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Frederic Palomino and Harald Uhlig

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Frederic Palomino, Tilburg University and CEPR
Harald Uhlig, Humboldt Universität and CEPR

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Centre for Economic Policy Research
90–98 Goswell Rd, London EC1V 7RR, UK
Tel: (44 20) 7878 2900, Fax: (44 20) 7878 2999
Email: cepr@cepr.org, Website: www.cepr.org

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ABSTRACT

Should Smart Investors Buy Funds with High Returns in the Past?*

Newspapers and weekly magazines catering to the investing crowd often rank funds according to the returns generated in the past. Aside from satisfying sheer curiosity, these numbers are probably also the basis on which investors pick a fund to invest in. In this article, we fully characterize the equilibrium in a game between a mutual fund manager of unknown ability who controls the riskiness of his portfolio and investors who only observe realized returns. We derive conditions under which (i) investors invest in the fund if the realized return falls within some interval, ie., is neither too low nor too high, (ii) an informed fund manager picks a portfolio of minimal riskiness, (iii) an uninformed mutual fund manager will pick a portfolio with higher risk, 'gambling' on a lucky outcome and (iv), when the fee structure is endogenous, both types of manager choose the same fraction-of-fund fee structure.

Our results are consistent with empirical evidence about the lack of persistence of top performance, and about the very wide use of fraction-of-fund fee structure among mutual funds.

JEL Classification: G11 and G23

Keywords: actively managed funds, fraction-of-fund fees, fund-picking strategy and index funds

Frederic Palomino
CentER
Tilburg University
P.O. Box 90153
NL-5000 LE Tilburg
THE NETHERLANDS
Tel: (31 13) 466 3262
Fax: (31 13) 466 3066
Email: f.palomino@kub.nl

Harald Uhlig
Department of Economics
Humboldt University
Spandauer Strasse 1
10178 Berlin
GERMANY
Tel: (49 30) 2093 5926
Fax: (49 30) 2093 5934
Email: uhlig@wiwi.hu-berlin.de

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1 Introduction

Newspapers and weekly magazines catering to the investing crowd often produce tables showing “rat races” of mutual funds. They rank funds according to the returns generated in the previous year or over a period of several years. Aside from satisfying sheer curiosity, these numbers are probably also the basis on which investors pick a fund to invest in. Empirical work shows that flows in and out of funds are indeed positively correlated with past performances (see See Ippolito (1992), Sirri and Tuffano (1998), Chevalier and Ellison (1997) and Lettau (1997)).

Should smart investors buy funds with high returns in the past? To answer this question, we shall build on the premise that mutual fund managers differ in their ability to generate high returns, and that these abilities are persistent at least in the short run so that returns in year t can be indicative of performances in year $t + 1$ (See Grinblatt and Titman (1992), Hendrick, Patel and Zeckhauser (1993), Brown and Goetzmann(1995) and Carhart (1997)¹). In such a situation, mutual fund managers with asset-based compensation schemes² are surely aware of the signaling function of their past performance, and will thus choose their portfolio strategies accordingly. For the sake of the argument, suppose, that investors always pick the fund which generated the highest return in the past. Knowing this, a fund manager with an inferior ability may be tempted to gamble, i.e. to invest in risky portfolios, hoping to generate the highest return in the crowd. But if that is indeed the case, high past returns are not indicative of high ability: rather, they indicate high risk and a lucky outcome. Smart investors should thus avoid the top-performing funds.

The contribution of this paper is not only to make this intuition precise, but to fully characterize the equilibrium in a game between a mutual fund manager and investors, when only returns are observable. We assume that mutual fund managers can be either bad or good, a good manager being informed with a high probability

¹Evidence of persistent under-performance seem to be stronger than that of over-performance

²See Khorana (1996, section 2)

while a bad manager is informed with low probability. Thus, a good manager realizes a larger return than a bad manager on average. Mutual fund managers control the riskiness of their portfolio. The investors observe a realization of the fund return, and invest in the fund, if it is sufficiently likely that the mutual fund manager is good. The manager will choose the riskiness of his to-be-observed portfolio return in order to maximize the chance that the investor will invest with him.

We obtain the following results, summarized in Theorem 1. Under some conditions on the parameters of our model, the investor will invest in the fund, if the realized return falls within some interval, i.e., is neither too low nor too high. An informed fund manager picks a portfolio of minimal riskiness. An uninformed mutual fund manager will pick a portfolio with higher risk, “gambling” on a lucky outcome.

Furthermore, this type of equilibrium is unique for sets of parameters such that ex-ante (i.e., before observing the return) investors are better off *not* investing in the active fund. Therefore, our results can be interpreted in the following way. If, ex-ante investors are better off *not* investing in the active fund, then a large return should always be interpreted as the return of an uninformed manager “gambling” rather than a sign of good information.

Such results are consistent with those of Carhart (1997) who shows that “the funds in the top decile differ substantially each year, with more than 80 percent annual turnover in the composition. In addition, last year’s winners frequently become next year’s losers and vice versa, which is consistent with gambling behavior by mutual funds”.

The paper proceeds as follows. Section 2 discusses related literature. Section 3 provides a simple two-quality model, and provides the key results in theorems 1 and 3. Section 4 provides analyzes a more general framework. Section 5 endogenizes the fee structure and section 6 concludes.

2 Related literature

Our paper is substantially different from the existing literature. In particular, the upper bound is the novel feature of our paper.

The most related model is that of Huddart (1999). He studies a two-period model in which, at the end of the first period, risk-averse investors reallocate their wealth between two funds after having observed the performance of each fund. It is shown that in the first, both an informed and an uninformed fund choose overly risky investment strategies; and in the second period investors should always invest in the fund that has realized the higher return in the first period.

There are several differences between Huddart's model and ours. First, Huddart assumes that portfolios are observable while we do not. Huddart's assumption implies that he studies a standard signaling model in which some additional information is given by realized returns. If portfolios are not observable, as in our model, managers' skills can only be derived from statistical inference. Furthermore, we do not find the assumption of observable portfolios very appealing since disclosure is not frequent³ and managers window-dress their portfolio around disclosure dates in practice (see Lakonishok, Thaler, Shleifer and Vishny (1991), Musto (1999) and Carhart, Kaniel, Musto and Reed (2001)). Second, Huddart assumes that returns are binomially distributed. Therefore, it is very "easy" for an uninformed manager to obtain exactly the same return as an informed manager. Hence, there is not much information an investor can gather from observed returns.

Other models have studied the game played by mutual fund managers (see, for example, Chen and Pennachi (1999), Gorjaev, Palomino and Prat (2000) and Taylor (2000)). However all these models take investors' fund picking rule as exogenous and study the consequences of such a rule on the game played by fund managers. Conversely, in our model, investors' picking rule is endogenous.

³In the United States, mutual fund portfolios have to be disclosed semiannually. However, other countries such as the Netherlands only require disclosure once a year.

Other papers such as Das and Sundaram (2002) have focussed on e.g. the fee structure as a way to signal skills: a good manager uses a performance fee structure with an appropriate benchmark to signal his quality. In our model, when the fee structure is endogenous, we show that for some sets of parameters, there exists a pooling equilibrium, which survives the Cho-Kreps intuitive criterion, such that both good and bad managers choose a fraction-of-fund fee. The main difference between Das and Sundaram's model and ours is that they assume that a good manager learns his information *before* choosing his fee while we assume that he receives his information *after* choosing the fee.

The pooling equilibrium with fraction-of-fund fees we find also provides a rational for the very extensive use of fraction-of-fund fees (rather than performance fee) in the mutual fund industry (See Golec(1992) and Elton, Gruber and Blake (2001)).

An other model focussing on the fee structure is the one of Heinkel and Stoughton who also analyze the interplay between a portfolio manager and an investor: there, the focus is to extract the right effort from the manager by conditioning a continued relationship on the observed return.

Finally, our model is a special case of problem studied by Crawford and Sobel (1982) in which a sender (the fund manager in our case) sends a *costless* (noisy) signal to a receiver (the investors) who makes inference about the sender's type and then, choose an action.

On the empirical side, investors' smartness in fund picking has been studied by Gruber (1996) and Zhen (1999). Both articles show that newly invested money performs better than the entire stock of money invested in actively managed funds. However, newly invested money in actively managed equity funds underperforms index funds. This suggests that investors' ability to select funds is rather limited. Patel Zeckhauzer and Hendricks (1994) and Massa (1997) show that the fund-picking by investors is dictated by their position on a relative ranking of funds performances rather than the absolute value of the return generated by the funds. This lends mild empirical support to Theorem 1: if returns are already sufficiently high to land you

on top of the heap, it does not help to achieve even higher returns⁴.

3 The Model

We consider a situation in the spirit of Ippolito (1992). The economy contains two funds, an actively managed fund which charges high “fraction-of-fund” management fees and an index fund which charges low fraction-of-fund management fees normalized to zero here. Denote $c \in (0, 1)$ the difference in fee and μ_o the expected return of the index fund.

The manager of the active fund is of good quality with some probability $\phi > 0$ and of bad quality with probability $1 - \phi$.

There are two periods: 0 and 1. At the beginning of period 0, the manager of the active fund picks a portfolio, which generates the return

$$R = \mu + \sigma\epsilon, \epsilon \sim \mathcal{N}(0; 1) \tag{1}$$

If the manager is of good quality, he is informed with probability θ_g while a manager of bad quality is informed with probability θ_b , with $\theta_b < \theta_g$. If the manager is informed, then $\mu = \mu_i$ and if the manager is uninformed, then $\mu = \mu_u < \mu_i$. In picking the portfolio, the mutual fund manager only controls the riskiness $\sigma \geq \underline{\sigma} > 0$ of the portfolio, i.e., the mutual fund manager cannot reduce the risk in his portfolio below some positive lower bound, but he can add as much risk as he wants. For simplicity, we have assumed that the market price of this risk is zero, i.e., that changing the riskiness of the portfolio does not affect its mean return. This is not restrictive in principle: if there is a market price for risk, rewrite everything in risk-adjusted terms, i.e., do a change of measure so that assets can be compared by comparing their expected returns only, using the new measure. Of course, while this is a standard and elegant procedure in theory, it may makes comparisons to the data tricky.

⁴We are grateful to Massimo Massa for pointing this out to us.

In order to make the problem interesting, we assume that

$$(1 - c)(\theta_b\mu_u + (1 - \theta_b)\mu_i) < \mu_o < (1 - c)(\theta_g\mu_i + (1 - \theta_g)\mu_u)$$

That is, if the manager is good, the net expected return from investing in the active fund is larger than the expected return from investing in the index fund. Conversely, if the manager is bad, the expected return from investing in the active fund is smaller than that from investing in the index fund.

Note that we may have $\mu_u = \mu_o$. In such a case, an uninformed manager does not underperform an index fund, but charges a larger management fee.

There is a continuum of identical risk-neutral investors. At the end of period 0, investors only observe the return, but not the choice σ or the quality of the manager. On aggregate, investors have K_1 units of capital to invest at the beginning of period 1.

Investors' problem

Given that all investors have the same information, they make the same decision. Investors choose the actively managed fund if they expect to earn a higher return after fees than with the index fund. I.e., if investors observe a return R realized by the actively managed fund, they invest in this fund if

$$(1 - c) \{P(\text{"good"} | R)[\theta_g\mu_i + (1 - \theta_g)\mu_u] + P(\text{"bad"} | R)[\theta_b\mu_i + (1 - \theta_b)\mu_u]\} > \mu_o \quad (2)$$

This is equivalent to

$$P(\text{"good"} | R) > \frac{\mu_o - (1 - c)[\theta_b\mu_i + (1 - \theta_b)\mu_u]}{(1 - c)(\theta_g - \theta_b)(\mu_i - \mu_u)} \quad (3)$$

We also have

$$\begin{aligned} P(\mu_i | R) &= P(\mu_i | \text{"good"})P(\text{"good"} | R) + P(\mu_i | \text{"bad"})P(\text{"bad"} | R) \\ &= \theta_g P(\text{"good"} | R) + \theta_b P(\text{"bad"} | R) \end{aligned}$$

Therefore,

$$P(\text{"good"} | R) = \frac{P(\mu_i | R) - \theta_b}{\theta_g - \theta_b}$$

Using equation (4), we deduce that investors invest in the actively managed fund if

$$P(\mu_i|R) > \frac{\mu_o - (1 - c)\mu_u}{(1 - c)(\mu_i - \mu_u)} \quad (4)$$

Let

$$\tau = \frac{\mu_o - (1 - c)\mu_u}{(1 - c)(\mu_i - \mu_u)} \quad (5)$$

Investors will choose to invest in the actively managed fund if the probability that the fund manager is informed exceeds some threshold $\tau \in (0, 1)$.

The fund manager's problem

We assume that the manager of the active fund is risk neutral and his objective is to maximize his expected revenue over periods 0 and 1. Denote K_0 , the size of the fund at the beginning of period 0.⁵ After knowing whether he is informed ($x = i$) or not ($x = u$) in period 0, the expected revenue of a manager of type $y = g, b$ is

$$Rev(c, x, y) = c \{K_0\mu_x + K_1P[P(\mu_i|R) > \tau | \mu = \mu_x] (\theta_y\mu_i + (1 - \theta_y)\mu_u)\}$$

Hence, after having set c and knowing whether he is informed, the objective of the active fund manager is to maximize the probability that investors will choose the active fund rather than the passive index fund. (The fee charged by the active fund will be endogenized in Section 5.)

We are interested in the sequential equilibria of the game played by the investors and the fund manager.

This is an extremely simple model aimed at providing a more precise underpinning of the intuition described in the introduction. For simplicity sake and on purpose,

⁵Here, we are considering an active fund manager who has not yet derived a reputation. So, K_0 should be viewed as small with respect to K_1 (i.e., the amount of money to be invested in period 1). The main objective of the manager is to signal good quality through the realized return so that investors choose to invest in the active fund at time 1.

the model ignores some potentially important aspects. First, the rather mechanical decision by the investors shortcuts a much lengthier derivation of the portfolio choice based on first principles in some multiperiod, multifund model. Such a derivation would likely complicate the analysis in an unnecessary way.

Second, we have completely ignored an important multiperiod aspect investors should take into account: the manager will have an incentive to signal good quality also in the *future*, using the investors resources to do so. The latter provides for additional interesting interactions which we hope to analyze in future work.

We analyze the game backwards, searching for the equilibrium (pure-strategy) portfolio risks σ_i^* and σ_u^* , to be picked by the informed manager and the uninformed manager, respectively, and the fund picking rule for the investors.

In the last stage of the game, having observed the realized return R and in knowledge of the equilibrium strategies (σ_i^*, σ_u^*) , the investors can calculate the likelihood ratio $L(R; \sigma_i^*, \sigma_u^*)$, that the manager is informed,

$$\begin{aligned} L(R; \sigma_i^*, \sigma_u^*) &= \frac{P(R | \mu_i)}{P(R | \mu_u)} \\ &= \frac{\sigma_u^*}{\sigma_i^*} e^{(R-\mu_i)^2/(2\sigma_i^{*2}) - (R-\mu_u)^2/(2\sigma_u^{*2})} \end{aligned}$$

Using Bayes rule, the probability of an informed manager having realized the return R computes to

$$\begin{aligned} P(\mu_i | R) &= \frac{P(R | \mu_i)P(\mu_i)}{P(R | \mu_u)P(\mu_u) + P(R | \mu_i)P(\mu_i)} \\ &= \frac{\psi}{\psi + (1 - \psi)L(R; \sigma_i^*, \sigma_u^*)} \end{aligned}$$

with

$$\psi = \phi\theta_g + (1 - \phi)\theta_b$$

Given that investors invest in the fund if $P(\mu_i | R) \geq \tau$ (see Equations (4) and (5)) and given the equilibrium strategies (σ_i^*, σ_u^*) , the fund will now pick σ so as to

maximize his chances of receiving investors,

$$\max_{\sigma} P\left(\frac{\psi}{\psi + (1 - \psi)L(R; \sigma_i^*, \sigma_u^*)} \geq \tau\right), \quad (6)$$

where the return distribution of R depends both on σ as well as on the information of the manager via the mean μ_i or μ_u . (See equation (1).)

Therefore, a strategy pair (σ_i^*, σ_u^*) is an *equilibrium*, if and only if

$$\sigma_x^* = \operatorname{argmax}_{\sigma} P\left(\frac{\psi}{\psi + (1 - \psi)L(R; \sigma_i^*, \sigma_u^*)} \geq \tau\right) \text{ s.t. } R = \mu_x + \sigma\epsilon, \epsilon \sim \mathcal{N}(0; 1)$$

for $x = i, u$.

We are now ready to state our main results.

Theorem 1 *There exists $\bar{\rho} < 1$ such that if the parameters $\mu_i, \mu_u, \mu_o, c, \underline{\sigma}, \theta_g, \theta_b, \phi$ satisfy*

$$0 > \log\left(\frac{\psi(1 - \tau)}{(1 - \psi)\tau}\right) > \operatorname{Max}\left(-\frac{1}{2} - \log\left(\frac{\mu_i - \mu_u}{\underline{\sigma}}\right), \log(\bar{\rho})\right) \quad (7)$$

with

$$\tau = \frac{\mu_o - (1 - c)\mu}{(1 - c)(\mu_i - \mu_u)}, \quad \psi = \phi\theta_g + (1 - \phi)\theta_b$$

then,

(i) *there exists an equilibrium (σ_i^*, σ_u^*) with the following features:*

1. *An informed manager picks the minimal feasible risk level, $\sigma_i^* = \underline{\sigma}$.*
2. *An uninformed manager picks a risk level, which is strictly greater than the feasible minimum, $\sigma_u^* > \underline{\sigma}$.*
3. *The investor invests in the fund, if the return R falls in the interval $[R_l, R_h]$ for some bounds R_l, R_h solving some quadratic equation, and satisfying*

$$\mu_u < R_l < \mu_i < R_h < \infty$$

(ii) There is no equilibrium, in which the smart investor will invest if $R \geq \underline{R}$ for some \underline{R} .

Proof: See Appendix

The following two theorems provide investors' equilibrium investment decisions for other sets of parameters.

Theorem 2 Suppose, the parameters $\mu_i, \mu_u, \mu_o, c, \underline{\sigma}, \theta_g, \theta_b, \phi$ satisfy

$$\frac{(\mu_i - \mu_u)^2}{2\underline{\sigma}^2} > \log \left(\frac{\psi(1 - \tau)}{(1 - \psi)\tau} \right). \quad (8)$$

Then, part (i) of Theorem 1 holds.

Proof: See Appendix.

Theorem 3 Suppose, the parameters $\mu_i, \mu_u, \mu_o, c, \underline{\sigma}, \theta_g, \theta_b, \phi$ satisfy

$$\log \left(\frac{\psi(1 - \tau)}{(1 - \psi)\tau} \right) > \frac{(\mu_i - \mu_u)^2}{2\underline{\sigma}^2} \quad (9)$$

Then there exists an equilibrium (σ_i^*, σ_u^*) with the following features:

1. Both managers pick the minimal feasible risk level, $\sigma_i^* = \sigma_u^* = \underline{\sigma}$.
2. The investor invests in the fund, if the return R is larger than some threshold \underline{R} with

$$\underline{R} = \frac{\mu_i + \mu_u}{2} - \frac{\underline{\sigma}^2}{\mu_i - \mu_u} \log \left(\frac{\psi(1 - \tau)}{(1 - \psi)\tau} \right) \leq \mu_u \quad (10)$$

Proof: See Appendix

Theorem 1 derives conditions under which the realization of a high return signals that the manager was uninformed and chose a large variance. Since this strategy is

more likely to be played by a bad manager (since $\theta_b < \theta_g$), a rational investor should not invest in the active fund in period 1.

The interpretation of Theorem 1 is then the following. The first inequality of (7) means that ex-ante (i.e., before observing the return) investors are better off *not* investing in the active fund. The theorem says that in such a case, a large return should always be interpreted as the return of a bad uninformed manager "gambling" for a lucky outcome, rather than a sign of good information. Such a result is consistent with those of Carhart (1997) on the lack of persistence of top performance and on the gambling behavior of mutual fund managers.

Figure 1 illustrates the results of Theorem 1. The grey area and the bell-shaped line represent the densities of return generated by an uninformed fund manager and an informed fund manager, respectively. The two vertical lines represent the lower and the upper bound (i.e., R_l and R_h) for realized returns between which investors decide to invest in the fund.

[Insert Figure 1]

The difference between Theorems 1 and 2 is that if the first inequality of (7) does not hold (but (8) holds), there also exists an equilibrium such that investors always invest in the active fund no matter what return is observed. This equilibrium is discussed in Appendix 8. Such an equilibrium does not exist under the conditions of Theorem 1

The equilibrium proposed by Theorem 3 is more "conventional": pick the fund which generates the highest returns. The inequality 9 thus provides the dividing line between a "naive" and a "sophisticated" reading of mutual fund return rankings. Such fund picking strategy should be implemented when difference between good and bad managers is small (inequality (8) does not hold).

From Theorem 1, we also derive some testable implications for young funds which have not yet derived a reputation, and mainly aim at attracting new money:

- The risk-return relation is U-shaped: top and very bad performance are the result of high risk portfolio while intermediate performances are the result of low risk portfolio.
- For top performers, the correlation between performances in two consecutive years is negative.
- For good (but not top) performers, the correlation between performances in two consecutive years is positive.

4 The general case

The preceding section concentrated on the highly restrictive case, that there are only two types of information for the manager. We shall now proceed to the general case of fairly arbitrary a priori expected returns. Our aim is now more modest: we want to state sufficient conditions under which we can *rule out*, that a smart investor should always pick the fund with the highest return, i.e. we want to state conditions, under which the answer to the question in the title is negative.

Assume, that the information of the manager is measured by the mean return μ he can achieve, which is drawn according to some prior probability measure $\pi(d\mu)$ with compact support $[\underline{\mu}, \bar{\mu}]$. The manager picks σ within a given interval $[\underline{\sigma}, \bar{\sigma}]$, $0 < \underline{\sigma} < \bar{\sigma}$ and realizes the return

$$R = \mu + \sigma\epsilon$$

where $\epsilon \sim N(0, 1)$. The investor observes R and forms posterior beliefs π_R for the managerial quality μ . He will invest in the fund iff

$$(1 - c)E_{\pi_R}[\mu] \geq \mu_o$$

Our general result is as follows.

Theorem 4 *Suppose that, a priori,*

$$(1 - c)E_\pi[\mu] < \mu_o$$

Then, for $\bar{\sigma}$ sufficiently large, there is no equilibrium, in which the smart investor will invest if $R \geq \underline{R}$ for some \underline{R} .

Proof: See Appendix

The theorem generalizes the result of Theorem 1 part (ii): whenever it is not a good idea to buy the active fund a priori, then investors should always interpret very large returns as a sign of a bad fund gambling rather than the sign of good ability.

5 Endogenous fee structure

In the previous sections, it was assumed that the fee structure is given: the active fund charges a fraction-of-fund type of fee. One might be concerned that the fee structure offers managers another avenue for signalling the type, thus undermining the results obtained above. Indeed, this is exactly what happens in Das and Sundaram (2002). They assume, however, that the fee structure is chosen *after* the manager has learned whether he is informed or not.

By contrast, assume here that the manager has to choose the fee charged to investors *before* knowing whether he will be informed (i.e., when choosing the fee structure, a manager of type $x = g, b$ only knows with that he will be informed with probability θ_x). In what follows, we will show that pooling equilibria exist for a range of parameters: in these equilibria, a good and a bad fund both charge the same fraction-of-fund fee.

The result in this section is meant to show that the mere possibility of signalling the type with the fee structure does not by itself imply that this method of signalling will be employed, and that the analysis above might still be applicable, i.e., Theorem

1 is robust to the possibility of signalling with incentive contracts. While it would be desirable to find out, whether or not signalling or pooling is going to be the more prevalent case to occur, a much more quantitative or empirical analysis would be needed to provide the answer.

As in Das and Sundaram (2002), we restrict our attention to linear fees allowed by the 1970 Amendment to the Investment Advisors Act of 1940. That is, we consider fraction-of-fund fees and linear performance fees which are symmetric around some chosen index (“fulcrum” fees). Therefore, feasible fees are of the following form

$$C(\alpha_1, \alpha_2, B, R) = \alpha_1 R + \alpha_2(R - B)$$

If $\alpha_2 = 0$, then the active fund charges a fraction-of-fund fee. If $\alpha_2 > 0$, the active fund charges a fulcrum fee where B is the chosen benchmark return.

We obtain the following result about the possibility of pooling equilibria.

Theorem 5 *Assume that the fund manager chooses the fee structure before knowing before whether his expected return is μ_i or μ_u . Then, there exist sets of parameters such that there exists a pooling equilibrium in which managers choose a fraction-of-fund fee structure (i.e., $\alpha_1 > 0$ and $\alpha_2 = 0$), and Theorem 1 holds.*

Proof: See Appendix

6 Conclusion

This paper provided a simple, highly stylized theory of the game played between a mutual fund manager and a collection of investors. The mutual fund managers can be either bad or good, and is thus able to realize either a low or a high return on investment on average. The mutual fund managers controls the riskiness of his portfolio; the investors observe a realization of the fund return, and invest in the fund, if it is sufficiently likely that the mutual fund manager is good. In such a situation,

the manager will choose the riskiness of his to-be-observed portfolio return in order to maximize the chance that the investor will invest with him.

We obtained the following results. Under some conditions on the parameters of our model, the investor will invest in the fund, if the realized return falls within some interval, i.e., is neither too low nor too high. An informed mutual fund manager picks a portfolio of minimal riskiness. A uninformed mutual fund manager will pick a portfolio with higher risk, “gambling” on a lucky outcome. Thus, smart investors may not want to buy the fund with the highest returns in the past.

This equilibrium holds for sets of parameters such that ex-ante (i.e., before observing the return) investors are better off *not* investing in the active fund. Therefore, our results can be interpreted in the following way. If, ex-ante, investors are better off *not* investing in the active fund, then a large return should always be interpreted as the return of an uninformed manager “gambling” rather than a sign of good information.

Our results on portfolio selection by fund managers are consistent with those of Carhart (1997) on the lack of persistence in performances by mutual funds. Hence, investment advisors making recommendations on the basis of funds’ past returns should tell investors to choose a good performer but not a top performer.

7 Appendix

Proof of Theorem 1

Proof of (i): The proof proceeds in several steps. We concentrate on finding equilibria, satisfying $\sigma_i^* \leq \sigma_u^*$. We will make ample use of this inequality, which obviously needs to be verified in the end.

1. The investment decision of the investors.

Given (5), the criterion (4) can be written as

$$(R - \mu_i)^2 / (2\sigma_i^{*2}) - (R - \mu_u)^2 / (2\sigma_u^{*2}) \leq \log \left(\frac{\sigma_u^*}{\sigma_i^*} \frac{\psi(1 - \tau)}{(1 - \psi)\tau} \right)$$

or, equivalently,

$$q(R; \sigma_i^*, \sigma_u^*) \leq 0 \quad (11)$$

where $q(R; \sigma_i^*, \sigma_u^*)$ is a quadratic function in R , given by

$$\begin{aligned} q(R; \sigma_i^*, \sigma_u^*) = & \left(\frac{1}{2\sigma_i^{*2}} - \frac{1}{2\sigma_u^{*2}} \right) R^2 \\ & - \left(\frac{\mu_i}{\sigma_i^{*2}} - \frac{\mu_u}{\sigma_u^{*2}} \right) R + \left(\frac{\mu_i^2}{2\sigma_i^{*2}} - \frac{\mu_u^2}{2\sigma_u^{*2}} \right) - \log \left(\frac{\sigma_u^*}{\sigma_i^*} \frac{\psi(1 - \tau)}{(1 - \psi)\tau} \right) \end{aligned} \quad (12)$$

Inequality (11) is satisfied, iff

$$R \in [R_l(\sigma_i^*, \sigma_u^*), R_h(\sigma_i^*, \sigma_u^*)] \quad (13)$$

where $R_l(\sigma_i^*, \sigma_u^*) \geq R_h(\sigma_i^*, \sigma_u^*)$ are the two solutions to the quadratic equation

$$q(R; \sigma_i^*, \sigma_u^*) = 0 \quad (14)$$

Let

$$\rho = \frac{\psi(1 - \tau)}{(1 - \psi)\tau} \quad (15)$$

A sufficient condition for the (14) to have two real solutions is that

$$\Delta = \left(\frac{\mu_i}{\sigma_i^{*2}} - \frac{\mu_u}{\sigma_u^{*2}} \right)^2 + \left(\frac{1}{2\sigma_i^{*2}} - \frac{1}{2\sigma_u^{*2}} \right) \left[\left(\frac{\mu_u^2}{2\sigma_u^{*2}} - \frac{\mu_i^2}{2\sigma_i^{*2}} \right) + \log \left(\frac{\sigma_u^*}{\sigma_i^*} \rho \right) \right] > 0 \quad (16)$$

Let

$$g(\sigma_i, \sigma_u) = \left(\frac{1}{2\sigma_i^2} - \frac{1}{2\sigma_u^2} \right) \log \left(\frac{\sigma_u}{\sigma_i} \rho \right)$$

and

$$\hat{g}(\rho) = \min_{(\sigma_i, \sigma_u), \sigma_u \geq \sigma_i} f(\sigma_i, \sigma_u, \rho)$$

$\hat{g}(\rho)$ is continuous in ρ and $\hat{g}(1) \geq 0$. Therefore, by continuity, there exists $\bar{\rho} < 1$ such that if $\rho > \bar{\rho}$, then $\Delta > 0$ and (14) has two real solutions. Furthermore, we note that for $\sigma_i^* = \sigma_u^*$, the coefficient on the lead quadratic term becomes zero. The solutions then take the form

$$\begin{aligned} R_h &= \infty \\ R_l &= \underline{R}(\sigma_i^*) \end{aligned}$$

where

$$\underline{R}(\sigma_i^*) = \frac{\mu_i + \mu_u}{2} - \frac{\sigma_i^{*2}}{\mu_i - \mu_u} \log \left(\frac{\psi(1 - \tau)}{(1 - \psi)\tau} \right) \quad (17)$$

2. A first-order condition for the fund manager.

With (13), the objective (6) of the fund manager with quality $x = i, u$ can be rewritten as

$$\max_{\sigma} f(\sigma; \sigma_i^*, \sigma_u^*) \quad (18)$$

where $f(\sigma; \sigma_i^*, \sigma_u^*)$ is the integral

$$f(\sigma; \sigma_i^*, \sigma_u^*) = \int_{R_l(\sigma_i^*, \sigma_u^*)}^{R_h(\sigma_i^*, \sigma_u^*)} \frac{1}{\sqrt{2\pi}\sigma} e^{-(R - \mu_x)^2 / (2\sigma^2)} dR$$

To find the optimum, it is useful to differentiate f with respect to σ ,

$$\frac{df}{d\sigma} = \frac{1}{\sigma} I(\sigma; R_l(\sigma_i^*, \sigma_u^*), R_h(\sigma_i^*, \sigma_u^*))$$

where $I(\sigma; R_l, R_h)$ is the integral

$$I(\sigma; R_l, R_h) = \int_{\frac{R_l - \mu_x}{\sigma}}^{\frac{R_h - \mu_x}{\sigma}} (\epsilon^2 - 1) \frac{1}{\sqrt{2\pi}} e^{-\epsilon^2/2} d\epsilon \quad (19)$$

Standard results about normal distributions imply immediately that

$$I(\sigma; -\infty, \infty) = I(\sigma; 0, \infty) = I(\sigma; -\infty, 0) = 0 \quad (20)$$

Furthermore, note that the integrand $(\epsilon^2 - 1)$ in (19) is negative if $|\epsilon| < 1$. These relationships will prove useful in the next step.

3. When do we have $\sigma_i^* = \underline{\sigma}$?

From the preceding analysis, we see that (18) is solved at $\sigma = \underline{\sigma}$, if

$$I(\sigma; R_l(\sigma_i^*, \sigma_u^*), R_h(\sigma_i^*, \sigma_u^*)) < 0$$

for all $\sigma \geq \underline{\sigma}$. With (20) and the remark following it, it is straightforward to check, that this is the case if $R_l \leq \mu_x \leq R_h$. This in turn is true iff $q(\mu_x; \sigma_i^*, \sigma_u^*) \leq 0$. For $x = i$, this can be rewritten as

$$q(\mu_i) = -\frac{(\mu_i - \mu_u)^2}{2\sigma_u^{*2}} - \log\left(\frac{\sigma_u^*}{\sigma_i^*} \frac{\psi(1 - \tau)}{(1 - \psi)\tau}\right) \leq 0.$$

The second inequality of (7) is sufficient for this inequality to hold. This shows, that $\sigma_i^* = \underline{\sigma}$, as claimed.

4. When do we have $\underline{\sigma} < \sigma_u^* < \infty$?

From the previous step, we deduce that we have an equilibrium with $\sigma_i^* = \underline{\sigma}$ and $\sigma_u^* > \underline{\sigma}$ if we can rule out $\sigma_u^* = \underline{\sigma}$ via

$$I(\underline{\sigma}, R_l(\underline{\sigma}, \underline{\sigma}), R_h(\underline{\sigma}, \underline{\sigma})) > 0 \quad (21)$$

and show, that for some $\sigma > \underline{\sigma}$, we have

$$I(\sigma, R_l(\underline{\sigma}, \sigma), R_h(\underline{\sigma}, \sigma)) < 0 \quad (22)$$

An interior solution must then exist by the mean value theorem.

Inequality (21) is equivalent to $\underline{R}(\underline{\sigma}) > \mu_u$ which is in turn equivalent to the first inequality of (7). For inequality (22), note that the integrand in equation (19) is negative for $|\epsilon| < 1$. Thus, (22) follows, if

$$-1 < \frac{R_l(\underline{\sigma}, \sigma) - \mu_u}{\sigma} \leq \frac{R_h(\underline{\sigma}, \sigma) - \mu_u}{\sigma} < 1$$

for some σ . Rewrite this as

$$\mu_u - \sigma < R_l(\underline{\sigma}, \sigma) \leq R_h(\underline{\sigma}, \sigma) < \mu_u + \sigma$$

which, in turn, is equivalent to

$$q(\mu_u - \sigma, \underline{\sigma}, \sigma) > 0, q(\mu_u + \sigma, \underline{\sigma}, \sigma) > 0$$

for some σ . However, inspecting (12), it is easy to see that

$$q(\mu_u - \sigma, \underline{\sigma}, \sigma) \rightarrow \infty$$

and

$$q(\mu_u + \sigma, \underline{\sigma}, \sigma) \rightarrow \infty$$

as $\sigma \rightarrow \infty$, completing the proof.

Proof of (ii): We consider the three possible types of equilibria: $\sigma_i < \sigma_u$, $\sigma_i > \sigma_u$ and $\sigma_i = \sigma_u$.

Let's consider first the case $\sigma_i < \sigma_u$. From the proof of Part (i), we know if $\rho > \bar{\rho}$, then $\Delta > 0$ and (14) has two real solutions. Hence, there is no equilibrium, in which the smart investor will invest if $R \geq \underline{R}$ for some \underline{R} .

Consider now the case $\sigma_i > \sigma_u$. Proceeding as in the proof of (i), one obtains that if the beliefs of the investor are such that $\sigma_i^* > \sigma_u^*$, then the objective of a manager is to minimize $f(\sigma; \sigma_i^*, \sigma_u^*)$ over σ

where $f(\sigma; \sigma_i^*, \sigma_u^*)$ is the integral

$$f(\sigma; \sigma_i^*, \sigma_u^*) = \int_{R_l(\sigma_i^*, \sigma_u^*)}^{R_h(\sigma_i^*, \sigma_u^*)} \frac{1}{\sqrt{2\pi}\sigma} e^{-(R-\mu_x)^2/(2\sigma^2)} dR$$

where $x = i$ if the manager is informed and $x = u$ if the manager is uninformed. It is straightforward that $f(\sigma; \sigma_i^*, \sigma_u^*)$ converges to 0 when σ goes to $+\infty$. Hence, there is no equilibrium, in which the smart investor will invest if $R \geq \underline{R}$ for some \underline{R} .

Last, consider the case $\sigma_i = \sigma_u$. Assume that the investor's beliefs are such that $\sigma_i^* = \sigma_u^*$. The objective of a manager is to maximize

$$Prob \left(R > -\frac{\sigma^2}{\mu_i - \mu_u} \log(\rho) + \frac{\mu_i + \mu_u}{2} \right)$$

over σ . From the proof of part (i), we know that a good manager chooses $\sigma_u = \underline{\sigma}$. The condition $\log(\rho) < 0$ implies that

$$-\frac{\sigma^2}{\mu_i - \mu_u} \log(\rho) + \frac{\mu_i + \mu_u}{2} > \mu_u$$

As a consequence, a bad manager chooses $\sigma_u > \underline{\sigma}$. Hence, there is no equilibrium, in which the smart investor will invest if $R \geq \underline{R}$ for some \underline{R} . \square

Proof of Theorem 2 If Inequality (8) holds then inequality (16) holds. Therefore, we have the desired result. \square

Proof of Theorem 3 Follow the proof of Theorem 1 above, except for the last step. Instead of inequality (21), we now get

$$I(\underline{\sigma}, R_l(\underline{\sigma}, \underline{\sigma}), R_h(\underline{\sigma}, \underline{\sigma})) < 0 \tag{23}$$

as a consequence of (9). It follows that $\sigma_u^* = \underline{\sigma}$ also. Finally, combine (9) and (10) to see that $\underline{R} < \mu_u$. \square

Proof of Theorem 4

Suppose to the contrary, that there was an equilibrium, where the investor always invests, if $R \geq \underline{R}$ for some \underline{R} . We will show, that there is some $R \geq \underline{R}$ (perhaps

requiring some sufficiently large $\bar{\sigma}$), so that

$$E_{\pi_R}[\mu] < \mu^* \quad (24)$$

which is a contradiction.

As in the proof of Theorem 1, the fund manager will maximize the probability that $R \geq \underline{R}$ via his choice of σ . It is easy to check, that he will choose $\sigma = \underline{\sigma}$, if $\mu \geq \underline{R}$ and $\sigma = \bar{\sigma}$, if $\mu < \underline{R}$. The posterior probability distribution therefore has the form

$$\pi_R(d\mu) = \phi(R, \bar{\sigma}) (m_A(d\mu; R, \bar{\sigma}) + m_B(\mu; R, \bar{\sigma}))$$

where m_A is a measure with support $[\underline{\mu}, \underline{R})$ and given by

$$m_A(d\mu; R, \bar{\sigma}) = \frac{1}{\bar{\sigma}} \exp\left(-\frac{(\mu - R)^2}{2\bar{\sigma}^2}\right) \pi(d\mu),$$

where m_B is a measure with support $[\underline{R}, \bar{\mu}]$ and given by

$$m_B(d\mu; R, \bar{\sigma}) = \frac{1}{\underline{\sigma}} \exp\left(-\frac{(\mu - R)^2}{2\underline{\sigma}^2}\right) \pi(d\mu),$$

and where $\phi(R, \bar{\sigma})$ is chosen so that π_R is a probability measure.

We now distinguish two cases, $\underline{R} > \underline{\mu}$ and $\underline{R} \leq \underline{\mu}$.

1. Case: $\underline{R} > \underline{\mu}$

Consider first the case $\underline{R} > \underline{\mu}$. In that case, the measure μ_A has positive mass, since $\pi(\mu < \underline{R}) > 0$.

Note that the lead term in the exponential expression in m_A is $-R^2/(2\bar{\sigma}^2)$, whereas it is $-R^2/(2\underline{\sigma}^2)$ in the exponential expression for m_B . Since $\bar{\sigma} > \underline{\sigma}$, it follows that m_B vanishes relative to m_A as $R \rightarrow \infty$, $\bar{\sigma} \rightarrow \infty$, $R/\bar{\sigma} \equiv \text{const.}$. More precisely, for any $\nu > 0$, one can find some sufficiently large R as well as some $\tilde{\sigma}$, so that for all $\bar{\sigma} > \tilde{\sigma}$, we have that

$$|E_{\phi(R, \bar{\sigma})m_A(\cdot; R, \bar{\sigma})}[\mu] - E_{\pi_R}[\mu]| < \nu,$$

holding $R/\bar{\sigma} \equiv \text{const}$. Fix $R/\bar{\sigma}$. One can see that

$$E_{\phi(R,\bar{\sigma})m_A(\cdot;R,\bar{\sigma})}[\mu] \rightarrow E_{\pi}[\mu \mid \mu < \underline{R}]$$

as $\bar{\sigma} \rightarrow \infty$. Put these two pieces together and use

$$E_{\pi}[\mu \mid \mu < \underline{R}] \leq E_{\pi}[\mu] < \mu^*$$

to demonstrate (24).

2. Case: $\underline{R} \leq \underline{\mu}$

Next, consider the case $\underline{R} \leq \underline{\mu}$. In that case, the fund manager will choose $\sigma = \underline{\sigma}$, regardless of his type μ . Now suppose that the investor observes $R = \underline{\mu}$: note that he will invest. In that case,

$$E_{\pi_R}[\mu] = \frac{\int \mu e^{-(\mu-\underline{\mu})^2/(2\underline{\sigma}^2)} \pi(d\mu)}{\int e^{-(\mu-\underline{\mu})^2/(2\underline{\sigma}^2)} \pi(d\mu)} \leq E_{\pi}[\mu] < \mu^*,$$

(where the first inequality can be seen to hold, since the posterior puts larger weight on smaller μ 's), yielding the contradiction. This contradiction does not require the probability-zero event of exactly observing $R = \underline{\mu}$: the inequality above also remains for R near $\underline{\mu}$ by continuity.

□

Proof of Theorem 5

Consider the following set of parameters: $\phi = 2/3$, $\theta_g = 0.55$, $\theta_b = 0.4$, $\mu_i = 1.2$, $\mu_u = 1.16$, $\mu_o = 1.18$, $\underline{\sigma} = 0.05$, $K_0 = 0$ and $K_1 = 1$. In such a case, $\psi = \phi\theta_g + (1 - \phi)\theta_b = 0.5$. If the manager chooses a fraction-of-fund fee structure, then for any $\alpha_1 > 0$, $\tau > 1$. Therefore, if α_1 is small enough, the conditions of Theorem 1 are met while if α_1 is large, the investor never invests in the fund. From Theorem 1, we know that R_l and R_h are function of the fee charged. Therefore, we rewrite them as $R_l(\alpha_1)$ and $R_h(\alpha_1)$.

Let

$$rev(\alpha_1) = \alpha_1 \{K_0 + K_1 Prob[R \in (R_l(\alpha_1), R_h(\alpha_1))]\} (\theta_g \mu_i + (1 - \theta_g) \mu_u)$$

and α_1^* be a solution of

$$\max_{\alpha_1} rev(\alpha_1) \quad \text{subject to} \quad (1 - \alpha_1)[\psi \mu_i + (1 - \psi) \mu_u] \geq \mu_o$$

Now, we show that for the set of parameters mentioned above, there exists a pooling equilibrium in which 1) both types of manager choose a fraction-of-fund fee ($\alpha_1 = \alpha_1^*$, $\alpha_2 = 0$) and 2) investors have the following out-of-equilibrium beliefs which satisfy the intuitive criterion (Cho and Kreps (1987)).

- If the manager chooses a fulcrum fee (i.e., $\alpha_2 > 0$) with $B \geq \bar{\mu}_b = \theta_b \mu_g + (1 - \theta_b) \mu_b$, then the manager is good.
- If the manager chooses a fulcrum fee (i.e., $\alpha_2 > 0$) with $B < \bar{\mu}_b$, then the manager is bad.
- If the manager choose a fraction-of-fund fee with $\alpha_1 \neq \alpha_1^*$, then the manager is bad.

First, assume that a bad manager chooses a fraction-of-fund fee with $\alpha_1 = \alpha_1^*$. If a good manager chooses a fraction-of-fund fee, he will also choose $\alpha_1 = \alpha_1^*$ since it maximizes its revenue among fraction-of-fund fees. Second, assume that a good manager deviates from the equilibrium strategy and chooses an incentive fee structure with $B \geq \bar{\mu}_b$. For this deviation to be profitable it must be the case that, given the incentive fee scheme he chooses, the investor invests in this fund, that is

$$\theta_g [\mu_g - \alpha_1 \mu_g - \alpha_2 (\mu_g - B)] + (1 - \theta_g) [\mu_b - \alpha_1 \mu_b - \alpha_2 (\mu_b - B)] \geq \mu_o, \quad (25)$$

It is straightforward that the revenue maximizing incentive fee is a triplet $(\hat{\alpha}_1, \hat{\alpha}_2, \hat{B})$ such that constraint (25) is binding. Given the set of parameters chosen, the revenue maximizing incentive fee gives an expected revenue

$$\theta_g \mu_g + (1 - \theta_g) \mu_b - \mu_o = 0.002$$

Now, consider the following fraction-of-fund fee: $\alpha_1 = 1 - 1.18/1.19 \approx 0.0084$. In such a case, $\sigma_u \approx 0.1685$, $R_l = 1.1754$, $R_h = 1.2322$, $P(R \in [R_l, R_h]|\mu_g) = 0.4289$ and $P(R \in [R_l, R_h]|\mu_b) = 0.1294$. The expected revenue of a good manager is then $rev(0.0084) = 0.00348$. By definition $rev(\alpha_1^*) > rev(0.0084)$. This implies that a good manager has no incentives to deviate from the fraction-of-fund fee α_1^* .

Now, we only need to show that a bad manager never deviates from the equilibrium. Given the out-of-equilibrium beliefs, a bad manager will never choose a fraction-of-fund fee with $\alpha_1 \neq \alpha_1^*$ nor an incentive scheme with $B < \bar{\mu}_b$ since this would give zero revenue while playing the equilibrium strategy gives him a strictly positive expected revenue. The last type of possible deviation is an incentive scheme with $B \geq \bar{\mu}_b$. Such a scheme gives a negative expected revenue. Hence, such a deviation is never chosen. This completes the proof.

□

8 Existence of other equilibria under the conditions of Theorem 2

There are also equilibria such that the investor always invests in the fund. Tracing through the proof above of theorem 1, one can see that such an equilibrium will result, if

$$q(R, \sigma_i^*, \sigma_u^*) < 0 \text{ for all } R \quad (26)$$

In that case, the fund manager is indifferent between all choices for σ , regardless of his type, since his choice does not influence the probability of the investor investing with him. So, any σ_i^* , σ_u^* satisfying (26) is an equilibrium.

Given ψ , τ , μ_u , μ_i and $\underline{\sigma}$, we shall now show, that such σ_i^* and σ_u^* can always be found, provided that

$$\log \left(\frac{\psi(1-\tau)}{(1-\psi)\tau} \right) > 0 \quad (27)$$

(note that this conditions is assumed to hold also for Theorems 2 and 3). To see this, first note that (26) requires $\sigma_i^* < \sigma_u^*$. Given this inequality, (26) can be rewritten as

$$(\mu_i - \mu_u)^2 - 2(\sigma_i^{*2} - \sigma_u^{*2}) \log \left(\frac{\psi(1 - \tau)\sigma_u^*}{(1 - \psi)\tau\sigma_i^*} \right) < 0 \quad (28)$$

after some algebra. Let $\zeta = \sigma_i^*/\sigma_u^*$: note that we need to keep $\zeta > 1$. Rewrite equation (28) as

$$(\mu_i - \mu_u)^2 - 2\sigma_u^{*2}(\zeta^2 - 1) \log \left(\frac{\psi(1 - \tau)}{(1 - \psi)\tau\zeta} \right) < 0 \quad (29)$$

With (27), find $\zeta > 1$ close enough to 1, so that

$$\log \left(\frac{\psi(1 - \tau)}{(1 - \psi)\tau\zeta} \right) > 0$$

Next, find σ_u^* large enough so that inequality (29) is satisfied. Calculate $\sigma_i^* = \zeta\sigma_u^*$ to find an equilibrium of the desired form.

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Figure 1

