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## AGGREGATE RISK, POLITICAL CONSTRAINTS AND SOCIAL SECURITY DESIGN

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## ABSTRACT

### Aggregate Risk, Political Constraints and Social Security Design\*

In a stochastic environment, with political constraints, we analyse the behaviour of a fully funded system whose portfolio is composed of a risk free and a risky asset. When an aggregate negative shock hits, a large share of the wealth of the elderly is wiped out and office-seeking policy-makers 'bail them out,' by instituting a long-lasting PAYG system. Under these political constraints, a fully funded system suffers from a moral hazard problem, since agents have an incentive to choose a riskier portfolio, which increases the wealth loss associated with the bad state. The introduction of a mixed system reduces the riskiness of the portfolio, which remains however higher than in the case of no policy-maker's intervention. Furthermore, the early adoption of a mixed system, previous to the occurrence of a negative shock, eliminates the policy-maker's incentive to intervene, albeit at a high cost. In fact, its unfunded pillar would be larger than the PAYG system introduced in the case of a bad shock. In our dynamically efficient economy, this would amount to imposing an extra loss on all future generations.

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## 1. Introduction

The recent debate on the social security design has focused on the costs and benefits of switching from an unfunded (pay as you go, PAYG) to a fully funded system. Advocates of fully funded systems suggest that these schemes provide higher average return on contributions than unfunded schemes, while being less subject to government intervention (Feldstein, 1995 and 1998). Others researchers have argued that these higher returns come at the cost of exposing the agents' portfolio to a higher volatility, which can not be cheaply insured against (Miles and Timmermann, 1999), and that funded systems entail larger administrative costs (Diamond, 1998a, 1998b and 2000). Meanwhile, several scholars have promoted a double pillar, or mixed system, which integrates features of both schemes (Orszag and Stiglitz, 2001). This intermediate position has been advocated because of risk diversification considerations (World Bank 1994, Shiller, 1999), and of efficiency arguments (Boldrin, Dolado, Jimeno, Peracchi 1999).

At least as important as economic costs and benefits of different systems is the politics of social security design. Redistribution within and across generations sets important political constraints on the policymaking process, while the status quo plays a crucial role in the process of reform. In particular, a shift from existing PAYG systems to fully funded schemes entails a strong element of redistribution, since current young generations would be required to sustain – at least part of – the cost of the transition by contributing both to their pensions and to the pension of the current elderly. To cope with this redistributive component, advocates of fully funded schemes have to consider the politics of transition, specifically how to share this cost among successive generations to raise the support for the transition. Mixed systems have often been proposed as a way to increase the political support in favor of a reform (World Bank 1994), and these considerations have guided some of the latest Latin American reforms (Argentina, Mexico, Peru).

In this paper, we argue that political considerations are relevant also for fully funded systems. We analyze the behavior of a fully funded system in a stochastic environment in the presence of political constraints. In this stochastic environment, an aggregate negative shock to the stock market may occur, which wipes out a large share of the wealth of the elderly, thereby reducing their consumption. As Orszag and Stiglitz (2001, see myth 9), we recognize that, if such negative shock takes place, politicians may decide to “bail the elderly out”. In particular, we argue that (i) office-seeking policy-makers will introduce an intergenerational transfer from the young to the old, which increases the consumption of the elderly; and (ii) once this PAYG system has been adopted, it will be hard (namely impossible) to dismantle. Albeit stylized, the behavior of our policy-makers, which we refer to as our political constraints, is consistent with several historical episodes.

As suggest by Feldstein and Liebman (2001), for instance, the US social security system was initially instituted as an unfunded scheme and only later on was it converted to a PAYG<sup>1</sup>.

We characterize the portfolio composition of a fully funded system and the aggregate savings. Our economic setting is similar to Diamond and Geanakoplos (1999): the portfolio of our fully funded social security system is composed of a risk free asset – e.g., long term government bonds – and a risky asset – e.g., claims to capital – which pays out a high return in the good state of the world, and a low return if the negative shock occurs. However, while their agents are divided among savers and non-savers, whose portfolio choice is delegated to a benevolent policy-maker, all our agents save for old age consumption, and they perfectly forecast the future actions of the policy-makers. In this environment, we analyze how the portfolio composition of the fully funded system is affected by our political constraints. The existence of policy-makers, who bail the elderly out in the case of a negative shock, creates a moral hazard problem. Agents, who in their youth enjoy a good state of the world, are effectively awarded a free insurance, which provides them with a pension if a bad state occurs in their old age. Thus, they are induced to reduce their savings and to choose a riskier portfolio composition than they would in case of no policy-maker’s intervention. The latter effect increases the wealth loss associated with the bad state.

The introduction of a mixed system – partially PAYG and partially funded – mitigates, although it does not eliminate, this moral hazard problem. Agents are required to contribute to the unfunded pillar when young and receive an old age transfer from the young, regardless of the state of the world. This mandatory transfer of resources from youth into old age changes the agents’ net income profile, and thus affects their savings and portfolio composition. The riskiness of the portfolio is reduced, although it remains larger than in the case of no political constraints.

In our politico-economic environment, the bailout of the elderly by the policy-makers – in the case of a negative shock – imposes a cost on all future generations. In fact, in our dynamically efficient economy, an unfunded system constitutes an inefficient instrument to transfer resources into the future. Besides the temporary gains to the first generation to receive a pension, a PAYG system is dominated, in rate of return, by the risk-free asset.

To avoid this long run cost of the bailout, we examine whether the early adoption – before a negative shock hits – of a mixed social security system could prevent the policy-makers from intervening once the negative shock has occurred. In other words, we ask whether the early introduction of a system that – through the un-

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<sup>1</sup>Analogously, the Italian funded social security scheme was turned into a PAYG system after that post II world war inflation had wiped out most of the wealth of the public trust fund.

funded pillar – guarantees a certain level of resources, and thus of consumption, even in the bad state could relax the policy-makers incentive to bail the elderly out. In particular, we characterize the mixed system with the smallest unfunded pillar, which avoids the institution of a PAYG system by the policy-makers, when a negative shock occurs. The size of this minimum-unfunded-pillar mixed system is then compared to the size of the PAYG system that the policy-makers would put in place in case of an aggregate negative shock. Somewhat surprisingly, we find the unfunded pillar associated to this mixed system is indeed larger than the PAYG system introduced by the policy-maker's intervention. Therefore, under our political constraints, the early adoption of this mixed system would entail an even larger cost on all future generations than the policy-maker's intervention to bail the elderly out.

This paper is related to two different strands of the literature on social security design. The economic considerations are surveyed in Feldstein and Liebman (2001). Among other reasons, the existence of an unfunded pillar has been advocated because of intergenerational risk sharing (Demange 2001, Bohn 1998, Ball and Mankiw, 2001) in the absence of intergenerational markets. In fact, a benevolent government may influence the allocation of risk among generations by using different instruments such as general taxation, public debt, or the unfunded pillar of the social security system. Analogously, if agents are myopic, a benevolent government will institute an unfunded system to guarantee them some old age consumption (see Diamond and Geanakoplos, 1999). Compared to this normative contributions, our paper is more positive in spirit. Several authors, on the other hand, have analyzed the political sustainability of the PAYG system (see Azariadis and Galasso, 2002, Boldrin and Rustichini, 2000, Cooley and Soares, 1998, Galasso, 1999, Mulligan and Sala-i-Martin, 1999), as well as the political sustainability of the reform of the existing PAYG systems (see Conesa and Kruger, 1999, Cooley and Soares, 1999). Less research has been devoted to how (partially or fully) funded system are robust to political manipulations<sup>2</sup>, which is instead the focus of our paper.

The paper proceeds as follows: Section 2 introduces the model and the political constraints. Section 3 examines the moral hazard problem in a fully funded and in mixed system. Section 4 analyses the minimum-unfunded-pillar mixed system, and compares it to the case of policy-maker's intervention. Section 5 concludes. All proofs are in the appendix.

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<sup>2</sup>For a survey of political economic models of social security see Galasso and Profeta (2002).

## 2. The Model

We consider a two period overlapping generations economy. Every period two generations are alive, we call them young and old. Population grows at a constant rate  $\mu$ . Agents born at time  $t$  receive an endowment,  $y_t > 0$ , in youth only. Endowments increase over time at a rate  $\lambda$ :  $y_t = (1 + \lambda) y_{t-1}$ . Agents evaluate their young and old age consumption according to a time separable utility function:  $u(c_t^t, c_{t+1}^t) = U(c_t^t) + \beta E_t U(c_{t+1}^t)$ , where  $U'(\cdot) > 0$ ,  $U''(\cdot) < 0$ , and  $\beta$  is the individual time discount rate. Additionally, we assume that the utility function is homothetic:

**Assumption 1**  $U'(\cdot)$  is homogeneous of degree  $\alpha \in \mathfrak{R}_+$ , and thus  $U(\cdot)$  is homothetic.

In this economy there exist two assets: a risk free asset,  $b$ , which pays a real interest rate  $r$  regardless of the state of the world, and a risky asset,  $k$ , which pays a real return  $R_i$ ,  $i = B, G$ , with  $R_G = \bar{R}$ , in the good state of the world, and  $R_B = \underline{R}$ , in the bad state. Every period, nature chooses the state of the world, which is good with probability  $p$ , and bad with probability  $1 - p$ . Returns are exogenous in our small open economy, and we assume that the risky asset pays more than the risk free asset in the good state, and less in the bad one.

**Assumption 2**  $\bar{R} > r > \underline{R}$ .

A fully funded social security system transfers the agent's resources from youth into old age, through an individual fund that is composed of the two available assets ( $b, k$ ). To simplify the analysis, we do not consider the optimal portfolio strategy by a more or less centralized trust fund. We analyze a completely decentralized funded system, in which the composition of the portfolio is unrestricted: agents may choose their most preferred portfolio. Thus, in our environment, a fully funded system is completely equivalent to private savings.

The optimization problem of a young agent at time  $t - 1$  consists of choosing the portfolio composition,  $(b_{t-1}, k_{t-1})$ , which maximizes her expected utility:

$$\underset{\{b_{t-1}, k_{t-1}\}}{\text{Max}} U(c_{t-1}^{t-1}) + \beta \{pU(c_{t,G}^{t-1}) + (1-p)U(c_{t,B}^{t-1})\} \quad (2.1)$$

subject to the budget constraints:

$$\begin{aligned} c_{t-1}^{t-1} &= y_{t-1}(1 - \tau) - b_{t-1} - k_{t-1} \\ c_{t,G}^{t-1} &= b_{t-1}(1 + r) + k_{t-1}(1 + \bar{R}) + P_{t,G} \\ c_{t,B}^{t-1} &= b_{t-1}(1 + r) + k_{t-1}(1 + \underline{R}) + P_{t,B} \end{aligned} \quad (2.2)$$

and to the non-negativity constraints:

$$b_{t-1} \geq 0, \text{ and } k_{t-1} \geq 0 \quad (2.3)$$

where  $c_{t,B}^{t-1}$  and  $c_{t,G}^{t-1}$  represent the old age consumption of a person born at time  $t - 1$  respectively in the bad and in the good state of the world, and  $\tau$  is the tax rate on the youth endowment, which would finance a PAYG system.  $P_{t,G}$  and  $P_{t,B}$  represent the pension received by the elderly in each state of the world at time  $t$  according to the social security. This maximization problem yields the following solution:

$$\frac{U'(c_{t,B}^{t-1})}{U'(c_{t,G}^{t-1})} = \frac{p}{1-p} \frac{\bar{R} - r}{r - \underline{R}} \quad (2.4)$$

which implicitly defines the optimal portfolio composition.

Our benchmark portfolio  $(b_{t-1}^*, k_{t-1}^*)$  refers to the case of a fully funded social security system, in which there is no policy-maker's intervention, even in a bad state of the world, that is  $P_{t,G} = P_{t,B} = 0$ . We call  $[b^*, k^*]$  the no policy-maker's intervention portfolio. In fact, as it will become clear in the next section, the policy-makers may have an incentive to intervene, if a bad state of nature occurs.

## 2.1. The Political Constraints

The behavior of any social security system – fully funded, PAYG or mixed – is subject to the actions of the policy-makers. In our model, policy-makers are office-seeking politicians, who set the current policy in order to win the election. We consider an economy in which there initially exists a fully funded system. Policy-makers may intervene in this economy and institute a PAYG social security system, which imposes a non-negative proportional tax rate,  $\tau_t$ , on the young's endowment,  $y_t$ , and transfers the revenue lump-sum,  $P_t = (1 + \mu) \tau_t y_t$ , to the old.

Our first political constraint describes the behavior of the policy-maker when a PAYG social security system has already been adopted. After a policy-maker has instituted a PAYG system, future politicians are to follow the same policy, and this scheme will never be scaled down in all future periods:

**Assumption 3**  $\tau_{T+s} \geq \tau_T$  if  $\tau_T > 0 \forall s > 0$ .

This assumption is meant to capture the strong persistence embodied in the PAYG social security system and in particular the large political success of a minority of voters, the elderly (see Mulling and Sala-i-Martin, 1999). In fact, although they do not constitute a majority of the voting population, the elderly seem to have enough political power to block policy reform that they do not endorse (see Esteban and Sakovics, 1993, and Azariadis and Galasso, 2002). To

summarize, assumption 3 constitutes our political constraint when a PAYG system already exists.

To model the behavior of the policy-makers, who is to decide when a fully funded system is in place, we use a probabilistic voting model (see Coughlin, 1992, and Persson and Tabellini, 2000). There exist two political candidates, who compete in a majoritarian election. Each candidate determines her political platform, which is represented by the size of the PAYG system,  $\tau$ , in order to maximize her probability of winning the election. The candidate who wins the election becomes the policy-maker, and implements the proposed policy. Elections take place every period, after the realization of the stochastic return. Therefore, the political candidates only need to choose the tax rate corresponding to the realized state of the world<sup>3</sup>. Every agent takes her voting decision according to the indirect utility associated to each candidate's platform, and to her degree of political ideology. Within any age group – young and old – all young individuals share the same economic preferences, and thus are equally affected by the candidates' platforms. The political ideology, on the other hand, has a common and an individual specific component. For lack of better assessment, we assume that the two groups, young and old, have the same uniform distribution of ideology across agents. This amounts to assume that, for each type ideology, there is the same mass of voters among the old as among the young.

It is straightforward to show (see Persson and Tabellini, 2000) that the two political candidates face the same optimization problem, and thus choose the same platform, i.e., the same tax rate,  $\tau$ . In this setting, maximizing the probability of winning the election at time  $t$ , when a fully funded system is in place, is equivalent to maximizing the following expression, which may be interpreted as the welfare function of the policy-maker at time  $t$ :

$$W_t = \frac{\Psi}{1 + \mu} U(c_t^{t-1}) + \Psi U(c_t^t) + \Psi \beta \{pU(c_{t+1,G}^t) + (1 - p)U(c_{t+1,B}^t)\} \quad (2.5)$$

where  $\Psi$  represents the density of the uniform ideology distribution function in the two groups. Notice that by imposing  $\Psi$  to be equal across groups, we are assuming that both groups have the same number of non-ideological, or swing voters, and therefore that they are equally important to the candidates.

Under the hypothesis that once introduced a PAYG cannot be abandoned (assumption 3), the optimization problem of a policy-maker at time  $t$  yields the following first order condition:

$$F(\tau|R_i) = y_t U'(c_i^{t-1}|R_i) - y_t U'(c_i^t) + \quad (2.6)$$

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<sup>3</sup>Notice that the results do not depend on the timing. If the elections were to take place before the realization, the candidates would simply have to indicate a tax rate for each possible state of the world.

$$+\beta(1+\mu)y_{t+1}\{pU(c_{t+1,G}^t)+(1-p)U(c_{t+1,B}^t)\}\leq 0.$$

The first term represents the marginal utility of increasing the consumption of the current old, while the remaining two terms define the expected marginal disutility to the current young of imposing a PAYG system.

To characterize the behavior of the policy-maker when a fully funded system is in place, and thus to obtain our second political constraint, we introduce the following assumption on the probability of having a good state:

**Assumption 4**  $p > \text{Max} \left\{ \frac{r-\underline{R}}{\bar{R}-\underline{R}} \frac{1}{\beta(1+\lambda)^\alpha(r-g)}, 1 - \frac{\bar{R}-r}{\beta(\bar{R}-\underline{R})(1+\lambda)^\alpha(r-g)} \right\}.$

where  $1+g = (1+\mu)(1+\lambda)$ . This assumption guarantees that a policy-maker, who inherits a fully funded system, will not introduce a PAYG system if a good state of the world occurs. However, the policy-maker will intervene to bail the elderly out, and to introduce a PAYG system, if the bad state occurs. This constitutes our second political constraint, which is formalized in the next proposition.

**Proposition 2.1.** *For  $\tau_{t-1} = 0$ , if assumption 4 holds then  $\tau_t = 0$  (i.e., the fully funded system is maintained) at  $R_t = \bar{R}$ , and  $\tau_t > 0$  (i.e., a PAYG system is introduced) at  $R_t = \underline{R}$ .*

It is important to notice that a necessary condition for assumption 4 to hold is that, at steady state, the return on the risk free asset is larger than the return on the PAYG system:

$$r > g. \tag{2.7}$$

This amounts to impose that in our economy the PAYG system is dominated, in rate of returns, by the risk free asset, and thus that our economy is dynamically efficient. Every young agent would prefer to transfer resources into the future with the risk-free asset, rather than with the PAYG system. The introduction of a PAYG system therefore represents a cost to them, although it entails a temporary gain to the initial generation of old, who receives a pension without having contributed to the system. The previous assumption guarantees that the consumption of the elderly in the bad state – but not in the good one – is low enough to induce the policy-makers to be willing to bail them out by introducing a PAYG system, despite its cost on the current (and future) young generation. In the numerical example provided in the next section, we shall argue that this assumption is satisfied for plausible values of the rate of return and parametrization of the utility function.

To summarize, we have introduced two political constraints, which completely characterize the behavior of the office-seeking politicians, for any current realization of the state of the world, and any history of past policies. Specifically, if there has always been a fully funded system, the policy-maker will keep this system if the current state of the world is positive, whereas it will introduce a PAYG scheme if the current state is negative. Once the PAYG has been adopted, however, future policy-makers will always support this system.

### 3. The Moral Hazard Problem

Consider an economy with a history of good realizations of the state of nature, in which a fully funded system is in place. Each successive generation of agents understands the existence of the political constraints described in the previous section. In other words, they know that, until an aggregate negative shock occurs, the fully funded system will be untouched. When the bad state of the world takes place, however, the policy-maker will intervene to institute a PAYG system, which will never be scaled down.

Agents anticipate the policy-maker's behavior, and adjust their savings decisions accordingly. At time  $t - 1$ , with a fully funded system in place,  $\tau_{t-1} = 0$ , young agents choose the savings and the portfolio composition that maximizes the expected utility at eq. 2.1 subject to the budget constraints at eq. 2.2 and to the non-negativity constraints at eq. 2.3. Our political constraints imply that  $P_t^G = 0$  and  $P_t^B = \tau_t y_t (1 + \mu) > 0$ . We indicate the optimal portfolio under policy-maker's intervention with  $[b_{t-1}^H(\tau), k_{t-1}^H(\tau)]$ .

For the current young, who do not contribute to the system, the old age transfer that they receive in the case of a bad shock constitutes a free insurance and increases their expected lifetime endowment. As the next proposition shows, this induces them to change their savings and their optimal portfolio composition by over-investing in risky assets.

**Proposition 3.1.** *Let  $(b^*, k^*)$  be the portfolio composition in the case of no policy-maker's intervention, and let  $[b^H(\tau), k^H(\tau)]$  be the portfolio composition under policy-maker's intervention. If  $\tau > 0$ , then (i)  $b^*/k^* > b^H(\tau)/k^H(\tau)$ , and (ii)  $b^* + k^* > b^H(\tau) + k^H(\tau)$ .*

The existence of a policy-maker, who is willing to institute a PAYG system in order to bail the elderly out in case of a negative aggregate shock hits the economy, has a cost. Agents are induced to hold a riskier portfolio, which in turn increases the wealth loss that takes place in a bad state. Moreover, the policy-maker's intervention raises the agents expected lifetime endowment by awarding a

transfer in a bad state. Thus, in order to smooth consumption, agents will reduce their savings with respect to the case of no policy-maker's intervention.

We now turn to the analysis of a social security system, which features two pillars – an unfunded and a funded one – to investigate whether its introduction may mitigate this moral hazard problem. In other words, we study to what extent this mixed system may induce the agents to decrease the holdings of risky assets in their portfolio.

The idea is simple. A mixed system is composed of an unfunded pillar – to which agents have to contribute to the system in their youth and from which they receive a constant pension benefit regardless of the state of the world – and a funded pillar – which we analyzed above, and which is subject to shocks to asset returns. The unfunded pillar of this scheme modifies the endowment profile of the agents. In particular, as compared to the fully funded system, it decreases the net endowment in youth, because it requires a contribution, and tends to equalize the wealth across states in old age, through the constant transfer. The portfolio decision is adjusted accordingly: every agent reduces her savings, and reshuffles her portfolio composition.

Let us now describe the agent's portfolio decisions. In a mixed social security system, agents face a mandatory young age contribution to the unfunded pillar, which takes the form of a proportional tax,  $\tau$ , levied on their endowment; and they receive a lump-sum old age transfer,  $P$ , regardless of the state of the world. We consider the unfunded pillar to be budget balanced. In every period, the amount of resource transferred to the retirees equals the revenues:  $P_t = (1 + \mu) \tau y_t$ . The agents' residual savings, after the social security contribution has been paid, are accumulated in a portfolio composed of the risk free and the risky asset. The size and the composition of this portfolio is determined by the young agents who maximize their expected utility at eq. 2.1 subject to the non-negativity constraint at eq. 2.3, and to the following budget constraints:

$$\begin{aligned} c_{t-1}^{t-1} &= y_{t-1} (1 - \tau) - b_{t-1} - k_{t-1} \\ c_{t,G}^{t-1} &= b_{t-1} (1 + r) + k_{t-1} (1 + \overline{R}) + P_t \\ c_{t,B}^{t-1} &= b_{t-1} (1 + r) + k_{t-1} (1 + \underline{R}) + P_t \end{aligned} \tag{3.1}$$

where  $P_t = \tau_t y_t (1 + \mu) > 0$  is the old age pension. We indicate the optimal portfolio which solves this maximization problem with  $[b^M(\tau), k^M(\tau)]$ .

The next proposition compares the moral hazard problem, which arises in a fully funded and in a mixed social security system, to the case of no policy-maker's intervention.

**Proposition 3.2.** *Let  $(b^*, k^*)$  be the optimal portfolio composition in the case of no policy-maker's intervention,  $[b^H(\tau), k^H(\tau)]$  be the optimal portfolio composition under policy-maker's intervention, and  $[b^M(\tau), k^M(\tau)]$  be the optimal*

portfolio in a mixed system. If  $\tau > 0$ , then (i)  $\frac{b^*}{k^*} > \frac{b^M(\tau)}{k^M(\tau)} > \frac{b^H(\tau)}{k^H(\tau)}$ ,  $\forall \tau \in \Upsilon$ , where  $\Upsilon = \{\tau | \tau > 0, b^M(\tau) > 0\}$ , and (ii)  $b^* + k^* > b^H(\tau) + k^H(\tau) > b^M(\tau) + k^M(\tau)$ .

This proposition suggests that a mixed system can indeed mitigate the moral hazard problem which arises in the fully funded case. In fact, it induces a reshuffling of the portfolio that reduces its riskiness, although this remains larger than in the case of no policy-maker's intervention.

The intuition is straightforward. When compared to a fully funded system with policy-maker's intervention, a mixed system equalizes the pension transfers across the two states, and thus increases the resources available to the old in the good state of the world. Agents optimally adjust their portfolio, and move more consumption towards the bad state, by substituting the risky asset with the risk free asset. At the same time, a mixed system reduces savings, since it decreases the expected lifetime endowment, due to the use of a dominated unfunded pillar, and increases the steepness of the net endowment profile.

When compared to the case of no policy-maker's intervention, a mixed system adds resources equally in both states of the world. For our homothetic utility function, the effect on the marginal utility is larger in the bad state than in the good one. In other words, these transfers are more beneficial in the bad state, when there are less resources, than in the good state, when resources are more abundant. Therefore, the agents modify their portfolio to compensate this effect, and move more consumption towards the good state, by substituting the risk free with the risky asset<sup>4</sup>. Once more, savings are lower under the mixed system than under no policy-maker's intervention, because of the reduced expected lifetime endowment and of the increased steepness of the net endowment profile.

It is interesting to notice that the same qualitative results also apply to the generation that benefits from a mixed system without having contributed in their youth. If these agents expected a mixed system to be introduced in the future, they would have chosen a less risky portfolio composition than in the case of policy-maker's intervention, albeit more risky than our benchmark case of no policy-maker's intervention.

Finally, to obtain a better grasp of these results, we turn to a simple numerical example.

### 3.1. A Numerical Example

To parametrize our simple model, we take one period to be 35 years, and consider a CES utility function:  $U(c) = c^{1-\gamma} / (1 - \gamma)$ , so that the degree of homogeneity

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<sup>4</sup>See Aura, Diamond and Geanakoplos (2002) for a general analysis of savings and portfolio decisions in a similar economic environment.

$\alpha$  equals  $-\gamma$ . The coefficient of risk aversion  $\gamma$  is set equal to 2.5, and the youth endowment,  $y$ , to 1. The annual rate of returns are 3.5% for the risk free asset and 1% and 7% – respectively in the bad and good state – for the risky asset. Population grows at an annual rate of 1%, while labor productivity grows at 1.5% a year. Finally, the probability that a good state occurs is equal to 85%. Under this parametrization, assumption 4 is satisfied, since  $p = 0.85 > \text{Max}\{0.8, -2\}$ .

In the case of no policy-maker’s intervention, agents save 27% of their youth endowment. Their portfolio is composed for 42% of risk free asset and for 58% of risky assets and thus  $b/k = 0.72$ . If agents expect the policy-maker to intervene to bail the elderly out in the case of a negative shock – in particular, by awarding a pension that is financed through a 10% social security tax rate – they will reduce their savings to 23% of their youth endowment. Moreover, they will increase the riskiness of their portfolio by holding only 16% of safe assets, and 84% of risky assets, thus yielding  $b/k = 0.19$ . In a mixed system – financed through a 10% tax rate – the savings equal 19% of the gross youth endowment, but the riskiness of the portfolio is slightly reduced, with 20% in safe assets and 80% of risky assets.

#### 4. Can Political Bailouts be Avoided?

Several authors (World Bank, ILO) have argued that the adoption of a mixed (double pillar) system composed of a funded and an unfunded scheme may provide some diversification of the risks that threaten each pillar. In particular, the returns from an unfunded scheme are often associated with demographic (e.g., a decrease in the population or labor force growth rate), economic (e.g., a reduction in the wage growth rate) and political risks (e.g., changes to the tax or benefit structure of the system). The main risk associated to a funded system is instead represented by the aggregate negative shocks to the returns of the assets that compose the portfolio of the funded system, and by the political responses that these shocks may trigger.

In our economy, however, the PAYG scheme faces no demographic, economic or political uncertainty, and thus yields a certain return; whereas the funded system is subject to an aggregate risk to the stock market, thereby providing a stochastic return, and to the political constraints. Moreover, at steady state, the return from the risk free asset is assumed to be larger than the return from the PAYG system (eq. 2.7). Thus, in our model, an unfunded pillar is a dominated asset that does not provide any risk diversification.

In our politico-economic environment, however, a mixed system may still play an important role. Since the bailout of the elderly by the policy-makers – in the case of a negative shock – imposes a cost on all future generations, besides the windfall gains to the initial elderly who receive the transfer, we examine

whether the early institution of a mixed system, previous to the occurrence of a negative shock, may prevent future policy-makers from intervening. Clearly, if a large unfunded pillar is already in place, so that the savings are almost entirely achieved through this intergenerational transfer scheme, no policy-maker would have an incentive to intervene, under our political constraints, even in a bad state of nature. In fact, the negative shock would have no impact on the wealth of the elderly, and thus on their consumption. Because of the cost to future generations from the unfunded scheme, we choose to concentrate on the smallest size of the unfunded pillar of a mixed system, which is needed to induce all the future policy-makers, who face a negative shock, not to intervene to bail the elderly out.

Consider an economy at time  $t$  with a history of good realizations of the state of nature, in which a fully funded system is in place, and call  $T > t$  the period when the first realization of the shock occurs<sup>5</sup>. We characterize the early adoption at time  $t < T$  – in a good state of the world – of the mixed system that features the smallest unfunded pillar to induce the policy-maker at time  $T$ , i.e., when the first negative shock hits, not to intervene, for instance to increase the unfunded pillar.

**Definition 4.1.** *Let  $\hat{\tau}$  be the payroll tax rate such that*

$$U' (c_T^{T-1}(\hat{\tau}) \mid R_T = \underline{R}) = \frac{r-g}{1+r} \beta U' (c_T^T(\hat{\tau})) \quad (4.1)$$

where  $c_T^{T-1}(\hat{\tau}) = b_{T-1}^M(\hat{\tau})(1+r) + k_{T-1}^M(\hat{\tau})(1+\underline{R}) + (1+\mu)\hat{\tau}y_T$ , and  $c_T^T(\hat{\tau}) = y_T(1-\hat{\tau}) - b_T^M(\hat{\tau}) - k_T^M(\hat{\tau})$ . The tax rate  $\hat{\tau}$  characterizes the size of the minimum-unfunded-pillar mixed system.

In words, the minimum-unfunded-pillar mixed system,  $\hat{\tau}$ , represents the tax rate that, at time  $T$ , i.e., at the first occurrence of a bad state of nature, equates the marginal utility to the elderly of increasing their consumption to the expected marginal disutility to the young of paying a contribution at time  $T$  and receiving a benefit at  $T+1$ . Notice that agents born at  $T-1$  contributed to the system in their youth, correctly anticipated the old age transfer, and made their savings decisions accordingly. The optimal portfolio composition associated to the minimum-unfunded-pillar mixed system is implicitly defined by applying to the optimal portfolio  $(b^M(\tau), k^M(\tau))$  the tax rate,  $\hat{\tau}$ , derived above.

How large is the minimum unfunded pillar of a mixed system compared to the PAYG system introduced by a policy-maker in a bad state of the world? In other words, how large is the cost to all future generations – except the first old to receive a pension – of these two alternative scenarios?

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<sup>5</sup>Notice that, since the shock follows a Markov process,  $T$  is a stochastic variable, which is equal to  $t+1$  with probability  $1-\rho$ , to  $t+2$  with probability  $\rho(1-\rho)$ , and so on.

To provide a quantitative answer to these questions, we compare the minimum-unfunded-pillar tax rate,  $\hat{\tau}$ , to the tax rate,  $\bar{\tau}$ , that finances the PAYG system which a policy-maker would put in place to bail the elderly out, in the case of a negative aggregate shock.

**Definition 4.2.** *Let  $\bar{\tau}$  be the PAYG payroll tax rate such that*

$$U' (c_T^{T-1}(\bar{\tau}) \mid R_T = \underline{R}) = \frac{r-g}{1+r} \beta U' (c_T^T(\bar{\tau})) \quad (4.2)$$

where  $c_T^{T-1}(\bar{\tau}) = b_{T-1}^H(\bar{\tau})(1+r) + k_{T-1}^H(\bar{\tau})(1+\underline{R}) + (1+\mu)\bar{\tau}y_T$ , and  $c_T^T(\bar{\tau}) = y_T(1-\bar{\tau}) - b_T^M(\bar{\tau}) - k_T^M(\bar{\tau})$ . The tax rate  $\bar{\tau}$  characterizes the size of the PAYG system introduced by the policy-maker's intervention.

As in the previous case, the tax rate under policy-maker's intervention,  $\bar{\tau}$ , represents the tax rate that, in a bad state of nature, equates the marginal utility to the current old of increasing their consumption to the expected marginal disutility to the current young of paying the current contribution and of receiving the future benefits. Notice, however, that in this case the current old did not contribute to the system in their youth, at  $T-1$ , although they correctly anticipated that they would have received a transfer  $(1+\mu)\bar{\tau}y_T$ , had a bad state of the world occurred, and they took their savings decisions accordingly.

The next proposition compares these two tax rates, and thus provides an assessment of the cost to the future generations of these alternative systems.

**Proposition 4.3.** *If  $r < g$ , then  $\hat{\tau} > \bar{\tau}$ .*

This proposition suggests that, unless the return on the PAYG scheme pays a positive premium over the return of the riskless asset, which we implicitly ruled out by assumption 4, the minimum-unfunded-pillar of a mixed system needed to dissuade the policy-maker from intervening in the case of a negative shock is even larger than the PAYG system that the policy-maker would institute in a bad state. This represents a surprisingly, yet powerful result, since it rules out a possible rationale for introducing a mixed system. The idea of instituting an ex-ante mixed system to prevent the introduction of a full fledged PAYG system in the case of a negative shock is not viable. The early introduction of this mixed system turns out to be more costly to all future generations than the intervention it is aiming to avoid. The reason is twofold. On one hand, a mixed system, by taxing the young age endowment and awarding an old age pension, modifies the net endowment profile, and thus decreases the savings. On the other hand, the (partial) use of a dominated asset – the unfunded pillar – rather than the risk free asset decreases the agents wealth. As a result, for a given tax rate,

agents would end up in a bad state of the world with less wealth under the mixed system than under the fully funded system. Therefore, the marginal utility to the old of additional consumption is larger under the mixed system case, and the policy-maker has an incentive to set a larger tax rate in this case.

## 5. Conclusion

Fully funded systems are known to be highly exposed to negative aggregate shocks to the return of the risky assets in their portfolio. Additionally, we suggest that, when a bad shock hits, office-seeking politicians may intervene to bail the elderly out by transferring resources from the young to the old. This occurrence resembles the institution of the PAYG social security system in several countries (e.g., in Germany, Italy and in the US).

We believe that the political decision to intervene in case of a negative shock is relevant, and should be taken into account in examining the behavior of a fully funded system under uncertainty. In fact, the young perceive the policy-maker's contingent intervention as a free insurance, and adjust their saving behavior accordingly. Specifically, they reshuffle the composition of their fully funded portfolio towards more risky assets. A double pillar, partially unfunded and partially funded, mixed system reduces the riskiness of their portfolio.

In interpreting our results, notice that under our fully funded – or mixed – system, the portfolio decision is completely decentralized, and thus the moral hazard problem due to the political bailout concerns each private individual's portfolio choice. However, our main results would not significantly change if the portfolio decision were to be managed by a (private or public) pension fund. In fact, to the extent that monitoring by governmental institutions is imperfect, and fund managers' pay schedules create incentives to deliver high returns in the good state, the same moral hazard problem arises. Further research should however be devoted to this issue.

Can we go further and design a double pillar mixed system, which prevents the policy-makers from tempering with the existing system, in the case of a negative shock? At which cost? We show that, if the return from the unfunded pillar does not pay a premium over the risk free rate – i.e., if the economy is dynamically efficient – the early introduction of a mixed system to prevent future policy-makers' intervention is not viable. In fact, the unfunded pillar of the mixed system would turn out to be even larger than the PAYG instituted by the policy-maker in a bad state. This represents an extra-cost to all future generations, a PAYG system being an inefficient saving device.

A similar line of reasoning may be applied to the recent “MIT solution” to the social security crisis put forward by Modigliani, Ceprini, and Muralidhar (1999).

They advocate the use of the social security payroll contributions to institute a fund composed of risky assets. The government would then provide some inter-generational risk sharing by guaranteeing a higher return to contributions than the return obtained by the risky portfolio in the bad state, at the cost of a lower return in the good state. This average return guaranteed by the government would clearly be lower than the average return of the risky portfolio, but less volatile. In other words, the government would provide an insurance at a premium, which is equal to the average difference between the return on the risky portfolio and the return paid out to the retirees. Unfortunately, this solution abstracts from the political behavior of the office-seeking policy-makers – our political constraints. In fact, could an office-seeking policy-maker credibly commit to swap risky with risk-free assets in a good state of the world? Or would she rather pay out the high return to the retirees in the current good state and leave future policy-makers to deal with the bad state?

## A. Appendix

### A.1. Proof of Proposition 2.1

Part (i). Since she faces a concave optimization problem, the policy-maker at time  $t$  to choose  $\tau = 0$  when  $R_t = \bar{R}$ , if  $F(\tau = 0 | R_t = \bar{R}) \leq 0$ , where  $F(\cdot)$  is defined in eq. 2.6, and  $c_t^{t-1}$ ,  $c_t^t$ , and  $c_{t+1}^t$  solve the optimization problem at eq. 2.1 and 2.2 (forwarded one period in the case of  $c_t^t$ , and  $c_{t+1}^t$ ), for  $P^G = 0$  and  $P^B \geq 0$ . The first order condition with respect to  $k_t$  for the agents' maximization problem is

$$U'(c_t^t) = \beta p U'(c_{t+1,G}^t) (1 + \bar{R}) + \beta (1 - p) U'(c_{t+1,B}^t) (1 + \underline{R}) \quad (\text{A.1})$$

>From the previous expression and from equations 2.4 and 2.6, we obtain:

$$\frac{F(\tau = 0 | R_t = \bar{R})}{\Psi y_t} = U'(c_{t,G}^{t-1}) - \beta p \left( \frac{\bar{R} - \underline{R}}{r - \underline{R}} \right) (r - g) U'(c_{t+1,G}^t) \leq 0.$$

Moreover, notice that for  $\tau = 0$ ,  $k_t = k_{t-1} (1 + \lambda)$ , and  $\frac{b_t}{k_t} = \frac{b_{t-1}}{k_{t-1}}$ . By dividing the eq. A.1 by  $(k_{t-1})^\alpha$ , and using the assumption of homotheticity, we have that

$$\begin{aligned} \frac{F(\tau = 0 | R_t = \bar{R})}{\Psi y_t} = & U' \left( \frac{b_{t-1}}{k_{t-1}} (1 + r) + (1 + \bar{R}) \right) \\ & - U' \left( \frac{b_t}{k_t} (1 + r) + (1 + \bar{R}) \right) \beta p (1 + \lambda)^\alpha \left( \frac{\bar{R} - \underline{R}}{r - \underline{R}} \right) (r - g) \end{aligned}$$

Since  $U' \left( \frac{b_{t-1}}{k_{t-1}} (1 + r) + (1 + \bar{R}) \right) = U' \left( \frac{b_t}{k_t} (1 + r) + (1 + \bar{R}) \right)$ , then  $F(\tau = 0 | R_t = \bar{R}) \leq 0$  if  $p > \frac{r - \underline{R}}{\bar{R} - \underline{R}} \frac{1}{\beta (1 + \lambda)^\alpha (r - g)}$  (assumption 4).

Part (ii): The proof is analogous to part (i). For the policy-maker at time  $t$  to choose  $\tau > 0$  when  $R_t = \underline{R}$ , we need to show that

$$\frac{F(\tau = 0 | R_t = \underline{R})}{\Psi y_t} = U'(c_{t,B}^{t-1}) - \beta (1 - p) \left( \frac{\bar{R} - \underline{R}}{\bar{R} - r} \right) (r - g) U'(c_{t+1,B}^t) > 0.$$

By using again the property of homotheticity, we have that  $F(\tau = 0 | R_t = \underline{R}) > 0$  if  $p > 1 - \frac{\bar{R} - r}{(\bar{R} - \underline{R}) \beta (1 + \lambda)^\alpha (r - g)}$ .

Finally, notice that for  $\beta (1 + \lambda)^\alpha (r - g) < 1$  the condition in part (i) implies the condition in part (ii), and viceversa for  $\beta (1 + \lambda)^\alpha (r - g) > 1$ . Thus, the two conditions can be summarized as follows:

$$p > \text{Max} \left\{ \frac{r - \underline{R}}{\bar{R} - \underline{R}} \frac{1}{\beta (1 + \lambda)^\alpha (r - g)}, 1 - \frac{\bar{R} - r}{\beta (\bar{R} - \underline{R}) (1 + \lambda)^\alpha (r - g)} \right\} \blacksquare$$

### A.2. Proof of Proposition 3.1

Recall that the portfolio  $(b^*, k^*)$  solves the optimization problem at equations 2.1, 2.2 and 2.3 in the case of no policy-maker's intervention, i.e., for  $P_t^B = P_t^G = 0$ . Whereas  $(b^H(\tau), k^H(\tau))$  indicates the portfolio that solves the optimization problem at equations 2.1, 2.2 and 2.3 when the policy-maker intervenes in the case of a negative aggregate shock, i.e., for  $P_t^B = \tau y_t (1 + \mu)$  and  $P_t^G = 0$ .

Suppose that  $b_{t-1}^H(\tau) = b_{t-1}^*$  and  $k_{t-1}^H(\tau) = k_{t-1}^*$ . Then, due to the strictly concavity of the utility function, from eq.2.4 we have the following inequality:

$$\frac{U'((1+r)b_{t-1}^* + (1+\overline{R})k_{t-1}^* + P_t^B)}{U'((1+r)b_{t-1}^* + (1+\underline{R})k_{t-1}^*)} < \frac{p(\overline{R}-r)}{(1-p)(r-\underline{R})},$$

which implies that  $(b^*, k^*)$  does not solve the agent's optimization problem in the case of policy-maker's intervention. Moreover, notice that the policy-maker's intervention increases the lifetime expected endowment. Because of separability and strict concavity of the utility function, consumption in all three states has to increase. However, for  $b_{t-1}^H(\tau) = b_{t-1}^*$  and  $k_{t-1}^H(\tau) = k_{t-1}^*$ , this increase in consumption only takes place in the bad state. Thus, agents will optimally reshuffle their portfolio to move resources from the bad state to the good state and to the first period. It is straightforward to see that this requires savings to decrease,  $b_{t-1}^H(\tau) + k_{t-1}^H(\tau) < b_{t-1}^* + k_{t-1}^*$ , the holding of risk free assets to decrease,  $b_{t-1}^H(\tau) < b_{t-1}^*$ , and the holding of risky assets to increase,  $k_{t-1}^H(\tau) > k_{t-1}^*$ . Thus,  $b_{t-1}^*/k_{t-1}^* > b_{t-1}^H/k_{t-1}^H$ . ■

### A.3. Proof of Proposition 3.2

Part (i). Let us first compare a mixed system to a fully funded system with no policy-maker's intervention. Recall that in a mixed system  $P_t^B = P_t^G = \tau y_t (1 + \mu) > 0$ , and the agents' optimal portfolio is given by:  $b_{t-1}^M(\tau)$  and  $k_{t-1}^M(\tau)$ , which maximizes eq. 2.1, subject to eq. 2.3 and eq. 3.1.

Moreover, notice that, for any positive  $\tau$ , the mixed system reduces the expected lifetime endowment with respect to the case of policy-maker's intervention. Thus, since the utility function is separable and strictly concave, in all three states – youth, bad and good state – consumption has to be lower under the mixed system than under policy-maker's intervention. Because of homotheticity, regardless of the expected lifetime endowment, the ratio of consumption in the bad and the good state has to be equal to a constant, which we call  $A$ :

$$\frac{(1+r)b_{t-1}^M(\tau) + (1+\underline{R})k_{t-1}^M(\tau) + P_t}{(1+r)b_{t-1}^M(\tau) + (1+\overline{R})k_{t-1}^M(\tau) + P_t} = \frac{(1+r)b_{t-1}^* + (1+\underline{R})k_{t-1}^*}{(1+r)b_{t-1}^* + (1+\overline{R})k_{t-1}^*} = A$$

Suppose now that  $b_{t-1}^M(\tau) = b_{t-1}^*$  and  $k_{t-1}^M(\tau) = k_{t-1}^*$ , then

$$\frac{(1+r)b_{t-1}^* + (1+\underline{R})k_{t-1}^* + P_t}{(1+r)b_{t-1}^* + (1+\overline{R})k_{t-1}^* + P_t} > A. \quad (\text{A.2})$$

Moreover, consumption in a bad and good state would be larger than with no policy-maker's intervention, while the consumption in youth would drop with the decrease in the net endowment.

Thus, agents reshuffle their portfolio to reduce consumption in both bad and good state and to re-establish the equality in eq. A.2. This requires a larger reduction of consumption in the bad state than in the good state – respectively the numerator and the denominator of the l.h.s. of the above equation – which is achieved by reducing the holding of the risk free,  $b_{t-1}^M(\tau) < b_{t-1}^*$ , and by increasing the holding of risky assets,  $k_{t-1}^M(\tau) > k_{t-1}^*$ , and thus  $b_{t-1}^*/k_{t-1}^* > b_{t-1}^M(\tau)/k_{t-1}^M(\tau)$ . Moreover, notice that to increase consumption in youth, the overall portfolio – the savings – has to decrease:  $b_{t-1}^M(\tau) + k_{t-1}^M(\tau) < b_{t-1}^* + k_{t-1}^*$ .

Part (ii). To compare the portfolio under mixed system  $(b_{t-1}^M(\tau), k_{t-1}^M(\tau))$  to the portfolio under policy-maker's intervention  $(b_{t-1}^H(\tau), k_{t-1}^H(\tau))$ , we follow the same strategy as in part (i). Notice first that the expected lifetime endowment is lower under mixed system, and so should consumption be in all three states.

Suppose that  $b_{t-1}^M(\tau) = b_{t-1}^H(\tau)$  and  $k_{t-1}^M(\tau) = k_{t-1}^H(\tau)$ , then

$$\frac{(1+r)b_{t-1}^H(\tau) + (1+\underline{R})k_{t-1}^H(\tau) + P_t}{(1+r)b_{t-1}^H(\tau) + (1+\overline{R})k_{t-1}^H(\tau) + P_t} < A. \quad (\text{A.3})$$

Moreover, consumption should decrease in both the bad and the good state, albeit more in the good state to re-establish the equality in eq. A.3, while it should increase in youth. This leads to a decrease in the holding of the risky assets,  $k_{t-1}^M(\tau) < k_{t-1}^H(\tau)$ , and to an increase in the holding of risk free assets,  $b_{t-1}^M(\tau) > b_{t-1}^H(\tau)$ , and thus to  $b_{t-1}^M(\tau)/k_{t-1}^M(\tau) > b_{t-1}^H(\tau)/k_{t-1}^H(\tau)$ , whereas overall savings decrease  $b_{t-1}^M(\tau) + k_{t-1}^M(\tau) < b_{t-1}^H(\tau) + k_{t-1}^H(\tau)$ . ■

#### A.4. Proof of Proposition 4.3

Consider the optimization problem of a policy-maker at time  $T$ , i.e., in the case of a bad shock. Notice that, by proposition 2.1, this problem yields a solution, which is characterized by the tax rate  $\bar{\tau}$  at eq. 4.2. Let us evaluate the optimization problem of the policy-maker for  $\tau = \hat{\tau}$ , where  $\hat{\tau}$  is defined at eq. 4.1. In particular, let us focus on eq. 4.2. Clearly, for  $\tau = \hat{\tau}$ , the r.h.s. of eq. 4.1 and of eq. 4.1 are equal. Moreover, since by Proposition 3.2 we have that  $b_{t-1}^M(\hat{\tau}) + k_{t-1}^M(\hat{\tau}) < b_{t-1}^H(\hat{\tau}) + k_{t-1}^H(\hat{\tau})$ , then the l.h.s. of eq. 4.2 is smaller than the l.h.s. of eq. 4.1. Then, for  $\tau = \hat{\tau}$ , eq. 4.2 yields the following inequality:

$$U' (c_T^{T-1}(\hat{\tau}) | R_T = \underline{R}) - \frac{r-g}{1+r} \beta U' (c_T^T(\hat{\tau})) < 0.$$

In order to re-establish the equality,  $\tau$  has to decrease, and thus  $\bar{\tau} < \hat{\tau}$ . ■

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