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Alejandro Cuñat and Marco Maffezzoli

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Alejandro Cuñat, London School of Economics (LSE) and CEPR
Marco Maffezzoli, Universita Bocconi

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Centre for Economic Policy Research
90–98 Goswell Rd, London EC1V 7RR, UK
Tel: (44 20) 7878 2900, Fax: (44 20) 7878 2999
Email: cepr@cepr.org, Website: www.cepr.org

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ABSTRACT

Neoclassical Growth and Commodity Trade*

We construct and numerically solve a dynamic Heckscher-Ohlin model in which the initial distribution of production factors in the world makes worldwide factor price equalization impossible, and leads countries to group in two diversification cones. We study the dynamics of income *per capita* and factor prices. Our results suggest that the Ramsey model under complete specialization overcomes several shortcomings of its autarky and factor price equalization counterparts. In comparison with the autarky model, for example, it can produce similar transitional dynamics and account for important cross-sectional differences in the levels and growth rates of income *per capita* while generating much smaller rental-rate differentials across countries. Moreover, it does not necessarily yield convergence in levels for identically parameterized economies. All in all, the Ramsey/Complete Specialization model seems to provide a better benchmark from which to depart when studying the dynamic behaviour of countries and cross-sectional differences in income *per capita* levels and growth rates.

JEL Classification: F10, F40 and O40
Keywords: convergence, economic growth, Heckscher-Ohlin, international trade and simulation

Alejandro Cuñat
London School of Economics (LSE)
Houghton Street
London
WC2A 2AE
Tel: (44 20) 7955 6961
Email: a.cunat@lse.ac.uk

Marco Maffezzoli
Università Bocconi
Via Salasco 3/5
20136 Milano
ITALY
Tel: (39 02) 5836 3314
Fax: (39 02) 5836 3309
Email: marco.maffezzoli@uni-bocconi.it

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1 Introduction

The neoclassical literature on economic growth considers the world as a collection of Solow-Ramsey economies, and explains differences in growth rates of income per capita on the basis of diminishing returns to capital. This view of the world implies the so-called convergence hypothesis, according to which differences in per capita income across countries tend to disappear as long as countries possess identical technologies, preferences, and population growth rates. In terms of growth rates, the Ramsey model delivers "absolute" convergence - poor economies grow faster than rich ones. This strong version of the neoclassical growth model seems to be at odds with the data, but a weaker one has become quite popular in the last 15 years.

Empirical work by Barro (1991), Barro and Sala-i-Martín (1992), and many others seems to support the existence of "conditional" convergence - economies grow faster the further away from their own steady states. Concerning the levels, Mankiw *et al.* (1992) explain the persistent cross-country differences in income per capita observed in the data on the basis of a Solow model with different savings rates across countries, which leads to cross-country differences in steady-state income per capita levels. The assumption that countries have different parameters ruling their saving rates is based on the observation of persistent cross-country differences in investment rates. These results are interpreted as evidence that the simple neoclassical growth model under autarky is consistent with observed growth patterns and cross-sectional income per capita differences, provided that countries are Solow-Ramsey economies with different parameters.

This framework has, however, many shortcomings, both theoretical and empirical. As pointed out by Lucas (1990), diminishing returns lead to large differentials in rates of return to capital between rich and poor countries that are hard to reconcile with a reasonable degree of international capital mobility, since the neoclassical growth model attributes differences in income per capita mainly to differences in aggregate capital-labor ratios. A related point, raised by King and Rebelo (1993), is that the Ramsey model "...cannot account for much growth without generating very large marginal products in the early stage of development." In other words, high growth rates are linked to high interest rates. Another important problem is that the role of international trade (or, in general, openness) is completely neglected. We think we need not justify our criticisms of the autarky assumption: countries trade in goods at length. Thus, assuming that the evolution of a country's prices depends exclusively on domestic factors seems to be a remarkable leap of faith.[1]

[1] Obviously, another important theoretical shortcoming of the neoclassical growth model is the exogeneity of technical progress, which does not help us understand why it takes place, and why it may vary across countries. We downplay this issue here, since it is not the scope of this paper. However, our

On the empirical side, Canova and Marcet (1995) show that the cross-country distribution of steady states is largely explained by the cross-sectional distribution of initial income per capita rather than controls such as government policies or education levels. That is, it is hard to find statistically significant variables that may reflect cross-country parameter differences but for the countries' own initial income per capita levels. On the other hand, Boldrin and Canova (2001) show that although regions within a country may exhibit growth rate convergence, income per capita differences across regions tend to be quite persistent. This fact also poses a problem to the neoclassical growth model, since it is hard to think about parameter differences across regions in the same country.

This paper addresses these shortcomings by allowing countries to trade in a complete specialization scenario. We combine the Ramsey model with a three-good, two-factor Heckscher-Ohlin model under general assumptions on the initial trade regime. In particular, we allow the initial cross-country distribution of production factors to render worldwide Factor Price Equalization (hereafter FPE) impossible and lead to Complete Specialization (hereafter CS), where by CS we mean that neither country is able to produce all goods in equilibrium due to comparative advantage considerations (*i.e.*, differences in factor prices across countries). Being no closed-form solution available, we solve the model numerically for a given benchmark parameterization and study the transitional dynamics and steady-state values of all variables of interest.[2]

Our choice of the CS framework is not arbitrary. A stream of recent research in international trade is casting serious doubts about the validity of the Heckscher-Ohlin model's "popular" FPE case. Davis and Weinstein (2001), for example, show that the Heckscher-Ohlin model's predictions on the net factor content of trade are much more in accordance with the data when they allow for CS.[3] A possible explanation for the lack of FPE in a large sample of countries is that capital-labor ratios are too different for the FPE condition to hold, leading to CS.[4] Besides being a more realistic scenario than autarky and FPE, the CS model[5] is capable of delivering a number of results that make it more appealing than its autarky counterpart for understanding the economics of capital accumulation and economic growth:

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analytical framework may easily generate cross-country differences in the levels and growth rates of total factor productivity on the basis of different specialization patterns.

[2] The model abstracts from human capital, differences in technologies, productivity growth, and all sorts of market imperfections, since our main goal is studying the influence of the trade scenario on the dynamics of countries' incomes through prices. Lucas (1988) and Grossman and Helpman (1991) provide suggestive insights on how international trade may affect the growth performance of countries through some of these alternative channels.

[3] See Harrigan (2001) for a review of this literature.

[4] This has been argued by Debaere and Demiroglu (1999) and Cuñat (2000).

[5] The Ramsey model under CS is identified by the initial conditions, *i.e.* by an initial cross-country distribution of production factors that makes FPE impossible. However, this does not rule out FPE in the long run. Stiglitz (1971) discusses conditions under which FPE holds in the long run.

1. During the transition, poor countries exhibit higher growth rates than rich countries. In the steady state, the model predicts equal growth rates for all countries.[6]

2. In comparison with the autarky model, ours accounts for important cross-sectional differences in the levels and growth rates of income per capita while generating smaller rental-rate differentials across countries.

3. Introducing international trade in a Ramsey model leads countries to different steady-state levels of income per capita in a framework in which countries only differ in their initial capital-labor ratios. Even when parameters are identical for all countries, poor countries do not manage to reach the steady-state income per capita levels of rich countries.

4. A related result implies that a country suffering a shock need not return to its previous growth path.

5. Another prediction of the model is that income per capita levels and investment rates are positively correlated across countries with identical parameters.

6. Our model can produce distribution dynamics in accordance with the club-convergence hypothesis, advocated for example by Quah (1996).

Part of our contribution is also methodological. Our model consists of two heterogeneous representative households (each representing a region or country) that accumulate capital and trade with each other under two possible trade regimes. The capital accumulation process leads eventually to a trade regime switch. This implies that the policy functions for consumption display a particular shape. We numerically approximate the optimal policy functions as implicitly defined by the Euler equations. To cope with their shape, we develop a two-step procedure. For that purpose we adapt to our needs the Galerkin projection method, as described in Judd (1992); the solution procedure seems to be a good compromise between numerical accuracy and computational complexity.

The closest reference to our paper is Ventura (1997), who remedies some of the shortcomings of the neoclassical growth model by studying a combination of the Ramsey model with the Heckscher-Ohlin model under a conditional version of worldwide FPE. The key insight in his model is that factor prices, which are equal across countries, are independent of domestic factor endowments and just depend on the world's aggregate factor endowments. However, as we noted above, Ventura's FPE model shares one problem with the closed-economy Ramsey model: the scenarios assumed in both models are counterfactual.

[6] The convergence speeds generated by our model are roughly in line with estimates in Canova and Marcet (1995) and Caselli *et al.* (1996).

Our setup enables us to frame CS as the "standard" trade regime of a model that has autarky and FPE for any cross-country distribution of factor endowments as limiting cases; what distinguishes these three cases is the varying dispersion in the relative factor intensities used in the production of different goods. An important difference among CS, autarky and FPE lies in the determination of factor prices. Whereas under autarky and FPE factor prices are determined exclusively by the path of the domestic capital-labor ratio and the world's capital-labor ratio, respectively, under CS factor prices depend on both domestic *and* foreign factor endowments.

A recent paper by Atkeson and Kehoe (2000) produces a dynamic two-sector Heckscher-Ohlin model that shows how a poor country's growth performance and steady state depend on its initial position relative to the rest of the world's diversification cone, which is assumed in steady state. We assume initial conditions that place all countries far from their steady states, and study the evolution of the trading equilibrium over time, allowing for changes in regime between CS and FPE. Thus, in comparison with Atkeson and Kehoe (2000), our model has the potential to describe the dynamics of the entire distribution of countries' per capita incomes, as well as the distribution of their steady states.

Another important precedent to our work is Deardorff (2001), who produces a model in which a steady-state equilibrium with two cones is possible in an overlapping generations framework in which savings are determined by wages. This leads countries to group in 'clubs.' In a sense, we are producing the Ramsey-counterpart to Deardorff's model. As we mentioned above, our model shows that a standard Ramsey model can also generate equilibria in which countries that only differ in initial capital stocks converge to different steady-state levels of income per capita. In comparison with Deardorff, we also study the dynamic behavior of countries within the same cone.

Finally, the importance of complete specialization has also been studied by Acemoglu and Ventura (2000) in an endogenous growth framework. They show that even in the presence of linear technologies, *de facto* diminishing returns to capital can occur because of changes in the terms of trade of completely specialized countries.

The rest of the paper is structured as follows: Section 2 presents a two-region static model of international trade with three possible regimes (autarky, FPE and CS). It discusses how the distribution of factor endowments across countries determines the trade regime in place and how factor prices are determined in each case. In Section 3 we combine the static model with a two-country Ramsey model, and discuss both the functional equations characterizing a recursive competitive equilibrium under perfect foresight and the model's steady state. In Section 4 we study the dynamics of our variables of interest. We compare the predictions of the CS model with those of the autarky model and the FPE model, and analyze the many-country case by splitting each region in two countries.

Section 5 concludes.

2 International Trade: FPE and CS

Our static trade model is a relatively simple version of the Heckscher-Ohlin model. Thus, economies trade in goods and differ only in their relative factor endowments. Let us initially assume that there are two regions in the world (North and South), indexed by $j \in \{N, S\}$, with identical technologies and preferences, and competitive markets. The world has $k = k_N + k_S$ units of capital and $l = l_N + l_S$ units of labor. We assume $l_N = l_S = 1$ constant. Without loss of generality, let us assume that the North is the capital-abundant region, and that both have positive capital stocks: $k_N > k_S > 0$.

Regions produce a final good y with a Cobb-Douglas production function of the form $y = \phi x_1^{\alpha/2} x_2^{1-\alpha} x_3^{\alpha/2}$, where $\alpha \in [0, 1]$ and ϕ is a positive constant. The final good, which is also the numeraire ($p_y = 1$), is produced out of three intermediate inputs x_z, with prices p_z, $z \in \{1, 2, 3\}$. Intermediate goods, in turn, are produced with the following technologies: $x_1 = l_1$, $x_2 = k_2^{1/2} l_2^{1/2}$, and $x_3 = k_3$. Let us assume that: (i) the final good y cannot be traded, whereas intermediates can be traded freely; (ii) there is no international factor mobility.[7]

The following results can be shown:

1. The solution to the model is unique: for any pattern of factor endowments, a unique pricing pattern of goods and factors is determined. Also, there is a unique equilibrium pattern of world and regional consumptions of intermediate goods, with consumption ratios the same in every region.[8]

2. The static model can lead to only two scenarios: (i) worldwide FPE, in which both North and South produce the three goods; (ii) CS, with capital-abundant North producing goods 2 and 3, and capital-scarce South producing goods 1 and 2. What scenario actually takes place depends on the distribution of factor endowments across North and South. If they are 'similar enough', we will have FPE. If they are 'too diverse', we will have CS.[9]

[7] In a recent paper, Kraay et al. (2000) find that only a small amount of capital flows from rich countries to poor countries. They argue in terms of sovereign risk to explain this phenomenon. Labor migration from poor countries to rich countries tends to be low for a number of reasons (e.g., legal and language barriers). To dispense with modelling difficulties, we simply assume factor mobility away.

[8] This is a standard result in international trade theory. See, for example, Dixit and Norman (1980) and Dornbusch et al. (1980).

[9] No other scenario is possible for the following reason: first, given that both N and S have positive amounts of capital and labor, full employment of resources implies they cannot specialize completely in good 1 one or good 3. Second, CS in good 2 is not possible either, since a region with comparative advantage in this good would also have a comparative advantage in either of the other goods, due to

The choice of such a restrictive model is not without loss of generality. In general, we do not know in how many cones the world sorts itself in the absence of worldwide FPE. Our assumption of only two cones is made first for simplicity. Secondly, it corresponds to the idea that we can roughly divide the world in rich and poor countries.

2.1 The Integrated Equilibrium

To understand what we mean by 'similar enough' and 'too diverse', let us review the concept of integrated equilibrium, which is defined as the resource allocation the world would have if both goods and factors were perfectly mobile internationally.[10] The FPE set is the set of distributions of factors among economies that can achieve the integrated equilibrium's resource allocation if we allow for free international trade, but no international factor mobility. Intuitively, the FPE set is the set of distributions of factors across economies that enable them to achieve full employment of resources while using the techniques implied by the integrated equilibrium. Thus, if the vector of production factors lies within the FPE set, the trading equilibrium will reproduce the integrated equilibrium's factor prices.

The world's integrated equilibrium behaves like a closed economy. Therefore factor prices in the integrated equilibrium depend on world aggregates. In terms of our model, the wage rate w and the rate of return to capital r depend, respectively, positively and negatively on the world's capital-labor ratio k/l: $w = \xi\,(k/l)^{1/2}$, $r = \xi\,(k/l)^{-1/2}$, where ξ is a positive constant.[11] Subsequently, the relationship between the factor-price ratio $\sigma \equiv w/r$ and the world's capital-labor ratio is positive: $\sigma = k/l$. The unitary elasticity is due to the Cobb-Douglas assumption.

The equilibrium sectorial allocation of production factors is as follows: $(k_1, l_1) = (0, \alpha l)$, $(k_2, l_2) = ((1-\alpha)k, (1-\alpha)l)$, and $(k_3, l_3) = (\alpha k, 0)$. This integrated equilibrium is depicted in Figure 1. The whole length of each axis represents the total amount of the corresponding factor in the world. The FPE set is delimited by the thick line, which is constructed by aligning the integrated equilibrium's sectorial allocation vectors from more to less capital-intensive. The slope of each vector reflects the capital-labor intensity of the corresponding sector, where k and l represent the world endowments of capital and labor, respectively. Notice that the proportions of capital and labor allocated to each sector are constant; this result is also due to the Cobb-Douglas assumption. Although it

$(w/r)_N \neq (w/r)_S$. This implies that in the absence of worldwide factor price equalization each region produces two goods. Moreover, in such a scenario we cannot have one region producing goods 1 and 3: with different factor prices across regions, a region cannot have a comparative advantage in the production of both of these two goods.

[10] See Dixit and Norman (1980).

[11] The entire solution of the static trade model is discussed in Appendix A.

is quite restrictive, it helps us obtain a very simple condition for FPE, as we show below.

Notice that our modelling strategy allows us to nest the three alternative trade regimes in the same framework. When $\alpha = 1$, only infinitely labor-intensive good 1 and infinitely capital-intensive good 3 are produced in the integrated equilibrium. This would grant FPE for any distribution of factors across regions.[12] In terms of Figure 1, the FPE set would be the entire box. The smaller α, the less likely FPE. Finally, when $\alpha = 0$, both regions produce only intermediate good 2, and therefore need not trade with each other. In fact, they behave as if they were closed economies with aggregate production function $y_j = \phi k_j^{1/2} l_j^{1/2}$, $j = \{N, S\}$. In this sense, the FPE and autarky cases are limiting cases of a more general model that allows for the possibility of CS as the "standard" case.

2.2 The Factor Price Equalization Condition[13]

Let us assume $\alpha \in (0, 1/2)$ for convenience. Consider the following distribution of capital stocks across North and South: $k_N = (1/2 + \varepsilon)k$, $k_S = (1/2 - \varepsilon)k$, $\varepsilon \in (0, 1/2)$. With each region having one unit of labor, ε determines differences in relative factor endowments across regions, and therefore whether the FPE condition holds. Figure 1 depicts two possible distributions of production factors across N and S. The factor endowments of North and South are measured with respect to O_N and O_S, respectively. The two regions are aligned from more to less capital-labor abundant. The vertical dotted line separates the two regions' labor endowments.

The variable k_N^{FPE} denotes the North's largest capital stock that allows for FPE. One can obtain k_N^{FPE} by realizing that it is the capital stock that enables the North to produce all of the integrated equilibrium's production of good 3 (from the integrated equilibrium's allocation, $k_3 = \alpha k$), and the part of the integrated equilibrium's production of good 2 that employs half the world's population (from the integrated equilibrium's allocation, $(k_2/l_2)(1/2)l = (k/l)(1/2)l = (1/2)k$).[14] Thus, $k_N^{FPE} = (1/2 + \alpha)\, k$.

Hence, for $\varepsilon \in (0, \alpha]$ the FPE condition holds. The short-dashed vectors of Figure 1 represent this case: factor endowments are 'similar enough' relative to the integrated equilibrium's technologies for the integrated equilibrium to be reproduced by the trading equilibrium. For $\varepsilon \in (\alpha, 1/2)$ the distribution of production factors across North and South instead violates the FPE condition. The long-dashed vectors of Figure 1 represent this case: regions are 'too diverse' in their capital-labor ratios and therefore specialize completely.

[12] This is the case analysed in Ventura (1997).

[13] See Deardorff (1994), Debaere and Demiroglu (1999) and Cuñat (2000) for formal discussions on how to assess the FPE condition.

[14] This is where the assumption that $\alpha < 1/2$ applies, since it guarantees $l_2 > (1/2)l$.

2.3 Complete Specialization

If the factor endowment vectors lie outside the FPE set (*i.e.*, if $\varepsilon \in (\alpha, 1/2)$), the trading equilibrium cannot reproduce the integrated equilibrium. This leads factor prices to differ across North and South, which specialize in different ranges of goods according to comparative advantage. As we mentioned above, under our assumptions there is only one equilibrium pattern of CS, which implies $x_{1N} = x_{3S} = 0$, and $x_{1S}, x_{2S}, x_{2N}, x_{3N} > 0$.

From the CS equilibrium conditions we obtain the following system of two equations, which yields the factor-price ratios σ_j as functions of the two capital stocks, $\sigma_j = \sigma_j(k_N, k_S)$:

$$(1-\alpha)\sqrt{\sigma_S} - \frac{k_S}{\sqrt{\sigma_S}} = \alpha\sqrt{\sigma_N} \tag{1}$$

$$(1-\alpha)\frac{k_N}{\sqrt{\sigma_N}} - \sqrt{\sigma_N} = \alpha\frac{k_S}{\sqrt{\sigma_S}} \tag{2}$$

Appendix A discusses the entire solution of the model. It also shows that by manipulating the equilibrium's pricing equations we can write factor prices as functions of the factor-price ratios, *i.e.* $w_j = w_j(\sigma_N, \sigma_S)$ and $r_j = r_j(\sigma_N, \sigma_S)$. Hence, factor prices are also functions of the capital stocks of North and South: $w_j = w_j(k_N, k_S)$, and $r_j = r_j(k_N, k_S)$.

The following results are worth mentioning:[15]

1. The North's factor-price ratio is greater than that of the South: $\sigma_N > \sigma_S$ ($w_N > w_S$ and $r_N < r_S$). Notice that otherwise the North would neither have a comparative advantage in the production of good 3 nor a comparative disadvantage in the production of good 1.

2. The ratio σ_N depends positively on k_N, and negatively on k_S. An increase in k_N creates an excess supply of good 3. This causes $p_3 = r_N$ to decrease. An increase in w_N and a decrease in r_N induce a higher capital-labor intensity in sector 2, helping achieve full employment of resources in the North. An increase in k_S implies a rise in income and spending on all goods for the South. This creates an excess demand of good 3, produced exclusively by the North, and an excess supply of good 2. The latter turns out to have a stronger effect, contributing to a fall in both r_N and w_N. The fall in w_N is more pronounced than the fall in r_N; this leads to a lower σ_N, a

[15] To obtain these results, we numerically approximate the solution to (1)-(2), construct the factor prices, and obtain their partial derivatives with respect to k_N and k_S. In particular, we approximate the solution over a rectangle $D \equiv [\underline{k}, \overline{k}] \times [\underline{k}, \overline{k}] \in R_+^2$ with a linear combination of multidimensional orthogonal basis functions taken from a *2*-fold tensor product of Chebyshev polynomials, and choose the coefficients using a simple collocation method. See Judd (1992) and Appendix B for more details.

lower capital-labor intensity in sector 2, and a subsequent transfer of capital from sector 2 to sector 3.

3. The ratio σ_S depends positively on both k_N and k_S. An increase in k_S requires a rise in σ_S (an increase in w_S and a decrease in r_S) that induces higher capital-labor intensities to achieve full employment of resources in the South. An increase in k_N causes an excess demand for good 1, produced exclusively by the South, and an excess supply for good 2. This leads to an increase in $p_1 = w_S$, and a decrease in r_S. The subsequent increase in σ_S produces a higher capital-labor intensity in sector 2, releasing labor towards sector 1.

The results of the CS case are, in a sense, halfway between the autarky and FPE results:

1. Under autarky, factor prices depend exclusively on domestic capital stocks: $r_j = r(k_j)$, $\frac{\partial r_j}{\partial k_j} < 0$; $w_j = w(k_j)$, $\frac{\partial w_j}{\partial k_j} > 0$; $j = N, S$.

2. In the FPE case, factor prices depend only on the world's capital stock: $r_j = r(k)$, $\frac{\partial r}{\partial k} < 0$; $w_j = w(k)$, $\frac{\partial w}{\partial k} > 0$; $j = N, S$.

3. In the CS case, each region's factor prices are affected by both the domestic and foreign capital stocks: $r_N = r_N(k_N, k_S)$, $\frac{\partial r_N}{\partial k_N} < 0$, $\frac{\partial r_N}{\partial k_S} < 0$; $w_N = w_N(k_N, k_S)$, $\frac{\partial w_N}{\partial k_N} > 0$, $\frac{\partial w_N}{\partial k_S} < 0$; $r_S = r_S(k_N, k_S)$, $\frac{\partial r_S}{\partial k_N} < 0$, $\frac{\partial r_S}{\partial k_S} < 0$; $w_S = w_S(k_N, k_S)$, $\frac{\partial w_S}{\partial k_N} > 0$, $\frac{\partial w_S}{\partial k_S} > 0$. Note that the CS case is not entirely symmetric in the signs of the derivatives. This is due to the fact that regions have different production structures.

3 The Dynamic Model

In this section we combine the static model discussed above with the discrete-time Ramsey model. Each region is populated by a *continuum* of identical and infinitely lived households, each of measure zero. Being identical, they can be aggregated into a single representative household. A unique homogeneous final good exists, that can be used for both consumption and investment. The preferences over consumption streams of the representative household in region j can be summarized by the following intertemporal utility function:

$$U_{j,t} = \sum_{s=t}^{\infty} \beta^{s-t} \ln c_{j,s} \tag{3}$$

where β is a subjective intertemporal discount factor and $c_{j,t}$ the per-capita consumption level in region j at date t.

The representative household maximizes (3) subject to the following intratemporal budget constraint:

$$c_{j,t} + \Delta k_{j,t} = w_{j,t} + (r_{j,t} - \delta)\, k_{j,t} \tag{4}$$

where $k_{j,t}$ is the current per-capita stock of physical capital in region j, $w_{j,t}$ the wage rate, $r_{j,t}$ the rental rate, and δ the depreciation rate. Factor prices are taken as given by the representative household. Depending on the distribution of capital across regions, factor prices $w_{j,t}$ and $r_{j,t}$ will be determined in the integrated or complete specialization equilibrium.

The first order conditions

$$\beta c_{j,t}(r_{j,t+1} + 1 - \delta) = c_{j,t+1} \tag{5}$$

$$k_{j,t+1} = w_{j,t} + (r_{j,t} + 1 - \delta)\, k_{j,t} - c_{j,t} \tag{6}$$

and the usual transversality conditions are necessary and sufficient for the representative household's problem.

3.1 Recursive Competitive Equilibrium

A recursive competitive equilibrium for this economy is characterized by equations (5)-(6) together with

$$w_{N,t} = w_{S,t} = \xi \left(\frac{k_t}{2}\right)^{\frac{1}{2}} \tag{7}$$

$$r_{N,t} = r_{S,t} = \xi \left(\frac{k_t}{2}\right)^{-\frac{1}{2}} \tag{8}$$

if $k_{N,t} \leq (1/2 + \alpha)\, k_t$, and

$$w_{N,t} = \xi \sigma_{N,t}^{\frac{2+\alpha}{4}} \sigma_{S,t}^{-\frac{\alpha}{4}} \tag{9}$$

$$w_{S,t} = \xi \sigma_{N,t}^{\frac{\alpha}{4}} \sigma_{S,t}^{\frac{2-\alpha}{4}} \tag{10}$$

$$r_{N,t} = \xi \sigma_{N,t}^{\frac{\alpha-2}{4}} \sigma_{S,t}^{-\frac{\alpha}{4}} \tag{11}$$

$$r_{S,t} = \xi \sigma_{N,t}^{\frac{\alpha}{4}} \sigma_{S,t}^{-\frac{(2+\alpha)}{4}} \tag{12}$$

if $k_{N,t} > (1/2 + \alpha)\, k_t$, where the values of $\sigma_{j,t}$ are implicitly defined by (1)-(2).

If a solution to (5)-(12) exists, the recursive structure of our problem guarantees the former can be represented as a couple of time-invariant policy functions expressing the optimal level of consumption in each region as a function of the two state variables, k_N

and k_S. These policy functions have to satisfy the following functional equations:

$$\beta c_j\left(k_N, k_S\right)\left(r_j' + 1 - \delta\right) = c_j\left(k_N', k_S'\right) \tag{13}$$

where:

1. $k_j' = w_j + (r_j + 1 - \delta)\, k_j - c_j\left(k_N, k_S\right)$;
2. $w_j = \xi\left(\frac{k}{2}\right)^{\frac{1}{2}}$, $r_j = \xi\left(\frac{k}{2}\right)^{-\frac{1}{2}}$ if $k_N \leq (1/2+\alpha)\,k$;
3. $w_N = \xi\sigma_N^{\frac{2+\alpha}{4}}\sigma_S^{-\frac{\alpha}{4}}$, $w_S = \sqrt{\frac{\sigma_S}{\sigma_N}}w_N$, $r_N = \xi\sigma_N^{\frac{\alpha-2}{4}}\sigma_S^{-\frac{\alpha}{4}}$, $r_S = \sqrt{\frac{\sigma_S}{\sigma_N}}r_N$ if $k_N > (1/2+\alpha)\,k$;
4. $r_j' = \xi\left(\frac{k'}{2}\right)^{-\frac{1}{2}}$ if $k_N' \leq (1/2+\alpha)\,k'$;
5. $r_N' = \xi\left(\sigma_N'\right)^{\frac{\alpha-2}{4}}\left(\sigma_S'\right)^{-\frac{\alpha}{4}}$, $r_S' = \xi\left(\sigma_N'\right)^{\frac{\alpha}{4}}\left(\sigma_S'\right)^{-\frac{(2+\alpha)}{4}}$ if $k_N' > (1/2+\alpha)\,k'$.

The values of σ_j and σ_j' are obtained by solving the nonlinear system (1)-(2) numerically. Furthermore, the policy functions have to generate stationary time series in order to satisfy the transversality conditions. To solve equation (13) numerically, we apply the projection methods described in Judd (1992). Appendix B describes our computational strategy in detail.

We discuss now our benchmark parameterization. In the Appendix we show that, under complete specialization, the parameter α corresponds to the volume of trade (the value of exports plus the value of imports) over income in the North. To make our choice of α less arbitrary, we parameterize it according to the average ratio of total trade over GDP, both at current prices, calculated for the US over the 1947:1-2001:4 time horizon, and set $\alpha = 0.15$. The initial values for the two regions' capital stocks are chosen arbitrarily: we set $k_N = 0.5$ and $k_S = 0.1$. Following Cooley and Prescott (1995), we assume $\beta = 0.949$ and $\delta = 0.048$.[16] Our parameterization is admittedly crude, since our main goal is a purely qualitative comparison among models under different trade regimes.[17]

[16] The scale parameter ϕ is calibrated to reproduce a world steady-state capital stock equal to unity in the CS model. Recall that in the long run the model switches to FPE. Our parameterization implies that the time unit is a year.

[17] In particular, the capital shares in income delivered by our parameterization are unrealistic. We chose a value of 1/2 for both coefficients in sector two's production function for symmetry reasons, in order to simplify both the algebra and the numerical computations. A coefficient for capital equal to 1/3 would yield an aggregate capital share in the neighborhood of 1/3 for both North and South, the benchmark value used by King and Rebelo (1993).

3.2 Steady State

Consider the Euler equation (5) and evaluate it at the steady state:

$$r_j = r \equiv \frac{1}{\beta} - 1 + \delta \tag{14}$$

Equation (14) pins down the steady-state interest rate as usual in Ramsey-type models. Evidently, FPE has to hold in steady state, since $r_N = r_S = r$. If FPE applies in steady state, then the equation

$$r = \xi \left(\frac{k}{2}\right)^{-\frac{1}{2}} \tag{15}$$

pins down the world-level steady-state capital stock uniquely. It is easy to show that (14) and (15) are enough to uniquely characterize the steady state at the *world level*.

However, any combination of k_N and k_S such that $k_N + k_S = k$ and FPE holds is compatible with the steady state, and hence (14) and (15) are unable to pin down the steady state at the *regional level*. Notice, however, that the multiplicity of steady states does not imply the indeterminacy of the optimal consumption plans: once the initial conditions k_{j0} are specified exogenously, the final steady state to which the system tends in the long run is fully determined and non-degenerate, because (*i*) the world as a whole is a standard stationary Ramsey economy with a well specified steady state; (*ii*) the regional policy functions for consumption are unique, and hence imply unique optimal paths for consumption, investment, and capital; (*iii*) the requirement that $k_N + k_S = k$ together with the FPE condition jointly imply that $k_{\min} \leq k_j \leq k_{\max}$ for some $k_{\max} > k_{\min} > 0$. In other words, the world reaches a steady state in which Equations (14) and (15) hold, and the cross-region distribution of capital stocks is not degenerate, *i.e.* both k_j's are strictly positive. Such a steady state may be characterized by different values of consumption, income, investment and capital across regions and countries.[18] Notice that in our framework the size of $k_{\min}$ and $k_{\max}$ depends exclusively on the size of the FPE region, and hence directly on the value of α.

[18] Krusell and Ríos-Rull (1999) obtain a similar result in a heterogenous-agent closed economy, driven essentially by the same economic mechanism: if all agents face the same rate or return, and this rate of return depends on the aggregate capital stock only, the steady-state wealth and income distributions are undetermined.

4 Results

4.1 Autarky

For comparison purposes, we first review the dynamics implied by the standard closed-economy Ramsey model under the same parameterization. The inability to trade yields an autarky model in which economies have the aggregate production function $y_j = \phi\alpha^{\alpha}(1-\alpha)^{1-\alpha} k_j^{1/2} l_j^{1/2}$, $j = \{N, S\}$.[19] Figures 3 and 4 illustrate the dynamic behavior of the levels and growth rates (denoted by γ_j) of income and capital for both North and South. Observe that, given our arbitrary choice of the initial conditions, the North is in steady state from the very beginning. Therefore it exhibits zero growth rates in all variables. The South displays positive growth rates initially, but the South-North growth differential falls over time until the South converges both in levels and growth rates to the North. Obviously, we observe σ-convergence, i.e. a decrease over time of the cross-region standard deviation of income and capital levels; in the long-run, the standard deviations tend to zero, as predicted by the absolute convergence hypothesis. Concerning the South's transitional dynamics, its speed of convergence is 6.1% per year. Figure 5 graphs the evolution of the rate of return to capital for both regions: notice the persistent differential in favour of the South.

The key role of diminishing returns to capital underlying these results is well known. A low capital stock (with respect to its steady-state level) implies a high marginal productivity of capital and a high rate of return, inducing an important process of capital accumulation that slows down over time, as the economy becomes richer. North and South exhibit the same parameterization, and consequently identical steady states. Figure 6 shows that, as expected, the capital-income ratios and the investment shares converge in both regions to the same value; however, during the transition the capital-income ratio remains consistently higher in the North and the investment share higher in the South.

As is well known, some of the counterfactual predictions of the neoclassical model (large interest rate differentials, apparent lack of convergence in levels) can be solved by assuming differences across economies leading them to different steady states. However, King and Rebelo (1993) show that similar shortcomings persist when one focuses on the "time-series" behavior predicted by the model for a single economy. Notice that in this model, if a country "loses" an important share of its capital stock, we should expect both its rate of return to capital and income per capita growth rate to increase dramatically, and its steady-state level of income per capita to remain the same. The predictions regarding the rate of return seem not to be borne by the growth experience of

[19] Notice that, but for the constant term $\alpha^{\alpha}(1-\alpha)^{1-\alpha}$, this is the production function of the autarky regime we obtained in Section 2.

countries that lost most of their physical capital stocks during World War II. Concerning the historical experience of "growth failures," to understand why some countries ceased to be among the richest in the world, in this framework one needs to assume that some of the parameters ruling their steady states changed over time.

4.2 Complete Specialization

With $k_N = 0.5$, $k_S = 0.1$ and $\alpha = 0.15$, we have $\alpha < \varepsilon$. Therefore the initial conditions imply the CS regime. Figures 7 through 10 summarize the dynamics of the CS case. Initial income levels turn out to be quite similar to those under autarky. Notice that the CS-regime lasts for 82 years before converging towards FPE, after which the world does not change regime any more. Unlike in the autarky model, North and South reach different steady-state levels of capital and income per capita despite being ruled by identical parameter values.[20]

Notice that the model also generates persistent cross-country differences in capital-output ratios and investment shares. Moreover, except for at the very beginning of the transition, there is a positive correlation between income per capita levels and investment ratios. Recall that a similar empirical finding in Mankiw *et al.* (1992) was interpreted in terms of the Solow model's steady-state predictions: cross-country (parameter) differences in saving rates lead to cross-country differences in steady-state income per capita levels in that model. Ours is an interesting counterpoint to this interpretation: differences in saving rates may arise endogenously simply due to the fact that countries trade and end up having strong differences in capital-labor ratios.

In spite of the lack of convergence in the levels, the model generates σ-convergence and convergence in growth rates under both trade regimes: convergence in growth rates is complete, as under autarky, whereas σ-convergence is now only partial, since the standard deviations of income and capital decrease over time during the transition but do not tend to zero in the long run. Table 1 reports differences in income levels over time yielded by the autarky and CS models. Notice that at time 1 both models deliver roughly the same difference. Over time, however, the autarky model washes away the whole difference quite rapidly, whereas income differences between North and South persist in the long run. Notice that under FPE convergence takes place despite North and South facing the same rate of return to capital.[21]

[20]The North reaches a steady-state level of income higher than under autarky, whereas the opposite holds for the South. This result can be related to the different rates of return yielded by the two models: whereas under FPE the North faces a higher rate of return than under autarky (for a given k_N/l_N), the South's rate of return is higher under autarky than under FPE (for a given k_S/l_S).

[21]The intuition can be found in Ventura (1997): North and South behave as permanent-income consumers. Given identical homothetic preferences across the two regions, they spend the same fraction of their wealth in each period, thus exhibiting identical rates of wealth accumulation. The growth rate of

		$t=1$	20	40	60	80	100
$\frac{y_N - y_S}{y_S}\%$	AU	123.61	19.78	4.87	1.33	0.37	0.11
	CS	*118.45*	*49.54*	*38.61*	*36.03*	*35.34*	*35.28*
$\gamma_S - \gamma_N$	AU	7.60	1.22	0.30	0.08	0.02	0.01
	CS	*4.60*	*0.71*	*0.17*	*0.04*	*0.01*	*0.00*
$(r_S - r_N)\,\%$	AU	12.58	2.01	0.50	0.14	0.04	0.01
	CS	*6.70*	*1.07*	*0.25*	*0.05*	*0.00*	*0.00*

Table 1: Income, growth, and interest rate differentials.

The variation in growth rates of income per capita across North and South displayed by the CS model is smaller than that of the autarky model. Table 1 summarizes the absolute differences between the growth rates of output in the North and the South at a few regularly spaced points in time: at the beginning of the transition, under CS $\Delta\gamma$ is 1.7 times lower than under autarky. In comparison with the autarky case, the North exhibits positive growth rates of income per capita over the transition. Finally, the convergence speed is 6.7% per year in both regions.

The evolution of factor prices implies $\sigma_N > \sigma_S$, $w_N > w_S$, and $r_N < r_S$ as long as CS applies. r_N decreases very slowly, whereas r_S falls rapidly in the first periods. Notice that the differential $r_S - r_N$ is smaller and less persistent over time than under autarky. Table 1 summarizes the differences between the interest rates in the North and the South, expressed in percentage points: at the beginning of the transition, when incomes are very similar both for autarky and CS, under CS $\Delta r\%$ is 2.2 times lower than under autarky. This suggests that reasonably low costs to capital mobility may rule out international movements of capital from North to South.[22] Furthermore, it shows that in our framework sizable cross-country differences in income per capital levels are compatible with small (possibly zero) interest rate differentials, during transition and in steady state.

Notice that at time 0 North and South have the same initial capital stocks and very similar levels of income per capita both under autarky and CS. On the other hand, the

wealth is a weighted average of the growth rates of its two components: the stock of capital and the net present value of wages. Under FPE the growth rate of the latter is the same for North and South, since wage growth is independent of domestic conditions and identical for all countries. If diminishing returns to capital at the world level are unimportant, wage growth will be slow as the world accumulates capital over labor. The capital-scarce region, the South, will need to accumulate more than the capital-abundant region for both to keep the same rate of wealth accumulation. Ventura (1997) shows that under FPE convergence in growth rates takes place as long as the elasticity of substitution in the final good aggregator y is sufficiently large. If the elasticity is sufficiently small, then the FPE model generates divergence in growth rates.

[22] Its is clear that increasing the initial gap in capital per capita rises the interest rate differential. Being the CS model a completely neoclassical model, diminishing returns to capital are still at work, and therefore high interest rates can be easily generated for low initial capital stocks. We keep a limited initial gap for numerical reasons, in order to guarantee the highest accuracy of the approximated solution. It should be clear that the discussion in our paper is based exclusively on a qualitative and quantitative *comparison* of two different models under the same parametrization.

U_1	AU	CS
North	-50.09	-49.65
South	-59.27	-58.55

Table 2: Welfare

inequality $r_S^{AU} > r_S^{CS} > r_N^{CS} > r_N^{AU}$ illustrates the effect of international trade on factor prices. Given that under autarky the interest rate is a function of only the country's own capital-labor ratio, large differences in capital-labor ratios across countries are translated into large rental-rate differentials. Under CS, in contrast, the equilibrium interest rates of North and South, even if different, must reflect relative scarcities of capital and labor at the world level, and are therefore closer to each other than under autarky.

Given that in the current framework (where only the intermediate goods are traded and factors are internationally immobile) regions reach steady-state levels different from those of the autarky model, we may wonder if international trade is beneficial for *both* North and South. Table 2 presents the simulated welfare levels under autarky and CS: both regions obtain the lowest welfare level under autarky.[23]

4.3 Many Countries

The division of the world in North and South helps us compare the dynamic behavior of poor and rich economies, but does not tell us about the dynamics of countries within each region. We address this issue by assuming that North and South consist each of two countries, *i.e.* $N = \{NW, NE\}$ and $S = \{SW, SE\}$, with identical preferences and technologies.[24] We consider an initial distribution of production factors across North and South identical to the one we assumed above: $l_N = l_S = 1$, $k_N = (1/2 + \varepsilon)k$, and $k_S = (1/2 - \varepsilon)k$, $\varepsilon \in (\alpha, 1/2)$. Without loss of generality we assume that in both regions the West is more capital-abundant than the East. Within each region j, production factors are initially distributed as follows: $k_{jW} = (1/2 + \varepsilon_j)k_j$, $k_{jE} = (1/2 - \varepsilon_j)k_j$, and $l_{jW} = l_{jE} = 1/2$, where $\varepsilon_j \in (0, 1/2)$. Assume ε_j small enough for FPE to hold within each region or diversification cone.

Rather than solving the corresponding dynamic general equilibrium, we make use of the results obtained for the CS model in the previous section. In particular, we take the paths of income, capital, consumption and prices of each cone and impose that the corresponding variables of East and West be consistent with their region's integrated equilibrium.[25] Obviously, we need to make sure that the distribution of factors across

[23] The numbers correspond to the discounted flow of utility, obtained from (3), over a 2000-year horizon.

[24] Alternatively, we may think of the East and West as smaller economic units (regions, provinces) within a country.

[25] Appendix B gives further details about this procedure.

East and West satisfies the condition for FPE within each cone, which is discussed in the Appendix.

We assume an initial distribution of factors across East and West that implies $\varepsilon_N = \varepsilon_S = 0.03$. After solving for the dynamic paths of East and West, we check whether the within-cone FPE condition holds. Figure 11 displays the growth rates of West and East. It is apparent there is convergence in growth rates within each cone. The intuition here is identical to that of the worldwide FPE case (see above). Notice that the within-cone partition of North and South produces additional cross-country variation in growth rates and income levels without higher interest-rate differentials. That is, $\gamma(y_{NW}) - \gamma(y_{SE}) > \gamma(y_N) - \gamma(y_S)$ and $y_{NW} - y_{SE} > y_N - y_S$, whereas $r_{SE} - r_{NW} = r_S - r_N$.

Figure 11 summarizes the intra-regional evolution of income levels: note that within each region we observe σ-divergence (cross-country differences in growth rates do not compensate cross-country differences in income per capita levels), whereas the world exhibits σ-convergence. Finally, Figure 12 plots the intra-regional output levels: the "poor" country in the South remains isolated, and a roughly bimodal distribution seems to emerge over time. These results suggest that countries with similar (but unequal) initial income per capita levels and production structures may not necessarily follow the same time path. The dynamics of factor prices, partly determined by the trade environment, can lead to diverging behavior and interesting distribution dynamics.

Notice that, unlike in the autarky model, shocks can lead countries to different steady states. Assume, for example, that at time 0 we redistribute some capital stock between the Southwest and the Southeast so that $k'_{SW} = k_{SE}$ and $k'_{SE} = k_{SW}$. In this case, we just need to relabel the paths of income per capita in Figure 12 correspondingly: the Southeast would now achieve a higher steady-state level of income per capita than the Southwest. Finally, notice also that in this framework a country that is sufficiently small compared to the region it belongs to and suffers a loss in its capital stock, might grow at a higher rate without necessarily experiencing a higher interest rate.

4.4 Factor Price Equalization

In the introduction, we discarded the FPE case on the basis that FPE does not seem to be a realistic scenario. For the sake of completeness, however, we now discuss the dynamics yielded by the FPE case with the proviso that the comparison with the CS model can not be perfect here: given our initial capital stocks $k_N = 0.5$ and $k_S = 0.1$, we need to enlarge the FPE set to ensure that FPE takes place from the very beginning. We do this by assuming $\alpha = 0.4$.[26]

[26] Hence, we compare the CS and FPE trade regimes in the same theoretical framework but under different parameterizations. To minimize the effects of this necessary but unpleasant choice, we recalibrate

Figures 13 through 16 plot the dynamic behavior of the relevant variables for both North and South. As in the autarky and CS cases, the FPE model yields convergence in growth rates in the sense that (*i*) the poor country has a higher growth rate than the rich country, and (*ii*) in the long run growth rates are equal across countries. The speed of convergence is slightly lower than under CS, and equal to 6.1% per year in both countries. Notice, however, that the growth rate differentials generated here are much smaller than under CS or autarky, while the interest rate differentials are, by construction, zero. Concerning the levels, the FPE model is similar to the CS model in the sense that it yields different steady-state income per capita levels for countries with different initial capital-labor ratios. However, it delivers σ-divergence: income per capita differences increase over time, since cross-country differences in levels are large relative to cross-country differences in growth rates.[27]

5 Concluding Remarks

Complete specialization is a more realistic trade scenario than autarky or factor price equalization for empirical reasons. Moreover, a very stylized dynamic macroeconomic model that implies complete specialization in its initial conditions yields more realistic dynamics or, at least, overcomes some shortcomings in the neoclassical growth model and the combination of the Ramsey model with FPE. In a strictly neoclassical framework in which countries only differ in initial capital-labor ratios, convergence in growth rates obtains without implying absolute convergence in levels. The CS model can also generate a sizable cross-country variation of growth rates of income per capita without yielding as large interest-rate differentials as in the autarky case. Moreover, it has the potential of producing realistic distribution dynamics. Thus, the Ramsey/CS model seems to be a better benchmark from which to depart when studying the economics of capital accumulation and income per capita growth: if one needs a starting point to assess the importance of human capital or technical progress, then it might be better to start from here rather than autarky.

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the value of the scale parameter ϕ to make again the model reproduce a steady-state capital stock in the world equal to one: this implies that the vale of ξ remains the same under both parametrizations. Given that under FPE the parameter α influences the dynamics of the system only trough the size of the FPE set and the value of ξ, in the experiment discussed here we are essentially isolating the first effect from the second.

[27] Ventura (1997) points out that the predictions of his two-good FPE model (that is, the case in which $\alpha = 1$ in our model) depend crucially on the value of the elasticity of substitution in the production function of the final good. We have simulated our three-good FPE model for a wide range of elasticities of substitution (0.1-4). In comparison with the results we report here, we find no remarkable qualitative differences concerning the time paths of both regions' income per capita levels and growth rates. These results are available upon request.

One may wonder whether the transition towards FPE can be long enough to sustain the relevance of the CS regime. In this respect, notice that we obtained a lengthy transition towards FPE despite assuming an initial relative capital-labor ratio between North and South of only five (or, alternatively, an initial relative income per capita ratio of roughly two). Higher relative factor-endowment ratios between rich and poor countries, a lower elasticity of intertemporal substitution, and positive depreciation rates would make the transition more protracted. The same would occur in the presence of an additional production factor such as human capital for several reasons: the FPE condition would imply an additional set of constraints, and human capital is very unevenly distributed across countries and tends to be accumulated more slowly than physical capital. Besides, major disruptions such as wars and technical progress are likely to increase the cross-country dispersion of factor endowments.

This paper underlines the importance of international trade for the understanding of economic growth in a world made of open economies and with imperfections in international capital markets. Further research on trade regimes may help us create more realistic scenarios to analyze the growth performance of countries more accurately.

6 References

Acemoglu, D. and J. Ventura (2000): "The World Income Distribution," *Quarterly Journal of Economics*.

Atkeson, A. and P. J. Kehoe (2000): "Paths of Development for Early- and Late-Bloomers in a Dynamic Heckscher-Ohlin Model," Federal Reserve Bank of Minneapolis Research Department Staff Report 256.

Barro, R.J. (1991): "Economic Growth in a Cross-Section of Countries," *Quarterly Journal of Economics*, 106, 407-443.

Barro, R.J. and X. Sala i Martin (1992): "Convergence," *Journal of Political Economy*,100, 223-251.

Boldrin, M. and F. Canova (2001): "Inequality and Convergence in Europe's Regions: Reconsidering European Regional Policies," *Economic Policy*, 32, 205-253.

Canova, F. and A. Marcet (1995): "The Poor Stay Poor: Non-Convergence across Countries and Regions," UPF Working Paper 137.

Caselli, F., G. Esquivel, and F. Lefort (1996): "Reopening the Convergence Debate: A New Look at Cross-Country Growth Empirics," *Journal of Economic Growth*, 1, 363-390.

Cooley, T. and E. Prescott (1995): "Economic Growth and Business Cycles," in T. Cooley, ed., *Frontiers of Business Cycle Research*, Princeton Univ. Press, Princeton, 1-38.

Cuñat, A. (2000): "Can International Trade Equalize Factor Prices?" IGIER WP

180.

Davis, D.R. and D. Weinstein (2001): "An Account of Global Factor Trade," *American Economic Review*.

Deardorff, A. (1994): "The Possibility of Factor Price Equalization, Revisited," *Journal of International Economics*, 36, 167-175.

Deardorff, A. (1997): "Rich and Poor Countries in Neoclassical Trade and Growth," *Economic Journal*, 111, 277-295.

Debaere and Demiroglu (1998): "On the Similarity of Country Endowments," University of Texas, Austin, manuscript.

Dixit, A.K. and V. Norman (1980): *Theory of International Trade*, Cambridge University Press.

Dornbusch, R., S. Fischer and P. Samuelson (1980): "Heckscher-Ohlin Trade Theory With a Continuum of Goods," *Quarterly Journal of Economics*, 95, 203-224.

Grossman, G. and E. Helpman (1991): *Innovation and Growth in the Global Economy*, MIT Press, Cambridge, Massachusetts.

Harrigan, J. (2001): "Specialization and the Volume of Trade: Do the Data Obey the Laws?" NBER Working Paper 8675.

Judd, K. (1992): "Projection Methods for Solving Aggregate Growth Models", *Journal of Economic Theory*, 58, 410-52.

King and Rebelo (1993): "Transitional Dynamics and Economic Growth in the Neoclassical Model," *American Economic Review*, 83, 908-931.

Kraay, A., N. Loayza, L. Servén and J. Ventura (2000): "Country Portfolios," NBER Working Paper.

Krusell, P. and J. Ríos-Rull (1999): "On the Size of the U.S. Government: Political Economy in the Neoclassical Growth Model," *American Economic Review*, 89, 1156-82.

Lucas, R.E. (1988): "On the Mechanics of Economic Development," *Journal of Monetary Economics*, 22, 3-42.

Lucas, R.E. (1990): "Why Doesn't Capital Flow from Rich to Poor Countries?" *American Economic Review*, Papers and Proceedings, 80, 92-96.

Mankiw, N.G., D. Romer and D.N. Weil (1992): "A Contribution to the Empirics of Economic Growth," *Quarterly Journal of Economics*, 407-437.

Quah, D.T. (1996): "Twin Peaks: Growth and Convergence in Models of Distribution Dynamics," *Economic Journal*, 106, 1045-1055.

Repetto, A. and J. Ventura (1998): "The Leontief-Trefler Hypothesis and Factor Price Insensitivity," MIT Working Paper.

Stiglitz, J. (1971): "Factor Price Equalization in a Dynamic Economy," *Journal of Political Economy*, 101, 961-987.

Ventura, J. (1997): "Growth and Interdependence," *Quarterly Journal of Economics*, 112, 57-84.

A Appendix: Static Equilibrium

A.1 The Integrated Equilibrium

The integrated equilibrium is obtained by considering that the entire world is a single closed economy. The equilibrium conditions can be summarized as follows:

1. Price equal to unit cost:

$$1 = c(p_1, p_2, p_3) = \theta p_1^{\frac{\alpha}{2}} p_2^{1-\alpha} p_3^{\frac{\alpha}{2}} \tag{16}$$

$$p_1 = c_1(w, r) = w \tag{17}$$

$$p_2 = c_2(w, r) = 2\sqrt{wr} \tag{18}$$

$$p_3 = c_3(w, r) = r \tag{19}$$

   where $\theta = \left[\phi \left(\frac{\alpha}{2}\right)^{\alpha} (1-\alpha)^{1-\alpha}\right]^{-1}$ is a positive constant.

2. Consumption shares:

$$p_1 x_1 = p_3 x_3 \tag{20}$$

$$p_1 x_1 = \frac{\alpha}{2(1-\alpha)} p_2 x_2 \tag{21}$$

3. Factor market clearing:

$$\sum_{z=1}^{3} \frac{\partial c_z(w, r)}{\partial w} x_z = l \tag{22}$$

$$\sum_{z=1}^{3} \frac{\partial c_z(w, r)}{\partial r} x_z = k \tag{23}$$

The unknowns of the problem are p_1, p_2, p_3, x_1, x_2, x_3, w, and r. We obtain the following solution for these variables:

1. Factor prices: $w = \xi \left(\frac{k}{l}\right)^{1/2}$, and $r = \xi \left(\frac{k}{l}\right)^{-1/2}$, where $\xi = \theta^{-1} 2^{\alpha-1}$ is a positive constant.

2. Goods prices: $p_1 = \xi \left(\frac{k}{l}\right)^{1/2}$, $p_2 = 2\xi$, and $p_3 = \xi \left(\frac{k}{l}\right)^{-1/2}$. The invariant behavior of p_2 is due to the symmetry we have imposed on the model.

3. Production of intermediate goods: $x_1 = \alpha l$, $x_2 = (1-\alpha) k^{\frac{1}{2}} l^{\frac{1}{2}}$, and $x_3 = \alpha k$.

4. Finally, we also compute the sectorial allocation of production factors, necessary to determine the FPE condition: $(k_1, l_1) = (0, \alpha l)$, $(k_2, l_2) = [(1-\alpha) k, (1-\alpha) l]$, and $(k_3, l_3) = (\alpha k, 0)$.

A.2 Complete Specialization

For $\varepsilon \in \left(\alpha, \frac{1}{2}\right)$, we know that $x_{1N} = x_{3S} = 0$. The equilibrium conditions for the two-cone case can be summarized as follows:

1. Price equal to unit cost:

$$1 = c(p_1, p_2, p_3) = \theta p_1^{\frac{\alpha}{2}} p_2^{1-\alpha} p_3^{\frac{\alpha}{2}} \tag{24}$$

$$p_1 = c_1(w_S, r_S) = w_S \tag{25}$$

$$p_2 = c_2(w_S, r_S) = 2\sqrt{w_S r_S} = c_2(w_N, r_N) = 2\sqrt{w_N r_N} \tag{26}$$

$$p_3 = c_3(w_N, r_N) = r_N \tag{27}$$

2. Consumption shares:

$$p_1 x_1 = \frac{\alpha}{2(1-\alpha)} p_2 (x_{2S} + x_{2N}) \tag{28}$$

$$p_3 x_3 = \frac{\alpha}{2(1-\alpha)} p_2 (x_{2S} + x_{2N}) \tag{29}$$

3. Factor market clearing:

$$\sum_{z=1}^{2} \frac{\partial c_z(w_S, r_S)}{\partial w_S} x_{zS} = 1 \tag{30}$$

$$\sum_{z=1}^{2} \frac{\partial c_z(w_S, r_S)}{\partial r_S} x_{zS} = k_S \tag{31}$$

$$\sum_{z=2}^{3} \frac{\partial c_z(w_N, r_N)}{\partial w_N} x_{zN} = 1 \tag{32}$$

$$\sum_{z=2}^{3} \frac{\partial c_z(w_N, r_N)}{\partial r_N} x_{zN} = k_N \tag{33}$$

The unknowns of the problem are p_1, p_2, p_3, x_1, x_{2S}, x_{2N}, x_3, w_S, r_S, w_N, and r_N. From the complete specialization equilibrium conditions we obtain the following system of two equations:

$$(1-\alpha)\sqrt{\sigma_S} - \frac{k_S}{\sqrt{\sigma_S}} = \alpha\sqrt{\sigma_N} \tag{34}$$

$$(1-\alpha)\frac{k_N}{\sqrt{\sigma_N}} - \sqrt{\sigma_N} = \alpha\frac{k_S}{\sqrt{\sigma_S}} \tag{35}$$

This yields the factor-price ratios σ_j as functions of the two capital stocks $\sigma_j = \sigma_j(k_N, k_S)$.

1. Factor prices: manipulating the equilibrium's pricing equations, we can write factor prices as functions of the factor-price ratios. This yields $w_N = \xi\sigma_N^{\frac{2+\alpha}{4}}\sigma_S^{-\frac{\alpha}{4}}$, $w_S = \xi\sigma_N^{\frac{\alpha}{4}}\sigma_S^{\frac{2-\alpha}{4}}$, $r_N = \xi\sigma_N^{\frac{\alpha-2}{4}}\sigma_S^{-\frac{\alpha}{4}}$, and $r_S = \xi\sigma_N^{\frac{\alpha}{4}}\sigma_S^{-\frac{(2+\alpha)}{4}}$. Hence, factor prices are also functions of the capital stocks of North and South: $w_j = w_j(k_N, k_S)$, $r_j = r_j(k_N, k_S)$.

2. Goods prices: plugging the solutions for w_j and r_j into the cost functions, we can also write goods prices as functions of the capital stocks. That is: $p_1 = w_S(k_N, k_S)$; $p_2 = c_2[w_j(k_N, k_S), r_j(k_N, k_S)]$, $j = N, S$; and $p_3 = r_N(k_N, k_S)$.

3. Production of intermediate goods: combining the solutions for σ_j and the factor-market clearing conditions yields $x_{zj} = x_{zj}(k_N, k_S)$.

Given the production structure implied by the CS equilibrium, the balanced trade condition, and our choice of numeraire, the net exports of region N are as follows:

$$e_{1N} = -\frac{\alpha}{2}\theta p_1^{\frac{\alpha}{2}-1}p_2^{1-\alpha}p_3^{\frac{\alpha}{2}}y_N < 0 \tag{36}$$

$$e_{2N} = \frac{\alpha}{2}\frac{(y_N - y_S)}{p_2} > 0 \tag{37}$$

$$e_{3N} = \frac{\alpha}{2}\theta p_1^{\frac{\alpha}{2}}p_2^{1-\alpha}p_3^{\frac{\alpha}{2}-1}y_S > 0 \tag{38}$$

Thus, the North's volume of trade v_N is

$$v_N \equiv |p_1e_{1N}| + |p_2e_{2N}| + |p_3e_{3N}| = \alpha y_N. \tag{39}$$

B Appendix: Computational Strategy

B.1 Policy functions

Following Judd (1992), we approximate the policy functions for consumption over a rectangle $D \equiv [\underline{k}, \overline{k}] \times [\underline{k}, \overline{k}] \in R_+^2$ with a linear combination of multidimensional orthogonal basis functions taken from a *2*-fold tensor product of Chebyshev polynomials. In other words, we approximate the policy function for cone $j \in \{N, S\}$ with:

$$\widehat{c}_j\left(k_N, k_S; \mathbf{a}_j\right) = \sum_{z=0}^{d} \sum_{q=0}^{d} a_{zq}^{j} \psi_{zq}\left(k_N, k_S\right) \tag{40}$$

where:

$$\psi_{zq}\left(k_N, k_S\right) \equiv T_z\left(2\frac{k_N - \underline{k}}{\overline{k} - \underline{k}} - 1\right) T_q\left(2\frac{k_S - \underline{k}}{\overline{k} - \underline{k}} - 1\right) \tag{41}$$

and $\{k_N, k_S\} \in D$. Each T_n represents an n-order Chebyshev polynomial, defined over $[-1, 1]$ as $T_n(x) = \cos(n \arccos x)$, while d denotes the higher polynomial order used in our approximation.

We defined the residual functions as:

$$R_j\left(k_N, k_S; \mathbf{a}_j\right) \equiv \beta \hat{c}_j\left(k_N, k_S; \mathbf{a}_j\right)\left(r_j' + 1 - \delta\right) - \hat{c}_j\left(k_N', k_S'; \mathbf{a}_j\right) \tag{42}$$

where:

1. $k_j' = w_j + (1 - \delta + r_j) k_j - \hat{c}_j\left(k_N, k_S; \mathbf{a}_j\right)$;
2. $w_N = \xi \sigma_N^{\frac{2+\alpha}{4}} \sigma_S^{-\frac{\alpha}{4}}$, $w_S = \sqrt{\frac{\sigma_S}{\sigma_N}} w_N$, $r_N = \xi \sigma_N^{\frac{\alpha-2}{4}} \sigma_S^{-\frac{\alpha}{4}}$, $r_S = \sqrt{\frac{\sigma_N}{\sigma_S}} r_N$ if $k_N > \left(\frac{1}{2} + \alpha\right) k$;
3. $r_j = r = \xi\left(\frac{k}{2}\right)^{-\frac{1}{2}}$ and $w_j = w = \left(\frac{k}{2}\right) r_j$ if $k_N \leq \left(\frac{1}{2} + \alpha\right) k$;
4. $r_N' = \xi\left(\sigma_N'\right)^{\frac{\alpha-2}{4}}\left(\sigma_S'\right)^{-\frac{\alpha}{4}}$, $r_S' = \xi\left(\sigma_N'\right)^{\frac{\alpha}{4}}\left(\sigma_S'\right)^{-\frac{(2+\alpha)}{4}}$ if $k_N' > \left(\frac{1}{2} + \alpha\right) k'$;
5. $r_j' = r' = \xi\left(\frac{k'}{2}\right)^{-\frac{1}{2}}$ if $k_N' \leq \left(\frac{1}{2} + \alpha\right) k'$.

The vectors $\mathbf{a}_j$ can be chosen efficiently using a projection method; in particular, the Galerkin method is well suited to our needs. This method identifies the $2(d+1)^2$ coefficients by imposing the following set of *orthogonality conditions* among $R_j\left(k_N, k_S; \mathbf{a}_j\right)$ and the *directions* $\left\{\psi_{zq}\right\}_{z,q=0}^{d}$:

$$\int_{\underline{k}}^{\overline{k}} \int_{\underline{k}}^{\overline{k}} R_j\left(k_N, k_S; \mathbf{a}_j\right) \psi_{zq}\left(k_N, k_S\right) W\left(k_N, k_S\right) dk_N dk_S = 0 \tag{43}$$

for $j \in \{N, S\}$ and $z, q \in \{0, 1, ..., d\}$, where:

$$W\left(k_N, k_S\right) \equiv \left[1 - \left(2\frac{k_N - \underline{k}}{\overline{k} - \underline{k}} - 1\right)^2\right]^{-\frac{1}{2}} \left[1 - \left(2\frac{k_S - \underline{k}}{\overline{k} - \underline{k}} - 1\right)^2\right]^{-\frac{1}{2}} \tag{44}$$

is the multivariate weighting function for which the ψ_{zq} are mutually orthogonal.

The multivariate integral in (43) can be approximated numerically by using a tensor-product extension of the univariate Gauss-Chebyshev quadrature method. Given $m > d+1$ Gauss-Chebyshev quadrature nodes in $\left[\underline{k}, \overline{k}\right]$, that correspond to the zeros of $T_m\left[2\left(x-\underline{k}\right)/\left(\overline{k}-\underline{k}\right)-1\right]$, we can organize them into two (identical) vectors $\{k_{N,i}\}_{i=1}^{m}$and $\{k_{S,i}\}_{i=1}^{m}$. The conditions in (43) can then be approximated by the following set of $2\left(d+1\right)^2$ nonlinear equations:

$$P_{zq}^{j}\left(\mathbf{a}_j\right) \equiv \sum_{i=1}^{m}\sum_{l=1}^{m} R_j\left(k_{N,i}, k_{S,l}; \mathbf{a}_j\right)\psi_{zq}\left(k_{N,i}, k_{S,l}\right) = 0 \tag{45}$$

where $z, q \in \{0, 1, ..., d\}$. The system (45) can be solved numerically by using any Newton-type algorithm; we adopt Broyden's method, a variant of the standard Newton's method that avoids explicit computation of the Jacobian at each iteration.

In our case, the general procedure outlined in the previous paragraph is not directly applicable. The policy functions turn out to be characterized by a particular shape. Under FPE, the policy functions look like a positively sloped hyperplane in R_{+}^{3}, while under CS they adopt a slightly more curved shape. Any attempt to approximate them using the same polynomials for both regions produces inaccurate results, in particular around the "regime"-switching region.

To bypass this problem, we develop a procedure that generates two different sets of coefficients, one for each region.First of all, we define two proper subsets of D: D_{FPE}, the FPE region, and D_{CS}, the CS region where the North is the capital-intensive cone (the policy functions in the other CS region are perfectly symmetric).

The first step in our procedure consists in finding $m_{FPE} \gg d_{FPE}+1$ quadrature nodes in $[\underline{k}, \overline{k}]$, and organizing them into the vectors $\{k_{N,i}\}_{i=1}^{m_{FPE}}$and $\{k_{S,i}\}_{i=1}^{m_{FPE}}$. Then, we isolate the subset of $\{k_{N,i}\}_{i=1}^{m_{FPE}} \times \{k_{S,i}\}_{i=1}^{m_{FPE}}$ that belongs to D_{FPE}, obtaining some $\tilde{m}_{FPE} < m_{FPE}$ values for k_N and k_S. Finally, we solve the following system of equations numerically for the $\left(d_{FPE}+1\right)^2$ elements of $\mathbf{a}_N^{FPE}$:

$$P_{zq}^{N}\left(\mathbf{a}_N^{FPE}\right) \equiv \sum_{i=1}^{\tilde{m}_{FPE}}\sum_{i=1}^{\tilde{m}_{FPE}} R_j\left(k_{N,i}, k_{S,l}; \mathbf{a}_N^{FPE}\right)\psi_{zq}\left(k_{N,i}, k_{S,l}\right) = 0 \tag{46}$$

where $z, q = 0...d_{FPE}$, since the symmetry of our model guarantees that, under FPE,

$$\widehat{c}_S\left(k_N, k_S\right) = \widehat{c}_N\left(k_S, k_N; \mathbf{a}_N^{FPE}\right) \tag{47}$$

Symmetrically, the second step of the procedure consists in finding $m_{CS} \gg d_{CS}+1$ quadrature nodes in $[\underline{k}, \overline{k}]$, and isolating the subset of $\{k_{N,i}\}_{i=1}^{m_{CS}} \times \{k_{S,i}\}_{i=1}^{m_{CS}}$ that belongs to D_{CS}, obtaining $\tilde{m}_{CS} < m_{CS}$ values for k_N and k_S. These values are used to solve the

	FPE	CS	
	North	North	South
$Avg.$	1.89e-9	1.59e-6	3.53e-6
$Med.$	1.47e-10	7.19e-7	1.46e-6
$Std.$	6.33e-9	3.14e-6	1.17e-5
$Max.$	7.00e-8	2.02e-5	1.06e-4

Table 3: Euler equation residuals

system

$$P_{zq}^{j}\left(\mathbf{a}_{j}^{CS}\right) \equiv \sum_{i=1}^{\tilde{m}_{CS}} \sum_{i=1}^{\tilde{m}_{CS}} R_{j}\left(k_{N,i}, k_{S,l}; \mathbf{a}_{j}^{CS}\right) \psi_{zq}\left(k_{N,i}, k_{S,l}\right) = 0 \tag{48}$$

where $z, q = 0...d_{CS}$, for the $2\left(d_{CS}+1\right)^{2}$ elements of $\left\{\mathbf{a}_{N}^{CS}, \mathbf{a}_{S}^{CS}\right\}$. A necessary condition is that $\tilde{m}_{FPE}^{2} > \left(d_{FPE}+1\right)^{2}$ and $\tilde{m}_{CS}^{2} > 2\left(d_{CS}+1\right)^{2}$.

Out two-step procedure deals with the curved shape of the policy functions while maintaining a good degree of numerical precision. Figure 2 shows the policy function for c_N under our benchmark parameterization (the policy function for c_S is perfectly symmetric), which is: $\alpha = 0.15$, $\beta = 0.949$, $\delta = 0.048$, $d_{FPE} = 8$, $d_{CS} = 4$, $\underline{k} = 0.1$, and $\overline{k} = 0.9$. Note that the scale parameter ϕ has been calibrated to reproduce a world steady-state capital stock equal to unity.

Table ?? summarizes the empirical distribution of the Euler equation residuals in absolute terms, i.e. the values of $\left|R_{j}\left(k_{N}, k_{S}, \mathbf{a}_{j}\right)\right|$, over 218 *equally spaced* points in D_{FPE} and 91 equally spaced points in D_{CS}. As we can see, the size of the residuals is extremely small in the FPE region, and reasonably small in the CS one. The functional equation residuals are of course only an indirect measure of the quality of our approximation, but still a very informative one. Another informative test of the approximation accuracy is the long-run stability of the solution: the approximated system remains in steady state even if the simulation horizon is extended to 10.000 years.

Once the approximated policy functions are available, we choose the initial conditions and simulate the system recursively to generate the artificial time series for all variables of interest by using the appropriate set of policy functions.

B.2 Disaggregation

To disaggregate each cone into East and West, we assume *ex-ante* that countries in each cone are under FPE. By iterating the country-level intratemporal budget constraint under this assumption, we obtain (for each cone):

$$k_{j,t} = \sum_{s=t}^{\infty} \frac{R_t^s}{r_t + 1 - \delta} \left(c_{j,s} - \frac{w_s}{2} \right) + \lim_{s \to \infty} \frac{R_t^s}{r_t + 1 - \delta} k_{j,s+1} \tag{49}$$

where $j \in \{W, E\}$, $R_t^s \equiv \prod_{i=t+1}^{s} (r_i + 1 - \delta)^{-1}$ and $R_t^t \equiv 1$.

Then we impose the transversality condition and transform the previous equation into a fully-fledged intertemporal budget constraint:

$$\sum_{s=t}^{\infty} R_t^s c_{j,s} = (r_t + 1 - \delta) k_{j,t} + \sum_{s=t}^{\infty} R_t^s w_s \tag{50}$$

Finally, by substituting the Euler equation we obtain

$$c_{j,t} = (1 - \beta) \left[(r_t + 1 - \delta) k_{j,t} + \sum_{s=t}^{\infty} R_t^s w_s \right] \tag{51}$$

The previous result implies that:

$$c_{W,t} - c_{E,t} = (1 - \beta)(r_t + 1 - \delta)(k_{W,t} - k_{E,t}) \tag{52}$$

or that:

$$c_{W,t} = \frac{1}{2} \left[(1 - \beta)(r_t + 1 - \delta)(k_{W,t} - k_{E,t}) + c_t \right] \tag{53}$$

$$c_{E,t} = c_t - c_{W,t} \tag{54}$$

where c_t is total consumption in each cone. The time series for c_t and r_t, and the initial conditions $k_{W,0}$ and $k_{E,0}$ are sufficient to recover the times series for $c_{W,t}$, $c_{E,t}$, $y_{W,t}$, $y_{E,t}$, $k_{W,t}$, and $k_{E,t}$. Once these series are at hand, we check *ex-post* that the corresponding static FPE conditions have been satisfied in all periods.

Checking the FPE conditions here is somewhat ellaborate. As long as worldwide FPE does not hold, we simply need to check the FPE condition within each cone:

The South's FPE condition can be written as follows:[28]

1. If $l_{SW} \leq l_{1S}$, $0 \leq k_{SW} \leq k_S$.
2. If $l_{SW} > l_{1S}$, $\frac{k_{2S}}{l_{2S}} (l_{SW} - l_{1S}) \leq k_{SW} \leq \frac{k_{2S}}{l_{2S}} l_{SW}$.

The corresponding North's FPE condition is:

For any $l_{NW} \in (0, l_N)$, $\frac{k_{2N}}{l_{2N}} l_{NW} \leq k_{NW} \leq k_{3N} + \frac{k_{2N}}{l_{2N}} l_{NW}$.

Once worldwide FPE holds, we combine results in Deardorff (1994) and Cuñat (2001)

[28] The variables indexed in ij denote the corresponding cone's resource allocation. $k_i = k_{iW} + k_{iE}$ is the cone's capital stock.

to check the many-country FPE condition. Define (k^n, l^n), $n = 1,2,3$, as follows:

$$\begin{aligned}(k^1, l^1) &\equiv (k_{NW}, l_{NW}) \\ (k^2, l^2) &\equiv (k_{NW} + k_{NE}, l_{NW} + l_{NE}) \\ (k^3, l^3) &\equiv (k_{NW} + k_{NE} + k_{SW}, l_{NW} + l_{NE} + l_{SW})\end{aligned}$$

Then, a necessary and sufficient condition for FPE is:[29] for $n = 1,2,3$,

1. If $l^n \in (0, l_1]$, $0 \leq k^n \leq k_3 + \frac{k_2}{l_2} l^n$.
2. If $l^n \in (l_1, l_2)$, $\frac{k_2}{l_2}(l^n - l_1) \leq k^n \leq k_3 + \frac{k_2}{l_2} l^n$.
3. If $l^n \in [l_2, l)$, $\frac{k_2}{l_2}(l^n - l_1) \leq k^n \leq k$.

B.3 Autarky

Under autarky, each region behaves as a closed Ramsey economy. The first order conditions for the representative household become

$$\beta c_t \left(\frac{1}{2} A k_{t+1}^{-\frac{1}{2}} + 1 - \delta \right) = c_{t+1} \tag{55}$$

$$k_{t+1} = (1 - \delta) k_t + A \sqrt{k_t} - c_t \tag{56}$$

where $A \equiv \phi \alpha^{\alpha} (1 - \alpha)^{1-\alpha}$.

We approximate the policy function for c_t over $[\underline{k}, \overline{k}] \in R_+$ with a linear combination of Chebyshev polynomials:

$$\widehat{c}(k, \mathbf{a}_A) = \sum_{i=0}^{d_A} a_i^A \psi_i(k) \tag{57}$$

where:

$$\psi_i(k) \equiv T_i \left(2 \frac{k - \underline{k}}{\overline{k} - \underline{k}} - 1 \right) \tag{58}$$

To choose a suitable vector $\mathbf{a}_A$, we apply the Galerkin method again, defining the residual function as:

$$R(k; \mathbf{a}_A) \equiv \beta \hat{c}(k; \mathbf{a}_A) \left[\frac{1}{2} A k'(k; \mathbf{a}_A)^{-\frac{1}{2}} + 1 - \delta \right] - \hat{c}\left[k'(k; \mathbf{a}_A); \mathbf{a}_A\right] \tag{59}$$

As before, we obtain $m_A > d_A + 1$ zeros of Chebyshev polynomials in $[-1, 1]$, find the corresponding values in $[\underline{k}, \overline{k}]$, and organize them into a vector $\{k_i\}_{i=1}^m$. Finally, we solve

[29] k and l denote the world's capital and labor endowments. Variables indexed in numbers refer to the integrated equilibrium's resource allocation.

the following system of equations numerically for $\mathbf{a}_A$:

$$P_i(\mathbf{a}_A) = \sum_{j=1}^{m_A} R(k_j; \mathbf{a}_A)\, \psi_i(k_j) = 0$$

where $i = 0...d_A$.

Once the approximated policy function is at hand, we simulate the system for each region, using the same parameterization and the same initial conditions as before.

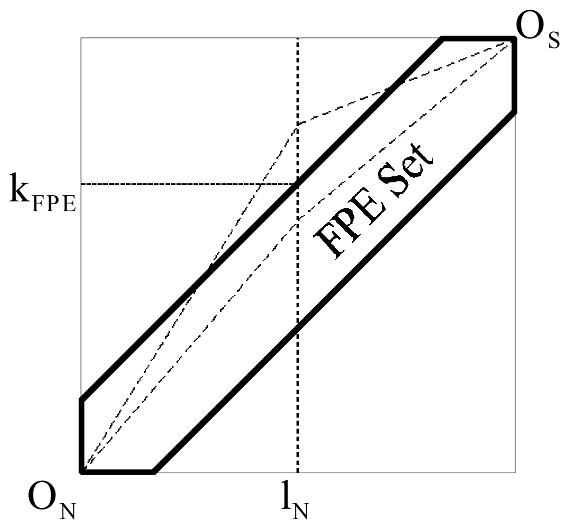

Figure 1: The integrated economy.

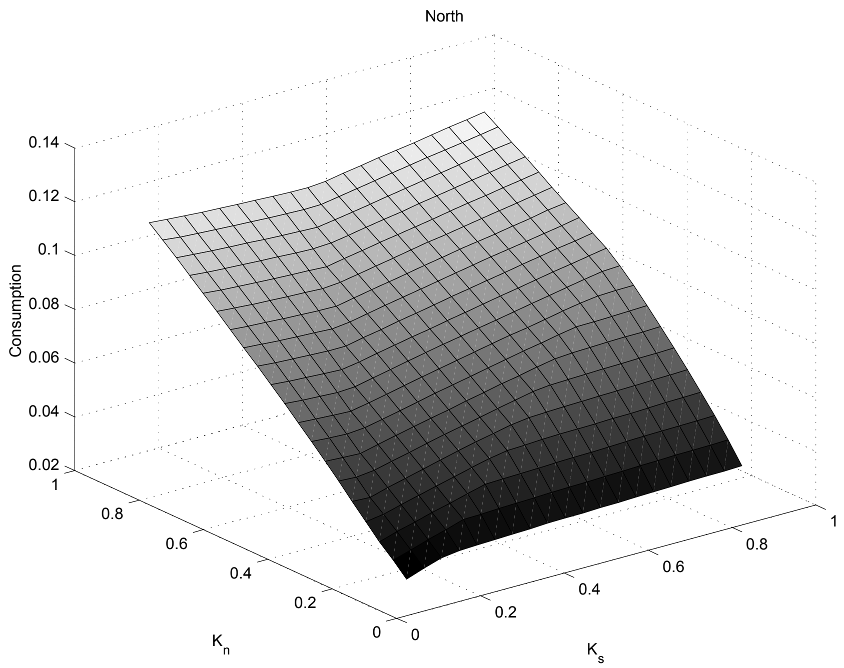

Figure 2: The policy function.

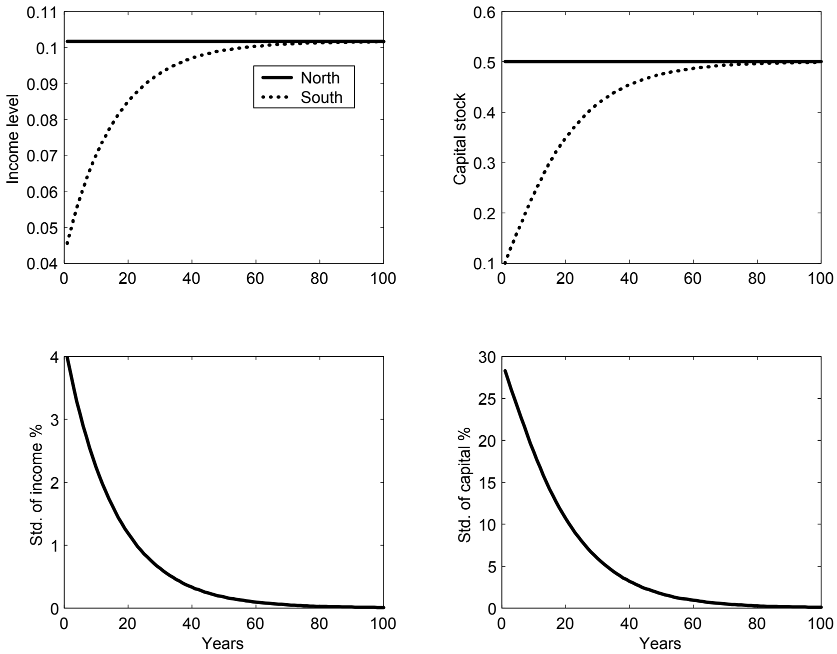

Figure 3: Income and capital under autarky.

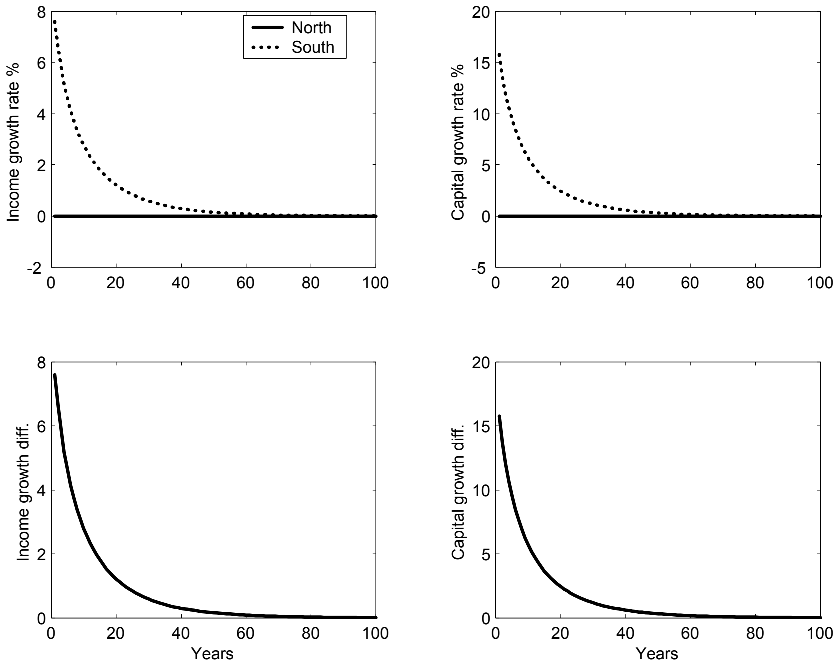

Figure 4: Growth rates under autarky.

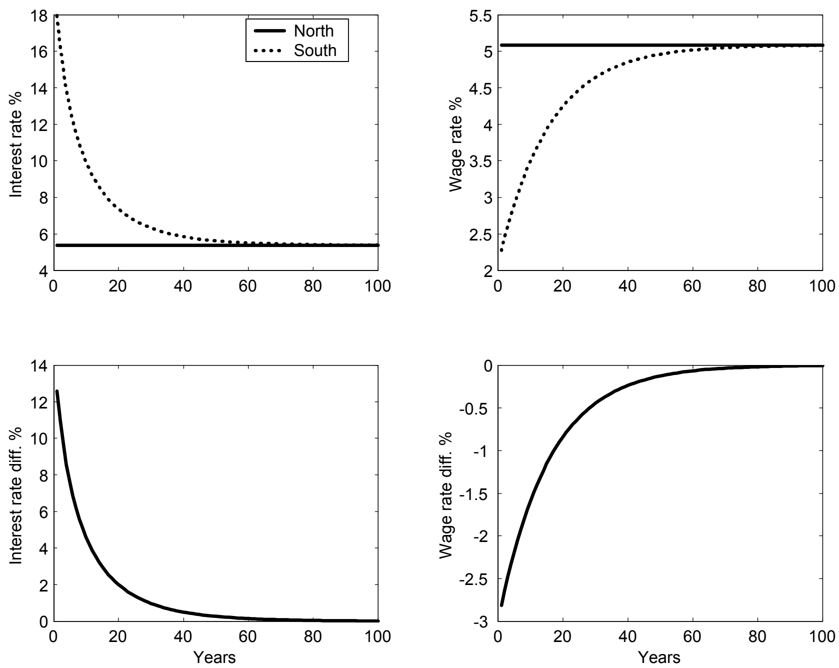

Figure 5: Factor prices under autarky.

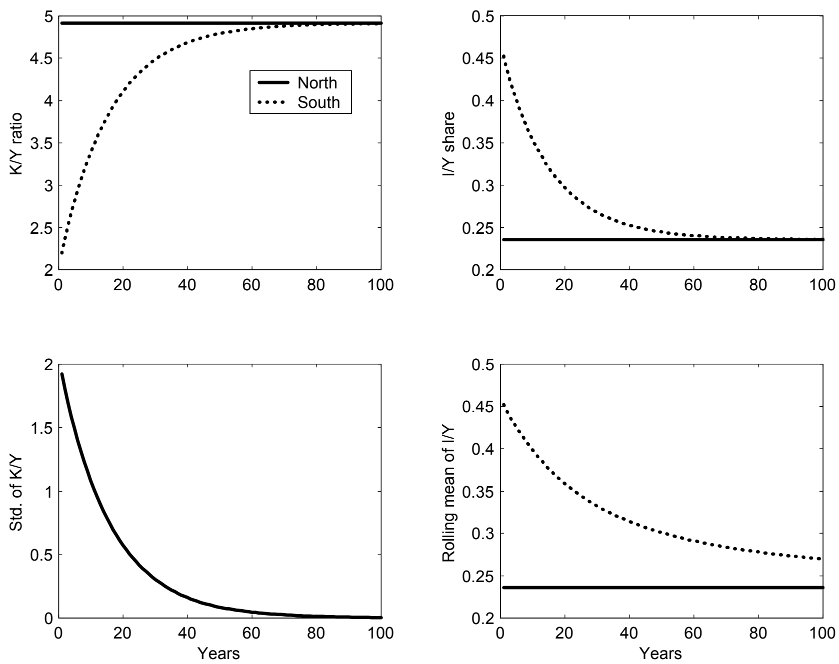

Figure 6: Capital/income ratios and investment shares under autarky.

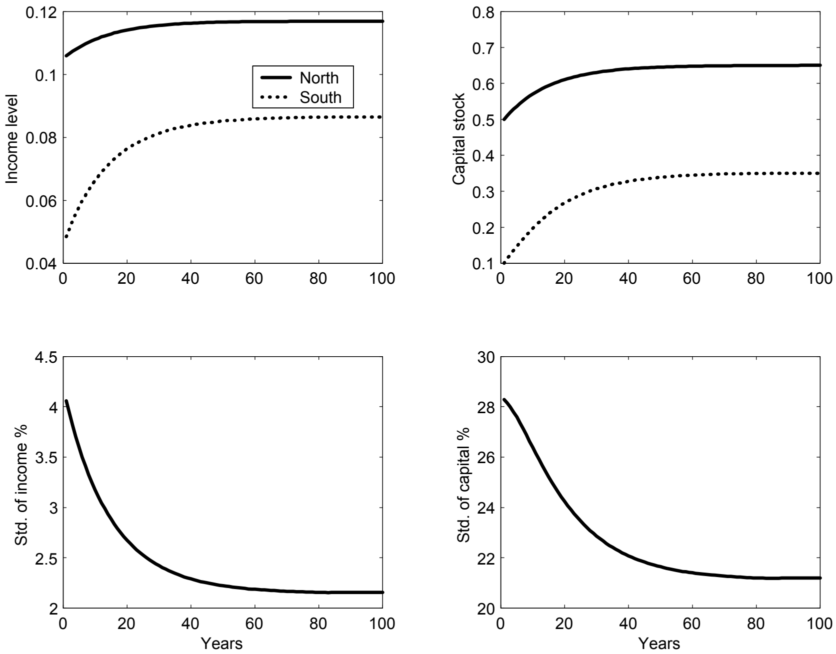

Figure 7: Income and capital under CS.

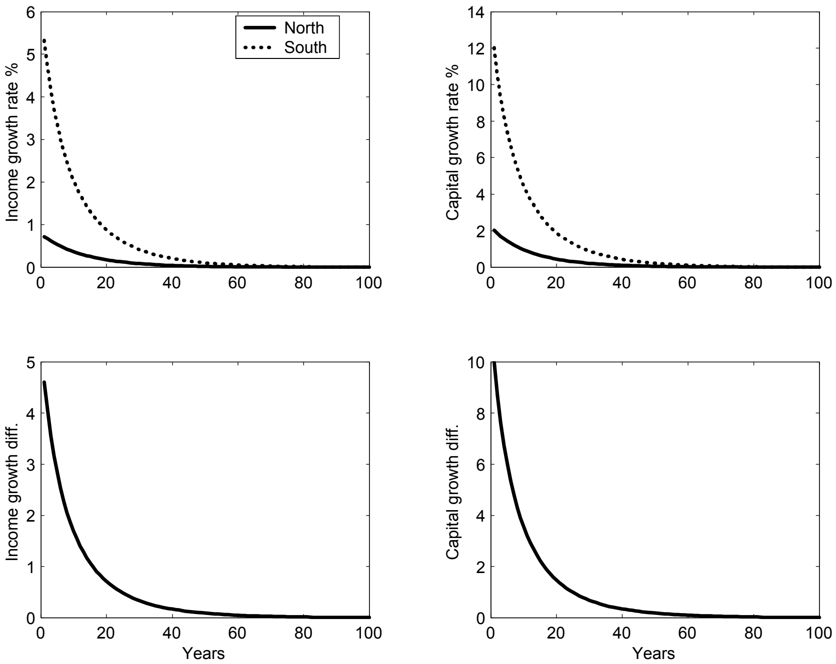

Figure 8: Growth rates under CS.

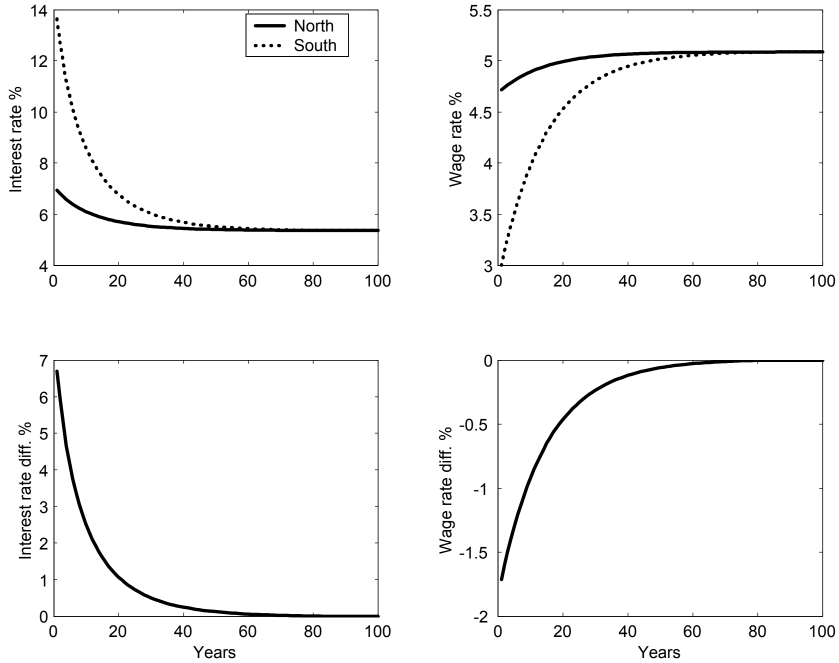

Figure 9: Factor prices under CS.

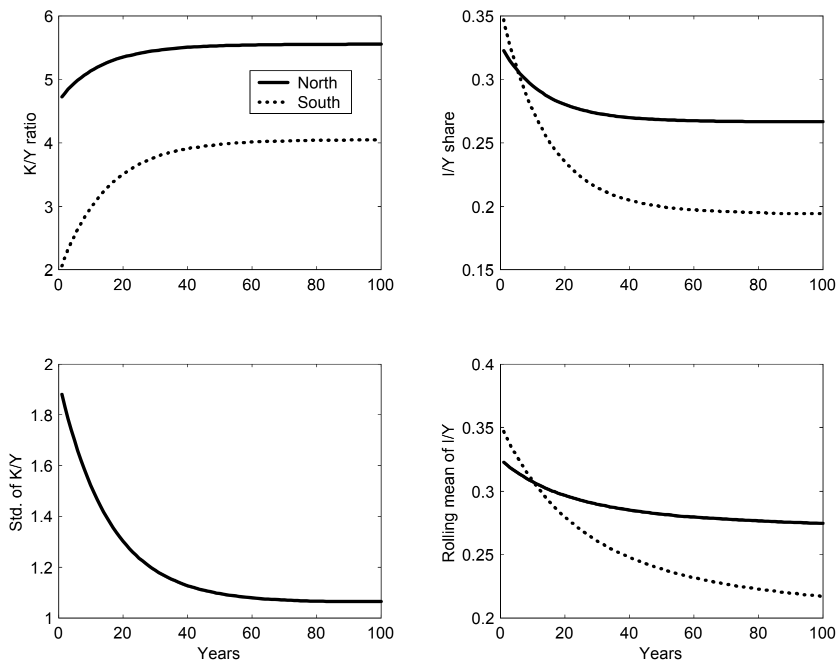

Figure 10: Capital/income ratios and investment shares under CS.

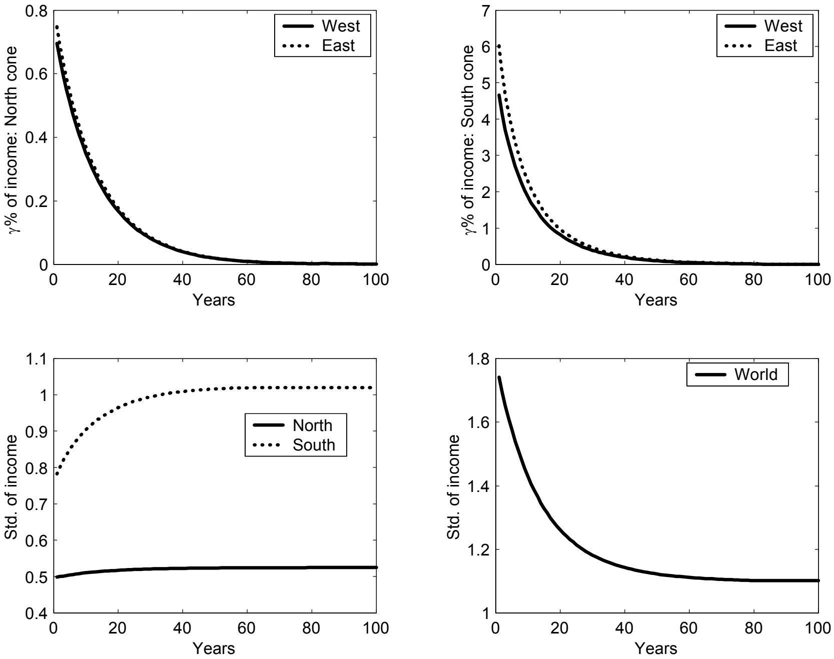

Figure 11: Disaggregation: convergence.

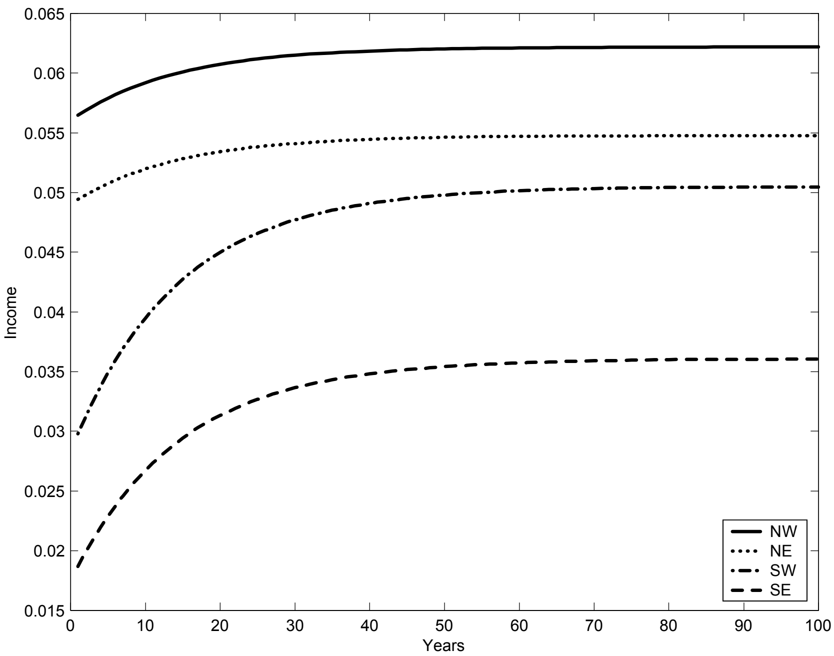

Figure 12: Disaggregation: income levels.

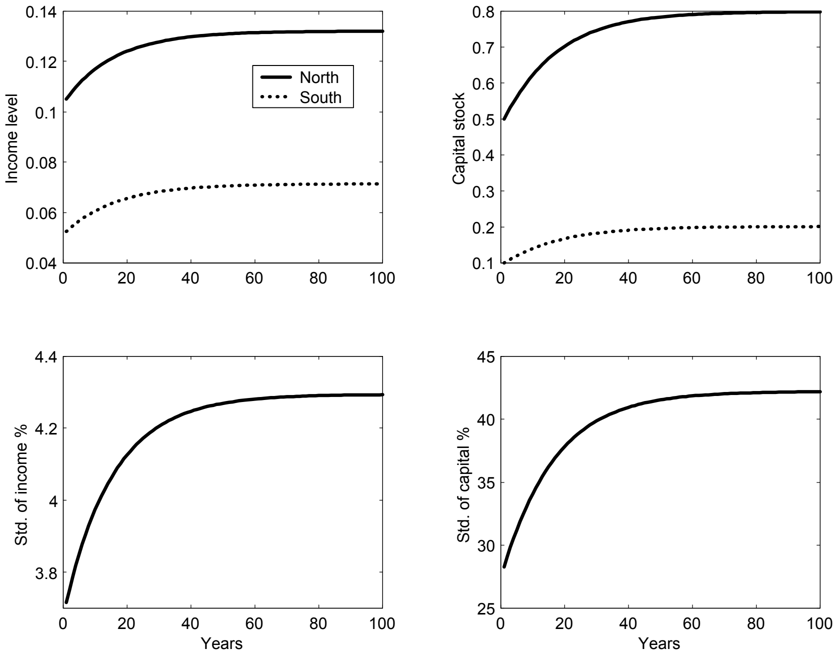

Figure 13: Income and capital under FPE.

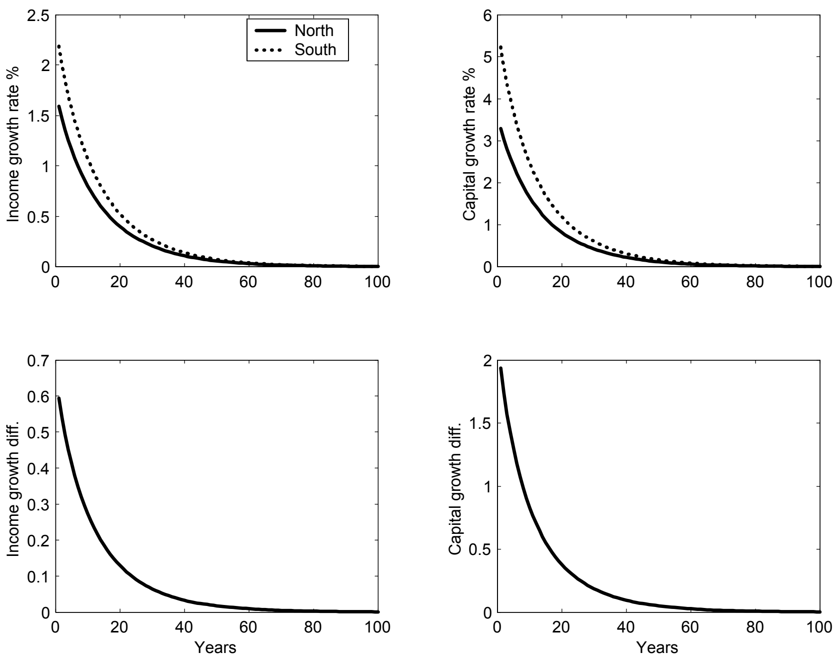

Figure 14: Growth rates under FPE.

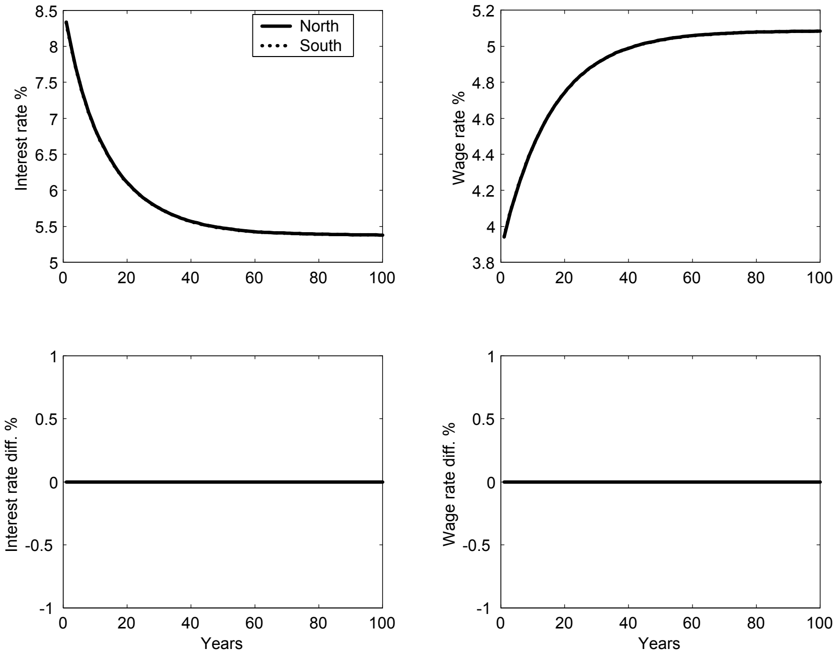

Figure 15: Factor prices under FPE.

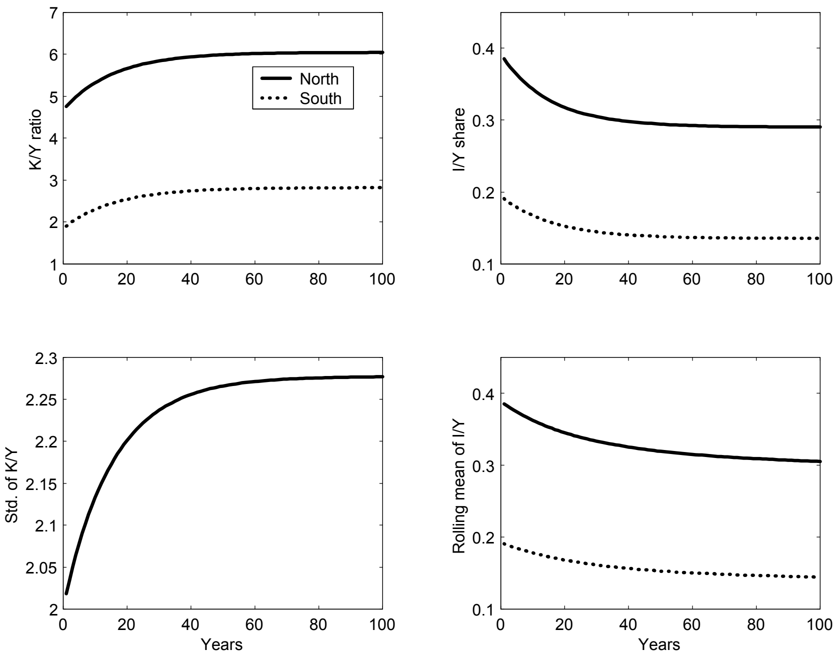

Figure 16: Capital/income ratios and investment shares under FPE.