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ABSTRACT

Economic Geography and the Role of Profits*

In modern economies, the amount of profits distributed to shareholders is far from being negligible. We show that the way profits are distributed among agents matters for the space-economy. For example, the existence of mobile rentiers is sufficient to make the symmetric configuration unstable for all transport cost values and to make partial agglomeration of firms stable. Obviously, to account for profits and for their distribution, the assumption of free entry must be abandoned. So doing, we ignore fixed costs and show that it is imperfect competition more than increasing returns that matters for the formation of agglomeration in economic geography.

JEL Classification: L13 and R13
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1 Introduction

Up to now, economic geography models have provided no insights into the role of profit distribution and firms' ownership on the location of economic activity.[1] Indeed, for such an issue to be properly addressed, firms must earn positive profits, which is inconsistent with the free-entry zero-profit condition imposed in monopolistic competition (Krugman, 1991; Fujita, Krugman and Venables, 1999; Neary, 2001; Ottaviano, Tabuchi and Thisse, 2002). As a result, most models consider wages as the unique source of individual incomes and do not account for the locational choices made by firms' owners. However, inasmuch as immobile "farmers" and mobile "workers" have been presented as the main actors of the core-periphery pattern in economic geography, it is our contention that, once it is recognized that firms make positive profits, *firms' owners must also be taken into account in the study of the agglomeration process*. Indeed, when choosing a location for themselves, firms' owners affect the size of demand within each region and, therefore, the location of firms and workers.[2]

Exploring the role of firm's ownership on the space-economy within an otherwise standard model of economic geography is the main objective of this paper. To this end, we follow Vives (1990) and retain the idea of competition within Chamberlin's (1933) "large group", whereas assuming that the size of the group is fixed (there is no entry and exit). The idea of using a fixed number of firms is far from being uncommon in economics. For example, it is central to the Arrow-Debreu model used in general equilibrium theory (Arrow and Hahn, 1971). The number of firms may also be fixed for various reasons. First, entrepreneurship may be a scarce resource that limits the proliferation of firms. Second, as argued in industrial organization, entry barriers may be built by firms to preserve their supranormal profits. Third, the number of possible varieties that firms may supply may be bounded because of legal (patents) and technological reasons.

As a natural by-product of this research strategy, we also intent to show that, once entry is restricted, increasing returns are no longer "the" necessary condition for agglomeration to arise. Rather, our analysis suggests that it is imperfect competition that matters for the formation of agglomeration in economic geography.[3] To be precise, using the canonical model

[1] According to Sørensen (2000), profits would account for about 15% of the GDP of many developed countries

[2] One could similarly study the impact of workers' remittances on the distribution of the industry.

[3] In saying so, we do not intend to deny the role of technological externalities which are

of economic geography with restricted entry, we show that *constant returns to scale are consistent with the emergence of an agglomeration provided that firms retain market power through the sale of a differentiated product.* By contrast, if the product they sell is homogenous, the only equilibrium involves dispersion with no interregional trade, a result that agrees with Starrett's (1978) spatial impossibility theorem.

In order to achieve our first objective, we use a framework involving several types of agents: *farmers* and *workers*, as in all economic geography models, but also *entrepreneurs* and *rent-holders*. Regional wages are determined on competitive labor markets and market power endows firms with a positive mark-up, thereby a positive profit can be distributed. Although we provide the analytical description of any ownership combination involving these agents, for the sake of clarity, we present and study a taxonomy of five particular ownership structures. In the first one, entrepreneurs are firms' owners - hence their earnings are equal to their firm's profits - and they must reside where their firm is established.[4] Second, we allow for the firms' owners to be rent-holders ("rentiers") who choose to reside wherever they want in the economy. In the third scenario, firms' owners live abroad, very much as the absentee landlords do in urban economics, in which case profits are not recycled within the economy.[5] In the fourth one, profits go to farmers who stand for immobile shareholders. Finally, profits are distributed to workers, and these earnings supplement their wages. In this case, workers' welfare depends on their real wage but also on the nominal profits made in each region. In accord with the spirit of economic geography, we assume that rent-holders, as well as other agents, never act as a group.

For all ownership structures, we show that agglomeration always arises when transport costs are sufficiently low (the sustain point). In other words, profit distribution may strengthen or weaken the agglomeration but never destroys it when transport costs are low. As expected, the sustain point varies with the structure of ownership. Specifically, we show that agglom-

crucial for the formation of urban clusters while being consistent with perfect competition (Henderson, 1974; Fujita and Ogawa, 1982; Anas *et al.*, 1998). However, we believe that this approach is more relevant in the study of micro-spaces than in studying the macro-spatial units considered in economic geography.

[4] A related model is developped by Forslid and Ottaviano (2001) who replace the mobile "entrepreneurs" with mobile skilled workers. Assuming that one unit of skilled labor is needed to start a firm, that the number of skilled workers is exogenous and that entry is free, they show that skilled workers earnings are equal to profits. A similar argument is made in Pflüger (2001).

[5] This is also a classical assumption in the literature on foreign direct investment; see, e.g. Haufler and Wooton (1999).

eration is the most likely when firms are owned by entrepreneurs and the least likely when profits go to the farmers or to absentee shareholders. By contrast, major differences occur when we come to the symmetric pattern: *the way profits are distributed may destroy the stability of the dispersed equilibrium for all values of the transport costs*

It is the entrepreneur model which bears the strongest resemblance with Krugman's core-periphery model: if varieties are very differentiated and/or the share of the manufacturing sector is large, there exists a "black hole" involving full agglomeration at any transport costs; otherwise, the industrial sector is dispersed (resp. agglomerated) for large (resp. small) transport costs. This strong similarity finds its origin in the fact that the regional income equals the regional product. Indeed, the entire value of production in a region is split between the workers and entrepreneurs living in this region. By contrast, the rentiers model, where rent-holders may displace their entire income, slightly differs from traditional core-periphery models. In this case, the symmetric equilibrium is never stable, thus implying that any stable equilibrium involves some degree of agglomeration. In other words, whatever its size, *the existence of a group of mobile rentiers is sufficient to trigger a process of* (partial or full) *agglomeration for all transport cost values.* This result is reminiscent of Cantillon who provided in 1755 the following explanation for the existence of cities: "landlords with large estates have the means to live at a distance from them to enjoy agreeable society with other gentlemen of the same condition". Their expenditure provides a living for craftsmen, merchants and servants, which in turn attract more landlords (Huriot and Perreur, 1992). It is also worth mentioning the case of the French Riviera where the move of rich and retired people has boosted the development of Nice (Eaton and Eckstein, 1997).[6]

Somewhere in between these two polar cases lie the remaining models. Farmer and absentee firms' owners models obey the same principles: there is never a black hole and industry is agglomerated (resp. dispersed) when transport costs are low (resp. large). Things are more subtle in the worker model: the symmetric equilibrium is always unstable when the share of industry is large and varieties are very differentiated. In the case where these two conditions are not met, we fall back on the traditional result: there is dispersion when transport costs are high. Whenever the symmetric equilibrium is unstable, the only stable equilibria are interior and involve a partial agglomeration of the manufacturing sector when transport costs are high.

[6]Florida is another point in case.

Regarding our second objective, the wide-spread belief is that increasing returns are needed for agglomeration to arise (see, e.g. Krugman, 1995). When dealing with increasing returns, it is nowadays common to appeal to Dixit and Stiglitz (1977), and models of economic geography are no exception. This means that increasing returns materialize at the firm's level under the form of positive fixed costs. Our claim is that these costs are no longer necessary once entry is restricted, thus allowing for constant returns to scale at the firm's level.[7] However, we should make it clear that our approach retains some form of indivisibility in that *each firm has a single address* (Fujita and Thisse, 1996). Hence, like in all economic geography models, agents are not ubiquitous and have a well-defined spatial identity. This should not be regarded as being very restrictive once it is understood that the two-region setting is a proxy for a model with a large number of regions. In this case, it is clear that managers, hence firms, and consumers cannot be all over the places without making the model totally unrealistic. The need for this assumption was already stressed by Koopmans (1957, p.154) long ago: "without recognizing indivisibilities - in human person, in residences, plants, equipment, and in transportation - urban location problems, down to those of the smallest village, cannot be understood".

Hence, we may conclude that what really matters for the existence of an agglomeration involving all mobile and nonubiquitous agents is the existence of positive price-cost margins.[8] This concurs with results obtained in spatial competition theory in which a sufficiently high degree of product differentiation is needed for a cluster of firms to arise in an oligopoly involving dispersed demand (de Palma *et al.*, 1985). In addition, our framework allows for studying asymptotic properties in which the elasticity of substitution among varieties tends to infinity (the product is homogeneous) or tends to one (varieties are independent). In the former case, we fall back on the competitive paradigm in which dispersion is the only equilibrium. In the latter, firms are independent and agglomeration is always an equilibrium when profits are distributed to mobile agents. Such an analysis cannot be performed within Krugman's model.

Although we use a standard model, our research objectives lead us to be very precise about the details of the model because, in contrast to many

[7]This is not inconsistent with Krugman's analysis because the level of fixed costs does not play any role in the conditions sustaining agglomeration as a stable equilibrium. Our conditions for agglomeration are also independent of the number of firms.

[8]Of course, our argument is developed within a specific model. This is because there exists no general model of general equilibrium with imperfect competition. A reflection's moment suffices to show that the same idea can be applied to other settings.

models of economic geography, we provide a full analytical solution. This is done in section 2 where we describe our various building blocks. The analysis of agglomeration is developed in section 3 for a general model encapsulating all ownership structures. However, the sustain point changes with the distribution of profits and the differences are made precise in Proposition 2. Because the stability of the symmetric pattern displays very different types of behavior according to the ownership structure, for clarity we consider each scenario separately in section 4. The various break points, when they exist, are compared in Proposition 7. Section 5 concludes.

2 The model

Our setting is standard in economic geography. The economic space is made of two regions $r = A, B$. There are two production factors, workers and farmers, and the economy is endowed with L farmers and M workers. Workers are perfectly mobile between regions, whereas farmers are immobile and equally split between the two regions. There are two sectors, farming and manufacturing. The former produces a homogeneous good, using farmers as the only input; the latter produces a continuum of varieties of a horizontally differentiated product, using workers as the only input.

2.1 Consumption

Preferences are represented by a Cobb-Douglas function with a share $\mu \in (0,1)$ of income spent on the manufactured good and a share $1-\mu$ spent on the agricultural good. The industrial good is a composite made of a continuum of differentiated varieties $i \in [0, N]$. Preferences for these varieties are represented by a CES function with an elasticity of substitution equal to $\sigma > 1$. Accordingly, the demand for variety i by households located in region r is as follows:

$$C_r(i) = \frac{\mu Y_r}{P_r}\left[\frac{\varsigma_r(i)}{P_r}\right]^{-\sigma} \qquad i \in [0, N] \tag{1}$$

where

$$P_r \equiv \left[\int_0^N \varsigma_r(i)^{-(\sigma-1)} di\right]^{-1/(\sigma-1)}$$

is the price index of the varieties sold in region r, $\varsigma_r(i)$ the price paid by a household located in region r for one unit of variety i, and Y_r the income available in region r.

The agricultural good is traded at no cost, whereas the proportion $0 < \tau < 1$ of a single unit of each variety shipped from one region arrives in the other region (the iceberg transport cost). Therefore, if variety i is consumed in the region where it is produced, the price paid by the households residing in this region, $\varsigma_r(i)$, is equal to the mill price $p_r(i)$. If variety i is consumed in the other region s, then the price paid by a household equals the delivered price, which exceeds the mill price and is such that

$$\varsigma_s(i) = p_r(i)/\tau > p_r(i) \qquad \text{with } s \neq r \tag{2}$$

Let n_r be the number of varieties produced in region r with $n_r + n_s = N$. Then, we have:

$$P_r = \left\{ \int_0^{n_r} p_r(i)^{-(\sigma-1)} di + \tau^{\sigma-1} \int_{n_r}^{N} p_s(i)^{-(\sigma-1)} di \right\}^{-1/(\sigma-1)}$$

Given (1), when firm i is located in region r, the demand for its output is

$$q_r(i) = \psi_r \left[\frac{p_r(i)}{P_r} \right]^{-\sigma} \tag{3}$$

where

$$\psi_r \equiv \mu \left[\frac{Y_r}{P_r} + \tau^{\sigma-1} \left(\frac{P_s}{P_r} \right)^{\sigma} \frac{Y_s}{P_s} \right]$$

is the total real income adjusted for transport costs (when $\tau \to 1$ and $P_r = P_s$, we have $\psi_r = \psi_s$).

For our analysis, the following ratios will be useful:

$$Y \equiv \frac{Y_r}{Y_s} \qquad P \equiv \frac{P_r}{P_s} \qquad \psi \equiv \frac{\psi_r}{\psi_s}$$

where it is readily verified that the ratio ψ of adjusted incomes is given by

$$\psi = \tau^{\sigma-1} \frac{1}{P^{\sigma}} \frac{1 + \tau^{-(\sigma-1)} P^{\sigma-1} Y}{1 + \tau^{\sigma-1} P^{\sigma-1} Y} \tag{4}$$

2.2 Production

The technology in the farming sector is such that one unit of output requires one farmer. The agricultural good is sold under perfect competition so that profits are zero. This good is chosen as the numéraire so that the wage prevailing in the farming sector equals 1 in each region.

Firms producing the differentiated product compete within a large group of firms, the total mass of which is fixed and equal to N. In other words, we have a market structure akin to monopolistic competition in which entry is restricted instead of being free. As a result, we may disregard the hypothesis of positive fixed costs and assume constant returns to scale in the manufacturing sector. Specifically, each variety is produced by a single firm and the production of one unit of variety i requires β workers; without loss of generality, we normalize $\beta = 1$ so that the number of units produced by a firm is equal to its number of workers. Because manufacturing firms have market power, they now earn *positive profits* regardless of their spatial distribution between regions. This is our main difference with Krugman. It should be clear that the way profits are distributed among economic agents affects individual incomes and, therefore, workers' as well as firms' location.

Suppose that firm i is located in region r. Its profits are defined as follows:

$$\pi_r(i) = [p_r(i) - w_r]q_r(i) \tag{5}$$

where w_r is the wage prevailing in region r whereas the demand $q_r(i)$ is given by (3). Since there is a continuum of manufacturing firms, each of them is negligible and accurately considers the price index P_r and the wage w_r as given. The firm producing variety i chooses its mill price to maximize its profits $\pi_r(i)$, which yields the same equilibrium price given by:

$$p_r \equiv p_r(i) = \frac{\sigma}{\sigma - 1} w_r \tag{6}$$

Using (3), it is easy to show that the profits made by a firm located in region r are such as

$$\pi_r = \frac{1}{\sigma}\left(\frac{\sigma}{\sigma - 1}\frac{w_r}{P_r}\right)^{1-\sigma} P_r \psi_r \tag{7}$$

These can be separated in revenues and costs:

$$\begin{aligned} p_r q_r &= \left(\frac{\sigma}{\sigma - 1}\frac{w_r}{P_r}\right)^{1-\sigma} P_r \psi_r = \sigma \pi_r \\ w_r q_r &= p_r q_r - \pi_r = (\sigma - 1)\pi_r \end{aligned} \tag{8}$$

They can also be used to compute the regional industrial product V_r:

$$V_r \equiv n_r p_r q_r = \sigma n_r \pi_r = \frac{\sigma}{\sigma - 1} n_r w_r q_r \tag{9}$$

2.3 Labor markets

In each region, the labor market is competitive. From (3) and (6), we may derive the demand for workers in region r:

$$n_r q_r = n_r \left(\frac{\sigma}{\sigma - 1}\frac{w_r}{P_r}\right)^{-\sigma} \psi_r \tag{10}$$

The demand changes with the adjusted income ψ_r, which depends itself on the spatial distribution of firms and workers. Hence, unlike Krugman (1991), *the firm/worker ratio changes with the distribution of activities.* Denoting m_r the number of workers in region r, (with $m_r + m_s = M$), labor market clearing in region r requires

$$m_r = n_r \left(\frac{\sigma}{\sigma - 1} \frac{w_r}{P_r} \right)^{-\sigma} \psi_r \tag{11}$$

Moreover, since labor market clearing requires that $m_r = q_r n_r$, we may relate the regional industrial product, the regional profit (Π_r) and the regional labor cost ($m_r w_r$) as follows:

$$V_r = \sigma \Pi_r = \frac{\sigma}{\sigma - 1} m_r w_r \tag{12}$$

2.4 Incomes

As discussed in the introduction, we allow for several types of agents in our economy: farmers and workers, but also entrepreneurs and rentiers. Denote by $R = r_r + r_s$ the number of rentiers, whereas the number of entrepreneurs is $N = n_r + n_s$ (one entrepreneur per firm). It is natural to assume that the rentiers live in the region in which their real income is the higher (or, equivalently, in the region in which the cost of living is the lower), whereas the entrepreneurs live in the region in which the firm is set up.

Let α be the share of profits distributed to the shareholders living in the economy (i.e. farmers, workers and rentiers) and γ the share attributed to the entrepreneurs, with $\alpha + \gamma \leq 1$; the share $1 - \alpha - \gamma$ goes to absentee shareholders, if any. We assume that shareholders (whoever they are) do not anticipate what the market outcome will be and buy fully diversified portfolio of firms' stocks. By contrast, entrepreneurs do not hold shares from other firms. When $\alpha > 0$, a fraction $\delta_F \geq 0$ of the corresponding total profits goes to the farmers, a fraction $\delta_W \geq 0$ to the workers, whereas the rentiers receive a fraction $\delta_R \geq 0$, with $\delta_F + \delta_W + \delta_R = 1$. As a result, the income of region r is made of the wages earned from the farming and manufacturing sectors, the dividends distributed by all firms to the local shareholders, and the profit share earned by the entrepreneurs working in this region:

$$Y_r = \frac{L}{2} + m_r w_r + \alpha \Pi \left(\delta_F \frac{1}{2} + \delta_W \frac{m_r}{M} + \delta_R \frac{r_r}{R} \right) + \gamma n_r \pi_r \tag{13}$$

where

$$\Pi \equiv \Pi_r + \Pi_s \equiv n_r \pi_r + n_s \pi_s$$

stands for the total profits in the economy. As discussed in the introduction, (13) is the main distinctive feature of our model. It implies that *regional income need not be equal regional product* because profits may flow from one region to the other according to the location of shareholders.

For simplicity, we will often focus on the following five institutional scenarios: the rentier model (case $R : \alpha = \delta_R = 1$), the entrepreneur model (case $E : \gamma = 1$), the worker model (case $W : \alpha = \delta_W = 1$), the farmer model (case $F : \alpha = \delta_F = 1$), and the absentee shareholders model (case $L : \alpha = \gamma = 0$).

2.5 The mobility of firms and workers

1. The locational decision of a firm depends on *who controls the firm.* In the entrepreneur model, each entrepreneur decides alone on the location of the firm and locates where his purchasing power is the larger, that is, where the real value of his profit share is the larger. Let G_r be the general price index of both the agricultural and manufactured goods consumed in region r:

$$G_r \equiv \left(\frac{1}{1-\mu}\right)^{1-\mu} \left(\frac{P_r}{\mu}\right)^{\mu}$$

The entrepreneur chooses region r if and only if

$$\frac{\pi_r}{G_r}\frac{G_s}{\pi_s} = \frac{w_r^{1-\sigma}P_r^{\sigma-\mu}\psi_r}{w_s^{1-\sigma}P_s^{\sigma-\mu}\psi_s} \geq 1 \quad \text{or} \quad w^{1-\sigma}P^{\sigma-\mu}\psi \geq 1$$

where

$$w \equiv \frac{w_r}{w_s}$$

By contrast, in the shareholder model, the location decision is driven by nominal profits regardless of where the shareholders reside because they fully diversify their assets. In this case, all firms locate in region r if and only if

$$\frac{\pi_r}{\pi_s} = w^{1-\sigma}P^{\sigma}\psi \geq 1$$

Using a dummy variable θ that is equal to one in the entrepreneur model and to zero otherwise, we find that firms always agglomerate in region r if, for $n_r = N$, we have:

$$w^{1-\sigma}P^{\sigma-\theta\mu}\psi \geq 1 \tag{14}$$

whereas an interior equilibrium ($n_r > 0$ and $n_s > 0$) implies that the equality holds in this expression. Hence, in all the institutional settings that we consider, (14) is the condition to satisfy for the agglomeration of firms in region r to be an equilibrium.

Using (2) and (6), the price ratio P in (14) is readily computed (see Appendix 1):

$$P = \tau \left(\frac{n + \tau^{\sigma-1} w^{\sigma-1}}{n + \tau^{-(\sigma-1)} w^{\sigma-1}} \right)^{\frac{-1}{\sigma-1}} \tag{15}$$

where

$$n \equiv \frac{n_r}{n_s}$$

2. Since workers can freely move between regions, they live in the region in which their real income is the higher. Clearly, a worker real income in region r is $(w_r + \alpha\delta_W \Pi/M)/G_r$. Hence, workers agglomerate in region r if, for $m_r = M$, we have:

$$\frac{w_r}{P_r^\mu} + \frac{\delta_W}{M}\frac{\alpha\Pi}{P_r^\mu} \geq \frac{w_s}{P_s^\mu} + \frac{\delta_W}{M}\frac{\alpha\Pi}{P_s^\mu} \tag{16}$$

whereas an interior equilibrium ($m_r > 0$ and $m_s > 0$) implies that the equality holds in this expression.

3. Since utilities and profits are continuous with respect to m_r and n_r, there always exists a spatial equilibrium (Ginsburgh *et al.*, 1985). We now turn to the study of these equilibria.

3 When does agglomeration arise?

Consider an agglomeration in which all firms, all workers and all rentiers are located within the same region r ($n_r = N$, $m_r = M$ and $r_r = R$). As usual, if such a configuration is an equilibrium, it is stable. Hence, we have to determine when the agglomeration conditions (14) and (16) hold.

To start with, we evaluate P and ψ at the agglomeration. As shown in Appendix 1, it is readily checked that $P = \tau < 1$. From (12), the following condition must also hold at the agglomeration:

$$\frac{\Pi}{Mw_r} = \frac{1}{\sigma - 1} \tag{17}$$

Using $P = \tau$ and (4), we get

$$\psi = \frac{1}{\tau}\frac{1+Y}{1+\tau^{2(\sigma-1)}Y} \tag{18}$$

which depends on the ratio of nominal incomes, Y. In Appendix 2, we show that this ratio is given by

$$Y = \frac{\sigma + \mu(\sigma - 1 + \alpha + \gamma) - \alpha\mu\delta_F}{\sigma - \mu(\sigma - 1 + \alpha + \gamma) + \alpha\mu\delta_F} > 1 \tag{19}$$

Since $P_r < P_s$ when $m_r = M$, rentiers choose to live in region r when firms agglomerate.

It remains to consider workers and firms. Substituting (17) in both the agglomeration condition for workers (16) and the agglomeration condition for firms (14) yields

$$P^{\frac{-\sigma+\theta\mu}{\sigma-1}} \psi^{\frac{-1}{\sigma-1}} \leq w^{-1} \leq \tau^{-\mu} + \alpha\delta_W \frac{1}{\sigma - 1}\left(\tau^{-\mu} - 1\right)$$

Let

$$\begin{aligned} K_1(\tau) &\equiv \tau^{-\frac{\sigma-1-\theta\mu}{\sigma-1}} \left(\frac{1+\tau^{2(\sigma-1)}Y}{1+Y}\right)^{\frac{1}{\sigma-1}} \\ K_2(\tau) &\equiv \tau^{-\mu} + \alpha\delta_W \frac{1}{\sigma-1}\left(\tau^{-\mu} - 1\right) \end{aligned}$$

Then, firms (resp.workers) cluster in region r if and only $K_1(\tau) \leq w^{-1}$ (resp. $w^{-1} \leq K_2(\tau)$). In other words, there is agglomeration in r if and only if

$$K_1(\tau) \leq w^{-1} \leq K_2(\tau) \tag{20}$$

When $K_1(\tau) > K_2(\tau)$ for some $\tau \in (0, 1)$, there exists no wage ratio w that satisfies simultaneously the equilibrium conditions (20). In other words, the agglomeration is not an equilibrium for the value of τ under consideration. By contrast, when $K_1(\tau) \leq K_2(\tau)$, the equilibrium wages satisfy (20) and agglomeration is an equilibrium for the corresponding τ. Indeed, if $w^{-1} < K_1(\tau)$, then workers want to agglomerate in region r whereas firms want to agglomerate in region $s \neq r$. In this case, wages in s (resp. r) must increase (resp. decrease). The same argument applies mutatis mutandis if $K_2(\tau) < w^{-1}$.

Let

$$\begin{aligned} F(\tau) &\equiv \left[\frac{\tau^{\mu}K_2(\tau)}{\tau^{\mu}K_1(\tau)}\right]^{\sigma-1} \\ &= \left[1 + \alpha\delta_W \frac{1}{\sigma-1}\left(1 - \tau^{\mu}\right)\right]^{\sigma-1} \tau^{(\sigma-1)(1-\mu)-\theta\mu} \frac{1+Y}{1+\tau^{2(\sigma-1)}Y} \end{aligned}$$

It is readily verified that $K_2(\tau) - K_1(\tau) \geq 0$ amounts to $F(\tau) \geq 1$. Therefore, the conditions for the agglomeration to be an equilibrium may be rewritten as follows:

$$F(\tau) \geq 1 \tag{21}$$

This condition is satisfied with equality when $\tau \to 1$.

If

$$(\sigma - 1)(1 - \mu) - \theta\mu \leq 0 \tag{22}$$

the condition (21) is always fulfilled so that there is always agglomeration. Hence, (22) is exactly what Krugman calls the black hole condition when $\theta = 1$. If $\theta = 0$, (22) never holds. In other words, *for a black hole to exist in our setting, firms must be controlled by entrepreneurs and not by shareholders whoever they are.* Such a restriction on firms' management does not arise in the standard core-periphery model in which the entire revenue of a firm is spent on wages.

When (22) is not fulfilled, (21) is never met when $\tau \to 0$, so that agglomeration is not an equilibrium. However, we have the following result.

Proposition 1 *(i) If $(\sigma - 1)(1-\mu) - \theta\mu \leq 0$, then the core-periphery structure is always a stable equilibrium. (ii) If $(\sigma - 1)(1-\mu) - \theta\mu > 0$, then there exists a unique value $\widehat{\tau} \in (0, 1)$ such that the core-periphery structure is a stable equilibrium for any $\tau \geq \widehat{\tau}$.*

Proof: The first part of the proposition is straightforward. The second part is proved by computing the first derivative of F with respect to τ:

$$F_\tau = \left[1 + \alpha\delta_W \frac{1}{\sigma - 1}(1 - \tau^\mu)\right]^{\sigma-1} \tau^{(\sigma-1)(1-\mu)-\theta\mu-1} \frac{1+Y}{1 + \tau^{2(\sigma-1)}Y}\Psi$$

where

$$\begin{aligned}\Psi \quad \equiv \quad & -\frac{\alpha\delta_W \mu \tau^\mu}{1 + \alpha\delta_W(\sigma - 1)^{-1}(1 - \tau^\mu)} - 2(\sigma - 1)Y \frac{\tau^{2(\sigma-1)}}{1 + \tau^{2(\sigma-1)}Y} \\ & + (\sigma - 1)(1 - \mu) - \theta\mu\end{aligned}$$

Since Y is here independent of τ by (19), it is readily verified that Ψ decreases with τ and is positive at $\tau = 0$ when $(\sigma - 1)(1 - \mu) - \theta\mu > 0$. Moreover, we have

$$F_\tau(1) = -\mu\alpha\delta_W - (\sigma - 1)\left(\frac{Y - 1}{Y + 1} + \mu\right) - \theta\mu < 0$$

because $Y > 1$. Thus, the derivative of F changes its sign once and only once over $(0, 1)$. The function F starts from 0 at $\tau = 0$; it is first increasing, then takes values larger than 1 and finally decreases to reach the value 1 at $\tau = 1$. We denote $\widehat{\tau}$ the single solution of $F(\tau) = 1$. Note that $F_\tau(\widehat{\tau})$ is positive. Q.E.D.

As discussed in the introduction, we see that *increasing returns are not necessary for the existence of agglomeration.* Yet, under constant returns but no entry, agglomeration arises under conditions similar to those obtained with the standard core-periphery model. When transportation costs are large ($\tau < \widehat{\tau}$), the competition effect dominates: shareholders and/or entrepreneurs prefer to avoid competition by choosing dispersed locations. When transportation costs are small ($\tau \geq \widehat{\tau}$), the home market effect dominates: firms prefer to agglomerate in the region that guarantees the higher earnings to their entrepreneurs and shareholders. Using the terminology of Krugman (1991), $\widehat{\tau}$ is called the *sustain point*.

Our result is very similar to the one obtained by Krugman (1991) in the case of the entrepreneur model ($\theta = 1$) since the black hole condition is the same, whereas a sustain point otherwise exists. In the other models, agglomeration is never a black hole, thus pointing to a first difference between the two frameworks.

Another distinctive feature of our model is that the emergence of the core-periphery pattern depends on the way profits are distributed across economic agents because the value of $\widehat{\tau}$ changes with the distribution parameters. When entrepreneurs choose location ($\theta = 1$), their decision to leave the core depends on the difference between real profits in the core and the periphery. In contrast, when managers choose location to maximize shareholders' earnings ($\theta = 0$), their decision is based on the difference between nominal profits. In the agglomeration equilibrium, the regional profit difference is larger in real terms than in nominal terms because $P_r < P_s$. As a consequence, *entrepreneurs are less enticed to leave the core than managers acting for shareholders.* Agglomeration is, therefore, sustained for a larger domain of transport costs in the entrepreneur model than in the shareholder model.

More precisely, given (19), it is readily verified that Y strictly increases with α and γ but strictly decreases with δ_F. Moreover, the left hand side of (21) strictly increases with δ_W, θ, Y and, hence, with α. Accordingly, we have the following properties. The domain of τ-values sustaining the agglomeration widens (i) when the share of profits distributed within the two regions increases (higher α and γ), (ii) when the share of profits distributed to mobile agents increases (higher δ_W and δ_R), and (iii) when firms are run by entrepreneurs. This implies that *the presence of mobile entrepreneurs and* *rentiers exacerbates the impact of workers' mobility.* When moving toward a particular region, these two types of agents also move their purchasing power, thus making the home market effect generated by workers stronger. The opposite holds when a larger fraction of profits goes to farmers or absentee shareholders.

Using the results above, it is possible to rank the sustain points corresponding to our five scenarios in which all profits are distributed to the entrepreneurs (E), the workers (W), the rentiers (R), the farmers (F), or the absentee shareholders (L) (see Appendix 3 for a proof).

Proposition 2 *We have* $\widehat{\tau}_E < \widehat{\tau}_W < \widehat{\tau}_R < \widehat{\tau}_F = \widehat{\tau}_L$.

Consequently, when firms belong to farmers or absentee shareholders, the agglomeration is less likely to occur. Nevertheless, even in this case, agglomeration is always sustainable for low transport costs since, in any case, workers have to spend their wage in the region where they live. *The mere existence of mobile workers is sufficient to generate a home market effect that triggers the agglomeration of the manufacturing sector.*

Our model supports the view that agglomeration is attributable to imperfect competition in the product market. Indeed, agglomeration never occurs for $\tau < 1$ when varieties are perfect substitutes ($\sigma \to \infty$) since

$$\lim_{\sigma \to \infty} F(\tau) = \exp\left(\alpha \delta_W \left(1 - \tau^{\mu}\right)\right) \cdot 0 \cdot (1 + Y) = 0 \qquad \text{for } \tau < 1$$

Likewise, agglomeration always occurs if products have unit elasticity of substitution ($\sigma \to 1$) and are, therefore, strongly differentiated:

$$\lim_{\sigma \to 1} F(\tau) = 1 \cdot \tau^{-\theta\mu} \cdot 1 = \tau^{-\theta\mu} \geq 1$$

since $\theta \geq 0$. Hence we have:

Proposition 3 *Agglomeration never occurs when firms sell a homogenous product ($\sigma \to \infty$). By contrast, when firms sell independent varieties ($\sigma \to 1$), agglomeration is always an equilibrium.*

4 When does symmetry arise?

The symmetric configuration is an equilibrium because, at $P = \psi = w = 1$, firms and workers are indifferent between locations, both the ratio of real and of nominal profits being equal to 1. However, the symmetric equilibrium may be unstable for all values of τ. This is so when there are mobile rentiers ($\delta_R > 0$). Indeed, starting from the symmetric configuration, any deviation by a nonnegligible set of firms toward a region decreases the price index in that region and attracts all rentiers ($\overline{\tau}_R = 0$). As a result, the regional income jumps up discontinuously, thus inducing more firms to relocate in this region. Moreover, when $\tau < \widehat{\tau}_R$, the agglomeration is not an equilibrium so that the only stable equilibria are interior and involve a partial agglomeration of the manufacturing sector. In order to find stable symmetric equilibria, we restrict ourselves to the case where $\delta_R = 0$ and $\delta_F + \delta_W = 1$.

We follow Puga (1999) and assume that workers adjust instantaneously across regions to a change in firms' distribution so that the equality in (16) always holds; on the contrary, firms react gradually to profit differences. The symmetric equilibrium is then unstable when the ratio of (real or nominal) profits decreases when a small but nonnegligible group of firms move to region r. To check the stability of this equilibrium, we totally differentiate the equilibrium condition for firms when $\theta = 1$ and $\theta = 0$. To this end, consider a relocation of firms $dn > 0$ around the symmetric equilibrium. This equilibrium is unstable if and only if

$$(1 - \sigma)\, dw + (\sigma - \theta\mu)dP + d\psi > 0 \tag{23}$$

Hence, studying the stability of the symmetric equilibrium amounts to characterizing the marginal variations of the variables w, P and ψ at this equilibrium.

Before proceeding, we present some relationships holding when $n_r = N/2$ and $m_r = M/2$. By (12), one gets

$$\frac{\Pi}{Mw_r} = \frac{1}{\sigma - 1} \tag{24}$$

Also, as shown in Appendix 1, the price index can be written as follows:

$$P_r = \frac{\sigma}{\sigma - 1} w_r \left(\frac{N}{2}\right)^{\frac{-1}{\sigma-1}} \left(1 + \tau^{\sigma-1}\right)^{\frac{-1}{\sigma-1}}$$

Using this expression, (11) and the value of ψ_r in (3) evaluated at the $n_r = N/2$, $m_r = M/2$, $Y_r = Y_s$, $P_r = P_s$, one can compute

$$\mu Y_r = \frac{\sigma}{\sigma - 1}\frac{M}{2} w_r = p_r q_r \frac{N}{2}$$

where, the last equality comes from (9) and labor market clearing ($M/2 = q_r N/2$). Thus, at the symmetric equilibrium, the expenditures on manufactures by individuals living in region r is equal to the value of manufactured production in that region. Moreover, applying the definition (13) of earnings to the symmetric equilibrium and using the equation (24), the earnings in region r can be written as

$$Y_r = \frac{L}{2} + \frac{M}{2} w_r \left(1 + \frac{\alpha + \gamma}{\sigma - 1}\right)$$

The last two equations allow us to derive the regional income and wage at the symmetric equilibrium:

$$Y_r = \frac{\sigma L/2}{\sigma - \mu(\sigma - 1 + \alpha + \gamma)} \tag{25}$$

$$w_r = \frac{\mu\,(\sigma - 1)}{\sigma - \mu(\sigma - 1 + \alpha + \gamma)} \frac{L}{M} \tag{26}$$

At the symmetric equilibrium, incomes and wages are independent of the distribution of profits between farmers and workers. However, both incomes and wages increase when more profits are distributed among agents within the economy, that is, when $\alpha + \gamma$ rises.

We are now able to analyze the variations of P, ψ and w at the symmetric equilibrium.

1. Taking the logarithm of the price index ratio (15) and differentiating the corresponding expression at $P = w = n = 1$ yields

$$(\sigma - 1)\, dP = -H[dn - (\sigma - 1)\, dw] \tag{27}$$

where

$$H \equiv \frac{1 - \tau^{\sigma-1}}{1 + \tau^{\sigma-1}} > 0$$

is decreasing with τ, is equal to 1 when $\tau \to 0$ and to 0 when $\tau \to 1$. The price index in one region decreases as more varieties are produced in that region and increases as the regional nominal wage rises.

2. Workers' mobility is given by condition (16) with equality. Differentiating the left hand side of this expression gives

$$\frac{dw_r}{P_r^{\mu}} - \left(\frac{\mu w_r}{P_r^{\mu+1}} + \alpha \frac{\mu \Pi}{P_r^{\mu+1}} \frac{\delta_W}{M}\right) dP_r + \alpha \frac{d\Pi}{P_r^{\mu}} \frac{\delta_W}{M} \tag{28}$$

whereas differentiating the right hand side gives a similar expression where r is replaced with s. Around the symmetric equilibrium, $\Pi / (w_r M) = 1/(\sigma - 1)$ (see (24)), $w_r = w_s$, $P_r = P_s$ while $dw_r = -dw_s$ and $dP_r = -dP_s$. Hence,

$$dw = \frac{2dw_r}{w_r} \qquad dP = \frac{2dP_r}{P_r}$$

thus implying that (28) can be rewritten as follows:

$$dw = \mu dP \left(1 + \alpha \delta_W \frac{1}{\sigma - 1}\right) \tag{29}$$

For this equation to hold, the distribution of workers must be interior ($m_A > 0$ and $m_B > 0$). It means that wages and prices vary together. When $\alpha\delta_W = 0$, wages vary less than price around the symmetric equilibrium. However, in the worker model ($\alpha\delta_W > 0$), a small decrease in the price index may generate a more than proportional decrease in wages due to an increase in real profits; clearly, the larger $1 + \alpha\delta_W/(\sigma - 1)$, the larger this impact.

Using (27) and (29), we obtain

$$dP = -\frac{H}{(\sigma - 1)(1 - \mu H) - \alpha\delta_W \mu H} dn \qquad (30)$$

In any model different from the worker model, we have $\alpha\delta_W = 0$ so that the denominator of (30) is always positive. This implies that P and n are inversely related. In the worker model, we have $\alpha\delta_W = 1$ so that the sign of the denominator may change. Yet, the price index and the number of firms are still inversely related if and only if the denominator of (30) is positive, that is,

$$(\sigma - 1) - \mu\sigma H > 0 \qquad (31)$$

This condition is satisfied for high σ, low μ and high τ. In this case, the relocation of a small number of firms triggers a small adjustment in the workers' distribution.

By contrast, when (31) does not hold, the adjustment of workers to dn is not marginal because the associated price decrease (see (30)) is amplified by the wage decrease; thus both effects combine to generate a snowball effect. As a result, the relocation of a small number of firms pull all workers into the region where these firms have relocated. This, in turn, means that more firms want to move into this region. Hence, when (31) does not hold, (29) is no longer valid, thus implying that the dispersed equilibrium is unstable. It is worth noting that *such a process is driven by the dividends that firms pay to the workers who therefore pay less attention to their wage*. In the other models considered in this paper, this never arises because workers care only about their wage.

3. The adjusted income ratio ψ is given by (4). The demand for manufactured goods rises as the income increases and as the price index decreases in the region under consideration. Taking the logarithm of (4) and differentiating the corresponding expression at $Y = P = \psi = 1$ shows that the marginal variation of the adjusted income ratio at the symmetric equilibrium is given by

$$d\psi = -\sigma dP + [(\sigma - 1) dP + dY] H \qquad (32)$$

so we must also consider dY.

4. Incomes are defined in (13) where we set $\delta_R = 0$. Using (9) and $\Pi = \pi_r n_r + \pi_s n_s$, incomes may be rewritten as follows:

$$Y_r = \frac{L}{2} + \frac{\sigma-1}{\sigma}V_r + \frac{\alpha}{\sigma}(V_r + V_s)\left(\frac{\delta_F}{2} + \delta_W \frac{\sigma-1}{\sigma}\frac{V_r}{w_r M}\right) + \gamma\frac{V_r}{\sigma} \tag{33}$$

Differentiating the income ratio at the symmetric equilibrium gives (see Appendix 4):

$$dY = \frac{\mu}{\sigma}\left\{(\sigma - 1 + \gamma + \alpha\delta_W)[(1-\sigma)dw + \sigma dP + d\psi + dn] - \alpha\delta_W dw\right\} \tag{34}$$

The values of dP, $d\psi$, dw, and dY may be obtained by solving the system made of (27), (29), (32) and (34) and are given in Appendix 5. Substituting those expressions into (23) and rearranging terms, we get the following general necessary and sufficient condition for the symmetric equilibrium to be unstable:

$$-\frac{\Lambda H - \mu(\sigma-1)\left[\gamma(\sigma-1) + (2\sigma-1)(\sigma-1+\alpha\delta_W) + \theta\sigma\right]}{(\sigma-1)\left[\mu\alpha\delta_W H - (1-\mu H)(\sigma-1)\right]\left[\mu\alpha\delta_W H - (1-\mu H)(\sigma-1) + \mu\gamma H - 1\right]} H dn > 0 \tag{35}$$

where

$$\begin{aligned}\Lambda \equiv\ & \alpha\mu^2\delta_W[(\sigma-2)\alpha\delta_W + (\sigma-1)(2\sigma+\gamma-3+\theta)] + \\ & (\sigma-1)\left[(\sigma-1)\sigma + \mu^2(\sigma+\gamma-1)(\sigma+\theta-1)\right]\end{aligned}$$

In the sequel, we focus on the following four special cases in which all profits are distributed to the entrepreneurs (E), the workers (W), the farmers (F), or the absentee shareholders (L).[9]

4.1 The entrepreneur model

We consider a setting in which $\theta = \gamma = 1$ and $\alpha = 0$. In this case, (35) becomes:

$$-\frac{(\sigma - 1 + \mu^2\sigma)H - \mu(2\sigma-1)}{(\sigma-1)(1-\mu H)^2} H dn > 0$$

Thus, the symmetric equilibrium is unstable if and only if

$$H < G_E \equiv \frac{\mu(2\sigma-1)}{\sigma - 1 + \mu^2\sigma}$$

[9]Recall that we have removed the case of rentiers.

If $(\sigma-1)(1-\mu)-\mu \leq 0$, the symmetric equilibrium is never stable because $G_E \geq 1$. Otherwise, G_E is smaller than 1 so that the equation $H = G_E$ has a unique solution, denoted $\overline{\tau}_E$, because H is decreasing in τ from 1 to 0. In this case, the symmetric equilibrium is unstable for all $\tau > \overline{\tau}_E$.

Proposition 4 *Let the entrepreneurs be the firms' owners. (i) If* $(\sigma-1)(1-\mu)-\mu \leq 0$*, the symmetric equilibrium is always unstable. (ii) If* $(\sigma-1)(1-\mu)-\mu > 0$*, then there exists a unique value* $\overline{\tau}_E \in (0,1)$ *such that the symmetric equilibrium is stable for any* $\tau \leq \overline{\tau}_E$*.*

Hence, the entrepreneur model yields results that are perfectly similar to those obtained by Krugman (1991) and Fujita *et al.* (1999) in the standard core-periphery model.

4.2 The worker model

We now turn to the case in which $\theta = \gamma = 0$ and $\alpha = \delta_W = 1$. Then, we know that the symmetric equilibrium is always unstable when (31) does not hold. In what follows, we therefore assume that (31) is satisfied. The condition (35) may be rewritten as follows:

$$-\frac{[(\sigma-1)^2 + \mu^2\left(\sigma^2-\sigma-1\right)]H - \mu(\sigma-1)(2\sigma-1)}{(\sigma-1)(\sigma-1-\mu H\sigma)(1-\mu H)} H dn > 0 \qquad (36)$$

Let

$$G_W \equiv \frac{\mu(2\sigma-1)(\sigma-1)}{\mu^2(\sigma^2-\sigma-1)+(\sigma-1)^2} = \frac{\mu(2\sigma-1)}{\sigma(1+\mu^2)-1-\frac{\mu^2}{(\sigma-1)}}$$

This can take negative values when the elasticity of substitution is very low (i.e., $-\left(\sigma^2-\sigma-1\right)/(\sigma-1)^2 > 1/\mu^2$). Hence, (36) may be rewritten as follows:

$$\frac{\mu(2\sigma-1)}{(1-H\mu)} \frac{1-HG_W^{-1}}{\sigma(1-H\mu)-1} H dn > 0$$

Since the first ratio of this expression is always positive, the stability property of the symmetric equilibrium depends on the second ratio. Consequently, we have the following two cases.

(i) When $G_W > 0$, that is, when the elasticity of substitution is not too low, the symmetric equilibrium is unstable if and only if $H < G_W$. Thus, there exists a $\overline{\tau}_W \in (0,1)$ such that all symmetric equilibria are unstable as long as $\tau > \overline{\tau}_W$. In other words, symmetry is unstable for low transport costs (high τ). Note, however, that there exists a non-empty set of parameters for which $G_W > 1 > H$ so that the symmetric equilibrium is always unstable ($\overline{\tau}_W = 1$).

(ii) When $G_W < 0$, the symmetric equilibrium is always unstable because $G_W^{-1} < 0 < H^{-1}$.

Proposition 5 *Let the workers be the firms' owners. (i) If $\sigma(1 - H\mu) < 1$, then the symmetric equilibrium is always unstable. (ii) If $\sigma(1 - H\mu) > 1$, then the symmetric equilibrium is unstable if and only if $HG_W^{-1} < 1$.*

Whenever the symmetric equilibrium is unstable, *the only stable equilibria are interior and involve a partial agglomeration of the manufacturing sector when $\tau < \widehat{\tau}_W$* because the agglomeration is not an equilibrium.

Thus, the case where profits are distributed to workers vastly differs from the standard core-periphery model since the symmetric equilibrium may be stable or unstable for all values of transport costs. Although the whole production value is given to workers in both Krugman's model and ours, the distribution of this value is very different. In Krugman, workers care only about the value of production within the firm they work for. Here, workers have a diversified portfolio of firms so that what matters to them is the level of total profits together with the wage paid by their firm. In addition, workers care about nominal profits here whereas they care about the real value of their production in Krugman.

4.3 The farmer or absentee shareholder model

When profits are distributed to farmers only ($\theta = \gamma = \delta_W = 0$ and $\alpha = 1$) or to absentee shareholders ($\theta = \gamma = \alpha = 0$), (35) becomes:

$$-\frac{(\sigma + \mu^2\sigma - \mu^2)H - \mu\,(2\sigma - 1)}{(\sigma - \mu H\sigma + \mu H)(1 - \mu H)} H dn > 0$$

Thus, the symmetric equilibrium is stable if and only if

$$H \geq G_F \equiv \frac{\mu\,(2\sigma - 1)}{\sigma + \mu^2\sigma - \mu^2}$$

It is readily verified that G_F is always positive and smaller than 1. Hence, the equation $H = G_F$ has a unique solution, denoted $\overline{\tau}_F$. In other words, we have:

Proposition 6 *Let the farmers be the firms' owners. Then, there exists a unique value $\overline{\tau}_F \in (0, 1)$ such that the symmetric equilibrium is stable if and only if $\tau \leq \overline{\tau}_F$.*

The symmetric equilibrium is stable if and only if $H \geq G_L \equiv G_F$, that is, if and only if $\tau \leq \overline{\tau}_L \equiv \overline{\tau}_F$. Stated differently, *the occurrence of dispersion is the same when profits are distributed to farmers or to absentee owners*, a result which agrees with intuition.

4.4 Comparative statics

It is readily verified that $G_F = G_L < G_E$ and that $G_L < G_W$. Hence, since $H(\tau) = G$ where H is strictly decreasing, we have the following proposition.

Proposition 7 *We have* $0 = \overline{\tau}_R \leq \overline{\tau}_E < \overline{\tau}_F = \overline{\tau}_L$ *and* $\overline{\tau}_W < \overline{\tau}_F$.

The range of transportation costs supporting the stability of the symmetric equilibrium is larger when profits are lost or distributed to farmers than when they are distributed to the entrepreneurs or to the workers. However, it is not possible to rank $\overline{\tau}_W$ unambiguously with respect to $\overline{\tau}_E$.

It remains to consider the case where $\sigma \to \infty$. It is readily verified that

$$\lim_{\sigma\to\infty} G_E = \lim_{\sigma\to\infty} G_W = \lim_{\sigma\to\infty} G_F = \frac{2\mu}{1+\mu^2}$$

Because

$$\lim_{\sigma\to\infty} H = \lim_{\sigma\to\infty} \frac{1-\tau^{\sigma-1}}{1+\tau^{\sigma-1}} = 1 > \frac{2\mu}{1+\mu^2}$$

(for $\tau < 1$ and $\mu \in (0,1)$). Hence, we have:

Proposition 8 *When the varieties sold by firms are perfect substitutes, symmetry is always a stable equilibrium.*

This agrees with the standard neoclassical model which predicts convergence between the two regions. More precisely, the market equilibrium is such that each region accommodates the same number of firms selling the same good at marginal cost pricing; wages are the same in the two regions and equal to the marginal productivity of labor and firms make zero profits. Furthermore, even though firms are indifferent about their locations since they earn zero profits regardless of their location, a partially agglomerated configuration is never an equilibrium. Hence, *symmetry is the unique equilibrium when varieties are perfect substitutes.*

When varieties become independent ($\sigma \to 1$), dispersion is always unstable in the entrepreneur and worker models. However, dispersion forces remain strong enough in the farmer and absentee owner models for the symmetric pattern to be stable when $\tau \leq \overline{\tau}_F$, because profits are evenly distributed across regions.

5 Concluding Remarks

We have shown that varieties sold by firms must be differentiated for agglomeration to emerge as a stable equilibrium. If they are not, there is always full dispersion. However, what really matters is the existence of a positive mark-up. To illustrate, agglomeration also arises in a setting in which firms produce a homogeneous good and compete in quantities (Combes, 1997).

According to the ownership structure, agglomeration occurs for different domains of transport cost values. For example, there is always some form of agglomeration when the shareholders are rentiers who are free to live where they want. Dispersion is more likely when profits are given to farmers or to absentee shareholders. This is because the spatial distribution of demand for the manufactured product is not affected by their additional purchasing power. On the other hand, the existence of a mobile work force is always sufficient to trigger an agglomeration for low transportation costs.

More surprising, perhaps, is the fact that the symmetric pattern may be an unstable equilibrium for all values of transport costs when varieties are differentiated. In this case, the stable equilibria involve partial or full agglomeration of firms and workers. The partially agglomerated configurations correspond to the stable equilibria in the standard core-periphery model, thus showing that the existence of pure profits may fundamentally affect the results of the core-periphery model. Hence, comparing the results of most economic geography models based on free-entry with our results, we may fairly conclude that the existence of positive profits tends to boost more (full or partial) agglomeration.

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Appendix

1. Using the relation $\varsigma_s(i) = p_r(i)/\tau$ for $s \neq r$ and (6), we get

$$\begin{aligned} P_r &= p_r \left[n_r + \left(\frac{1}{\tau} \frac{w_s}{w_r} \right)^{-(\sigma-1)} n_s \right]^{\frac{-1}{\sigma-1}} A.1 \qquad (37) \\ P_s &= p_s \left[n_s + \left(\frac{1}{\tau} \frac{w_r}{w_s} \right)^{-(\sigma-1)} n_r \right]^{\frac{-1}{\sigma-1}} \\ &= \frac{1}{\tau} p_r \left[n_r + \left(\tau \frac{w_s}{w_r} \right)^{-(\sigma-1)} n_s \right]^{\frac{-1}{\sigma-1}} \end{aligned}$$

which implies that the price ratio is

$$P \equiv \frac{P_r}{P_s} = \tau \left(\frac{n + \tau^{\sigma-1} w^{\sigma-1}}{n + \tau^{-(\sigma-1)} w^{\sigma-1}} \right)^{\frac{-1}{\sigma-1}}$$

In case of agglomeration in region r, we have $n_r = N$, $n_s = 0$ and

$$P_r = p_r N^{\frac{-1}{\sigma-1}} = \tau P_s = \frac{\sigma}{\sigma - 1} w_r N^{\frac{-1}{\sigma-1}} \qquad (A.2)$$

where the last equality uses (6). Also, $P = \tau$.

In case of symmetric locations, $n_r = N/2 = n_s$. Also, by (6) and (A.1), the price index can be written as follows:

$$P_r = p_r \left(\frac{N}{2}\right)^{\frac{-1}{\sigma-1}} \left(1+\tau^{\sigma-1}\right)^{\frac{-1}{\sigma-1}} = \frac{\sigma}{\sigma-1} w_r \left(\frac{N}{2}\right)^{\frac{-1}{\sigma-1}} \left(1+\tau^{\sigma-1}\right)^{\frac{-1}{\sigma-1}} \tag{A.3}$$

2. Proof of (19). In order to evaluate Y, we need the value of w_r at the agglomeration. Using (13) and (17), we then find

$$\begin{aligned} Y_r &= \frac{L}{2} + Mw_r + \frac{Mw_r}{\sigma-1}\left[\alpha\left(\frac{\delta_F}{2} + \delta_W + \delta_R\right) + \gamma\right] A.4 \qquad (38) \\ Y_s &= \frac{L}{2} + \alpha\frac{\delta_F}{2}\frac{Mw_r}{\sigma-1} \end{aligned}$$

which can be evaluated once the wage w_r is known. This wage is computed from labor market equilibrium (11) when $m_r = M$ and $n_r = N$:

$$M = N\left(\frac{\sigma}{\sigma-1}\frac{w_r}{P_r}\right)^{-\sigma} \psi_r$$

Using this expression, the last equality in (A.2) and, from (3), the value of ψ_r evaluated at agglomeration

$$\psi_r = \frac{\mu}{P_r}(Y_r + Y_s)$$

we find the wage in region r under agglomeration:

$$w_r = \mu\frac{\sigma-1}{\sigma}\frac{Y_r+Y_s}{M}$$

Substituting this expression in (A.4) and solving for Y_r and Y_s, we find the corresponding value of Y:

$$Y \equiv \frac{Y_r}{Y_s} = \frac{\sigma + \mu(\sigma-1+\gamma) + \alpha\mu(\delta_W + \delta_R)}{\sigma - \mu(\sigma-1+\gamma) - \alpha\mu(\delta_W+\delta_R)} = \frac{\sigma+\mu(\sigma-1+\alpha+\gamma) - \alpha\mu\delta_F}{\sigma - \mu(\sigma-1+\alpha+\gamma) + \alpha\mu\delta_F}$$

which is the desired equality.

3. The sustain point is such that $F(\widehat{\tau}) = 1$. We evaluate F in the five cases under consideration. First, note that

$$\begin{aligned} Y_R &= Y_W = Y_E = \frac{1+\mu}{1-\mu} \\ Y_L &= Y_F = \frac{\sigma+\mu(\sigma-1)}{\sigma-\mu(\sigma-1)} \end{aligned}$$

which can be used to compute F in the five cases. Set

$$K(\tau) \equiv 1 - \mu + \tau^{2(\sigma-1)}(1+\mu)$$

After some algebraic manipulations, the condition $F(\tau)=1$ can be shown to be equivalent to each of the following conditions:

$$\begin{aligned}
2\tau^{(\sigma-1)(1-\mu)}-\frac{\mu}{\sigma}\left[1-\tau^{2(\sigma-1)}\right] &= K(\tau) \quad \text{(Lost profits)}\\
2\tau^{(\sigma-1)(1-\mu)}-\frac{\mu}{\sigma}\left[1-\tau^{2(\sigma-1)}\right] &= K(\tau) \quad \text{(Farmers)}\\
2\tau^{(\sigma-1)(1-\mu)}\left[1+\frac{1}{\sigma-1}\left(1-\tau^{\mu}\right)\right]^{\sigma-1} &= K(\tau) \quad \text{(Workers)}\\
2\tau^{(\sigma-1)(1-\mu)-\mu} &= K(\tau) \quad \text{(Entrepreneurs)}\\
2\tau^{(\sigma-1)(1-\mu)} &= K(\tau) \quad \text{(Rentiers)}
\end{aligned}$$

We have already shown that, in each model, there is a single value of τ that satisfies these conditions over the interval $(0,1)$. Note that $K(\tau)$ increases in τ and is positive at $\tau=0$, whereas the left-hand-sides (LHS) are negative or equal to zero at $\tau=0$. Hence, each LHS must be increasing when it crosses the corresponding $K(\tau)$. Therefore, if, for all $\tau\in(0,1)$, the LHS of a model is larger than the LHS of another model, the sustain point of the former must be lower than the sustain point of the latter.

It is straightforward to check that $\text{LHS}_L=\text{LHS}_F$, $\text{LHS}_R>\text{LHS}_F$ and $\text{LHS}_W>\text{LHS}_R$. Hence, $\widehat{\tau}_W<\widehat{\tau}_R<\widehat{\tau}_F=\widehat{\tau}_L$.

It is also possible to prove that $\text{LHS}_E>\text{LHS}_W$ (that is, $\widehat{\tau}_E<\widehat{\tau}_W$). Indeed,

$$\text{LHS}_W-\text{LHS}_E=2\tau^{(\sigma-1)(1-\mu)}\left\{\left[1+\frac{1}{\sigma-1}\left(1-\tau^{\mu}\right)\right]^{\sigma-1}-\tau^{-\mu}\right\}$$

The sign of this expression is the same as the sign of

$$1+\frac{1}{\sigma-1}\left(1-\tau^{\mu}\right)-\tau^{-\frac{\mu}{\sigma-1}}=\frac{\sigma}{\sigma-1}-\tau^{\mu}\left(\frac{1}{\sigma-1}+\tau^{-\frac{\mu\sigma}{\sigma-1}}\right)$$

which increases with τ. Indeed, the derivative of this expression with respect to τ is

$$\frac{\mu}{\sigma-1}\tau^{\mu-1}\left(\tau^{-\frac{\mu\sigma}{\sigma-1}}-1\right)>0$$

Hence, the largest value of $\text{LHS}_W-\text{LHS}_E$ occurs at $\tau=1$, which is equal to 0, thus showing that $\text{LHS}_E>\text{LHS}_W$ for all $\tau\in(0,1)$. Putting all these results together, we obtain:

$$\widehat{\tau}_E<\widehat{\tau}_W<\widehat{\tau}_R<\widehat{\tau}_F=\widehat{\tau}_L$$

4. From (33), we have

$$dY=\frac{2dY_r}{Y_r}$$

Since $dY_r = -dY_s$ and since, by (9), $V_r + V_s = \sigma\Pi = Mw_r\sigma/(\sigma-1)$, we obtain

$$dY_r = \frac{\sigma - 1 + \gamma}{\sigma} dV_r + \frac{\alpha\delta_W}{\sigma}\left(dV_r - V_r\frac{dw_r}{w_r}\right)$$

From (8), it is straightforward to check that

$$\begin{aligned}\frac{dV_r}{V_r} &= (1-\sigma)\frac{dw_r}{w_r} + \sigma\frac{dP_r}{P_r} + \frac{d\psi_r}{\psi_r} + \frac{dn_r}{n_r} \\ &= \frac{1}{2}\left[(1-\sigma)\,dw + \sigma dP + d\psi + dn\right]\end{aligned}$$

Using this expression, the value of V_r (9), the expression $dw_r = w_r dw/2$ and the values of Y_r and w_r evaluated at the symmetric equilibrium ((25) and (26) give $w_r/Y_r = \mu\left(\sigma-1\right)/M\sigma$) yields after simplifications:

$$dY = \frac{\mu}{\sigma}\left\{(\sigma - 1 + \gamma + \alpha\delta_W)[(1-\sigma)dw + \sigma dP + d\psi + dn] - \alpha\delta_W dw\right\}$$

5. The solution of (27), (32), (29) and (34) is given by:

$$\begin{aligned}dP &= \tfrac{-H}{(1-\mu H)(\sigma-1)-\mu\alpha\delta_W H}dn \\ d\psi &= -H\left\{H\tfrac{\mu\alpha\delta_W[(\sigma-1)(\sigma-\mu)-\alpha\delta_W\mu]+\sigma(\sigma-1)[(1+\mu)(\sigma-1)+\mu\gamma]}{(\sigma-1)[\mu\alpha\delta_W H-(1-\mu H)(\sigma-1)][\mu\alpha\delta_W H-(1-\mu H)(\sigma-1)+\mu\gamma H-1]}\right. \\ &\quad \left. - \tfrac{\left[\sigma^2+\mu(\sigma-1)(\sigma-1+\gamma+\alpha\delta_W)\right]}{[\mu\alpha\delta_W H-(1-\mu H)(\sigma-1)][\mu\alpha\delta_W H-(1-\mu H)(\sigma-1)+\mu\gamma H-1]}\right\}dn \\ dw &= -H\mu\tfrac{\sigma-1+\alpha\delta_W}{[(1-\mu H)(\sigma-1)-\mu\alpha\delta_W H](\sigma-1)}dn \\ dY &= \mu\tfrac{(1-H^2)(\sigma-1)^2(\sigma-1+\alpha\delta_W+\gamma)+\mu\alpha\delta_W(\sigma-1+\alpha\delta_W)H}{(\sigma-1)[\mu\alpha\delta_W H-(1-\mu H)(\sigma-1)][\mu\alpha\delta_W H-(1-\mu H)(\sigma-1)+\mu\gamma H-1]}dn\end{aligned}$$