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COMMON CURRENCIES VERSUS MONETARY INDEPENDENCE

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ABSTRACT

Common Currencies versus Monetary Independence*

We study the optimal monetary policy in a two-country open-economy model under two monetary arrangements: (a) multiple currencies controlled by independent policy-makers; (b) common currencies controlled by a centralized policy-maker. Our findings suggest that: (i) monetary policy competition leads to higher long-term inflation and interest rates with large welfare losses; (ii) the inflation bias and the consequent losses are larger when countries are unable to commit to future policies; (iii) in both cases, the welfare losses from higher inflation dominates the welfare costs of losing the ability to react optimally to business cycle shocks. Therefore, the coordination of policies implicit in the adoption of a common currency or dollarization has positive welfare consequences.

JEL Classification: E00, E50 and F00
Keywords: common currency, international coordination and optimal monetary policy

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Introduction

The study of monetary policy interaction in open economies has a long tradition in international economics. The conventional view that emerges from this literature is that coordination of policies is in general preferable to monetary competition (see Persson & Tabellini (1995) for an overview). The problem with policy coordination is that it can be difficult to achieve. Independent monetary authorities have incentives to deviate from agreed policies and undertake competitive devaluation policies with beggar-thy-neighbor consequences. The key issue is not whether monetary policies should be coordinated, but how to make coordination feasible. One way to achieve coordination is through currency unification: either through the creation of a currency union or through the unilateral adoption of a foreign currency as in the case of dollarization.

In contrast to the conventional view of policy coordination, the recent literature based on variations of the "new open economy macroeconomic models" pioneered by Obstfeld & Rogoff (1995) reaches more pessimistic conclusions about the benefits of coordinated policies. Papers by Benigno & Benigno (2001), Corsetti & Pesenti (2001a), Devereux & Engel (2000), Obstfeld & Rogoff (2000a) and Pappa (2000) conclude that either the allocation with coordinated policies does not differ from the allocation with independent policies, or the gains from coordination are small. Moreover, in these models where monopolistic competition and nominal rigidities are important, coordinated policies need not imply lower inflation. Earlier work by Rogoff (1985) reached an even more pessimistic conclusion. According to that paper, policy coordination may actually reduce welfare.

In this paper we evaluate the consequences of using a common currency as a means to achieve monetary policy coordination. Although a common currency can bring some of the benefits of policy coordination, for example by eliminating possible inflation biases, it also implies a loss of monetary independence. The loss of monetary independence means that an individual country can no longer have a long-term inflation rate that differs from the inflation rate of the currency area and, as stressed by the theory of "optimal currency areas", an individual country can no longer use the instruments of monetary policy to adjust to asymmetric internal or external shocks.

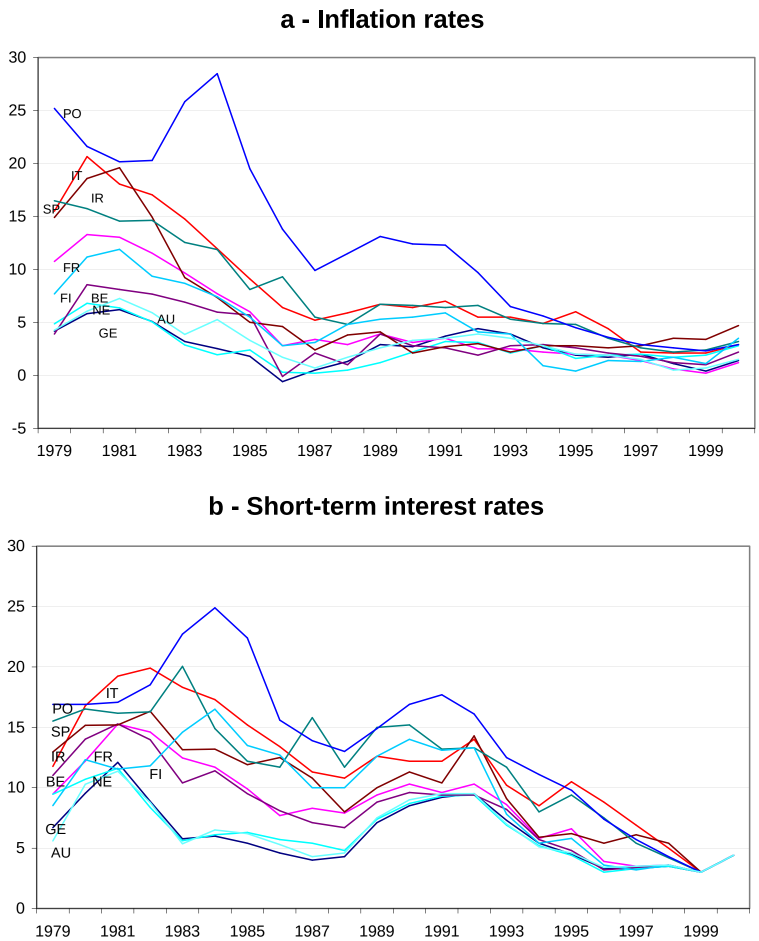
The questions that concern us in thinking about the use of common currencies are these. First, does monetary policy competition lead to systematic inflation biases? Second, does the adoption of a common currency eliminate the inflation bias and increase welfare? Third, abstracting from systematic inflationary biases, is the loss of the ability to react optimally to shocks quantitatively important in terms of welfare? Finally, are the economic consequences of creating a currency union different from the passive adoption of an existing currency (for example, dollarization)?

We address these questions in the context of a simple two-country open economy model where countries are technologically integrated. The production activity in each country requires two inputs: one is domestically produced and the other imported from abroad. Each country is affected by a productivity shock. In the model, monetary policy interventions have liquidity effects, that is, a monetary contraction induces an increase in the domestic nominal interest rate which in turn has a negative impact on the real sector of the economy. In a closed economy, where both inputs are produced domestically, this would be the only effect of a monetary contraction. Because high interest rates reduce the level of economic activity, the optimal monetary policy would keep the nominal interest rate at the lowest level, zero, consistent with the Friedman rule. In an open economy, however, monetary policy also affects the terms of trade. Specifically, an increase in the interest rate induces an appreciation of the real exchange rate that, controlling for the negative liquidity effect described above, generates an expansion in the domestic level of activity. This terms of trade channel, which is also emphasized in other open economy models as shown in Corsetti & Pesenti (2001b) and Corsetti, Pesenti, Roubini, & Tille (2000), generates a conflict of interest between the two countries. This conflict is the basis for the policy competition. In our model, this policy competition generates an inflationary bias with sizable adverse welfare consequences.

We characterize the equilibrium policies in two different competitive environments. In the first environment countries choose a sequence of interest rates and this sequence will not be changed in future periods (policy commitment). In the second environment, instead, the interest rates are chosen in every period (policy discretion) and the equilibrium policies are time-consistent. By comparing these two different environments we show that when countries are unable to commit to future policies, competition leads to higher inflation, higher interest rates and lower welfare. The key element that generates these results is the assumption of a difference between the short-term and the long-term elasticity of substitution between domestic and foreign inputs. Because firms find it more difficult to readjust their input composition in the short-term, changes in the current interest rates will have a larger impact on the terms of trade than future changes. This implies that, when the policy makers commit to future policies, they understand that the interest rates they choose today for the future will have a smaller impact on the real exchange rate: the negative liquidity effect becomes more important than the positive terms of trade effect and the monetary authorities will have less incentive to choose higher interest rates. In future periods, however, when firms become inflexible, the monetary authorities will have an incentive to change the pre-set interest rates. This implies that the policy with commitment is time-inconsistent and the time-consistent policies will generate higher long-term interest rates, higher long-term inflation and lower welfare. Because this inflation bias will be eliminated when countries coordinate their policies, the currency unification solves both the problems of policy coordination and commitment.

This theoretical result finds empirical support in the experience of the European countries. As the recent history of the "European Monetary System" and the subsequent experience of the "European Monetary Union" have shown, the increasing monetary integration of Europe has been accompanied by falling and convergent inflation and interest rates as shown in Figure 1.

In addition to studying the implications of a common currency for inflation and



Source: OECD historical statistics 1970-2000

Figure 1: Inflation and interest rates in the EMU countries, 1979-2000

welfare in the long-term, we also evaluate the welfare costs of losing the short-term ability to react optimally to internal and external shocks (cyclical independence). We find that the cost of losing cyclical monetary policy independence is extremely small, almost insignificant. Importantly, neither this finding nor the estimated welfare gains associated with inflation reduction in the long run depend on the weights given by the policy maker to the welfare of the two countries. Therefore, from an economic point of view, the creation of a currency union and the unilateral adoption of a foreign currency are equivalent. Currency unions and dollarization lead to the same long-term inflation reduction and to similar cyclical policies. Of course, dollarization and currency unions may also have redistributional effects through seigniorage as well as political implications. However, we abstract from these considerations.

Most of the recent literature on international monetary policy coordination has used variations of the "new open economy macroeconomic models" (Obstfeld & Rogoff (2000b)). Contrary to the results of our paper, most of these studies do not conclude that international monetary policy competition leads to higher inflation. Furthermore, as observed above, they cast doubt on the ability of monetary policy coordination to yield large welfare gains. See for example, Benigno & Benigno (2001), Corsetti & Pesenti (2001a), Devereux & Engel (2000), Obstfeld & Rogoff (2000a) and Pappa (2000). One exception is Canzoneri, Cumby, & Diba (2001). Overall, however, the conclusion of these recent contributions is that international monetary policy coordination leads to modest or zero gains.

Our model differs from the new open economy macroeconomic models in three important ways. First, we do not assume that producers operate in monopolistic competitive markets. Accordingly, production is not necessarily inefficient. A second difference is that money does not enter the utility function but is held for transactions purposes. By further assuming limited participation in financial markets, monetary policy interventions have liquidity effects as in the two country open economy models of Grilli & Roubini (1992) and Schlagenhauf & Wrase (1995). The third difference is that prices are perfectly flexible. Thus, the channel through which a monetary expansion affects the real sector of the economy is not by increasing the nominal aggregate demand but through the reduction in the financing cost of firms.

These modeling features lead to the conclusion that policy competition has large welfare consequences contrary to the conclusion from many of the recent papers. This is because these papers focus on the strategic interaction that derives from optimal response to shocks (policy stabilization) and devote little attention to the problem of systematic inflationary biases, which instead is central for our results. Rogoff (1985) does focus on systematic inflationary bias arising from strategic interaction but reaches a completely different conclusion from ours. He concludes that, abstracting from the gains of policy stabilization, coordination may lead to higher inflation and lower welfare. The same conclusion is also likely to follow from many of the new open economy macroeconomic models, when they consider discretionary policies (without commitment). The intuition is simple. As discussed in Benigno & Benigno (2001), in a coordinated equilibrium the distortions induced by monopolistic competition create an incentive for the monetary authorities to inflate. When policies are chosen competitively, however, this incentive is mitigated by the negative impact of inflation on the terms of trade. Assuming that an equilibrium exists, coordinated policies can lead to higher inflation and lower welfare.

Plan of the paper

To facilitate the understanding of the theoretical framework, we introduce the model sequentially in three different stages. Section I presents the simplest version in which there is only one period and decisions are static. This simplified framework is useful to illustrate how "policy competition" leads to an inflation bias among the competing countries and reduces welfare. Section II makes the model dynamic by adding a second period. The two-period model allows us to distinguish the elasticity of substitution between domestic and foreign inputs in the short-run and in the long-run. This distinction is important because it differentiates the case of "policy commitment" where the monetary authorities choose their policies in advance from the case of "policy discretion". Within this framework we show that there is a time-consistency problem in the choice of policies. We also show that the lack of policy commitment exacerbates the inflationary bias induced by policy competition. After showing the problems of policy coordination and commitment in the two period-model, Section III generalizes the analysis to an infinite horizon setting. The quantitative properties are studied in Section IV. The analysis conducted in the main body of the paper assumes international financial autarky. The model with international mobility of capital will be studied in Appendix B.

I One-period model: the coordination problem

We begin by describing a simple model with only one period. This model conveys the simple intuition about the mechanism through which policy competition leads to an inflation bias.

Households: There are two symmetric countries populated by a continuum of households and a continuum of firms. For simplicity we assume that the mass of households, normalized to 1, is equal to the mass of firms. Therefore, each firm employs one household-worker. Households' utility is a function of consumption and is denoted by $u(c)$. In this simple framework, households supply their labor services inelastically and consume the surplus generated by the firm. Because they are the owners of the firms, the division of the surplus between wages and profits is irrelevant in this model.

Firms and technology: The main action comes from the decisions of firms. Firms produce output with two intermediate inputs: a domestically produced input and an imported input. In country 1 the production technology is:

$$y_1 = A_1 x_1^{\nu} \qquad\qquad x_1 = \Big(x_{11}^{\epsilon} + \phi \cdot x_{12}^{\epsilon}\Big)^{\frac{1}{\epsilon}} \tag{1}$$

where A_1 is the technology level of country 1 and x_1 is a composite input. The composite input results from the aggregation of the intermediate input produced domestically, x_{11}, and the intermediate input produced abroad, x_{12} (import). Throughout the paper we will use the first subscript to denote the country that uses the input and the second subscript for the country that produces the input. The same production function, with technology level A_2, is used by firms in country 2. We assume that $\nu < 1$ and $\epsilon < \nu$.[1]

[1] In the calibration exercise of Section IV we will show that this condition is not very restrictive given the empirical estimates of the Ermington elasticities.

Notice that the intermediate inputs are used in production in the same period in which they are produced. This assumption is also made in Alesina & Barro (2002) for analytical simplicity.

Firm financing and monetary policy: Firms need to finance the purchase of the intermediate inputs by borrowing from a financial intermediary. The nominal interest rate on loans in country 1 is R_1 and the interest rate in country 2 is R_2. Denote by e the nominal exchange rate (units of currency of country 1 to purchase one unit of currency of country 2). The real exchange rate is denoted by $\bar{e}$ and is equal to eP_2/P_1, where P_1 is the nominal price in country 1 and P_2 is the nominal price in country 2 (both expressed in their respective currencies). Because the price of the final goods must be equal to the price of the intermediate goods produced at home, the loan contracted by a firm in country 1 is equal to $P_1x_{11} + eP_2x_{12} = P_1(x_{11} + \bar{e}x_{12})$ and the loan contracted by a firm in country 2 is $P_2x_{22} + P_1x_{21}/e = P_2(x_{22} + x_{21}/\bar{e})$.

The monetary authorities of the two countries maximize their own country's welfare by choosing the nominal interest rates. Therefore, R_1 and R_2 are the policy instruments and they are chosen strategically according to a Nash scheme of policy competition. In this simplified version of the model it is not necessary to specify the whole monetary sector because all the relevant problems can be expressed as functions of real variables and interest rates. The full specification of the monetary sector will be provided in Section III when we consider the infinite horizon model.

I.1 The firm's problem and general equilibrium for given interest rates

Let's start by taking as given the interest rates chosen by the two countries. After solving for the general equilibrium we will determine these interest rates as the solution to the policy game played by the monetary authorities. The optimization problem solved by a firm in country 1 is:

$$\max_{x_{11},x_{12}} \left\{ P_1 \left[A_1 \left(x_{11}^{\epsilon} + \phi x_{12}^{\epsilon} \right)^{\frac{\nu}{\epsilon}} - (x_{11} + \bar{e} \cdot x_{12})(1 + R_1) \right] \right\} \tag{2}$$

The solution to the firm's problem is:

$$x_{11} = \left(\frac{\nu A_1}{1 + R_1} \right)^{\frac{1}{1-\nu}} \left[1 + \phi \left(\frac{\phi}{\bar{e}} \right)^{\frac{\epsilon}{1-\epsilon}} \right]^{\frac{\nu-\epsilon}{\epsilon(1-\nu)}} \tag{3}$$

$$x_{12} = \left(\frac{\phi}{\bar{e}} \right)^{\frac{1}{1-\epsilon}} x_{11} \tag{4}$$

Equations (3) and (4) are the demands for the domestic and foreign inputs. These inputs depend positively on the level of technology, and negatively on the domestic interest rate. Moreover, if $\epsilon < \nu$, as we assume, a lower value of $\bar{e}$ has a positive impact on both inputs. Therefore, a policy that is able to induce an appreciation of the real exchange rate—that is, a fall in $\bar{e}$—might have an expansionary effect in the economy. The firm's solution in country 2 is also given by (3) and (4) but with A_2, R_2 and $1/\bar{e}$ replacing A_1, R_1 and $\bar{e}$.

The general equilibrium is derived by imposing market clearing conditions in the goods markets and the foreign exchange market. The equilibrium conditions in the goods markets are:[2]

$$Y_1 = C_1 + X_{11} + X_{21} \tag{5}$$
$$Y_2 = C_2 + X_{22} + X_{12} \tag{6}$$

The gross production (Y_1 or Y_2) must be equal to the demand for domestic consumption, (C_1 or C_2), and the demand for intermediate inputs from domestic firms, (X_{11} or X_{22}), and foreign firms, (X_{21} or X_{12}).

We assume that there is not international mobility of financial assets. Consequently, the trade account is always balanced, that is, $eP_2X_{12} = P_1X_{21}$. Dividing both sides by P_1, the equilibrium condition in the exchange rate market can then be written as:

$$\bar{e} \cdot X_{12} = X_{21} \tag{7}$$

The case with international financial transactions will be studied in Appendix B after extending the model in an infinite horizon setting.

Conditions (5), (6) and (7), together with the demands for the intermediate inputs (3) and (4), define the general equilibrium of this economy. Notice that these conditions do not depend on the nominal prices. Therefore, in the determination of the equilibrium we can ignore the full specification of the monetary sector once we know the two nominal interest rates. Using these conditions we can derive:

$$C_1 = \left(\frac{\nu A_1}{1+R_1}\right)^{\frac{1}{1-\nu}} \left(\frac{1+R_1-\nu}{\nu}\right) \left[1 + \phi\left(\frac{\phi}{\bar{e}}\right)^{\frac{\epsilon}{1-\epsilon}}\right]^{\frac{\nu(1-\epsilon)}{\epsilon(1-\nu)}} \tag{8}$$

$$C_2 = \left(\frac{\nu A_2}{1+R_2}\right)^{\frac{1}{1-\nu}} \left(\frac{1+R_2-\nu}{\nu}\right) \left[1 + \phi\left(\phi\bar{e}\right)^{\frac{\epsilon}{1-\epsilon}}\right]^{\frac{\nu(1-\epsilon)}{\epsilon(1-\nu)}} \tag{9}$$

$$\left[\frac{A_1(1+R_2)}{A_2(1+R_1)}\right]^{\frac{1}{1-\nu}} = \bar{e}^{\frac{1+\epsilon}{1-\epsilon}} \left[\frac{1+\phi(\phi\bar{e})^{\frac{\epsilon}{1-\epsilon}}}{1+\phi\left(\frac{\phi}{\bar{e}}\right)^{\frac{\epsilon}{1-\epsilon}}}\right]^{\frac{\nu-\epsilon}{\epsilon(1-\nu)}} \tag{10}$$

These three equations, derived in Appendix A.1, characterize the general equilibrium, given the interest rates R_1 and R_2. Equation (8) defines the net production and consumption in country 1. Equation (9) defines the net production and consumption in

[2]We use capital letters to denote aggregate variables and prices, and lowercase letters to denote individual variables. The only exception is the exchange rate that we denote by e to distinguish it from the expectation operator E.

country 2. Equation (10) defines the equilibrium in the exchange rate market (the value of imports must be equal to the value of exports). These three equations are all we need to determine how an interest rate change affects the country's welfare. To understand the interest rates effects, it will be convenient to emphasize the following properties of the equilibrium.

(a) **Liquidity effect**: Keeping constant the terms of trade ($1/\bar{e}$ for country 1 and $\bar{e}$ for country 2), an increase in the domestic interest rate decreases domestic consumption.

(b) **Terms of trade effect**: Keeping constant the interest rate in the other country, an increase in the domestic interest rate improves the terms of trade. Moreover, keeping constant the domestic interest rate, an improvement in the terms of trade increases consumption.

These are the two channels through which the interest rates affect production and consumption. As can be seen from equations (3) and (4), if we keep the real exchange rate constant, an interest rate increase reduces the intermediate inputs, which in turn reduces production and consumption. This is because the intermediate inputs must be financed in advance and a higher interest rate increases the cost of financing these inputs. This is the usual *liquidity* channel of limited participation models as in Christiano & Eichenbaum (1995), Fuerst (1992) and Lucas (1990).

If we were in a closed economy and both inputs were produced domestically, the liquidity channel would be the only mechanism through which the nominal interest rate affects the real sector of the economy. In this case the optimal policy would set a zero interest rate consistent with the Friedman rule. With trade, however, there is an additional channel which works through the terms of trade. As can be seen from equation (10), a higher value of R_1 must be associated with a lower value of $\bar{e}$ or better terms of trade for country 1. The lower value of $\bar{e}$ reduces the cost of the foreign input and increases the demand for this input. This can be seen from equations (3) and (4) if we keep the interest rates constant. The complementarity between the two intermediate inputs then implies that the domestic input will also increase.

The total effect of an interest rate increase depends on whether the liquidity channel dominates the terms of trade channel. The tension created by the terms of trade channel has been emphasized in recent papers by Corsetti & Pesenti (2001b) and Corsetti et al. (2000). It turns out that the terms of trade channel dominates when the interest rate is low while the liquidity channel dominates when the interest rate is high. Therefore, consumption is first increasing and then decreasing as shown in Figure 2. This figure plots the level of consumption for country 1 as a function of its interest rate, R_1, for given values of the interest rate in country 2.[3] In this model the objective function of the policy

[3] After fixing R_1 and R_2, the value of C_1 is determined by the solutions of equations (8) and (10).

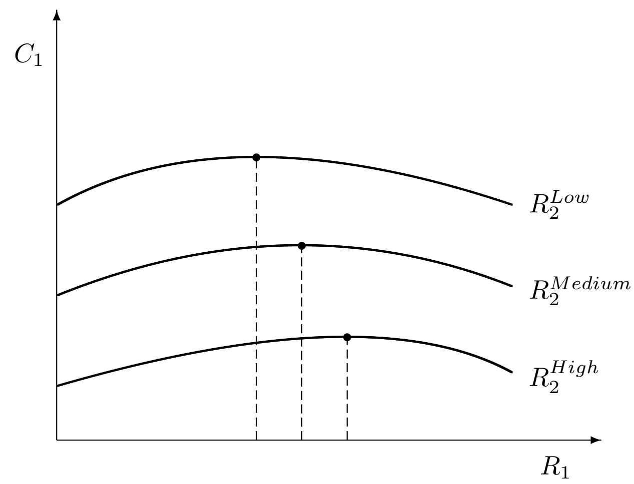


Figure 2: Consumption of country 1 as a function of the domestic interest rate for given interest rates in country 2.

maker is a monotone transformation of consumption. Therefore, the maximization of the policy objective is equivalent to the maximization of consumption.

I.2 Nash policy equilibrium in the one-period model

If we assume that the two countries set their interest rates competitively according to a Nash strategic scheme, then the maximizing value of R_1 constitutes a point in the reaction function of country 1 to the interest rate of country 2. By determining the optimal R_1 for each possible value of R_2, we construct the whole reaction function for country 1. Similarly, we can also construct the reaction function of country 2. The intersection of the reaction functions determines the Nash equilibrium as shown in Figure 3.

The parameter ϵ plays a key role in determining the equilibrium interest rates. As we reduce the value of ϵ, that is, we increase the degree of complementarity between domestic and foreign imports, the reaction function of country 1 moves to the right and the reaction function of country 2 moves up. Therefore, the equilibrium will be characterized by higher nominal interest rates. In a world where long term interest rates are determined by a Fisherian rule, the higher nominal interest rates will be associated with higher inflation rates. The link between the nominal interest rate and the inflation rate will be made precise when we describe the infinite horizon model in Section III.

The higher interest rate would be welfare improving if the other country were to keep the interest rate low. However, because the other country also increases the interest rate, both countries cannot benefit from this competitive interaction. In particular, when countries are symmetric, that is, $A_1 = A_2$, neither of them would benefit from this

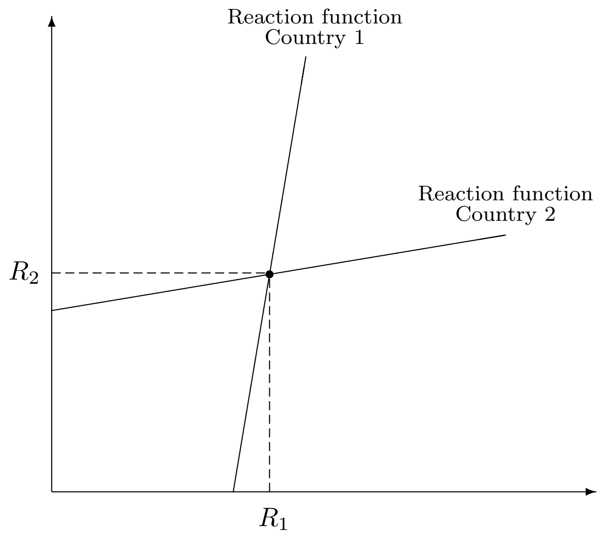


Figure 3: Reaction functions and Nash equilibrium.

strategic interaction and the equilibrium will be characterized by higher interest rates and lower welfare. In fact, in equilibrium, the real exchange rate will be 1 and only the negative *liquidity* effect of the interest rate operates. If the countries set their policies cooperatively, for example by creating a currency union, the optimal interest rates would be zero in both countries and they would experience higher welfare. The zero interest rate (Friedman rule) is optimal because the interest rate acts as a tax on production and creates a distortion in the allocation of resources.

For illustrative purposes, we show here the consequences of monetary policy competition with a numerical example. We set $A_1 = A_2 = 1$, $\nu = 0.9$ and consider alternative values of ϵ which determines the elasticity of substitution between domestic and foreign inputs. More specifically, the elasticity of substitution is given by $1/(1-\epsilon)$. For each value of ϵ we then choose ϕ so that the imports of each country is 25 percent of GDP. Table 1 reports the equilibrium interest rates and the welfare losses from policy competition for this parameterization of the model.

In the case of policy coordination the equilibrium interest rates will be zero while with policy competition they are strictly positive. Moreover, as we reduce the degree of substitutability between domestic and foreign inputs, that is, we reduce ϵ, the equilibrium interest rates and the welfare losses increase. For the moment these numbers are just an example. The quantitative analysis of the model will be conducted in Section IV when we perform a full calibration exercise.

We summarize the main result of the one-period model as follows:

Result 1 (Policy competition and inflationary bias) *Policy competition leads to higher interest (and inflation) rates. The higher the complementarity between domestic*

Table 1: Equilibrium interest rates and consumption losses relative to a common currency for different elasticities of substitution.

	Elasticity = 1.6	Elasticity = 1.1	Elasticity = 0.7
Common currency			
Interest rate	0.00	0.00	0.00
Policy competition			
Interest rate	5.31	9.67	24.40
Consumption loss	0.70	2.12	10.57

and foreign inputs, and the higher is the interest rate and the inflation bias.

II A two-period model: the commitment problem

Because of the static nature of the model developed in the previous section, there are no time-consistency issues in the choice of policies. In order to illustrate how the ability to commit to policies in advance may affect the equilibrium outcome, we extend the previous analysis in two ways. First, we add a second period. Second, we distinguish between short-term and the long-term elasticity of substitution between domestic and foreign inputs.

Households: The characteristics of the household sector do not change. Although now there are two periods, households do not take any action in the first period. They supply labor and consume only in the second period. The utility from second period consumption is still $u(c)$.

Firms and technology: As in the previous model, firms produce output with two intermediate inputs: a domestically produced input and an imported input. However, we now assume that the short term elasticity of substitution between these two inputs differs from the long term elasticity. As emphasized in empirical studies of international trade (see, for example, Gallaway, McDaniel, & Rivera (2000)), the elasticity of substitution between domestically produced goods and foreign imports depends on the length of the period: the longer is the period and higher is the elasticity. To capture the idea of a different elasticity of substitution between the short-term and the long-term, we assume that the production plan of the firm takes place in two stages. In the first period the firm chooses the optimal production plan according to the long-term technology. In the second period the firm considers the possibility of changing the original plan. At this stage, however, the technology available to the firm is characterized by a lower degree of substitutability between inputs. The modeling idea is similar to the putty-clay model of Atkeson & Kehoe (1999), although it differs in the details.

Consider a firm in country 1. The long-term technology faced in the first period is:

$$y_1 = A_1 x_1^{\nu} \qquad\qquad x_1 = \left(x_{11}^{\epsilon} + \phi \cdot x_{12}^{\epsilon}\right)^{\frac{1}{\epsilon}} \tag{11}$$

Denote by $(\hat{x}_{11}, \hat{x}_{12})$ the plan chosen in the first period. This can be interpreted as the choice of a production technique. The actual purchase of the inputs will take place in the second period. After choosing the optimal plan, the firm can change this plan but at a cost. This is formalized by assuming that in the second period there is a lower degree of substitutability between the domestic and the foreign inputs. Given the commitment to the plan $(\hat{x}_{11}, \hat{x}_{12})$, the short-term technology is:

$$y_1 = A_1 x_1^{\nu} \qquad\qquad x_1 = \lambda(\hat{x}_{11}, \hat{x}_{12}) \cdot \left[x_{11}^{\eta} + \omega(\hat{x}_{11}, \hat{x}_{12}) \cdot x_{12}^{\eta}\right]^{\frac{1}{\eta}} \tag{12}$$

where $\eta < \epsilon$ and the parameters λ and ω are functions of the previously chosen plan $(\hat{x}_{11}, \hat{x}_{12})$. At this stage the firm chooses x_{11} and x_{12} but cannot change $\hat{x}_{11}$ and $\hat{x}_{12}$. We will refer to (11) as the "long-term technology" and to (12) as the "short-term technology".

The condition $\eta < \epsilon$ captures the idea that the elasticity of substitution between the domestic and the foreign inputs is smaller in the short-term than in the long-term. The link between the short-term technology and the previously chosen plan $(\hat{x}_{11}, \hat{x}_{12})$ is given by the fact that the two parameters $\lambda(\hat{x}_{11}, \hat{x}_{12})$ and $\omega(\hat{x}_{11}, \hat{x}_{12})$ are functions of the plan. These two parameters are determined by imposing two conditions. First, if the firm does not change the original plan, it will produce the same amount of goods planned in advance, that is:

$$\left[\hat{x}_{11}^{\epsilon} + \phi \cdot \hat{x}_{12}^{\epsilon}\right]^{\frac{1}{\epsilon}} = \lambda \cdot \left[\hat{x}_{11}^{\eta} + \omega \cdot \hat{x}_{12}^{\eta}\right]^{\frac{1}{\eta}} \tag{13}$$

The second condition is that the marginal rate of substitution between the domestic and the foreign inputs are the same in the short-term and in the long-term technologies when evaluated at the original plan. Because the marginal rates of substitution are given by $\phi \cdot (x_{12}/x_{11})^{\epsilon-1}$ and $\omega \cdot (x_{12}/x_{11})^{\eta-1}$, this condition can be written as:

$$\phi \cdot \left(\frac{\hat{x}_{12}}{\hat{x}_{11}}\right)^{\epsilon-1} = \omega \cdot \left(\frac{\hat{x}_{12}}{\hat{x}_{11}}\right)^{\eta-1} \tag{14}$$

Denote by $\kappa_1 = (\hat{x}_{12}/\hat{x}_{11})$ the ratio between the domestic and the foreign inputs chosen in the original plan. It can be verified that the two parameters of the short-term technology λ and ω depend on this ratio, not the absolute values of the planned inputs. Therefore, we will express these parameters as a function of κ_1, that is, $\lambda(\kappa_1)$ and $\omega(\kappa_1)$.

The structure of the production technology is illustrated in Figure 4. This figure plots the inputs requirement (isoquant) for a given level of the composite input x_1, for the two production technologies. The isoquant of the long-term technology (darker curve) is flatter than the short-term isoquants. In the first period, the firm chooses a point along the long-term isoquant. After choosing a point along this isoquant, the firm can change this point in the second period. However, if the firm decides to do so, the relevant isoquant becomes more curved and the elasticity of substitution between domestic and foreign inputs declines. The distinction between these two stages is key to differentiating the time-consistent policies from the policies with commitment.

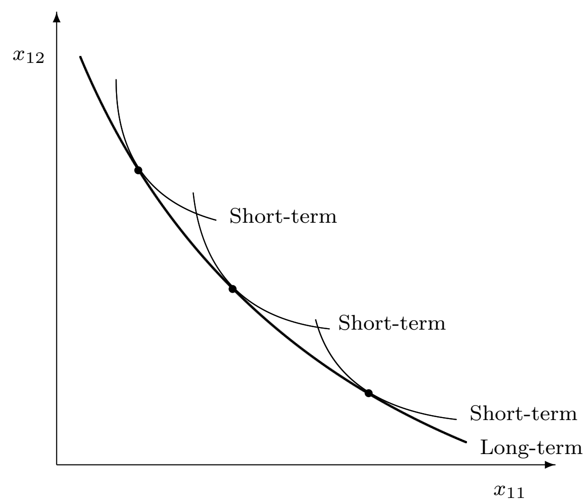


Figure 4: Isoquants for long-term and short-term production technologies.

Monetary policy: The monetary authorities control the interest rates as in the previous model. However, now it becomes important to specify when the interest rates are chosen. We will characterize the equilibrium in two policy environments:

- **Discretion**: The monetary authorities decide the interest rates in the second period, after the firms have committed to the long-term plan.
- **Commitment**: The monetary authorities decide the interest rates in the first period before the firms have chosen the long-term plan. In the second period they simply implement the policy targets decided in the first period.

II.1 Equilibrium with policy discretion

Consider the environment where the monetary authorities choose the interest rates in the second period (policy discretion). Because there are two periods, we have to distinguish the problem solved in the first period from the one solved in the second period. We start with the characterization of the second period equilibrium.

Second period equilibrium for given κ_1 and κ_2: At the beginning of the period firms are already committed to the input ratios $\kappa_1 = \hat{x}_{12}/\hat{x}_{11}$ and $\kappa_2 = \hat{x}_{22}/\hat{x}_{21}$. The parameters of the short-term production function are $\lambda(\kappa_1)$ and $\omega(\kappa_1)$ in country 1 and $\lambda(\kappa_2)$ and $\omega(\kappa_2)$ in country 2. Given the interest rates and the real exchange rate, firms in country 1 solve the problem:

$$\max_{x_{11},x_{12}} \left\{ A_1\lambda(\kappa_1)^\nu \Big[x_{11}^\epsilon + \omega(\kappa_1)\cdot x_{12}^\epsilon \Big]^{\frac{\nu}{\epsilon}} - (x_{11} + \bar{e}\cdot x_{12})(1+R_1) \right\} \tag{15}$$

Taking the first order conditions and rearranging, the solution is:

$$x_{11} = \left(\frac{\nu A_1 \lambda(\kappa_1)^\nu}{1+R_1} \right)^{\frac{1}{1-\nu}} \left[1 + \omega(\kappa_1) \left(\frac{\omega(\kappa_1)}{\bar{e}} \right)^{\frac{\epsilon}{1-\epsilon}} \right]^{\frac{\nu-\epsilon}{\epsilon(1-\nu)}} \tag{16}$$

$$x_{12} = \left(\frac{\omega(\kappa_1)}{\bar{e}} \right)^{\frac{1}{1-\epsilon}} x_{11} \tag{17}$$

Equations (16) and (17) are the demands for the domestic and foreign inputs. These demand functions take the same form as the demand functions derived in the one-period model. The only difference is that now the parameters of the production function depend on the previous choice of κ_1. Similar demand functions are derived for country 2.

Given the current states κ_1 and κ_2, and the interest rates chosen by the two monetary authorities, we can derive the equilibrium conditions in the goods markets and the foreign exchange market. These equilibrium conditions are the same conditions we have seen in the static model, that is, (5), (6) and (7). Using these conditions and the input demands (16) and (17), the general equilibrium is defined by the following three equations:

$$C_1 = \left(\frac{\nu A_1 \lambda(\kappa_1)^\nu}{1+R_1} \right)^{\frac{1}{1-\nu}} \left(\frac{1+R_1-\nu}{\nu} \right) \left[1 + \omega(\kappa_1) \left(\frac{\omega(\kappa_1)}{\bar{e}} \right)^{\frac{\eta}{1-\eta}} \right]^{\frac{\nu(1-\eta)}{\eta(1-\nu)}} \tag{18}$$

$$C_2 = \left(\frac{\nu A_2 \lambda(\kappa_2)^\nu}{1+R_2} \right)^{\frac{1}{1-\nu}} \left(\frac{1+R_2-\nu}{\nu} \right) \left[1 + \omega(\kappa_2) \left(\omega(\kappa_2)\bar{e} \right)^{\frac{\eta}{1-\eta}} \right]^{\frac{\nu(1-\eta)}{\eta(1-\nu)}} \tag{19}$$

$$\left[\frac{A_1\lambda(\kappa_1)^\nu (1+R_2)}{A_2\lambda(\kappa_2)^\nu (1+R_1)} \right]^{\frac{1}{1-\nu}} = \bar{e}^{\frac{1+\eta}{1-\eta}} \left[\frac{1+\omega(\kappa_2)(\omega(\kappa_2)\bar{e})^{\frac{\eta}{1-\eta}}}{1+\omega(\kappa_1)\left(\frac{\omega(\kappa_1)}{\bar{e}}\right)^{\frac{\eta}{1-\eta}}} \right]^{\frac{\nu-\eta}{\eta(1-\nu)}} \tag{20}$$

These equations are similar to equations (18)-(20) in the analysis of the one-period model and they are derived in a similar fashion (see Appendix A.1). Some of the parameters, however, are now functions of the input ratios κ_1 and κ_2. These ratios were chosen in the previous period and they are given in this stage.

Given the input ratios κ_1 and κ_2, the policy equilibrium can be derived as in the analysis of the static model through the intersection of the reaction functions of the two countries. The important point, here, is that the equilibrium depend on the input ratios

κ_1 and κ_2 and on the level of technology A_1 and A_2. Therefore, we can express the equilibrium interest rates as a function of these variables, that is, $(R_1, R_2) = \Psi(A_1, A_2, \kappa_1, \kappa_2)$. We will refer to this function as the "policy function". Similarly, we can express the equilibrium real exchange rate as a function of the same variables, that is, $\bar{e}(A_1, A_2, \kappa_1, \kappa_2)$. With this in mind, we can now study the equilibrium in the first period when κ_1 and κ_2 are determined.

First-period equilibrium: In the first period firms choose the input ratio by maximizing next period profits which depend on the next period interest rates and real exchange rate. The next period interest rates will be determined by the policy function $(R_1, R_2) = \Psi(\kappa_1, \kappa_2, A_1, A_2)$ and the real exchange rate by the function $\bar{e} = \bar{e}(\kappa_1, \kappa_2, A_1, A_2)$. These functions where defined above.

Let $V_i(A_i, R_i, \bar{e}, k)$ be the surplus of a firm in country i, where k denotes the input ratio chosen by this firm. A brief inspection of the objective (15) shows that this surplus depends on the level of technology, the interest rate, the real exchange rate and the input ratio. The problem solved by the firm in the first period can then be written as:

$$h_i(A_1, A_2, \kappa_1, \kappa_2) = \arg\max_k V_i\Big(A_i, \Psi_i(A_1, A_2, \kappa_1, \kappa_2), \bar{e}(A_1, A_2, \kappa_1, \kappa_2), k\Big) \tag{21}$$

The function $h_i(A_1, A_2, \kappa_1, \kappa_2)$ gives the input ratio that a firm in country i would choose if the levels of technology are A_1 and A_2 and all other firms choose κ_1 in country 1 and κ_2 in country 2. Because firms in each country are homogeneous, they will all choose the same input ratio. This implies that in the policy equilibrium $h_i(A_1, A_2, \kappa_1, \kappa_2)$ must be equal to κ_i. We then have the following definition of policy equilibrium:

Definition 1 (Discretionary equilibrium in two-period model) *A discretionary policy equilibrium is defined by a couple (κ_1^*, κ_2^*) satisfying:*

$$\begin{aligned} \kappa_1^* &= h_1(A_1, A_2, \kappa_1^*, \kappa_2^*) \\ \kappa_2^* &= h_2(A_1, A_2, \kappa_1^*, \kappa_2^*) \end{aligned}$$

Therefore, the equilibrium of the two-period model is defined as the fixed point of the mapping h.

II.2 Equilibrium with commitment

With policy commitment, the interest rates are chosen in the first period. When the monetary authorities decide their policies, they understand that firms are not committed yet to the production plan. Because there is not uncertainty between the first and the second period, firms will always choose the input ratio based on the long-term technology. In this case we can ignore the short-term technology and the equilibrium is characterized in the same manner we characterized it in the one-period model using the long-term technology.

II.3 Lack of commitment and inflation bias

The important point of this section is that without the ability to commit, policy competition will induce higher interest rates and inflation. A numerical example illustrates this point. We assume that $A_1 = A_2 = 1$, $\nu = 0.9$ and we consider alternative values for the long-term elasticity $1/(1-\epsilon)$ and the short-term elasticity $1/(1-\eta)$. After choosing ϵ, the parameter ϕ is chosen such that in the equilibrium with common currencies imports are 25 percent of domestic net production (GDP). Table 2 reports the equilibrium interest rates and welfare losses relative to the common currency.

Table 2: Equilibrium inflation, interest rates and consumption losses relative to a common currency with and without policy commitment.

	LT elasticity = 1.4 ST elasticity = 1.1	LT elasticity = 1.6 ST elasticity = 0.9	LT elasticity = 1.8 ST elasticity = 0.7
Common currency			
Interest rate	0.00	0.00	0.00
Policy commitment			
Interest rate	6.49	5.30	4.48
Consumption loss	1.02	0.70	0.51
Policy discretion			
Interest rate	9.67	14.14	24.40
Consumption loss	2.12	4.17	10.57

As shown in the table, the interest rates and the consumption losses increase when countries are not able to commit to future policies (discretion). The interest rate bias from policy discretion is larger when the short-term elasticity is more different from the long-term elasticity.

The intuition for these results can be illustrated with the help of Figure 5. Consider first the equilibrium with commitment. Assume that this equilibrium is at point B. This point will be associated with interest rates R_1^B and R_2^B. For this to be an equilibrium, a unilateral change in the interest rate from one country can not increase the welfare of that country. In evaluating the welfare consequences, the monetary authorities use the flatter isoquant because firms are not committed yet to the production plan. Because firms are flexible at this stage, an increase in the interest rate does not have a large impact on the real exchange rate (i.e. the *terms of trade* effect is small). As a result, the equilibrium interest rates must be relatively low.

Now consider the case of policy discretion and assume that we start from the same point B, which is the equilibrium with policy commitment. Do the monetary authorities have an incentive to increase the interest rate? The answer is yes. This is because firms are now committed to the production plan and face the more curved isoquant. In the short run they are less flexible. Consequently, a "unilateral" increase in the interest rate

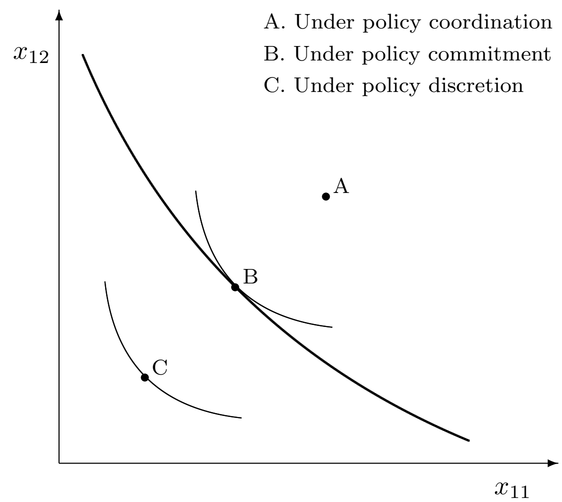


Figure 5: Equilibrium under policy commitment and policy discretion.

will have a large impact on the real exchange rate and will increase the welfare of the deviating country. Only after the interest rates have reached certain levels, they no longer have an incentive to deviate. At this point the terms of trade effect is counterbalanced by the liquidity effect. The equilibrium with policy discretion will be at point C which is characterized by higher interest rates, lower production and lower welfare.

Finally, we can compare the equilibrium with discretionary policies to the equilibrium that would prevail if the monetary authorities coordinate their policies. In this case the interest rates would be zero independently of whether they choose these interest rates in the first period (policy commitment) or in the second period (policy discretion). In Figure 5 the equilibrium with policy coordination is represented by the point A. This point is characterized by higher production and consumption.

We summarize the results of the two-period model as follows:

Result 2 (Lack of policy coordination and commitment) *Policy competition induces higher interest (and inflation) rates. The interest rate bias is exacerbated by the lack of policy commitment.*

III The general model

In this section we extend the model to an infinite horizon setting and introduce aggregate shocks. In extending the model, we also specify the whole monetary/financial sector. This allows us to make precise the relationship between the nominal interest rate and the inflation rate. We retain the assumption that capital is not mobile internationally. The role of international financial transactions will be discussed in Appendix B.

The results of this section show that the equilibrium in the infinite horizon model is equivalent to a sequence of equilibria in the two-period model. To solve the infinite horizon model it is sufficient to solve for the two-period model studied in the previous section. Readers not interested in the derivation of this result can go directly to Section IV where we conduct the quantitative analysis.

Households: Households are infinitely lived and they maximize the lifetime utility $E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$ where β is the discount factor and c_t is consumption. Households hold financial assets in domestic banks. Henceforth we will refer to these financial assets as deposits and they are denoted by d. Deposits are decided at the end of the period and households have to wait until the end of the next period before being able to change these deposits. This is the assumption usually made in the class of "Limited Participation" models as in Christiano & Eichenbaum (1995), Fuerst (1992) and Lucas (1990).

In addition to bank deposits, households also own liquid assets for transaction purposes as they face a cash-in-advance constraint in the purchase of consumption goods. The cash-in-advance constraint in country i is $P_i c_i \leq n_i$, where n_i is the liquid funds retained at the end of the previous period and P_i is the nominal price. The beginning-of-period financial assets in country i are equal to the retained liquidity plus the nominal value of domestic deposits, that is, $n_i + d_i$.

Households solve a dynamic problem in their portfolio choice. As shown in Appendix A.2, the optimality condition for the choice of next period deposits in country i is:

$$E\left(\frac{u_c(c_i')}{P_i'}\right) = \beta E\left(\frac{(1+R_i')u_c(c_i'')}{P_i''}\right) \tag{22}$$

where u_c is the marginal utility of consumption and the prime denotes the next period variable (double prime denotes the variable two periods from now). From this equation is now easy to see the connection between the nominal interest rate and the inflation rate. In particular, if we abstract from uncertainty and consider a steady state in which consumption is constant, the above condition can be written as $P_i'/P_i = \beta(1+R_i)$. Therefore, higher is the inflation rate between today and tomorrow, and higher is the nominal interest rate in the current period.

Production technology and the financial sector: Firms are also infinitely lived and they operate the same production technology described in the two-period model. At the end of each period they choose the optimal input ratio κ_i based on the long-term technology (11), and at the beginning of next period they choose the intermediate inputs based on the short-term technology (12). The level of technology is allowed to change stochastically over time. More specifically, we assume that $A_{i,t} = \bar{A}e^{z_{i,t}}$ where $z_{i,t}$ is a country-specific shock that follows a first order Markov process. The shock becomes known at the end of the previous period. At the moment of choosing the long-term plan, firms already know the level of technology. The alternative assumption is that the shock is revealed after the choice of the production plan. This would complicate the analysis but would not change the main results of the paper.

As in the simplified version of the model, firms need to finance the purchase of the intermediate inputs. Loans are contracted only in domestic currency. As in the simplified version of the model, the loan contracted by a firm in country 1 is $P_1(x_{11} + \bar{e}x_{12})$ and the loan contracted by a firm in country 2 is $P_2(x_{22} + x_{21}/\bar{e})$. Banks make loans only in the currency in which they receive deposits. Because firms contract loans that are denominated in domestic currency, this implies that domestic firms borrow only from domestic banks.

Monetary policy: The monetary authority of each country chooses the policy instrument (interest rate) optimally in the sense of maximizing the welfare of the domestic households, taking as given the policy strategy of the other country. We first consider the case of time-consistent policies meaning that the monetary authorities cannot commit to future policies. In the class of time-consistent policies we restrict the analysis to Markov strategies, that is, policies that depend only on the current (physical) states of the economy. The equilibrium of this environment will be contrasted with two alternatives: an environment where the monetary authorities choose the whole sequence of policy instruments today (i.e. it can commit) and one where the monetary authorities coordinate their policies (i.e. where they have a common currency).

The monetary authorities choose the interest rates by controlling the domestic monetary aggregates. The monetary aggregate, in turn, is controlled by making transfers to households in the form of bank deposits. The pre-transfer monetary aggregate in country i is denoted by M_i and the monetary transfer by $T_i = g_i M_i$, where g_i is the growth rate of money. Because transfers are in the form of bank deposits, higher transfers increase the liquidity available to domestic intermediaries to make loans. By limiting the ability of households to readjust their portfolio of deposits, the increase in liquidity induces a fall in the nominal interest rate.[4]

Equilibrium conditions for given κ_1 and κ_2. Before studying the policy problem of the two monetary authorities, let's derive first the equilibrium conditions for given input ratios κ_i and interest rates R_i. The problem faced by the firm in the choice of the intermediate inputs is the same problem faced in the two-period model (see Section II.1). Given the current states and the interest rates chosen by the two monetary authorities, we can derive the equilibrium conditions in all markets: the goods markets, the loan

[4]As we will show below, the specification of the policy tool as an interest rate or as a growth rate of money is equivalent because in the model there is a unique correspondence between the growth rate of money and the nominal interest rate.

markets, the money markets and foreign exchange market. The current states are given by the stocks of money, the stock of deposits and the input ratios chosen by firms in the previous period.

The equilibrium conditions in the goods markets do not change and they are still given by equations (5) and (6). Because we keep the assumption that there is not international financial transactions, the equilibrium condition in the exchange rate market is still given by (7). The equilibrium conditions in the loan markets are:

$$P_1(X_{11} + \bar{e} \cdot X_{12}) = D_1 + T_1 \quad (23)$$

$$P_2(X_{22} + X_{21}/\bar{e}) = D_2 + T_2 \quad (24)$$

The left-hand-side is the demand for loans from domestic firms and the right-hand-side is the supply of loans from domestic banks. The supply of loans is given by the deposits of domestic residents, D_i, and the monetary injection, T_i.

These equilibrium conditions can be used to derive the relation between the interest rate and the growth rate of money. This relation is characterized in the following lemma.

Lemma 1 *The nominal interest rate in country i is determined by the domestic growth rate of money according to:*

$$1 + R_i = \frac{1 + g_i}{D_i/M_i + g_i} \quad (25)$$

Proof: *Appendix A.3.*

Equation (25) tells us that, given the stocks of deposits, there is a unique relationship between the domestic growth rate of money and the domestic interest rate. Moreover, the domestic interest rate is not affected by the interest (and money growth) rate in the other country. This implies that the specification of the monetary policy instrument in terms of money growth rate or interest rate is equivalent. Because we have chosen the interest rate as the policy instrument, equation (25) can be used to determine the growth rate of money consistent with the chosen interest rate.

III.1 Policy equilibrium with discretion: Time-consistent policies

Because we limit the set of policy strategies to be Markov, the policy equilibrium is defined by the function $(R_1, R_2) = \Psi(\mathbf{s})$ which returns the current interest rates as a function of the aggregate states $\mathbf{s}$. After normalizing all the nominal variables by the domestic stock of money, the aggregate states are given by the levels of technology, the normalized stock of deposits and the input ratios chosen by firms in the previous period. Formally, $\mathbf{s} = (A_1, A_2, \kappa_1, \kappa_2, D_1, D_2)$.

The policy function $\Psi(\mathbf{s})$ is the solution of the repeated policy game played by the two monetary authorities. The strategy played by country i determines the current interest rate as a function of (i) the aggregate states $\mathbf{s}$; (ii) the other country's interest rate R_{i^*}; and (iii) a policy function $\Psi(\mathbf{s})$ determining future interest rates. We denote the policy strategy of country i as $R_i(\mathbf{s}, R_{i^*}; \Psi)$. Using these strategies we can define an equilibrium for a given policy function determining the future interest rates as follows:

Definition 2 (Policy equilibrium for given Ψ) *A policy equilibrium for given Ψ is defined by a "current" policy function $R_i = \psi_i(\mathbf{s}; \Psi)$ and countries' policy strategies $R_i(\mathbf{s}, R_{i^*}; \Psi)$, such that:*

(a) The policy strategies $R_i(\mathbf{s}, R_{i^*}; \Psi)$ maximize the welfare of country i;

(b) The current policy function ψ_i and the strategies R_i satisfy:

$$\begin{aligned} \psi_1(\mathbf{s}, \Psi) &= R_1(\mathbf{s}, \psi_2(\mathbf{s}; \Psi); \Psi) \\ \psi_2(\mathbf{s}, \Psi) &= R_2(\mathbf{s}, \psi_1(\mathbf{s}; \Psi); \Psi) \end{aligned}$$

The current policy function $\psi_i(\mathbf{s}, \Psi)$ is the solution of the policy game played by the two countries in the current period when future interest rates are determined by the policy function Ψ. Consistent with the definition of a Nash equilibrium, the strategies played by each country must be the optimal responses to the strategy played by the other country (condition (a)), and the equilibrium interest rates are such that countries do not have an incentive to deviate from these strategies (condition (b)).

The above definition imposes optimality in the choices of current policies but it takes as given the function that determines future policies, the function Ψ. This function is also determined as a solution to the policy game played by the two countries in future periods. Therefore, we have to make sure that future policies are also the results of optimal strategies played by the two countries. We then have the following definition of a discretionary policy equilibrium:

Definition 3 (Discretionary equilibrium) *A discretionary Markov perfect equilibrium is defined as a fixed point $\Psi^*(\mathbf{s})$ of the following mapping:*

$$\Psi(\mathbf{s}) = \psi(\mathbf{s}; \Psi)$$

In other words, the policy function Ψ determining future policies must be exactly equal to the current policy function ψ. When this condition is satisfied, the policy function that agents assume to determine future policies is the solution of the policy game played by the two countries in future periods. This definition will be useful in the characterization of the equilibrium.

Proposition 1 *The Markov perfect equilibrium of the repeated policy game is the equilibrium of the two-period model defined in Section II.1.*

Proof: *Assume that future interest rates are determined by the policy function* $\Psi(\mathbf{s}')$, *where* $\mathbf{s}' = (A_1', A_2', \kappa_1', \kappa_2', D_1', D_2')$. *In this economy the current equilibrium does not affect the future states* $\mathbf{s}' = (A_1', A_2', \kappa_1', \kappa_2', D_1', D_2')$. *In fact, even though the variables* κ_1', κ_2', D_1' *and* D_2' *are chosen in the current period, they only depend on the expectation of future policies. This can be easily seen by inspecting the firms' problem (15) and the households' first order condition (22). Therefore, the current interest rates do not affect next period states and next period equilibrium. This implies that the policy makers solve a static problem and the optimal strategies are independent of the function* Ψ *determining future policies.*

The key feature of this economy is that, whatever is done in the current period, this is not going to affect the future states. The new input ratios and the new stocks of bank deposits will be affected only by future policies but are not affected by the current states and current policies. Because the current policies cannot affect the next period states, they also do not impact on the next period policies. Therefore, from the prospective of the policy makers, the function that determines future policies is irrelevant for the choice of the current optimal strategies.

III.2 Equilibrium with commitment: Ramsey policies

The usual approach to the study of optimal policies with commitment in an infinite horizon setting is the Ramsey allocation. In this setting, the monetary authorities of the two countries choose the whole sequence of state contingent interest rates in the initial period, taking as given the sequence of state contingent interest rates chosen by the other country. Interest rates are state contingent because the two economies are subject to technology shocks.

Let $\mathbf{h}^t = \{z_{1,0}, z_{2,0}, ..., z_{1,t}, z_{2,t}\}$ be the history of shock realizations from time zero up to time t and let $\mathbf{H}^t$ be the collection of all possible histories. Denote by $R_i(\mathbf{h}^t)$, the sequence of state contingent interest rates in country i for all $\mathbf{h}^t \in \mathbf{H}^t$ and $t \geq 0$. A policy strategy with commitment for country i can then be expressed as $R_i(\mathbf{h}^t) = r_i(\mathbf{h}^t; R_{i^*}(\mathbf{h}^t))$. Using this notation, an equilibrium with commitment is defined as:

Definition 4 (Equilibrium with commitment) *An equilibrium with commitment is a sequence of state contingent interest rates* $R_i^*(\mathbf{h}^t)$ *and countries' strategies* $r_i(\mathbf{s}; R_{i^*}(\mathbf{h}^t))$ *such that:*

(a) The policy strategies $r_i(\mathbf{s}; R_{i^*}(\mathbf{h}^t))$ *maximize welfare in country* i;

(b) The state contingent interest rates $R_i^*(\mathbf{h}^t)$ *satisfy:*

$$\begin{aligned} R_1^*(\mathbf{h}^t) &= r_1(\mathbf{s}; R_2^*(\mathbf{h}^t)) \\ R_2^*(\mathbf{h}^t) &= r_2(\mathbf{s}; R_1^*(\mathbf{h}^t)) \end{aligned}$$

for all $\mathbf{h}^t \in \mathbf{H}^t$ *and* $t \geq 0$.

This is the standard definition of a Nash equilibrium. The only complication derives from the fact that the state space for the players' action is a sequence of functions. The next proposition characterizes the policy equilibrium with commitment.

Proposition 2 (Nash Ramsey equilibrium) *The Ramsey policy equilibrium is the equilibrium under policy commitment of the two-period model defined in Section II.2.*

Proof: *Because all the interest rates are chosen in advance, firms are not committed yet to the long-term plans. Therefore, the short-term technology becomes irrelevant. The same argument used in the proof of proposition 1 then shows that the interest rates chosen in a particular contingency do not affect future equilibria. Therefore, the equilibrium in the two-period model is also the equilibrium in the infinite horizon model.*

III.3 Equilibrium with policy coordination: Common currencies

A primary goal of this paper is to evaluate the gains from adopting a common currency. The adoption of a common currency is only the visible and perhaps the least important aspect of a more complex process of capital market integration and liberalization. The process leading to a common currency is associated with increasing financial integration of the countries adopting the currency. This process of financial integration is not only a consequence of legal liberalization, but also the result of market reactions (the elimination of the exchange rate risk, for example, facilitates foreign financial investments).

In the model, the process of financial integration following the adoption of a common currency is captured by the fact that firms borrow indifferently from domestic and foreign banks. With multiple currencies, banks have relationships only with domestic firms because they lend only in the currency in which their deposits are denominated and firms borrow only in domestic currency. With a common currency, however, the nationality of the currency disappears and the interest rate in the two countries will be equalized.

The other important aspect of the process leading to a common currency is how this changes policy decisions. Although monetary policy becomes centralized, the objective of the policy maker can be different depending on whether it is a currency union, as in the EMU, or the unilateral adoption of another countries currency, as in dollarization. In the first case we assume the policy maker maximizes the weighted welfare of the households in the two countries. We interpret this to be what happens in a "currency union". In the second case, we assume that the policy maker maximizes only the welfare of the first country. We interpret this to be the practice with dollarization.

After the adoption of the common currency, the households in both countries receive transfers from the unified monetary authority. This is unlikely to be the case, especially when one country unilaterally adopts the currency of the other country. However, the assumption that monetary policy interventions take the form of monetary transfers is made only for analytical convenience. We should think of these interventions as open market operations conducted by the monetary authority(s) with banks. To maintain the neutrality of monetary policy interventions in redistributing wealth, we assume that monetary transfers are distributed proportionally to the households of the two countries. In this paper we neglect possible gains and losses of seigniorage that occur when one country adopts the currency of another.

In the environment with a common currency, the equilibrium conditions in the goods markets do not change and they are still given by equations (5) and (6). The loans markets, instead, become unified and conditions (23) and (24) are replaced by the new equilibrium condition:

$$P_1 X_{11} + P_2 X_{12} + P_2 X_{22} + P_1 X_{21} = D_1 + T_1 + D_2 + T_2 \tag{26}$$

where now P_1 and P_2 are denominated in the same currency. Although prices have the same denomination, they are not necessarily the same. The next proposition establishes the optimal policy with common currencies.

Proposition 3 (Optimal policy with common currencies) *With common currencies the optimal policy is the Friedman rule of a zero interest rate independently of the weight assigned by the monetary authority to the two countries and independently of whether the monetary authority can commit to future policies.*

Proof: *See appendix A.4.*

The intuition for this result derives from the cost nature of the interest rate. Because the interest rate acts as a production tax, a positive value of R is inefficient. For given values of the shocks, the net production of both countries is affected negatively by a higher interest rate. Therefore, independently of the weights given to the welfare of the two countries, the policy maker will set the interest rate at the lowest possible value, that is, zero.

This result has an important implication: there is equivalence between a currency union and the unilateral adoption of a foreign currency as in the case of dollarization. The financial integration and the consequent unification of the interest rates of the two countries imply that monetary policy has a similar impact on the welfare of the two countries. Consequently, pursuing the maximal welfare of one country automatically maximizes the welfare of the other. The maximal welfare is obtained by minimizing the financial costs of firms, that is by setting $R = 0$.

IV Quantitative analysis

We parameterize the model using data from the 11 European countries that became members of the European Monetary Union in 1999. This is motivated by the fact that the EMU is one of the most important recent examples of monetary unification.[5] For simplicity we interpret the two-country interaction as representative of the strategic interaction among the original member countries of the European Monetary Union. We then use average values between these countries to calibrate the model.

We parameterize the model assuming a period is a quarter and the discount factor β is 0.995. We assume logarithmic utility, that is, $u(c) = \log(c)$. The specification of the utility function, however, is not important.

The production technology is characterized by the parameters ν, ϕ, ϵ, η, and by the level of technology $A_1 = e^{z_1}$ and $A_2 = e^{z_2}$, where z_1 and z_2 are the technology shocks in country 1 and country 2 respectively. To calibrate ν we observe that the fraction of liquid funds used by households for transaction purposes is approximately equal to $1 - \nu$.[6] If we take the monetary aggregate M1 as the measure of liquid funds used for transaction by households and M3 as the measure of their total financial assets, then the average ratio of these two monetary aggregates will determine $1 - \nu$. Therefore, we set ν =1-M1/M3=0.9 which is in the order of values for the EMU countries.

The parameters ϵ and η affect the degree of substitutability between domestic and foreign inputs in the long-run and in the short-run. Reinert & Roland-Holst (1992) estimates the Ermington elasticities at the industry level for the U.S. manufacturing sector. The average elasticity is 0.9. These are short-term elasticities of substitution. Gallaway et al. (2000) also estimate the long-term elasticities and they find that on average the long-run elasticities are about twice as large as the short-run elasticities. Although these estimates are not for the European countries, there is not reason to believe that they are dramatically different.[7] Based on these numbers we set the short-term elasticity $1/(1 - \eta)$ to 0.9 and the long-term elasticity $1/(1 - \epsilon)$ to 1.6. These elasticities are not very different from the range of numbers used in international business cycles studies. For example, Backus, Kehoe, & Kydland (1994) use an elasticity of 1.5 and Stockman & Tesar (1995) use an elasticity of 1. We will also conduct a sensitivity analysis to show how these parameters affect the results of the paper.[8]

After fixing η and ϵ, the value assigned to ϕ is such that in the steady state with discretionary policies the value of imports plus export (the openess index) is 50 percent the value of net output (GDP). This is the average value of imports plus export for the

[5] The group of countries that became part of the EMU in 1999 includes Austria, Belgium, Germany, Finland, France, Ireland, Italy, Luxembourg, Netherlands, Portugal, Spain. We do not include Greece because it became a member three years later. The exclusion of Greece does not affect the results given that the size of the Greek economy is small relative to the size of the whole Union.

[6] It will be exactly $1 - \nu$ if the interest rate is zero.

[7] If there are differences, it is more likely that European elasticities are smaller than the U.S. elasticity. See chapter 3 in Deardoff & Stern (1990) and chapter 5 in Whalley (1985). This will reinforce the results of the paper.

[8] Notice that given our calibration of $\nu = 0.9$, the condition $\epsilon < \nu$ imposes that the elasticity of substitution between domestic and foreign inputs is smaller than 10. Because all the empirical estimates of the Ermington elasticities are much smaller than 10, the $\epsilon < \nu$ is not very restrictive.

EMU countries weighted by the relative size of these economies at the end of the 1970s as reported in Table 3. We consider the end of 1970s as the starting point of the current unification. In fact, 1979 was the starting date of the European Monetary System. The value of ϕ that generates the desired openess target is 0.117.

Table 3: Trade openess of EMU countries in the periods 1974-79, 1980-89 and 1990-2000.

	Imports+Exports %GDP, 1974-79	*Imports+Exports %GDP, 1980-89*	*Imports+Exports %GDP, 1990-00*
Austria	65.0	74.1	81.1
Belgium	111.8	133.2	139.6
Finland	54.9	56.7	62.2
France	39.3	43.8	45.7
Germany	46.6	55.0	53.0
Ireland	98.5	106.9	138.1
Italy	44.5	42.7	45.4
Luxembourg	192.5	211.9	225.8
Netherlands	95.4	107.0	110.1
Portugal	49.2	65.9	67.4
Spain	29.5	37.3	45.9
EMU average	49.2	55.3	58.3

Source: *OECD historical statistics 1970-2000*

We assume that in both countries the technology shock follows the autoregressive process $z' = \rho_z z + \varepsilon$ with $\rho_z = 0.95$. The innovations ε_1 and ε_2 are jointly normal with mean zero. Specifically we assume that $\varepsilon_1 = \rho_\varepsilon \varepsilon_2 + \upsilon$ where $\varepsilon_1 \sim N(0, \sigma_\varepsilon^2)$ and $\upsilon \sim N(0, \sigma_\upsilon^2)$. The parameter ρ_ε determines the correlation structure of the shocks in the two countries. We will consider several cases: the case of positive correlation, independence and negative correlation. Once we have fixed ρ_ε, the other two parameters, σ_ε and σ_υ, are calibrated so that the volatility of aggregate output in the model is equal to the average GDP volatility in the EMU countries.

IV.1 Loss of long-term monetary independence

We start by analyzing the quantitative properties of the model when there is not uncertainty, that is, $z_1 = z_2 = 0$ at all times. Table 4 reports the equilibrium inflation rates, interest rates and welfare gains for the two countries in absence of technology shocks. Three different environments are considered. The first environment is when countries have adopted the common currency. In the second environment countries choose their policies strategically and they commit to future policies (policy commitment). In the third environment, countries also act strategically but they are unable to commit to future policies (policy discretion). The equilibrium is computed for different values of η and ϵ which determine the short-term and long-term substitutability between domestic and foreign inputs. The welfare consequences are computed as the percentage decrease (or increase if negative) in consumption respect to the common currency.

Table 4: Equilibrium inflation, interest rates and consumption losses relative to a common currency. Values are in annual percentage.

	(*Baseline model*) ST elasticity = 0.9 LT elasticity = 1.6	ST elasticity = 0.7 LT elasticity = 1.8	ST elasticity = 1.1 LT elasticity = 1.4
Common currency			
Inflation	-1.98	-1.98	-1.98
Interest rate	0.00	0.00	0.00
Policy commitment			
Inflation	3.22	2.41	4.37
Interest rate	5.31	4.48	6.49
Consumption loss	0.70	0.51	1.02
Policy discretion			
Inflation	11.90	21.93	7.49
Interest rate	14.10	24.40	9.67
Consumption loss	4.17	10.57	2.12

We observe first that with common currencies the equilibrium nominal interest rate is zero and the inflation rate negative. As stated in proposition 3, the Friedman rule of a zero nominal interest rate is the optimal policy. However, without coordination, each country has an incentive to deviate from this policy which explains the higher inflation (and interest) rates in the competitive environments. Because both countries will deviate from the Friedman rule in absence of coordination, neither of them will benefit from the higher interest rates. The inflation bias and the welfare losses are larger when the monetary authorities cannot commit to future policies. This is because in this case an interest rate increase has a larger impact on the real exchange rate—given the inflexibility of firms—which induces the monetary authorities to choose higher interest rates. In this case the welfare gains from policy coordination are quite large, about 4 percent of consumption.

When the monetary authorities commit to future policies, instead, the interest rates are chosen before the firms commit to the long-term plan. Given the higher flexibility of firms, an interest rate increase does not have a large impact on the real exchange rate and the incentive to raise the interest rates is lower. In this case the welfare gain from policy coordination is 0.7 percent.

The second and third columns of Table 4 repeat the calculation for different elasticities of substitution. The inflation bias is larger the less substitutable domestic and foreign imports are, that is, the smaller the value of η and ϵ. The inflation bias almost vanishes when domestic and foreign imports are good substitutes and the monetary authorities can commit to future policies.

Although the EMU countries now have coordinated policies within the Union, they are still trading with countries that are not part of the Union. Therefore, they are still facing strategic problems with countries that are not part of the monetary union. This may explain why the interest rates in the EMU countries have declined but they are still greater than zero. This observation also points out that the numbers reported in Table 4 over-estimates the real welfare impact of the introduction of the Euro.

An important consequence of the monetary union is that the area as a whole becomes less dependent on trade. This is because the trade dependence of all member countries is smaller than the external trade dependence of the whole area. Before the monetary integration, the relevant trade dependence for the choice of policies was the trade dependence of each individual country. After the monetary union, however, it becomes relevant the *external* trade dependence of the whole area. Therefore, a better evaluation of the impact of the European Monetary Union would compare the equilibrium under policy competition after reducing the dependence on trade. More specifically, the openness index (imports plus export as a percentage of GDP) of the EMU area was about 50 percent with the inclusion of infra-EMU trade, but only half of this is with countries outside the Union.

Table 5: Steady state equilibrium inflation, interest rates and consumption gains before and after the monetary integration of Europe. Values are in annual percentage.

	With policy commitment	With policy discretion
Pre-integration		
Inflation	3.21	11.87
Interest rate	5.30	14.14
Post-integration		
Inflation	0.56	5.05
Interest rate	2.59	7.17
Consumption gain	0.52	2.51

To account for this lower trade dependence we compute the welfare gains from reducing the interest and inflation rate to the levels that would prevail when the trade dependence is only half the trade dependence of the European countries (including external trade). This is obtained by reducing the value of the parameter ϕ such that in the steady state the volume of imports (and exports) is 12.5 percent of GDP in the competitive equilibrium with discretionary policies. Table 5 reports the inflation rates, interest rates and welfare gains when the degree of trade dependence drops. We interpret these welfare gains as the possible gains from the monetary integration of countries similar to the EMU countries.

IV.2 Loss of long-term monetary independence: heterogeneous countries

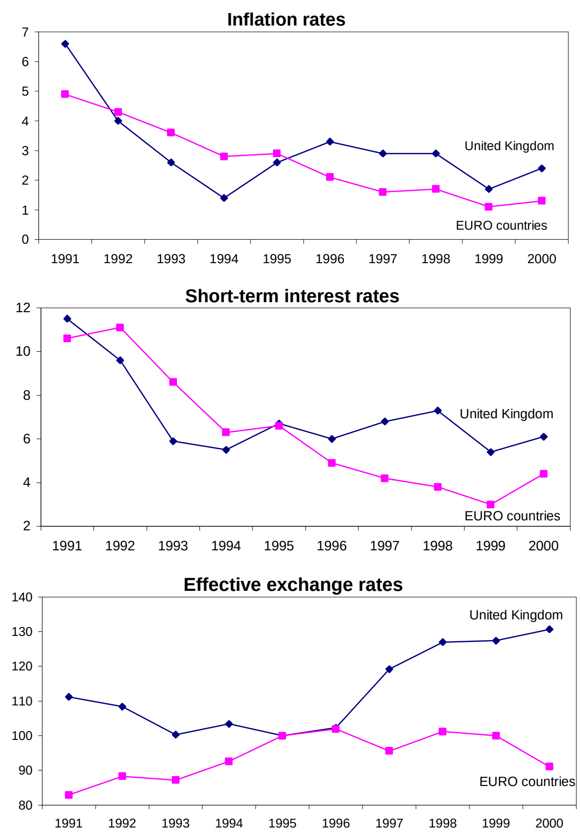
The analysis conducted up to this section assumes symmetry in the technologies and size of the two countries. However, many cases of monetary integration involves countries that are in different stages of economic development and are of different size. The economic size of a country is likely to affect its degree of dependence on the other country. In this section we assume that countries have different population size. We denote by μ the population size of country 2 relative to the population size of country 1. So, for example, $\mu = 2$ means that in country 2 there are twice as many households as in country 1. We assume that countries have the same level of per-capita output. This implies that the import dependence of the larger country is smaller. This is obtained by assuming that countries are also heterogeneous in the parameter ϕ (but they have the same ϵ and η).

Table 6: Steady state equilibrium inflation, interest rates and consumption losses relative a common currency when countries are heterogeneous. Values are in annual percentage.

	$\mu = 2$		$\mu = 5$	
	Country 1	Country 2	Country 1	Country 2
Common currency				
Inflation	-1.98	-1.98	-1.98	-1.98
Interest rate	0.00	0.00	0.00	0.00
Policy commitment				
Inflation	3.44	1.47	3.64	0.06
Interest rate	5.53	3.53	5.74	2.08
Consumption loss	0.24	0.67	-0.34	0.33
Policy discretion				
Inflation	14.23	6.85	16.84	3.06
Interest rate	16.55	9.02	19.21	5.15
Consumption loss	3.30	3.28	2.55	2.11

Table 6 reports the equilibrium values when the population of the country 2 is twice and five times bigger than country 1. The first point to observe is that the interest and inflation rates are higher in the country that is more dependent on foreign inputs, that is, the smaller country. Moreover, as could be anticipated from the previous section, the inflation and interest rates are higher when the countries are unable to commit to future policies. The second point to observe is that when the first country is sufficiently small and it is able to commit to future policies, then it may actually loose from adopting the common currency. This is because for the larger country the negative liquidity effect of a higher interest rate is more important relative to the positive terms of trade effect. Therefore, it will have a lower incentive to increase the interest rate. Because a higher interest rate of the smaller country is not accompanied by a higher interest rate from the larger country, the former could gain from policy competition. However, if the small country is not able to commit to future policies, with policy competition it will hand up with a too high interest rate. In this case, the adoption of a common currency would be preferable.

The case $\mu = 5$ can be interpreted as being like the position of the UK vis-a-vis the EMU. The total population of the EMU countries is in fact about 5 times the population of the UK. At the same time, the per-capita income of the UK is not very different from the EMU average, as assumed in the numerical experiment. Therefore, if we believe that the UK does not have problems of policy commitment, the above exercise suggests that the UK would not gain from adopting the Euro. Notice that the model predicts that the UK should have higher interest and inflation rates than the EMU countries and an appreciated real exchange rate. This seems to be the case in the second half of 1990s as shown in Figure 6. For other countries that have commitment problems, instead, the adoption of the Euro could be the optimal strategy. We may think that the Greek choice to adopt the Euro in 2001 can be rationalized in these terms.



Source: OECD historical statistics 1970-2000

Figure 6: Inflation, interest rates and effective exchange rates for the UK and the EMU countries, 1991-2000

IV.3 Loss of cyclical monetary policy independence

One of the objective of this paper is to quantify the welfare costs of losing the ability to respond optimally to internal and external shocks, that is the costs of losing cyclical monetary policy independence in the presence of shocks.

Figures 7 reports the impulse responses of several variables after a positive technology shock in country 1 when country-specific shocks are not correlated. Therefore, the level of technology in country 2 remains constant. Three different policy environments are considered. In the first environment the two countries adopt a common currency. The second environment is when countries choose policies competitively and they are able to commit to future policies (policy commitment). In the third environment countries cannot commit to future policies (policy discretion). Each column of Figure 7 refers to one of the three policy environments.

With a common currency the Friedman rule is the optimal policy, and therefore, the interest rate is kept constant. When countries have different currencies and they act strategically, they respond to the shock by changing the interest rates. Country 1 responds by lowering the nominal interest rate while the reverse takes place in country 2. The interest rate responses are larger when the countries conduct discretionary monetary policy. However, in both cases of discretion and commitment, the response of the interest rates are quantitatively negligible.

In terms of money supply, both countries increase the growth rate of money (except in the first period and for the second country). This monetary expansion is necessary to allow output to expand. Plots (c.1)-(c.3), in fact, shows that the outputs of both countries increase. Although technology shocks are not correlated between countries, the shock in country 1 is transmitted to country 2 through the real exchange rate. Due to the higher demand of foreign goods, the real exchange rate in country 1 depreciates ($\bar{e}$ increases) as can be seen from figures (d.2) and (d.3). The depreciation of country 1 currency corresponds to an appreciation in country 2 which makes foreign inputs cheaper. This allows country 2 to expand.

In order to compute the welfare costs of losing cyclical monetary policy independence, we compare the expected welfare reached under monetary independence with the welfare level reached when the nominal interest rate is kept constant at its long-term value in the strategic equilibrium. The constancy of the interest rate captures the cyclical properties of the optimal monetary policy under a common currency and characterizes the situation that the two countries would face if they lose the ability to react optimally to shocks. These welfare consequences are extremely small, almost insignificant. In all calculations they are less than 0.01 percent of consumption, independently of the correlation structure between the shocks of the two countries.

This result is not surprising given that the policy responses when countries interact strategically are not very different from the Friedman rule (see Figure 7). Because the constancy of the interest rate is the optimal policy when countries adopt a common currency, the cyclical allocations under the two environments do not differ in a substantial way. This is clearly shown in the impulse responses of output under the three policy environments (see plots (c.1)-(c.3)). This result, when evaluated in conjunction with the results of the previous section shows that the possible gains or losses from a common currency derive from the changes in the long-term inflation and interest rates. The loss of the ability to react optimally to internal or external shocks is of secondary concern.

V Conclusion

In this paper we have analyzed the welfare consequences of adopting a common currency either by creating a currency union or by unilaterally adopting a foreign currency. By studying these policies in a two-country open economy model in which monetary policies are decided competitively, we have shown that the lack of coordination may lead to an inflation bias. This inflation bias is further exacerbated by the inability of the monetary authorities to commit to future policies.

In this economy the inflation rate is not neutral even if it is perfectly predicted. In fact, higher systematic inflation rates are associated with higher nominal interest rates. Higher interest rates, in turn, distort the allocation of resources by increasing the financing cost of firms. Consequently, the elimination of the inflation bias following the adoption of a common currency will have large welfare consequences. Assuming that the competing countries are sufficiently homogeneous, they all benefit from coordinating their policies. However, if countries are not homogeneous (for example they have different population sizes) and they do not have problems of policy commitment, then it is not necessarily the case that they all benefit from adopting the same currency. On the other hand, for countries that find difficult to commit to future policies, monetary integration is more likely to bring benefits to all countries even if they are heterogeneous. In all cases, however, the net gains from policy coordination are positive.

Our model (and conclusion) differs from the "new open economy macroeconomic models" in several dimensions. Firstly, in our framework producers do not operate in monopolistic competitive markets and production is not necessarily inefficient. Secondly, money does not enter the utility function and monetary policy interventions have liquidity effects. Thirdly, prices are perfectly flexible. Therefore, the channel through which monetary expansions affect the real sector of the economy is not by increasing nominal aggregate demand but through the reduction in the financing cost of firms. These modeling differences lead to the result that international monetary policy coordination generates significant welfare gains.

A Appendix: Technical derivations and proofs

A.1 Derivation of equations (8)-(10)

The final goods production in country 1 and country 2, and the equilibrium in exchange rate market are given by:

$$C_1 = A_1[X_{11}^{\epsilon} + \phi X_{12}^{\epsilon}]^{\frac{\nu}{\epsilon}} - X_{11} - \bar{e}X_{12} \tag{27}$$

$$C_2 = A_2[X_{22}^{\epsilon} + \phi X_{21}^{\epsilon}]^{\frac{\nu}{\epsilon}} - X_{22} - X_{21}/\bar{e} \tag{28}$$

$$\bar{e}X_{12} = X_{21} \tag{29}$$

The solutions of the firms' problem in country 1 and 2 (see problem (2)) are:

$$X_{11} = \left(\frac{\nu A_1}{1+R_1}\right)^{\frac{1}{1-\nu}} \left[1 + \phi\left(\frac{\phi}{\bar{e}}\right)^{\frac{\epsilon}{1-\epsilon}}\right]^{\frac{\nu-\epsilon}{\epsilon(1-\nu)}} \tag{30}$$

$$X_{12} = \left(\frac{\phi}{\bar{e}}\right)^{\frac{1}{1-\epsilon}} X_{11} \tag{31}$$

$$X_{22} = \left(\frac{\nu A_2}{1+R_2}\right)^{\frac{1}{1-\nu}} \left[1 + \phi\left(\phi\bar{e}\right)^{\frac{\epsilon}{1-\epsilon}}\right]^{\frac{\nu-\epsilon}{\epsilon(1-\nu)}} \tag{32}$$

$$X_{21} = (\phi\bar{e})^{\frac{1}{1-\epsilon}} X_{22} \tag{33}$$

Using these conditions to eliminated X_{11}, X_{12}, X_{21} and X_{22} in equations (27)-(29) we get equations (8)-(10).

A.2 Derivation of condition (22)

Assume that the interest rates are determined by the policy function $\Psi(\mathbf{s})$ where $\mathbf{s}$ are the aggregate states of the economy. In order to use a recursive formulation, we normalize all nominal variables by the pre-transfer stock of money in which the variables are denominated (either M_1 or M_2). After this normalization the aggregate states of the economy are $\mathbf{s} = (A_1, A_2, D_1, D_2)$. The individual households' states are n_i and d_i, where n_i are the liquid assets kept for transactional purposes and d_i are the bank deposits. The problem solved by a household in country i is:

$$\Omega_i(\mathbf{s}, n_i, d_i) = \max_{d_i'} \left\{u(c_i) + \beta\, E\, \Omega_i(\mathbf{s}', n_i', d_i')\right\} \tag{34}$$

subject to

$$c_i = \frac{n_i}{P_i} \tag{35}$$

$$n_i' = \frac{(d_i + g_i)(1+R_i) + P_i\pi_i}{(1+g_i)} - d_i' \tag{36}$$

$$R_i = \Psi_i(\mathbf{s}) \tag{37}$$

$$\mathbf{s}' = H(\mathbf{s}) \tag{38}$$

The derivation of the first order condition for this problem gives (22).

A.3 Proof of lemma 1

Because $M_1 = D_1 + N_1$, using the cash-in-advance constraint and equations (5) and (23), we get:

$$P_1 Y_1 + P_1 (\bar{e} \cdot X_{12} - X_{21}) = M_1 + T_1 \tag{39}$$

which expresses the equality between the volume of transactions executed with the use of domestically denominated liquid funds, and the total quantity of these funds. For country 2, the analog of condition (39) is:

$$P_2 Y_2 - \frac{P_2}{\bar{e}} (\bar{e} \cdot X_{12} - X_{21}) = M_2 + T_2 \tag{40}$$

Using the equilibrium condition in the exchange rate market (7), equations (39) and (40) become:

$$P_1 Y_1 = M_1 + T_1 \tag{41}$$

$$P_2 Y_2 = M_2 + T_2 \tag{42}$$

After eliminating P_1 in equation (41) using the equilibrium condition in the loans market (equation (23)) we get:

$$\frac{Y_1}{X_{11} + \bar{e} X_{12}} = \frac{M_1 + T_1}{D_1 + T_1}$$

Using the production function (12) and the solutions for the firm problem (equations (16) and (17)), it can be verified that the left-hand-side of the above equation is equal to $(1 + R_1) M_1$. After dividing by M_1, we get (25).

A.4 Proof of proposition 3

From the cash-in-advance constraints we have:

$$P_1 C_1 = M - D_1 - D_2 - N_2 \tag{43}$$

$$P_2 C_2 = N_2 \tag{44}$$

Because all the right-hand-side variables are determined at the beginning of the period, the nominal value of consumption is also determined, that is:

$$\frac{P_2 C_2}{P_1 C_1} = \bar{e} \frac{C_2}{C_1} = \frac{N_2}{M - D_1 - D_2 - N_2} \tag{45}$$

Consumption in the two countries is still given by (18) and (19). Because the interest rate in the two country is the same, the ratio C_2/C_1 is independent of R, and given the technology levels, this ratio only depends on $\bar{e}$. Given this result, equation (45) uniquely determines the real exchange rate (or price ratio) $\bar{e}$ and this rate does not depend on R. Therefore, the common interest rate does not affect consumption through the real exchange rate. This implies that we can determine the effect of R on C_1 and C_2 using equations (18) and (19), keeping $\bar{e}$ constant. It can be easily verified that R has a negative effect on the consumption of both countries. Therefore, the optimal policy is to choose the minimum value of the interest rate, that is $R = 0$, independently of the welfare weights.

B Appendix: International mobility of capital

In this section we generalize the infinite horizon model by allowing for international mobility of capital. We do this by allowing households to hold financial assets (deposits) in foreign banks. To simplify the analysis, we assume that foreign deposits are always denominated in the currency of the first country. By making this assumption, we need to keep track only of the net foreign position of country 2. The net stock of foreign deposits of households in country 2 (foreign deposits if positive and foreign debt if negative) is denoted by $\tilde{d}_2$.

For households in country 1 the cash-in-advance constraint is still $P_1 c_1 = n_1$. Households in country 2, however, also hold foreign currency that they retained at the end of the previous period. Denoting by $\tilde{n}_2$ the liquid funds retained in foreign currency, the cash-in-advance constraint is $P_2 c_2 \leq n_2 + \tilde{n}_2/e$. Notice that $\tilde{n}_2$ derives from the previous liquidation or acquisition of foreign deposits and can be negative.[9] The beginning-of-period financial assets in country 1 are equal to the retained liquidity plus the nominal value of domestic deposits, that is, $n_1 + d_1$. In country 2, the beginning-of-period financial assets, denominated in country 2 currency, are given by $n_2 + d_2 + (\tilde{n}_2 + \tilde{d}_2)/e$.

The choice variables for households in country 2 now also include the deposits in foreign currency. The optimality conditions for the choice of $\tilde{d}_2$ is:

$$E\left(\frac{u_c(c_2')}{P_2'}\right) \quad = \quad \beta E\left(\frac{(1+R_1')u_c(c_2'')P_1'\bar{e}'}{P_2'P_1''(1+g_1')\bar{e}''}\right) \tag{46}$$

The optimality condition for the choice of the domestic deposits is still given by (22).

As before, banks make loans only in the currency in which they receive deposits. Because firms contract loans that are denominated in domestic currency, this implies that domestic firms borrow only from domestic banks.[10]

The equilibrium conditions in the goods markets do not change. In the loan and exchange markets, instead, they are slightly different. More specifically, the equilibrium condition in the loan market of country 1 is:

$$P_1(X_{11} + \bar{e} \cdot X_{12}) = \tilde{D}_2 + D_1 + T_1 \tag{47}$$

where now the supply of loans from domestic banks also includes the deposits of foreign residents $\tilde{D}_2$. In country 2, instead, we still have:

$$P_2(X_{22} + X_{21}/\bar{e}) = D_2 + T_2 \tag{48}$$

Finally, the equilibrium condition in the exchange rate market is:

$$P_1 \cdot \bar{e} \cdot X_{12} + \tilde{N}_2 = P_1 \cdot X_{21} \tag{49}$$

[9] Households in country 2 use all the foreign currency owned at the beginning of the period to buy consumption goods. Therefore, their end-of-period currency position is equal to the payments received from foreign deposits, that is, $\tilde{d}_2(1+R_1)$. Given this position, households decide the new foreign deposits $\tilde{d}_2'$. The difference is the new foreign currency position, that is, $\tilde{n}_2' = \tilde{d}_2(1+R_1) - \tilde{d}_2'$. In the next period this currency will be sold (or purchased if $\tilde{n}_2' < 0$) in the exchange rate market.

[10] We allow households to hold foreign deposits but banks can not make loans to foreign firms. Allowing banks to make loans to foreign firms would not change the results as long as the balance sheet position of the banks is covered.

The exchange rate market takes place at the beginning of the period after the government transfers. The supply of country 1's currency derives from the purchase of the input produced in country 2 from firms in country 1 (imports of country 1) and the foreign currency retained by households in country 2. The demand derives from the purchase of the input produced in country 1 from firms in country 2 (exports of country 1).

It can be verified that lemma 25 still holds with international mobility of capital and there is a unique correspondence between the domestic interest rate and the domestic growth rate of money. Therefore, it is still true that the specification of the monetary policy instrument in terms of money growth rate or interest rate does not affect the results.

B.1 Policy equilibrium with international mobility of capital

With international mobility of capital the set of state variables also includes $\tilde{N}_2$ and $\tilde{D}_2$. The whole set of state variables for the normalized model is $\mathbf{s} = (A_1, A_2, \kappa_1, \kappa_2, D_1, D_2, \tilde{N}_2, \tilde{D}_2)$. One important difference respect to the case of financial autarky is that now monetary policy affects the real value of the financial wealth that households own in foreign currency. In our model, households in country 2 owns deposits in country 1 (or borrows if those are negative) and they hold foreign currency from the previous liquidation of foreign deposits. The foreign currency will be converted in domestic currency and will affect the nominal exchange rate. To see this, consider the equilibrium condition in the exchange rate market which for simplicity we rewrite below:

$$\bar{e} \cdot X_{12} + \frac{\tilde{N}_2}{P_1} = X_{21} \tag{50}$$

From this equation we see that the determination of the real exchange rate depends on the volume of currency owned by foreigners, $\tilde{N}_2$, and on the nominal price P_1. Therefore, the policy equilibrium will depend on the state variable $\tilde{N}_2$. However, an important property of this model is that the next period value of $\tilde{N}_2$ does not depend on the current policies R_1 and R_2. To see this notice that $\tilde{N}_2'$ is determined by the equation:

$$\tilde{N}_2' = \tilde{D}_2(1 + R_1) - \tilde{D}_2' \tag{51}$$

Although R_1 enters in this equation, whatever the value of the term $\tilde{D}_2(1 + R_1)$, the value of $\tilde{D}_2'$ will be chosen to satisfy the first order condition (46). This condition depends only on the next period states A_1', A_2', $\tilde{N}_2'$. Therefore, $\tilde{D}_2'$ will be chosen such that $\tilde{N}_2'$ satisfy the first order conditions independently of the value of $\tilde{D}_2(1 + R_1)$. In other words, the equilibrium policies do not depend on $\tilde{D}_2$.

If the next period states do not depend on the current policies, then the current policies can not affect the next period policies. These implies that, when we limit our analysis to Markov policy strategies, the equilibrium interest rates do not depend on the policy function $\Psi(\mathbf{s})$ assumed to determine future interest rates. The determination of the policy fixed point $\Psi^*(\mathbf{s})$ is then trivial. This fix point is a function of $(A_1, A_2, \kappa_1, \kappa_2, \tilde{N}_2)$. Therefore, the only difference respect to the economy without mobility of capital is that the policy function also depends on $\tilde{N}_2$.

Although the policy function Ψ takes a simple form, the international mobility of capital makes the dynamic system derived from equations (22) and (46), unstable. There is a stationary steady state, but a small perturbation from this steady state will push the economy in a non converging path in the variable $\tilde{D}_2$. However, despite the potential for an explosive path in the financial position of the two countries, the real allocation of resources (consumption) is not affected by the presence of capital mobility. This is because production and consumption are fully determined by the nominal interest rates controlled by the two countries. The growth rates of money and the financial flows will adjust so that the chosen interest rates are consistent with the general equilibrium.

One way to eliminate the non-stationarity in the financial positions of the two countries, is by assuming that there are some frictions in international financial markets. One possibility is to assume that the expected return from foreign investments (foreign deposits) decreases when the international position of the country increases. We can justify this assumption with the possibility that the country may default on the foreign debt. Because the properties of the model do not change in a substantial way when we allow for the international mobility of capital, the analysis of this paper has been limited to the case of financial autarky.

References

Alesina, A. & Barro, R. J. (2002). Currency unions. *Quarterly Journal of Economics*, *117*(2), 409–36.

Atkeson, A. & Kehoe, P. J. (1999). Models of energy use: putty-putty versus putty-clay. *American Economic Review*, *89*(4), 1028–43.

Backus, D. K., Kehoe, P. J., & Kydland, F. E. (1994). Dynamics of the trade balance and the terms of trade: the j curve?. *American Economic Review*, *84*, 84–103.

Benigno, G. & Benigno, P. (2001). Price stability in open economies. Unpublished manuscript, London School of Economics and New York University.

Canzoneri, M. B., Cumby, R. E., & Diba, B. T. (2001). The need for international policy coordination: what's old, what's new, what's yet to come?. Unpublished manuscript,Georgetown University.

Christiano, L. J. & Eichenbaum, M. (1995). Liquidity effects, monetary policy, and the business cycle. *Journal of Money Credit and Banking*, *27*(4), 1113–36.

Corsetti, G. & Pesenti, P. (2001a). International dimensions of optimal monetary policy. FRBNY Staff Report No. 124.

Corsetti, G. & Pesenti, P. (2001b). Welfare and macroeconomic interdependence. *Quarterly Journal of Economics*, *116*(2), 421–46.

Corsetti, G., Pesenti, P., Roubini, N., & Tille, C. (2000). Competitive devaluations: toward a welfare-based approach. *Journal of International Economics*, *51*(1), 217–41.

Deardoff, A. V. & Stern, R. M. (1990). *Computational Analysis of Global Trading Arrangements*. University of Michigan Press, Ann Arbor, Michigan.

Devereux, M. B. & Engel, C. (2000). Monetary policy in the open economy revised: price setting and exchange rate flexibility. NBER Working Paper No. 7655.

Fuerst, T. S. (1992). Liquidity, loanable funds, and real activity. *Journal of Monetary Economics*, *29*(4), 3–24.

Gallaway, M. P., McDaniel, C. A., & Rivera, S. A. (2000). Long-run industry-level estimates of u.s. armington elasticities. Office of Economics Working Paper, U.S. International Trade Commission.

Grilli, V. & Roubini, N. (1992). Liquidity and exchange rates. *Journal of International Economics*, *32*(2), 339–52.

Lucas, R. E. (1990). Liquidity and interest rates. *Journal of Economic Theory*, *50*(2), 273–64.

Obstfeld, M. & Rogoff, K. (1995). Exchanged rate dynamics redux. *Journal of Political Economy*, *103*(3), 624–60.

Obstfeld, M. & Rogoff, K. (2000a). Do we really need a new international monetary compact?. NBER Working Paper No. 7864.

Obstfeld, M. & Rogoff, K. (2000b). New directions for stochastic open economy models. *Journal of International Economics*, *50*(1), 117–53.

Pappa, E. (2000). Do the ecb and the fed really need to cooperate? optimal monetary policy in a two-country world. Unpublished manuscript, London School of Economics.

Persson, T. & Tabellini, G. (1995). Double-edged incentives: institutions and policy coordination. In Rogoff (Ed.), *Handbook of International Economics*, Vol. III, chap. 7. North Holland, Amsterdam.

Reinert, K. A. & Roland-Holst, D. W. (1992). Armington elasticities for united states manufacturing sectors. *Journal of Policy Modeling*, *14*, 631–39.

Rogoff, K. (1985). Can international monetary cooperation be counterproductive?. *Journal of International Economics*, *181*(1), 199–217.

Schlagenhauf, D. E. & Wrase, J. M. (1995). Liquidity and real activity in a simple open economy model. *Journal of Monetary Economics*, *35*(3), 431–461.

Stockman, A. C. & Tesar, L. L. (1995). Tastes and technology in a two-country model of the business cycle: explaining international comovements. *American Economic Review*, *85*(1), 168–85.

Whalley, J. (1985). *Trade Liberalization Among Trading Areas.* MIT Press, Cambridge, Massachussets.

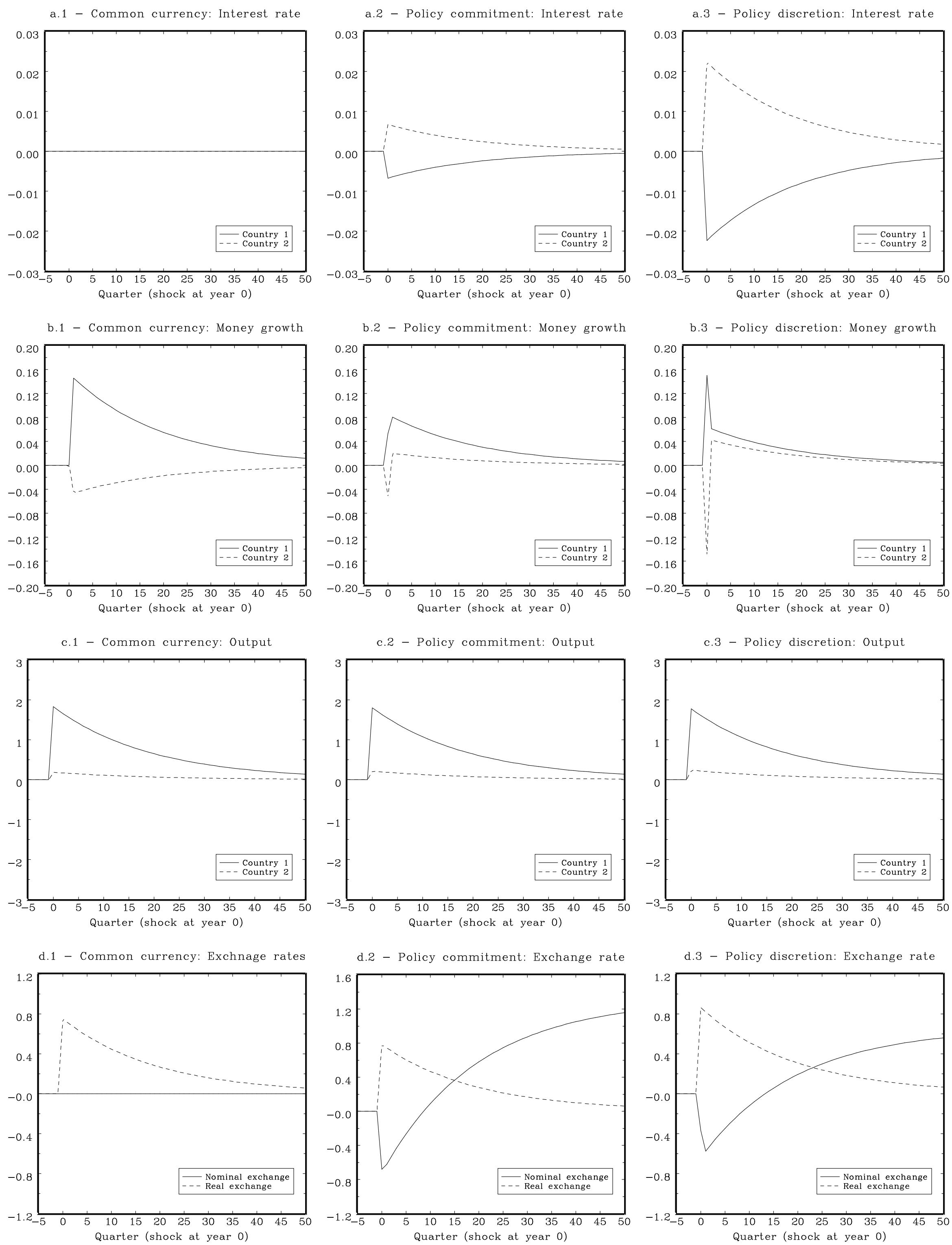


Figure 7: Impulse responses after a positive technology shock in country 1 when country-specific shocks are not correlated. Values in percent deviations from steady state.