

# DISCUSSION PAPER SERIES

No. 3366

## TRADE AND INDUSTRIAL LOCATION WITH HETEROGENOUS LABOUR

Mary Amiti and Christopher Pissarides

*INTERNATIONAL TRADE  
and LABOUR ECONOMICS*



**C**entre for **E**conomic **P**olicy **R**esearch

[www.cepr.org](http://www.cepr.org)

Available online at:

[www.cepr.org/pubs/dps/DP3366.asp](http://www.cepr.org/pubs/dps/DP3366.asp)

# TRADE AND INDUSTRIAL LOCATION WITH HETEROGENOUS LABOUR

Mary Amiti, University of Melbourne and CEPR  
Christopher Pissarides, London School of Economics (LSE) and CEPR

Discussion Paper No. 3366  
May 2002

Centre for Economic Policy Research  
90–98 Goswell Rd, London EC1V 7RR, UK  
Tel: (44 20) 7878 2900, Fax: (44 20) 7878 2999  
Email: [cepr@cepr.org](mailto:cepr@cepr.org), Website: [www.cepr.org](http://www.cepr.org)

This Discussion Paper is issued under the auspices of the Centre's research programme in **INTERNATIONAL TRADE and LABOUR ECONOMICS**. Any opinions expressed here are those of the author(s) and not those of the Centre for Economic Policy Research. Research disseminated by CEPR may include views on policy, but the Centre itself takes no institutional policy positions.

The Centre for Economic Policy Research was established in 1983 as a private educational charity, to promote independent analysis and public discussion of open economies and the relations among them. It is pluralist and non-partisan, bringing economic research to bear on the analysis of medium- and long-run policy questions. Institutional (core) finance for the Centre has been provided through major grants from the Economic and Social Research Council, under which an ESRC Resource Centre operates within CEPR; the Esmée Fairbairn Charitable Trust; and the Bank of England. These organizations do not give prior review to the Centre's publications, nor do they necessarily endorse the views expressed therein.

These Discussion Papers often represent preliminary or incomplete work, circulated to encourage discussion and comment. Citation and use of such a paper should take account of its provisional character.

Copyright: Mary Amiti and Christopher Pissarides

May 2002

## ABSTRACT

### Trade and Industrial Location with Heterogenous Labour\*

We show in the framework of a new economic geography model that when labour is heterogenous and productivity depends on the quality of the match between job and worker, trade liberalization may lead to industrial agglomeration and inter-industry trade. The agglomeration force is the improvement in the quality of matches when firms recruit from a bigger pool of labour. The forces against agglomeration are the existence of trade costs and monopoly power in the labour market. We show that more heterogeneity in skills attracts both firms and workers to bigger markets and supports agglomeration at higher trade costs.

JEL Classification: F12, J41, R12 and R13

Keywords: agglomeration, inter-regional trade, matching and spacial mismatch

Mary Amiti  
Department of Economics  
University of Melbourne  
Parkville  
Victoria 3010  
Australia  
Tel: (1 613) 8344 7652  
Fax: (1 613) 8344 6899  
Email: m.amiti@unimelb.edu.au

Christopher Pissarides  
London School of Economics  
Houghton Street  
LONDON  
WC2A 2AE  
Tel: (44 20) 7955 7513  
Fax: (44 20) 7831 1840  
Email: c.pissarides@lse.ac.uk

For further Discussion Papers by this author see:  
[www.cepr.org/pubs/new-dps/dplist.asp?authorid=137757](http://www.cepr.org/pubs/new-dps/dplist.asp?authorid=137757)

For further Discussion Papers by this author see:  
[www.cepr.org/pubs/new-dps/dplist.asp?authorid=101163](http://www.cepr.org/pubs/new-dps/dplist.asp?authorid=101163)

\*We thank Gilles Duranton, Gene Grossman and Tony Venables for their comments. A previous version of this Paper was presented at the CEPR Workshop 'The Economic Geography of Europe: Measurement, Testing and Policy Simulations,' Villars, 18/19 January 2002. This Paper is produced as part of a CEPR research network on 'The Economic Geography of Europe: Measurement, Testing and Policy Simulations', funded by the European Commission under the Research Training Network Programme (Contract No: HPRN-CT-2000-00069).

Submitted 25 April 2002

# Trade and Industrial Location with Heterogeneous Labor

Mary Amiti

University of Melbourne and CEPR

Christopher A. Pissarides\*

Centre for Economic Performance, London School of Economics,  
and CEPR

This version, April 2002

## Abstract

We show in the framework of a new economic geography model that when labor is heterogenous and productivity depends on the quality of the match between job and worker, trade liberalization may lead to industrial agglomeration and inter-industry trade. The agglomeration force is the improvement in the quality of matches when firms recruit from a bigger pool of labor. The forces against agglomeration are the existence of trade costs and monopoly power in the labor market. We show that more heterogeneity in skills attracts both firms and workers to bigger markets and supports agglomeration at higher trade costs.

In this paper we demonstrate that when labor is heterogenous and the matching of skills with jobs below first-best, the introduction of trade may lead to industrial agglomeration and inter-industry trade. Agglomeration takes place because the average quality of matches improves when the local market has a bigger pool of workers and firms to choose from. The force against agglomeration is the

---

\*We thank Gilles Duranton, Gene Grossman and Tony Venables for their comments. A previous version of this paper was presented at the CEPR Workshop "The Economic Geography of Europe: Measurement, Testing and Policy Simulations," Villars, 18/19 January 2002.

existence of trade costs. At zero trade costs regions that can trade always specialize, whereas when there are positive trade costs they may or may not specialize, depending on the values taken by several other parameters.

Our model resembles other models in new trade theory, except that there are heterogeneities in the performance of tasks done by apparently similar workers. *Ex ante* workers appear identical and no worker is more productive than another across the whole range of jobs. But some workers are more productive in some jobs and other workers are more productive in other jobs. Labor productivity in the model depends on technology, training and the other factors of conventional production theory, but also on the quality of the match between the job and the worker. We postulate that even when workers are allocated to the jobs where they are most productive, companies that have specialized skill requirements can recruit better-matched workers if they recruit in larger markets. We show that when this feature is combined with features commonly assumed in new trade theory, in particular increasing returns, differentiated goods and transport costs, it alone can explain agglomeration by industries that use specialized skills.

Anecdotal evidence in favor of our hypothesis is easy to find. To give two examples, one by an employer and one by an agent looking for employers, the general manager of Sony UK recently explained why his company remains in high-wage Britain: “What keeps us here is the quality of the staff and the research and development capacity” (*Financial Times*, January 19, 2002). On the other side of the market, Gavin Clarkson, the owner of a software company but speaking as a member of the Choctaw Nation of Oklahoma, was reported as planning to set up a technical training centre in Oklahoma to attract companies because “Having a critical mass of people who are highly skilled and resourced is what attracts business to any location or community.” (*Australian Financial Review*, 11 April 1999). The writers of the *Financial Times* article went further in arguing that “The survival of struggling volume producers may or may not prove directly vital to the economy as a whole but their role in providing skilled staff and components infrastructure for higher-margin niche manufacturers is hard to ignore.” The contribution of “components infrastructure” to industrial agglomeration was the theme of Krugman and Venables (1995). Our focus is on the role of “skilled staff.”

Formal econometric evidence in favor of labor pooling was recently provided by Dumais et al. (1997). Making use of the LRD manufacturing data base for the United States, they examined the relative importance of Marshall’s three reasons for agglomeration for the location of manufacturing plants.<sup>1</sup> They found that

---

<sup>1</sup>Marshall’s often quoted three reasons are proximity to suppliers and customers, labor pooling

labor pooling was by far the most important force for agglomeration, at least at the metropolitan area level. Examining the location decisions of new entrants, as well as the location of expanding and exiting firms, they found that the most robust results obtained were for new entrants and that new entrants tended to locate in areas where existing firms had similar labor requirements to their own.

The Dumais et al. (1997) research gives support to labour pooling as an agglomeration force but does not differentiate between different reasons that might make it important. Indirect evidence, however, supports our matching reasons. Match differentiation is likely to be more important for more advanced skills. The routine tasks that dominate production in less advanced economies do not afford much scope for differentiation or creativity. But with the invention of more complicated tasks, the routine nature of agricultural and industrial work gives way to work situations which allow different and varied types of performance. If we are correct in claiming that this is a reason for agglomeration, agglomeration should increase with economic growth and should be more prevalent in industries that require more high-tech labor. Dumais et al. (1997) find that labor pooling is especially strong as an agglomeration force in high technology industries.<sup>2</sup>

Our paper is related to three strands of literature. First is the “new economic geography” literature, which shows that agglomeration of manufacturing industries can arise when combined with inter-regional labour mobility (Krugman, 1991); or when combined with input/output linkages between vertically linked firms (Krugman and Venables, 1995). The labor market is perfectly competitive in both these models, so they do not share our reasons for agglomeration, but we share the same new trade theoretical framework.

The second strand is the “labor pooling” literature. Krugman (1991) formalizes Marshall’s reasons for emphasizing labor pooling. He claims that labor pooling is a way of achieving more efficiency when firms are exposed to idiosyncratic risk, because when the number of firms in a region is large the law of large numbers ensures that on average idiosyncratic shocks wash out.<sup>3</sup> Rotemberg and Saloner (2000) examine the case where skilled labor has to get trained, and identify a hold-up problem in the absence of labor pooling. A worker is more likely

---

and information spillovers.

<sup>2</sup>The main industries they list are fabricated metals, industrial machinery, electronic and electrical equipment and instruments. Knowledge spillovers are also relatively more important for these industries but not as important as labor pooling.

<sup>3</sup>Dumais et al. (1997) also find some evidence supporting this hypothesis. Labor pooling appears to be a more important agglomeration force in industries with more volatile employment.

to pay the up-front cost of training if she knows that there are many firms in her town that will compete for her services. Competition ensures that she will recoup the cost of her training through higher wages.

Our reason for agglomeration is different both from Krugman's and from Rotemberg and Saloner's. It is more closely related to the reasons for external economies discussed by Henderson (1988), and to the agglomeration reason invoked by Helsley and Strange (1990) in a Henderson-type model of city size. Trade is absent from this literature strand. Helsley and Strange (1990) show that the existence of matching externalities is a force pushing for the agglomeration of production but the scarcity of land prohibits the agglomeration of all industry in a single location. As more people gather in one location the marginal worker has to live further away from the business center, and her travel costs offset the gains from the agglomeration. In contrast, land in our model is a free commodity and location decisions are made by differentiated firms that can move from one region to another.

Section 1 introduces our formal definition of heterogeneity and its connection with labor skills. Sections 2-4 derive the microfoundations: the decisions of households and firms in the labor and goods markets. Section 5 derives the labor market equilibrium and section 6 the aggregate equilibrium with and without trade. It shows that with zero trade costs agglomeration dominates the symmetric equilibrium. In section 7 we report simulations that illustrate the relative importance of heterogeneity and trade costs in the choice between symmetric and agglomeration equilibrium. In section 8 we calculate the welfare effects of the agglomeration and symmetric equilibria, and show that agglomeration can impose welfare costs on the country that fails to attract the specialist firms, even in the absence of trade costs.

## **1. Differentiated workers**

When an employer requires an unskilled worker to perform routine tasks, the main concern is the wage rate at which labor is available. But as the work requirements become more complicated, differentiation in jobs introduces the possibility of good and bad matches between the requirements of the job and seemingly qualified job applicants. Employers devote resources to targeting good applicants, screening and training them, in their search for good matches. Our focus in this paper are the implications of this conjecture for the location decisions of firms and their consequences for market size.

We make four critical assumptions about the mobility of goods and factors: (a) there is perfect inter-sectoral and inter-regional mobility of capital, (b) there is perfect inter-sectoral mobility of labor, (c) there is no inter-regional mobility of labor, and (d) there is inter-regional trade. To facilitate modeling we make the further assumption that there are only two types of jobs - one requiring simple tasks, “agriculture,” and one requiring complicated tasks, “manufacturing.” All workers employed in agriculture have the same productivity. We refer to these workers as the unskilled and to those working in manufacturing as the skilled. The productivity of skilled workers is on average higher than that of unskilled workers, but varies with the quality of their match.

Other things equal (mainly wages and final demand), manufacturing employers will want to locate in the market that improves their chances of a good match; agricultural employers will be indifferent about where they locate. Skilled workers will also want to be in markets that give them a better chance of a good match. Companies like the ones in our introductory remarks do not locate where land and labor are cheaper and spend resources training their own employees, because they do not know how good the quality of the match between the trained employee and their job opening will turn out to be. It can be expensive to train workers until a good match is found, especially in more advanced technologies, where labor skills are more differentiated. But moving to a region where there are many trained workers, each with his or her own idiosyncrasies, makes the selection of a good match cheaper - and improves the expected productivity of the work force.<sup>4</sup>

All workers in our model are capable of performing unskilled tasks and this option is always open to them. They can perform skilled tasks only by acquiring training, for a cost. The agricultural sector has a constant returns to scale production technology and is competitive, with a fixed wage rate that is normalized to unity. In manufacturing there is a distribution of wages dictated by the quality of matches and mean productivity. By the assumption that all workers can perform unskilled (agricultural) tasks the lower support of the wage distribution in manufacturing is not less than 1, but its shape and actual limits are unknowns of the model.

Training costs are measured in utility units and are borne by the worker. This assumption simplifies the derivations, because it allows us to ignore the

---

<sup>4</sup>These ideas have implications for increasing returns to scale in search markets and how to test for them that we do not explore here. For example, search time may actually be longer in larger markets characterized by a variety of skills, because there is more selection, but the productivity (and hence wage) of the match may also be larger.



cost of training in the calculation of national income. Because labor is mobile between sectors, in the equilibrium of an economy with active manufacturing and agricultural sectors the distribution of wages in the manufacturing sector equates the expected utility of entering manufacturing with the utility of entering the agricultural sector. If the economy produces only manufacturing goods the wage distribution is determined by marginal productivity conditions and market clearing.

We now model skill differentiation and the quality of the match. Skilled workers are horizontally differentiated because of idiosyncratic characteristics that reveal themselves after training. No worker can vary these characteristics. Workers can decide to train or not train, say go to school, without knowing exactly what they will be good at once the training is complete. But once the training is complete, both the worker and potential employers know exactly the worker's skill attributes. Matching then takes place such that the productivity of each worker is the maximum possible given the revealed attributes.<sup>5</sup>

We assume that the quality of a match is unidimensional and is measured by the “distance” of the worker from the firm.<sup>6</sup> Skills are distributed along a circle, whose circumference is of length  $2H$ .  $H$  is a measure of the unobserved heterogeneity of skills, of how far match-specific productivities can vary from each other. If  $H = 0$  there is no heterogeneity and all skilled workers have the same productivity in all firms. When workers acquire their skill they are allocated randomly on this circle, with all locations equally likely. This implies that the density of workers on this circle is uniform, so if there are  $L_s$  workers on the circle there will be  $dL_s/2H$  workers in an arc of length  $d$ .

Unlike workers, firms can choose their location on the skills circle. A firm can decide what specialization to seek and it recruits on that basis. The closer a firm and a worker are on the circle the better the quality of their match. In equilibrium, each firm and worker will be matched to the agent from the other side located closest to them.

We assume that productivity is measured by “effective” or “efficiency” units of labour and that the productivity of a match depends linearly on the distance

---

<sup>5</sup>The introduction of matching frictions (e.g. imperfect information about potential employers) will add another dimension to the costs of the heterogeneities that we study. Although we have not worked out formally the implications of frictions, we suspect that they will add to the advantages of agglomeration, because a larger market is likely to reduce the mismatches due to frictions.

<sup>6</sup>Helsley and Strange (1990) and Thisse and Zenou (2000) employ similar assumptions to represent heterogeneity.

between the firm and the worker on the skills circle. A worker who has a firm located at exactly the same point as herself, inputs one effective unit of labour into that firm's production function. If the worker is located a distance measure  $d$  away, she inputs either  $1 - d$  or zero units. We ignore negative or zero inputs because they will never be equilibrium outcomes.<sup>7</sup>

## 2. The location of firms

Firms choose their location and post a wage for each effective unit of labor supplied by a worker and a price for each unit of output sold. They can therefore exploit monopoly powers both in labor and in output markets. The labor input is both observable and verifiable. Once the wage is posted, workers arrive and the firm decides how many to recruit and how much to produce, given its production function and the demand for its product. We assume that competition and the assumption that the labor input can be monitored at zero cost lead to full employment, although generally not at the competitive wage.

Although the model is static and the firm's decisions - location, wage posting, demand for labor and output pricing - maximize the unique profit function, it is convenient to derive optimal actions in steps. We first derive the optimal location decision for an arbitrary posted wage and show that the optimal location is independent of the wage rate. We then use the location outcomes to derive the supply of labor and the demand for output by households. Finally, we return to the decisions facing firms and complete the derivation of equilibrium.

Intuitively, firms are more likely to get workers the further away they are from other firms. Consider now the choice of location on the circle for given wages. The criterion that we use for firm decisions is the constrained maximization of profit. The labor-market constraint is the supply of effective units to the firm and the product-market constraint is the demand for its output. Therefore, given an arbitrary wage posting  $w_i$ , firm  $i$  will choose the location that maximizes the effective units of labor that it can attract, i.e., it will push the supply-of-labor constraint as far out as possible.

In figure 1, two firms are located at points  $a$  and  $b$  on the circle and pay wages  $w_a$  and  $w_b$  per effective unit of labor respectively. A new firm  $i$  enters and for given wage  $w_i$  chooses its location  $s$ . A worker located at some point  $x$  to the left

---

<sup>7</sup>The coefficient on  $d$  can be set equal to unity without loss of generality by appropriate choice of units of measuring  $H$ . For example, because maximum distance is  $H$ , we can ensure that the productivity of a match is never negative by restricting the range of  $H$  to the  $[0, 1]$  interval.

of  $s$  is attracted to firm  $i$  if

$$w_i(1 - (s - x)) \geq w_a(1 - (x - a)),$$

because if this worker goes to firm  $a$  she will supply  $1 - (x - a)$  effective units at  $w_a$  each, whereas if she goes to firm  $i$  she will supply  $1 - (s - x)$  units at  $w_i$  each. Similarly, the firm attracts a worker located at point  $y$  to the right of  $s$  if

$$w_i(1 - (y - s)) \geq w_b(1 - (b - y)).$$

It follows that firm  $i$  attracts all the workers between points  $s_{-1}$  and  $s_{+1}$  which satisfy

$$w_i(1 - (s - s_{-1})) = w_a(1 - (s_{-1} - a)) \quad (2.1)$$

$$w_i(1 - (s_{+1} - s)) = w_b(1 - (b - s_{+1})). \quad (2.2)$$

With  $L_s$  workers uniformly distributed on the circle, the total number of workers who travel from the left of  $s$  to join the firm is  $(s - s_{-1})L_s/2H$  and the average number of effective units of labor contributed by each one is  $(1 - (s - s_{-1})/2)$ , because on average each worker travels a distance  $(s - s_{-1})/2$  to reach the firm. Similarly,  $(s_{+1} - s)L_s/2H$  workers travel from the right of  $s$  to work for the firm and the average contribution of each worker to the firm's labor input is  $(1 - (s_{+1} - s)/2)$ . It follows that if the firm locates at  $s$  its total effective labor input is

$$L_{si}^E = \frac{L_s}{2H} \left[ (s - s_{-1}) \left( 1 - \frac{s - s_{-1}}{2} \right) + (s_{+1} - s) \left( 1 - \frac{s_{+1} - s}{2} \right) \right]. \quad (2.3)$$

The firm chooses its location to maximize (2.3) subject to (2.1) and (2.2), given its rivals' wages, and for any arbitrary  $w_i$ .

The maximization conditions with respect to  $s$ ,  $s_{-1}$  and  $s_{+1}$  satisfy

$$\frac{L_s}{2H}(s_{+1} - s - (s - s_{-1})) - w_i(\lambda_1 - \lambda_2) = 0, \quad (2.4)$$

$$-\frac{L_s}{2H}(1 - (s - s_{-1})) + \lambda_1(w_i + w_a) = 0, \quad (2.5)$$

$$\frac{L_s}{2H}(1 - (s_{+1} - s)) - \lambda_2(w_i + w_b) = 0, \quad (2.6)$$

where  $\lambda_1$  and  $\lambda_2$  are Lagrangian multipliers associated with (2.1) and (2.2) respectively.

It follows immediately from conditions (2.4)-(2.6) that  $\lambda_1 w_a = \lambda_2 w_b$ . We focus on symmetric Nash equilibria, i.e., the equilibria implied by profit-maximizing wage posting when all other firms post a common wage per effective unit of labor. We show later that such equilibria exist, indeed it is difficult to see what the alternative can be, given the assumptions of uniform skills distribution and the consumption preferences assumed in the next section. Setting  $w_a = w_b$ , we derive  $\lambda_1 = \lambda_2$  for any positive  $w_i$ , and so (2.4), (2.1) and (2.2) immediately yield that the firm will locate exactly halfway between firm  $a$  and firm  $b$  :  $b - s = s - a$ .

It follows that if there are  $N$  manufacturing firms in the market, in equilibrium the distance between any two firms is  $2H/N$  and so the maximum distance that workers have to cover to gain employment (i.e., the worst case of mismatch) is  $H/N$ .<sup>8</sup> With a uniform distribution of workers on the circle, the mean measure of mismatch is  $H/2N$ .

To derive the supply of labor function that constrains firms, note that (2.1) implies

$$s - s_{-1} = \frac{w_i - w_a + w_a(s - a)}{w_i + w_a} \quad (2.7)$$

and (2.2) similarly implies

$$s_{+1} - s = \frac{w_i - w_b + w_b(b - s)}{w_i + w_b}. \quad (2.8)$$

We have just proved that if  $w_a = w_b \equiv w$ ,  $s - a = b - s = 2H/N$ . Substitution into (2.7) and (2.8) and subsequently into (2.3) gives the supply of labor constraint facing firm  $i$  for any arbitrary choice of  $w_i$  :

$$L_{si}^E = \frac{L_s}{2H} \frac{(w_i - w + 2wH/N)(w_i + 3w - 2wH/N)}{(w_i + w)^2}. \quad (2.9)$$

The partial derivative of this with respect to the own wage  $w_i$  is a complicated expression that cannot in general be signed, but at the symmetric equilibrium it

---

<sup>8</sup>When the rivals' wages are not equal, firms locate closer to the low-wage rival than the high wage rival. Perhaps unintuitively, this implies that they recruit over a longer range of the skill distribution on the side of the high-wage rival than on the side of the low-wage rival. As a consequence, workers who are located closer to high-wage firms but are not recruited by those firms suffer bigger mismatches on average than workers located closer to low-wage firms. It would appear from this that high-wage firms impose an externality on the workers that they do not recruit. This property may have interesting implications for non-symmetric equilibrium, which we do not pursue in this paper.

simplifies to

$$\frac{\partial L_{si}^E}{\partial w_i} \Big|_{w_i=w} = \frac{L_s}{2Hw} > 0. \quad (2.10)$$

The firm is facing an upward-sloping supply of labor curve. Restricting again attention to the symmetric Nash equilibrium, we derive the elasticity of the labor supply curve as the constant

$$\frac{\partial L_{si}^E}{\partial w_i} \frac{w_i}{L_{si}^E} \Big|_{w_i=w} = \left( \frac{H}{2N} \left( 1 - \frac{H}{2N} \right) \right)^{-1} > 0. \quad (2.11)$$

Recalling that  $H/2N$  is the mean measure of mismatch in this economy, we derive that in symmetric equilibrium the larger the mean mismatch the less elastic the firm's labor supply curve. With firms located further apart, competition for workers in the labor market is less intense.

### 3. Households

Households supply one unit of labor each, either in the manufacturing sector or in the agricultural sector. If they decide to enter agriculture, they know with certainty that the wage rate will be 1. If they decide to enter manufacturing, they have to train first for a cost and then discover their location on the skills circle, and hence their realized wage. Once the wage is known they decide if they want to remain in manufacturing or revert to agriculture, and make their consumption decisions. Both household decisions, labor allocation and demand, are governed by the maximization of a single utility function.

#### 3.1. Demand functions

We assume that manufacturing goods are differentiated and each firm produces its own variety  $i$ . Consumers have Dixit-Stiglitz preferences over these goods, which are aggregated into a composite denoted by  $C_x$ . The output of agriculture is a single good denoted by  $C_a$ . The utility function of individual  $k$  is

$$U_k = v_k C_{xk}^\mu C_{ak}^{1-\mu}, \quad 0 < \mu < 1 \quad (3.1)$$

where  $v_k = t^{-1}$  if the individual decides to train and seek a manufacturing job and  $v_k = 1$  otherwise.  $t > 1$  is a parameter that measures the utility cost of training. Higher  $t$  implies more expensive training. Our formulation makes training costs

akin to “iceberg” costs. The skilled worker loses a fraction of her utility when transporting herself to the skilled labor pool, from which she is recruited by high-wage firms.

We introduce traded goods in preparation for the subsequent analysis. Thus, let  $N$  be the number of domestically produced manufacturing goods (the same as the number of domestic manufacturing firms) and  $N^*$  the number of foreign firms. The sub-utility function for manufacturing goods is

$$C_{xk} = \left[ \sum_{i=1}^N c_{ik}^{\frac{\sigma-1}{\sigma}} + \sum_{j=1}^{N^*} \left( \frac{m_{jk}}{\tau} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \quad \sigma > 1, \tau \geq 1, \quad (3.2)$$

where  $c_{ik}$  is the consumption of the domestically-produced manufacturing good  $i$ ,  $m_{jk}$  is the demand for the imported manufacturing good  $j$ ,  $\tau$  are iceberg transportation costs and  $\sigma$  is the elasticity of substitution between varieties.

Let  $y_k$  represent the income of individual  $k$ . The budget constraint is

$$\sum_{i=1}^N p_i c_{ik} + \sum_{j=1}^{N^*} p_j^* m_{jk} + P_a C_{ak} = y_k \quad (3.3)$$

where  $p_i$  is the price of good  $i$ ,  $p_j^*$  is the price of the imported good and  $P_a$  is the price of the agricultural good. Define the price index of the manufacturing composite good by

$$P_x = \left[ \sum_{i=1}^N p_i^{1-\sigma} + \sum_{j=1}^{N^*} (\tau p_j^*)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}. \quad (3.4)$$

Maximization of the utility function for a given income level gives the following demand functions for the manufacturing composite and the agricultural good:

$$P_x C_{xk} = \mu y_k \quad (3.5)$$

$$P_a C_{ak} = (1 - \mu) y_k, \quad (3.6)$$

and for each manufacturing good:

$$c_{ik} = \mu \left( \frac{p_i}{P_x} \right)^{-\sigma} \frac{y_k}{P_x} \quad (3.7)$$

$$m_{jk} = \mu \left( \frac{\tau p_j^*}{P_x} \right)^{-\sigma} \frac{\tau y_k}{P_x} \quad (3.8)$$

Although utility functions are different for skilled and unskilled workers, the proportionality assumptions made on the cost of training ensure that demand functions are identical, except for income differences. Aggregation over incomes therefore gives the aggregate demand functions for each good and for the composites. The aggregate demand functions have the same linear-expenditure form as the individual functions and they are not written explicitly to save space. We denote domestic aggregate income by  $Y$ .

### 3.2. Occupational choice

Workers are mobile between sectors and choose their sector to maximize their utility function (3.1). By substitution from the demand functions into (3.1) we derive the indirect utility function of individual  $k$

$$U_k = v_k \mu^\mu (1 - \mu)^{1-\mu} P_x^{-\mu} P_a^{-(1-\mu)} y_k. \quad (3.9)$$

Individuals who join the agricultural sector are characterized by  $v_k = 1$  and  $y_k = 1$  and those who train to join the manufacturing sector are characterized by  $v_k = t^{-1}$  and their income is a random draw from the uniform distribution of wages. In agriculture, a fixed coefficients production function and competition ensures that there are zero profits (hence, given the normalization for wages in agriculture,  $P_a$  must also be equal to 1 in equilibrium and acts as the numeraire in this economy). In manufacturing, free entry and exit of firms also ensures that there are zero profits in equilibrium.

To derive the distribution of wages in manufacturing, we note that the maximum wage rate earned, by those with a perfect match, is  $w$ , the wage posted per effective unit in symmetric equilibrium. The lowest wage is earned by those who have to travel maximum distance, which, with  $N$  firms, is  $H/N$ . Therefore, the lowest wage rate is  $w(1 - H/N)$  or 1, whichever is higher. We will derive the solution under the assumption that  $w(1 - H/N)$  is always above unity, i.e., that no trained workers are forced back into agriculture because of poor matches. In the simulations of section 7 this assumption is always satisfied.

Wages in manufacturing are uniformly distributed between  $w(1 - H/N)$  and  $w$ , so the expected utility of a worker who chooses to get trained is

$$\begin{aligned} \bar{U}_k &= t^{-1} \mu^\mu (1 - \mu)^{1-\mu} P_x^{-\mu} P_a^{-(1-\mu)} \int_{w(1-H/N)}^w y_k dF(y_k) \\ &= t^{-1} \mu^\mu (1 - \mu)^{1-\mu} P_x^{-\mu} P_a^{-(1-\mu)} w(1 - H/2N), \end{aligned} \quad (3.10)$$

where  $F(\cdot)$  is the uniform distribution. Inter-sectoral mobility of labor requires that the expected utility of those who train themselves and join manufacturing be equal to the utility of those who join agriculture, at least when both sectors are active. From (3.9) and (3.10), and given that in agriculture  $v_k = y_k = 1$ , we derive the condition implied by equality of the utility levels:

$$w(1 - H/2N) = t > 1. \quad (3.11)$$

With both sectors active, condition (3.11) gives a negative “compensating differentials” relation between the wage rate for each effective unit of labor and the number of firms. The intuition behind it is that for a given effective wage rate, the wage distribution in manufacturing with many firms dominates one with fewer firms, because of better matches. The effective wage rate then has to be lower to compensate for the increased attractiveness of entering manufacturing.

#### 4. Firms

Firms are wage and price setters. Their choice variables are their own wage rate  $w_i$ , which is constrained by (2.9), and their output price  $p_i$ , which is constrained by the individual demand functions (3.7), aggregated over all individuals.

The (inverted) production function for firm  $i$  is a linear function of the effective units of labor:

$$L_{si}^E = \alpha + \beta x_i, \quad \alpha, \beta > 0, \quad (4.1)$$

where  $x_i$  is the firm’s output and  $L_{si}^E$  is the number of effective units supplied to the firm at its posted wage  $w_i$ .

Firms maximize profit

$$\pi_i = p_i x_i - w_i L_{si}^E. \quad (4.2)$$

Aggregation of demand in the domestic and foreign country yields, from (3.7) and (3.8),

$$\begin{aligned} x_i^d &= c_i + m_i^* \\ &= \mu p_i^{-\sigma} (P_x^{\sigma-1} Y + \tau^{1-\sigma} P_x^{*\sigma-1} Y^*) \end{aligned} \quad (4.3)$$

where  $x_i^d$  is the total demand for good  $i$ ,  $c_i$  and  $m_i$  are respectively home and export demand and  $Y$  is aggregate income.



Maximization of profit requires that the firm's controls  $w_i, p_i, x_i$  and  $L_{si}^E$  satisfy the equality  $x_i = x_i^d$ , and the mark-up equation (under the symmetric equilibrium assumption  $w_i = w$ )

$$p_i = w \left( 1 + \frac{2H}{L_s} L_{si}^E \right) \frac{\sigma\beta}{\sigma - 1}. \quad (4.4)$$

But in symmetric equilibrium (2.9) implies

$$L_{si}^E = \left( 1 - \frac{H}{2N} \right) \frac{L_s}{N}, \quad (4.5)$$

which simplifies the markup equation to

$$p_i = w \left( 1 + 2\frac{H}{N} - \frac{H^2}{N^2} \right) \frac{\sigma\beta}{\sigma - 1}. \quad (4.6)$$

We note that  $H/N < 1$ , because  $1 - H/N$  is the number of effective units supplied by the most mismatched worker. Higher  $H/N$  implies that firms are located further away from each other and so competition in the labor market is less intense (as shown in (2.11), the elasticity of the supply of labor in this case is smaller). It is therefore not surprising to find that higher  $H/N$  implies a higher markup of prices over wages.

Freedom of entry and exit of manufacturing firms eliminates profits in equilibrium, which, when applied to (4.2) gives, by virtue of (4.6) and (4.1),

$$x = \frac{\alpha(\sigma - 1)}{\beta} \left( 1 + 2\sigma\frac{H}{N} - \sigma\frac{H^2}{N^2} \right)^{-1}. \quad (4.7)$$

Unlike the typical Dixit-Stiglitz symmetric equilibrium found in trade models, each firm's output here is not constant, because the markup of prices on costs is not constant. Market-size effects work through both channels, variations in the number of varieties (firms) and variations in firm size. By substituting (4.7) into (4.1) we obtain the demand for effective labor units by each manufacturing firm,

$$L_{si}^E = \alpha\sigma \frac{1 + 2\frac{H}{N} - \frac{H^2}{N^2}}{1 + 2\sigma\frac{H}{N} - \sigma\frac{H^2}{N^2}}. \quad (4.8)$$

As before, higher  $H/N$  implies less competition in the labor market, the price markup is higher and so each firm's demand and output are lower and the demand for labor is consequently also lower. This introduces a link from the number of firms to each firm's output: higher  $N$  implies more competition in the labor market and so yields higher output per firm and more demand for labor.

## 5. Labor-market equilibrium

The market in effective units of labor clears. Because the equilibrium is symmetric, we find the equilibrium by equating the supply of effective labor units to each firm with the demand for labor by the firm. Both expressions were derived in the preceding section, the supply in (4.5) and the demand in (4.8). It is convenient to introduce the notation  $n \equiv N/H$ , noting that  $n/2$  is the density of firms on the circle. We loosely refer to  $n$  as the density of firms or as the number of firms. We re-write the supply and demand functions for labor to each individual firm as

$$L_{si}^E = \frac{2n-1}{n^2} \frac{L_s}{2H} \quad (5.1)$$

$$L_{si}^E = \alpha\sigma \frac{1 + (2n-1)/n^2}{1 + \sigma(2n-1)/n^2}. \quad (5.2)$$

Larger  $n$  implies that firms are closer together in the manufacturing sector, and so the supply of effective labor units to each individual firm is larger:  $\partial L_{si}^E / \partial n > 0$ . For given  $n$ , the supply of labor to each firm increases in  $L_s/2H$ , the density of workers in manufacturing, for obvious reasons. On the demand side, larger  $n$  implies more competition in the labor market, and so the demand for labor is less.

Labor-market clearing requires equality between demand and supply. Equality gives, for each  $H$ , a relation between the number of manufacturing firms and the total supply of skilled labor, which we use later to derive the number of firms for a given labor allocation to manufacturing. Equating (5.1) and (5.2) we obtain, after some rearranging of terms,

$$\sigma \frac{L_s}{2H} \left( \frac{2n-1}{n^2} \right)^2 + \left( \frac{L_s}{2H} - \alpha\sigma \right) \left( \frac{2n-1}{n^2} \right) - \alpha\sigma = 0. \quad (5.3)$$

It is apparent from the signs of the coefficients that there is only one positive root for the “unknown”  $(2n-1)/n^2$ . Let this root be denoted by  $r > 0$ .<sup>9</sup> This unique positive root is a decreasing function of the density of workers,  $L_s/2H$ .

To obtain  $n$ , we solve the quadratic equation

$$rn^2 - 2n + 1 = 0. \quad (5.4)$$

---

<sup>9</sup>We can also see this by noting that supply in (5.1) increases uniformly in  $n$ , whereas demand in (5.2) decreases, so there is a unique intersection in supply demand space.

Because  $r > 0$ , there are either two positive solutions for  $n$  or none. The condition that ensures that there are non-trivial solutions is  $r < 1$ . Making use of (5.3) this imposes a feasible range on the density of workers

$$\frac{L_s}{2H} > \frac{2\alpha\sigma}{\sigma + 1}. \quad (5.5)$$

Intuitively, labor market equilibrium is consistent with the existence of a positive number of firms only if the number of workers in the sector exceeds a critical value given by the parameters of the production and demand functions. Below this critical value no firm will find it optimal to enter the sector. We show in the next section that there is a unique solution for  $L_s$  in terms of the model's parameters, so (5.5) can be interpreted as a restriction on the feasible range of the parameters.

## 6. Aggregate equilibrium

To derive the aggregate equilibrium we need to derive an expression for national income and the allocation of labor between sectors. National income is

$$Y = W_s L_s + W_a L_a, \quad (6.1)$$

where  $W_s$  is the mean manufacturing wage per person, which, by the argument of section 3.2 is equal to  $w(1 - H/2N)$ . With the normalization  $W_a \equiv 1$  and noting that  $L_a = L - L_s$ , national income becomes

$$Y = L + (W_s - 1)L_s. \quad (6.2)$$

Our next task is to relate aggregate demand to labor income and here results depend on whether there is trade or not.

### 6.1. No trade equilibrium

Suppose first there is no trade in equilibrium, for example, let  $\tau \rightarrow \infty$  in all expressions. Output market clearing and zero manufacturing profits then imply that the domestic demand for manufacturing is equal to total manufacturing income, i.e.,  $\mu Y = W_s L_s$ . Substitution into (6.2) yields

$$L_s = \frac{\mu}{(1 - \mu)W_s + \mu} L. \quad (6.3)$$

In this economy, because both agricultural and manufacturing goods are produced in equilibrium, the market clearing condition (3.11) must hold. Therefore the equilibrium number of skilled workers is

$$L_s = \frac{\mu}{(1 - \mu)t + \mu} L. \quad (6.4)$$

With knowledge of  $L_s$  the other variables are immediately obtained. The key variable  $n$  is obtained from (5.3) and there are two feasible solutions.

We illustrate the autarkic equilibrium with a diagram that will be useful in the later analysis. The diagram, figure 2, gives the equilibrium relationship between per-person wages in manufacturing and the number of firms. The horizontal line  $SS$  is the market clearing condition (3.11). It plays the role of a labor supply curve. The  $DD$  lines show the number of firms at given wage rate derived from the demand side of the model. We saw that (5.4) implies that there are two solutions for  $n$ . By differentiation, and noting that  $r < 1$  and  $n > 1$  in the feasible range, we find that the larger solution decreases in  $r$  but the smaller one increases in it. Therefore, since  $r$  decreases in  $L_s$  and  $L_s$  decreases in  $W_s$ , the larger solution must be decreasing in  $W_s$  but the smaller solution increasing. The  $DD$  lines show the two solutions as functions of  $W_s$ . The larger solution is well behaved, in the sense that a lower wage rate attracts more firms into the market in each case. But the smaller solution is “unstable,” in the sense that a lower wage rate makes firms exit the manufacturing sector. We choose the larger solution as the equilibrium, i.e., in equilibrium the density of firms  $n$  satisfies

$$n = r^{-1} + r^{-1}(1 - r)^{1/2}, \quad (6.5)$$

where  $r$  is the unique positive root of (5.3). In the neighborhood of equilibrium the density of firms increases in the labor force and in the share of manufacturing goods in consumer expenditure but decreases in the degree of heterogeneity, as measured by  $H$ .

## 6.2. Trade equilibrium

With trade, the output of each manufacturing firm has to satisfy both domestic consumption and exports. Substitution into the demand functions (4.3) of aggregate income for each country from (6.2), and the aggregate price indices from (3.4), yields

$$x = \mu p^{-\sigma} \left[ \frac{(W_s - 1)L_s + L}{Np^{1-\sigma} + N^*(\tau p^*)^{1-\sigma}} + \frac{\tau^{1-\sigma}[(W_s^* - 1)L_s^* + L^*]}{N(\tau p)^{1-\sigma} + N^*p^{*1-\sigma}} \right], \quad (6.6)$$

$$x^* = \mu p^{*-\sigma} \left[ \frac{\tau^{1-\sigma} [(W_s - 1)L_s + L]}{Np^{1-\sigma} + N^*(\tau p^*)^{1-\sigma}} + \frac{(W_s^* - 1)L_s^* + L^*}{N(\tau p)^{1-\sigma} + N^*p^{*1-\sigma}} \right]. \quad (6.7)$$

Under free trade,  $\tau = 1$  and the terms in the square brackets of (6.6) and (6.7) become identical. Therefore, free trade equilibrium yields, after dividing (6.6) by (6.7),

$$\frac{x}{x^*} = \left( \frac{p}{p^*} \right)^{-\sigma}. \quad (6.8)$$

Making use of (4.7) and (4.6) we re-write (6.8) as

$$\frac{1 + \sigma(2n^* - 1)/n^{*2}}{1 + \sigma(2n - 1)/n^2} = \left( \frac{w(1 + (2n - 1)/n^2)}{w^*(1 + (2n^* - 1)/n^{*2})} \right)^{-\sigma} \quad (6.9)$$

The mean manufacturing wage in each country is given by the product  $w(1 - H/2N)$ , so

$$\frac{W_s}{W_s^*} = \frac{w(1 - 1/2n)}{w^*(1 - 1/2n^*)}. \quad (6.10)$$

Substitution of  $w/w^*$  from (6.10) into (6.9) yields

$$\frac{W_s}{W_s^*} = \left( \frac{1 + \sigma(2n - 1)/n^2}{1 + \sigma(2n^* - 1)/n^{*2}} \right)^{1/\sigma} \frac{1 + (2n^* - 1)/n^{*2}}{1 + (2n - 1)/n^2} \frac{1 - 1/2n}{1 - 1/2n^*}. \quad (6.11)$$

Equation (6.11) replaces the demand for labor function  $DD$  of figure 2, to give the free trade equilibrium. Although it is in relative terms, we show that it can determine equilibrium in the two economies in the same way as figure 2 did for a single economy under autarky. We plot (6.11) in figure 3 for the domestic economy when foreign wages are at their equilibrium value  $t$  and the number of firms is at the right-hand intersection of figure 2, denoted as  $N^*$  in figure 3. Holding foreign wages and the number of foreign manufacturing firms constant, we differentiate (6.11) with respect to the domestic number of firms  $n$  to obtain  $\partial W_s / \partial n > 0$ . The relation between domestic wages and the domestic number of firms is shown as  $DD$  in figure 3.  $DD$  cuts the equilibrium at  $N^*$ , because a symmetric free-trade equilibrium always exists. But the domestic equilibrium is “unstable”. If a small number of firms is shifted from the foreign to the home economy wages in the home economy rise, attracting more workers into manufacturing, and foreign wages fall, discouraging the movement into manufacturing.

The intuition for the instability with free trade is this. When a firm moves from the foreign country to the home country, the world demand for its output is

unaffected. But in the labor market two forces are at work. The larger number of firms increases competition for workers and the markup of prices on wages falls. This force works against agglomeration. But because there are now more firms in the domestic economy the quality of matches improves. The firm could pay the same wage for each effective unit of labor and pass on the improvement in the quality of matches to workers, attracting more workers into the manufacturing sector. With more workers attracted, the negative effects from the increased competition for workers are offset. A complementarity comes into force: when a manufacturing firm leaves the foreign country and joins the home one, more workers at home train and join the manufacturing sector. With completely free trade the complementarity offsets the negative effects of the competition for workers and leads to complete specialization; i.e., in the case where the manufacturing sector is less than half the world economy, all manufacturing firms leave the foreign country and join the home one.

The complementarity may fail with sufficiently high trade costs. If there are trade costs, when the firm leaves the foreign country to join the home one foreign consumers have to pay the trade cost to buy its output and it has to compete with other home firms for domestic aggregate demand. This induces a fall in the firm's output price and in the effective wage rate. The lower effective wage acts as a disincentive to training, which, if sufficiently strong, offsets the incentive coming from the better matches. In the diagram, the critical point where the two forces exactly balance each other out takes place when the  $DD$  curve becomes a vertical line.

## 7. Numerical simulations

We use numerical simulations to illustrate both the nature of free-trade equilibrium and the consequences of intermediate trading costs when (a) countries are of equal size ( $L = L^*$ ) and (b) countries differ in size ( $L < L^*$ ).

### 7.1. Equal country size

At intermediate levels of trade costs, the equilibrium outcome will either be a diversified (symmetric) equilibrium with both countries producing manufactures and  $N = N^*$ , or an agglomerated equilibrium. In the agglomeration case we suppose that manufacturing agglomerates in the “home” country. An agglomerated equilibrium involves both countries producing agricultural goods if the share of

manufactures in consumption,  $\mu$ , is less than a half; in this case the mean skilled wage is constrained to equal  $W_s = t$ . However, if  $\mu > 1/2$ , an agglomerated equilibrium involves the home country producing only manufactures and the foreign country producing both agriculture and manufacturing. In this case, the skilled wage distribution in the home country depends on the value of marginal product of labor in manufactures, which is greater than  $t$ . Although we discuss both cases, we continue to focus on the case  $\mu < 1/2$  (hence, our “manufacturing” is a small specialized sector of the economy that employs skilled labor).

We investigate the existence of an agglomerated equilibrium in two ways. First, we compute an equilibrium when trade costs are sufficiently high to give symmetry and no trade. We then compute the equilibrium implied by lower levels of trade costs, holding all other parameters fixed. The point at which the equilibrium switches from symmetry and no trade to agglomeration with trade is known in the new economic geography literature as the “break point.”<sup>10</sup> In a second experiment our initial equilibrium is an agglomerated one with low trade costs, and investigate whether there are trade costs that imply profitable deviations for one or more firms. The point at which this is observed is known as the “sustain point.”

To find the break point,  $\tau(B)$ , we ensure that at the initial value of trade costs firms make zero profits in symmetric equilibrium. Firms make zero profits when equations (6.6) and (6.7) are satisfied (with prices substituted out by making use of equation (3.11) and (4.6)). If there are trade costs at which demand in a country exceeds the zero-profit output level, given by equation (4.7), firms find it profitable to enter that country, whereas when demand falls short of (4.7) firms exit. To see if there are any profitable deviations at lower trade costs we check whether total demand for goods produced in the home country, given by equation (6.6), is higher than the zero profit output level. If so, there are profitable entry opportunities and the symmetric equilibrium breaks. This critical value for the benchmark case is found to be  $\tau(B) = 1.19$  and illustrated in figure 4<sup>11</sup>. The implication is that when trade costs fall below 19% of the producer price, a small group of firms could break the symmetric equilibrium by leaving their country and locating in the other, inducing agglomeration.

To find the sustain point (in the case  $\mu < 1/2$ ) we set  $N > 0$ ,  $N^* = 0$  with  $N$  chosen such that firms earn zero profit. This agglomeration is an equilibrium when no firm can make positive profits by moving from the home to the foreign

---

<sup>10</sup>See Fujita et al (1999), p. 9.

<sup>11</sup>The benchmark parameter values for all of the simulations are as follows:  $\sigma = 4$ ,  $\alpha = 1/\sigma$ . In Figure 4, we set  $L = L^* = 100$ ,  $\mu = 0.40$ ,  $H = 1.0$ ,  $t = 1.1$  and  $L = L^* = 100$ .

country. Figure 4 shows that for trade costs below  $\tau(S) = 2.48$  no firm will find it optimal to deviate but at higher trade costs the agglomeration equilibrium is not sustainable. In general, the sustain point is higher than the break point, i.e., an agglomeration can be an equilibrium at levels of trade costs that are too high to render a symmetric equilibrium unstable. At such levels of trade costs, between  $\tau(B)$  and  $\tau(S)$ , there are multiple equilibria.

Table 1 summarizes the results for different values of  $H$  and  $t$  for the case when (a) countries are of equal size; and (b) the home country is smaller than the foreign country. The results indicate that reducing either  $t$  or  $H$  reduces both the critical break and sustain points. The lower is the heterogeneity, the weaker are the agglomeration forces, hence agglomeration requires a lower level of trade costs. For example, when  $t = 1.1$  and  $L = L^* = 100$  reducing  $H$  from 1.5 to 0.5 reduces the break point from  $\tau(B) = 1.24$  to  $\tau(B) = 1.13$  and the sustain point from  $\tau(S) = 3.37$  to  $\tau(S) = 1.76$ . Agglomeration is an equilibrium at all levels of trade costs equal to or less than these critical values. The results are quantitatively the same when  $\mu > 1/2$ . However, in all cases both the critical and sustain points are lower for higher values of  $\mu$ . When  $\mu$  is high the share of income spent on the cost of importing goods is high hence lower trade costs are necessary for an agglomeration.

## 7.2. Different Country size

Allowing countries to differ in market size ( $L < L^*$ ) creates an additional tension as firms are drawn to the large market to save on trade costs. In new economic geography models generally, a country that has the initial head start in terms of size is the more profitable location for an agglomeration.<sup>12</sup> Interestingly in our model, the agglomeration does not necessarily locate in the large country because the source of agglomeration arises in the skilled labor market. So the benefit of locating in a country with a larger pool of skilled labor could outweigh the benefit of locating in a large market for goods. For  $L = 100$  and  $L^* = 120$ , the critical and sustain points are reported in Table 1. For  $H = 1.0$  and  $t = 1.10$ , the break point in the larger country is  $\tau(B) = 1.30$  and all firms agglomerate in it at lower trade costs. But at trade costs below  $\tau(B) = 1.01$  (noting that  $\tau \geq 1$ ),

---

<sup>12</sup>The statement is correct under the assumption that the economy moves to the nearest stable equilibrium, ignoring global stability properties: agents are assumed to be myopic with workers only moving in response to current wage differences and firms responding to current differences in profitable opportunities. See Fujita et al. (1999, p.7) for justification of these assumptions.



the agglomeration can locate in the smaller home country too. A larger skilled labour pool can tip the scales in favour of the home country. Agglomeration in the smaller country is more likely to be the case the higher the heterogeneity and the cost of training. In free trade, agglomeration will always be an equilibrium in the country with the larger initial pool of skilled labor: the market size effect becomes irrelevant, except to the extent that it influences the initial pool of skilled labor.

Agglomeration in the small country is also an equilibrium at higher levels of trade costs. Starting with an agglomeration, we find that the critical sustain points in the smaller country are always less than in the larger country, as we expect, but at sufficiently low trade costs an agglomeration equilibrium can be sustainable in the smaller country too. At the benchmark  $H = 1.0$  and  $t = 1.10$ , these trade costs can be as high as  $\tau \leq 2.38$ .

With different country sizes, an equilibrium with agglomeration in the home country may consist of the home country only producing manufactures even when  $\mu < 0.5$ , in which case workers would be paid an amount that is proportional to the value of their marginal product in manufactures. The smaller country could end up with a higher total income than the larger country.

## 8. Welfare

The welfare effects of trade in our model depend on the level of trade costs and the size of the manufacturing sector. We illustrate only the case of “trade liberalization” when the two countries are of equal size, but both for a small and a large manufacturing sector. We interpret trade liberalization as meaning a gradual reduction of trade costs. The relevant critical point in this case is the break point: starting from high costs and symmetric equilibrium we reduce trade costs to zero and compute each country’s and the world’s welfare as the equilibrium changes from the symmetric to the agglomeration one. We compute welfare by making use of equation (3.9) and aggregating across all individuals  $k$ . The results are shown in figure 5, panel (a) for  $\mu < 1/2$  and panel (b) for  $\mu > 1/2$ .

Aggregate utility levels are equal in the two countries in the symmetric equilibrium. As trade costs fall, both countries’ welfare improves because they have access to more varieties through trade. When the break point is reached manufacturing firms agglomerate in the home country. At this point, the welfare in the home country jumps up because its consumers pay less for transportation (nothing in the case of a small manufacturing sector, when all manufactures are produced in the home country) but the welfare in the foreign country jumps down because

consumers have to pay more transport costs. In the case of small manufacturing sector world welfare rises with the agglomeration, but in the case of a large manufacturing sector, the fact that there is a lot more concentration of activity and trade now reduces world welfare. As trade costs continue to fall welfare increases in the foreign country.

In the case of small  $\mu$  welfare in the foreign country and in the world eventually reach a higher level than the level of welfare reached at the symmetric equilibrium. In free trade, the utility levels are the same in both countries. Even though the home country has all the skilled workers, the utility of skilled and unskilled workers is the same by the assumption of inter-sectoral mobility of labor and because both countries have an active agricultural sector.

But when  $\mu > 1/2$  the mean skilled wage in the home country is no longer constrained to equal  $t$  in the agglomerated equilibrium. The initial jump in welfare in the home country is higher but this does not necessarily compensate for the losses in the foreign country. The sum of the two countries' utilities may be lower in free trade than for intermediate levels of trade costs, because of the price effects of free trade. In the home country, the price of manufacturing goods jumps up with agglomeration because prices are a mark-up on marginal cost, which increase because of the increase in the skilled wage rate. The increase in wages in the home country more than offsets the price effects but not in the foreign country, where wages are tied down by productivity in the agricultural sector. Welfare in the foreign country suffers further from the lower productivity in manufacturing, which follows the exit of firms after agglomeration. The combination of higher prices in the home country and lower productivity in the foreign country may actually make foreign consumers worse off even with zero trade costs.<sup>13</sup>

## 9. Conclusions

The availability of large quantities of qualified labor in a local market is often invoked as a force for the agglomeration of economic activity. In this paper we showed how this could arise in a market characterized by heterogeneity of skills, even when the heterogeneity gives monopoly power to firms. The key to our model

---

<sup>13</sup>Clearly, in the case of a large manufacturing sector a welfare-improving move would be to allow international migration. International migration would allow all manufacturing firms to locate in one country, which would improve welfare for two reasons. It would keep manufacturing wages and prices low through the more abundant supply of labor and it would improve the quality of matches through the larger pool of both firms and labor

is that firms prefer to enter a market that already has a large pool of labor and firms, and so forego some of their monopoly power, than enter an isolated market and train their own workers, because in the former case they can choose the most suitable employees, whereas in the latter they have to rely on luck to find a good match. Agglomeration fails in our model only if trade costs are sufficiently high to make it more profitable for firms to locate in the market that they supply, rather than in the one that their labor productivity is highest.

We cited anecdotal and econometric evidence by others on the advantages of labor pooling. Our analysis has narrowed the types of labor and the reasons that labor pooling is advantageous, and a necessary next step is to look for these in the data. Two forces in particular appear to be consistent with casual observation: that there should be more agglomeration of small high-tech industries than of the larger and heavier type of industry and that agglomeration and wage inequality should increase as trade costs come down. Testing whether this is true and whether the reasons are related to the matching of skills is a theme for future research.

## References

- [1] Dumais, G., G. Ellison and E. L. Glaeser (1997). "Geographic Concentration as a Dynamic Process." Working Paper 6270, NBER (forthcoming in the *Review of Economics and Statistics*).
- [2] Fujita, M., P. Krugman and A. J. Venables (1999). *The Spacial Economy: Cities, Regions and International Trade*. Cambridge: MIT Press.
- [3] Henderson, J. V. (1988). *Urban Development: Theory, Fact and Illusion*. Oxford: Oxford University Press.
- [4] Krugman, P. R. (1991). *Geography and Trade*. Cambridge: MIT Press.
- [5] Helsley, R. W. and W. C. Strange (1990). "Matching and Agglomeration Economies in a System of Cities." *Regional Science and Urban Economics* 20, 189-212.
- [6] Krugman, P. R. and A. J. Venables (1995). "Globalization and the Inequality of Nations." *Quarterly Journal of Economics* 110, 857-880.
- [7] Rotemberg, J. J. and G. Saloner (2000). "Competition and Human Capital Accumulation: A Theory of Interregional Specialization and Trade." *Regional Science and Urban Economics* 30, 373-404.

- [8] Thisse, J.-F. and Y. Zenou (2000). "Skill mismatch and Unemployment." *Economics Letters* 69, 415-420.

Table 1  
“Break” and “sustain” points for trade costs at different values of parameters

$H$ $t$		$L = L^* = 100$				$L = 100 < L^* = 120$			
		$\mu = 0.4$		$\mu = 0.6$		$\mu = 0.4$			
		Both countries		Both countries		Home	Foreign	Home	Foreign
		$\tau(B)$	$\tau(S)$	$\tau(B)$	$\tau(S)$	$\tau(B)$	$\tau(B)$	$\tau(S)$	$\tau(S)$
1.5	1.15	1.25	3.43	1.21	2.71	1.01	1.37	3.29	3.50
1.5	1.10	1.24	3.37	1.19	2.67	1.01	1.36	3.24	3.45
1.5	1.05	1.22	3.32	1.18	2.63	1.01	1.35	3.19	3.40
1.0	1.15	1.20	2.51	1.17	2.00	1.01	1.32	2.41	2.57
1.0	1.10	1.19	2.48	1.15	1.97	1.01	1.30	2.38	2.53
1.0	1.05	1.18	2.44	1.14	1.94	1.00	1.29	2.34	2.49
0.5	1.15	1.14	1.79	1.12	1.39	1.00	1.24	1.72	1.83
0.5	1.10	1.13	1.76	1.11	1.37	1.00	1.23	1.69	1.80
0.5	1.05	1.12	1.73	1.09	1.34	1.00	1.22	1.66	1.77

Notes. The values taken by the other parameters are  $\sigma = 4$ , and  $\alpha = 1/\sigma$ .  $H$  measures the degree of heterogeneity and  $t$  the training cost. At trade costs above  $\tau(B)$  the symmetric equilibrium can be broken and at points below  $\tau(S)$  the agglomeration equilibrium becomes unsustainable.

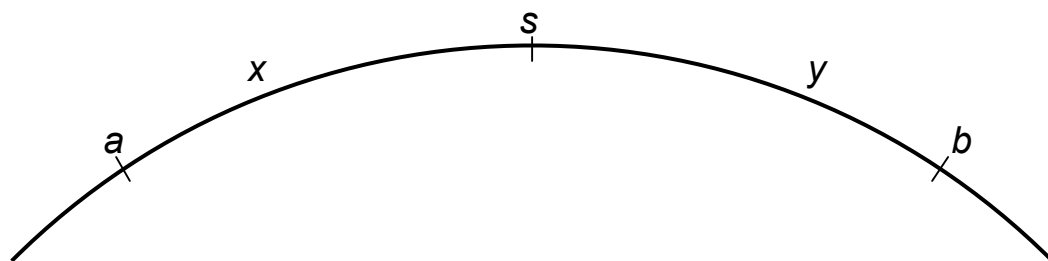


Figure 1  
The firm's choice of location on  
the skills circle

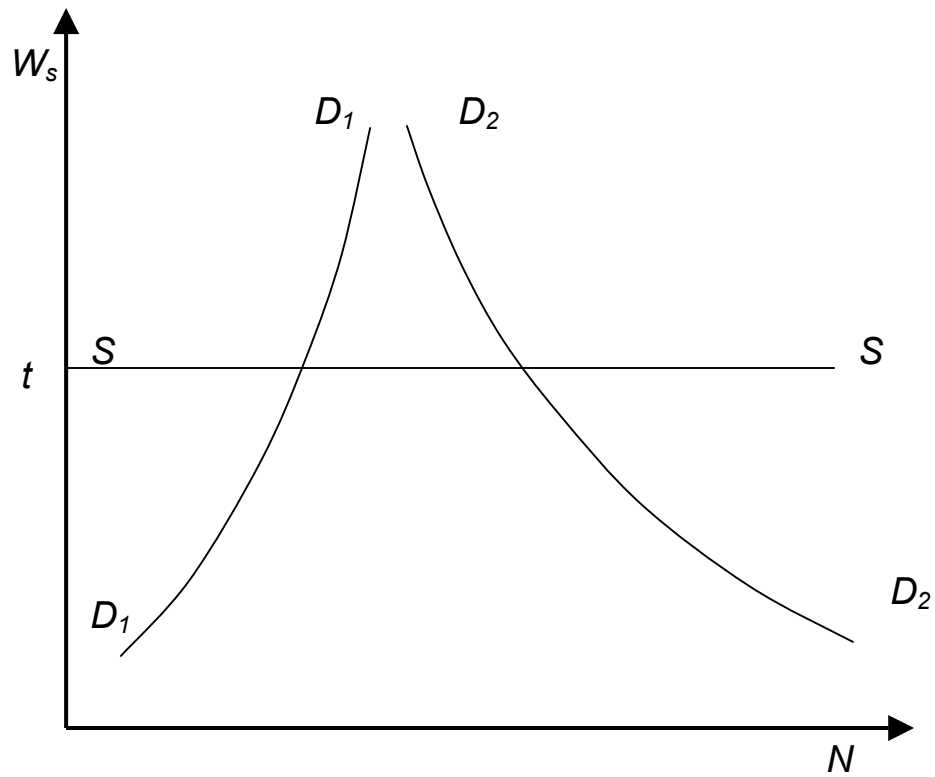


Figure 2  
The equilibrium number of firms in the  
closed economy

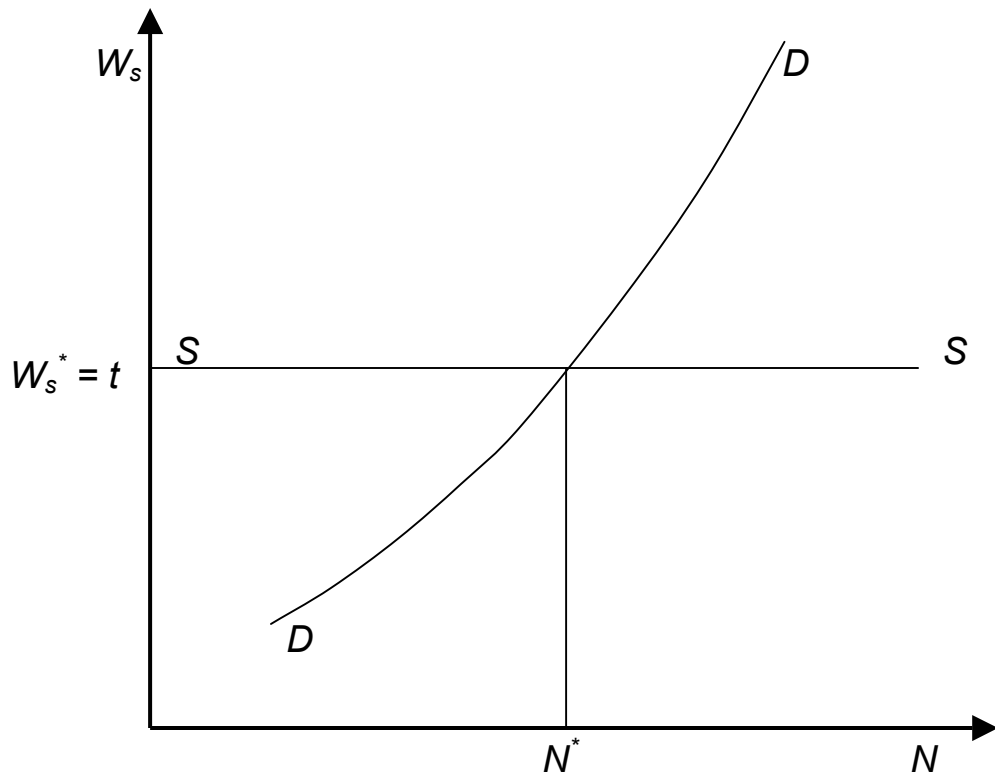


Figure 3  
The equilibrium number of firms in the  
domestic economy for given foreign  
wages and firms, free trade



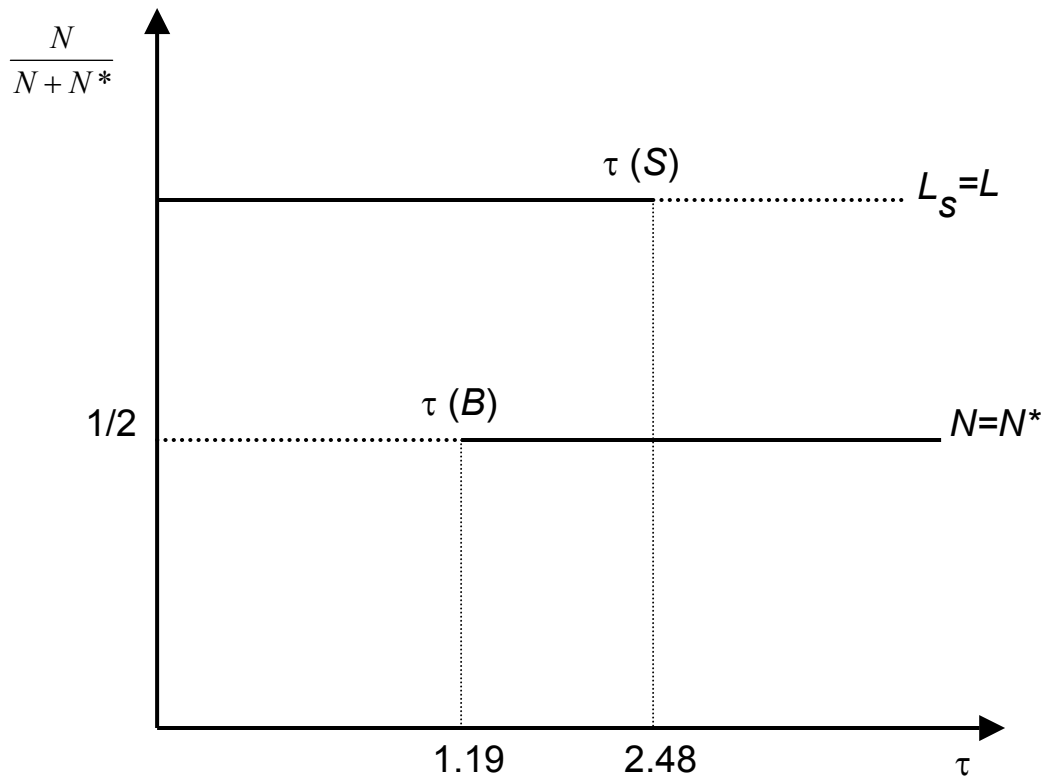
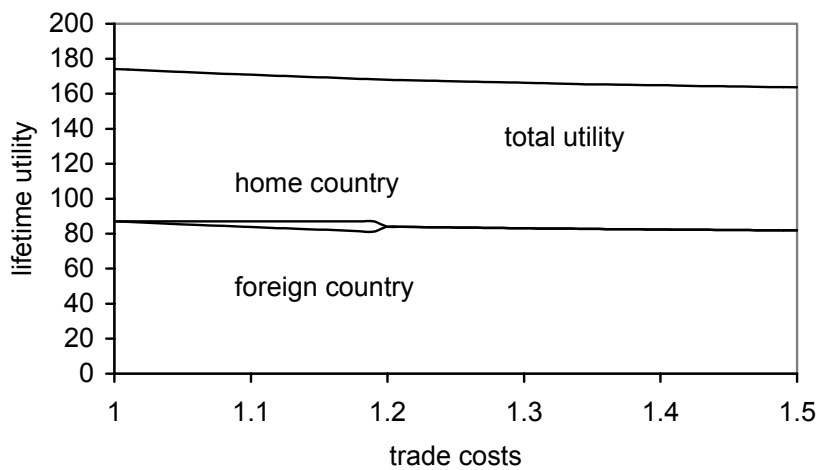


Figure 4  
 Agglomeration and symmetric equilibrium at different  
 trade costs  
*Unbroken lines indicate stable equilibria and dashed  
 lines indicate unstable equilibria*

panel (a), small  $\mu$



panel (b), large  $\mu$

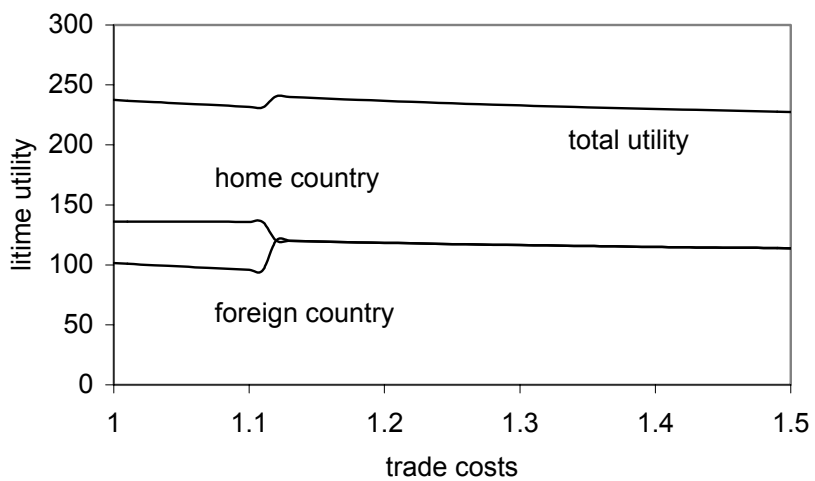


Figure 5

The effect of trade liberalization on welfare