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ABSTRACT

Spatial Mismatch and Skill Accumulation*

Increasing returns in matching between skilled workers and firms create a local thick-market externality when labour markets are geographically segmented. This generates an agglomeration force that can offset the dispersion force due to local competition in a segmented product market. When this is the case, only some regions specialize in high-skill productions while others are caught in a low-skill trap.

JEL Classification: F12, L13 and R13
Keywords: agglomeration, dual labour markets, skill accumulation and spatial mismatch

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1 Introduction

In the European Union high-skill job opportunities are concentrated in central areas (European Commission, 1996). Contrary to expectations, easier transportation is not altering this pattern, which not only has shown a high degree of persistence but is also strengthening through time (Magrini, 1999). A similar center-periphery pattern can be found also in the United States where high-skill jobs tend to cluster in few metropolitan areas (Ellison and Glaeser, 1997).

We argue that one cause of such geographical disparities in high-skill job opportunities is local skill accumulation in the presence of matching frictions in the labor market. The central idea is a sort of large-scale *spatial mismatch* (Kain, 1968) whereby people stuck in peripheral areas see little reward in accumulating skills that are not valuable locally due to the absence of high-skill jobs. At the same time firms have little incentive to locate in those areas due to the shortage of skilled labor ('low-skill bad-job trap', Snower, 1996).[1]

Our argument rests on three key assumptions grounded on three separate pieces of evidence: (i) the geographical concentration of skills is better explained by local accumulation than by migration; (ii) labor market interactions are central in explaining the spatial clustering of firms; (iii) there are increasing returns to matching in local labor markets.

As to the first assumption, Glaeser and Mare (2001) show that spatial clustering is associated with both better matching in the labor market and enhanced skill accumulation. In particular, local skill accumulation dominates skilled immigration as an explanation of urban wage premia. As to the second, Dumais, Ellison, and Glaeser (1997) find that plant location choices appear to be dominated by the local availability of a labor force with the right mix of skills. Though significant, the role of alternative explanations of agglomeration - namely, demand and cost linkages (Fujita, Krugman, and Venables, 1999) as well as informal information flows (Henderson, 1974) - seems to be fairly modest. As to the third assumption,

[1] At city level the *spatial mismatch* hypothesis by Kain (1968) states that access to employment is not independent from residential location. For example, in the US (black) workers who, due to discrimination and high rents, are segregated in areas that are distant from and poorly connected to business districts, face high geographic barriers to finding and keeping skill-intensive well-paid jobs. This reduces their incentive to accumulate skills so that they end up being trapped in the ghetto. In departing from Kain's urban perspective, we remove workers' commuting and endogenize firms location.

Blanchard and Diamond (1989) as well as Burda and Profit (1996) find evidence in favor of constant or slightly increasing returns to scale. Anderson and Burgess (1995) as well as Warren (1996) reject the hypothesis of constant returns in favor of increasing ones.[2]

So far these three facts have not been integrated into a unified model of geographical agglomeration. To provide such a model is the main contribution of the present paper.

In our model the spatial distributions of jobs and skills are jointly determined by the interaction between 'centripetal' and 'centrifugal' forces. Centripetal forces foster agglomeration and originate from the labor market when matching between high-skill workers and firms exhibits increasing returns to scale. Such forces are due to the presence of two mutually reinforcing localized externalities: a 'vacancy supply externality' and a 'training supply externality' (Snower, 1996). As to the former, when a firm posts a high-skill job vacancy in a local labor market, it increases the probability of a local high-skill worker to find a good job and thus her expected return from upgrading her skills. As to the latter, when a worker chooses to invest in skill accumulation, she raises the probability of a local high-skill firm to fill in its vacancies and thus its expected profits.[3] Centrifugal forces foster dispersion and originate from the product market due to the presence of a 'pecuniary externality' generated by transport costs and plant-level increasing returns (Fujita, Krugman, and Venables, 1999). When a firm decides to operate in a certain location, it has a negative impact on the profits of existing firms and thus on the expected return from posting a vacancy in the corresponding local market.

As a result we are able to assess how the geography of high-skill jobs arises from the interactions between transport costs and labor market frictions. The level of transport costs plays a critical role. For high transport costs, the geographical distributions of jobs and skills is closely related to exogenous differences between areas. In particular, if locations have equally functioning labor markets, those with better access to product markets generate better jobs and higher skills. The same happens to locations with better functioning labor

[2] Overall, increasing returns seem to emerge from analysis relying on a low level of aggregation in the data and/or relying on more flexible functional forms than the widely used log linear one.

[3] We assume that skill accumulation is a costly private activity of workers. This assumption is made here for simplicity. More generally, as surveyed by Acemoglu and Pischke (1999), firms also contribute to the cost of training investment. However, taking this into account would not alter the insights of the present analysis.

markets when product market access is the same everywhere. Things turn out to be quite different if transport costs are low. As even a small idiosyncratic shock can cause good jobs and high skills to agglomerate in space, ex-ante identical locations may end up being very different. Some of them are characterized by good jobs and high skills. Other are caught in a low-skill bad-job trap.

The paper is organized in five additional sections. Section 2 relates our contribution to the existing literature. Section 3 develops the model. Section 4 characterizes its equilibrium. Section 5 studies the dependence of economic geography on product and labor market imperfections and shows how increasing returns to matching may lead to catastrophic outcomes. Section 6 concludes.

2 Related literature

Our modeling of the labor market rests on two distinct traditions, labor dualism and search theory. We build on *labor dualism* by considering two employment sectors: a 'primary' sector where firms offer relatively high wages and a stimulating working environment and a 'secondary' sector where such features are absent (Doeringer and Piore, 1971). However, we depart from the standard approach in dual labor market theory that differentiates the two sectors in terms of the relative difficulty of monitoring workers performance (Smith and Zenou, 1997). The departure is twofold. On the one hand, we base sectoral wage differences on the participation of workers to the oligopolistic rents of firms in the modern sector (Abdel-Rahman and Wang, 1997). On the other, we capture the origin of high-skill unemployment in the modern sector by a *search theoretic approach* whereby labor market frictions generate imperfect firm-worker matching (Pissarides, 1990) that in turn discourages investment in skills (Snower, 1996). In particular, we assume that the search efficacy of workers and firms depends on their respective locations (Seater, 1979).

In terms of geographical economics our contribution is related to the microfoundation models of the black-box scale economies discussed by Henderson (1974). In particular, it shares with Helsley and Strange (1990) and Abdel-Rahman and Wang (1997) the modeling of centripetal forces as scale economies in the processes of search and matching in local labor markets. However, it differs from those works under three major respects. First, while in those works the spatial distribution of skills is the outcome of the migration decisions of a mobile population with given skill heterogeneity, in our model that distribution is determined by the training decisions of an initially homogenous immobile population (Benabou, 1993).[4] Second, while in those works centrifugal forces arise from the internal limitations to city size, in our model they come from an immobile dispersed demand in the presence of transport costs (Krugman, 1991). Third, while those works abstract from the spatial distribution of cities in order to focus on their number and sizes, we abstract from the inner spatial dimension of locations as well as their number in order to focus on the role of inter-location distance in the emergence of core-periphery patterns.

Our explanation of geographical imbalances is also different from 'new economic geography' models (Fujita, Krugman and Venables, 1999) in that we stress labor market interactions rather than demand and cost linkages. Related works are Krugman (1991) as well as Rotemberg and Saloner (2000). The former formalizes Marshall's idea of labor pooling according to which firms want to locate near each other to hedge workers against the risk of firm-specific shocks, the reason being that risk-averse workers are willing to accept lower wages in locations where there are many firms ready to hire them. The latter argue that workers accept lower wages in locations with many firms because with incomplete contracts the presence of many potential employers protects labor against ex post expropriation of investments in industry-specific human capital. Differently, in our model firms and workers colocate because this increases the probability of successful matches.

3 The model

The economy consists of two regions, West and East, of equal immobile population L. Things pertaining to the West bear no label while those pertaining to the East are labeled by a star. We describe the West and the description of the East is symmetric.

[4] For the empirical reasons discussed in the introduction, we rule out migration to focus on local skill accumulation. However, allowing for migration would simply add a centripetal force (namely, a 'demand linkage') to our model without altering its main insights.

3.1 Consumers

Preferences are encapsulated by the quasi-linear utility function:

$$W = A + \alpha c - \frac{\beta}{2}c^2 + z \tag{1}$$

where A, α, and β are positive constants, c is consumption of a *modern* good C and z is consumption of a *traditional* good Z which we choose as numeraire. For simplicity utility (1) removes income effects. This is not crucial for future results since in equilibrium expected income will be the same across all individuals no matter where they reside.

Following Snower (1996), each consumer is endowed with two units of working time. All consumers are initially low-skill. However, they can choose to sacrifice a unit of their potential working time to enhance their human capital and become high-skill. We call S the number of those who decide to become high-skill and U the number of those who remain low-skill with $S + U = L$. According to their skills, workers face different employment opportunities. While low-skill workers have access only to jobs in the traditional sector, high-skill workers are also demanded by the modern sector. The traditional and modern labor markets are segmented and differ in terms of their efficiency. While the low-skill market is frictionless, the high-skill one is characterized by *imperfect matching* between firms and workers, i.e. high-skill workers might not find a job in the modern sector even if simultaneously some jobs remain vacant.[5] Those high-skill workers who are not matched with any modern firm fall back into the traditional sector, which therefore acts as a buffer.[6] Thus, potentially, there are three groups of workers in the economy: the high-skill who work in the modern sector (S_s), the high-skill who work in the traditional sector (S_u, with $S_s + S_u = S$), and the low-skill who can be employed only in the traditional sector (U, with $S_s + S_u + U = S + U = L$).

The frictions in the high-skill labor market are modeled by an *aggregate matching function* (Pissarides, 1990), which captures the idea that in the West the number of local suc-

[5] This is reminiscent of Helsley and Strange (1990) who propose a model where workers with heterogenous skill endowments face firms with heterogeneous skill requirements. Under incomplete information workers and firms may end up with the wrong matches.

[6] Alternatively, unmatched workers could end up being unemployed and earn some associated benefit. This would not affect our main results as the value of the benefit would simply multiply $(1 - \rho)$ in (23).

cessful matches (i.e. filled vacancies) S_s increases with the number of local high-skill workers S and the number of vacancies V posted locally by firms in the modern sector:

$$S_s = BV^{\nu} S^{\sigma} \tag{2}$$

where B, ν, $\sigma > 0$. If $\nu + \sigma = 1$ the matching technology exhibits constant returns to scale. On the other hand, if $\nu + \sigma < (>)1$ returns are decreasing (increasing). We assume that firms can post their vacancies in both regions but workers are able to fill in only those vacancies that are posted in the region where they reside.

Function (2) implies that the probabilities of a successful match for a firm and a high-skill worker are respectively:

$$\theta = \frac{S_s}{V} = BV^{\nu-1} S^{\sigma} \tag{3}$$

$$\rho = \frac{S_s}{S} = BV^{\nu} S^{\sigma-1} \tag{4}$$

where, by definition, $0 \leq \theta \leq 1$ and $0 \leq \rho \leq 1$. An adequate choice of the scaling parameter B ensures that this is always the case.

Expressions (3) and (4) reveal the existence of a 'training supply externality' and a 'vacancy supply externality' (Snower, 1996). When workers train, they raise the probability θ of modern firms to fill in their vacancies. At the same time, when modern firms post vacancies, they increases the probability ρ of local high-skill workers to find a modern job. Clearly, the probabilities θ and ρ are positive only if firms post vacancies and workers acquire skills. We assume that both actions are costly. More specifically, each firm bears a cost of γ units of the numeraire for each vacancy it posts. As to workers, skills acquisition requires training which costs δ units of the numeraire and 1 unit of working time. Therefore, if a worker does not train, she spends both her units of working time in the traditional sector and earns twice the corresponding hourly wage w_u. If she does train, she spends one unit of time training and the remaining unit working. If she finds a job in the modern sector, she earns the corresponding hourly wage w_s. However, if she cannot be employed in the modern sector, she falls back into the traditional one and earns w_u.

As a result, we must distinguish between three alternative budget constraints. The first pertains to a worker who decides not to train and thus spends both her units of time working in the traditional sector:

$$2w_u + \overline{z} = pc + z \tag{5}$$

The second belongs to a worker who decides to train and eventually finds a job in the modern sector:

$$w_s + \overline{z} = pc + z + \delta \tag{6}$$

The third refers to a worker who trains but is forced to work in the traditional sector:

$$w_u + \overline{z} = pc + z + \delta \tag{7}$$

The left hand sides of (5), (6) and (7) are the incomes of the three groups. They consist of wages, equal shares of firms' profits/losses and an initial endowment of the numeraire, which is assumed to be large enough to imply always an interior solution of the consumer optimization problem. Accordingly, $\overline{z}$ labels the initial endowment of the numeraire net of profits/losses. Thus, for simplicity, we do not consider separate firms owners but, instead, assume public ownership of firms by workers. As a matter of fact, this choice will turn out to be immaterial because, as explained below, the equilibrium is characterized by the erosion of any profit in excess of fixed costs.

The right hand sides represent the corresponding expenditures in the two goods and possibly in training with p labeling the price of the modern good.

Maximization of (1) given (5), (6) and (7) yields the aggregate inverse demand function for the modern good:

$$p = \alpha - \beta \frac{C}{L} \tag{8}$$

where $C = c(S_s + S_u + U) = cL$ is the aggregate demanded quantity. As usual with quasi-linear utility, at an interior solution all workers consume the same amount c of the modern good independently from their level of income while the consumption of the traditional good is determined as a residual.

3.2 Firms

The traditional good Z is supplied by a perfectly competitive sector which employs low-skill labor U under constant returns to scale. The technology is assumed to require one unit of low-skill labor for each unit of output so that U also measures the output of the traditional sector. This good is freely traded between regions and, assuming incomplete specialization, this implies that its price is the same everywhere. The choice of such good as the numeraire entails that in equilibrium the wage of the low-skill workers is equal to unity everywhere, i.e. $w_u = 1$ in both regions.

The modern good C is produced by an imperfectly competitive sector which uses high-skill labor S under increasing returns. In particular, it is assumed that each firm needs one unit of high-skill labor to produce any amount of output. In such setting, a high-skill worker can be viewed as the manager appointed by owners to run the firm. Therefore, in equilibrium S_s will turn out to be not only the stock of high-skill labor employed in the modern sector but also the number n of active firms in that sector, i.e. $n = S_s$. Differently from the traditional good, the modern good can be traded only at a cost: a positive transportation cost of τ units of the numeraire has to be paid for each unit shipped from one region to the other.[7]

In each regional market firms are assumed to be engaged in Cournot competition with both local and foreign rivals. Following Brander and Krugman (1983), firms are assumed to be able to price discriminate between local and distant consumers ('segmented markets') as it is usually the case even within a unified economic area (Head and Mayer, 2000). Furthermore, the fact that in equilibrium, as we will see, firms absorb part of the transportation costs makes 'grey market' sales impossible.

Analogously to Helsley and Strange (1990), before paying the cost of posting a vacancy a firm is not sure to be matched with a worker whose skills exactly fit her job. Consequently it forms expectation about its profits under (3). Then, the typical western firm maximizes expected profits:

$$\Pi = \theta \left[p\left(\frac{X+Y}{L}\right) x + p^*\left(\frac{X^*+Y^*}{L^*}\right) x^* - \tau x^* - w_s \right] - \gamma \tag{9}$$

[7] One may think of the modern good as relatively intensive in business services that are typically difficult to trade in space.

where x and x^* are the quantities sold by the western firm, X and X^* are the quantities sold by all western firms, Y and Y^* are the quantities sold by all eastern firms in the western and eastern markets respectively so that $X+Y=C$ and $X^*+Y^*=C^*$. Because of market power, when active the firm generates operating profits $\pi = px + p^*x^* - \tau x^* > 0$. In the wake of Abdel-Rahman and Wang (1997), such profits are assumed to be split between the firm and its labor force by decentralized Nash bargaining so that:

$$w_s = \arg\max(w_s - 1)^{\lambda}(\pi - w_s)^{1-\lambda} \tag{10}$$

where factors $(w_s - 1)$ and $(\pi - w_s)$ represent the net surpluses that a high-skill worker (given $w_u = 1$) and a firm gain respectively from a successful match while $0 \leq \lambda \leq 1$ measures the bargaining power of the high-skill worker.

4 The equilibrium location of firms

We focus on equilibria such that all firms and workers belonging to the same region behave in the same way. Since, due to product market segmentation, regional markets can be analyzed separately, we concentrate on the western market. Conditions pertaining to the eastern one can be derived by symmetry.

After substituting (8) into (9) and the symmetric expression for the East, the typical western and eastern firms maximize expected profits in the western market if their sales satisfy respectively:

$$\theta[\alpha - 2\beta x/L - \beta(n-1)\overline{x}/L - \beta n^* y/L] = 0 \tag{11}$$

$$\theta^*[\alpha - 2\beta y/L - \beta(n^*-1)\overline{y}/L - \beta n x/L - \tau] = 0 \tag{12}$$

where y is the output of the eastern firm sold in the West and an upper bar labels the competitors quantity choices that each firm takes as given.

Since our attention is restricted to symmetric outcomes, we can impose $\overline{x} = x$ and $\overline{y} = y$ in (11) and (12) to obtain:

$$\alpha - \beta(n+1)x/L - \beta n^* y/L = 0 \tag{13}$$

$$\alpha - \beta(n^* + 1)y/L - \beta nx/L - \tau = 0 \tag{14}$$

which can be solved together for the equilibrium outputs:

$$x = \frac{\alpha + n^*\tau}{\beta(n^* + n + 1)}L \tag{15}$$

$$y = \frac{\alpha - (n+1)\tau}{\beta(n^* + n + 1)}L \tag{16}$$

The comparison between (15) and (16) shows how obstacles to interregional trade increase the output of local firms and decrease the output of distant firms.

Using (15) and (16) the aggregate supply of the modern good to western consumers $X + Y = nx + n^*y$ turns out to be:

$$X + Y = \frac{\alpha(n + n^*) - n^*\tau}{\beta(n^* + n + 1)}L \tag{17}$$

so that by (8) the equilibrium consumer price can be calculated as:

$$p = \frac{\alpha + n^*\tau}{(n^* + n + 1)} \tag{18}$$

i.e., the larger the transportation cost τ the higher the price because higher barriers to trade shelter local firms from distant competitors thus increasing their market share. Indeed, eastern firms sell at all in the West ($y > 0$) if and only if $p - \tau > 0$ i.e. $\alpha > \tau(n + 1)$. In the same way, western firms export to the East if and only if $p^* - \tau > 0$ i.e. $\alpha > \tau(n^* + 1)$. In addition, (18) shows that the price increases if the number of local and/or foreign firms falls and it is lower in the region with the larger number of local firms.

Plugging (17) and (18) into (9) the typical western firm expects to earn:

$$\Pi = \theta \left\{ \frac{(\alpha + n^*\tau)^2 L + [\alpha - (n^* + 1)\tau]^2 L^*}{\beta(n^* + n + 1)^2} - w_s \right\} - \gamma \tag{19}$$

In equilibrium potential firms must be indifferent between posting a vacancy and being inactive which is the case if and only if $\Pi = 0$, that is:

$$BV^{\nu-1}S^{\sigma} \left\{ \frac{(\alpha + n^*\tau)^2 L + [\alpha - (n^* + 1)\tau]^2 L^*}{\beta(n^* + n + 1)^2} - w_s \right\} - \gamma = 0 \tag{20}$$

where we have used (19) and the definition of θ given by (3). Two comments about (20) are in order. On the one hand, the expected profits are increasing in the probability of a successful match θ. Due to the mutually reinforcing effect of the training-supply and vacancy-supply externalities, this foster firms agglomeration (*centripetal forces*). On the other hand, the operating profits of a successfully matched western firm (i.e., the ratio inside the curly bracketed term) is decreasing in n. An increase in the number of local firms increases local competition and depresses local profits, which hampers firms agglomeration (*centrifugal force*).

Turning now to workers, before paying the training cost a worker is not sure to be matched with a modern firm. Therefore, a worker's choice is to pick the option that pays off the highest expected utility between acquiring skills and not acquiring them. Given (1), if a worker chooses to train, then she can expect to reach the (indirect) utility level:

$$W_s = D - \frac{\alpha}{\beta}p + \frac{1}{2\beta}p^2 + \rho w_s + (1-\rho) - \delta + \overline{z} \tag{21}$$

where D is a positive constant. Since, by (4), ρ is the probability of finding a job in the modern sector and w_u equals 1, $\rho w_s + (1-\rho) - \delta$ is her expected income net of the training cost. If she decides not to train, her utility is:

$$W_u = D - \frac{\alpha}{\beta}p + \frac{1}{2\beta}p^2 + 2 + \overline{z} \tag{22}$$

where, given that $w_u = 1$, 2 is her certain income.

At an interior equilibrium the worker has to be indifferent between the two choices, which is the case if and only if:

$$\rho w_s + (1-\rho) - \delta = 2 \tag{23}$$

that is, if and only if the expected hourly salary for a high-skill worker net of the training cost equals twice the hourly salary of a low-skill one.[8]

Using the definition of ρ given by (4), the indifference condition (23) can be rewritten as:

$$w_s = \frac{1+\delta}{BV^{\nu}S^{\sigma-1}} + 1 \tag{24}$$

[8] Notice that in equilibrium the expected income is the same for all workers no matter where they reside and what level of skills they attain. Thus, given quasi-linear utility, people are better off in the region with more firms since, as already argued, it offers a lower price.

A final condition expresses the outcome of the Nash bargaining over the division of operating profits:

$$w_s = \lambda \frac{(\alpha + n^* \tau)^2 L + [\alpha - (n^* + 1)\tau]^2 L^*}{\beta(n^* + n + 1)^2} + (1 - \lambda) \tag{25}$$

Using (25) to substitute for w_s in (24) and (20), it is straightforward to show that the equilibrium number of vacancies posted by firms in the West as a function of n and n^* is:

$$V = \left[\frac{1+\delta}{B \left(\frac{\lambda}{1-\lambda} \frac{\gamma}{1+\delta} \right)^{\sigma - 1} \lambda \left\{ \frac{(\alpha + n^* \tau)^2 L + [\alpha - (n^* + 1)\tau]^2 L^*}{\beta(n^* + n + 1)^2} - 1 \right\}} \right]^{\frac{1}{\nu + \sigma - 1}} \tag{26}$$

and the corresponding equilibrium number of workers who decide to train is:

$$S = \left(\frac{\lambda}{1-\lambda} \frac{\gamma}{1+\delta} \right) V \tag{27}$$

which states the natural result according to which the ratio between the number of workers that decide to train and the number of vacancies posted by firms increases with workers' bargaining power λ as well as the cost of posting a vacancy γ while it decreases with the cost of training δ.

Plugging (26) and (27) in the matching function (2) and remembering that $S_s = n$, we find the equilibrium relation between the number of posted vacancies V and the number of active firms n. Then (3) implies that a firm is matched with probability:

$$\theta = \left[B \left(\frac{\lambda}{1-\lambda} \frac{\gamma}{1+\delta} \right)^{\sigma} \right]^{\frac{1}{\nu+\sigma}} n^{\frac{\nu + \sigma - 1}{\nu + \sigma}} \tag{28}$$

which is increasing (decreasing) in n as long as there are increasing (decreasing) returns to matching $[\nu + \sigma > (<) 1]$. Furthermore, when returns are constant, (28) is independent of n. Accordingly, using (25) and (28) in (19) we get that the typical western firm expects to earn:

$$\Pi = \left[B \left(\frac{\lambda}{1-\lambda} \frac{\gamma}{1+\delta} \right)^{\sigma} \right]^{\frac{1}{\nu+\sigma}} n^{\frac{\nu + \sigma - 1}{\nu + \sigma}} (1 - \lambda) \left\{ \frac{(\alpha + n^* \tau)^2 L + [\alpha - (n^* + 1)\tau]^2 L^*}{\beta(n^* + n + 1)^2} - 1 \right\} - \gamma \tag{29}$$

Finally, imposing $\Pi = 0$ on (29) allows us to obtain the locus of combinations (n, n^*) such that western firms break even:

$$n^{\chi}\left[\frac{(\alpha+n^*\tau)^2L+[\alpha-(n^*+1)\tau]^2L^*}{\beta(n^*+n+1)^2}-1\right]-\kappa=0 \tag{30}$$

where

$$\chi\equiv\frac{\nu+\sigma-1}{\nu+\sigma}\text{ and }\kappa\equiv\left[\frac{\gamma^{\nu}(1+\delta)^{\sigma}}{B\lambda^{\sigma}(1-\lambda)^{\nu}}\right]^{\frac{1}{\nu+\sigma}} \tag{31}$$

Symmetric expressions characterize the break even locus of eastern firms:

$$n^{*\chi}\left[\frac{(\alpha+n\tau)^2L^*+[\alpha-(n+1)\tau]^2L}{\beta(n^*+n+1)^2}-1\right]-\kappa^*=0 \tag{32}$$

where

$$\chi^*\equiv\frac{\nu^*+\sigma^*-1}{\nu^*+\sigma^*}\text{ and }\kappa^*\equiv\left[\frac{\gamma^{*\nu^*}(1+\delta^*)^{\sigma^*}}{B^*\lambda^{*\sigma^*}(1-\lambda^*)^{\nu^*}}\right]^{\frac{1}{\nu+\sigma}} \tag{33}$$

The solution to the system of equations (30) and (32) determines the equilibrium number of modern firms n and n^* that operate in the two regions as functions of their sizes L and L^*, their degree of integration as inversely measured by τ and the efficiency of the underlying labor market institutions as captured by the parameters χ, κ, χ^*, and κ^*.

5 Explaining geographical disparities

Equation (29) clarifies that an increase in the number of firms in the West has two effects on their expected profits. On the one side larger n depresses the surplus from successful matches. The reason is that a larger number of local competitors cuts into firms operating profits (i.e. the ratio inside the curly bracket). Ceteris paribus, this *competition effect* fosters the dispersion of firms. This effect is weaker the lower transport costs are and the more local firms are, because in such cases local competition is tougher and the appearance of a new local firm is less relevant. On the other side, unless returns to matching are constant, larger n affects the chances of successful matches between skilled workers and firms (*matching effect*). When returns are decreasing, a rise in n lowers the matching probability thus fostering firms dispersion. On the contrary, with increasing returns a rise in n enhances the matching probability and, ceteris paribus, fosters agglomeration. In other words, while the competition effect acts as a centrifugal force, the matching effect is centrifugal under decreasing returns, neutral under constant returns and centripetal under increasing returns.

As a result, when returns to matching are non-increasing ($\chi \leq 0$), regional disparities will originate from exogenous differences between regions such as product market acccess or labor market efficiency. On the contrary, when returns to matching are increasing ($\chi > 0$), even regions that are ex ante identical may end up facing very different economic fortunes. The aim of what follows is to spell out this argument.

5.1 A stable symmetric benchmark case

The first step is to characterize a benchmark case in which the two regions are identical in terms of the underlying parameters ($L = L^*$, $\chi = \chi^*$, $\kappa = \kappa^*$) and returns to scale are non-increasing ($\nu + \sigma \leq 1$, i.e. $\chi \leq 0$). The benchmark case is represented in Figure 1 where the steeper curve is the zero profit locus for western firms (30) while the flatter one is the zero profit locus for eastern firms (32). The equilibrium is at the crossing between those two curves. It lies on the 45-degree line along which $n = n^*$ and it is unique. For combinations (n, n^*) above (30) western profits are negative so that some western firms are forced to exit the market and n decreases, while the reverse is true below it. This is shown by the horizontal arrows in the picture. For combinations (n, n^*) above (32) eastern profits are negative so that some eastern firms have to leave the market and n^* decreases, while the reverse is true below it. This is shown by the vertical arrows in the picture. Reading all the arrows together, we can conclude that the unique equilibrium is stable.[9] Therefore our benchmark case depicts a situation in which, absent regional differences in the underlying parameters, whatever the initial condition on (n, n^*), the spatial distribution of firms evolves towards a *dispersed* outcome where both regions host the same number of firms because the same number of workers decide to train and eventually find a job in the modern sector. The reason why is that with decreasing returns to matching, the efficiency of the labor market falls with size so that both product and labor market externalities act as centrifugal forces. Differently, with constant returns only the product market externality operates. As a result,

[9] Existence, uniqueness and stability of free-entry Cournot equilibrium is a general problem extensively discussed, for instance, in Tirole (1988).

in both cases agglomeration never takes place.

The equilibrium number of firms, which can be measured by the distance of the equilibrium point (n, n^*) from the origin of the axes, depends on all the underlying parameters in an intuitive way. In particular, such distance increases with the size of the market (L), with total input matching productivity (B) as well as lower costs of training (δ) and posting vacancies (γ).

Less clearcut is the impact of changes in the bargaining power of high-skill workers on labor market efficiency and hence on the equilibrium number of firms. Simple algebra reveals that, all the rest equal, the distance of the equilibrium point from the origin of the axes reaches a maximum for $\lambda = \sigma/(\nu + \sigma)$, namely when workers' bargaining power mirrors their weight in the matching function (which corresponds to the *Hosios condition*, Hosios, 1990). The intuition behind such result is that a firm's expected share of operating profits is a non-monotonic function of λ. This is due to a trade-off between two opposite effects. On the one hand, a larger bargaining power of workers reduces the share of operating profits accruing to the firm. On the other hand, it also reinforces the incentive for workers to train, thus increasing the probability of a successful match. It is easy to show that $\lambda = \sigma/(\nu + \sigma)$ maximizes the firm's expected share of operating profits and thus the number of firms that can break even in equilibrium.

5.2 Exogenous asymmetries

Maintain now the assumption of non-increasing returns to matching and consider what happens when one parameter changes in one region only. The impact can be assessed by studying the impact of parameter changes on the position of the two zero profit loci. We focus on two thought experiments. First, we study what happens when regions differ in terms of product market access. Second, we investigate the implications of different labor market efficiency.

5.2.1 Market size and market access

An increase in the West market size ($\Delta L > 0$) pushes up the profits of both western and eastern firms, thus increasing the number of workers that decide to train as well as the number of firms posting vacancies. Both (30) and (32) shift away from the origin. However, due to trade costs, a western location provides a better production site to exploit the increased western demand (*home-market effect*, Helpman and Krugman, 1985). As a result, profits and the number of vacancies rise more in the West than in the East leading to more firm creation in the former than in the latter ($n > n^*$). The explanation is that, given better product market access, for any given number of active firms competition is weaker in the West than in the East so that in equilibrium a larger number of firms are able to break even in the former than in the latter.

A strictly related result appears if we move to a multi-region setting. As a simple example, we consider the case of three regions, say East, West and North, which are identical except for their mutual distances. In particular, we assume that to ship one unit of the modern good between West and East as well as between West and North costs τ units of the numeraire, while to ship one unit between East and North costs $\tau' > \tau$. This implies that West ('center') has a better access to the world markets than the other two regions ('periphery'). In this setting the zero profit condition for firms in the West is:

$$n^{\chi}\left[\frac{(\alpha+2n^*\tau)^2L+[\alpha-(2n^*+1)\tau]^2 2L}{\beta(2n^*+n+1)^2}-1\right]-\kappa=0 \tag{34}$$

where n^* is the number of firms in each of the other two regions, whose firms break even if and only if:

$$n^{*\chi}\left[\frac{(\alpha+n\tau+n^*\tau')^2L+[\alpha-(n+n^*+1)\tau]^2L+[\alpha-n\tau-(n^*+1)\tau']^2L}{\beta(2n^*+n+1)^2}-1\right]-\kappa^*=0 \tag{35}$$

In terms of Figure 1 the introduction of a third northern region which is better connected to the West than to the East shifts the intersection between (30) and (32) in such a way that in equilibrium $n > n^*$, that is, even if all regions have identical sizes, the West supports a larger number of firms than the other two regions. This result points out the importance of economic geography for the creation of firms. All the rest equal,

Proposition 1 *By raising firm profits, better access to product markets fosters local skill accumulation and firm creation in the modern sector.*

5.2.2 Labor market institutions

Labor market institution are also a crucial determinant of the spatial distribution of firms. Whatever enhances the working of the high-skill labor market has a positive impact on the number of firms created in the modern sector. For example, as already argued, if labor market efficiency improves in the West, for example through an increase in the degree of returns to scale χ or in the total matching productivity B, or a decrease in the costs of training δ or posting vacancies γ, its zero profit curve moves away from the origin of the axes. As a result, the intersection between (30) and (32) takes place for a combination (n, n^*) such that $n > n^*$. All the rest equal,

Proposition 2 *By improving the expected match between workers and firms, a better functioning high-skill labor market fosters local skill accumulation and firm creation in the modern sector.*

Skill accumulation and firm creation are also affected by the way the operating profits are split between workers and firms. Since the zero profit conditions are positioned as far as possible away from the origin when the Hosios condition is met (i.e., $\lambda = \sigma/(\nu + \sigma)$), skill accumulation and firm creation are more intense in the region where the bargaining power of the two parties more closely mirror their weights in the matching function.

5.3 Low-skill bad-job traps

So far we have focused on non-increasing returns in matching and we have seen that they generate a unique and stable equilibrium spatial distribution of modern firms.[10] Regional differences in the underlying parameters bias this distribution in favor of the location which boasts a larger local market, a better access to world product markets, a more efficient high-skill labor market, and bargaining powers that more closely mirror firms' and workers' weights in the matching technology.

[10] The only crucial qualitative difference between decreasing and constant returns to matching appears when, absent any regional difference in underlying parameters, there are no trade costs in the modern sector ($\tau = 0$). In that case, decreasing returns still lead to a unique stable equilibrium with $n = n^*$, while constant returns to scale produce a continuum of stable equilibria along (30) and (32) which coincide.

All these results do not necessarily hold in the case of increasing returns to scale ($\chi > 0$). In particular, their validity depends on the level of trade costs τ. We start the analysis with assuming the same parameters values for the two regions. For high trade costs the qualitative features of system (30)-(32) are represented by Figure 1 with a unique and stable equilibrium E. For low trade costs however things are quite different. As shown in Figure 2, (32) becomes steeper than (30) so that the equilibrium E is still unique but unstable. Two other equilibria, E_1 and E_2, appear in which the modern sector is active in one region only, West ($n^* = 0$) and East ($n = 0$) respectively. Because of increasing returns, any perturbation of the dispersed outcome $n = n^*$ gets amplified by a process of cumulative causation due to the fact that, if in a region there are more high-skill workers, firms post more vacancies and the number of successful matches increases more than proportionately. In other words, in the case of increasing returns to scale, better matching in the labor market generates an agglomeration force that opposes the dispersion force stemming from competition in the product market. With high trade costs, firms' sales are essentially local so that the entry of new firms in a certain region increases the intensity of local competition a lot with respect to the other region. As a result, the dispersion force dominates. On the contrary, with low trade costs, the location of new firms does not affect much the relative intensity of competition across regions so that the agglomeration force prevails. Thus,

Proposition 3 *Under increasing returns in matching and low trade costs, if one region gets an initial lead in the modern sector, that lead becomes self-sustaining. Eventually all high-skill firms and workers will reside in that region while the other region will be trapped in a situation of no skill accumulation and no firm creation in the modern sector ('low-skill bad-job trap').*

There exists also a third possible configuration which is represented in Figure 3 and arises for intermediate values of τ. It exhibits five equilibria: the dispersed outcome E, the agglomerated configurations, E_1 and E_2, and two additional configurations, E_3 and E_4, with partial agglomeration. While E, E_1 and E_2 are all locally stable, the additional E_3 and E_4 are not. Differently from Figure 2 regions that initially are not so different might end up being identical in equilibrium. On the contrary, regions that are initially very different diverge and one of them is caught once more in a low-skill bad-job trap.

When formally analyzing the impact of transport costs on the stability of the symmetric equilibrium, $n = n^* = n_s$, it is possible to show (see the Appendix for details) that the condition for this particular outcome to be stable is:

$$\frac{\chi\kappa\beta}{2} < \tau^2 \frac{n_s^{\chi+1}}{1+2n_s} \tag{36}$$

It immediately appears that, absent transport costs ($\tau = 0$), the symmetric equilibrium is always unstable. The reason is that free transportation eliminates the centrifugal force since firms interactions in the product market are not affected by their locations. At the other extreme, when the level of transport costs is so high that interregional trade vanishes ($\tau = \tau_{\max} = \frac{\alpha}{n_s+1}$), (36) cannot be unambiguously signed and stability or instability depends on all the model's parameters. The effect of transport costs on the stability of the symmetric equilibrium can be summarized by Figure 4, which displays the right hand side of (36) as a function of τ. The figure shows that, by lowering transport costs from $\tau_{\max}$, *integration weakens the competition effect* thus fostering agglomeration. Specifically, we must distinguish between two cases with regard to the left hand side of (36) (referred to in the figure as the $(\chi\kappa\beta/2)^a$ and $(\chi\kappa\beta/2)^b$ cases). In the $(\chi\kappa\beta/2)^a$ case there exists a threshold value τ_1 such that for $\tau_1 < \tau < \tau_{\max}$ the symmetric equilibrium is stable while it is unstable for $0 < \tau < \tau_1$. In the $(\chi\kappa\beta/2)^b$ case, (36) is always violated for any τ in the interval $[0, \tau_{\max}]$ and the symmetric equilibrium is always unstable: the matching effect always dominates the competition effect.

What about the impact of differences in the other underlying parameters? With increasing returns in matching, a larger local market, a better access to world product markets, a more efficient high-skill labor market of the West shift the crossing between (30) and (32) towards the $n = 0$ axis. This means that a wider set of initial conditions on (n, n^*) leads towards the agglomeration of the modern sector in the western region ($n > 0$ and $n^* = 0$). Therefore, larger home market size, better world market access and better labor market efficiency reduce the risk for a region of incurring a low-skill bad-job trap.

6 Conclusion

We have proposed a spatial matching model where the geographical distribution of a high-skill sector emerges as a result of the interactions between the location choices of firms and the training decisions of an initially unskilled population. A training supply externality meets a vacancy supply externality to generate centripetal forces that support agglomeration. These are opposed by a pecuniary externality through which the spatial concentration of firms depresses their profits.

The results of the analysis can be summarized as follows. With non-increasing returns in matching, if locations have equally functioning labor markets, the locations with better access to product markets attract larger shares of the high-skill jobs. The same happens to the location with the better functioning labor market when locations have equal access to product markets. Catastrophic agglomeration processes are however absent in this case. On the contrary, when returns are increasing, while for large transport costs there is no crucial difference with respect to the previous case, for low transport costs the modern sector may catastrophically agglomerate in the location which has even a small initial advantage, for instance, in terms of market access and/or labor market functioning. In such case, the integration of product markets increases the likelihood of the agglomeration of high-skill jobs even if locations are initially very similar in their fundamentals.

With respect to alternative explanations of geographical disparities, the practical relevance of our model rests on the availability of positive answers to three key empirical questions. Is the geographical concentration of skills better explained by local accumulation than by migration? Are labor market interactions central in explaining the spatial clustering of firms? Are there increasing returns to matching in local labor markets? Based on state-of-the-art evidence, the answers to all those questions are likely to be positive. This may suggest that a relevant fraction of the observed uneven geographical distribution of high-skill jobs is attributable to the type of circular causality described in the present paper. As more empirical work is certainly needed, the proposed model yields precise falsifiable implications that could be tested through disaggregated analyses. In particular, it suggests that high-skill sectors, characterized by more relevant matching problems, should display a stronger propensity to geographical concentration than low-skill sectors, in which labor market frictions are likely to be less important.

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Appendix

This appendix analyzes the role of trade costs in determining the stability of the symmetric equilibrium in the case of increasing returns in matching. To simplify the presentation, we assume that both regions are similar in terms of the underlying parameters and normalize regional populations to 1.

We begin by deriving the necessary condition for stability of the symmetric equilibrium. If it is stable, starting from it, a transfer of firms from one region to the other, say from West to East should generate a decrease (increase) in western (eastern) expected profits. Specifically, say we transfer ε, i.e. $(dn = -dn^* = \varepsilon)$. Then we have:

$$sign(d\Pi_{|Trsfr.}) = sign(\chi n_s^{\chi-1}(\pi - 1)\varepsilon + n_s^{\chi} d\pi_{|Trsfr.})$$

where $\pi = \left\{(\alpha + n^*\tau)^2 + [\alpha - (n^* + 1)\tau]^2\right\} / \left[\beta(n^* + n + 1)^2\right]$ is western operating profits and $d\pi_{|Trsfr.} = -2\tau^2\varepsilon / [\beta(1 + 2n_s)] < 0$ is the change in such profits caused by the transfer. Accordingly, for the symmetric equilibrium to be stable, the following condition should be satisfied:

$$\chi n_s^{\chi-1}(\pi - 1) - \frac{2\tau^2}{\beta(1 + 2n_s)} n_s^{\chi} < 0$$

or equivalently, by using the zero profit condition:

$$\frac{\chi\kappa\beta}{2} - \tau^2 \frac{n_s^{\chi+1}}{1 + 2n_s} < 0 \tag{A.1}$$

It is easy to check that for $\tau = 0$, this condition is never met and therefore the symmetric equilibrium is unambiguously unstable. On the contrary, if τ reaches the limit above which trade vanishes i.e. $\tau_{\max} = \alpha / (n_s + 1)$ (which implies $n_s = \alpha/\tau_{\max} - 1$), we have no unambiguous sign for (A.1). Thus, for $\tau = \tau_{\max}$, the symmetric equilibrium is stable or unstable depending on parameter values.

What happens between these two extreme values of trade costs? Let us first examine the relationship between τ and n_s as implicitly described by the zero profit condition. Using its total derivative, we have:

$$\frac{\partial n_s}{\partial \tau} = \frac{-n_s^\chi \frac{\partial \pi}{\partial \tau}}{\frac{\chi}{n_s}\kappa + n_s^\chi \frac{\partial \pi}{\partial n_s}}$$

with

$$\frac{\partial \pi}{\partial \tau} = \frac{2(-\alpha + \tau + 2\tau n_s + 2\tau n_s^2)}{\beta(1+2n_s)^2}$$
$$\frac{\partial \pi}{\partial n_s} = \frac{-2(-2\alpha + \tau)^2}{\beta(1+2n_s)^3} < 0$$

While the zero profit condition cannot be solved analytically for n_s, it is possible to do so for τ which yields two roots (their tedious expressions are not reported here). Note first that, for $\tau = 0$, we have $\partial\pi/\partial\tau = -2\alpha/\left[\beta(1+2n_s)^2\right] < 0$, which implies $\partial n_s/\partial\tau < 0$. Second, there exists a value of trade costs, $\overline{\tau}$, which is the solution of $n_s^{-1}(\tau) = \alpha/\left[1 + 2n_s + 2n_s^2\right]$, such that for $\tau = \overline{\tau}$ we have $\partial\pi_s/\partial\tau = 0$ implying $\partial n_s/\partial\tau = 0$. Taken together, these observations suggest the following characteristics for the function $n_s(\tau)$. Low values of τ yield $\partial n_s/\partial\tau < 0$ until transport costs reach the level $\overline{\tau}$ for which $\partial n_s/\partial\tau = 0$. This means that $n_s(\tau)$ has reached a minimum. We therefore have $\partial n_s/\partial\tau < 0$ for $\tau < \overline{\tau}$, $\partial n_s/\partial\tau = 0$ for $\tau = \overline{\tau}$, and $\partial n_s/\partial\tau > 0$ for $\tau > \overline{\tau}$.

We can now turn to the analysis of the stability condition (A.1). Let us rewrite it as:

$$\frac{\chi\kappa\beta}{2} < \tau^2 \frac{n_s^{\chi+1}}{1+2n_s} \tag{A.2}$$

Taking the derivative of the right hand side (RHS) of (A.2), we obtain:

$$\frac{\partial RHS}{\partial \tau} = [\frac{(1+\chi)n_s^\chi + 2\chi n_s^{1+\chi}}{(1+2n_s)^2}]\tau^2 \frac{\partial n_s}{\partial \tau} + 2n_s^{1+\chi}\tau$$

One can easily check that, $\tau = 0$ implies $\partial RHS/\partial\tau = 0$, while, using the information collected on $n_s(\tau)$, $\tau \geq \overline{\tau}$ implies $\partial RHS/\partial\tau \geq 0$. On the contrary, for $\tau < \overline{\tau}$, we cannot unambiguously sign $\partial RHS/\partial\tau$ and we must rely on numerical results. Among the large number of simulations realized, we have always observed $\partial RHS/\partial\tau > 0$. These results are depicted in Figure 4.

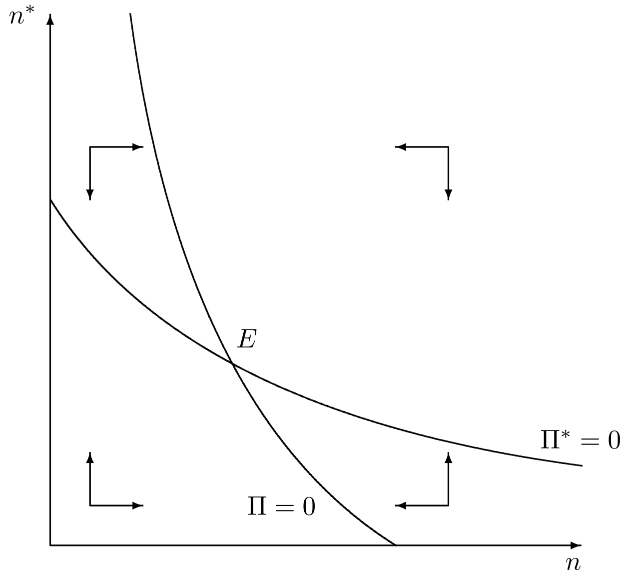

Figure 1: Decreasing returns to scale

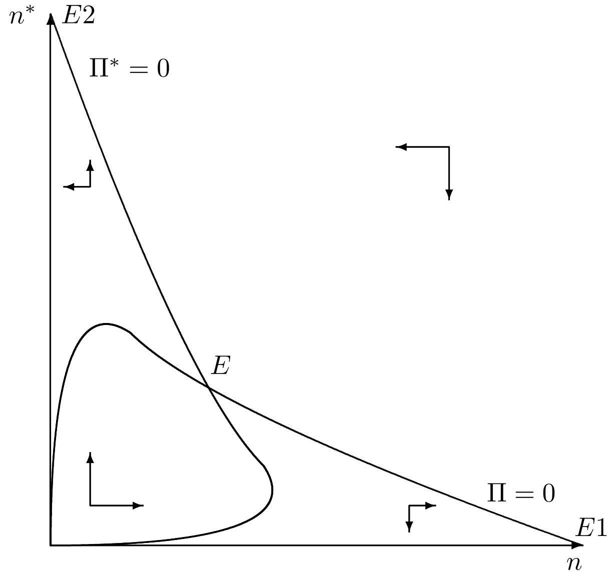

Figure 2: Increasing returns to scale (low transaction costs)

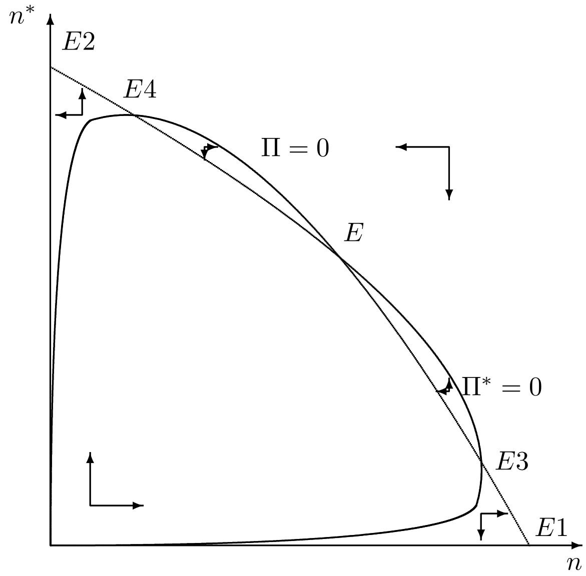

Figure 3: Increasing returns to scale (intermediate transaction costs)

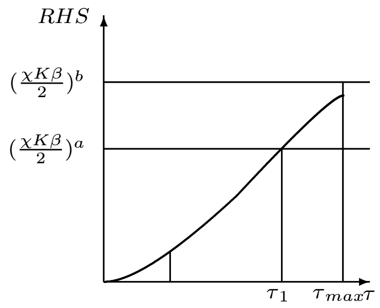

Figure 4: Transport costs and stability of the symmetric equilibrium