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ABSTRACT

Reputation-Based Pricing and Price Improvements in Dealership Markets*

In many security markets, dealers trade with their regular clients at a discount relative to prevailing bid and ask quotes. In this article we provide an explanation to this phenomenon. We consider a dealer and an investor engaged in a long-term relationship. The dealer assigns a reputational index to his client. This index increases (reputation decreases) when the client conducts trades which results in a loss for the regular dealer. The dealer grants a price improvement if and only if the client's index is smaller than a threshold and suspends price improvements otherwise. We show that this pricing strategy induces the investor to refrain from exploiting private information against their regular dealer. We also find that it worsens the quotes posted by other dealers. For this reason, there are cases in which the investor is better off if long-term relationships are impossible (for instance, if trading is anonymous). Our model predicts that a dealer's decision to grant a price improvement depends on their past trading profits with the trader requesting the improvement.

JEL Classification: D82, G14 and L14
Keywords: market microstructure, non-anonymous trading and reputation and implicit contracts

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Reputation-Based Pricing and Price Improvements in Dealership Markets

1 Introduction

The relation between security prices and trading protocols is a central issue (see Biais et al.(2002) for a survey). Theoretical models usually assume that trading in financial market takes place place between faceless traders. This view of the trading process is not always adequate. In dealership markets (e.g. the London Stock Exchange), some market-makers engage in one-to-one negotiations and forge long-term trading relationships with their clients.[1] Empirical findings (e.g. Reiss and Werner (1993), Hansch, Naik and Viswanathan (1999) or Bernhardt et al. (1999)) suggest that these relationships affect dealers' pricing strategies in ways which are *not* predicted by extant theories. In particular dealers frequently offer to trade within the prevailing bid and ask prices. These price improvements do not occur in the standard model of price formation in dealership markets (e.g. Glosten and Milgrom (1985)).[2] Our purpose is to analyze dealers' pricing strategies in presence of long-term relationships.

Seppi (1990) and Benveniste, Marcus and Wilhelm (1992) argue that long-term relationships serve to alleviate adverse selection problems. In Benveniste et al. (1992), investors commit to honestly reveal their trading motivation. In Seppi (1990), investors agree to not conduct subsequent trading after a transaction (so called "no bagging commitment"). Forgoing the option to retrade is more costly to informed investors. Hence the "no bagging commitment" offers a way to screen informed and non-informed traders. Long-term relationships help to enforce these commitments since investors can be deterred from reneging by the threat of future sanctions.

We also analyze a model in which a risk-averse investor and her relationship dealer make use of repeated interactions to mitigate asymmetric information. In our model, the investor trades either to hedge endowment shocks or to exploit private information on the security's payoff. The investor implicitly agrees to not contact her regular dealer when she has private information. In turn, the relationship dealer sometimes (but not always) grants a price improvement to the investor. Our approach is distinct from Benveniste et al. (1992) or Seppi (1990) because the

[1] Other prominent examples include the Nasdaq or the upstairs market in the NYSE.

[2] Price improvements have been documented in different markets: NYSE (Petersen and Fialkowski (1994)), Nasdaq (Huang and Stoll (1996)), London Stock Exchange (e.g. Reiss and Werner (1993)) and Frankfurt Stock Exchange (Theissen (2000)).

relationship dealer is unable (at any point in time) to observe violations of the implicit contract by his client. We show that the dealer can enforce the contract by making the decision to grant price improvements contingent on his past trading profits with the investor. Specifically, these profits are used to construct a reputational index which increases each time the dealer books a loss after a transaction. The price improvement is offered if the value of the index is smaller than a threshold.

This pricing policy confronts the investor with the following dilemma when she possesses private information. She can earn a large return by exploiting her information at the dealer's expense. But the dealer records a loss and the investor's reputation is impaired (the index increases). Thus informed trading increases the likelihood that the investor will be denied price improvements in the future. This is a concern to the investor because price improvements allow more efficient hedging when she is hit by liquidity shocks. For this reason, a loss in reputation is costly. For some values of the parameters, the reputational cost is larger than the immediate gain from informed trading. Accordingly there exist 'cooperative' equilibria in which the investor does not trade with her relationship dealer when she has information. In turn, the dealer offers a price improvement if the investor's reputation is good enough. Interestingly, in equilibrium, the regular dealer obtains strictly positive expected profits while other dealers (who do not observe the investor's trading history) make zero profits.

The regular dealer's prices are contingent on his (past) trading profits with the investor because they are indicators of the investor's unobserved behavior.[3] Actually trading losses are more frequent when the latter violates the no informed trading agreement. Nevertheless the dealer experiences losses even when the investor honors the agreement. For this reason, price improvements are not permanently granted in equilibrium. Rather, there are phases during which the investor receives a price improvement and phases during which she does not. The model indicates which sequence of transactions leads to a suspension of price improvements. Thus it explains why and when dealers trade within prevailing quotes.

We also study the interactions between the relationship dealer's pricing strategy and the quotes posted by the dealers who have no relationships with the investor (that we call 'once-off

[3] There are anecdotal evidences that dealers use pricing strategies which depend on the profitability of their past transactions with a given client. For instance, Smith (1985) describes the dealers' behavior in the upstairs market as follows: 'Dealers try to keep track of who bags them [...]. They try to maintain a ratio of $6 in commissions for every $1 in trading losses generated by a specific customer. [...] if the ratio drops to 3-to-1, someone from the firm may have to chat with the customer'.

dealers'). This is another important difference with Benveniste et al (1992) and Seppi (1990). First we show that the bid-ask spread posted by once-off dealers widens when the frequency of price improvements increases. Actually, the investor never routes her uninformed trades to once-off dealers when she qualifies for a price improvement. Hence the implicit contract between the investor and the relationship dealer exacerbates the adverse selection problem faced by once-off dealers. We find that this effect can even lead to a collapse of the once-off dealers' market. Second, we obtain that cooperative equilibria are easier to sustain when once-off dealers's bid-ask spread is large. Intuitively, the investor benefits more from the implicit contract with the regular dealer when her outside option deteriorates. In some cases, cooperative equilibria exist only if the negative externality exerted by the relationship dealer is strong enough.

Do investors benefit from long-term relationships and price improvements? For given quotes posted by once-off dealers, the answer is positive since price improvements generate more efficient risk sharing. It is not so clear-cut when we account for the damaging impact of price improvements on once-off dealers' bid-ask spread. Actually, the investor sometimes trades at prevailing quotes (in particular during periods in which she does not receive improvements). In this case, she is hurt by the worsening of once-off dealers' quotes. Building on this effect, we provide an example in which the investor would be better off if she could commit to not establish a relationship.

Situations in which an anonymous and a non-anonymous trading venue operate in parallel for the same securities abound. For instance, on the NYSE, traders can operate anonymously on the floor of the exchange or/and non-anonymously in the upstairs market. Non-anonymous trading is usually viewed as beneficial because it mitigates asymmetric information (See Benveniste et al. (1992) for instance). Our findings uncover the dark side of non-anonymous trading. It worsens the quotes that an investor can obtain when she trades anonymously. This effect imposes a welfare loss on all investors, including those who sometimes receive price improvements.

Two recent papers (Rhodes-Kropf (1998) and Bernhardt et al.(1999)) provide theories of price improvements. Rhodes-Kropf (1998) advances two different explanations. The first explanation is that informed traders are less likely to negotiate a price improvement than uninformed traders. Hence dealers can offer good prices to those who request a price improvement without losing money. Rhodes-Kropf (1998) argues that informed traders are deterred from requesting price improvements by the threat of future sanctions. In our model we explicitly model these sanctions and we show that they consist in sometimes refusing price improvements. In the second explanation, dealers offer price improvements to clients with bargaining power. Bernhardt et al.(1999) endogenize the bargaining power by considering repeated relationships between a dealer and his clients. Their central result is that more regular traders obtain larger price improvements. Asymmetric information plays no role in their model whereas it is a key ingredient of the present article. One distinctive prediction of our model is that a dealer's decision to grant a price improvement depends on the profitability of his past transactions.

In our framework there is no persistence in the investor's trading motive (hedging or private information) from one period to the next. This modeling approach is natural for the problem at hand but it prevents learning effects. This feature distinguishes our model from reputation models like Kreps and Wilson (1982) or Benabou and Laroque (1992).[4] Our approach is more closely related to Radner (1985)'s repeated principal-agent game. He considers incentives schemes which depend on the agent's past performance. Here the incentive scheme is based on the alternance of phases with and without price improvements. The switch from a phase with price improvements to a phase without price improvements is contingent on the regular's dealer past trading profits.

The paper is organized as follows. In the next section, we describe the model. Section 3 analyzes the benchmark case of short-term (or anonymous) trading relationships. Section 4 delineates the set of parameters for which the no informed trading commitment can or cannot be enforced. Section 5 focuses on the interactions between the relationship dealer and the once off dealers. Section 6 discusses some implications of the model and Section 7 concludes. The proofs which are not in the text appear in the Appendix.

2 The Model

We consider the market for a risky security. The model features a risk averse investor, a risk neutral dealer (henceforth the relationship or regular dealer) with whom the investor is engaged in enduring relationships and many risk-neutral dealers (henceforth once-off dealers) who have no relationships with the investor. In the rest of this section, we describe the repeated game which is analyzed in the article and the actions that can be chosen by the different agents in this game. The investor can be seen as an institution. For instance, on the London Stock Exchange, institutional investors frequently bypass brokers and directly negotiate with their regular dealer.

[4] In these models agents' type is determined by Nature and is persistent throughout the relationship.

They receive the bulk of price improvements for large orders (see Hansch, Naik and Viswanathan (1999)).

The Investor.

The investor has trading opportunities at dates $1, 2, ...t, ...$ A 'period' is the interval of time between opportunities. The risky security can be thought of as a derivative contract. In the middle of each period, the security pays $\tilde{\epsilon}$ where $\epsilon = +1$ or $\epsilon = -1$ with probabilities $\frac{\mu}{2} > 0$ or $\epsilon = 0$ with probability $(1 - \mu)$ $(E(\tilde{\epsilon}) = 0)$.[5] Parameter μ measures the asset volatility since $Var(\tilde{\epsilon}) = \mu$. The investor can also invest in a riskless asset. The (intra period) risk free rate is set to zero, for simplicity.

There are two types of trading opportunities. With probability $\alpha > 0$, the investor receives perfect information on the final value of the security. With probability $(1 - \alpha)$, she receives a risky endowment which has a payoff $\tilde{z} = \tilde{h}\tilde{\epsilon}$, at the end of the period and she has no privileged information on the security. We assume that $\tilde{h} = +Q$ or $\tilde{h} = -Q$ with equal probabilities. Furthermore $\tilde{h}$ and $\tilde{\epsilon}$ are independent. If h is positive (negative) the investor must sell (buy) the security in order to reduce her risk exposure. She is perfectly hedged if she trades Q shares. The investor privately learns the direction of her hedging need (h) at the beginning of each period. After learning the direction of the hedging need or receiving information, the investor can trade the security.

Te investor starts each period with a constant endowment W_0 in the riskless asset. For tractability, we assume that she entirely consumes her wealth at the end of each period (no savings from one period to the next). Without affecting the results, we normalize W_0 to zero. As in Dow (1998), the investor's *per period* utility function is

$$U(W) = \gamma W \quad for \quad W > 0, \quad and \quad U(W) = W \quad for \quad W \leq 0, \tag{1}$$

where W is the investor's end of period wealth and $\gamma \in (0, 1)$. Notice that the *lower* is γ, the larger is the investor's risk aversion. Our results require the investor to be risk averse. The piecewise linear specification for the investor's per period utility yields simple explicit solutions. At the end of each period, there is a probability $(1 - \beta)$ that the investor exits the trading game forever.

[5]The date of the payoff in a period is not important.

To sum up, the investor can trade either to hedge or to benefit from her private information (as it is standard in the market microstructure literature). More precisely, in every period, the investor has one of five possible types (trading motives):

1. Positive hedging need ($h = +1$): the investor needs to sell the security in order to hedge.
2. Negative hedging need ($h = -1$): the investor needs to buy the security in order to hedge.
3. Informed with bad news ($\epsilon = -1$).
4. Informed with good news ($\epsilon = +1$).
5. Informed with neutral news ($\epsilon = 0$).

We denote each possible type by θ_j, $j \in \{+, -, -1, +1, 0\}$. When she has type θ_+, the investor has a positive hedging need, etc.... It is worth stressing that the type randomly changes from one period to the next. We denote the investor's type at date t by $\tilde{\theta}^t$. The dealer is unable to observe the investor's trading motive, at any point in time and statements regarding this motive cannot be verified. This precludes pricing policies contingent on the investor's true trading motive (as in Benveniste et al. (1992)).

Relationship Dealer and Once-Off Dealers.

The investor has access to *two* trading venues. She can trade with her regular dealer. She can also send an order to dealers with whom she has no established relationships. Trading with these once-off dealers is anonymous and once-off dealers do not observe the orders received by the regular dealer. Thus they cannot monitor the investor's past transactions. Two implications follow. First the investor cannot build a relationship with once-off dealers. Second once-off dealers cannot differentiate an investor with a relationship and an investor without a relationship. The investor can opt to trade both with once-off dealers and with the relationship dealer. The latter does not observe once-off dealers' trades. Thus he cannot constrain the investor to trade in only one trading venue as in Seppi (1990).

The Trading Process.

In every period, we model the trading process as a two stage game. In the first stage, the investor observes her type and chooses the size of her trades with (a) her regular dealer and (b) the once-off dealers. We denote by q^c the order sent to the regular dealer and by q^{nc} the order sent to the once-off dealers.[6] A positive (negative) quantity indicates an order to buy (sell). The investor's trading strategy, $q(\theta, S) = (q^c(\theta, S), q^{nc}(\theta, S))$, depends on her type and a state variable (S) which is determined by her trading history with the regular dealer (see Section 4). If this dealer is contacted by the investor ($q^c \neq 0$), he chooses the price at which he accommodates the investor's order. We refer to $p^c(q, S)$ as the regular dealer's bidding strategy. The dealer's offer depends on his belief regarding the payoff of the security. We denote by $\phi^c(q, S)$ the probability distribution of this payoff conditional on the order size and the state variable. The regular dealer's expected profit when he trades q shares is

$$\pi(p^c(q, S), q) = q\left[p^c(q, S) - E_{\phi^c}(\tilde{\epsilon})\right]. \tag{2}$$

The trading process with once-off dealers follows the same steps. However once-off dealers' bidding strategies are not contingent on the investor's trading history since they do not observe it. Their posterior belief conditional on the investor's trade size is denoted $\phi^{nc}(q)$. The investor's order for once-off dealers is eventually routed to the dealer who posts the best price. If several dealers post the best price, one is randomly chosen to execute the order. These assumptions imply that once-off dealers choose quotes which yield zero expected profits.

The regular dealer and the investor choose their strategies so as to maximize their long-run payoffs in the relationship (see Section 4). Figure 1 summarizes the sequence of events in each period.

3 Equilibria with Short-Term Trading Relationships

In this section, we analyze the equilibria which arise when investors cannot establish enduring relationships (only once-off dealers exist). This analysis constitutes a building block to solve for the equilibria when the regular dealer is active as well. The findings are qualitatively similar to those obtained in standard market microstructure models (e.g. Glosten (1989)). In absence of the regular dealer, our framework differs from these models essentially because we use different parametric assumptions in order to obtain closed-form solutions in the repeated game.

[6] Superscripts 'c' and 'nc' stand for 'cooperative' and 'non-cooperative' respectively for reasons which will become clear below.

Once-off dealers charge prices which result in zero expected profit. Hence once-off dealers' bidding strategy is

$$p^{nc}(q) = E_{\phi^{nc}}(\tilde{\epsilon} \mid q). \tag{3}$$

We focus on trading strategies and bidding strategies that form a Perfect Bayesian Equilibrium (PBE) of the trading game. A PBE of the once-off dealers' market is a set $(q^{nc*}(.), p^{nc*}(.), \phi^{nc*}(.))$ such that

1. The trading strategy $q^{nc*}(.)$ maximizes the investor's expected utility given the once-off dealer's bidding strategy, $p^{nc*}(.)$.
2. The bidding strategy $p^{nc*}(.)$ is such that $p^{nc*}(q) = E_{\phi^{nc*}}(\tilde{\epsilon} \mid q)$.
3. The dealers' posterior belief $\phi^{nc*}(.)$ is derived from the investor's trading strategy using Bayes rule where possible.

Equilibria always exist as shown in the next two propositions. First we establish some properties common to all equilibria.[7]

Lemma 1 : *All the equilibria in the once-off dealers' market have the following properties:*

- *The investor buys the same quantity when she has type θ_- or θ_{+1}, i.e. $q^{nc*}(\theta_-) = q^{nc*}(\theta_{+1}) \geq 0$.*
- *The investor sells the same quantity when she has type θ_+ or θ_{-1}, i.e. $q^{nc*}(\theta_+) = q^{nc*}(\theta_{-1}) \leq 0$.*
- *The investor does not trade when she knows that there will be no change in the asset value, i.e. $q^{nc*}(\theta_0) = 0$.*
- *In the equilibria for which $q^{nc*}(\theta_-) > 0$, the once-off dealers charge a price equal to $p^{nc*}[q^{nc*}(\theta_-)] = s^{nc}(\alpha, \mu)$ when they receive the equilibrium buy order. In the equilibria for which $q^{nc*}(\theta_+) > 0$ the dealers charge a price equal to $p^{nc*}[q^{nc*}(\theta_+)] = -s^{nc}(\alpha, \mu)$ when they receive the equilibrium sell order, with $s^{nc}(\alpha, \mu) \stackrel{def}{=} \frac{\alpha\mu}{\alpha\mu + (1-\alpha)}$.*

[7] We consider equilibria in pure strategies only. Moreover we assume that the investor does not trade when she is indifferent between trading and not trading.

The investor sells the security in order to decrease her risk exposure when she has type θ_+. She also sells the security when she has bad information (type θ_{-1}). She sells the same quantity in the two states in order to avoid detection by the dealers when she is informed. For the same reason she buys the same quantity in states θ_- and θ_{+1}. In order to break-even, the dealers must price the security at a discount (a markup) relative to the security unconditional expected value when the investor chooses to sell (buy) the security. This wedge, s^{nc}, determines the bid-ask spread charged by the dealers. If the investor is informed that the innovation in the asset value is zero, she is left with no better choice than not trading.

Figure 2a) gives the investor's hedging demand when she receives a risky endowment as a function of the spread (s^{nc}) charged by the once-off dealers.[8] When this spread is larger than $\bar{s}(\gamma,\mu) = \frac{(1-\gamma)\mu}{2-(1-\gamma)\mu} < 1$, the hedging demand is zero. This remark yields the following proposition.

Proposition 1 *: When $s^{nc}(\alpha,\mu) \geq \bar{s}(\gamma,\mu)$, the once-off dealers' market is shut down: The unique equilibrium is such that (i) the once-off dealers sell (buy) at price $p^{nc*} = 1\ (-1)$ and (ii) the investor never trades, i.e. $q^{nc*}(\theta) = 0, \forall\theta$. In this case the investor's ex-ante expected utility (i.e. before learning her type) can be written as $E\bar{U}^{nc}(\bar{s}) = -(1-\alpha)\frac{\bar{s}Q}{1+\bar{s}}$.*

By substituting s^{nc} and $\bar{s}$ with their expressions in the condition $s^{nc}(\alpha,\mu) \geq \bar{s}(\gamma,\mu)$, we rewrite the condition under which a market breakdown occurs in terms of the exogenous parameters.

Corollary 1 *: A market breakdown occurs in the once-off dealers' market if and only if $\mu \leq \mu^{nc}(\alpha,\gamma)$, where $\mu^{nc}(\alpha,\gamma) \stackrel{def}{=} \frac{1}{1-\gamma} + \frac{1}{2} - \frac{1}{2\alpha}$.*

Figure 3 illustrates the corollary, for a fixed value of γ. As γ increases, the curve which separates the two areas (trading/no trading) shifts to the left. Hence a market breakdown occurs when (i) the probability of informed trading is large or (ii) the investor's hedging need is weak because the asset is not very volatile or she is not very risk averse (μ small or γ large). The occurrence of a market breakdown is of course not surprising given the adverse selection problem faced by the dealers. Other authors (e.g. Glosten (1989), Bhattacharya and Spiegel (1991) and Madhavan (1992)) have developed models where market breakdowns occur under similar conditions. There is a difference, however. In these models a market breakdown designates a situation in which there is no equilibrium. In our model, it refers to a situation in which the equilibrium exists but

[8] Lemma 7 in the Appendix formally derives the investor's hedging demand.

it features no trading. Now consider the following trade size for the investor:

$$\bar{q}(\alpha, \mu) \stackrel{def}{=} \frac{Q}{1 + s^{nc}(\alpha, \mu)}. \tag{4}$$

There is an equilibrium in which the investor sends an order of size $\bar{q}(\alpha, \mu)$ to the once-off dealers.

Proposition 2 *: When $s^{nc}(\alpha, \mu) < \bar{s}(\gamma, \mu)$ (that is $\mu > \mu^{nc}$), there is an equilibrium in which*

1. *the investor's trading strategy is (a) $q^{nc*}(\theta_1) = q^{nc*}(\theta_{-1}) = -\bar{q}(\alpha, \mu)$, (b) $q^{nc*}(\theta_2) = q^{nc*}(\theta_{+1}) = \bar{q}(\alpha, \mu)$, and (c) $q^{nc*}(\theta_5) = 0$,*
2. *and the once-off dealers' bidding strategy is : (a) $p^{nc*}(-\bar{q}) = -s^{nc}(\alpha, \mu)$, (b) $p^{nc*}(\bar{q}) = s^{nc}(\alpha, \mu)$, (c) $p^{nc*}(-q) = -1 \quad \forall q \neq \bar{q}, q > 0$, and (d) $p^{nc*}(q) = +1 \quad \forall q \neq \bar{q}, q > 0$.*

In this equilibrium the investor's ex-ante expected utility is:

$$E\bar{U}^{nc}(s^{nc}) = \alpha \left(\frac{\gamma\mu(1 - s^{nc})Q}{1 + s^{nc}} \right) - (1 - \alpha) \left(\frac{s^{nc}Q}{1 + s^{nc}} \right). \tag{5}$$

Order sizes different from $\bar{q}(\alpha, \mu)$ can be supported in equilibrium.[9] However the spread is the same in all equilibria in which there is trading between the investor and the once-off dealers. The equilibrium described in Proposition 2 appears more sensible than other equilibria for two reasons. First, $\bar{q}$ is the investor's hedging demand given the equilibrium spread s^{nc} (see Figure 2b). Second, the equilibrium under consideration is the investor's favorite equilibrium (the investor's ex-ante expected utility in this equilibrium is larger than her ex-ante expected utility in any other equilibrium).[10] As for the once-off dealers, they are indifferent between all equilibria since they obtain zero expected profits in any case. Consequently, when there is no market breakdown, we focus on this equilibrium of the once-off dealers' market in the rest of the paper.

[9]As usual in signalling games, the multiplicity of equilibria reflects the absence of restrictions imposed by the PBE concept on dealers' belief, ϕ^{nc*}, for trade sizes out-of-the equilibrium path. It can be shown that any trade size in $[0, Q]$ can be sustained as the outcome of a PBE.

[10]This claim is an immediate consequence of Lemma 7 in Appendix. We skip its proof for brevity. Details can be obtained upon request.

4 Enforcing a No Informed Trading Agreement

Now we consider the case in which the relationship dealer co-exists with the once-off dealers. We analyze the equilibrium strategies of the relationship dealer and the investor. For expositional purposes, we assume for the moment that once-off dealers' bidding strategy is as described in the previous section. In Section 5 we shall take into account the interactions between once-off dealers' quotes and the relationship dealer's pricing strategy.

This section is organized as follows. We first describe one specific pricing policy (the scoring policy) for the relationship dealer (Section 4.1). Then we analyze the investor's optimal response to this pricing policy (Section 4.2). Eventually we establish that there exist equilibria in which the regular dealer uses a scoring policy and the investor abstains from trading with her relationship dealer when she has information (Section 4.3). We call them cooperative equilibria.

4.1 Reputation-Based Pricing

We restrict our attention to a specific type of pricing policy for the regular dealer. This policy is such that the relationship between the dealer and the investor alternates between cooperative phases and non-cooperative phases.

1. In *cooperative phases*, the dealer charges a spread $s^c < Min\{s^{nc}, \bar{s}\} \leq 1$ for a trade of size $\frac{Q}{1+s^c}$. This is the investor's optimal order when she is uninformed and the dealer's spread is s^c (Figure 2a). The dealer refuses to accommodate other orders (he charges a price of $+1$ for buy orders and -1 for sell orders).

2. After each encounter, the dealer assigns a 'score', $S \geq 0$, to the investor. This score depends on the profitability of his trades with the investor. When the dealer loses money, he increases the score by one unit. Since $s^c < 1$, this happens when the investor sells the asset and subsequently the asset price decreases or when the investor buys the asset and subsequently the asset price increases . For all the other trades, the dealer earns his posted spread (if the asset payoff is zero) or even more. For these profitable trades or when there is no trade, the score is unchanged.

3. When the score reaches the threshold, S^*, a non-cooperative phase begins.

4. *In non-cooperative phases*, the regular dealer refuses to trade with the investor (i.e. he charges a price of $+1$ for buy orders and -1 for sell orders). In each period, he increases the score by 1 unit. When the score is equal to $S^* + T$, the relationship dealer starts a new cooperative phase and the score is reset at zero. Thus a non-cooperative phase lasts $T \geq 1$ periods.

We refer to this pricing policy as *a scoring policy* and we denote it $p^{lt}(T, S^*, s^c)$ (superscript 'lt' stands for 'long-term'). A scoring policy is characterized by three parameters: (i) the 'trigger value' of the score (S^*), (ii) the length of the non cooperative phase (T) and (iii) the size of the spread during cooperative phases (s^c). In every period, the dealer's pricing strategy is determined by the investor's score only, i.e. the score serves as state variable. During cooperative phases, the relationship dealer offers a price improvement relative to the price posted by once-off dealers since $(s^{nc} - s^c) > 0$.[11] For simplicity, the relationship dealer's policy is such that he does not trade during non-cooperative phases. This is not crucial. The important point is that the investor has no access to price improvements during these phases.

The scoring policy can be viewed as an incentive scheme: it deters the investor from trading with the regular dealer when she has information. The intuition is as follows. The investor can behave in two different ways: 'cooperatively' (she contacts her regular dealer only when she is uninformed) or 'non-cooperatively' (she trades with the regular dealer whether she is informed or not). In order to induce cooperation, the regular dealer must punish the investor when she conducts informed trades. This is not straightforward since the investor's trading motivation is never observed. Punishment must therefore be based on market data related to the investor's unobserved trading motivation. This is the case of trading profits. To see this point, observe that the probability of a trading loss for the dealer, conditional on a trade taking place, is

$$Prob^c(Loss) = \frac{\mu}{2},$$

if the investor behaves cooperatively. If the investor behaves non-cooperatively, this probability is

$$Prob^{nc}(Loss) = \frac{(\alpha + 1)\mu}{2(\alpha\mu + (1 - \alpha))},$$

[11] Recall that if $s^{nc} \geq \bar{s}$, once-off dealers' equilibrium spread is 1 and there is no trading with once-off dealers.

so that the associated likelihood ratio is

$$\frac{Prob^{nc}(Loss)}{Prob^{c}(Loss)} = \frac{\alpha + 1}{\alpha\mu + (1-\alpha)} > 1. \tag{6}$$

Hence the distribution of trading losses for the regular dealer depends on the investor's behavior. The likelihood of a trading loss is larger when she exploits her private information at the dealer's expense.

The dealer must therefore punish the investor when he books a loss. The penalty takes the form of a decrease in the expected length of the cooperative phase. Actually, if the dealer bears a loss, he increases the investor's score. This shortens the expected duration of the cooperative phase since it stops when the score hits the threshold S^*. This penalizes the investor since hedging is more efficient when she receives price improvements (they are granted during cooperative phases only). We interpret the score as a *reputational index*. The investor's reputation deteriorates as the score increases. The scoring policy is designed in such a way that the investor is more likely to lose her reputational capital if she misbehaves than if she does not.[12]

4.2 Cooperative Trading

Now we derive the conditions under which cooperation is optimal for the investor when her regular dealer uses a scoring policy. To this end, we introduce the notion of *cooperative trading policy* which is denoted $q^{lt}(T, S^*, s^c)$. It is defined as follows:

1. During Cooperative Phases, the investor behaves cooperatively. That is she contacts the relationship dealer when she is uninformed and the once-off dealers when she is informed. More formally, when her score is $S < S^*$, the investor's trading strategy is:

$$\begin{cases} q^*(\theta, S) \overset{def}{=} (\frac{Q}{1+s^c}, 0) \quad if \quad \theta \in \{\theta_+, \theta_-\}. \\ q^*(\theta, S) \overset{def}{=} (0, q^{nc*}(\theta)) \quad if \quad \theta \in \{\theta_{-1}, \theta_{+1}, \theta_0\}. \end{cases} \tag{7}$$

   where $q^{nc*}(.)$ is defined as in Section 3. The first (resp.second) entry in $q^*(.,.)$ describes the order sent to the regular dealer (resp. once-off dealers).

2. During Non-Cooperative Phases, the investor only trades with once-off dealers. Her trading

[12] This interpretation is valid for cooperative phases only. During non-cooperative phases, the score is just a measure of the time elapsed since the beginning of the phase.

strategy is therefore

$$q^*(\theta, S) \stackrel{def}{=} (0, q^{nc*}(\theta)) \quad \forall \theta. \tag{8}$$

Recall that if $s^{nc} \geq \bar{s}$, then the once-off dealers' market is shut down. In this case $q^{nc*} = 0$ and the investor never trades with once-off dealers.

When the investor follows the cooperative trading policy, her long-run *expected payoff* at date τ is

$$V(\theta^\tau, S_\tau) = E\left[U(q^*(\theta^\tau, S_\tau))\right] + \sum_{t=\tau+1}^{+\infty} \beta^{t-\tau} E\left[U(q^*(\tilde{\theta}^t, \tilde{S}_t)) \mid S_\tau\right]. \tag{9}$$

Let $V(in, S)$ be the investor's expected payoff conditional on the investor being informed. In the same way, let $V(he, S)$ be the investor's expected payoff when she has a hedging need.[13] The investor's ex-ante expected payoff when she follows the cooperative trading policy is

$$V(S) = \alpha V(in, S) + (1-\alpha) V(he, S). \tag{10}$$

The next lemma shows that the value of the relationship for the investor ($V(.)$) decreases with her score *during the cooperative phase*. This means that a loss in reputation is costly for the investor.

Lemma 2 : *(A loss in reputation is costly) The value of the relationship, $V(.)$, decreases with S for $S \leq S^*$. Furthermore*

$$V(S+1) - V(S+2) \geq V(S) - V(S+1) \quad for \quad S \leq S^* - 2.$$

The larger is the score, the shorter is the expected length of the cooperative phase. This effect reduces the value of her relationship for the investor since hedging is less efficient during non cooperative phases.

Consider the investor when she is informed during a cooperative phase. If she contacts the relationship dealer and masquerades as being uninformed, she trades $(\frac{Q}{1+s^c})$ shares and earns a

[13] Formally $V(in, S) = \frac{\mu}{2} V(\theta_{-1}, S) + \frac{\mu}{2} V(\theta_{+1}, S) + (1-\mu) V(\theta_0, S)$ and $V(he, S) = \frac{1}{2} V(\theta_+, S) + \frac{1}{2} V(\theta_-, S)$.

per unit profit of $(1 - s^c)$. This transaction increases the investor's expected utility by $\frac{\gamma(1-s^c)Q}{1+s^c}$. However, following this transaction her reputation deteriorates (her score is increased by one unit) and she expects a lower long-run payoff from the relationship. If the investor abstains from contacting the relationship dealer, she bears the opportunity cost of not exploiting her information against the relationship dealer but she does not impair her reputation. The investor is better off **not** contacting the relationship dealer iff

$$\bar{U}_{in}^{nc} + \beta V(S) \geq \frac{\gamma(1 - s^c)Q}{1 + s^c} + \bar{U}_{in}^{nc} + \beta V(S + 1) \quad \forall S \leq S^* - 1,$$

where $\bar{U}_{in}^{nc}$ is the investor's utility when she is informed and trades only with once-off dealers.[14] The previous condition is equivalent to

$$\beta(V(S) - V(S + 1)) \geq \frac{\gamma(1 - s^c)Q}{1 + s^c} \quad \forall S \leq S^* - 1. \tag{11}$$

The L.H.S of this inequality is the reputational cost borne by the investor if she trades with her relationship dealer when she is informed. The R.H.S is the utility gain that the investor obtains if she exploits information at the expense of her relationship dealer. Informed trading with the relationship dealer is suboptimal if its reputational cost is larger than its utility gain.

From Lemma 2, we know that $V(.)$ decreases at an increasing rate for $S \leq S^* - 1$. This means that if the previous inequality is satisfied for $S = 0$, then it is satisfied for every $S \leq S^* - 1$. This condition is necessary for the cooperative trading policy to be optimal. The condition is also sufficient: if it holds true then the cooperative trading policy yields the largest possible long-run expected payoff to the investor. That is it dominates any other trading policy (cooperative or not) at any stage of the relationship (that is for all values of the score). This claim is established in the proof of the next lemma.

Lemma 3 : *The cooperative trading policy $q^{lt}(T, S^*, s^c)$ is optimal when the dealer follows the scoring policy $p^{lt}(T, S^*, s^c)$ if and only if*

$$\beta(V(0) - V(1)) \geq \frac{\gamma(1 - s^c)Q}{1 + s^c}. \tag{12}$$

[14] If the once-off dealers' market is shut-down, the investor does not trade when she is informed. In this case $\bar{U}_{in}^{nc} = 0$.

We refer to this incentive compatibility constraint as the 'No Informed Trading' condition. It remains to identify the set of parameters for which it holds true. Clearly, it cannot be satisfied when β is too small. In the rest of the paper, in order to reduce the number of parameters, we focus on the limit case in which the investor's β goes to 1. Let $E\bar{U}^c$ and $E\bar{U}^{nc}$ be the investor's ex-ante per period expected utility during cooperative and non cooperative phases, respectively. We denote by $\Delta U(s^c)$, the expected utility differential between cooperative and non-cooperative phases. This is a measure of the per period welfare gain brought about by cooperation for the investor. We obtain

$$\Delta U(s^c) \stackrel{def}{=} E\bar{U}^c - E\bar{U}^{nc} = \begin{cases} \frac{(\bar{s}-s^c)(1-\alpha)Q}{(1+\bar{s})(1+s^c)} \quad if \quad s^{nc}(\alpha,\mu) \geq \bar{s}(\gamma,\mu), \\ \frac{(s^{nc}-s^c)(1-\alpha)Q}{(1+s^{nc})(1+s^c)} \quad if \quad s^{nc}(\alpha,\mu) < \bar{s}(\gamma,\mu)), \end{cases} \tag{13}$$

where expressions for $\bar{s}$ and s^{nc} are given in Section 3.[15] Solving for the value function $V(.)$ we obtain the following result.

Lemma 4 *: The No Informed Trading condition can be written as*

$$\Delta U(s^c) \geq \frac{(1-s^c)\gamma Q}{1+s^c}\left(Prob(\Delta S = +1) + \Lambda\right), \tag{14}$$

with $Prob(\Delta S = +1) = \frac{(1-\alpha)\mu}{2}$ *and* $\Lambda = \frac{S^*}{T}$.

The probability $Prob(\Delta S = +1)$ is the probability of an increase in the score during the cooperative phase when the investor hnors the no informed trading agreement.

Observe that the R.H.S of the No Informed Trading condition is strictly positive. Hence when the condition holds, we have $\Delta U(s^c) > 0$ which requires $s^c < Min\{s^{nc}, \bar{s}\}$. Thus the regular dealer *must* grant a price improvement to the investor during cooperative phases. Otherwise there is no gain to being cooperative for the investor and the No Informed Trading condition cannot hold. The size of the spread improvement has an ambiguous effect on incentives. On the one hand, a large price improvement (a small value of s^c) results in more efficient hedging for the investor. This effect increases ΔU and disciplines the investor. On the other hand, a large price improvement makes informed trading more profitable ($\frac{(1-s^c)\gamma Q}{1+s^c}$ decreases with s^c). This effect reduces the investor's incentive to behave. The first effect always dominates the second effect in

[15]The expression for ΔU depends on the position of s^{nc} relative to $\bar{s}$ because this position determines whether the once-off dealers market is opened or not. The expression for $\Delta U(s^c)$ is derived in the proof of Lemma 4.

our model.

Proposition 3 *: For the investor to cooperate with the relationship dealer, she must receive a price improvement during cooperative phases. Furthermore larger price improvements stiffens the investor's incentive to cooperate: If the No Informed Trading condition holds for a given price improvement (say $s^c = s_0$) then it holds true for larger price improvements ($s^c < s_0$), other things equal.*

We denote by $s^*(\Lambda)$ the largest spread (smallest price improvement) that can be charged (offered) by the regular dealer during cooperative phases. This threshold is the spread s^c for which the No Informed Trading Condition is binding for a given ratio $\Lambda = \frac{S^*}{T}$. Clearly, the expected length of a cooperative phase increases with S^*. This remark motivates the next result.

Corollary 2 *: The maximal spread $s^*(\Lambda)$ decreases with Λ. Hence there is a positive (negative) relationship between the minimal size of a price improvement and the average length of a cooperative phase (non-cooperative phase).*

When cooperative phases are expected to last longer or when non-cooperative phases are shorter, the investor is more tempted to breach the implicit no-informed trading agreement (the R.H.S of Condition (14) increases with S^* and decreases with T). In order to maintain incentives, the regular dealer must grant a larger price improvement (s^* decreases).

The dealer does not directly observe the investor's behavior but past trading profits serve as an indicator of this behavior. This indicator is imperfect since the dealer can make a trading loss whether the investor behaves or not. For this reason, the score can increase even though the investor honors the no informed trading agreement (i.e. $Prob(\Delta S = +1)$ is not zero). This possibility of 'mistakenly' punishing the investor has a damaging effect on her incentive: the R.H.S of the No Informed Trading condition increases with $Prob(\Delta S = +1)$ and is strictly larger than zero. This last remark has an important implication: there are values of the exogenous parameters for which the No Informed Trading condition does not hold even if the incentive to behave is maximal ($s^c = 0$ and $T = \infty$). The set of parameters for which this problem does not occur is derived in the next proposition.

Proposition 4 *: There exist $s^c \geq 0$ such that when the dealer follows the scoring policy*

$p^{lt}(T, S^*, s^c)$ *then* $q^{lt}(T, S^*, s^c)$ *is optimal if and only if* $\gamma \leq \frac{1}{2}$ *and* $\mu \leq \mu^c(\alpha, \gamma)$ *with* $\mu^c(\alpha, \gamma) \stackrel{def}{=} \frac{1}{\gamma} + \frac{1}{2} - \frac{1}{2\alpha}$.

4.3 Cooperative Equilibria

We first study the dealer's optimal behavior when the investor follows the cooperative trading policy $q^{lt}(T, S^*, s^c)$. Observe that the regular dealer is the only dealer who can establish a relationship in our setting. The dealer might be tempted to exploit this feature to renege on the promised spread, s^c and charges a larger spread. The investor has some leverage because the dealer benefits from cooperation as well, however. In particular she can threaten to terminate her relationship with the dealer if he reneges on their implicit agreement.

In order to formalize this idea, we add the following provision to the implicit contract between the investor and the regular dealer. The dealer and the investor cooperates (he uses a scoring policy and she uses the associated cooperative policy) as long as the dealer follows the promised scoring policy. Otherwise, the investor stops contacting the regular dealer forever and the regular dealer charges a price equal to $+1$ (-1) for a buy (sell) order if he is contacted by the investor. This course of actions following a breach of the implicit contract by the dealer forms an equilibrium. In particular the specification of the dealer's quotes implies that implementing her threat is optimal for the investor.[16]

The regular dealer's long-run expected profit at date τ when he and the investor follow the cooperative policies $p^{lt}(T, S^*, s^c)$ and $q^{lt}(T, S^*, s^c)$ is

$$\Pi(S_\tau) = E\left[\pi(p^{c*}(\tilde{q}_\tau, S_\tau), \tilde{q}_\tau) \mid S_\tau\right] + \sum_{t=\tau+1}^{+\infty} \beta^{t-\tau} E\left[\pi(p^{c*}(\tilde{q}_t, \tilde{S}_t), \tilde{q}_t) \mid S_\tau\right], \qquad (15)$$

where $p^{c*}(.,.)$ is the regular dealer's bidding strategy in each period.

The investor's threat obliges the dealer to balance the one shot profit from charging a supra-normal spread against the future profits of his relationship with the investor. Consider the dealer in a cooperative phase when the investor's score is S. If he charges a spread equal to s^c, his total

[16] Given the investor's strategy, the dealer does not expect to receive an order if he breaches the implicit contract. Thus, after a breach of the implicit contract, the dealer's beliefs (and then his pricing strategy) conditional on receiving an order can be chosen arbitrarily.

expected profit is

$$s^c \frac{Q}{1+s^c} + \beta \left[\frac{\mu}{2} \Pi(S+1) + (1 - \frac{\mu}{2}) \Pi(S) \right] \quad for \quad S \leq S^* - 1.$$

If instead the dealer deviates and charges a spread equal to $s^d > s^c$, he obtains a total expected profit equal to s^d since from the next period onward, the investor will abstain from trading with the dealer. Hence the dealer is better off charging the promised spread iff

$$s^d \frac{Q}{1+s^c} \leq s^c \frac{Q}{1+s^c} + \beta \left[\frac{\mu}{2} \Pi(S+1) + (1 - \frac{\mu}{2}) \Pi(S) \right]. \tag{16}$$

The value of the future profits earned with the investor (the term inside the brackets) grows with β. When β goes to 1, this value become infinite **if** $s^c > 0$ and $T < \infty$.[17] Since s^d is bounded (otherwise the investor would simply refuse to trade), the condition for no opportunistic behavior on the dealer's side is satisfied for β large enough. This yields the following result.

Lemma 5 *: For every S^* and $T < \infty$, the scoring policy $p^{lt}(T, S^*, s^c)$ is the optimal pricing policy for the relationship dealer when the investor uses the cooperative trading policy $q^{lt}(T, S^*, s^c)$ if and only if $s^c > 0$.*

We call *a cooperative equilibrium*, a situation in which the course of actions prescribed by a scoring policy and the associated cooperative trading policy are self-enforcing.

Proposition 5 *: There exists a cooperative equilibrium if and only if $\gamma < \frac{1}{2}$ and $\mu < \mu^c(\alpha, \gamma)$.*

This result directly follows from combining Proposition 4 and Lemma 5. The only difference with the conditions in Proposition 4 is that here the inequalities are strict. This reflects the fact that the regular dealer must obtain strictly positive expected profit ($s^c > 0$). In Section 5, we will show that the existence conditions given in this proposition are still sufficient when once-off dealers' quotes are affected by the implicit contract between the investor and the regular dealer.

Figure 4 describes the possible equilibrium outcomes for each value of (α, μ) (γ being fixed and smaller than 1/2). There are 3 possible cases: (i) trading takes place only with once-off dealers

[17] When $s^c = 0$, the dealer's total expected profit is obviously equal to zero for every β. Thus it is optimal for the dealer to renege on the promised spread. In contrast, for $s^c > 0$, there always exists a value of β (large enough) such that Condition (16) is satisfied. When $T = \infty$, a non-cooperative phase lasts forever. This implies that $\Pi(S^*) = 0$ and for this reason $\Pi(S^* - 1)$ is finite for every β.

because a cooperative equilibrium does not exist (Region A), (ii) trading takes place both with once-off dealers and the relationship dealer (Region B) and (iii) trading takes place only with the relationship dealer (Region C). In the two last cases, a cooperative equilibrium exists.

Interestingly the no informed trading agreement cannot be enforced when α is small or μ is large. This observation is counter-intuitive. One would expect that it is easy to discipline the investor when she is rarely in possession of private information (α small) or her risk exposure is high (μ large). The key reason for our counter-intuitive finding is that the investor's trading motivation is never observed in our model. The relationship dealer must therefore rely on market data to detect misbehavior. Unfortunately, these data (here the dealer's profits or losses) can be poor indicators of the investor's true behavior. For instance when α is small, the investor is uninformed most of the time. This means that the likelihood of a trading loss for the regular dealer is not significantly larger when the investor does not honor the no informed trading agreement.[18] Hence, cooperative phases are not substantially shorter if the investor uses her information against the regular dealer. Accordingly a decrease in α reinforces the investor's temptation to breach the agreement. The same effect is at play when μ is large. Actually, in this case, the likelihood of a trading loss for the regular dealer is large, whether the investor is informed or not.

Finally consider the effect of risk aversion (γ). Risk aversion does not affect the likelihood of trading profits (losses) for the dealer. But risk sharing gains from cooperation become larger as risk aversion increases. For this reason, the investor assigns a larger value to her relationship with the dealer when she is more risk averse. Accordingly, the set of parameters for which cooperation arises is larger when the investor becomes more risk-averse (see Figure 4).

Figure 5 describes the set of possible equilibrium values for the parameters of the scoring policy, that is s^c and $\Lambda = S^*/T$. In particular it depicts the maximal spread frontier, that is the pairs (s^c, Λ) for which the no informed trading condition is binding. For a given value of Λ, any spread strictly larger than zero and below this frontier can be the spread charged by the regular dealer during cooperative phases, in equilibrium.

The multiplicity of possible cooperative equilibria raises the question of which scoring policies are likely to be chosen. A complete study of this issue is beyond the scope of the paper. We just point out that the maximal spread frontier is the Pareto frontier of the set of cooperative equilibria (when each equilibrium is characterized by s^c and Λ). This suggests that the regular

[18] The likelihood ratio $Prob^{nc}(Loss)/Prob^{c}(Loss)$ (see Section 4.1) increases with α.

dealer and the investor should endeaveor to select s^c and λ on this frontier. To see this point, consider $(1-\beta)V(0)$ and $(1-\beta)\Pi(0)$. These variables measure the ex-ante expected utilities of the investor and the dealer, respectively.[19] For a given cooperative equilibrium, we obtain

$$lim_{\beta\to 1}(1-\beta)V(0) = (1-\omega^*)E\bar{U}^{nc} + \omega^* E\bar{U}^{c} = E\bar{U}^{nc} + \omega^*\Delta U(s^c), \tag{17}$$

where

$$\omega^* = \left(\frac{1}{1+\frac{Prob(\Delta S=+1)}{\Lambda}}\right). \tag{18}$$

The investor average per-period expected utility is a weighted average of her expected utility during cooperative phases ($E\bar{U}^c$) and her expected utility during non-cooperative phases ($E\bar{U}^{nc}$). The weight ω^* can be interpreted both as the long run probability of being in a cooperative phase.[20] The investor's expected utility is larger during cooperative phases ($\Delta U(s^c) > 0$). Therefore the investor is better off when the frequency (ω^*) of cooperative phases enlarges. This frequency increases with $\Lambda = S^*/T$, that is when cooperative phases last longer on average and/or non-cooperative phases are shorter. For the dealer, we obtain

$$\lim_{\beta\to 1}(1-\beta)\Pi(0) = \omega^*(1-\alpha)\frac{s^c Q}{1+s^c}. \tag{19}$$

The average per period profit of the dealer increases with the frequency of cooperative phases as well. We conclude that, *for given spreads charged by the regular dealer and the once-off dealers*, the welfare maximizing cooperative equilibrium for both the investor and the dealer is obtained by choosing the largest possible value of $\Lambda = \frac{S^*}{T}$, that is the value for which the 'No Informed Trading' condition is binding.

Finally, it is worth stressing that there is no equilibrium in which cooperative phases last forever. This would require $\Lambda = \infty$ and the No Informed Trading condition would not hold. Consequently penalty phases necessarily occur in equilibrium. This implies that the probability of the investor being in a non-cooperative phase (ω^*) is strictly smaller than 1. We shall exploit

[19] $(1-\beta)V(0)$ is the average 'per period' ex-ante expected utility obtained by the investor in a cooperative equilibrium since this is the total expected payoff ($V(0)$) divided by the expected length of the relationship. The same interpretation is given to $(1-\beta)\Pi(0)$. A formal justification to this interpretation is provided in the proof of Lemma 4. In the proof of this lemma, we also derive the expression for $lim_{\beta\to 1}(1-\beta)V(0)$. The expression for $(1-\beta)\Pi(0)$ is obtained using the same type of arguments that we omit for brevity.

[20] See the proof of Lemma 4 for a formal justification to this interpretation.

this fact in the next section.

5 The Effect of Long-Term Relationships on Once-Off Dealers

The results in the previous section have been derived for given quotes posted in the once-off dealers' market. They do not depend on the exact value of these quotes, except the existence conditions for a cooperative equilibrium. In order to compute these conditions, we have assumed that once-off dealers' quotes were not affected by the trading arrangements between the investor and the regular dealer. In this section we allow for interactions between the quotes posted in the two trading venues.

We proceed as follows. We assume that the investor's β can take two values: 0 with probability $(1-\Psi)$ or very close to 1 with probability Ψ. The investor's β is drawn at the beginning of the game. The relationship dealer knows the investor's β but the once-off dealers do not (unless $\Psi = 1$ or $\Psi = 0$). The No Informed Trading condition cannot be satisfied if $\beta = 0$. Thus the investor has a relationship only if her β is very close to 1. Our approach aims at capturing in a simple way the idea that dealers who occasionally trade with an investor do not necessarily know whether she is engaged in a relationship or not. We show below that the trading arrangements between the regular dealer and the investor have a negligible impact on once-off dealers' quotes when Ψ is close to zero but not otherwise. In this way we generalize the analysis pursued in Section 4.

First we study the equilibrium in the once-off dealers' market for a given scoring policy of the relationship dealer. The investor chooses to trade with the relationship dealer if (1) she has a relationship, (2) she is in a cooperative phase and (3) she is uninformed. This event occurs with probability $\Psi\omega^*(1-\alpha)$ (recall that ω^* is the long run probability of being in a cooperative phase for the investor). Thus, the investor selects the the once-off dealer's market with probability $(1-\Psi\omega^*(1-\alpha))$. Conditional on this choice of trading venue, she is informed with probability:

$$\alpha^*(\Psi,\omega^*) \stackrel{def}{=} Prob\left[\theta \in \{\theta_{-1},\theta_{+1},\theta_0\} \mid q(\tilde{\theta},\tilde{S}) = (0, q^{nc*}(\tilde{\theta}))\right] = \frac{\alpha}{1-(1-\alpha)\Psi\omega^*} < 1. \qquad (20)$$

The implicit contract offered by the relationship dealer increases the risk of adverse-selection faced by once-off dealers ($\alpha^* > \alpha$ if $\omega^* > 0$). Actually, the relationship dealer diverts uninformed orders during cooperative phases. We pointed out that $\omega^* < 1$ at the end of the previous section. This explains why α^* is always strictly smaller than 1 (even if $\Psi = 1$).

The equilibrium in the once-off dealers' market is as described in Section 3 but the role of α is now played by α^*, which is endogenous.[21]. It is straightfoward from Propositions 1 and 2 that (i) there is a market breakdown if $s^{nc}(\alpha^*, \mu) \geq \bar{s}(\gamma, \mu)$ and (ii) otherwise, the spread charged by once-off dealers is $s^{nc}(\alpha^*, \mu)$. Given that s^{nc} increases with α^*, we obtain the next proposition.

Proposition 6 *: Suppose $s^{nc}(\alpha^*, \mu) < \bar{s}(\gamma, \mu)$. For a given scoring policy, the risk of informed trading (α^*) and the spread charged by once-off dealers ($s^{nc}(\alpha^*, \mu)$) become larger when*

- *The probability that the investor has a relationship (Ψ) increases.*
- *The frequency of price improvements (ω^*) increases.*

Thus the trading arrangements between the relationship dealer and the investor exert a negative externality on the liquidity of the once-off dealers' market. Notice that when Ψ goes to zero, the negative externality vanishes and $s^{nc}(\alpha^*, \mu)$ goes to $s^{nc}(\alpha, \mu)$, as we assumed in the previous section.

Now we re-examine the existence conditions for a cooperative equilibrium between the investor and the regular dealer (when the investor's β is large of course). Obviously, we can reiterate the analysis of Section 4. For a given value of $s^{nc}(\alpha^*, \mu)$, the investor chooses the cooperative trading policy if and only if the No Informed Trading condition (Equation (14)) is satisfied. The only alteration is that $s^{nc}(\alpha^*, \mu)$ substitutes for $s^{nc}(\alpha, \mu)$ in the definition of $\Delta U(s^c)$, namely

$$\Delta U(s^c) = \begin{cases} \frac{(\bar{s}-s^c)(1-\alpha)Q}{(1+\bar{s})(1+s^c)} & if \quad s^{nc}(\alpha^*, \mu) \geq \bar{s}(\gamma, \mu), \\ \frac{(s^{nc}(\alpha^*,\mu)-s^c)(1-\alpha)Q}{(1+s^{nc}(\alpha^*,\mu))(1+s^c)} & if \quad s^{nc}(\alpha^*, \mu) < \bar{s}(\gamma, \mu) \end{cases}. \tag{21}$$

Notice that the utility differential between cooperative and non-cooperative phases ($\Delta U(s^c)$) increases in the spread posted by once-off dealers. Intuitively, other things equal, cooperative phases are more attractive for the investor when trading conditions during non-cooperative phases

[21] Once-off dealers do not know whether the investor is (i) in a cooperative or (ii) a non-cooperative phase. It is natural to assume that they affect a probability ω^* ($resp.(1-\omega^*)$ to the first (resp. second) event since this is the (unique) stationary probability of this event (we derive this probability from the invariant measure of the Markov chain followed by the score S in the proof of Lemma 4).

worsen. For this reason, an increase in once-off dealers' spread strengthens the incentive to cooperate for the investor. Now the spread posted by once-off dealers increases with Ψ. This remark (together with Proposition 5) yields the following result.

Proposition 7 *: If the strategies $q^{lt}(T, S^*, s^c)$ and $p^{lt}(T, S^*, s^c)$ form a cooperative equilibrium when $\Psi = \Psi_0$ then they still form an equilibrium for larger values of Ψ. Thus, for every Ψ, a sufficient condition for the existence of a cooperative equilibrium is $\gamma < 1/2$ and $\mu < \mu^c(\alpha, \gamma)$.*

The first part of the proposition implies that an increase in Ψ cannot reduce the set of parameters for which cooperative equilibria exist. Hence the existence condition for a cooperative equilibrium when Ψ is negligible is sufficient for all values of Ψ. This means that the set of parameters for which cooperative equilibria exist include Regions B and C in Figure 4. For Ψ large enough, the inclusion is strict: there are parameters values in Region A for which cooperative equilibria exist, as well. Intuitively a large Ψ magnifies the negative externality of scoring policies on the once-off dealers's quotes and thereby facilitates the emergence of cooperation. This is the next result.

Proposition 8 *: There are values of the parameters for which a cooperative equilibrium exists when Ψ is large but not when when Ψ is small.*

Proof. Suppose $\alpha = 0.35$, $\mu = 1$, $\gamma = 0.3$ and $Q = 1$. For these values of the parameters, $\bar{s}(\gamma, \mu) = 0.53$. For $\Psi = 0$, the spread charged by once-off dealers is $s^{nc}(\alpha, \mu) = 0.35$. For this spread, there is no scoring policy such that the No Informed Trading condition is satisfied (the pair (α, μ) is in Region A in Figure 4).

Now suppose that $\Psi = 1$ and consider the following scoring policy: $\Lambda = S^*/T = 0.3$ and $s^c = 0.1$. For this scoring policy, $\omega^* = 0.48$. It follows from Equation (20) that the probability of informed trading in the once-off dealers' market is $\alpha^* = 0.5$. The spread charged by once-off dealers is $s^{nc}(\alpha^*, \mu) = 0.5$. As this is smaller than $\bar{s} = 0.53$, the once-off dealers' market does not break down. The L.H.S of the No Informed Trading Condition is $\Delta U(s^c) = 0.16$ and the R.H.S is equal to 0.15. The condition is satisfied. Thus the scoring policy $\Lambda = 0.3$ and $s^c = 0.1$ sustains a cooperative equilibrium if $\Psi = 1$ but not if $\Psi = 0$. ∎

Hence the relationship dealer can create his own 'raison d'être' by undermining the liquidity of the once-off dealers' market. Worst, this effect may even lead to a collapse of the once-off dealers' market in situations where this market would be viable if the investor had no enduring relationships, as shown below.

Proposition 9 *: There exist values of the parameters for which the once-off dealers' market is viable when Ψ is small but collapses when Ψ is large, while a cooperative equilibrium exists in both cases.*

Proof: Suppose $\alpha = 0.5$, $\mu = 1$, $\gamma = 0.2$ and $Q = 1$. This is the case considered in Figure 5. Figure 5 shows that $\Lambda = S^*/T = 0.5$ and $s^c = s^*(0.5) = 0.354$ is an equilibrium scoring policy when Ψ is negligible. Proposition 7 implies that this is also the case for $\Psi >> 0$. For this scoring policy, $\omega^* = 0.66$. Now we observe that for $\Psi = 0$, $s^{nc} = 0.5 < \bar{s}(\gamma, \mu) = 0.66$. Hence, by continuity, the once-off dealers' market is opened and co-exists with the relationship dealer for Ψ small enough. However for $\Psi = 1$, we have $\alpha^* = 0.75$ and $s^{nc}(\alpha^*, \mu) = 0.75$. This spread is larger than the threshold $\bar{s}(\gamma, \mu) = 0.66$. Hence for $\Psi = 1$, the once-off dealers' market breaks-down when the relationship dealer uses the equilibrium scoring policy $\Lambda = 0.5$ and $s^c = 0.354$.$\blacksquare$

The intuition is straightforward. As Ψ enlarges, the spread posted in the once-off dealers' market widens. In some cases, this spread becomes larger than the cut-off $\bar{s}(\gamma, \mu)$ and the once-off dealers' market collapses. It is worth stressing that this scenario does *not necessarily* occur, even when once-off dealers know that they trade with an investor engaged in long-term relationship ($\Psi = 1$). In the proof of Proposition 8, we have given an example in which $\Psi = 1$ and the once-off dealers co-exist with the relationship dealer in equilibrium.

6 Implications

6.1 Implications for Market Design

One important question for market organizers is whether the coexistence of anonymous and non-anonymous trading venues is desirable or not. Our model has some implications for this debate because the situation of once-off dealers vis-à-vis the relationship dealer is akin to the position of an anonymous trading venue vis-à-vis a non-anonymous trading venue.

A celebrated argument in favor of non-anonymous trading is that it mitigates adverse selection. Our results partially concur with this view. The relationship between the regular dealer and his client is indeed immune to adverse selection in cooperative equilibria. However the regular dealer's pricing policy reinforces the exposure to adverse selection for the once-off dealers. Through this channel, this pricing policy raises the spread posted by once-off dealers. This is detrimental to the welfare of investors who cannot establish a relationship. This is particularly obvious in the case in which the relationship dealer crowds out the once-off dealers (Proposition 9).

More surprisingly, even investors who can establish long-term relationships may vote against non-anonymous trading. Consider the case in which $\Psi = 1$. On the one hand, the investor benefits from non-anonymity because price improvements during cooperative phases lead to better risk sharing. On the other hand, during non-cooperative phases, the investor trades at worst prices than those obtained when anonymous trading is impossible. In some cases, she does not even trade during non-cooperative phases, whereas she could if non-anonymous trading was forbidden. This worsening of trading conditions during non-cooperative phases has a negative impact on the investor's welfare. In some cases, this impact dominates the positive impact of price improvements.

Proposition 10 *: Suppose $\Psi = 1$. There exist values of the parameters for which the investor is better off if she can commit to not trade with the relationship dealer.*

Proof. We consider again the numerical example used in the proof of Proposition 9. When the investor trades only with the once-off dealers, the equilibrium spread is $s^{nc} = 0.5$ and there is no market breakdown. The investor's ex-ante per period expected utility is equal to -0.15. If the investor establishes a relationship, there is an equilibrium in which the dealer uses the following scoring strategy: $\Lambda = 0.5$ and $s^c = 0.35$. The probability of being in a cooperative phase is $\omega^* = 0.66$. In this equilibrium, the once-off dealers' market collapses. During cooperative phases, the investor's ex-ante expected utility is equal to -0.129. During non-cooperative phases, the investor cannot trade and her ex-ante expected utility is -0.225.[22] The average ex-ante per period expected utility is

$$lim_{\beta \to 1}(1-\beta)V(0) = (1-\omega^*)(-0,225) + \omega^*(-0,129) = -0.16 < -0.15.$$

Hence the investor is better off when she only trades with once-off dealers. ∎

Interestingly a commitment of not trading with the relationship dealer is not credible. Suppose

[22] Expressions for the investor's expected utilities in the various cases are given in Propositions 1, 2 and by Equation (13).

that the investor makes such an announcement and that she is believed by the once-off dealers. Then it becomes optimal for the investor to secretely enter into a no-informed trading agreement with the relationship dealer. In this way she better hedges during cooperative phases and her outside option does not deteriorate since once-off dealers believe she has no relationship. Thus the investor has no incentive to honor the commitment. A crude way to overcome this problem is to ban non-anonymous trading. Such a policy prevents reputation-based pricing and thereby enforces the commitment of not establishing a relationship.

Recall that the once-off dealers' market can break down even in absence of the negative externality exerted by scoring policies (even if $\Psi = 0$). For such 'natural' breakdowns, non-anonymous trading venues can be particularly useful since they may enable trades which cannot occur otherwise. As shown by the next proposition, this is the case in our model.

Proposition 11 *: For $\gamma < \frac{1}{2}$ and $\Psi \geq 0$, the set of parameters for which a cooperative equilibrium exists includes the set of parameters for which the once-off dealers' market breaks down when $\Psi = 0$ (Region C in Figure 4).*

Thus, paradoxically, non-anonymous trading venues can both resolve and exacerbate market breakdowns problems.

6.2 Empirical Implications

The main and novel prediction of our model is that the decision to grant a price improvement by a dealer depends on the profitability of his past transactions with the client requesting the improvement. Madhavan and Cheng (1996) show empirically that prices in the NYSE block ("upstairs") market are trader specific. They interpret this finding as evidence of reputation pricing in the upstairs market. However, they do not identify the variables upon which reputation is based. Our paper suggest to consider the profitability of the trader's past transactions as a determinant of her reputation and thereby the prices she will receive. This seems a reasonable hypothesis given that dealers keep track of this profitability. For instance Sirri (2000) notices that:

> *Technology is such that some broker-dealers have invested in systems that let them monitor the profitability of trading as principal against their retail customers on a*

name by name basis".

Battalio et al. (2001) study the order flow routed to a major Nasdaq dealer. They find that the dealer's gross trading revenues vary among routing brokers, after controlling for trade size. They do not however analyze the relationships between dealer's prices and the profitability of his past transactions with each broker. According to our model, such a relationship could well exist and explain the pattern of price improvements received by each broker.

7 Conclusions

Our model offers an explanation to the practice of price improvements in dealership markets (e.g. the London Stock Exchange or Nasdaq). We have argued that price improvements can be part of an implicit contract between an investor and his regular dealer. This contract is such that the investor does not conduct informed trades against the regular dealer. In turn the dealer sometimes offers price discounts relative to prevailing quotes. As the dealer does not observe the investor's trading motivation, there is an enforcement problem. The dealer enforces the contract by making his decision to grant price improvements contingent on the profitability of his past transactions with the investor. Intuitively, this profitability is indicative of the investor's unobserved behavior because trading losses are more likely if the investor breaches the implicit contract.

Our approach complements the previous literature on long-term relationships in financial markets for two reasons. First, in a realistic manner, we assume that the dealer has no way to observe violations of the implicit contract by the investor. This is an important difference with Benveniste et al.(1992) and Seppi (1990). This feature explains why the dealer relies on market data (his trading profits) to determine his quotes. It also implies that in equilibrium, the dealer sometimes refuses to grant price improvements. Second, we analyze the impact of the implicit contract on the quotes posted by dealers who have no relationship with the investor. We show that the implicit contract worsens these quotes. In our model, the investor does not exclusively trade with her relationship dealer in equilibrium. For this reason, there are cases in which she would be better off if she could commit to not establish a relationship.

There are several possible extensions which could be considered in future work. For instance, one could consider more general probability distributions for the security's payoff. Furthermore, it is worth stressing that we have made no attempt to characterize the most efficient contract between the investor and the dealer. It is very likely that there exist equilibria with more complex strategies for the dealer and the investor which yield larger expected payoffs to both parties. Finally, in our model, the purpose of the implicit contract between the relationship dealer and the investor is to eliminate the risk of informed trading. The relationship dealer could also seek to acquire the investor's information. The form of the implicit contract which would enforce truthful revelation of her information by the investor is another fascinating question.

8 Bibliography

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9 Appendix

Preliminary Results.

We first give two Lemmas providing general results regarding once-off dealers' bidding strategy (Lemma 6) and the investor's optimal order size when she is uninformed (Lemma 7).

Lemma 6 *: The dealers accommodate equilibrium sell orders (resp. buy orders) at a discount (resp. at a markup) relative to the security's expected value. Formally, on the equilibrium path,* $p^{nc*}(q) \in [0,1]$ *for every* $q > 0$ *and* $p^{nc*}(q) \in [-1,0]$ *for every* $q < 0$.

Proof. We prove the lemma for a buy order only. For a sell order, the argument is similar. As explained in the text (see Equation (3)), the price posted by once-off dealers is equal to the expected value of the asset. Hence, it belongs to $[-1,+1]$. Thus an investor with bad news (resp. good news) never submits a buy (sell) order. It follows that a buy order signals that the investor's type is different from θ_{-1}. Consequently, if $q > 0$, we have $p^{nc*}(q) = E_{\phi^{nc}}(\epsilon|q) \geq 0$ (where this expectation is well defined since q is on the equilibrium path).■

The next lemma analyzes the investor's optimal order at a given price when she is uninformed. We restrict our attention to the optimal buy order at price $s \geq 0$ and the optimal sell order at price $-s \leq 0$. Knowledge of the optimal buy and sell orders in these cases will prove sufficient in what follows. We make use of the following notation:

$$\bar{s}(\gamma,\mu) = \frac{(1-\gamma)\mu}{2-(1-\gamma)\mu} < 1.$$

Lemma 7 *:*

1. *At price* $0 \leq s \leq 1$*, the investor's optimal buy order when she has type* θ_- *is: (a)* $\frac{Q}{1+s}$ *if* $s < \bar{s}(\gamma,\mu)$*; (b) zero if* $s > \bar{s}(\gamma,\mu)$*; (c) any quantity in* $\left[0, \frac{Q}{1+s}\right]$ *if* $s = \bar{s}(\gamma,\mu)$.

2. *At price* $-1 \leq -s < 0$*, the investor strictly prefers to not trade rather than to submit a sell order when she has type* θ_-.

3. *At price* $-1 \leq -s \leq 0$*, the investor's optimal sell order when she has type* θ_+ *is: (a)* $-\frac{Q}{1+s}$ *if* $s < \bar{s}(\gamma,\mu)$*; (b) zero if* $s > \bar{s}(\gamma,\mu)$*; (c) any quantity in* $\left[-\frac{Q}{1+s}, 0\right]$ *if* $s = \bar{s}$.

4. *At price $0 < s \leq 1$, the investor strictly prefers to not trade rather than to submit a buy order when she has type θ_+.*

To sum up, if buy (sell) orders are executed at a premium (discount) then the investor buys (sells) the security or does not trade when she has type θ_- (θ_+). The exact order size is a function of the spread (s) charged by the dealer. This function is depicted in Figure 2a) where the order size is given in absolute value for a given spread.

Proof. The final wealth of an investor with type θ_- when she trades q shares at price $p(q)$ is

$$W_\epsilon(q) = -Q\epsilon + q(\epsilon - p(q)). \tag{22}$$

Suppose first that $q \geq 0$ and $p(q) = +s$. In this case (i) $W_{-1}(q) \leq 0$ if and only if $q \geq \frac{Q}{1+s}$ and (ii) $W_{+1}(q) \leq 0$ if and only if $q \leq \frac{Q}{1-s}$. Finally, $W_0(q) \leq 0$. The expected utility of the investor, $E(U(W_\epsilon(q))$, is continuous and piecewise linear in q and

$$E(U(W_\epsilon(q))) = \begin{cases} \gamma\frac{\mu}{2}W_{-1}(q) + (1-\mu)\,W_0(q) + \frac{\mu}{2}W_{+1}(q) & if \quad 0 \leq q \leq \frac{Q}{1+s}, \\ \frac{\mu}{2}W_{-1}(q) + (1-\mu)\,W_0(q) + \frac{\mu}{2}W_{+1}(q) & if \quad \frac{Q}{1+s} \leq q \leq \frac{Q}{1-s}, \\ \frac{\mu}{2}W_{-1}(q) + (1-\mu)\,W_0(q) + \gamma\frac{\mu}{2}W_{+1}(q) & if \quad q \geq \frac{Q}{1-s}. \end{cases} \tag{23}$$

It is now straightforward that (i) when $0 \leq q \leq \frac{Q}{1+s}$, the slope of $E(U(W_\epsilon(q)))$ is positive if $s < \bar{s}$, zero if $s = \bar{s}$, negative if $s > \bar{s}$; and (ii) when $q \geq \frac{Q}{1+s}$, the slope of $E(U(W_\epsilon(q)))$ is negative. Point 1 follows.

Now suppose that $q < 0$ and $p(q) = -s$. Since $q < 0$, it immediately follows that (i) $W_{-1}(q) \geq 0$, (ii) $W_{+1}(q) \leq 0$ and $W_0(q) \leq 0$. Thus

$$E(U(W_\epsilon(q))) = \gamma\frac{\mu}{2}W_{-1}(q) + (1-\mu)\,W_0(q) + \frac{\mu}{2}W_{+1}(q). \tag{24}$$

Some computations give

$$E(U(W_\epsilon(q))) = (\gamma-1)\frac{\mu}{2}Q - (\gamma-1)\frac{\mu}{2}q - \left(1 + (\gamma-1)\frac{\mu}{2}\right)qs. \tag{25}$$

Finally, using Equation (23) written for $q = 0$, we obtain

$$E(U(W_\epsilon(0))) = (\gamma - 1)\frac{\mu}{2}Q. \tag{26}$$

One easily checks that $E(U(W_\epsilon(q))) < E(U(W_\epsilon(0)))$ always holds if $q < 0$(even if $s = 0$). An analogous argument shows that Point 3 and 4 hold true.$\blacksquare$

Proof of Lemma 1.[23] This Lemma follows from Lemmas 6 and 7. First, it is immediate from Lemma 6 that a trader who learns that the security's payoff is zero does not trade, that is $q^{nc*}(\theta_0) = 0$. Second, given that the price of a sell order is negative (Lemma 6), a trader with a negative hedging need never submits a sell order (Lemma 7), namely $q^{nc*}(\theta_-) \geq 0$. Analogously, $q^{nc*}(\theta_+) \leq 0$. Third, the investor never buys (sells) the security when she learns good (bad) news since once-off dealers' prices belong to $[-1, +1]$. Thus $q^{nc*}(\theta_{+1}) \geq 0$ $(q^{nc*}(\theta_{-1}) \leq 0)$. In equilibrium, there are only two possibilities for the order submitted by an investor with type θ_{+1}: **(i)** $q^{nc*}(\theta_{+1}) \neq q^{nc*}(\theta_-)$ or **(ii)** $q^{nc*}(\theta_{+1}) = q^{nc*}(\theta_-)$. Suppose $q^{nc*}(\theta_{+1}) > 0$. In Case (i), $p^{nc*}[q^{nc*}(\theta_{+1})] = E(\tilde{\epsilon} \mid \theta = \theta_{+1}) = +1$. Thus the investor obtains a zero expected profit when she has type θ_{+1}. In this case she is indifferent between trading and not trading and we have assumed that she does not trade in such a situation. This contradicts $q^{nc*}(\theta_{+1}) > 0$. Thus Case (i) cannot obtain if $q^{nc*}(\theta_{+1}) > 0$. Now assume that $q^{nc*}(\theta_{+1}) = 0$ but $q^{nc*}(\theta_-) > 0$. In equilibrium, $p^{nc*}[q^{nc*}(\theta_-)] = E(\tilde{\epsilon} \mid \theta \neq \{\theta_{-1}, \theta_{+1}, \theta_0\}) = 0$. But then $q^{nc*}(\theta_{+1}) = 0$ cannot be optimal since the investor can make a strictly positive expected profit by submitting an order for $q^{nc*}(\theta_-) > 0$ shares when she has type θ_{+1}. It follows that Case **(ii)** is the only possibility in equilibrium. The same argument proves that $q^{nc*}(\theta_+) = q^{nc*}(\theta_{-1})$.

If $q^{nc*}(\theta_-) > 0$, we deduce (using Bayes rule) that

$$p^{nc*}[q^{nc*}(\theta_-)] = E(\tilde{\epsilon} \mid \theta = \{\theta_-, \theta_{+1}\}) = s^{nc}.$$

In the same way, if $q^{nc*}(\theta_+) < 0$, we obtain

$$p^{nc*}[q^{nc*}(\theta_+)] = E(\tilde{\epsilon} \mid \theta \in \{\theta_+, \theta_{-1}\}) = -s^{nc}.\blacksquare$$

Proof of Proposition 1.

[23] Recall that if a trader is indifferent between trading or not trading then we assume that she does not trade.

Step 1. No trading is an Equilibrium Outcome

The investor's best response to the once-off dealers' strategy is to not trade when she is uninformed (Lemma 7). When she is informed, the investor is indifferent between trading or not given once-off dealers' pricing strategy. We assume that she does not trade. Hence, not trading is optimal for the investor in all possible states. Given the investor's trading strategy, there is a zero probability that once-off dealers receive an order. Thus, conditional on such an event, we can choose dealers' beliefs ϕ^{nc} (and then dealers' pricing behavior) arbitrarily. Here dealers' beliefs are chosen in such a way that they assign a probability one to the event $\{\epsilon = 1\}$ (resp. $\{\epsilon = -1\}$) when they receive a buy (resp. sell) order. This specification always sustains a PBE.

Step 2. No Trading is the Unique Equilibrium Outcome when $s^{nc} \geq \bar{s}$.

Lemma 1 implies that if there is an equilibrium where the investor trades then $q^{nc*}(\theta_+) < 0$ or/and $q^{nc*}(\theta_-) > 0$. Furthermore, the lemma states that these orders are executed at price s^{nc} (buy order) or $-s^{nc}$ (sell order). But, when $s^{nc} \geq \bar{s}$, the uninformed investor is better off not trading (see Lemma 7). This implies that $q^{nc*}(\theta_+) < 0$ ($q^{nc*}(\theta_-) > 0$) cannot be optimal. Thus the only possible equilibrium outcome involves no trade when $s^{nc} \geq \bar{s}$.

Step 3. Investor's Expected Utility when She does not Trade.

When the investor is informed and she does not trade, it is straightforward that her expected utility is

$$\bar{U}_{in}^{nc}(\bar{s}) = 0. \tag{27}$$

When the investor is uninformed and she does not trade, we use Equation (23) written for $q = 0$ in order to obtain the investor's expected utility:

$$\bar{U}_{he}^{nc}(\bar{s}) = -\frac{\mu(1-\gamma)Q}{2} = -\frac{\bar{s}Q}{1+\bar{s}}. \tag{28}$$

(the second equality follows from the definition of $\bar{s}$). Hence the investor's ex-ante expected utility when she does not trade is $E\bar{U}^{nc}(\bar{s}) = (1-\alpha)\bar{U}_{he}^{nc}(\bar{s})$.■

Proof of Corollary 1. Proposition 1 states that a market breakdown occurs if and only if $s^{nc} \geq \bar{s}$. Using the expressions for $\bar{s}$ and s^{nc}, this inequality rewrites $\mu \leq \mu^{nc}$. ■

Proof of Proposition 2.

Step 1. Strategies $q^{nc*}(.)$ and $p^{nc*}(.)$ form an equilibrium.

Once-off dealers' prices for trade sizes on the equilibrium path follow from Lemma 1. For trade sizes out-of-the equilibrium path, dealers' beliefs ϕ^{nc} (and then dealers' pricing behavior) can be chosen arbitrarily. Here dealers' beliefs are chosen in such a way that they assign a probability 1 to the event $\{\epsilon = 1\}$ (resp. $\{\epsilon = -1\}$) when they receive a buy (resp. sell) order.

Consider now the problem faced by the investor. It follows from Lemma 7 that submitting a buy (sell) order of size $\bar{q}$ is the investor's best response when she has type θ_- (θ_+). The investor's strategy when she is informed follows from Lemma 1. Hence we have proved that the strategies given in Proposition 2 form an equilibrium when $s^{nc} \leq \bar{s}$.

Step 2. Investor's Ex-ante Expected Utility in Equilibrium.

When the investor has a negative hedging need (type θ_-) or when she receives good news, she buys $(\frac{Q}{1+s^{nc}})$ shares at price $s^{nc} < 1$. Hence, her expected utility in equilibrium is

$$\bar{U}^{nc}_{he}(s^{nc}) = \frac{\mu}{2}\left(-Q + \frac{Q(1-s^{nc})}{1+s^{nc}}\right) - (1-\mu)\frac{s^{nc}Q}{1+s^{nc}} = -\frac{s^{nc}Q}{1+s^{nc}}, \tag{29}$$

when she has a negative hedging need, and

$$\bar{U}^{nc}_{in}(s^{nc}) = \frac{\gamma(1-s^{nc})Q}{1+s^{nc}}, \tag{30}$$

when she receives good news. By symmetry of the buy and sell sides, these expected utilities are the same when the investor has a positive hedging need (type θ_+) or receives bad news. Lastly, when the investor learns that the security's payoff is zero, she does not trade and gets a zero expected utility. Hence the investor's ex-ante expected utility is

$$E\bar{U}^{nc}(s^{nc}) = (1-\alpha)\bar{U}^{nc}_{he}(s^{nc}) + \alpha\mu\bar{U}^{nc}_{in}(s^{nc}). \blacksquare \tag{31}$$

Proof of Lemma 2. In cooperative phases, when the investor receives good or bad news, she only trades with once-off dealers. Thus

$$V(in, S) = \mu\bar{U}^{nc}_{in} + \beta V(S) \quad for \quad S \leq S^* - 1, \tag{32}$$

($\bar{U}^{nc}_{in}$ is given in Equations (27) and (30)). In cooperative phases, when she nedds to hedge, the investor trades $\frac{Q}{1+s^c}$ shares with her relationship dealer. In this case, she obtains an expected utility

$$\bar{U}^c_{he} = \frac{-Qs^c}{1+s^c}. \tag{33}$$

(this follows from Equation (29), substituting s^c for s^{nc}). Following this transaction, the investor's score deteriorates if the relationship dealer loses money. The probability of this event is $\frac{\mu}{2}$. Otherwise her score is unchanged. Therefore,

$$V(he,S) = \bar{U}^c_{he} + \beta\left(\frac{\mu}{2}V(S+1) + (1-\frac{\mu}{2})V(S)\right) \quad for \quad S \leq S^* - 1. \tag{34}$$

Substituting Equation (10) in the expressions of $V(in,S)$ and $V(he,S)$ given by Equations (32) and (34) respectively and rearranging, we obtain

$$V(S+1) = kV(S) - \frac{2E\bar{U}^c}{\mu\beta(1-\alpha)} \quad for \quad S \leq S^* - 1, \tag{35}$$

with $k \overset{def}{=} \left(1 + \frac{2(1-\beta)}{\beta(1-\alpha)\mu}\right)$ and

$$E\bar{U}^c \overset{def}{=} (1-\alpha)\bar{U}^c_{he} + \alpha\mu\bar{U}^{nc}_{in}. \tag{36}$$

Notice that $E\bar{U}^c$ is the investor's per period expected utility during the cooperative phase. We deduce

$$V(S+1) - V(S) = k(V(S) - V(S-1)) \quad for \quad 1 \leq S \leq S^* - 1. \tag{37}$$

Furthermore, Equation (35) for $S = 0$ implies

$$V(1) - V(0) = (k-1)\left(V(0) - \frac{E\bar{U}^c}{1-\beta}\right). \tag{38}$$

Then, notice that $E\bar{U}^c \geq E\bar{U}^{nc}$ ($E\bar{U}^{nc}$ is given in Equation (31)) if and only if $\bar{U}^c_{he} \geq \bar{U}^{nc}_{he}$. As $s^c < Min\{s^{nc}, \bar{s}\}$, it is straightforward that this inequality is always satisfied (see Equations (28), (29) and (33) for the values of $\bar{U}^c_{he}$ and $\bar{U}^{nc}_{he}$). Recall that $V(0)$ is the (discounted at rate β) sum of the investor's per period payoff (which is either $E\bar{U}^c$ or $E\bar{U}^{nc}$). This implies that

$$V(0) \leq \sum_{t=0}^{+\infty} \beta^t E\bar{U}^c = \frac{E\bar{U}^c}{1-\beta}. \tag{39}$$

Equation (38) implies then that $V(1) - V(0) \leq 0$. In turn, this yields (iterating Equation (37))

$$V(S+1) - V(S) \leq 0 \quad for \quad 1 \leq S \leq S^* - 1. \tag{40}$$

Hence $V(.)$ decreases with S. Furthermore, since $k > 1$ and $V(0) - V(1) \geq 0$, Equation (37) implies that $(V(S-1) - V(S))$ increases for $1 \leq S \leq S^*$. ∎

Proof of Lemma 3. According to the Optimality Principle of Dynamic Programming, the investor has no incentive to deviate from the cooperative trading strategy q^{lt} if and only if there is no circumstance in which a one shot deviation is profitable.[24] Hence, we just need to show that the condition given in Lemma 3 is necessary and sufficient to deter a one shot deviation from the trading strategy q^{lt}. It derives immediately from the analysis of Section 3 that $q^{nc*}(.)$ gives the investor's optimal order with once-off dealers. It follows that no one shot deviation is profitable during non-cooperative phases. During cooperative phases, it is obvious that the investor must buy or sell $\frac{Q}{1+s^c}$ shares when she trades with her relationship dealer. The one shot deviations which remain to analyze are:

1. The investor is informed with good (bad) news. She sends a buy (sell) order to her relationship dealer (instead of contacting only the once-off dealers).

2. The investor is uninformed. She trades with the relationship dealer *and* the once-off dealer (instead of trading only with the relationship dealer).

3. The investor is uninformed and she trades only with the once-off dealer (instead of trading with the relationship dealer).

We consider each of these actions in turn.

Deviation 1. We have already explained in the text that the first deviation is not optimal if and only if Condition (12) holds true.

[24]As the relationship dealer's strategy is Markovian (the state variable S_t follows a Markov chain, see Lemma 4), the investor's equilibrium strategy is Markovian and contingent on S_t as well.

Deviation 2. When the investor is uninformed, the quantity traded with the regular dealer is the investor's optimal trade size given the price charged by this dealer (see Lemma 7). This means that trading additional size at the regular dealer's spread (s^c) is suboptimal. This in turn implies that trading additional size at the larger spread quoted by once-off dealers ($s^{nc} > s^c$) is suboptimal. Hence deviation 2 is suboptimal.

Deviation 3. When the investor is uninformed and she has score $S \leq S^* - 1$, she is better off trading with her relationship dealer if and only if

$$\bar{U}_{he}^{nc} + \beta V(S) \leq V(he, S), \tag{41}$$

($\bar{U}_{he}^{nc}$ is given in Equations (28) and (29)). Using Equation (34), we rewrite Condition (41) as

$$\bar{U}_{he}^{nc} \leq \bar{U}_{he}^{c} + \frac{\beta\mu}{2}\left(V(S+1) - V(S)\right), \tag{42}$$

($\bar{U}_{he}^{c}$ is given in Equation (33)). Using Equation (35), after straightforward manipulations, we can rewrite Equation (42) as

$$\frac{E\bar{U}^{nc}}{1-\beta} \leq V(S), \tag{43}$$

($E\bar{U}^{nc}$ is given in Equation (31)). Notice that $V(S)$ is the discounted sum of the investor's per period payoff (which is either $E\bar{U}^c$ or $E\bar{U}^{nc}$). As $E\bar{U}^c \geq E\bar{U}^{nc}$ (see the proof of Lemma 2), we deduce that

$$V(S) \geq \sum_{t=0}^{+\infty} \beta^t E\bar{U}^{nc} = \frac{E\bar{U}^{nc}}{1-\beta}, \tag{44}$$

which means that Equation (43) holds true. Hence deviation 3 is suboptimal. ∎

Proof of Lemma 4.

Denote by $E\tilde{U}_t$ and by S_t the investor's expected utility and score in period t when she follows the cooperative trading policy. $E\tilde{U}_t$ depends on whether the relationship is in a cooperative or a non-cooperative phase. If $S_t \in [0, S^* - 1]$ then $E\tilde{U}_t = E\bar{U}^c$. If $S_t \in [S^*, S^* + T - 1]$ then $E\tilde{U}_t = E\bar{U}^{nc}$ ($E\bar{U}^c$ and $E\bar{U}^{nc}$ are given in Equations (36) and (31)). Notice that S_t is a Markov chain with state space $\{0, 1, ..., S^* + T - 1\}$. Denote by $\Pr_{SS'}$ the transition probability from state S to state S'. We have (i) for $S \le S^*-1$, $\Pr_{S(S+1)} = (1-\alpha)\frac{\mu}{2}$ and $\Pr_{SS} = 1-(1-\alpha)\frac{\mu}{2}$; (ii) for $S^* \le S < S^*+T-1$, $\Pr_{S(S+1)} = 1$ and $\Pr_{(S^*+T-1)0} = 1$; (iii) the other transition probabilities are zero. Let $o(S)$ be the long-run frequency of state S. By definition, the probabilities $o(S)$ form the invariant measure of the Markov chain, namely the solution of the linear system:

$$\forall S', o(S') = \sum\nolimits_{S=0}^{S=S^*+T-1} \Pr_{SS'} o(S). \tag{45}$$

Given that most of the $\Pr_{SS'}$ are zero, solving for $o(S)$ is easy. In particular, we obtain

$$o(S) = \frac{1}{S^* + T\frac{(1-\alpha)\mu}{2}}, \quad \forall S \in [0, S^*-1].$$

We deduce that the probability, ω^*, of a cooperative phase is

$$\omega^* \overset{def}{=} \sum_{S=0}^{S=S^*-1} o(S) = \frac{\frac{S^*}{T}}{\frac{S^*}{T} + \frac{(1-\alpha)\mu}{2}}.$$

Observe that ω^* and $1-\omega^*$ give the long-run frequencies of $E\bar{U}^c$ and $E\bar{U}^{nc}$ respectively. An ergodic theorem for Markov chains (Theorem I.15.2 in Chung (1967)) implies that

$$\lim_{T\to+\infty} \frac{\sum_{t=0}^{t=T} E\tilde{U}_t}{T+1} = \omega^* E\bar{U}^c + (1-\omega^*)E\bar{U}^{nc} \quad with \quad probability \quad 1. \tag{46}$$

Denote by $E\bar{U}_t$ the expected value of $E\tilde{U}_t$ conditional on the score being equal to zero at a given date $\tau < t$ (say $\tau = 0$), i.e. $E\bar{U}_t = E[E\tilde{U}_t|S_0 = 0]$. For $T \ge 0$ and $\beta \in [0,1]$, define $f_T(\beta) \overset{def}{=} \frac{\sum_{t=0}^{t=T} \beta^t E\bar{U}_t}{\sum_{t=0}^{t=T} \beta^t}$. Notice that

$$f_\infty(\beta) = (1-\beta)V(0). \tag{47}$$

Every f_T is a continuous function of β. Furthermore, when T tends to $+\infty$, we have pointwise convergence of $f_T(\beta)$ to $f_\infty(\beta) \overset{def}{=} \frac{\sum_{t=0}^{t=\infty} \beta^t E\bar{U}_t}{\sum_{t=0}^{t=\infty} \beta^t}$. Dini's Theorem implies then that convergence of f_T to f_∞ is uniform on the compact set $[0,1]$. Hence, f_∞ is continuous at $\beta = 1$. Using this remark, Equation (46) and Equation (47), we conclude that[25]

$$\lim_{\beta\to 1}(1-\beta)V(0) = f_\infty(1) = \omega^* E\bar{U}^c + (1-\omega^*)E\bar{U}^{nc}. \tag{48}$$

[25]The result can also be obtained by solving for the value function. Computations are longer however.

Now recall that $(V(1) - V(0))$ is given (38). Substituting $(V(1) - V(0))$ by its expression in the No Informed Trading condition gives

$$\frac{2}{\mu(1-\alpha)}\left(E\bar{U}^c - (1-\beta)V(0)\right) \geq \frac{\gamma(1-s^c)Q}{1+s^c}. \tag{49}$$

Substituting $(1-\beta)V(0)$ by its expression given in Equation (48) (recall that we consider the limit case in which β goes to 1), we rewrite the previous equation as

$$E\bar{U}^c - E\bar{U}^{nc} \geq \frac{\gamma(1-s^c)Q}{1+s^c}\left(\frac{(1-\alpha}{2} + \frac{S^*}{2}\right). \tag{50}$$

Since $\Delta U(s^c) = E\bar{U}^c - E\bar{U}^{nc}$ (by definition), the result is proved. Recall that $E\bar{U}^c = (1-\alpha)\bar{U}^c_{he} + \alpha\mu\bar{U}^{nc}_{in}$ and $E\bar{U}^{nc} = (1-\alpha)\bar{U}^{nc}_{he} + \alpha\mu\bar{U}^{nc}_{in}$. Hence $\Delta U(s^c) = (1-\alpha)(\bar{U}^c_{he} - \bar{U}^{nc}_{he})$. Using the expressions for $\bar{U}^c_{he}$ and $\bar{U}^{nc}_{he}$ (given by Equations (28), (29) and (33)), straightforward computations show that $\Delta U(s^c)$ is as given in Equation (13). ∎

Proof of Proposition 3. The first claim follows from the discussion which precedes the proposition. In order to prove the second claim, we define

$$G(s^c, \Lambda) \stackrel{def}{=} \Delta U(s^c) - \frac{(1-s^c)\gamma Q}{1+s^c}\left(Prob(\Delta S = +1) + \Lambda\right). \tag{51}$$

Condition (14) is $G(s^c, \Lambda) \geq 0$. Using Equation (13) for $\Delta U(s^c)$, we obtain that $G(s^c, \Lambda)$ decreases with s^c. This shows that the proposition. ∎

Proof of Corollary 2. Given that $G(s^c, \Lambda)$ (given by Equation (51) decreases with s^c, the maximal spread $s^*(\Lambda)$ such that $G(s^c, \Lambda) \geq 0$ satisfies $G(s^*(\Lambda), \Lambda) = 0$. As G decreases with Λ as well, $s^*(\Lambda)$ decreases with Λ. ∎

Proof of Proposition 4. There exists s^c such that when the dealer follows a policy $p^{lt}(T, S^*, s^c)$ then the trading policy $q^{lt}(T, S^*, s^c)$ is optimal if and only if there are s^c, T and S^* satisfying Condition (14), that is $G(s^c, \frac{S^*}{T}) \geq 0$ ($G(.,.)$ is defined in Equation (51). As G decreases with s^c and $\frac{S^*}{T}$, this is equivalent to $G(0, 0) \geq 0$. We distinguish between two cases.

Case 1:$s^{nc} < \bar{s}$. Using Equation (13), we obtain

$$G(0,0) = \frac{s^{nc}(1-\alpha)Q}{1+s^{nc}} - \gamma Q Prob(\Delta S = +1). \tag{52}$$

Using the expressions of s^{nc} and $Prob(\Delta S = +1)$, $G(0,0) \geq 0$ is equivalent to $\mu \leq \mu^c$.

Case 2:$s^{nc} \geq \bar{s}$**.** Using Equation (13), we obtain

$$G(0,0) = \frac{\bar{s}(1-\alpha)Q}{1+\bar{s}} - \gamma Q Prob(\Delta S = +1). \tag{53}$$

Using the expressions of $\bar{s}$ and $Prob(\Delta S = +1)$, $G(0,0) \geq 0$ is equivalent to $\gamma \leq 1/2$.

Recall that $s^{nc} \geq \bar{s}$ is equivalent to $\mu \leq \mu^{nc}$ (see the proof of Corollary 1). Thus the previous analysis (Cases 1 and 2) show that the cooperative trading policy is optimal if and only if one of the two following properties holds: (i) $\mu^{nc} < \mu$ and $\mu \leq \mu^c$; or (ii) $\mu \leq \mu^{nc}$ and $\gamma \leq 1/2$**.** Some computations show that $\mu^{nc} \leq \mu^c$ is equivalent to $\gamma \leq 1/2$. Hence, (i) is equivalent to $\mu^{nc} < \mu \leq \mu^c$ and $\gamma \leq 1/2$. It follows that Property (i) or Property (ii) are equivalent to $\mu \leq \mu^c$ and $\gamma \leq 1/2$**.** ∎

Proof of Lemma 5. The investor's trading policy is Markovian (it is determined by S_t which follows a Markov chain). Consequently the dealer's equilibrium strategy is Markovian and determined by S_t as well.[26] Therefore, according to the Optimality Principle of Dynamic Programming, it is sufficient to check that, for each value of the score, no one-shot deviation is profitable. For $S < S^*$, we have shown in the text that the dealer should not charge a spread different from s^c for an order of size on the equilibrium path. For trade sizes out-of-the equilibrium path, the regular dealers' beliefs ϕ^c can be chosen arbitrarily. Here the beliefs are specified in such a way that the dealer assigns a probability 1 to the event $\{\epsilon = 1\}$ (resp. $\{\epsilon = -1\}$) when he receives a buy (resp. sell) order. The same reasoning yields the dealer's pricing strategy when $S \geq S^*$ (in this case any order is out-of-the equilibrium path since the investor does not contact the dealer during non-cooperative phases).∎

Proof of Proposition 5. A necessary and sufficient for the existence of a cooperative equilibrium is that the no-informed trading condition is satisfied for $s^c > 0$ and $\Lambda > 0$ (that is $T < \infty$) This claim immediately follows from Lemma 4 and 5. The no-informed trading condition is satisfied for $s^c > 0$ and $\Lambda > 0$ if and only if $G(0,0) > 0$ (G is given by Equation (51)). Actually if $G(0,0) = 0$ then the no-informed trading condition cannot be satisfied for $s^c > 0$ and $\Lambda > 0$ since G decreases with s^c and Λ. It follows from the proof of Proposition 4 that $G(0,0) > 0$ is equivalent to $\gamma < \frac{1}{2}$ and $\mu < \mu^c$.∎

[26]The trading history does not provide information on the investor's type since this type is not persistent over time.

Proof of Propositions 6 and 7. Immediate from the arguments in the text. ∎

Proof of Proposition 11. For $\Psi = 0$, a market breakdown occurs when $\mu < \mu^{nc}(\alpha, \gamma)$ (see Corollary 1). For every Ψ and for $\gamma < \frac{1}{2}$, a cooperative equilibrium exists if $\mu < \mu^{c}(\alpha, \gamma)$ (see Proposition 5). As $\mu^{nc} < \mu^{c}$ for $\gamma < \frac{1}{2}$ (see proof of Proposition 4), the result follows. ∎

Figure 1: Sequence of Events

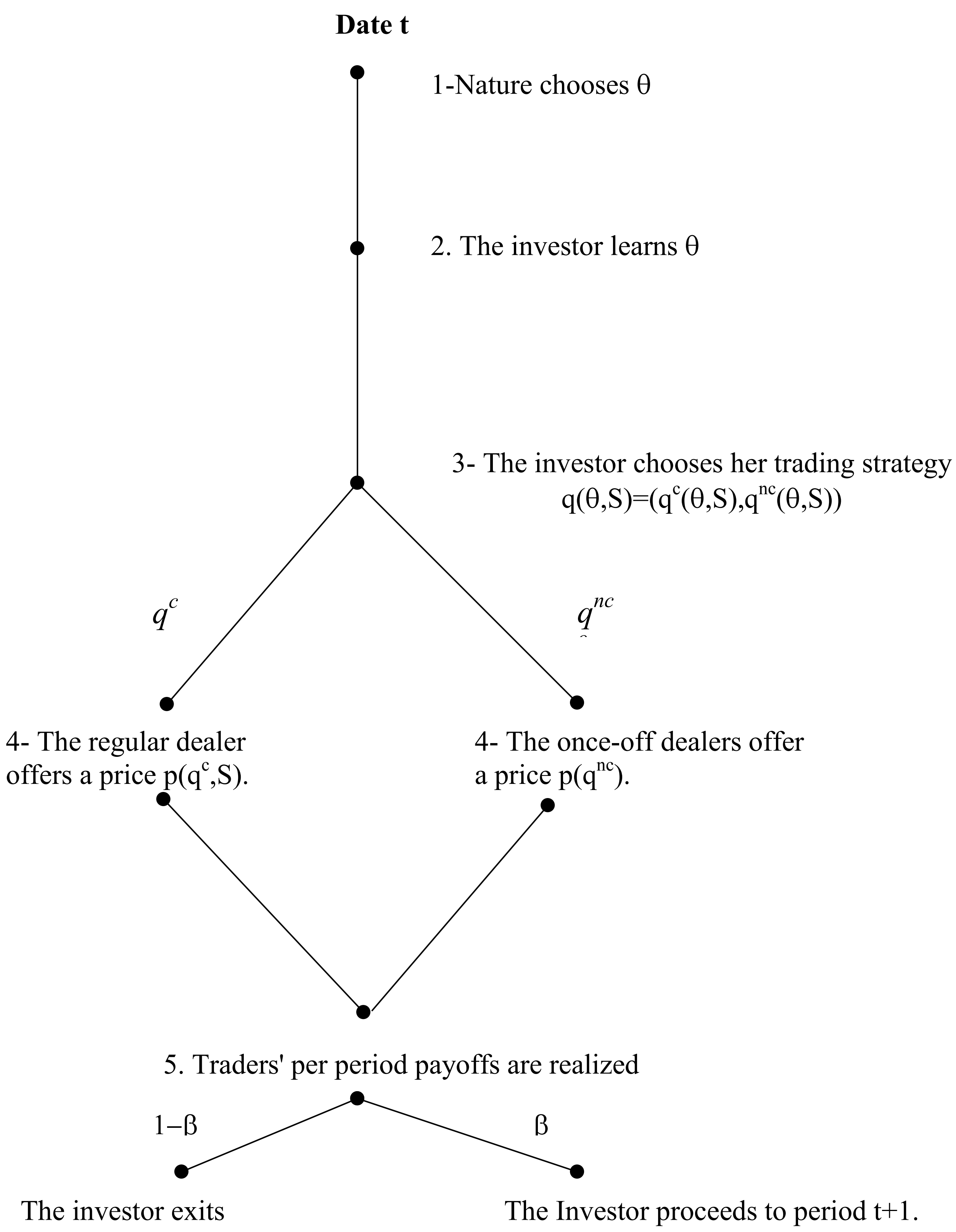

Figure 2: The Hedging Demand

Figure 2a)

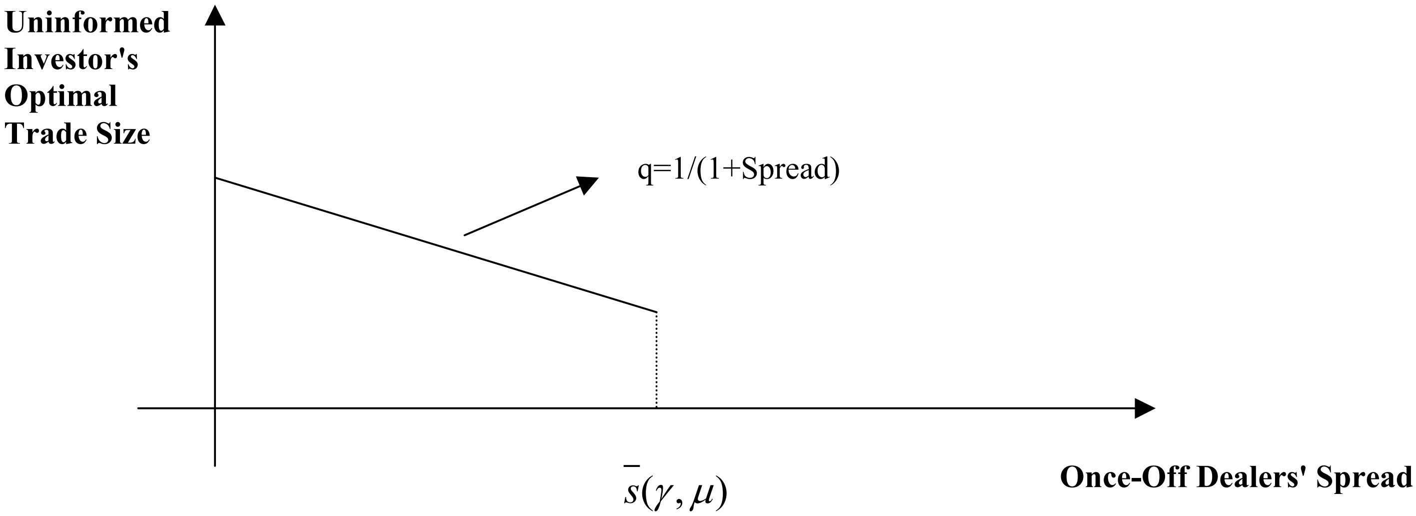

Figure 2a: Investor's optimal trade size (in absolute value) when she is uninformed as a function of the once-off dealers' spread.

Figure 2b)

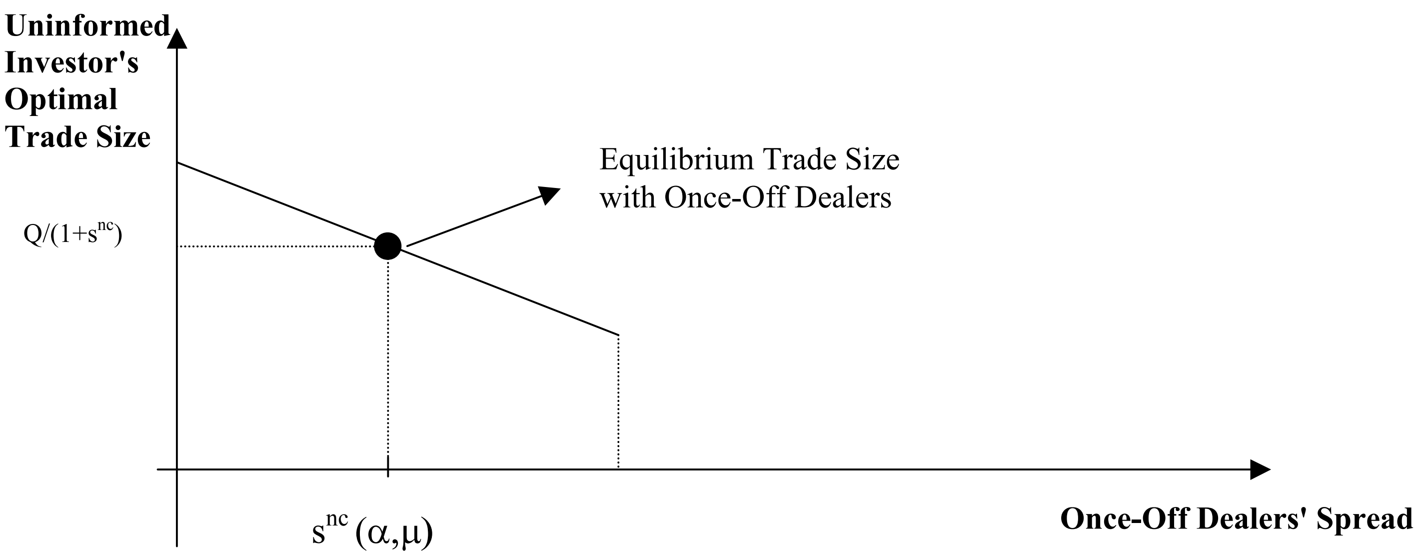

Figure 3

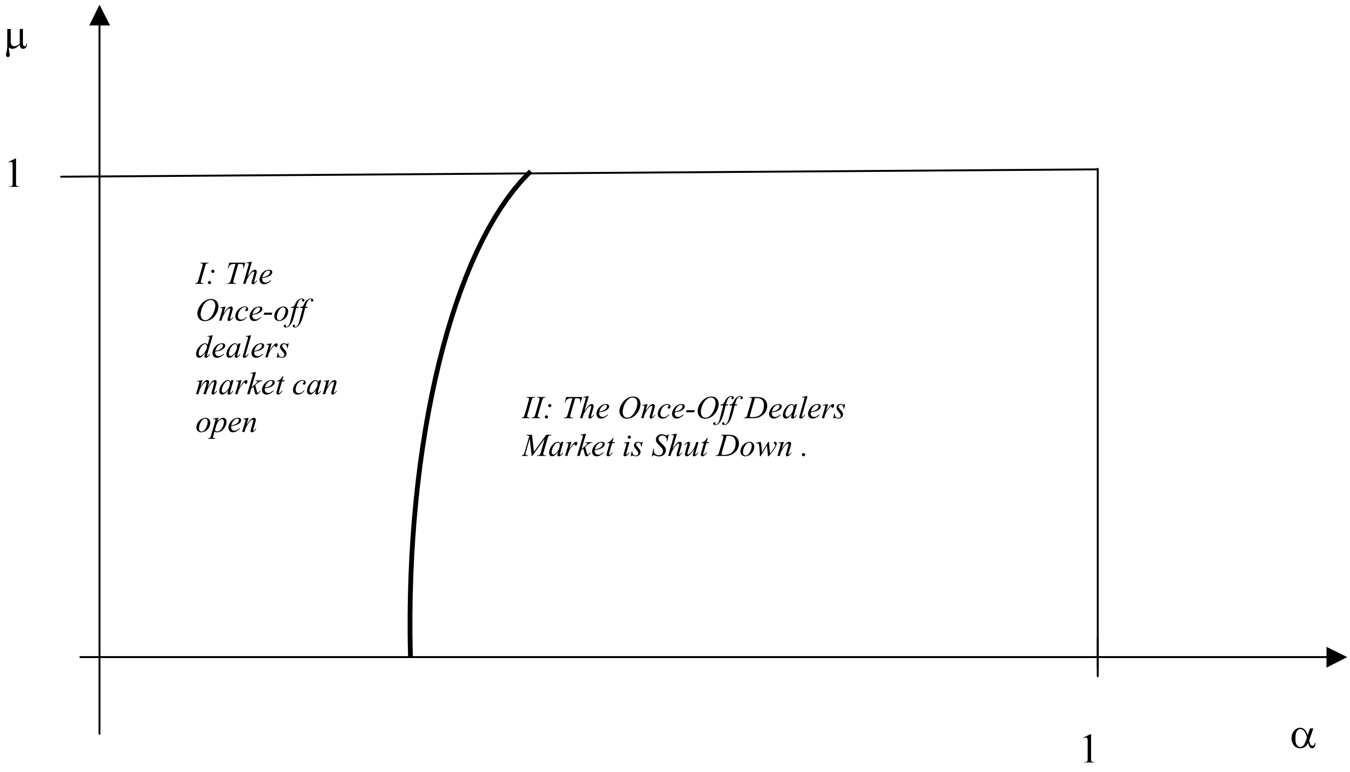

Figure 3: The curve which separates the two areas represents the function $\mu^{nc}(\gamma,\alpha)$.

Figure 4

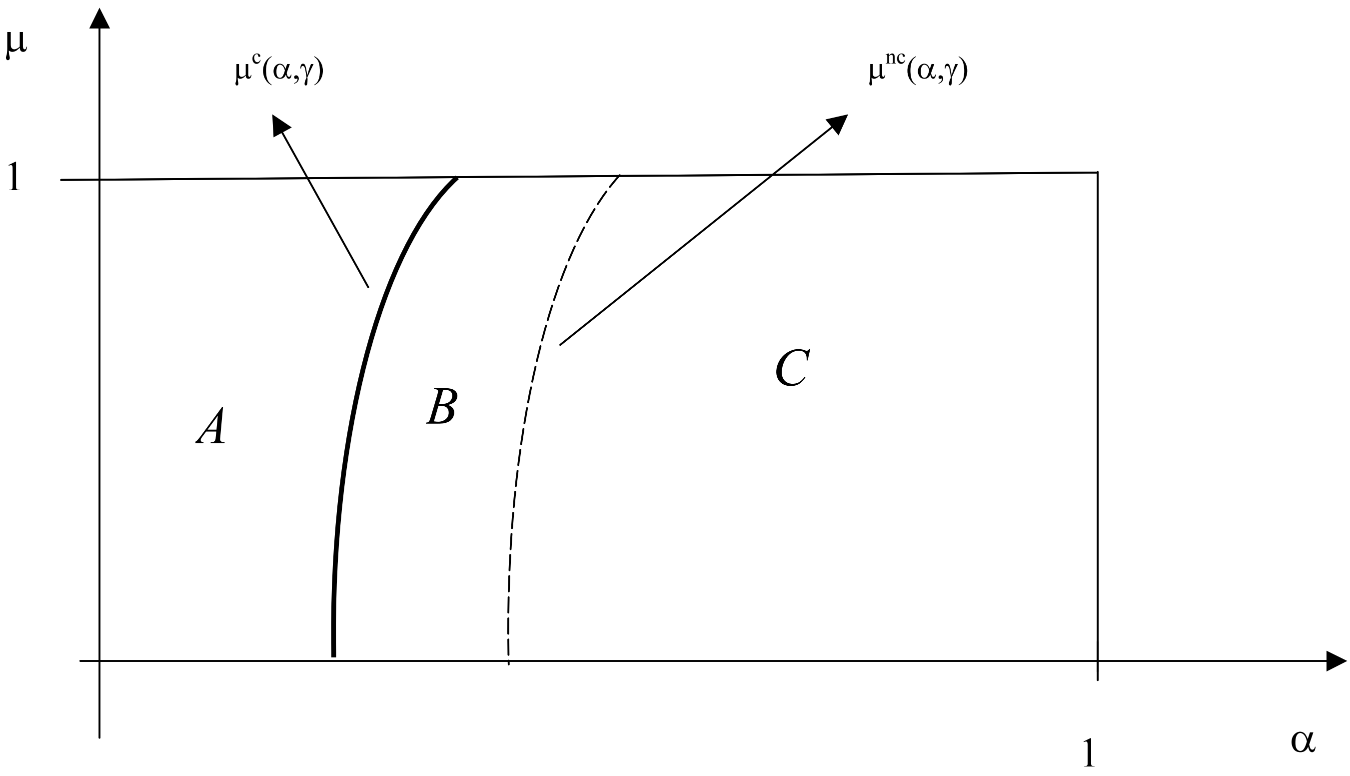

Figure 4: In Region A, it is impossible to design a scoring policy which induces the investor to honor the no-informed trading agreement. In equilibrium, only once-off dealers are active. In Region B and C, there are scoring policies which induce the investor to honor the agreement. Hence cooperative equilibria exist. In Region B, the once-off dealers and the relationship dealer co-exist. In Region C, the once-off dealers' market cannot open and the relationship dealer is the only active dealer. When the investor's risk aversion increases (γ decreases), Region B enlarges (the plain curve shifts to the left and the dotted curve shifts to the right).

Figure 5: Set of Equilibrium Values for s^c,T and S*
Numerical Example: α=0.5, μ=1, γ=0.2,Q=1