero, the proportional distortion, $\frac{\psi U_h(k-1)}{\lambda_t}$, does not approach zero, and so it is still optimal to set a non zero proportional tax. However, from (20), the optimal price to be charged for the use of money, i_t, is zero. It is the costless nature of money that explains the robustness of the optimality of the Friedman rule.

Under the standard transactions technology, (4), it is generally optimal to tax money even when the technology is constant returns to scale. In this case, the optimal taxation rules of Diamond and Mirrlees (1971) do not apply. Again we modify the Ramsey problem in Section 3 in order to incorporate a cost of producing real money. The resources constraints are

$$c_t + g_t \leq 1 - h_t - (1+\theta_t)^k \, l\,(c_t, m_t) - \alpha m_t,\ t \geq 0. \tag{30}$$

We consider the case when the income tax is restricted to be zero. The restrictions of the Ramsey problem are (30), (13) and the restriction that $\theta_t = \theta\,(t)$, which is defined implicitly by (7). The planner's marginal condition for real balances is

$$-\left[\psi U_h(t)(k-1) + \lambda_t\right]\left\{(1+\theta_t)^k l_m\,(c_t, m_t) + k(1+\theta_t)^{k-1} l\,(c_t, m_t)\,\theta_m(t)\right\} - \alpha\lambda_t = 0. \tag{31}$$

When $k = 1$, this becomes

$$-(1+\theta_t) l_m\,(c_t, m_t) = \alpha + l\,(c_t, m_t)\,\theta_m(t) \tag{32}$$

The first best is characterized by $-(1+\theta_t)l_m\,(c_t, m_t) = \alpha$. As long as $l\,(c_t, m_t)\,\theta_m(t) \neq 0$ at that point, it is optimal to distort production and to tax or subsidize money when there are constant returns to scale in production. When $\alpha \to 0$, the first best coincides with the point of full liquidity. At that point, it is reasonable to assume that $l\,(c_t, m_t)\,\theta_m(t) = 0$, as Kimbrough (1986) does.[8] Hence, when $\alpha \to 0$, the Friedman rule may be optimal. When $\alpha > 0$, the first best allocation is not at full liquidity and it is not reasonable to assume that $l\,(c_t, m_t)\,\theta_m(t) = 0$ at that point. Thus, if money were costly, and $k = 1$, the optimal tax on money would be different from zero. This is inconsistent with Diamond and Mirrlees (1986) optimal taxation rules of intermediate goods. Thus Kimbrough's (1986) interpretation of the optimality of a zero tax on money, based on money being an intermediate good, is not the appropriate one. Kimbrough's (1986) result is due to the assumption that money is costless.

The reason why the Diamond and Mirrlees' (1971) optimal taxation rules fail to apply under Kimbrough's (1986) specification of the transactions technology is the odd structure of the economy, where the consumption tax affects the technology used to produce the consumption good. According to

[8] Kimbrough (1986) assumes that transactions time at full liquidity is zero.

this production structure, the consumption good, c, is produced using time, n_1, and transactions in fixed coefficients. Transactions are performed according to the function $f(\frac{s}{(1+\theta)^k}, m)$, which is obtained from the transactions function $s = (1+\theta)^k l(c,m)$. The Leontief technology is thus described by $c = \min\left(n_1,\ f(\frac{s}{(1+\theta)^k}, m)\right)$. Now consider a reduction in the consumption tax, θ. For a given amount of labor supplied, n_1, it is possible to produce the same amount of the consumption good, c, using less time, s, while keeping constant money, m, and the time devoted to the production of money, n_2. Thus, a reduction in the consumption tax allows to save on resources that can be used for consumption or leisure. When the choice of the government is between a tax on money and a tax on consumption, it is optimal to set a positive tax on money in order to lower the consumption tax and to save on the resources used for transactions.

6 Concluding Remarks

In this paper, we derive the optimal inflation tax in a monetary model where money and time are necessary for transactions. We show that the Friedman rule is the optimal solution when the fiscal choice is between an income tax and an inflation tax, a consumption tax and an inflation tax or all three tax instruments. In doing so, we object to recent claims in the literature that the optimality of the Friedman rule is a fragile result because it crucially hinges upon the fiscal instrument used by the government as an alternative to the inflation tax.

We show that the claimed fragility is due to the specification of the transactions technology traditionally used in the literature, when consumption taxes are considered. In particular, the elasticity of money with respect to the price level gross of taxes has been assumed to be non-unitary. We argue that a unitary elasticity is the appropriate assumption. In that case, the Friedman rule is optimal for all homogeneous transactions technologies, irrespective of the choice of the alternative tax instrument.

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A Proposition 1

Here we show that, at the optimum,

$$[\psi U_h(t)(k-1)+\lambda_t]>0. \tag{33}$$

When $k\geq 1$, the term is strictly positive, because the multipliers are strictly positive. When $k\in[0,1)$, we use an argument similar to the one used in Correia and Teles (1996), to ensure that the term is still strictly positive. The first order conditions of the Ramsey problem with respect to h is given by

$$U_h(t)+\psi\left\{U_{ch}(t)c_t+U_h(t)-U_{hh}(t)\left[1-h_t+(k-1)(1+\theta_t)^k l(t)\right]\right\}=\lambda_t \tag{34}$$

Suppose that, for some $k\in[0,1)$, $[\psi U_h(t)(k-1)+\lambda_t]=0$. Then, we can use this to substitute λ_t in (34). Since the problem is stationary, manipulating the implementability condition we obtain

$$D(t)c_t=\left[1-h_t+(k-1)(1+\theta_t)^k l(t)\right],$$

where $D(t)\equiv\frac{U_c(t)}{U_h(t)}$ is the marginal rate of substitution between consumption and leisure. Using this expression and rearranging terms, we can reformulate condition (34) as

$$U_h(t)+\psi\left[U_h(t)D_h(t)c_t+kU_h(t)\right]=0.$$

As shown in De Fiore (2000), if consumption is a non inferior good, $D_h(t) \geq 0$. Therefore, the term on the LHS of this expression is strictly positive since $U_h(t) > 0$, and the equation cannot be satisfied. This implies that $[\psi U_h(t)(k-1) + \lambda_t] = 0$ cannot be a solution to the Ramsey problem. We thus conclude that the term in square brackets in (33) is strictly positive for $k \geq 0$.