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ABSTRACT

Do the ‘Three-Point Victory’ and ‘Golden Goal’ Rules Make Soccer More Exciting? A Theoretical Analysis of a Simple Game

This note argues that a rigorous application of simple game theory may provide unambiguous yet non-trivial theoretical insights about the behaviour of players in simple games. This contrasts with a commonly held view that many predictions in applied game theory are either obvious or inconclusive. To illustrate our point, we analyse the merits of two controversial changes in soccer rules, namely the ‘three-point victory’ and the ‘golden goal’. Starting from standard premises, we present some original conclusions that are neither trivial nor the result of a twisted argument. We feel that soccer is a particularly good example for our exercise due to the simplicity of its main rules, but also to the proliferation of *ad-hoc* reasoning among soccer fans.

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1 Motivation

A major criticism of applied Game Theory is that it often generates results that are either obvious or inconclusive. Most games played in real life are complex, with multidimensional strategies, and incomplete information. Besides, they are often not fully specified ex-ante. The assumptions needed to have a well-defined game and avoid a multiplicity of equilibria tend to make the model excessively simplistic and/or the theoretical conclusions trivial.

One area in which simple game theoretic tools can easily be used to predict behavior is sport. There are concrete situations in sport where the game is simple and well-defined, and players have a limited number of strategies. Few authors have recognize this advantage of sport. Notable exceptions are the recent papers by Walker and Wooders (2001), Chiappori et al. (2000), Palacios-Huerta (2001) and Palomino et al. (1999). The first paper looks at serve-and-return play at Wimbledon whereas the second and third analyze penalty kicks in European soccer leagues. All three show that the data supports the mixed strategy Nash equilibrium prediction of the theory. The fourth paper also deals with soccer. It shows that, although the behavior of teams is roughly consistent with rationality (losing teams adopt more offensive strategies than winning teams), there is still a substantial component of irrationality or "passion", illustrated by the fact that teams perform better at home than they do away.

Testing predictions of Game Theory that are basic though not fully intuitive (like the concept of mixed strategy Nash equilibrium) is extremely important. It is also reassuring to notice that the observed behavior is roughly consistent with the predictions. However, these papers are still subject to the criticism that the theoretical conclusions are rather straightforward. If anyone can come up with these predictions, why do we need game theory and theoretical models?[1]

Overall, these papers do not attempt to counter the idea that, for simple games, game theory is useless as it makes points that are either obvious or twisted. The goal of this paper is to challenge that agnostic view of applied theory. The method we propose is to analyze a simple game, start with standard and widely accepted premises, and present a theory which is *original* but, once stated, *clear* and *simple*. Soccer is a suitable candidate for this exercise. It is extremely popular and its main rules are simple. Most importantly, the following conditions hold for virtually every soccer fan: (i) (s)he has spent a fair amount of leisure time thinking about the effects and

[1] We want to make a couple of clarifications. First, as far as we understand, the purpose of these papers is not to propose a new theory and test it, but rather to test the empirical relevance of *basic* game theory concepts. To do this, they all spend a substantial amount of effort precisely in finding games with the simplest possible strategies. It is therefore quite natural that knowledge of game theory is not required to determine what is the (roughly) optimal behavior. In other words, criticizing these papers on the grounds that predictions are straightforward is, in a sense, saying that the papers are "too good" at identifying simple games. Second, the paper by Chiappori et al. (2001) considers a 3×3 payoff matrix with some parameter restrictions so, even if the mixed-strategy equilibrium is relatively easy to compute, some of the predictions are not straightforward.

suitability of rule changes, (ii) (s)he has come up with a strong ad-hoc opinion about it, and (iii) (s)he believes that economic modelling cannot add anything to this debate.[2]

To focus the discussion, we concentrate on the effects of the two major changes in rules introduced in international events such as the World Cup during the 90s: the "three-point victory" (3PV) and the "golden goal" (GG). The 3PV system is employed in league tournaments. Under this rule, the winner of a match obtains three points and the loser obtains zero points. In case of a draw, each team obtains one point. The main argument in favor of this system –instead of the traditional two-point victory (2PV)– is simple. Adopting an offensive strategy increases the team's chances of scoring but also of conceding a goal. Therefore, teams are encouraged to play more offensively if the expected payoff of breaking a tie is raised. The GG rule is employed in elimination tournaments. Before its adoption, if two teams were tied at the end of the regular time they would play for a fixed 30 minutes extra time and, if the draw persisted, they would proceed to the penalty kicks. With the GG rule, the first team to score within the 30 minutes of extra time wins the match. If no one scores, the penalty kick method again determines the winner. Thus, the GG rule decreases the expected time of play and, other things equal, the probability of reaching the penalty kick stage.

In this note, we provide the simplest possible model of a soccer game. Using basic game theory, we qualify the ideas stated above in favor of 3PV and GG. We show that, although correct, the arguments are excessively simplistic because they only capture one effect of the rule on the behavior of teams. More specifically, we show in Proposition 1 that, conditional on the game being tied, increasing the value of a victory may induce teams to adopt a more offensive strategy towards the end of the game (so as to break the tie in one direction or another late in the match) but a more defensive strategy towards the beginning (so as to avoid being led early in the match). As a result, teams may on average play more defensively under 3PV than under 2PV. In other words, just by accounting for the possibility of changing the strategy over time (which seems quite natural in this game), we show that a rule established to favor certain objectives (in this case, more offensive behavior) may in fact be counter-productive *using the same criterion.* In Proposition 2 we show that, in the context of elimination tournaments, the GG rule modifies the payoff of scoring (it prevents the team that concedes a goal to come back on the score) but not the incentives of teams to play offensively. Therefore, the popular idea that under GG fewer matches will be decided by penalty kicks is supported by our model. However, the major benefits are obtained when the 3PV and GG rules are put together. In Proposition 3 we argue that the combination of an increase in the expected value of breaking a tie (3PV rule) together with a reduction in the ability to come back in the game when the opponent scores (GG rule) is unambiguously beneficial; it induces

[2] Our guess is that all popular collective sports share these same characteristics.

teams to play more offensively than under 3PV alone. Interestingly, this possibility has never been considered in practice, even though its implementation would be quite simple.

To sum up, this paper shows that basic game theory principles can be a powerful tool to obtain non-trivial theoretical insights about the behavior of players in simple games. Moreover, a careful modelling can deliver unambiguous recommendations for the improvement of existing rules.

2 A simple model of the three-point victory (3PV) rule

2.1 Strategies of teams and timing

We consider the simplest model able to capture the main effects of the scoring system on the strategy of teams. Two teams $i \in \{A, B\}$ play a match against each other. The winner of the game obtains x points and the loser gets zero points. In case of a draw, they both get one point. Teams are risk-neutral and play in a league tournament. Their objective is to maximize the expected number of points collected in the game.[3]

For simplicity, we assume that each team i decides at the beginning of the game (date $t = 1$) and at half-time (date $t = 2$) the strategy θ_t^i employed during the upcoming half-period. Date $t = 3$ denotes the end of the match, in which final payoffs are realized. The parameter $\theta_t^i \in [\underline{\theta}, \overline{\theta}]$ denotes how 'offensive' the strategy selected by team i at date t is, with higher values of θ denoting a more offensive strategy.[4] Playing more offensively increases the chances of scoring (and therefore of winning the match) but also the chances of conceding a goal (and therefore of losing it). Naturally, the optimal strategies selected by teams will be contingent on the current score of the game.

Denote by $\tau \in \{1, 2\}$ the two half-periods of play, i.e. $\tau = 1$ refers to the first half-period (between $t = 1$ and $t = 2$) and $\tau = 2$ refers to the second half-period (between $t = 2$ and $t = 3$). Suppose that during each half-period τ only three events $e_\tau \in \{a, o, b\}$ concerning the score of the game may occur: team A scores either one more goal (a), or the same number of goals (o), or one less goal (b) than team B. The probability of these events will depend on the strategies (θ^A, θ^B) selected by both teams. From now on, we will call 'dates' ($t \in \{1, 2, 3\}$) the beginning, half-time and end of the match, and 'half-periods' ($\tau \in \{1, 2\}$) the intervals of play going from beginning to half-time and from half-time to end of the match. The timing of the game can thus be summarized as follows.

[3] It is obviously not the same to compare 2PV vs. 3PV ($x = 2$ vs. $x = 3$) in a league of 4 teams than in a league of 18 teams. The objectives also play an important role (e.g. to win vs. to be among the top teams in the pool so as to advance into the next round). These considerations can be included just by re-scaling the value of a victory.

[4] In a more realistic model, teams should be able to change their strategy at every moment in time (and in particular whenever they score or concede a goal). This more complicated structure is out of the scope of the present paper since it is not necessary for the point we want to make.

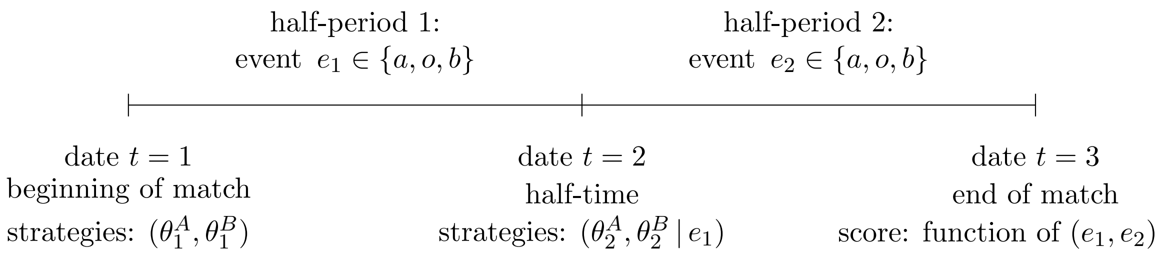

Figure 1. Timing of the game

We will use a reduced-form model. Instead of defining the probability that each team scores a goal given both teams' strategies, we work directly with the probability that each team scores *one more goal* than its rival (events $e_\tau = a$ and $e_\tau = b$) given both teams' strategies. Denote:

$$\alpha(\theta^A, \theta^B) = \Pr(a \mid \theta^A, \theta^B) \quad \text{and} \quad \beta(\theta^A, \theta^B) = \Pr(b \mid \theta^A, \theta^B)$$

We use the subscript n in $\alpha(\cdot)$ and $\beta(\cdot)$ to denote the partial derivative with respect to the n^{th} argument. These probabilities satisfy the following assumptions.

Assumption 1 $\alpha_1(\theta', \theta'') > 0$*;* $\alpha_2(\theta', \theta'') > 0$*;* $\beta_1(\theta', \theta'') > 0$*;* $\beta_2(\theta', \theta'') > 0 \;\; \forall\, \theta',\, \theta''$*.*

Assumption 1 is quite uncontroversial. It simply states that choosing a more offensive strategy (higher θ) increases both the chances of scoring and of conceding one more goal than the rival.[5]

Assumption 2 $\alpha(\theta', \theta'') = \beta(\theta'', \theta') \;\; \forall\, \theta',\, \theta''$*.*

Assumption 3 $\alpha_{11}(\theta', \theta'') < 0$*;* $\alpha_{22}(\theta', \theta'') > 0$*;* $\alpha_{12}(\theta', \theta'') = 0 \;\; \forall\, \theta',\, \theta''$*.*

Assumption 2 states that teams are homogeneous. This implies in particular that, if both teams play the same strategy, they have the same chances of winning and of losing ($\alpha(\theta', \theta') = \beta(\theta', \theta')$). According to Assumption 3, the marginal probability of scoring (resp. conceding) one more goal than the rival is decreasing (resp. increasing) in the level of offensive play. Also, the marginal effect of one team's degree of offensive behavior is independent of the strategy of the rival. Assumptions 2 and 3 are debatable, to say the least. Homogeneity is more the exception than the rule: few matches are played by teams of equal strength. As for the marginal effect of one team's strategy,

[5] We attract the attention of the reader to the fact that the assumption is not that 'the probabilities of scoring and conceding one goal are increasing in the level of attack'. The distinction makes sense only because strategies are chosen at $t = 1$ and $t = 2$ exclusively. Suppose for example that strategies were selected every 30 seconds. In that case and given that both teams cannot possibly score in the same interval of time, there would be no distinction between the events 'scoring' and 'scoring one more goal than the rival'. Note also that if the score changes during a half-period, teams are most likely to modify their strategy within that half-period. Our three-period model is not able to capture this property (see also footnote 4).

it will often depend on its opponent's behavior.[6] However, there are two interconnected reason for maintaining these assumptions. First, simplicity: under these assumptions we will obtain a unique and easy to compute symmetric equilibrium, which will allow us to perform clear cut comparative statics about the effect of the reward system on the strategy of parties. Second, and more importantly, transparency: strategies can always have perverse indirect effects in payoffs if we include some suitably chosen asymmetries in the teams and/or if we "twist sufficiently" the second and cross derivatives of the scoring probabilities. This is not the purpose of our paper. Instead, we present the simplest and most standard model of a soccer game and look, as our main departure, at the ability of teams to change their strategy during the game (e.g. at half-time).

2.2 Teams' value functions and payoffs

The payoff of teams depends exclusively on the final score at date $t = 3$, that is on the realization of the events e_1 and e_2. If e_1 and e_2 are such that a occurs more often than b (resp. both equally often, resp. b more often than a), then team A wins (resp. ties, resp. loses) the match, in which case its payoff is x (resp. 1, resp. 0) and the payoff of team B is 0 (resp. 1, resp. x).

Denote by $v_1^i(\theta_1^A, \theta_1^B)$ the value function of team i at the beginning of the match ($t = 1$) if teams select strategies θ_1^A and θ_1^B for half-period 1. Similarly, $v_2^i(\theta_2^A, \theta_2^B \,|\, e_1)$ is the value functions of team i at half-time ($t = 2$) given the current difference of goals (i.e. the realization of event e_1 during the first half-period) if strategies θ_2^A and θ_2^B are selected for half-period 2. In order to solve this game, we first study the value functions at half-time. We have:

$$\begin{aligned} v_2^A(\theta_2^A, \theta_2^B \,|\, a) &= \left[1 - \beta(\theta_2^A, \theta_2^B)\right] x + \beta(\theta_2^A, \theta_2^B) \\ v_2^A(\theta_2^A, \theta_2^B \,|\, b) &= \alpha(\theta_2^A, \theta_2^B) \\ v_2^A(\theta_2^A, \theta_2^B \,|\, o) &= \alpha(\theta_2^A, \theta_2^B)\, x + \left[1 - \alpha(\theta_2^A, \theta_2^B) - \beta(\theta_2^A, \theta_2^B)\right] \end{aligned}$$

We can then determine the optimal strategies of teams in the second half-period (θ_2^A, θ_2^B), conditional on the event realized in the first half-period ($e_1 \in \{a, o, b\}$):

$$\begin{aligned} \underline{\theta} &= \arg\max_{\theta} v_2^A(\theta, \theta_2^B \,|\, a) &= \arg\max_{\theta} v_2^B(\theta_2^A, \theta \,|\, b) \\ \overline{\theta} &= \arg\max_{\theta} v_2^A(\theta, \theta_2^B \,|\, b) &= \arg\max_{\theta} v_2^B(\theta_2^A, \theta \,|\, a) \\ \theta^{**} &= \arg\max_{\theta} v_2^A(\theta, \theta_2^B \,|\, o) &= \arg\max_{\theta} v_2^B(\theta_2^A, \theta \,|\, o) \end{aligned}$$

where, given Assumptions 1, 2 and 3, the strategy θ^{**} is unique and solves:

$$\frac{\alpha_2(\theta^{**}, \theta^{**})}{\alpha_1(\theta^{**}, \theta^{**})} = x - 1 \tag{1}$$

[6] Palomino et al. (1999) have an extensive discussion of this point and the controversy about what seems to be the plausible sign.

In words, a team which is leading at half-time will choose the most defensive strategy ($\underline{\theta}$) during the second half-period in order to minimize the probability of conceding a goal. Conversely, the team which is losing at half-time will only be interested in maximizing its probability of scoring one more goal than the rival, and therefore will play as offensively as possible ($\overline{\theta}$).[7] The most interesting situation arises when the score is tied at half-time. In this case, the optimal second period strategy of both teams is given by (1). The idea behind this equation is very simple. A team sets its optimal level of offensive play for the second half-period where the marginal increase in the probability of a victory $[\alpha_1(\cdot)]$ weighted by the increase in the payoff $[x-1]$ equals the marginal increase in the probability of a defeat $[\beta_1(\cdot)]$ weighted by the decrease in the payoff $[\,1\,]$. Given teams' homogeneity, $\beta_1(\theta', \theta'') \equiv \alpha_2(\theta'', \theta')$ and θ^{**} follows. Differentiating (1), one notices that:

$$\frac{\partial \theta^{**}(x)}{\partial x} \;=\; \frac{\alpha_1(\theta^{**}, \theta^{**})}{\alpha_{22}(\theta^{**}, \theta^{**}) - (x-1)\alpha_{11}(\theta^{**}, \theta^{**})} \;>\; 0 \tag{2}$$

Other things equal, if teams are tied at half-time, they will play more offensively during the second half-period the greater the value of a victory x. In fact, (2) formalizes the standard (static) argument in favor of the 3PV relative to the 2PV system: by increasing the expected payoff of breaking a tie, teams are induced to adopt relatively more offensive strategies.

As stated in the introduction, our goal is not to refute this argument by adopting some suitably chosen utility functions or winning probabilities. On the contrary, we take this same theory as our starting point. However, we claim that using a static argument may not be appropriate for the game we are analyzing. In other words, we ask whether the conclusion presented in (2) holds when we assume that the game is *dynamic* and that strategies can be modified over time. The simplest way to answer this question is to study the two half-periods model depicted in Figure 1. Having analyzed the optimal strategy of teams selected at half-time contingent on the current score, we are now left to determine their optimal strategy for the first half-period selected at the beginning of the game. Naturally, teams are tied when the match starts. The value function of team A is then:

$$v_1^A(\theta_1^A, \theta_1^B) = \alpha(\theta_1^A, \theta_1^B) v_2^A(\underline{\theta}, \overline{\theta}|a) + \Big[1 - \alpha(\theta_1^A, \theta_1^B) - \beta(\theta_1^A, \theta_1^B)\Big] v_2^A(\theta^{**}, \theta^{**}|o) + \beta(\theta_1^A, \theta_1^B) v_2^A(\overline{\theta}, \underline{\theta}|b)$$

So we get:

$$\theta^* \quad = \quad \arg\max_{\theta}\; v_1^A(\theta, \theta_1^B) \quad = \quad \arg\max_{\theta}\; v_1^B(\theta_1^A, \theta)$$

where, given Assumptions 1, 2 and 3, the optimal strategy θ^* is unique and solves:

$$\frac{\alpha_2(\theta^*, \theta^*)}{\alpha_1(\theta^*, \theta^*)} = \frac{v_2^A(\cdot\,|\,a) - v_2^A(\cdot\,|\,o)}{v_2^A(\cdot\,|\,o) - v_2^A(\cdot\,|\,b)} \quad \equiv \frac{(x-1)[1 - \beta(\underline{\theta}, \overline{\theta}) - \alpha(\theta^{**}, \theta^{**})] + \beta(\theta^{**}, \theta^{**})}{1 - \beta(\underline{\theta}, \overline{\theta}) - \alpha(\theta^{**}, \theta^{**}) + (x-1)\beta(\theta^{**}, \theta^{**})} \tag{3}$$

[7] The fact that teams play a fully offensive and a fully defensive strategy when they are losing and winning respectively is excessively simplistic. However, it is not a necessary ingredient for our analysis.

Consider the following function:

$$F(\theta^{**},\theta^{**}) = \Big(\alpha(\theta^{**},\theta^{**}) + \beta(\theta^{**},\theta^{**})\Big) - \Big(1 - \beta(\underline{\theta},\overline{\theta})\Big)$$

Note that $F(\cdot)$ is increasing in the likelihood of breaking a tie in the second period. At this stage, we can state our first result.[8]

Proposition 1 (The 3PV rule) (i) $\theta^*(2) = \theta^{**}(2)$ *(under 2PV teams do not change their strategy during the whole match as long as they are tied).*

(ii) $\partial\theta^{**}(x)/\partial x > \partial\theta^*(x)/\partial x$ *for all* $x > 2$ *(under 3PV teams play relatively more offensively towards the end of the match than towards the beginning).*[9]

(iii) *If* $F(\theta^{**},\theta^{**}) > 0$*, then* $\partial\theta^*(x)/\partial x < 0 < \partial\theta^{**}(x)/\partial x$ *(under 3PV teams play more defensively in the first half-period and more offensively in the second half-period than under 2PV).*

Proof. Part (i) is trivial. Using (1), (3) and the monotonicity of $\alpha_2(\cdot)/\alpha_1(\cdot)$, one can notice that:

$$\theta^{**}(x) - \theta^*(x) \ \propto\ \frac{\alpha_2(\theta^{**},\theta^{**})}{\alpha_1(\theta^{**},\theta^{**})} - \frac{\alpha_2(\theta^*,\theta^*)}{\alpha_1(\theta^*,\theta^*)} = \left(x - 1 - \frac{1}{x-1}\right)\frac{1}{1 + \frac{1-\beta(\underline{\theta},\overline{\theta})-\alpha(\theta^{**},\theta^{**})}{(x-1)\beta(\theta^{**},\theta^{**})}}$$

Given that $\partial\alpha(\theta^{**},\theta^{**})/\partial x = \partial\beta(\theta^{**},\theta^{**})/\partial x > 0$, then for all $\alpha(\cdot)$ and $x > 2$, $\frac{\partial\theta^{**}(x)}{\partial x} - \frac{\partial\theta^*(x)}{\partial x} > 0$. Also, $\frac{\partial\theta^{**}(x)}{\partial x} > 0$ is shown in (2). Last, taking the derivative in the r.h.s. of equation (3) we get:

$$\frac{\partial\theta^*(x)}{\partial x} \ \propto\ -F(\theta^{**},\theta^{**}) - \Big[(x-1)^2 - 1\Big]\frac{\partial\beta(\theta^{**},\theta^{**})}{\partial x}$$

Hence, $F(\theta^{**},\theta^{**}) > 0 \ \Leftrightarrow\ \alpha(\theta^{**},\theta^{**}) + \beta(\theta^{**},\theta^{**}) > 1 - \beta(\underline{\theta},\overline{\theta})$ is a *sufficient* condition which ensures that $\partial\theta^*(x)/\partial x < 0$ for all $x \geq 2$. □

The idea behind parts (i) and (ii) rests on a standard *option value* argument. A team that concedes an early goal may still tie or win the match, and a team that scores an early goal may still tie or lose the match. Naturally, the absolute change in payoff from victory to defeat and from defeat to victory is symmetric ($|x|$). The crucial issue is that the absolute change in payoff between a tie and a defeat is $|1|$. When $x > 2$, this is smaller than $|x-1|$, the absolute change in payoff between a victory and a tie. As a result, for $x > 2$, the benefits of scoring a goal early in the match (in terms of the increase in the expected final payoff) are smaller than the costs of conceding an early goal (in terms of the decrease in the expected final payoff). Stated differently, when receiving an early goal the team can mostly hope to move from 0 to 1 point, whereas after scoring an early goal the team has still chances of moving from x to 1. Since $x - 1 > 1$, teams

[8] The technical result of Proposition 1 parts (ii) and (iii) is given for all $x \geq 2$. However, for expositional purposes, the explanation is given in terms of 3PV vs. 2PV ($x = 3$ vs. $x = 2$).

[9] Note that the combination of (i) and (ii) implies that $\theta^{**}(x) > \theta^*(x)$ for all $x > 2$.

prefer to play relatively more defensively at the beginning of the match so as to avoid conceding an early goal, even if it comes at the expense of decreasing also the chances of scoring. When $x = 2$, leading and being led are symmetric events in terms of expected payoffs. In that case, the same strategy is maintained by both teams as long as the match is tied.

Building on the previous argument, part (iii) shows that increasing the reward of a victory may have the perverse effect of increasing the incentives of teams to adopt a defensive strategy during the first half-period ($\partial\theta^*/\partial x < 0$). Indeed, suppose that the likelihood of breaking a tie during the second half-period ($\alpha(\theta^{**}, \theta^{**}) + \beta(\theta^{**}, \theta^{**})$) is greater than the probability that the team leading at half-time does not win the match ($1-\beta(\underline{\theta}, \overline{\theta})$), which technically corresponds to $F(\theta^{**}, \theta^{**}) > 0$. In this case, the optimal strategy of teams is to play very defensively at the beginning of the game (so as to avoid being led early in the match) and very offensively towards the end (so as to break the tie in one direction or another late in the match). Overall, a static reasoning suggests that increasing the payoff x of a victory always increases the incentives of teams to play offensively (see (2)). However, keeping the same reasoning and the same analysis, we conclude that turning this zero-sum game (2PV) into a non-zero sum game (3PV) has ambiguous effects on the average level of offensive play as soon as we account for the dynamic nature of the game and the possibility of changing strategies over time.

Our result deserves some further comments. It would certainly be more realistic to include some strategic interaction (e.g. $\alpha_{12}(\theta^A, \theta^B) > 0$, which means that a team's marginal probability of scoring is greater the more offensive the strategy of its rival) and the possibility of scoring two more goals than the opponent in a given half-period. These considerations would affect the equilibrium strategies and probabilities of scoring of both teams. However, the insights of Proposition 1 would remain unaffected. More importantly, if dynamic considerations are key, then choosing a strategy only twice during the match is still too simplistic. One may wonder what would be the equilibrium in a model where teams could select their optimal strategy during a discrete but arbitrarily large number of periods. The present paper cannot answer that question. Last, the usual argument against an "excessively high" x builds on a fairness consideration: since soccer is an inherently stochastic game, high distortions in the payoff of a victory may reward luck in excess. Our model argues that the optimal reward for a victory can be bounded above even if we consider exclusively the incentives of teams to play offensively.

An empirical test of our theory is, although interesting, out of the scope of this paper. However, we may obtain some insights from previous empirical analyses. The paper by Palomino et al. (1999, Figure 1) suggests that, under 3PV, more goals are scored towards the end of the matches than towards the beginning. This seems consistent with the idea that, under 3PV, teams adopt more defensive strategies early in the game than late in the game. The paper by Palacios-Huerta (1999) shows that the 3PV rule has not affected significantly the average number of goals in the English Premier League, which seems to indicate that the average level of offensive play is similar under 2PV than under 3PV.[10]

3 A simple model of the golden goal (GG) rule

With a very simple extension of our framework it may be possible to analyze the effect of the "golden goal" (GG) rule in the strategy of teams. The GG rule has been recently used in the World Cup and other tournaments at the elimination stage (not for matches in a pool). Before its introduction, two teams finishing the match tied played during a (fixed) 30 minutes extra time. If the draw persisted, the winner was selected by penalty kicks. According to the new GG rule, the first team to score within the 30 minutes extra time wins the match. If there are no goals, then the penalty kicks determine the winner.

The GG rule has two obvious effects: it reduces the expected time of play and, other things equal, it decreases the probability of deciding the winner in the penalty kicks. However, one may wonder if teams adopt more offensive strategies under the GG rule or under the traditional system. In fact, this is important not only because maximizing the level of offensive play is part of the objective function, but also because it determines whether *in equilibrium* fewer matches reach the penalty kick phase. To answer this question, consider the following extension of the model presented in the previous section. Denote by $t \in \{1, 2, 3\}$ the beginning, half and end of the extra-time. One can easily account for the main property of the GG rule by assuming that if by half-time ($t = 2$) one team has scored one more goal than its rival, then it is declared the winner of the match.[11] In this setting, denote by $(\gamma_t^i, \gamma^*, \gamma^{**})$ the analogue of $(\theta_t^i, \theta^*, \theta^{**})$ to the new case. For any given x, team A's value function at half-time of the extra-time when the match is tied is:

$$v_2^A(\gamma_2^A, \gamma_2^B \,|\, o) = \alpha(\gamma_2^A, \gamma_2^B)\, x + \Big[1 - \alpha(\gamma_2^A, \gamma_2^B) - \beta(\gamma_2^A, \gamma_2^B)\Big]$$

The optimal strategy γ^{**} selected by both teams for the second half-period in case of a tie at half-time is unique and solves:

$$\frac{\alpha_2(\gamma^{**}, \gamma^{**})}{\alpha_1(\gamma^{**}, \gamma^{**})} = x - 1 \tag{4}$$

At the beginning of the extra-time the game is, by definition, tied. Team A's value function is:

$$v_1^A(\gamma_1^A, \gamma_1^B) = \alpha(\gamma_1^A, \gamma_1^B)\, x + \Big[1 - \alpha(\gamma_1^A, \gamma_1^B) - \beta(\gamma_1^A, \gamma_1^B)\Big]\, v_2^A(\gamma^{**}, \gamma^{**} \,|\, o)$$

[10] We should be cautious when interpreting these results. First, the analysis of Palomino et al. (1999) does not say anything about the evolution of the number of goals under 2PV. Second, our theory with 3PV predicts an increase in the probability of scoring as time elapses, but only conditional on the match being tied.

[11] As for the analysis of the 3PV rule, this three-period model captures imperfectly the effects of the GG rule. In particular, the match should be stopped as soon as a team scores rather than at half-time if one team is leading. However, we think once again that it is a reasonable approximation for the purpose of our study.

which gives the following optimal strategies γ^* of both teams for the first half-period:

$$\frac{\alpha_2(\gamma^*,\gamma^*)}{\alpha_1(\gamma^*,\gamma^*)} = \frac{x - v_2^A(\gamma^{**},\gamma^{**}\,|\,o)}{v_2^A(\gamma^{**},\gamma^{**}\,|\,o)} \equiv \frac{(x-1)(1-\alpha(\gamma^{**},\gamma^{**})) + \beta(\gamma^{**},\gamma^{**})}{1-\alpha(\gamma^{**},\gamma^{**}) + (x-1)\beta(\gamma^{**},\gamma^{**})} \tag{5}$$

One should immediately notice that in the context of elimination tournaments (the only cases where the GG rule is currently applied), it makes no sense to consider variations on the prize of a victory. Since both teams are equally strong, it seems reasonable to assume that each of them wins at the penalty kicks with probability 1/2. Therefore, the expected value of a tie is exactly half-way between the value of winning and the value of losing. Formally, this translates into $x \equiv 2$. We can now state our second result.

Proposition 2 (The GG rule) $\gamma^*(2) = \gamma^{**}(2) = \theta^*(2) = \theta^{**}(2)$ *(in elimination tournaments –i.e. for $x = 2$– the GG rule does not affect the incentives of teams to play offensively).*

Proof. Immediate by setting $x = 2$ in equations (1), (3), (4) and (5). □

In elimination tournaments, adopting the golden goal rule increases the variance in the payoff of playing offensively. Indeed, once team A has scored a goal (event $e_1 = a$), its rival B does not have the opportunity to come back in the game. That is, team A gets a payoff of 2 ($> v_2^A(\cdot\,|\,a)$) and team B a payoff of 0 ($< v_2^B(\cdot\,|\,a)$). However, as long as $x = 2$, the increase in the variance of payoffs is symmetric. Formally, $2 - v_2^A(\gamma^{**},\gamma^{**}\,|\,a) = v_2^B(\gamma^{**},\gamma^{**}\,|\,a) - 0$. As a result, and exactly for the same reasons as in part (i) of Proposition 1, the incentives of teams to play offensively do not change with the introduction of the GG rule. Overall, Proposition 2 shows that the GG rule fulfills its mission: it reduces both the equilibrium probability of deciding the winner with the (unsatisfactory) penalty kick method and the expected extra-time of play without inducing teams to adopt more defensive strategies.

4 A positive analysis of 3PV and GG

Given the previous analysis, one could consider the possibility of designing a simple combination of the 3PV and GG rules in league tournaments. This would be relatively easy to implement. For example, instead of a fixed 90 minutes play, a regular match could last only 80 minutes, with the winner obtaining x points and the loser 0. In case of a draw, teams would play for an extra 20 minutes under the GG rule. If at the end of the extra time the draw persisted, then each team would get one point.[12] Given these considerations, we have the following unambiguous recommendation.

[12] The National Hockey League (NHL) has a similar rule during its regular season: in case of a draw, teams play extra time under the GG rule and, if no team scores in the extra time, they both get 1 point. However, the value of a victory is always 2 points, i.e. the GG is not combined with the 3PV.

Proposition 3 (The 3PV and GG rules) $\gamma^*(x) > \theta^*(x)$ *and* $\gamma^{**}(x) = \theta^{**}(x)$ *for all* $x > 2$ *(the average incentives of teams to play offensively is higher with a combination of 3PV and GG than with 3PV alone).*

Proof. Immediate given (1), (3), (4), (5) and $\beta(\underline{\theta}, \overline{\theta}) > 0$. □

The greatest benefits of the GG rule are best highlighted when we combine it with the 3PV system. Proposition 3 shows that an increase in the variance of payoffs (obtained via the GG rule) together with an increase in the expected benefit of breaking a tie (obtained via the 3PV rule) encourages teams to adopt more offensive strategies at the beginning of the extra-time, without affecting their behavior towards the end. In other words, with the combination of 3PV and GG, teams have relatively more to gain than to lose if they adopt riskier strategies early in the extra time. Formally,

$$x - v_2^A(\gamma^{**}, \gamma^{**} \,|\, a) \;\equiv\; \beta(\gamma^{**}, \gamma^{**})(x-1) \;>\; \beta(\gamma^{**}, \gamma^{**}) \;\equiv\; v_2^B(\gamma^{**}, \gamma^{**} \,|\, a) - 0 \quad \forall\, x > 2$$

Overall, our result suggests that even if sport rules are necessarily part of a second-best world, if we understand the main underlying effects of the rules, then it is possible to design *simple* changes that generate *unambiguous* improvements.

5 Conclusion

Professional sport has become an important part of our everyday life. Many individual decisions (including not only allocation of leisure and money but even social conduct) are affected by sport events. Optimizing the rules of sport can therefore be of interest. Surprisingly, economists have almost completely neglected this possibility, even though sports seems to be one of the most natural applications of game theory. The first goal of this paper was to analyze the merits of two controversial changes in soccer rules: the ‘three-point victory’ and the ‘golden goal’. However, the second objective of this exercise was more ambitious: contrary to the common view that most results of applied game theory are either obvious or inconclusive, we argued in this paper that with a rigorous application of simple game theory it is possible to obtain unambiguous yet non-trivial theoretical insights about the effects of game rules on the behavior of participants. From a positive viewpoint, these conclusions can be used to shape simple and clear recommendations for the modification of rules. Given the availability of data on soccer, a natural next step will be to test our predictions. This is left for future work.

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