DISCUSSION PAPER SERIES

No. 3275

THE INTERGENERATIONAL STATE: EDUCATION AND PENSIONS

Michele Boldrin and Ana Montes

PUBLIC POLICY

Centre for Economic Policy Research

www.cepr.org

Available online at: **www.cepr.org/pubs/dps/DP3275.asp**

ISSN 0265-8003

THE INTERGENERATIONAL STATE: EDUCATION AND PENSIONS

Michele Boldrin, University of Minnesota and CEPR
Ana Montes, Universidad de Murcia

Discussion Paper No. 3275
March 2002

Centre for Economic Policy Research
90–98 Goswell Rd, London EC1V 7RR, UK
Tel: (44 20) 7878 2900, Fax: (44 20) 7878 2999
Email: cepr@cepr.org, Website: www.cepr.org

CEPR Discussion Paper No. 3275

March 2002

ABSTRACT

The Intergenerational State: Education and Pensions*

When credit markets to finance investment in the human capital of young people are missing, the competitive equilibrium allocation is inefficient. When generations overlap, this failure can be mitigated by properly designed social institutions such as public education and public pensions. We show that, when established jointly, they implement an intergenerational transfer scheme supporting the complete market allocation. Through the public financing of education, the young borrow from the middle aged to invest in human capital. When employed, they pay back their debt via a social security tax, the proceedings of which finance pension payments to the now elderly lenders. We consider other, allocationally equivalent, financing schemes. In all cases, when the complete market allocation is achieved a certain equality should be observed among implicit rates of return and the market rate of return. We test this prediction by using micro and macro data from Spain. The results are, surprisingly, good. We also use the model to quantify the impact of undergoing demographic change on the implicit rates of return. The results point, unsurprisingly, to dramatic changes in generational rates of return. Contrary to what was predicted by earlier studies in the generational accounting tradition, our findings suggest that future generations are not necessarily going to be worse than current ones.

JEL Classification: H11, H30, H42, I20 and O11
Keywords: efficient intergenerational arrangements, public education and public pensions

Michele Boldrin
University of Minnesota
Department of Economics
Minneapolis MN 55455
USA
Tel: (1 612) 624 4551
Fax: (1 612) 624 0209
Email: mboldrin@econ.umn.edu

Ana Montes
Universidad de Murcia
Departamento de Fundamentos del Análisis Económico
Campus de Espinardo
30100 Murcia
SPAIN
Tel: (34 968) 36 3881
Email: anmontes@fcu.um.es

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=111922

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=147824

* Financial support from the NSF, the University of Minnesota Grants-in-Aid Program, the Fundación BBVA and the Spanish DGES is gratefully acknowledged. Earlier versions of this Paper have circulated since 1997 under the title ‘Intergenerational Transfer Institutions. Public Education and Public Pensions’. Our thanks to Gary Becker, Patrick Kehoe and Timothy Kehoe for useful suggestions. What remains is our doing.

Submitted 03 March 2002

1. Introduction

A well established tradition in public economics argues that a number of government policies and, in particular, most institutions comprising the Welfare State are either due or justified to the inability of decentralized markets to deliver a Pareto efficient allocation. This approach is positive and normative at once. It explains the existence of certain arrangements as cooperative remedies to allocational inefficiencies and it provides, at least in principles, guidance to the socially optimal design for such institutions.

In our work we apply this point of view to the study of public education and public pensions. We build a simple dynamic environment in which the lack of certain specific credit markets leads to suboptimal accumulation of human capital. By construction, a policy of public education financing is therefore desirable. This is all fine, and not very surprising. More surprising, or at least less obvious, is the fact that introducing a scheme for the public financing of education cannot, by itself, restore the complete market allocation. An additional institutional arrangement, closely resembling a public pension system, is also needed to achieve efficiency. Further, we show that an extremely simple (but so far altogether ignored) link between the two systems must be established to replicate the complete market allocation. Such "efficiency link" is captured by the equality between two rates of return (implicit in the public financing of education and pensions) and the market rate of return on capital. After this characterization of the optimal intergenerational arrangement we discuss a number of practical ways to implement it. Finally, we take the predictions (or, prescriptions) of our model to the data. We use Spanish micro and macro data to show that the intergenerational transfers implemented by the current Spanish system are not very far away from those predicted by our model at the complete market allocation. Nevertheless, the computed rates of return are far enough from the model's prescriptions to suggest that reforming the two systems by making the linkage between education and pension explicit would improve economic efficiency and social welfare. This is especially true when the impact of the ongoing demographic change is taken into due account.

The choice of bringing such a simple and stylized model to the data is motivated by the fact that, prima face, none of the practical solutions we discuss resemble the way in which public education and pensions are organized in the countries we are familiar with. The results are surprising, in the sense that, under the assumption of demographic stationarity, current arrangements come extremely close to satisfy the predicted equality. According to our criterion, this means both that the efficient allocation is being approximated and that a negligible amount of intergenerational redistribution is taking place. To test the robustness of this finding, we also simulate a number of reasonable future demographic and policy scenarios. We find that, under the current rules, the combined Spanish education and pension system would violate the desired rate of return equality and, therefore, become both inefficient and highly redistributive across generations.

In an early paper (Becker and Murphy (1988)) Gary Becker and Kevin Murphy argued that the current Welfare State should be understood as replicating or replacing previous social arrangements which, within either the family or small communities, have implemented efficient intergenerational allocations for a long time past. This paper can be seen as a formal and quantitative general equilibrium investigation of their conjecture. We develop a model in which the existence of a public education and pension system are justified by technologies, preferences and market structure. The environment we model is simple, still it seems to capture most features of the real world which one may deem relevant when thinking about public education and pensions. Young generations would like to accumulate productive human capital, but have impediments at financing it via credit markets. Middle age individuals would like to diversify their retirement portfolio by investing in the human capital of younger people, but financial instruments to do so are unavailable. When the assumptions of our model are verified a benevolent social planner would want to explicitly link education and pensions via an intergenerational contract which mimics certain traditional family arrangements: parents take care of young children in exchange for children taking care of elderly parents.

The model we study is, on purpose, very simplified. Still, we believe our main results would be maintained, and indeed strenghtened, if a number of realistic features were added. In particular, it seems straightforward that introducing population growth and a realistic number of periods of life would leave the results unaltered. Adding some form of parental (or filial, as in Boldrin and Jones (2001)) altruism would only modify the quantitative but not the qualitative prescriptions, unless one adopts the fully dynastic model of familial relation proposed in Barro and Becker (1989). Adding uncertainty, in the form of unexpected shocks to the productivity of the two kinds of capital, would most likely strenghten our normative prescriptions on the grounds of portfolio diversification. This is akin to the point already made in Merton (1983), in a different formal context but with similar implications for policy. Finally, and aside for the redistributive concerns this may or may not create for public policy, the introduction of heterogeneity within a generation would also not alter the prescriptions we derive for aggregate fiscal policy.

While Becker and Murphy (1988) argues convincingly for a positive explanation of the growth of the welfare state on the ground that it replicates a number of functions previously accomodated within the (extended) family, neither them nor, a fortiori, us, are the first to argue that a link between the accumulation of human capital by the young and the payment of public pensions to the old does or should exist. Pogue and Sgontz (1977), for example, make this point in the context of a simple model of social security taxation. While they do not fully develop the dynamic implications of their argument, nor they bring it to the data, they stress that "the investment incentive provided by [pay-as-you-go payroll tax] financing is for *collective* investment by each generation in capital that will enhance the income of persons who will be working during the generation's years of retirement" (p. 163, italics in the original). Richman and Stagner (1986) also argues, albeit even more informally, that the very existence of a pay-as-you-go pension system should generate an incentive for the adult cohorts to invest in the younger ones. Further, a very large demographic, sociological and anthropological literature has argued since long ago that such intergenerational linkages (either within the family, the clan, the village or the entire society) are critical for understanding both fertility choices and parental investments in their progenies. Caldwell (1978) and Nugent (1985) are recent references, while Neher (1971) is a very early economic paper in which fertility choices are linked to the parental desire to draw a pension when old.

Relative to the earlier literature our contribution is therefore twofold. First, we build a dynamic general equilibrium model of intergenerational choice and economic growth in which the relation between investment in human capital and pension payments can be formally investigated and the impact of such policies on growth rates, welfare and intergenerational distribution quantified. Second, we take our model to the data to check, on the one hand, how badly it fares as a positive explanation of existing arrangements and, on the other, how far removed from the complete markets allocation current policies are.

In recent years other authors have addressed similar or related issues in the context of various versions of the overlapping generations model. The general question asked by this literature is: if present generations are strictly selfish, why should they invest in assets that are valuable only to future ones? Simmetricaly, what does lead the young generations to transfer resources to the old ones who will not be around tomorrow? Kotlikoff, Persson and

Svensson (1988) is an earlier reference: they cast the problem in term of time-consistency of the optimal policy. The solution proposed involves a social contract which is "sold" by the old to the young generation in exchange for tax revenues. Boldrin (1994) and Boldrin and Rustichini (2000) analyze public education and pensions respectively. In the first case, education is financed because it increases the future productivity of private physical capital, which provides the old generation with a channel to collect a return on their investment. In the second case, pensions are paid because this allows the working generation to act as a "monopolist" in the supply of savings and therefore earn a higher total return on their investment. Subgame perfectness is used to show that an equilibrium with social security can be sustained by simple trigger strategies. Rangel (1999) and Conley (2001) also use game theoretical arguments to study the problem in the context of voting models. The general message is: establish intergenerational arrangements such that future marginal payoffs are transferred backward to the generation making the investment. Rangel (1999), in particular, derives an interesting theory of "backward" and "forward" public goods on the basis of these premises. He uses game theoretical arguments, not dissimilar from those used in earlier versions of this paper, to show that an equilibrium exists in which all generations play a trigger strategy which guarantees that the appropriate amount of (backward) public goods is purchased. While Rangel's argument is developed in the context of a stationary exchange economy, it is clear that it can be generalized to one with production and endogenous growth. Conley (2001) shows that when the public goods in question are durable and there is land, the "Tiebout solution" of providing the public goods locally achieves the efficient allocation. Finally, Belettini and Berti Cerroni (1999) also use an overlapping generations model with production to argue that the existence of pay-as-you-go pensions which are financed by labor income taxation may not necessarily reduce growth. They do so by introducing public capital in the production function and using game theoretical arguments to show that, when pensions are financed by taxes on future labor income, there exists a subgame perfect equilibrium in which investment in the public good (and growth) are higher than otherwise.

While the positive predictions of our model are interesting and may provide valuable in trying to understand the historical origins and the linkage between public education and public pensions, it is on the normative prescriptions that we like to put more emphasis. Should the public education and the public pension systems be designed according to the simple rules presented here? We believe they should. Would this be practically feasible?

We discuss three possible implementations, all of which use fairly traditional tools of public policy: taxes, subsidies, transfers and public debt. We find the third method, based on the issuance of public debt specifically targeted at financing education and paying pensions, to be the most palatable and interesting. The empirical analysis of the Spanish data shows that, indeed, the intergenerational flows implied by our criteria would not be very different from those that the current system originates and could, therefore, be implemented without generating major resistance from the affected parties.

2. The Basic Model

2.1 Complete Markets

Consider an overlapping generations economy in which agents live for three periods. Within each generation individuals are homogenous and, to simplify, each generation has constant size of one. Adding population growth would not alter the results, while the case of stochastic fertility and mortality rates are considered in Boldrin and Montes [2001a].

Physical, k_t, and human, h_t, capital are owned, respectively, by the old and the middle age individuals. Output of the homogeneous commodity is $y_t = F(h_t, k_t)$, where $F(h, k)$ is a constant return to scale neoclassical production function. Young agents are born with an endowment h_t^y of basic knowledge, which is an input in the production of their future human capital $h_{t+1} = h(d_t, h_t^y)$. With d_t we denote the physical resources invested in education, which comprise both direct and opportunity costs. We assume that competitive markets exist in which young agents can borrow such resources. The function $h(d, h^y)$ is also a constant return to scale neoclassical production function. During the second period of life, individuals work and carry out consumption-saving decisions. When old, they consume the total return on their savings before dying. We assume agents draw utility from (c_t^m, c_{t+1}^o) denoting, respectively, consumption when middle age and old. Neither consumption when young, nor leisure, nor the welfare of descendants affect lifetime utility. Adding such considerations would only increase the notational burden without contributing additional insights.

Let the homogenous commodity be the numeraire. Output y_t is allocated to three purposes: aggregate consumption ($c_t = c_t^m + c_t^o$), accumulation of physical capital for next period (k_{t+1}) and investment in education (d_t). Human and physical capital are purchased by firms at competitive prices equal, respectively, to $w_t = F_1(h_t, k_t)$ and $1+r_t = F_2(h_t, k_t)$. Aggregate saving finances investment in physical and human capital ($s_t = k_{t+1} + d_t$), accruing a total return equal to $(1 + r_{t+1})s_t = R_{t+1}s_t$.

The life-cycle optimization problem for an agent born in period $t - 1$ is

$$U_{t-1} = \max_{d_{t-1}, s_t} \; \text{½} \; u(c_t^m) + \delta u(c_{t+1}^o) \; \text{¾} \qquad (2.1)$$

$$\text{subject to} : 0 \leq d_{t-1} \leq \frac{w_t h_t}{R_t}$$
$$c_t^m + s_t + R_t d_{t-1} \leq w_t h_t$$
$$c_{t+1}^o \leq R_{t+1} s_t$$
$$h_t = h(d_{t-1}, h_{t-1}^y).$$

Its first order conditions can be simplified to yield (subscripts of functions indicate partial derivatives):

$$u'\left[w_t h(d_{t-1}, h_{t-1}^y) - s_t - R_t d_{t-1}\right] = \delta R_{t+1} u'\left[s_t R_{t+1}\right] \tag{2.2a}$$

$$\left[w_t h_1(d_{t-1}, h_{t-1}^y) - R_t\right] = 0 \tag{2.2b}$$

The first is the usual equality between interest factor and marginal rate of substitution in consumption. The second equates the private return from investing in human capital to the cost of financing it via the credit market.

Competitive equilibrium is defined by the following set of equalities:

$$F(h_t, k_t) = c_t + s_t \tag{2.3a}$$
$$F_1(h_t, k_t) = w_t \tag{2.3b}$$
$$F_2(h_t, k_t) = R_t = w_t h_1(d_{t-1}, h_{t-1}^y) \tag{2.3c}$$
$$s_t = d_t + k_{t+1} \tag{2.3d}$$

Given a sequence $\{h_t^y\}_{t=0}^{\infty}$, one can solve equations (2.2) and (2.3) for (d_t, h_{t+1}, k_{t+1}), $t = 0, 1, \ldots$ to obtain a dynamical system $\Phi : (d_{t-1}, h_t, k_t) \mapsto (d_t, h_{t+1}, k_{t+1})$. Given initial conditions (d_{-1}, h_0, k_0), Φ induces the equilibrium path $\left\{(d_t, h_{t+1}, k_{t+1})\right\}_{t=0}^{\infty}$.

In our setting, the equilibrium rental-wage ratio R/w is a decreasing function of the factor intensity ratio $x = k/h$, i.e.

$$\frac{R}{w} = \frac{f'(x_t)}{f(x_t) - x_t f'(x_t)} = \frac{R(x_t)}{w(x_t)} = \omega(x_t)$$

where $f(x) = F(1, k/h)$. Without loss of generality, the algebra leading from (2.2) and (2.3) to Φ can be simplified by means of three technical assumptions.

Assumption 1 The function $h : \Re_+^2 \mapsto \Re_+$ is smooth. The function $g : \Re_+^2 \mapsto \Re_+$ satisfying $h_1[g(x, h^y), h^y] - \omega(x) = 0$ exists and it is well defined and continuous.

Assumption 2 The utility function $u : \Re_+ \mapsto \Re_+$ is strictly increasing, strictly concave and smooth. Given numbers I and R both larger than zero, the function $V(I-z, Rz) = u(I-z) + \delta u(Rz)$ is such that $\arg\max_{0 \le z \le I} V(I-z, Rz) = S(R, I)$ has the form $S(R, I) = s(R) \cdot I$, with $s(\cdot)$ monotone increasing.

Assumption 3 For all period $t = 0, 1, 2, \ldots$ the initial endowment, h_t^y, of the young generation satisfies $h_t^y = \mu h_t$, $\mu > 0$.

Under these hypotheses, tedious but straightforward algebra shows that, given d_{t-1}, the two-dimensional implicit function problem

$$h_{t+1} - h\left[g(x_{t+1}, h_t), h_t\right] = 0$$
$$s\left[R(x_{t+1})\right]\left[w(x_t)h_t - R(x_t)d_{t-1}\right] - k_{t+1} - g(x_{t+1}, h_t) = 0$$

has a well-defined solution

$$h_{t+1} = \Phi^1(h_t, k_t) \quad (2.4a)$$

$$k_{t+1} = \Phi^2(h_t, k_t) \quad (2.4b)$$

Standard methods can be used to show that, given (h_t, k_t) and d_{t-1}, the equilibrium choice of (h_{t+1}, k_{t+1}) is unique and induces a Pareto efficient allocation of resources in period t. This amounts to "static efficiency": in each period aggregate savings are allocated to equalize rates of return between the investments in physical and human capital. Dynamic efficiency is more subtle. In this case one asks if, given (d_{-1}, h_0, k_0), there exists a feasible path $\left\{(\hat{k}_t, \hat{h}_t)\right\}_{t=0}^{\infty}$, other than the competitive equilibrium, which delivers more consumption during some periods without requiring less consumption during any other period. In our setting, one can use the characterization of dynamically efficient paths obtained by Cass [1972]. To apply the original argument one must account for the possible unboundedness of consumption paths, which requires normalizing all variables by a factor growing at the balanced growth rate. †

Under our assumptions, the technology set is a convex cone and unbounded paths are feasible. They are an equilibrium if the utility function allows for enough intertemporal elasticity of substitution in consumption. In this case, the dynamical system (2.4) does not have any fixed point

$$h^* = \Phi^1(h^*, k^*)$$
$$k^* = \Phi^2(h^*, k^*),$$

† Technical details are available from the authors upon request.

other than the origin and equilibrium paths converge to a (unique) balanced growth path characterized by a constant growth rate and a constant ratio $x^* = k^*/h^*$. To illustrate these results, consider two examples.

Example 1. Let $u(c) = \log c$, $F(h,k) = A \cdot k^\alpha h^{1-\alpha}$ and $h(d, h^y) = B \cdot \lambda(h^y) d^\beta$, $\alpha \in (0,1)$, $\beta \in (0,1)$, $A \geq 1$, $B \geq 1$ and $\lambda : \Re_+ \mapsto \Re_+$ continuous and monotone increasing. Manipulating the first order conditions yields

$$s_t = \frac{\delta}{1+\delta}\left[w_t h_t - (1+r_t)d_{t-1}\right]$$

$$d_{t-1} = \frac{\beta(1-\alpha)}{\alpha} k_t$$

which, setting $\frac{\beta(1-\alpha)}{\alpha} = \gamma$ and using the market clearing condition for saving and investment, gives

$$d_{t-1} = \frac{\gamma s_{t-1}}{1+\gamma}.$$

Aggregate saving is therefore equal to

$$ss_t = \left(A \frac{\delta(1-\alpha)(1-\beta)}{1+\delta}\right)\left(k_t^\alpha h_t^{1-\alpha}\right),$$

which implies

$$k_{t+1} = A\eta \left[k_t^\alpha h_t^{1-\alpha}\right] \tag{2.5a}$$

$$h_{t+1} = B\lambda(h_t^y)(A\gamma\eta)^\beta \left[k_t^\alpha h_t^{1-\alpha}\right]^\beta \tag{2.5b}$$

where $0 < \eta = \frac{\delta}{1+\delta}\frac{(1-\alpha)(1-\beta)}{1+\gamma} < 1$. Now let $h_t^y = h_t$. Picking specific functional forms for $\lambda(\cdot)$, consistent with our technical assumption, yields different patterns of long-run behavior. One, none or more than one interior steady states may exist and they may be either asymptotically stable or unstable. Similarly, balanced growth may or may not be an equilibrium. A conveniente specification is $\lambda(h) = h^{1-\beta}$. Then, the dynamical system (2.5) reads:

$$k_{t+1} = A\eta \left(k_t^\alpha h_t^{1-\alpha}\right) \tag{2.6a}$$

$$h_{t+1} = B(A\gamma\eta)^\beta \left(k_t^{\alpha\beta} h_t^{1-\alpha\beta}\right) \tag{2.6b}$$

The only rest point of (2.6) is the origin. The ray

$$x^* = \frac{k_t}{h_t} = \left[\frac{A\eta}{B\left(A\gamma\eta\right)^\beta}\right]^{\frac{1}{1-\alpha(1-\beta)}} \tag{2.7}$$

in the (h_t, k_t) plane defines a balanced growth path. Straightforward algebra shows that for all initial conditions $(h_0, k_0) \in \Re^2_+$ iteration of (2.6) leads (h_t, k_t) to the ray x^*.

Along the balanced growth path the two stocks of capital expand (or contract) at the factor

$$1 + g^* = A\eta \left(\frac{B(A\gamma\eta)^\beta}{A\eta} \right)^{\frac{1-\alpha}{1-\alpha(1-\beta)}}$$

which is larger than one when

$$\eta > \frac{1}{A} \cdot \left[\frac{1}{B^{1/\beta}\gamma} \right]^{(1-\alpha)}$$

A sufficient condition for the equilibrium path to be dynamically efficient is that the gross rate of return on capital be larger or equal than one plus the growth rate of output. With linearly homogenous production functions the rate of return on capital is determined by the factor intensity ratio. Hence we need

$$(1 + g^*) < \alpha A \left(x^* \right)^{-(1-\alpha)}.$$

The latter reduces to $\alpha > \eta$ which is equivalent to

$$\frac{(1-\alpha)(1-\beta)}{\alpha + \beta(1-\alpha)} < \frac{1+\delta}{\delta}$$

For reasonable values of α and β, the latter is satisfied, as long as $\delta > 0$.

Example 2. The use of a linear utility function may help building intuition about the technological side of the model. Hence, we briefly work out the case $u(c) = c$. The first order conditions for households and firms give

$$d_{t-1} = \frac{\beta(1-\alpha)}{\alpha} k_t$$

and s_t is $[w_t h_t - (1 + r_t)d_{t-1}]$, or the interval $[0, w_t h_t - (1 + r_t)d_{t-1}]$, or 0, if $-1 + \delta(1 + r_t)$ is positive, zero, or negative respectively. As before, market clearing gives

$$d_{t-1} = \frac{\gamma s_{t-1}}{1+\gamma}$$

and

$$s_t = A(1-\alpha)(1-\beta) \left[k_t^\alpha h_t^{1-\alpha} \right],$$

where γ is as before. The law of motion for the two stocks of capital is

$$k_{t+1} = A\overline{\eta}\left[k_t^{\alpha} h_t^{1-\alpha}\right] \tag{2.8a}$$

$$h_{t+1} = B(A\gamma\overline{\eta})^{\beta}\left[k_t^{\alpha\beta} h_t^{1-\alpha\beta}\right] \tag{2.8b}$$

where $0 < \overline{\eta} = \frac{(1-\alpha)(1-\beta)}{1+\gamma} < 1$. Once again, the only rest point of (2.8) is the origin. The ray

$$\overline{x}^* = \frac{k_t}{h_t} = \left[\frac{A\overline{\eta}}{B\left(A\gamma\overline{\eta}\right)^{\beta}}\right]^{\frac{1}{1-\alpha(1-\beta)}} \tag{2.9}$$

in the (h_t, k_t) plane defines the balanced growth path. For all initial conditions $(h_0, k_0) \in \Re^2_+$ solutions to (2.8) converge to the ray $\overline{x}^*$.

Along the balanced growth path the two stocks of capital expand (or contract) at the rate

$$1 + \overline{g} = A\overline{\eta}\left[\frac{B(A\gamma\overline{\eta})^{\beta}}{A\overline{\eta}}\right]^{\frac{1-\alpha}{1-\alpha(1-\beta)}}$$

which, as in the previous example, is larger than one when

$$\overline{\eta} > \frac{1}{A} \cdot \left[\frac{1}{B^{1/\beta}\gamma}\right]^{(1-\alpha)}.$$

Also in this case, dynamic efficiency obtains if $\alpha \geq \overline{\eta}$, that is

$$2\beta > \frac{(1-2\alpha)}{(1-\alpha)},$$

which is easily satisfied for reasonable values of α and β.

2.2 Equilibrium when Credit Markets are Missing

In reality, competitive markets to finance private educational investments are rare. The reasons for such lack of privately provided credit are various and widely studied (see e.g. Becker [1975] for a classical discussion). In our model, lack of borrowing opportunities for the young generation implies that $d_t = 0$ for all t and, therefore, $h_{t+1} = h(0, h_t^y)$. This makes the complete market allocation (CMA, from now on) unachievable and, by eliminating investment in human capital, leads the economy to an inefficient equilibrium. The specific properties of such equilibrium would depend upon the assumptions one is willing to make about $h(0, h^y)$. This is not our concern here.

Our interest lies, on the one hand, with the CMA as a theoretical benchmark and, on the other hand, with the class of intergenerational transfer policies that are capable of replicating the CMA when credit markets to finance education are unavailable. We now turn to this issue.

3. Introducing the Intergenerational State

Consider the situation in which d_t is constrained to zero in all periods. In general, condition (2.3c) is violated and $F_2(h_t, k_t) = R_t < w_t h_1(0, h_{t-1})$ will hold. Profitable investment opportunities exist in the educational sector which cannot be exploited. As a consequence, too much saving is invested in the physical stock of capital, the k/h ratio is too high and the rate of return on capital too low with respect to the benchmark case. The allocation is inefficient.

Young individuals can increase their lifetime income $w_t h_t$ by investing some d_{t-1} in the accumulation of human capital, but are prevented to do so. Similarly, middle age individuals are unable to increase their retirement income by shifting some of their savings from k_t to d_{t-1}. Apparently, such inefficiency could be alleviated by making resources available for educating the young.

Assume, then, that the government levies a lump-sum tax on the middle age individuals to finance education for the young. Assume that, in period t, the initial conditions (h_t, k_t) are the same as in the case of complete markets and an amount d_t equal to the one that solves (2.3c) is transferred from middle age to young individuals. If repeated in each period from t onward, is this "public education financing" scheme enough to restore efficiency? The answer is: no.

While in period t the young individuals are accumulating the "correct" amount of human capital and the middle age individuals have performed the "correct" investment in the human capital of the future generation, things are no longer "correct" one period later. The reason is simple: middle age individuals do not expect to be compensated, in $t+1$, for their investment in education. The $R_{t+1}d_t$ component is missing from their income next period. To compensate for this, they must invest in k_{t+1} more than they would have done otherwise, thereby lowering R_{t+1} below its CMA level. The other side of the coin is that in period $t+1$ middle age individuals have the correct amount of human capital h_{t+1} but "too much income". Contrary to (2.1) above, they are not requested to pay back $R_{t+1}d_t$. Again, this will induce overinvestment (relative to the CMA benchmark) in the stock of physical capital.

The solution is simple: make sure that in each period the middle age pay back their debt to the people who, via the public education system, lent them the money in the first place. In such a way, old people will be collecting the amount $R_{t+1}(k_{t+1} + d_t)$ as in the CMA, and the incentive to overinvest in k_{t+1} will disappear. Notice that, while the $R_{t+1}k_{t+1}$ comes from privately issued financial securities, the portion $R_{t+1}d_t$ corresponds to an intergenerational transfer mediated by the government. In other words: by itself, public financing of education is not enough, even when carried out in the correct amount, to restore the CMA. A mechanism that taxes the working middle age and transfers the proceeds to the old retirees is also needed. What is crucial, though, is that in this scheme the two intergenerational transfers are not independent but, instead, are tied up together by a very precise rate of return restriction.

3.1 Public Financed Education and Pay-As-You-Go Pensions

Consider the following scheme. In each period t two lump-sum taxes are levied to provide resources for two simultaneous intergenerational lump-sum transfers. Both taxes are levied on the middle age generation, and the proceedings used to finance, respectively, pensions for the old age and education for the young individuals. We assume a period-by-period balanced budget. With obvious notation we write

$$T_t^p = P_t \tag{3.1}$$

$$T_t^e = E_t \tag{3.2}$$

The period-by-period budget constraint for the representative member of the generation born in period $t-1$ becomes

$$0 \leq d_{t-1} \leq E_{t-1} \tag{3.3a}$$

$$c_t^m + s_t \leq w_t h_t - T_t^p - T_t^e \tag{3.3b}$$

$$c_{t+1}^o \leq R_{t+1}s_t + P_{t+1} \tag{3.3c}$$

For given initial conditions, let starred symbols, e.g. d_t^*, w_t^*, etc., denote the CMA quantities. Comparison of equations (3.3) with the budget restrictions of problem (2.1) shows that, if the lump-sum amounts satisfy

$$E_t = d_t^*, \qquad\qquad P_t = d_{t-1}^* R_t^*, \tag{3.4}$$

the competitive equilibrium achieves the CMA. A benevolent planner can restore efficiency and improve long-run growth by establishing publicly financed education and pay-as-you-go pensions and by linking the two flows of payments via the market interest rate.

Efficiency properties aside, a Public Education and Public Pension scheme (PEPP) satisfying restrictions (3.1), (3.2) and (3.4) would also be actuarially fair in the following sense. The pension payment (contribution) that a typical citizen receives (pays) during the third (second) period of his life corresponds to the capitalized value of the educational taxes (transfers) he contributed (received) during the second (first) period of his life. These quantities are capitalized at the appropriate market rate of interest.

$$E_t R^*_{t+1} = T^p_{t+1} \qquad (3.5a)$$

$$T^e_t R^*_{t+1} = P_{t+1} \qquad (3.5b).$$

In the applied literature on contribution-based Social Security systems the issue of actuarial fairness between contributions paid and pensions received is an actively debated topic. Our model suggests that one should look for actuarial fairness somewhere else, that is between contributions paid and amount of public funding for education received on the one hand, and between taxes devoted to human capital accumulation and pension payments on the other.

3.2 Distortionary Taxation

We have assumed so far that the benevolent planner has access to lump-sum instruments of taxation. This is seldom the case. In this subsection we take a brief look at the case of linear income taxes. Once again, we assume that the period by period budget constraint must be satisfied and ask if the CMA can be implemented as a competitive equilibrium with linear income taxes. The novel result presented here is that, in fairly general circumstances, taxing the purchases of physical capital to finance education while also subsidizing the return from physical capital is the way to support the complete market allocation.

The case in which only labor income may be taxed is easy. While, in the absence of credit markets for education, it may still be beneficial to introduce a PEPP system, there is no reason to expect that the CMA will be supported as a competitive equilibrium. The tax on labor income reduces the rate of return on human capital investment and distorts the borrowing/lending decisions of both young and middle age agents.

The fact that the CMA cannot be achieved by taxing labor income, suggests we may want to consider different sources of tax revenues. We begin by noticing that, in our model, it is not unreasonable to treat the quantity T_t^p as effectively lump-sum. This is because of two features of our environment, which we consider particularly realistic. In the first period of their life, agents cannot do anything but acquire human capital. The subsidy d_{t-1} is therefore lump-sum: if you attend school you borrow that amount, if not you do not. In the second period, the amount $R_t d_{t-1}$ to be repaid does not depend upon the labor supply of the middle age individual either. Hence, it can be treated as a lump-sum amount to be detracted from $w_t h_t$. [†]

The quantity $T_t^e = d_t$ is harder to treat as a lump-sum amount. One may notice that the reason for which we need to collect T_t^e from middle age agents is that they are unable to invest in human capital. Once T_t^e is taxed away, we want to pay a pension to the old agents because otherwise their return from physical capital is too little. This suggests that the following, somewhat unusual, scheme may work in practice. The benevolent planner taxes purchases of physical capital in period t (to lower $s_t = k_{t+1}$ and bring it to the CMA level) and subsidizes the return from physical capital in period $t+1$ (to increase third period income to the CMA level). To avoid confusion with notation, denote with $\hat{s}_t$ the saving (consisting only of purchases of physical capital) that obtains in a competitive equilibrium with the proposed tax and subsidy scheme. Recall that, if the CMA is achieved, $\hat{s}_t = k_{t+1}^*$, where starred symbols still denote CMA quantities and prices. The government budget constraint requires $\tau_t \in [0,1]$ to satisfy

$$\tau_t \hat{s}_t = \tau_t k_{t+1}^* = E_t^* = d_t^*.$$

The household budget constraints become

$$0 \leq d_{t-1} \leq E_{t-1}^* \tag{3.6a}$$

$$c_t^m + (1+\tau_t)\hat{s}_t \leq w_t h_t - T_t^p \tag{3.6b}$$

$$c_{t+1}^o \leq R_{t+1}(1+\tau_t)\hat{s}_t. \tag{3.6c}$$

The first order condition determining d_{t-1} is identical to the one for complete markets, eqn. (2.2b). The one determining $\hat{s}_t$ becomes

$$u'\left[w_t h_t - (1+\tau_t)\hat{s}_t - T_t^p\right](1+\tau_t) = \delta R_{t+1} u'\left[(1+\tau_t)\hat{s}_t R_{t+1}\right](1+\tau_t). \tag{3.7}$$

† Introducing individual heterogeneity and income uncertainty, complicates but does not alter the main qualitative conclusions, see Boldrin and Montes [2001a] for details.

Cancelling $(1+\tau_t)$ on both sides, replacing the lump-sum value T_t^p with $R_t d_{t-1}$ and setting $\tau_t = d_t^*/k_{t+1}^*$ yields $(2.2a)$, which has the unique solution $\hat{s}_t = k_{t+1}^*$, as desired. Further

$$\tau_t \hat{s}_t = d_t^*; \qquad \text{and } R_{t+1}\tau_t \hat{s}_t = R_{t+1} d_t^*$$

which corresponds to the CMA's investment in, and return from human capital assets.

For the simple economies considered in Examples 1 and 2 the choice of a constant $\tau_t = \gamma/(1+\gamma)$ suffices to implements the CMA. In general, a constant tax rate suffices along a balance growth path when the production functions are linearly homogenous. For production functions that are not linearly homogeneous or outside the balanced growth path, the tax rate cannot be constant as the composition d_t^*/k_t^* of the CMA investment portfolio is neither in those circumstances.

3.3 Using Debt

We note briefly that a third, in our view more compelling, implementation of our transfer scheme is possible. In this interpretation, the government issues one-period debt in the amount d_t in each period. Given the demographic structure assumed, this debt will be purchased only by the middle age people. The resources collected this way are used to finance education for the young. In the following period, the government pays back $R_{t+1}d_t$ to the, now aged, debt holders. Such repayment is financed by an income tax on the middle age individuals. This tax should be proportional to the past usage of public funding for education.

This scheme is not exempt from the usual distortionary effects of labor income taxation. We should stress, though, that (even in an environment with heterogenous individuals and labor income uncertainty) the amount $R_{t+1}d_t = T_{t+1}^p$ can be usefully broken down into two parts. The first, and most likely larger, portion should be proportional to public school usage and therefore lump sum. A second portion may have to be collected for intragenerational insurance or redistributive purposes. It is only this second share of T_t^p which, being proportional in nature, distorts labor supply. To the extent that current social security contributions achieve some degree of redistribution or intragenerational risk-sharing, the suggested scheme cannot do any worse in terms of economic efficiency.

3.4 Testable Predictions of the Model

The normative implications of the model are clear. When private competitive markets for financing education are not available, a properly designed PEPP scheme may restore the efficient CMA as a competitive equilibrium. Next, we ask if there is any evidence supporting a positive interpretation of our model as a description of existing systems of public education financing and public pension provision.

In the real world, benevolent planners are probably harder to come across than credit instruments for financing education. A priori, there are very few reasons to expect that existing public education and pension systems should strive to replicate the complete market allocation and achieve the efficiency gains we have outlined here. As a matter of fact, in none of the countries we are aware of the welfare state legislation is explicitly organized around the principles advocated in this paper. In general, social security contributions are levied as a percentage of labor income and bear no clear relation to the previous usage of public education. Pension benefits received are related, in a form or another, to past social security contributions but never to some measure of lifetime contributions to aggregate human capital accumulation.

Still, there are intuitive reasons to believe that intergenerational transfers that are either grossly inefficient or openly unfair (in the sense that some generations collect rates of return systematically higher than those of other generations) would be subject to strong public pressure to be either dismantled or improved upon. This is the intuition set forth by Becker and Murphy [1988], which is captured in our model by conditions (3.5). In particular, as those equations show, fairness and replication of the CMA are both summarized by a simple present value calculation that uses the market rate of return as a yardstick.

Further, in a recursive environment in which the middle age generations decide if and how to implement a PEPP system, an equilibrium satisfying (3.5) may easily arise. In previous versions of this paper we presented a dynamic game of generational voting, along the lines of Boldrin and Rustichini [2000], which possesses a subgame perfect equilibrium implementing the CMA. We refer the interested reader to Boldrin and Montes [2001b] for this result, a discussion of the circumstances under which the political equilibrium implementing the CMA is the unique subgame perfect and, finally, for extensions to other notions of recursive equilibrium and more general OLG environments. Results along the same lines have independently been derived by Rangel [1999] and, to a smaller degree, by Bellettini and Berti-Cerroni [1999].

In the sequel of this paper we take a less theoretical approach and use available micro and macro Spanish data to check the extent to which observable allocations satisfy restrictions such as (3.5). Before moving to this task, we state here the empirical prediction that follows from a positive interpretation of our model.

Conjecture If the set of intergenerational transfers induced by the public education and the public pension systems support the complete market allocation, the following should be observed. For a given generation, the implicit rate of return i_t which, along the life cycle, equalizes the discounted values of educational services received and social security contributions paid, is equal to the market rate of interest r_t. Similarly, the implicit rate of return π_t that, along the life cycle, equalizes the discounted values of educational taxes paid and pension payments received, is also equal to the market rate of interest r_t.

4. The Spanish Case

In this section we use Spanish data to compute the values of i and π faced by Spanish citizens, under the rules in place and the taxes and transfers implemented in the years 1990-1991. To carry out our computations, the stationarity assumptions made in the model are first taken verbatim and then relaxed as we move along. We proceed in three stages. In the first we abstract from demographic change and economic growth. It will be shown that, as long as growth takes place at a constant rate, it makes only a quantitative but not a qualitative difference in the results. In the second stage, we incorporate the forecasted demographic evolution for the period 1990-2089 and consider a number of reasonable policy scenarios. In the third, we use the same demographic predictions to evaluate the quantitative impact that economic growth at a constant rate would have on the implicit rates of return faced by different generations.

More specifically, in our empirical exercise we assume that the rules of the Spanish public education and public pension systems will not be changed for the very long future and that all individuals currently alive have also lived under those same rules in the past. This is obviously false, as both education and pension systems underwent large and frequent changes in the period 1960-1985. In the latter year the pension system was reformed once more and, since then, it has kept its basic rules. The same applies to the public education system, which achieved its current structure in the early 1980s and has not changed much since then. Hence, while our assumption of stationarity is only an approximation to the reality, it is a a good approximation for the last 20 years and it appears as a reasonable one for the foreseeable future.

In the first stage we assume that the aggregate burden of taxation and its age distribution have not and will not vary over the lifetime of the individuals alive in 1990-91. In the second and third we let aggregate public expenditure change according to specific scenarios. As for income, in the first and second exercise we assume it remains constant, for each age group, over the whole simulation horizon. In the third, we let age specific per-capita income grow at a constant rate and adjust aggregate taxation accordingly, under the assumption of a constant age distribution of taxes and transfers. Notice that, if it were not for the changing demographic structure, this would imply constant tax and transfer rates for each age group and function. Finally, in all of our simulations we make the assumption that, for each function, the yearly budget is balanced.

We have made the choice of ignoring deficit financing and the generational burden of public debt for a variety of reasons. First, the Spanish public sector deficit has varied a lot during the last 15 years and was much higher in the early 1990s than it is now. In fact, partly because of the EMU implementation, the fiscal deficit has decreased steadily since 1994 reaching very low values in the last four years. The same applies to the Social Security Administration budget, which is often manipulated by changing accounting criteria and has generated a surplus since 1997. Secondly, we do not have a reliable method to allocate the debt burden over different age groups either for the last ten years or for the future. The intergenerational distribution of the debt burden remains, nevertheless, an important issue to be be addressed. It requires an explicit model of stochastic demographic change and of optimal fiscal policy. A first step in this direction is taken in Boldrin and Montes [2001a].

Ours are, indeed, relatively strong assumptions. Stationarity and balanced-growth assumptions are often made in most empirical applications of dynamic models, and our case makes no exception. Given the available micro data, we find our approach to be a reasonable starting point.

Data †

To compute the implicit rates i and π we use several kinds of micro and macro data. The choice of the reference year is dictated by the availability of information about individual behavior along the life cycle. At present, there is only one reliable source of microeconomic observations of the allocation of personal time between school, work and retirement at various stages of the life cycle. This is available only for the years 1980-81

† Further details about the data sets we use are in the Appendix and in Montes [1998].

and 1990-91 via the Spanish household budget survey (Encuesta de Presupuestos Familiares, or EPF). We have used the 1990-91 EPF because the Spanish public pension system underwent a major reform in 1985 and because the 1980-81 EPF contains only a severely limited subset of the information we need.

For each individual in the sample, conditional upon its age and occupational status, the information in the EPF allows for the estimation of (1) the amount and value of public school services received, (2) the amount of direct and indirect taxes paid, (3) the amount of pension contributions paid and, (4) the amount of public contributive pensions received. The information in the EPF also affords the computation of the share of the population which, at each age, is studying, working, unemployed or retired. Such life-time distribution of activities is reported, in percentage terms and for each age group, in Figure 1. Together with quantities (1)-(4) it allows computation of the implicit rates of return.

4.1 The base case

Consider an individual living for a maximum of A periods and let p_a denote the (conditional) probability of survival between age a and $a+1$. Denote with i the interest rate at which young people "borrow" through public education, and with π the rate of return the elderly receive from their "investment" in public education. For a given sequence of taxes and transfers, the rates i and π (time invariant, because of the stationarity assumptions) are defined implicitly by

$$\sum_{a=1} \left[\left(\Pi_{j=1}^{a} p_j \cdot \Pi_{j=a}^{A} \left(1 + i_j\right) \right) \left(E_a - T_a^p \right) \right] = 0 \tag{4.1a}$$

$$z \sum_{a=1} \left[\left(\Pi_{j=1}^{a} p_j \cdot \Pi_{j=a}^{A} \left(1 + \pi_j\right) \right) \left(T_a^e - P_a \right) \right] = 0 \tag{4.1b}$$

The representative agent for our base case is then defined by the following assumptions:

(a) At each age $a = 1, \ldots, 99$ (there are no individuals older than 99 years in the EPF) the probability p_a of being alive at age $a+1$ is the one reported by INE for that age group in 1990.

(b) At each age $a = 1, \ldots, 99$ the representative individual is working, studying, unemployed or retired with a probability equal to the frequency of each activity in the EPF sample of people of age a.

(c) At each age $a = 1, \ldots, 99$ an individual receives or pays tranfers and taxes equal to the average, in the EPF, for those individuals that at age a were in the same occupational status.

Assumptions (a)-(c) can be used to extract from the EPF the amounts E, P, T^e and T^p that an individual of age a would pay or receive. Such estimation uses the age- and status-specific information contained in the EPF, according to assumptions (b) and (c). Let X be a stand-in for any of the four quantities. For each $a = 1, 2, \ldots, 99$ we use population data to compute the amounts X_a attributable to the representative agent of that age. Let L_a be the number of individuals of age a in the Spanish population in 1990 (INE (1991)). A four-tuple of weights x_a can be computed by setting

$$x_a = \frac{X_a}{\sum_{a=1}^{A} X_a L_a}.$$

Write this four-tuple of x_a as $[\alpha_a, \beta_a, \gamma_a, \delta_a]$. They denote, respectively, the share of total T^e and T^p paid and of total P and E received (according to the EPF) by the representative individual of age a.

Next, from the Government and Social Security Administration budgets for 1990 we compute the quantities X^{90}, corresponding to the effective total tax or transfer relative to each function. We allocate these amounts over the life-cycle of the representative agent by means of the weights x_a. The life-time distribution of these four flows, in thousands of 1990 Pesetas, is reported in Figure 2. Equations (4.1) become

$$\sum_{a=1}^{99} \left(\Pi_{j=1}^{a} p_j\right) \left(1+i\right)^{99-a} \left[\delta_a \cdot E^{90} - \beta_a \cdot T^{90,p}\right] = 0 \qquad (4.2a)$$

$$\sum_{a=1}^{99} \left(\Pi_{j=1}^{a} p_j\right) \left(1+\pi\right)^{99-a} \left[\alpha_a \cdot T^{90,e} - \gamma_a \cdot P^{90}\right] = 0 \qquad (4.2b)$$

Notice in passing that, had we assumed a constant annual growth rate of $g > 0$ for both taxes and transfers, equations (4.2) would be modified by multiplying each annual entry by a factor $(1+g)^a$. Dividing through by $(1+g)^{99}$ and replacing $(1+i)$ and $(1+\pi)$ by $(1+i)/(1+g)$ and $(1+\pi)/(1+g)$ respectively, leads back to (4.2). This implies that adding a constant growth rate changes only the quantitative but not the qualitative conclusions of our exercise. We come back to this point at the end of the section, when we quantify the joint impact of economic growth and demographic change.

We solve expressions (4.2) numerically. Our point estimate of the implicit rate of return on educational investment is

$$\pi = 4.238\%.$$

Our point estimate of the implicit rate of interest at which young people borrow is more ambiguous. It depends upon the convention with which one handles the yearly surpluses and deficits of the various Spanish social security administrations. In 1990, the social security administration for workers of the private sector (INSS) realized a surplus of pension contributions over pension outlays [‡], while the social security administration for public employees (RCP) realized a deficit. The latter was covered by a transfer of funds from the general government budget. Our model assumes year by year balanced budget. One possibility is to get rid of both the INSS surplus and of the RCP deficit by assuming that the total amount of social security contributions was in fact equal to the public contributive pension payments made in that year (P^{90}). In this case our point estimate is

$$i_1 = 3.6307\%.$$

A second possibility is to use the actual social security contributions paid to INSS and RCP in 1990 ($T^{90,p}$). In this case we have

$$i_2 = 3.772\%.$$

Finally, a third alternative is to add to the total contributions paid in 1990 ($T^{90,p}$) the amount transferred from the general government budget to cover the RCP deficit. Adopting this wider definition of social security contributions, the implicit rate of interest is computed to be

$$i_3 = 4.2601\%.$$

We believe that i_3 is a better estimate of the true implicit rate. This is because the RCP "deficit" is a misnomer attributable to accounting practices. The government portion of the social security contributions for its employees is highly forecastable and, de facto, it is always recorded as a transfer from the general budget to cover the RCP deficit. In other words, the transfers included in the computation of i_3 are functionally equivalent to

† The INSS is divided further in six different funds, some of which exhibited a deficit and other a surplus during the same year. Our micro-data do not allow to consider this finer partition.

the employers' contributions paid by private sector firms to the INSS. To us, this means they are part of the gross labor income of public employees and, for this reason, should be treated as part of the total social security contributions they pay.

Accounting for changes in mortality rates

In our definition of the Spanish representative agent we have used the mortality rates reported by INE for the year 1990 (assumption (a) above). This is not very reasonable, as those rates were computed using observations prior to 1990; survival probabilities have changed greatly since then, and are still changing. To correct for this, we run the base case simulation using the updated survival probabilities implicit in Fernandez-Cordon [1996] demographic forecasts. The new estimates are respectively equal to $i = 4.4565\%$ (from now on, only i_3 will be reported) and $\pi = 4.7853\%$, reversing the order of our previous findings. As it should be expected, current pensions are a good deal at lower mortality rates.

The population forecasts of Fernandez-Cordon [1996] are obtained by simultaneously using new mortality rates and expected immigration flows. Unfortunately, the two sources of change cannot be disentagled, so that for certain age groups the estimated probability of survival are slightly higher than one. Immigrants are concentrated in the 20 to 40 age groups so, if we force all probabilities of survival to be bounded above by one, we get $\pi = 4.859\%$ and $i = 4.388\%$, making pensions an even better deal.

4.2 Impact of non stationarity

To provide a fuller account of the impact that demographic change and economic growth may have on i and π we simulated a number of alternative scenarios, which are illustrated next.

We begin with the impact of expected demographic change. To do this, we replace the assumption of demographic stationarity with the projections of Fernandez-Cordon [1996], which take into account variations in both mortality and fertility rates. Removing demographic stationarity makes i and π dependent upon the date of birth. Each cohort faces different rates (i_a, π_a), depending on its age $a = 1, 2, \ldots, A$ in the first period. Here we report results only for two particular cohorts that were alive in 1990: the just born and those aged 16 (corresponding to the Spanish legal working age). Results for other cohorts are available upon request.

Impact of demographic change

Changing the demographic structure while keeping the balanced budget requirement satisfied in each period, requires making assumptions on the policies followed to distribute taxes and transfers across individuals belonging to different generations. In particular, one needs to make assumption as to which features of the PEPP system that were observable in 1990 will be maintained by the future policies. Many different scenarios are conceivable. We have selected four, which we consider most likely or, at least, most informative. In the first one, Scenario A, we assume that (age specific) per capita expenditure in education and pension will remain at its 1990 level, in real terms. Symmetrically, in Scenario B we assume that (age specific) per capita education taxes and social security contributions will remain at their 1990 values. In Scenario C, it is the borrowing rate realized in 1990 that we take as fixed and apply it to the generation born in that same year. Finally, Scenario D considers the case in which the lending rate is kept at its base case level for the generation born in 1990. In all cases, for the generation aged sixteen in 1990 we use 1990 age-specific quantities for the amounts they had already paid or received in the years previous to 1990.

The careful reader will notice that, for each scenario, the "constant policy" adopted plus year-by-year balanced budget are not enough, for given demographic, to determine all the remaining variables. Consider, for example, Scenario A. Using per capita expenditure in education and demographic data one can compute total educational expenditure E_t in each future year. The balanced budget constraint implies that $E_t = T_t^e$, for all t. This determines the total educational tax to be levied in each year, but leaves its distribution, across generations, still open. The same is true, in Scenario A, for the distribution of T_t^p across generations, and for other quantities in the other scenarios. To address this problem we proceed as follows.

For each of the four flows, let X_a be the amount paid or received by the representative individual of age a according to the 1990 data. For each $x = E, T^e, P, T^p$ and age $a, a' = 1, 2, \ldots, A$ define the constants

$$k^x(a, a') = \frac{X_a}{X_{a'}}.$$

Then, for all future years and in all four scenarios we assume that, for each function x whose distribution over cohorts is to be determined endogenously, the payments from or transfers to the average individuals of age a and a' will yield the same $k^x(a, a')$ as in 1990. In other words, we assume that, while a certain policy may favor or disfavor a certain cohort over its entire life time, it will not do so by charging different taxes to people of different ages in any given year. Same for transfers.

Scenario A

We start by fixing real per capita expenditure in E and P to remain at the 1990's level at each age. The demographic projections allow the computation of aggregate expenditures, E_t and P_t, for each year $t = 1990, \ldots, 2089$. Use balanced budget in each year to compute T_t^e and T_t^p. Use the assumption of constant $k^{T^e}(a, a')$ and $k^{T^p}(a, a')$, again together with demographic data, to compute the distribution of taxes across individuals in each year. Given this, we compute the rates of return. For the generation that was born in 1990 we obtain $i = 6.0643\%$ and $\pi = 6.8918\%$, while we have $i = 4.7620\%$ and $\pi = 6.3121\%$ for the generation aged sixteen in 1990.

Scenario B

Here we fix real per capita taxation (T^e and T^p) to remain at the 1990's level at each age. Then we proceed like in Scenario A, using the assumption of constant $k^E(a, a')$ and $k^P(a, a')$ to compute the yearly distribution of E_t and P_t across individuals of different age. For the generation born in 1990, this policy gives $i = 3.0018\%$ and $\pi = 2.2021\%$, while we have $i = 4.2322\%$ and $\pi = 2.1912\%$ for the generation aged sixteen in 1990.

Scenario C

In this case we take the borrowing rate $i = 4.4565\%$ for the generation born in 1990 as given. On the basis of the 1990 data, we fix the per capita expenditure in E for each age group. We use this per capita expenditure to project total life time transfers to each generation alive in 1990. This gives total educational expenditure in each fiscal year between 1990 and 2089. This is E_t. Next, we use 1990 per capita social security contributions (for each age group) to compute how much will be available to pay pensions during each fiscal year between 1990 and 2089. This is T_t^p. Notice that, by doing this, we guarantee that the generation born in 1990 will pay the same $i = 4.4565\%$ as the representative agent in the base case. Finally, we use the yearly balanced budget restrictions together with the assumption of time-invariant $k^{T^e}(a, a')$ and $k^P(a, a')$ to determine endogenously the amount of taxes $T_t^{e,a}$ paid and pensions P_t^a received by an individual of age a in year t. For the generation born in 1990, this yields a lending rate of $\pi = 4.3843\%$, while the cohort aged 16 in 1990 faces implicit rates equal to 4.3843% and 3.9804% respectively, under this policy.

Scenario D

This case takes as given the lending rate $\pi = 4.7853\%$ for the generation born in 1990. Again, we start from the 1990 data for real per capita T^e and P and use demographic projections to compute future E_t and T_t^p. The yearly balanced budget restrictions together with the constants $k^E(a, a')$ and $k^{T^p}(a, a')$ determine the other two flows. For the generation born in 1990, this yields a borrowing rate of $\pi = 4.7497\%$, while the cohort aged 16 in 1990 faces implicit rates of $i = 4.6312\%$ and $\pi = 4.7323\%$ in this scenario.

Impact of Economic Growth

In considering equations (4.2) we have already pointed out that the addition of a constant growth rate cannot change the qualitative conclusions of our exercise. In each scenario, introducing a constant growth rate g implies that $(1+g)(1+i)$ and $(1+g)(1+\pi)$ would be the new rates, where i and π are the implicit rates reported in the previous simulations. Hence, for example, in the base case and accounting for recent mortality rates, a balanced growth rate of about 3% (which is pretty close to the historical experience of the last 15 years) would yield a borrowing rate of $i^* = 7.51\%$ and a lending rate of $\pi^* = 8.00\%$. Adjusting for growth does not change the qualitative conclusions, while making the comparison to historical rates of return on capital more meaningful. [†]

5. Conclusions

We have studied a three period overlapping generation model with production and accumulation of physical and human capital. When the young generation cannot borrow to finance investment in human capital, the competitive equilibrium outcome does not satisfy either static or dynamic efficiency and the aggregate growth rate of output and consumption is lower than under the complete market allocation. We have shown that a simple intergenerational transfer agreement could eliminate this problem and induce an efficient allocation.

The intergenerational transfer agreement we study is inspired by the argument advanced in Becker and Murphy (1988). Accordingly, we interpret public funding for education as a loan from the middle age to the young generation. The latter uses this loan to finance its accumulation of human capital. Symmetrically, the pay-as-you-go public pension system can be seen as a way for the former borrowers to repay the capitalized

† We thank Tim Kehoe for pointing this out to us.

value of their educational debt to the previous generation. In this interpretation the two institutions of the welfare state, public education and public pensions, support each other and achieve a more efficient allocation of resources over time.

There are important normative implications of this analysis. Our model suggests that utilization of either public or publicly financed education should be treated as accumulation of debt toward the older generations. Such debt, capitalized at the market rate of interest should be paid back, during one's working life, by means of a tax levied upon labor income. Repayment of the educational debt can be achieved by means of a voluntary mortgage plan or by means of a compulsory tax. Either choice has some obvious incentive and redistributive implications, which are, nevertheless, not dissimilar from those faced by current arrangements for financing public education. On the side of retirement pensions, the model requires earmarking some tax (paid by individuals) as a source of funding for the public financing of education and to capitalize at the market rate of interest, the amounts paid by each single citizen. The capital so accumulated should then be paid out, in form of annuities, to the same citizen once retirement age is reached.

While a benevolent planner could easily implement such a system of lump-sum taxes and transfers, it is not obvious that a benevolent planner is behind the design of modern welfare state institutions. Hence it is worth investigating if existing systems are very far away from the predictions of a positive intepretation of such a normative model. We do so by computing the "borrowing" and "lending" rates implicit in the Spanish public education and public pension system. We use both microeconomic and aggregate data for the years 1990-91. The model predicts that the borrowing and the lending rates should equal each other and be equal, in turn, to the rate of return on capital. For the base line case, our point estimates of borrowing and lending rates are relatively close to 4.0%, which corresponds to the risk free real rate of return on Spanish Treasury bonds during the last fifteen years or so. This optimistic finding, though, is based upon the assumptions of demographic and policy stationarity.

Once the assumption of demographic stationarity is replaced by realistic projections of the future evolution of the Spanish population, results change dramatically. We carry out a number of simulations based on such projections, each scenario being characterized by different assumptions about the form in which public policy may react to the demographic change. While the policies we consider are obviously hypothetical, common sense suggest they are a reasonable starting point for this kind of analysis. In each one of the four cases we consider, the implicit rates we estimate move apart from each other. In particular, unless it is held fixed by the assumptions underlying the policy scenario being considered, pensions tend to yield a rate of return (on the previous educational investment) higher than the rate of interest the working cohorts are expected to pay (via social security contributions) on the educational services they received.

A second finding is that the rates of interest paid by or accrued to generations born in different years move quite apart from each other when the demographic evolution is taken into account. Nevertheless, and contrary to a widespread presumption, such movements are not monotone; in particular they do not seem to necessarily favor the older relative to the younger generations. In other words, rebus sic stantibus, the expected demographic evolution should not necessarily lead to a huge redistribution of resources away from the younger or not-yet-born generations and toward the older ones. Most previous findings, based on the Generational Accounting methodology pioneered by Auerbach and Kotlikoff, see e.g. Auerbach, Kotlikoff and Leibfritz (1999), have instead shown that the interaction between demographic change and current fiscal policies (in particular, current welfare policies) is likely to engender a large intergenerational redistribution in favor of the older cohorts. While our findings cannot rule out this conclusion and, in fact, lends support to it under certain policy scenarios. we believe our estimates have independent value and should shed some additional light on the intricancies of intergenerational public policy.

Bibliography

Auerbach, Alan, Larry Kotlikoff and Willi Leibfritz (eds.) (1999), *Generational Accounting Around the World*, Chicago University Press.

Barro, Robert and Gary S. Becker (1989), "Fertility Choice as a Model of Economic Growth," *Econometrica*, **57** (2), 481-501.

Becker, Gary S. (1975), *Human Capital*, Chicago, IL : The University of Chicago Press.

Becker, Gary S. and K. Murphy (1988), "The Family and the State", *Journal of Law and Economics* **XXI**, 1-18 .

Bellettini, Giorgio and Carlotta Berti Cerroni (1999), "Is Social Security Bad for Growth?", *Review of Economic Dynamics* **2**.

Boldrin, Michele (1994), "Public Education and Capital Accumulation", C.M.S.E.M.S. Discussion Paper 1017, Northwestern University.

Boldrin, Michele and Aldo Rustichini (2000), "Political Equilibria with Social Security", *Review of Economic Dynamics* **3**, 41-78.

Boldrin Michele and Ana Montes (2001a), "Optimal Intergenerational Debt", mimeo, University of Minnesota and Universidad de Murcia, in progress.

Boldrin Michele and Ana Montes (2001b), "Games Generations Play", mimeo, University of Minnesota and Universidad de Murcia, in progress.

Boldrin Michele and Larry E. Jones (2001), "Mortality, Fertility and Savings in a Malthusian Economy", mimeo, October, University of Minnesota.

Caldwell, John C. (1978), "A Theory of Fertility: From High Plateau to Destabilization", *Population and Development Review*, **4**, 553-577.

Cass, David (1972), "Distinguishing Inefficient Competitive Growth Paths: A Note on Capital Overaccumulation and Rapidly Diminishing Future Value of Consumption in a Fairly General Model of Capitalistic Production", *Journal of Economic Theory* **4** 224-240.

Conley, John P. (2001), "Intergenerational Spillovers, Decentralization and Durable Public Goods", mimeo, December, University of Illinois, Champaign.

Fernández Cordón, Juan A. (1996), *Demografia, Actividad y Dependencia en España*, Bilbao, Fundación Banco Bilbao Vizcaya.

Kotlikoff, Laurence J., Torsten Persson and Lars E.O. Svensson (1988), "Social Contracts as Assets: A Possible Solution to the Time Consistency Problem", *The American Economic Review*, **78**, 662-677.

Instituto Nacional de Estadística (INE) (1991) "Tablas de Mortalidad de la Población Española con referencia a los años 1990-91", Madrid.

Instituto Nacional de Estadística (INE) (1992a), *Encuesta de Presupuestos Familiares 1990-91. Metodología*, Madrid, Instituto Nacional de Estadística.

Instituto Nacional de Estadística (INE) (1992b), *Encuesta sobre Financiación y Gasto de la Enseñanza en Espa na*, Madrid, Instituto Nacional de Estadística.

Intervención General de la Administración del Estado (IGAE) (1991a), *Actuación Económico Financiera de las Administraciones Públicas 1990*, Madrid, Ministerio de Economía y Hacienda.

Intervención General de la Administración del Estado (IGAE) (1991b), *Cuentas de las Administraciones Públicas 1990*, Madrid, Ministerio de Economía y Hacienda.

Merton, Robert C. (1983) "On the Role of Social Security as a Means for Efficient Risk-Bearing in an Economy Where Human Capital is Not Tradeable." in *Financial Aspects of the U.S. Pension System*, edited by Z. Bodie and J. Shoven. Chicago: University of Chicago Press.

Ministerio de Educación y Ciencia (1995), *Estadística del Gasto Público en Educación*, Madrid, Ministerio de Educación y Ciencia.

Montes, Ana (1998), "Educación para los jovenes y pensiones para los mayores. ¿'Existe alguna relación? Evidencia para España?", mimeo, Universidad Carlos III de Madrid, October.

Neher, Philip A. (1971), "Peasants, Procreation, and Pensions", *American Economic Review*, **61**, 380-389.

Nugent, James B, (1985), "The Old Age Security Motive for Fertility", *Population and Development Review*, **11**, 75-98.

Pogue, Thomas F. and Larry G. Sgontz (1977), "Social Security and Investment in Human Capital", *National Tax Journal* **30**, 157-169.

Rangel Antonio (1999), "Forward and Backward Intergenerational Goods: A Theory of Intergenerational Exchange", mimeo, Stanford University, November.

Richman, Harold A. and Matthew W. Stagner (1986), "Children: Treasured Resource or Forgotten Minority", in A. Pifer and L. Bronte (eds.) *Our Aging Society: Paradox and Promise.* New York, NY: Norton, 161-179.

Samuelson, Paul (1958), "An Exact Consumption Loan Model with or without the Social Contrivance of Money", *Journal of Political Economy 66*, 467-482.

Appendix: Data sources and their treatment

A.1 Data sources

Our sources of data are the following.

We obtain the aggregate expenditure on public education from the Estadística del Gasto Público en Educación (EGPE 1995, in Ministerio de Educación y Ciencia [1995]) and the Encuesta sobre Financiación y Gasto de la Enseñanza Privada (EFGEP 1990-91, in INE [1992b]). The first data base contains public expenditure for each schooling level, the second reports the amount of public funding going to private schools (centros concertados). Aggregate tax revenues are obtained from the Cuentas de las Administraciones Públicas (IGAE [1991b]). From this we extract the share of total tax revenues allocated to financing public expenditure on education, excluding the fraction that is covered with public debt. We assume that the fraction of public expenditure covered by debt financing is equal to the average share of public expenditure financed by debt during the years 1990-1991.

Aggregate flows of public pension payments are obtained from the Cuentas de las Administraciones Públicas (IGAE [1991b]) and Actuación Económica y Financiera de las Administraciones Públicas (IGAE [1991a]).

The conditional survival probabilities at each age are equal to those obtained by the latest mortality tables published by the National Statistical Institute (INE, INE [1991]) with reference to the year 1990.

The aggregate data do not allow the study of individual life-cycle behavior. To do this we use a Spanish household budget survey (Encuesta de Presupuestos Familiares, or EPF (INE [1992a]) carried out by INE in 1990-91. This survey contains data on individual income, expenditure, personal characteristics and demographic composition for 21,155 households and 72,123 Spanish citizens. This survey is representative of the entire Spanish population and is calibrated on the Spanish Census data.

A.2 Treatment of the data

A.2.1 Life-time distributions

We now detail how, using the data in the EPF, we calculated the life-time distribution of the four flows associated to the two public systems.

The information in the EPF allows the estimation of the contributions and payments associated to the two public systems for each individual in the sample. These contributions and payments depend upon the labor market condition of the individual. Thus, we have considered five states in which each individual can be. For each state we compute contributions and payments the individual receives or makes. These five states are:

($\mathcal{E}$) Student. If the individual is enrolled in a school or university receiving public funds. The individual is then receiving a transfer (E^i_a), of an amount equal to the average cost of a pupil of his/her age attending a school of the kind he/she specifies, during the fiscal year 1990-91. The same individual contributes toward financing of public education through a portion of his/her direct and indirect taxes, (T^i_a).

($\mathcal{W}$) Worker. In this class we include all employed individuals. Such individuals pay direct or indirect taxes to support public education, (T^i_a) and also pay social security contributions, $\left(T^{pi}_a\right)$.

($\mathcal{R}$) Retired. We consider as retired only those individuals receiving a contributive pension $\left(P^i_a\right)$. Retired individuals are also financing the public educational system with a portion of their taxes (T^i_a).

($\mathcal{U}$) Unemployed. If an individual receives unemployment benefits he/she is financing the public pension system through the social security contributions paid, $\left(T^{pi}_a\right)$. Once again, the unemployed are also financing the public education system with a portion of their taxes (T^i_a).

($\mathcal{I}$) Inactive. Here we include all the individuals that are not in any of the previous four states. If these individuals pay some income taxes, this is recorded in the EPF. Otherwise we attribute them a share of the indirect taxes based on their reported expenditure. The total gives (T^i_a).

These five states are mutually exclusive. For the very rare cases in which the same individual in the EPF reports to be in two or more of them, we create two or more "artificial" individuals and increase correspondingly the sample size. We define the universe of states to be $\mathcal{S} = \{\mathcal{E}, \mathcal{W}, \mathcal{P}, \mathcal{U}, \mathcal{I}\}$. The total population at each age $a = 1, \ldots, A$ is $\sum_{s \in \mathcal{S}} L_a(s)$, with $L_a(s)$ equal to the number of individuals of age a that are in state s. Denote the share of the population of age a in state s as $\mu_a(s) = L_a(s) / \sum_{s \in \mathcal{S}} L_a(s)$, with $\sum_{s \in \mathcal{S}} \mu_a(s) = 1$. For each a and $s \in \mathcal{S}$, $\mu_a(s)$ is the probability that an individual be in state s at age a.

A.2.1 Public education system

In Spain, public financing of education is allocated in part to public schools and in part to a special kind of private schools, centros concertados, by means of school vouchers to students. At the compulsory school level (up to age 14 in 1990, 16 in the current legislation) schooling is completely free. After that, students attending public institutions pay only a small fraction of the total cost, the rest being born by general tax revenues. Students attending private institutions bear the full cost.

Cost of public schooling

For each educational level (primary, secondary, higher and other) we have computed the real, per-pupil public expenditure on education for various types of schools (public and concertados) and for the public universities.

The EPF reports if an individual is enrolled at school, the type of school (public or private) and the level he/she is attending. This information is enough to compute the total number of students in each level, type of school and age group.

The criterion we followed to compute the cost of schooling for each "kind" of student (age a, level j, type k of school) is the following. From the EGPE and the EFGEP we obtain the actual total amount of public expenditures for each kind (kj) of school. We divide these amounts by the total number of pupils attending each. This gives us the effective per-student cost for each kind kj of school, E^{jk}. From the EPF we compute how many students of age a are attending a school of kind kj. Using this, we estimate public school expenditure on the representative individual at each age a as

$$E_a = \mu_a\left(\mathcal{E}\right) \sum_{k \in TC} \sum_{j \in NE} \mu_a\left(\mathcal{E}^{jk}\right) E_a^{jk} = \mu_a\left(\mathcal{E}\right) \overline{E}_a$$

where $\mu_a(\mathcal{E})$ denotes the fraction of the population of age a which is attending school, NE is the universe of educational levels, TC is the universe of types of schools. Finally $\mu_a\left(\mathcal{E}^{jk}\right)$ is the portion of students of age a enrolled in the educational level j in a school of type k.

The age distribution of public education "borrowing" is

$$\delta_a = \frac{E_a}{\sum_{a=1}^{A} E_a L_a}$$

Hence, δ_a is the share of (life-time total) education-related transfers the representative individual receives at age a.

Financing of the public education system

On the financing side we need to compute the amount of education-related taxes paid by the representative individual at age a. The taxes we consider are the following: personal income tax (Impuesto sobre la Renta de las Personas Físicas, or IRPF), Value Added Tax (VAT), special and other local taxes.

The EPF provides detailed information about the income flow of each individual, and the wealth and consumption baskets of each household. This allows a detailed reconstruction of the various taxes paid by an individual, which we then aggregate in a total burden of taxation $\left(T_a^i\right)$, for individual i of age a. We calculate the average tax paid by a person of age a as

$$T_a = \sum_{s \in \mathcal{S}} \mu_a\left(s\right) \frac{\sum_{i \in s} T_a^i}{L_a\left(s\right)} = \sum_{s \in \mathcal{S}} \mu_a\left(s\right) \overline{T}_a^s$$

where $\overline{T}_a^s$ is the average tax paid by an individual in state s, at age a.

Given the values T_a for $a = 1, \cdots, A$ the computation of the life-time distribution of the total investment in public education is straightforward,

$$\alpha_a = \frac{T_a}{\sum_{a=1}^{A} T_a L_a}$$

Hence α_a represents the relative burden of taxation charged upon the representative individual at age a, for $a = 1, \cdots, A$, Call this the age distribution of the total tax burden.

To impute the flow of real expenditures in education to the various years of one's life we need to scale the coefficients α_a by the actual public expenditure in education. We retrieve this from IGAE [1991b], call it T_{90}^e. Then we compute as $T_a^{e*} = \alpha_a \cdot T_{90}^e$ for $a = 1, \cdots, A$, the investment in public education for the representative agent.

A.2.2 Public pensions

Public contributory pensions are provided by the following programs. The "General Social Security Regime" (Régimen General de la Seguridad Social, or RGSS) is the main one and cover most private sector employees plus a (small but growing) number of public employees. The five plans included in the "Special Social Security Regimes" (Regímenes Especiales de la Seguridad Social, or RESS) are, respectively, for the self-employed (Régimen Especial de Trabajadores Autónomos or RETA), the agricultural workers and small farmers (Régimen Especial Agrario or REA), the domestic employees (Régimen Especial de Empleados de Hogar or REEH), the sailors (Régimen Especial de Trabajadores de Mar or RETM) and the coal miners (Régimen Especial de la Minería del Carbón or REMC). Finally, there exists a seventh, special pension system for the public employees (Régimen de Clases Pasivas, or RCP.

Financing the public contributive pension system

All seven pension regimes are of the pay-as-you-go-type and, presumably , self-financing †. To estimate the life-time distribution of social security payments we identified all individuals in the EPF paying social security contributions, and split them among the seven plans. For each individual we have enough information, either from the EPF or from current legislation (e.g. for public employees) to compute the "fictitious income" (bases de cotización and haberes reguladores) upon which pension contributions are being charged. To each of the fictitious incomes we apply the social security contribution rate, as specified by the 1990-91 legislation, for the pension regime in which the individual was enrolled. Aggregating these amounts over all the individuals of age a, we obtain, for each $a = 1, \ldots, A$, the amount of social security contributions paid by individuals in state $\mathcal{W}$ ($T_a^{\mathcal{W}}$) and state $\mathcal{U}$ ($T_a^{\mathcal{U}}$). The social security contribution paid by the representative agent at age a is then

$$T_a^p = \mu_a \left(\mathcal{W}\right) \cdot T_a^{\mathcal{W}} + \mu_a \left(\mathcal{U}\right) \cdot T_a^{\mathcal{U}}$$

† The RGSS shows a surplus. The five special regimes show small deficits.

Also in this case we compute weights by setting

$$\beta_a = \frac{T_a^p}{\sum_{a=1}^{A} T_s^p L_a}$$

Finally, from IGAE [1991a] and IGAE [1991b] we obtain the total amount of social security contributions paid to the seven plans during the year 1990, T_{90}^p. In our simulation we use

$$T_a^{p*} = \beta_a \cdot T_{90}^p$$

Benefits of the public pension system

The Spanish social security system provides five types of contributive pensions: old-age, disability, widowers, orphans, and other relatives. We have not considered payments of non-contributive pensions as part of our scheme, as they are not financed by means of social security contributions.

In the EPF we are told if an individual is a pension recipient, what kind of pension he or she receives and in which amount. The average contributive pension received at each age a is therefore easily computed as

$$P_a = \mu_a\left(\mathcal{P}\right) \cdot \sum_{k \in TP} \mu_a(\mathcal{P}^k) \cdot \frac{\sum_{i \in k} P_a^i}{L_a\left(\mathcal{P}^k\right)} = \mu_a\left(\mathcal{P}\right)\overline{P}_a$$

where $\mu_a\left(\mathcal{P}\right)$ is the fraction of the population of age a receiving a contributive pension, TP is the universe of kinds of contributive public pensions, $\mu_a(\mathcal{P}^k)$ is the portion of pensioners at age a receiving a pension of type k , P_a^i is the actual pension received by individual i of age a and $L_a\left(\mathcal{P}^k\right)$ is the number of individuals of age a receiving a pension of type k.

As in the previous cases, the life-time weights are computed as

$$\gamma_a = \frac{P_a}{\sum_{a=1}^{A} P_a L_a}$$

Finally, from IGAE [1991a] and [1991b] we obtain the total contributive pension payments effectively made, by the seven regimes, during the year 1990, P_{90}. The amounts used in our calculations are, therefore, $P_a^* = \gamma_a \cdot P_{90}$.