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ABSTRACT

Optimal Contracting in Agency with Verifiable *Ex Post* Information*

The Paper studies a straightforward adverse selection problem in which an informative but imperfect signal on the agent's type becomes public *ex post*. The agent is protected by limited liability, which rules out unboundedly high penalties. Analysing the consequences of the additional information and the corresponding extension of the space of feasible contracts, the following conclusions emerge. First, the qualitative effects of the signal can be unambiguously tied to the nature of the underlying problem (e.g., whether the agent is in a 'buyer' or a 'seller' position). Second, the joint surplus of the relationship may well fall, i.e., the distortions due to informational asymmetries can become more pronounced although more information is now available. Finally, the agent may benefit from releasing additional signals because of an associated increase in expected informational rents.

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1 Introduction

Informational asymmetries are prevalent in many economic transactions. Often, the situation at hand can be characterized as one with adverse selection: at the time two contracting parties meet, one party possesses superior information about a parameter that is relevant to the transaction and, by extension, is of interest for the other party. Examples include an employee who knows more about his abilities than the employer, a debtor whose creditworthiness is unknown to the bank, a customer who can assess his risks better than the insurance company, or a producer whose products are of a quality unknown to the consumer. These and many more instances are formalized in the corresponding literature by a simple agency framework in which a principal contracts with a privately informed agent. The properties of this workhorse model are by now well-understood: first, all agents other than the 'worst' type can command a rent and second, because of this fact, the contracted decisions for all agents other than the 'best' type are distorted downwards, away from their first-best levels.

The present paper studies a natural extension of the standard principal-agent model. We analyze a situation where information that is correlated with the agent's type becomes available after a contractually specified action has been chosen. In many applications, additional ex post information appears to be the rule rather than the exception. One set of prominent examples that fit our model are those where the principal actively monitors the agent. In employment or procurement relationships, ex post audits reveal information on the agent's productivity or production costs. Tax authorities will conduct audits to verify taxpayers' reports. Also, franchise contracts are usually contingent not only on sales data, but also on the franchisee's behavior (cleanliness, product quality, friendly service) which is regularly monitored by the franchisor.[1] But further information is frequently available even without the principal taking any explicit action. If the agent sells a durable good to the principal, product failure or malfunctioning after the purchase will reveal additional

[1]See Lafontaine and Slade (1996). In an empirical study of automobile franchising, Arruñada, Garicano, and Vázquez (2001) for instance report that manufacturers condition their contracts on monitoring evidence related to the dealer's sales, customer satisfaction, machinery, personnel, and financial performance, obtained by direct inspections and auditing.

information about the quality of the product. In credit (insurance) market contexts, changes in independent credit ratings (an accident) will constitute a verifiable ex post signal about a customer's true risk type on which his interest payment (insurance premium) can – and indeed will – be based. Or consider an upstream firm that contracts as a principal with a downstream firm as an agent over the supply of an intermediate good. The agent's willingness to pay for the input will then be related to the price which is later charged on the downstream market.[2] Finally, ex post information is common in situations where the decisions of other parties are based on information correlated with the agent's type as will be the case in the context of, e.g., auctions, team production, or product market competition.

Obviously, if the agent's private information is perfectly revealed ex post, he could simply be sufficiently punished whenever he did not report his type truthfully [Nalebuff and Scharfstein (1987)]. Furthermore, even a weakly informative ex post signal is sufficient to eliminate the agent's rent and, hence, ensure the first-best outcome, provided the signal can be contractually employed and the agent can be subjected to unboundedly high penalties at no effective cost.[3] Since the latter condition seems rather unrealistic, however, a natural question is whether the favorable effect of additional information is preserved under the more reasonable assumption that the agent cannot be held unboundedly liable. For this reason, we assume in what follows that payments to (respectively, from) the agent are constrained by a liability limit, which may be due to the agent having only finite wealth or arise from legal practice restricting the enforceability of contractual penalty clauses. At first glance, the finding that perfectly revealing signals eliminate the principal's information problem (a first best allocation can be achieved) would appear to qualitatively carry over in the sense that signals with more informational content lead to smaller rents for the agent, which in turn mitigates the induced distortions. Indeed, this intuitive reasoning is well in line with the results of the corresponding literature, which we discuss in more detail below.

Yet, our results show that this natural conjecture turns out to be false in a range of

[2]A related point is made by Riley (1988) who argues that the seller of an oilfield will want to condition the buyer's payment on the quantity of oil extracted, which is readily observable and a noisy signal of oilfield profitability.

[3]See Riordan and Sappington (1988), Crémer and McLean (1988), and McAfee and Reny (1992).

situations. As we will see, the total expected surplus of the relationship frequently *falls* as an informative signal becomes publicly available. Moreover, the direction of the additional distortions that arise crucially depends on the nature of the underlying problem. In particular, we demonstrate that if the agent's utility is increasing in the contracted action (e.g., the quantity of a good or service he buys from the principal), existing downward distortions will be amplified (the volume of trade further decreases) for signals that are not too informative. Conversely, if the agent's utility is decreasing in the contracted action (e.g., the amount of output he is to produce), the distortions may go in the opposite direction, i.e., the agent's actions may exceed their first-best level. In this second scenario, the agent also does *not* necessarily command a lower rent relative to a situation where the ex post signal about his type is not available. But if a (more informative) signal increases the agent's expected utility, he will have an incentive to ex post reveal or generate signals about his privately held information.

An important new insight to be drawn from our analysis is thus that the specific situation under consideration matters if one allows for ex post information: the qualitative conclusions differ substantially depending on whether the agent's utility is increasing or decreasing in the contracted action (whether he is in a 'buyer' or in a 'seller' position). Note that this is not true in the standard model without ex post information where both scenarios lead to less than efficient quantities traded for all types but the 'top' agent.[4] In the light of the widespread application of the standard adverse selection model, and given that our extension is quite natural, this new insight seems to be a significant aspect to be taken into account when modelling, say, labour, credit, or insurance markets. Intuitively, the dichotomy between 'buyers' and 'sellers' we identify arises for the following reason. As will become clear shortly, a penalty is optimally imposed and set to its maximal level whenever the signal ex post contradicts what the agent has claimed to be true. Consider first an agent in a 'buyer' position and, for concreteness, suppose he is privately informed about his willingness to pay and has only limited wealth. Now, the smaller the quantity that

[4] What is important is that the single crossing property holds; i.e., absolute and marginal utility move in the same direction as we replace better types (high valuations or low costs) by worse types. This implies that a good-type agent suffers more (or benefits less) from a reduction of the quantity than a bad type, so that distorting the quantity downwards makes it less attractive to mimic worse types. Obviously, this logic does not depend on whether the agent is a 'buyer' or a 'seller'.

a low-valuation agent purchases from the principal, the lower the price he has to pay, and the more money is left in the pocket of an untruthful high-valuation agent that can serve as a penalty if the signal indicates non-compliance. Therefore, if the principal wants to increase potential penalties, she decreases the quantity traded beyond the level that would have been optimal in a situation where no ex post information is available. The converse is true if the agent is in a 'seller' position: the higher the amount of output a high-cost agent is to produce, the more he receives in compensation and the more severely can an untruthful low-cost agent be punished if the signal indicates that he has been untruthful. Therefore, the principal optimally increases production and, as we will see, this effect may be sufficiently strong to yield production above the first-best level, so that total surplus may again fall.[5] The agent may then even enjoy a higher rent when information about his type is available ex post. As we will see, this is because a low-cost agent is affected less than a high-cost agent if the quantity produced rises, i.e., it becomes more attractive for a low-cost type to mimic a high-cost type, *ceteris paribus*. Finally, note that the extent to which the principal actually wants to increase potential penalties depends on the informativeness of the ex post signal. If the signal is very precise, small penalties are sufficient and therefore the distortion will be weakened (i.e., additional information is welfare enhancing) in both cases.

Our paper is not the first to investigate agency situations with additional information. Apart from those contributions previously mentioned, most of the related literature considers signals obtained through auditing as a means to mitigate the prevailing incentive problem. To our knowledge, these papers restrict attention to a setting where the agent's utility is decreasing in the induced action, i.e., where he is in a 'seller' position in our terminology. Kofman and Lawarrée (1993) study a framework where the principal hires a supervisor to collect information that is correlated with the agent's unknown productivity. It is shown that the principal uses the signal to extract informational rents, leaving the downward distorted action of

[5] Importantly, this occurs even though we ignore any commitment problems. It is well known that additional information can be harmful if commitment not to use the information is ruled out [Riordan (1990), Dewatripont and Maskin (1995), or Crémer (1995)]. Upward distortions are also known from the literature on countervailing incentives where the agent's reservation utility is type dependent. See Lewis and Sappington (1989) and Jullien (2000) for further references.

a low-type agent largely unaffected.[6] The crucial difference is that the authors represent limited liability as an exogenous upper bound on the maximum punishment. As a result, the penalty that can be inflicted on the agent when the signal indicates non-compliance does not vary with the transfer, and hence, does not depend on the specified action. Laffont and Tirole (1993, ch. 12) interpret limited liability in the same way we do, namely, as the inability of the principal to extract money from her agent.[7] In their model, the signal is perfectly revealing but observed only with a certain probability, and the agent cannot be punished if no signal is observed. While Laffont and Tirole argue that monitoring simply reduces the agent's rent and mitigates the downward distortion, we show that additional ex post information can render it optimal to raise output above the first-best level, so that total surplus is reduced. Khalil (1997) analyzes a setting where the principal cannot commit to costly auditing. He finds that whenever auditing occurs with positive probability, the agent receives no rent, and there is an upward distortion which increases the probability that the agent complies with the contract. The driving force behind this result is that the principal must be given sufficient incentives to audit. While the mechanism at work is thus different, it also requires transfer-dependent penalties. Finally, the model most closely related to ours is Demougin and Garvie (1991) who conclude that a more informative signal always mitigates the distortions and is thus welfare improving. Although contradictory at first sight, their result is consistent with the findings laid out above as we will see below. In particular, their model may be seen as a special case of ours (but with a continuous type space) with specifications of the agent's utility function and his limited liability constraint ensuring that neither of the counterintuitive effects we identify can occur.

The remainder of the paper is organized as follows. Section 2 introduces the model. The optimal contract in the presence of a verifiable ex post signal is derived and discussed in Section 3. A final section concludes. All proofs are relegated to the Appendix.

[6] See also Baron and Besanko (1984). Only if the signal is very accurate and the informational rent is zero, the action is adjusted upward and eventually approaches its first-best level.

[7] This representation of a limited liability constraint can also be found in Sappington (1983), Border and Sobel (1987) and Melumad and Mookherjee (1989), among others.

2 The Model

Consider the following principal-agent model with adverse selection. The utility functions of the principal (P) and the agent (A) are, respectively, $u_P = v(x,\theta) - t$ and $u_A = u(x,\theta) + t$, where $x \in [\underline{x}, \bar{x}] \subset \mathbb{R}$ is a contractible action, θ is a random parameter, and $t \in \mathbb{R}$ is a (possibly negative) transfer payment from P to A. The parameter θ is private information to the agent and can take only two values, $\theta \in \{\theta_h, \theta_l\}$. The ex ante probability that $\theta = \theta_h$ is common knowledge and denoted by $q \in (0,1)$.

To fix ideas, it will be helpful in what follows to keep in mind two applications of the above model that have been extensively studied in the literature. The first example is a vertical relationship between an upstream firm (a manufacturer) and a downstream firm (a retailer), where the principal is the monopolistic supplier who sells a quantity x of some good or service to the agent at cost $-v(x,\theta) \equiv c(x) \geq 0$. The agent's utility from the good is $u(x,\theta) \geq 0$. It depends on his intrinsic preference parameter θ (e.g., downstream market characteristics) and is strictly increasing in the quantity bought; i.e., $u'(x,\theta) > 0$, primes denoting partial derivatives with respect to x. The second example is a procurement situation where the principal hires the agent to produce a quantity x of a good with $v(x,\theta) \equiv v(x) \geq 0$ as her willingness to pay. The supplier's production cost is $-u(x,\theta) = c(x,\theta) \geq 0$. It depends on a cost parameter θ (e.g., the agent's ability to carry out the task) and is strictly increasing in the quantity supplied; i.e., $u'(x,\theta) < 0$.

Although the principal does not observe θ directly, she has access to a verifiable signal $s \in \{s_h, s_l\}$ which is realized after the action x has been taken, and which is imperfectly correlated with θ. In the manufacturer-retailer example, s could be the final price that can be charged by the retailer as an intermediate on the downstream market.[8] In the procurement example, s could be information related to the agent's

[8] In franchising relationships, for instance, the franchisor usually takes the right to terminate the contract at will if the franchisee is not maintaining quality standards (which is subject to random monitoring). Since franchise contracts typically require franchisees to make highly specific investments in equipment, these investments are (at least partially) lost when the franchisor receives a bad signal [Dnes (1996)]. More generally, contracts between a manufacturer and its exclusive retailers are often contingent on the retailers' behavior that supplements the information contained in the sales data [Lafontaine and Slade (1996)]. Similarly, royalties to be paid by coal mining

cost parameter that the principal can infer from the characteristics of the finished product or obtain through conducting audits. Let π_{ij} be the probability that the signal $s = s_i$ is realized, conditional upon a parameter value $\theta = \theta_j$, $i, j \in \{h, l\}$ and assume $\pi_{ll} = \pi_{hh} \equiv \pi$ for simplicity. Hence, the signal is 'correct' with probability π and 'incorrect' with probability $1 - \pi$. In what follows, we use π as a measure of the informativeness of the signal s and without loss of generality let $\pi > \frac{1}{2}$.

Assumption 1. The functions $v(\cdot)$ and $u(\cdot)$ are twice continuously differentiable, monotone, and concave in x. Total surplus $S(x, \theta_i) \equiv v(x, \theta_i) + u(x, \theta_i)$ is strictly concave in x. Furthermore,

a) $u(x, \theta_h) > u(x, \theta_l)\ \forall x \in (\underline{x}, \bar{x}]$,

b) $u'(x, \theta_h) > u'(x, \theta_l)\ \forall x \in [\underline{x}, \bar{x}]$,

c) $x_i^{FB} = \arg\max S(x, \theta_i)$ satisfies $x_l^{FB} \leq x_h^{FB}$ and $x_i^{FB} \in (\underline{x}, \bar{x})$, $i \in \{l, h\}$,

d) $\hat{x}_l(\pi) = \arg\max S(x, \theta_l) - \frac{q}{1-q}\left[u(x, \theta_h) - \frac{1-\pi}{\pi} u(x, \theta_l)\right]$ is unique and satisfies $\hat{x}_l(\pi) \in (\underline{x}, \bar{x})$ for all values $\pi \in [\frac{1}{2}, 1)$.

Assumptions 1 a) and b) state that the agent's possible types θ have a natural ordering both in absolute and marginal utilities (single crossing property). Parts c) and d) are made for convenience and allow us to focus on first-order conditions.[9]

The timing is as follows. Before contracting takes place, nature chooses θ and the agent learns his type. Then, the principal proposes a contract to the agent which the latter can accept or reject. Focusing on the most interesting case, we assume throughout that the principal wants to contract with both types. If the agent accepts, the action x is taken in accordance with the contract. Next, the signal s is realized and the contractually specified transfer is paid. If the agent rejects, the parties obtain their reservation utilities. The sequence of events is summarized in Figure 1.

companies for operating on publicly owned land will generally vary with the price of coal from the same general area, its quality, and other factors the commissioner in charge deems 'relevant' [see, e.g., the Alaska Administrative Code].

[9]Note that $x_l^{FB} \leq x_h^{FB}$ is already implied by the single crossing property if the principal's utility $v(\cdot)$ does not depend on θ. Also observe that for any given $q \in (0, 1)$ the objective function in Assumption 1d) is concave if $v(\cdot)$ is sufficiently concave. Moreover, it is sufficient for our analysis that Assumption 1d) is satisfied for $\pi \leq \pi^{FB}$, which is defined below.

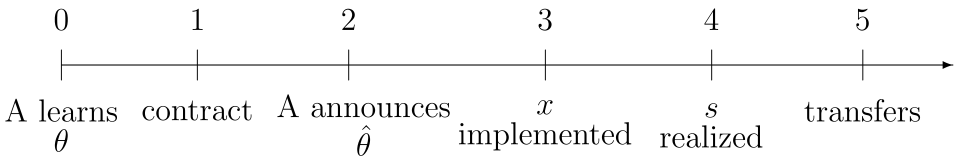

Figure 1

Assuming the principal can commit herself not to renegotiate the contract, we can invoke the Revelation Principle and confine attention to contracts $\{x(\hat{\theta}), t(\hat{\theta}, s)\}$ that specify an action x and a transfer t as a function of the agent's report $\hat{\theta}$ and the realization of the signal s. As future points of reference, we first characterize the cases where information is symmetric and where the signal is not available, respectively.

<u>*First Best:*</u> If θ is publicly observable, the principal maximizes her utility, $u_P = v(x, \theta_i) - t$, subject to the agent's participation constraint $u_A = u(x, \theta_i) + t \geq \bar{u}$, where $\bar{u}$ denotes his reservation utility. For a type-θ_i agent, the first-best action is uniquely characterized by $S'(x_i^{FB}, \theta_i) = 0$ and the transfer payment is $t_i^{FB} = \bar{u} - u(x_i^{FB}, \theta_i)$.

<u>*Second Best:*</u> Next, suppose θ is known only to the agent but the signal is not available or, equivalently, completely uninformative ($\pi = \frac{1}{2}$). Let (x_i, t_i) denote the contract designed for an agent who claims to be of type θ_i. The individual rationality constraints are

$$u(x_i, \theta_i) + t_i \geq \bar{u} \qquad i \in \{l, h\}. \tag{1}$$

In addition, truthful revelation requires

$$u(x_i, \theta_i) + t_i \geq u(x_j, \theta_i) + t_j \qquad i, j \in \{l, h\},\ i \neq j. \tag{2}$$

The principal maximizes her expected utility $q[v(x_h, \theta_h) - t_h] + (1-q)[v(x_l, \theta_l) - t_l]$, subject to (1) and (2). It is easy to see that the incentive constraint for the low-type agent as well as the individual rationality constraint for the high-type agent are slack at the optimum and can be ignored. From the first-order conditions, the second best actions are determined by

$$\begin{aligned} S'(x_h^{SB}, \theta_h) &= 0 \quad \Rightarrow \quad x_h^{SB} = x_h^{FB}, \quad \text{and} \\ S'(x_l^{SB}, \theta_l) &= \frac{q}{1-q}\phi'(x_l^{SB}) \quad \Rightarrow \quad x_l^{SB} < x_l^{FB}, \end{aligned}$$

where $\phi(x) \equiv u(x, \theta_h) - u(x, \theta_l) > 0$ with $\phi' > 0$. The corresponding transfers are $t_l^{SB} = \bar{u} - u(x_l^{SB}, \theta_l)$ and $t_h^{SB} = \bar{u} - u(x_h^{FB}, \theta_h) + \phi(x_l^{SB})$. Hence, under the no-signal second best contract, the action of the high-type agent is first-best and he earns an informational rent equal to $\phi(x_l^{SB})$. In contrast, the action of a low-type agent is distorted downward and he obtains his reservation utility. Evidently, this result does not depend on whether the agent's payoff is increasing or decreasing in x.[10]

3 Optimal Contracts with a Verifiable Signal

We now turn to the case where the principal can condition the transfers specified in the contract on the verifiable (and informative) signal s. Clearly, the principal may now want to lower the transfer when the realization of s contains evidence that contradicts the agent's claim $\hat{\theta}$. As explained above, unboundedly high penalties are ruled out in the remainder:

Assumption 2. The agent is protected by limited liability,

$$t_i(s_j) \geq -W \qquad \forall\, i, j \in \{l, h\}. \tag{LL$_{ij}$}$$

Furthermore,

$$\bar{u} + W \geq \max\left\{u(x_h^{FB}, \theta_h), u(x_l^{SB}, \theta_l), u(x_l^{FB}, \theta_h)\right\}. \tag{3}$$

The first part of Assumption 2 imposes an exogenous lower bound on the feasible transfers between principal and agent, which we assume to be commonly known.[11] The most prominent interpretation of (LL$_{ij}$) is that of a wealth constraint where $W \geq 0$ denotes the initial wealth of the agent, thus justifying our notation. But similar limits on transfers would arise from legal restrictions such as minimum wage laws (in which case $-W \geq 0$ is the minimum wage) or if the agent's preferences

[10] Note, though, that the conclusion is a matter of convention given the properties of the underlying problem as summarized in Assumption 1. For instance, we could use the transformation $y := \bar{x} - x$ which reversed the single crossing property. The optimal contract would then specify $y_l^{SB} > y_l^{FB}$ and our results in the remainder of the paper would change accordingly.

[11] For an analysis of situations where the agent alone knows W, see Lewis and Sappington (2000, 2001). The relevance of limited liability constraints has long been stressed in the literature on moral hazard [see, e.g., Sappington (1983) on hidden information and Brander and Spencer (1989) on hidden action].

exhibit infinite risk aversion below a certain transfer level. The second part of Assumption 2 ensures that the limited liability constraints do not bind at the first and second-best benchmark solutions determined previously, so that those optimal contracts are unaffected by (LL$_{ij}$).[12] This requirement also allows us to focus attention on the interesting case where the agent's liability is large enough for the first-best to be implementable at $\pi = 1$, i.e., if the agent's type is perfectly revealed ex post.[13]

Given the distribution of the signal as described above, the agent's participation constraints can be written as

$$\pi t_i(s_i) + (1-\pi)t_i(s_j) + u(x_i, \theta_i) \geq \bar{u} \quad i, j \in \{l, h\},\ i \neq j. \tag{IR$_i$}$$

In addition, incentive compatibility now reads for $i, j \in \{l, h\},\ i \neq j$,

$$\pi t_i(s_i) + (1-\pi)t_i(s_j) + u(x_i, \theta_i) \geq \pi t_j(s_i) + (1-\pi)t_j(s_j) + u(x_j, \theta_i). \tag{IC$_i$}$$

The principal's problem is to choose $\{x_i, t_i(s_j)\}$ so as to

$$\begin{aligned} \text{maximize} \quad & q\Big[v(x_h, \theta_h) - \pi t_h(s_h) - (1-\pi)t_h(s_l)\Big] \\ & \qquad + (1-q)\Big[v(x_l, \theta_l) - \pi t_l(s_l) - (1-\pi)t_l(s_h)\Big] \\ & \text{s.t. (IR}_i\text{),(IC}_i\text{) and (LL}_{ij}\text{).} \end{aligned} \tag{P}$$

As can easily be seen, the principal's return from the relationship is now strictly higher than under the optimal second-best contract where no informative signal is available. Also, if the signal is sufficiently precise, the possibility of contracting on s enables her to implement the first-best. The relevant cut-off value is given by (see the Appendix)

$$\pi^{FB} \equiv \frac{\bar{u} + W - u(x_l^{FB}, \theta_l)}{2\left[\bar{u} + W - u(x_l^{FB}, \theta_l)\right] - \phi(x_l^{FB})} \in \left(\frac{1}{2}, 1\right).$$

In the following, we first focus our investigation on how additional ex post information affects the feature of the optimal contract that concerns the contracted action

[12] Note that in the procurement example, (3) is trivially satisfied if $\bar{u} = W = 0$. The vertical relation example obviously requires $W > 0$ if $\bar{u} = 0$, because otherwise the principal could never sell anything to the agent.

[13] To see this, suppose $\pi = 1$ and consider the first-best contract with the transfer equal to $-W$ in case the agent did not tell the truth. Under Assumption 2, this contract is incentive compatible because $\overline{u} \geq u(x_l^{FB}, \theta_h) - W$ and $\overline{u} \geq u(x_h^{FB}, \theta_l) - W$.

x, turning to the question of informational rents later. As the effect on the efficiency of the contractually induced allocation will crucially depend on whether the agent's utility is increasing or decreasing in x, we consider each of these two possibilities in turn. Let us start with a situation where the agent's utility is increasing in x, which corresponds to the monopolistic supplier example laid out above. Other situations where the agent is in a 'buyer' position are loan contracting (the agent is granted a credit of size x by the principal) or environmental regulation (the agent prefers to produce more output x, which is harmful for the environment).

Proposition 1. *Suppose $u(x, \theta_i)$ is a monotonically increasing function of x. Under the optimal contract with a verifiable ex post signal, the action taken by the low-type agent, x_l^S, is continuous in the informativeness π of the signal and efficient for $\pi \geq \pi^{FB}$. Otherwise, we have $x_l^S \leq x_l^{FB}$ and there exists a value $\underline{\pi} \in (\frac{1}{2}, \pi^{FB})$ such that*

$$\pi < \underline{\pi} \Leftrightarrow x_l^S < x_l^{SB}.$$

The action taken by the high-type agent is efficient, i.e., $x_h^S = x_h^{FB}$ irrespective of π.

The main result of Proposition 1 is illustrated in Figure 2, which depicts the equilibrium action in state θ_l as a function of the precision of the signal, π.

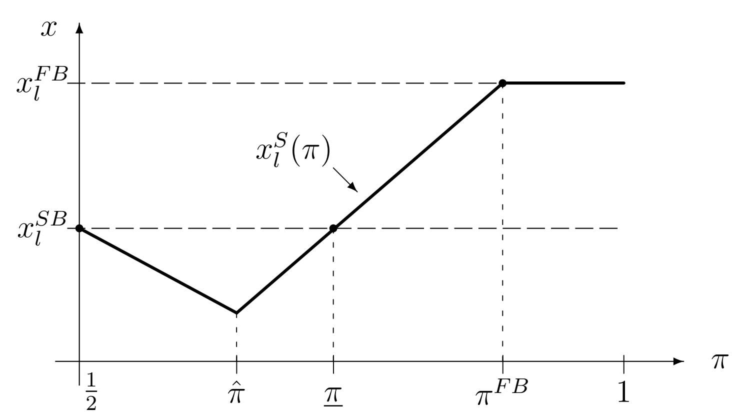

Figure 2

We see that the induced distortions are alleviated relative to a situation where no signal is available *only if* the ex post signal is sufficiently precise. In particular, additional information reinforces the inefficiency whenever $\pi < \underline{\pi}$. Moreover, as the signal becomes more and more informative, the total surplus falls over the range

$\pi \in (1/2, \hat{\pi})$. This finding seems to lack intuition at first glance; after all, we know that the optimal contract balances the principal's concern for surplus maximization with her desire to extract rents. Since an informative signal alleviates the latter problem, the extent of this trade-off should change in favor of surplus maximization. This intuition is flawed, however, because it ignores the fact that additional information may be used most effectively to reduce the agent's rent by introducing further distortions. To see this more clearly, recall from the second-best benchmark that for an uninformative signal with $\pi = \frac{1}{2}$ only (IR_l) and (IC_h) bind and the informational rent is strictly positive. For π not too large, this will continue to be the case and we can use (IR_l) and (IC_h) to write the informational rent of a high-type agent as

$$\begin{aligned} R(x_l, \pi) &= u(x_h, \theta_h) + \pi t_h(s_h) + (1-\pi) t_h(s_l) - \bar{u} \\ &= [u(x_l, \theta_h) + \pi t_l(s_h) + (1-\pi) t_l(s_l)] - [u(x_l, \theta_l) + \pi t_l(s_l) + (1-\pi) t_l(s_h)] \\ &= \phi(x_l) - (2\pi - 1)\big[t_l(s_l) - t_l(s_h)\big], \end{aligned} \tag{4}$$

where $\phi(x_l) = R(x_l, \frac{1}{2})$ is the corresponding rent in the no-signal case. The last term on the right hand side of (4) reflects the fact that a dishonest high-type agent faces a different probability distribution of the signal than a truthful low-type agent. In particular, since the probability of a signal $s = s_h$ is higher for the former, it is optimal for the principal to set the agent's compensation in case of contradicting evidence as low as possible. Hence, we must have $t_l(s_h) = -W$ and (4) becomes

$$R(x_l, \pi) = \phi(x_l) - (2\pi - 1)\left[t_l(s_l) + W\right]. \tag{5}$$

The term in square brackets is the net punishment inflicted on an agent who untruthfully claims to be of type θ_l. It consists of the difference between the transfer $t_l(s_l)$ that the agent receives if the signal falsely indicates that he was honest and the minimum transfer when the signal (correctly) indicates non-compliance, $t_l(s_h) = -W$. If the transfer payment is negative as would be the case in a vertical (or credit) relationship with the agent buying (or borrowing) from the principal, we have $-t_l(s_h) = W > -t_l(s_l) > 0$.[14] Inspection of (5) reveals that in order to reduce the rent, the principal will want to raise the payment $t_l(s_l)$ in comparison to the

[14] Note that the first inequality is a consequence of Assumption 2.

no-signal case. What does increasing $t_l(s_l)$ mean in terms of the action x_l? From (IR_l), we have

$$t_l(s_l) = [\bar{u} - u(x_l, \theta_l) + (1 - \pi)W]/\pi, \tag{6}$$

i.e., $u'(x, \theta) > 0$ implies that $t_l(s_l)$ decreases in x_l. Hence, a *reduction* in x_l diminishes the agent's rent relative to the no-signal second-best case by the above argument. In our monopolistic supplier example, the smaller the quantity purchased from the principal, the lower the price a truthful agent has to pay, and the more money is left in the pocket of an untruthful agent that can serve as a penalty if the signal indicates non-compliance.

Consequently, the availability of additional information through s aggravates the welfare reduction caused by the downward distortion of x_l, provided π is relatively small. Indeed, this effect becomes stronger the more precise the signal is for values $\pi \leq \hat{\pi}$.[15] At $\hat{\pi}$, the agent's rent drops to zero and (IR_h) becomes binding. Then, the above effect is no longer operative and the principal optimally increases x_l until the (IC_h) constraint becomes slack at $\pi = \pi^{FB}$.

Next, we turn to the case where the agent's utility is decreasing in x, which corresponds to the procurement example detailed above. Another obvious situation where the agent is in a 'seller' position is a trade relationship where the agent sells x units of a product of unknown quality to the principal. Finally, note that $u'(x, \theta) < 0$ also holds in a variant of the adverse selection model which includes unobservable effort and is often encountered in the literature.[16] Let

$$\bar{\pi} \equiv \frac{u'(x_l^{FB}, \theta_l)}{u'(x_l^{FB}, \theta_l) + u'(x_l^{FB}, \theta_h)} \in \left(\frac{1}{2}, 1\right).$$

Proposition 2. *Suppose $u(x, \theta_i)$ is a monotonically decreasing function of x. Under the optimal contract with a verifiable ex post signal, the low-type agent's action x_l^S is continuous in π and efficient for $\pi \geq \pi^{FB}$. Otherwise, $x_l^S \geq x_l^{SB}$ and*

a) if $\bar{\pi} \geq \pi^{FB}$, we have $x_l^S < x_l^{FB}$ for all values $\pi \geq \frac{1}{2}$;

[15] Formally, $R'(x_l, \pi) = \phi'(x_l) + \frac{2\pi - 1}{\pi} u'(x_l, \theta_l)$, i.e., the principal's marginal benefit from reducing x_l, is increasing in π.

[16] For example, we could have $x = \theta + e$ where x is an output or cost related variable and e the effort exerted by the agent. Effort is costly, so that the agent's utility can be written as $u_A = -C(e) + t = -C(x - \theta) + t$ with $u'(x, \theta) = -C_e < 0$. Some well-known applications of this framework are the regulation of a firm with unknown cost as formalized by Laffont and Tirole (1993), or the manager-shareholder model by Kofman and Lawarrée (1993).

b) if $\bar{\pi} < \pi^{FB}$, we have $x_l^S > x_l^{FB} \Leftrightarrow \pi > \bar{\pi}$.

The action taken by the high-type agent is efficient, i.e., $x_h^S = x_h^{FB}$ irrespective of π.

Again, the content of this proposition is best illustrated graphically. This is done in Figure 3, which displays x_l^S as a function of π.

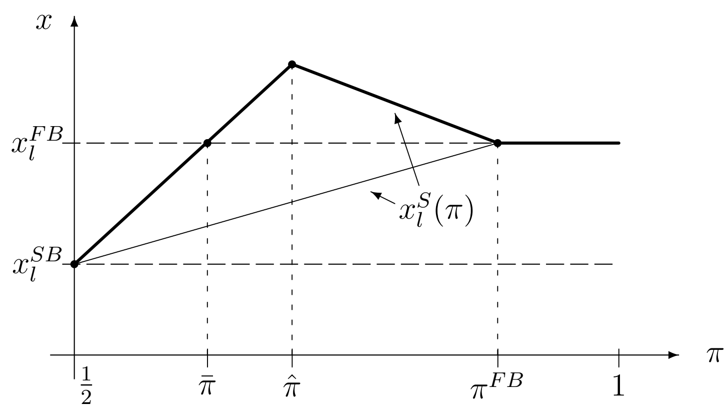

Figure 3

According to the proposition, two possibilities have to be distinguished. First, if $\bar{\pi} \geq \pi^{FB}$, the downward distortion of x_l is simply alleviated so that total welfare increases as additional ex post information is present; i.e., the more precise the signal, the smaller is the distortion, as has been suggested by the literature discussed in the Introduction. This is case a), as indicated by the thin line. Importantly, case a) always applies if $\bar{u} = W = 0$ and $u(x, \theta) = -x\theta$, implying $\bar{\pi} = \pi^{FB}$ from the definitions of $\bar{\pi}$ and π^{FB}. This particular specification, which is completely innocuous in the standard model without ex post information, has been used in Demougin and Garvie (1991) as well as in Khalil (1997). Both papers, therefore, correctly conclude that a transfer dependent penalty by itself does not generate overproduction and the well-known result of downward distorted production continues to hold.[17] However, $\bar{\pi} < \pi^{FB}$ is easily possible if u is strictly concave (see the Appendix for a simple example) or if $W > 0$. This is case b), which is depicted by the thick line in the figure. If $\bar{\pi} < \pi^{FB}$, the availability of noisy ex post information on the agent's type

[17] For a model where overproduction may occur despite the normalization $\bar{u} = W = 0$, see Khalil and Lawarrée (2001). The authors consider a very different framework in which the principal ex post chooses between input and output monitoring, thereby taking advantage of the possibility that the agent can be 'caught on the wrong foot'.

will lead the principal to distort x_l in the *opposite* direction. To see why, recall that the informational rent of the high-type agent is given by (5), provided R is strictly positive. As before, we have $t_l(s_h) = -W$ and the principal wants to raise $t_l(s_l)$ above the amount that is optimal in the no-signal case in order to reduce the rent. In contrast to the previous case, however, $t_l(s_l)$ is now increasing in x_l due to $u'(x, \theta) < 0$ [see eq. (6)]. In our procurement example, the higher the amount of output a high-cost agent is to produce, the more he receives in compensation and the more severely can an untruthful low-cost agent be punished if the signal indicates non-compliance. The same reasoning applies if the principal is a consumer who buys a product of unknown quality from the agent, in which case the penalty in case of non-compliance has the natural interpretation of a warranty that covers malfunctioning (a signal indicating low quality).[18]

Therefore, x_l always unambiguously exceeds x_l^{SB}, regardless of whether we are in case a) or case b). Also, as long as π is sufficiently small, the effect again becomes stronger the more informative the signal is.[19] But since the driving force behind raising x_l is the *marginal* – rather than the absolute – effect on the agent's rent, the effect may even lead the principal to raise x_l above x_l^{FB}.[20] Specifically, this situation emerges if the marginal effect on $-u(x_l^{FB}, \theta_l)$ and, hence, on $t_l(s_l)$ is sufficiently stronger than the marginal effect on $\phi(x_l^{FB}) = u(x_l^{FB}, \theta_h) - u(x_l^{FB}, \theta_l)$, i.e., for $\bar{\pi} < \pi^{FB}$. Again, if the agent's rent R drops to zero and (IR$_h$) becomes binding at $\hat{\pi}$, this effect vanishes. For high values of π, the (downward or upward) distortion is thus alleviated and eventually, the first-best is implemented for $\pi \geq \pi^{FB}$.

Having analyzed the allocative consequences of additional information, let us now

[18]See Emons (1989) for a survey of warranty contracts and the rationales behind them. A slightly different formulation of the problem would also allow the penalty to be interpreted as a money-back guarantee (with $W = 0$).

[19]Formally, $R(x_l, \pi)$ is now decreasing in π.

[20]This result should be contrasted to Laffont and Tirole (1993, ch. 12), who conclude that "monitoring of effort [...] consequently leads to a smaller distortion of effort for the inefficient type" (p. 529). In their model, monitoring generates a signal which is perfectly revealing but observed with less than full probability, $z < 1$. The authors implicitly assume that i) the agent cannot be punished if no signal has been observed (no evidence and favorable evidence cannot be distinguished) and that ii) z is smaller than some threshold value z^* (which corresponds to $\bar{\pi}$ in our context). If one maintains i) but admits $z > z^*$, a positive output distortion would emerge for the same reason as in the present model. However, if the signal is verifiable [i.e., assumption i) does not hold], it is easy to see that the first-best can be achieved in their framework.

turn to the question of how individual payoffs will be affected. Clearly, the principal's expected return never falls as an informative signal becomes available: because she could always offer the optimal second-best contract that does not condition on the signal, this observation follows trivially from revealed preferences. In light of the previous results, however, it is not *a priori* obvious how the agent's informational rent $R(x_l^S, \pi)$ is affected by additional ex post information.

Corollary. *If $u'(x, \theta_i) > 0$, $R(x_l^S, \pi)$ is weakly decreasing in the precision of the signal, π. If $u'(x, \theta_i) < 0$, $R(x_l^S, \pi)$ weakly decreases in π for all values $\pi \geq \bar{\pi}$, but it may increase in π for values $\pi < \bar{\pi}$. In particular, we may have $R(x_l^S, \pi) > R(x_l^{SB}, \frac{1}{2}) = \phi(x_l^{SB})$ for some $\pi \in (\frac{1}{2}, \bar{\pi})$.*

The first part of the corollary is straightforward. Consider Figure 2 where $u'(x, \theta_i) > 0$. For values of $\pi < \hat{\pi}$, the distortion of x_l is aggravated and the total surplus decreases in π. Hence, the agent cannot be better off if additional ex post information becomes available. For $\pi \geq \hat{\pi}$, his rent is zero, so that a more precise ex post signal cannot help him either. But a similar reasoning need not apply for $u'(x, \theta_i) < 0$ as is indicated in the second part of the corollary: the agent may actually *profit* from an observable ex post signal about his type. From Figure 3, we see that the informational rent can only decrease for values $\pi \geq \bar{\pi}$. In case a), the agent earns no rent for those values of π. In case b), x_l increases above x_l^{FB}, so that the total surplus again decreases (if $\pi \leq \hat{\pi}$) or the agent's rent is again zero (if $\pi > \hat{\pi}$). However, if $\pi < \bar{\pi}$ more precise ex post information reduces the distortion, so that total surplus increases. Then, not only the principal's profit, but also the agent's rent can increase. In particular, one can easily construct examples where the negative effect on $R(\cdot)$ due to a more precise signal is overcompensated by the positive effect on $R(\cdot)$ caused through the corresponding raise in x_l.[21]

4 Conclusion

In this paper, we have studied a straightforward variant of a simple principal-agent adverse selection problem in which information that is imperfectly correlated with

[21]See the Appendix for a simple example.

the agent's type becomes public ex post. Because we have assumed that the agent is protected by limited liability, the principal could not implement the first-best allocation with signals of arbitrary informativeness. Our first main result was that the expected surplus of the relationship need not increase as the signal becomes available or more informative, respectively. Rather, we have identified a range of situations in which more contractually employable information leads to less efficiency and lower welfare overall. Second, it was demonstrated that the standard result of a downward distortion, which is possibly mitigated as additional information of the agent's type becomes available (e.g., through audits), is not necessarily robust with regard to our plausible modification. Interestingly, we have also seen that the qualitative effects of additional information can unambiguously be tied to the nature of the underlying problem, namely, whether the agent is in a 'buyer' or a 'seller' position. If the agent's utility is strictly increasing in the contractually specified action, we found that downward distortions will be strengthened if the signal is not too informative. In some applications, for instance, this would imply that the equilibrium volume of trade drops if manufactures use more information in their contractual relations with their retailers. Similarly, the equilibrium volume of loans may fall as banks make use of additional information on their customer's creditworthiness. Conversely, additional information can result in upward (rather than downward) distortions if the agent's utility is strictly decreasing in the contractually specified action. This situation encompasses many applications in trade and procurement transactions, employment contracts, or regulatory relations. Moreover, the agent may command a higher informational rent in the presence of an informative signal relative to the case where no additional information is available.

Although we have used a very simple framework to highlight the economic forces at work, several extensions of our model could be considered. In particular, we have treated the ex post signal itself as well as its informativeness as exogenously given. A natural extension of our model would allow for the information to be observed only if the principal conducts costly audits, or for an endogenous choice of the informativeness of the signal. As the principal's payoff is always higher if the signal is available and monotonically increasing in its precision, neither extension is likely to change our qualitative insights.[22] Another and perhaps more promising

[22]This argument notwithstanding, taking the choice of monitoring as exogenous may lead to

extension concerns the possibility that the agent himself can affect the signal that is received by the principal. In the manufacturer-retailer example, for instance, the agent's private information may relate to demand conditions and he may be able to influence the price of final output that is produced using the principal's input. Then, the equilibrium signal itself will depend on the contracting terms and thus be chosen endogenously. More generally, suppose the agent can manipulate ex post information at some personal cost. Our findings then suggest that, surprisingly, total surplus could actually be enhanced if the agent's cost of manipulating ex post information decreases.[23] Moreover, the agent may be better off if he can release some public information ex post, which may be an interesting topic for future research.

wrong conclusions regarding the effects of variations in the informativeness of the signal on the variables of the model [see, e.g., Demougin and Fluet (2001)]. Also observe that since the agent is punished with positive probability in equilibrium, the principal might find it optimal to conduct costly audit even if she cannot commit to do so ex ante.

[23]See Maggi and Rodríguez-Clare (1995) for a related result in a model based on countervailing incentives.

Appendix

The optimal contract $\{(t_i^S(s_j), x_i^S)\}_{i,j=l,h}$ solves the program (P), the solution of which is characterized in the lemma below. Define $\Phi(x,\pi) \equiv u(x,\theta_h) - \frac{1-\pi}{\pi}u(x,\theta_l) - \frac{2\pi-1}{\pi}(\bar{u}+W)$ and note that

$$\hat{x}_l(\pi) = \arg\max_{x\in[\underline{x},\overline{x}]} S(x,\theta_l) - \frac{q}{1-q}\Phi(x,\pi). \tag{7}$$

Lemma. *The optimal contract $\{x_i^S, t_i^S\}$ always prescribes $x_h^S = x_h^{FB}$. For values of $\pi \geq \pi^{FB}$, we have $x_l^S = x_l^{FB}$ and $\Phi(x_l^S, \pi) \leq 0$. For $\pi < \pi^{FB}$, x_l^S is characterized by*

a) $\Phi(x_l^S,\pi) = 0$ *if* $\Phi(\hat{x}_l(\pi),\pi) \leq 0$,

b) $x_l^S = \hat{x}_l(\pi)$ *otherwise.*

The transfers under this contract are $t_l^S(s_h) = -W$, $t_l^S(s_l) = [\bar{u} - u(x_l^S,\theta_l) + (1-\pi)W]/\pi$ *and* $t_h^S(s_h) = t_h^S(s_l) = \bar{u} - u(x_h^{FB},\theta_h) + \max\left\{\Phi(x_l^S,\pi), 0\right\}$.

Proof. Note first that by invoking the Maximum Punishment Principle [Baron and Besanko (1984)], we can set $t_l(s_h) = -W$ and let $t_l(s_l) \equiv t_l$ for brevity of exposition. Second, provided the (IC$_l$) constraint is slack, we can without loss of generality assume that $t_h(s_h) = t_h(s_l) \equiv t_h$. In what follows, we will ignore the (IC$_l$) constraint and later verify that it is indeed not binding in the optimum. Furthermore, since $t_h^S \geq \bar{u} - u(x_h^{FB},\theta_h) \geq -W$ by (3), the wealth constraint for the θ_h-type agent will not be binding and can be ignored. Similarly, for values $\pi \geq \pi^{FB}$ we have $x_l^S = x_l^{FB}$, implying $t_l^S \geq -W$ by (3). An argument that the wealth constraint $t_l \geq -W$ is not binding for $\pi < \pi^{FB}$ is deferred to the proofs of Propositions 1 and 2 below. Rewriting the principal's payoff and the remaining constraints, the Lagrangian of the principal's problem is

$$\begin{aligned}\mathcal{L} = {} & q\left[v(x_h,\theta_h) - t_h\right] + (1-q)\left[v(x_l,\theta_l) - \pi t_l + (1-\pi)W\right] \\ & + \lambda_l\left\{\pi t_l - (1-\pi)W + u(x_l,\theta_l) - \bar{u}\right\} + \lambda_h\left\{t_h + u(x_h,\theta_h) - \bar{u}\right\} \\ & + \mu\left\{t_h + u(x_h,\theta_h) - (1-\pi)t_l + \pi W - u(x_l,\theta_h)\right\}.\end{aligned}$$

The first-order conditions are

$$\frac{\partial \mathcal{L}}{\partial x_h} = qv'(x_h, \theta_h) + (\lambda_h + \mu)u'(x_h, \theta_h) = 0 \tag{8}$$

$$\frac{\partial \mathcal{L}}{\partial t_h} = -q + \lambda_h + \mu = 0 \tag{9}$$

$$\frac{\partial \mathcal{L}}{\partial x_l} = (1-q)v'(x_l, \theta_l) + \lambda_l u'(x_l, \theta_l) - \mu u'(x_l, \theta_h) = 0 \tag{10}$$

$$\frac{\partial \mathcal{L}}{\partial t_l} = -(1-q)\pi + \lambda_l \pi - \mu(1-\pi) = 0. \tag{11}$$

Inserting (9) into (8), using $S(x, \theta) = v(x, \theta) + u(x, \theta)$, yields

$$S'(x_h, \theta_h) = 0 \qquad \Rightarrow \quad x_h^S = x_h^{FB}.$$

Next, (11) implies $\lambda_l = (1-q) + \mu \frac{1-\pi}{\pi} > 0$. Thus, ($\text{IR}_l$) is always binding. We can substitute λ_l in (10) to obtain

$$S'(x_l, \theta_l) = \frac{\mu}{1-q} \left[u'(x_l, \theta_h) - \frac{1-\pi}{\pi} u'(x_l, \theta_l) \right]. \tag{12}$$

Consider first $\mu = 0$. Then, $x_l = x_l^{FB}$ by (12) and $\lambda_h = q > 0$ from (9), so (IR_h) is satisfied with equality. Since (IR_l) is binding, in order for (IC_h) to hold in this case, we must have

$$u(x_l^{FB}, \theta_h) - \frac{1-\pi}{\pi} \left[u(x_l^{FB}, \theta_l) - \bar{u} \right] - \frac{2\pi - 1}{\pi} W \leq \bar{u},$$

which is equivalent to $\Phi(x_l^{FB}, \pi) \leq 0$ or $\pi \geq \pi^{FB}$ as defined in the text. Conversely, for $\pi < \pi^{FB}$, $\mu = 0$ yields a contradiction, so (IC_h) is binding. There are two cases to distinguish:

a) $\lambda_h > 0$ implies that both (IR_h) and (IC_h) are binding at the optimum. x_l^S is then implicitly characterized by

$$\Phi(x_l^S, \pi) = u(x_l^S, \theta_h) - \frac{1-\pi}{\pi} u(x_l^S, \theta_l) - \frac{2\pi - 1}{\pi} (\bar{u} + W) = 0.$$

b) $\lambda_h = 0$ yields $\mu = q$ from (9) and x_l^S can be recovered from equation (12),

$$S'(x_l^S, \theta_l) = \frac{q}{1-q} \left[u'(x_l^S, \theta_h) - \frac{1-\pi}{\pi} u'(x_l^S, \theta_l) \right], \tag{13}$$

implying $x_l^S = \hat{x}_l(\pi)$. Note that $\lambda_h = 0$ requires $\Phi(\hat{x}_l(\pi), \pi) \geq 0$.

To complete the proof, it remains to show that (IC_l) is not binding. Using (IR_l) and (IC_h), this constraint can be written as

$$u(x_h^{FB}, \theta_l) - u(x_h^{FB}, \theta_h) + \max\left\{\Phi(x_l^S, \pi), 0\right\} \leq 0. \tag{14}$$

Observe first that if $\Phi(x_l^S, \pi) \leq 0$, inequality (14) is implied by Assumption 1 a). Thus, we can confine attention to case b) where (IR_h) is not binding and $x_l^S = \hat{x}_l(\pi)$. Next, note that for (14) to hold it suffices to prove that $\Phi(x_l^S, \pi) \leq \Phi(x_l^{FB}, \frac{1}{2})$, because $\Phi(x_l^{FB}, \frac{1}{2}) = u(x_l^{FB}, \theta_h) - u(x_l^{FB}, \theta_l)$, so (14) will again be satisfied by Assumptions 1 b) and c). Consider the following alternative contract: $x_i = x_i^{FB}$, $t_l(s_l) = t_l(s_h) = \bar{u} - u(x_l^{FB}, \theta_l)$ and $t_h = \bar{u} - u(x_h^{FB}, \theta_h) + \Phi(x_l^{FB}, \frac{1}{2})$. Since this contract satisfies all the constraints, the principal's expected utility under the optimal contract must weakly exceed her expected utility under the alternative contract. Hence,

$$S(x_l^S, \theta_l) - \frac{q}{1-q}\Phi(x_l^S, \pi) \geq S(x_l^{FB}, \theta_l) - \frac{q}{1-q}\Phi(x_l^{FB}, \frac{1}{2}),$$

by revealed preferences. The claim follows from $S(x_l^{FB}, \theta_l) \geq S(x_l^S, \theta_l)$ by definition of x_l^{FB}. □

Proof of Proposition 1: As Lemma 1 already characterizes x_h^S as well as x_l^S for $\pi \geq \pi^{FB}$, we focus on how x_l^S varies with π for $\pi < \pi^{FB}$. Suppose $u'(x, \theta_i) > 0 \; \forall x \in [\underline{x}, \bar{x}]$ and recall from Section 2 that at $\pi = \frac{1}{2}$, $x_l^S = x_l^{SB} = \hat{x}_l(1/2)$ and $\Phi(x_l^{SB}, 1/2) > 0$. Since the objective function in (7) has decreasing marginal returns, $\hat{x}_l(\pi)$ is a strictly decreasing function of π [see, e.g., Theorem 1 in Edlin and Shannon (1998)].[24] Also note that

$$\begin{aligned}\frac{d\Phi(\hat{x}_l(\pi), \pi)}{d\pi} &= \frac{\partial\Phi(\hat{x}_l(\pi), \pi)}{\partial\pi} + \frac{\partial\Phi(\hat{x}_l(\pi), \pi)}{\partial\hat{x}_l}\frac{\partial\hat{x}_l}{\partial\pi} \\ &= -\frac{1}{\pi^2}\left[\bar{u} + W - u(\hat{x}_l, \theta_l)\right] + \frac{1-q}{q}S'(\hat{x}_l, \theta_l)\frac{\partial\hat{x}_l}{\partial\pi},\end{aligned} \tag{15}$$

where we have used the definition of $\Phi(\cdot)$ and the fact that $\hat{x}_l$ satisfies the first-order condition (13) to program (7). Due to $\partial\hat{x}_l(\pi)/\partial\pi < 0$, we must have $\hat{x}_l < x_l^{SB}$ and, hence, $S'(\cdot) > 0$ and $\bar{u} + W - u(\hat{x}_l, \theta_l) > 0$ using (3) and $u'(\cdot) > 0$.

[24] A function $f(x,t)$ is said to display increasing (decreasing) marginal returns, if $\frac{\partial f}{\partial x}$ is strictly increasing (decreasing) in t. This property is sometimes also referred to as (strict) supermodularity.

Therefore, $\frac{d\Phi}{d\pi} < 0$, i.e., the agent's rent is decreasing in π as one would expect. Since $\Phi(\hat{x}_l(\pi^{FB}), \pi^{FB}) < \Phi(x_l^{FB}, \pi^{FB}) = 0$, there exists a unique $\hat{\pi} \in (\frac{1}{2}, \pi^{FB})$ such that $\Phi(\hat{x}_l(\hat{\pi}), \hat{\pi}) = 0$. Now consider increasing π from $1/2$ to $\hat{\pi}$, i.e., over the range where $\Phi(x_l^S, \pi) > 0$ so that the (IR$_h$) constraint is not binding. Then, we are in case b) of Lemma 1 and $x_l^S = \hat{x}_l(\pi)$. Hence, x_l^S is decreasing in π for $\pi \in [1/2, \hat{\pi})$. At $\hat{\pi}$, the (IR$_h$)- constraint becomes binding. For values $\hat{\pi} \leq \pi \leq \pi^{FB}$, we are in case a) of Lemma 1, where x_l^S is implicitly determined by $\Phi(x_l^S, \pi) = 0$. By the implicit function theorem,

$$\frac{\partial x_l^S}{\partial \pi} = -\frac{\partial \Phi(\cdot)/\partial \pi}{\partial \Phi(\cdot)/\partial x_l^S} = -\frac{-\frac{1}{\pi^2}\left[\bar{u} + W - u(x_l^S, \theta_l)\right]}{\frac{1-q}{\mu} S'(x_l^S, \theta_l)}, \tag{16}$$

which is strictly positive as $x_l^S \leq x_l^{FB}$ using a similar argument as above. Continuity follows from the Theorem of the Maximum. To summarize, x_l^S at first decreases in π below x_l^{SB}, obtains a minimum at $\hat{\pi}$ and then increases again until $x_l^S = x_l^{FB}$ at $\pi = \pi^{FB}$. The existence of the critical value $\underline{\pi}$ as stated in Proposition 1 now follows from an intermediate value argument. Finally, note that (3) and $x_l^S \leq x_l^{FB}$ together with $u' > 0$ ensure that $\bar{u} + W - u(x_l^S, \theta_l) \geq 0$ over the entire range. This implies $t_l^S \geq -W$, so that the contract characterized in Lemma 1 also satisfies the wealth constraint for the θ_l-type agent. $\square$

Proof of Proposition 2: Suppose $u'(x, \theta_i) < 0 \ \forall x \in [\underline{x}, \overline{x}]$. Again, we confine attention to the case where $\pi < \pi^{FB}$ and investigate how x_l^S varies with π, starting from $x_l^S = x_l^{SB} = \hat{x}_l(\frac{1}{2})$ at $\pi = \frac{1}{2}$. Due to $u' < 0$, the objective function in (7) has increasing marginal returns and $\hat{x}_l(\pi)$ is therefore a strictly increasing function of π. Since $\Phi(\hat{x}_l(\frac{1}{2}), \frac{1}{2}) > 0$, case b) of Lemma 1 again applies for π sufficiently close to $\frac{1}{2}$, implying $x_l^S = \hat{x}_l(\pi)$. Moreover, note that $\bar{\pi}$ as defined in the text is such that $\hat{x}_l(\pi) \geq x_l^{FB} \Leftrightarrow \pi \geq \bar{\pi}$. There are two possibilities to consider.

First, if $\pi^{FB} < \bar{\pi}$, then $\hat{x}_l(\pi) < x_l^{FB}$ for all $\pi \leq \pi^{FB}$. Since $\frac{\partial \Phi}{\partial \hat{x}_l} > 0$ for $\hat{x}_l < x_l^{FB}$, we have $\Phi(\hat{x}_l(\pi^{FB}), \pi^{FB}) < \Phi(x_l^{FB}, \pi^{FB}) = 0$, so that by continuity there must exist a $\hat{\pi} \in (1/2, \pi^{FB})$ with $\Phi(\hat{x}_l(\hat{\pi}), \hat{\pi}) = 0$ such that x_l^S is characterized as in Lemma 1 a) for $\pi \geq \hat{\pi}$. Over this range, $\Phi(x_l^S, \pi) = 0$, so that x_l^S continues to be increasing [see (16) and note that $x_l^{SB} < x_l^S < x_l^{FB}$]. Hence, x_l^S monotonically increases in this case from x_l^{SB} to x_l^{FB} as π varies from $\frac{1}{2}$ to π^{FB}. Second, we may have $\bar{\pi} < \pi^{FB}$, so that $\hat{x}_l(\pi) > x_l^{FB}$ for all $\pi \in (\bar{\pi}, \pi^{FB}]$. From $\frac{\partial \Phi}{\partial \hat{x}_l} < 0$ for

$\hat{x}_l > x_l^{FB}$, it follows again that $\Phi(\hat{x}_l(\pi^{FB}), \pi^{FB}) < \Phi(x_l^{FB}, \pi^{FB}) = 0$. Furthermore, $\Phi(\hat{x}_l(\bar{\pi}), \bar{\pi}) = \Phi(x_l^{FB}, \bar{\pi}) > \Phi(x_l^{FB}, \pi^{FB}) = 0$, because $\frac{\partial \Phi}{\partial \pi} < 0$. By continuity there thus exists a $\hat{\pi} \in (\bar{\pi}, \pi^{FB})$ with $\Phi(\hat{x}(\hat{\pi}), \hat{\pi}) = 0$ such that x_l^S is characterized as in Lemma 1 a) for $\pi \geq \hat{\pi}$. Over this range, x_l^S is decreasing in π as can be seen from (16) using $x_l^{FB} < x_l^S$. To summarize, for $\bar{\pi} < \pi^{FB}$, x_l^S at first increases in π above x_l^{FB} for $\pi > \bar{\pi}$, obtains a maximum at $\hat{\pi}$, and then decreases again until $x_l^S = x_l^{FB}$ at $\pi = \pi^{FB}$. Again, it is straightforward to show that $\bar{u} + W - u(x_l^S, \theta_l) \geq 0$ over the entire range, which implies $t_l^S \geq -W$, so that the contract characterized in Lemma 1 is indeed optimal. □

Proof of the Corollary: The first part of the corollary follows immediately from the proof of Proposition 1. In order to prove the second part, suppose $u'(x, \theta_i) < 0$. The agent's rent is given by $\max\{\Phi(\hat{x}_l(\pi), \pi), 0\}$, where $\Phi(\hat{x}_l(\pi), \pi)$ varies with π according to (15). The first term in (15) is negative due to $\hat{x}_l \geq x_l^{SB}$ and (3). The second term is also negative if $\hat{x}_l(\pi) \geq x_l^{FB}$, which is equivalent to $\pi \geq \bar{\pi}$. For $\hat{x}_l(\pi) < x_l^{FB}$, however, the second term is positive. It is straightforward to construct examples where the second effect overcompensates the first effect for some $\pi < \bar{\pi}$. For instance, let $u(x, \theta) = -\frac{1}{2}(x - \theta)^2$, $v(x, \theta) = x$, $q = .5$, $\theta_l = .5$, $\theta_h = 1$, $\underline{x} = 1$, $\bar{x} = 3$, and $W = \bar{u} = 0$. It can easily be verified that $\bar{\pi} = 2/3$ and $\pi^{FB} = .8$, and that the agent's rent is increasing for $\pi \in [.5, .6]$. □

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