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SUSPENSE

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ABSTRACT

Suspense*

In a dynamic model of sports competition, we show that when spectators care only about the level of effort exerted by contestants, rewarding schemes that depend linearly on the final score difference provide more efficient incentives for efforts than schemes based only on who wins and loses. This result is puzzling because rank order schemes are the dominant forms of reward in sports competitions. The puzzle can be explained if one takes into account the fact that spectators also care about the suspense in the game. We define the spectators' demand for suspense as a greater utility derived from contestants' efforts when the game is closer. As the demand for suspense increases, so does the advantage of rank order schemes relative to linear score difference schemes. This relative advantage is realized by suitably increasing the winner's prize in rank order schemes. When the demand for suspense is sufficiently high, the optimal rank order scheme dominates all linear score difference schemes, and in a limit case, it is optimal among all incentive schemes that reward contestants on the basis of the final score difference.

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1. Introduction

Contestants in sports events are typically rewarded on the basis of who wins and who loses. More informative measures of performance such as score differences rarely matter, even though they are readily available. For example, in a boxing match, the winner's purse and the loser's purse are independent of whether the match ends with a knockout in the first round, or turns out to be a drawn-out fight to the last round. This is in spite of the fact that quite objective scores are assessed throughout the match. When there are several contestants in a sports event, rewards depend on who wins the most games (round robin tournament), or on the sequence of games won (elimination tournament). Total scores and other performance measures matter only in terms of determining who the winner is, not how much the winner gets.[1] Why do win-lose rank order incentive schemes prevail in sports?

One answer is that spectators simply derive great utility from watching rank order contests (see, e.g., O'Keeffe, Viscusi and Zeckhauser, 1984, pp 28-29). Such preference for rank order contests presumably arises from the notion that winner-take-all tournaments increase the stakes that contestants face through payoff discontinuity, and create the drama that somehow makes the games more exciting for the spectators to watch. This notion is intuitively appealing, but it begs the fundamental question of what makes a "good" sports competition.

Using a rank order incentive scheme is not a sure recipe to arouse spectators' interests. Rank order tournaments can get boring when the game becomes lope-sided. Intuition suggests that whether a sports event is good or not depends on how the game is played out from the beginning to the end. In this paper, we present an explanation of the dominance of rank order schemes in sports which rests on an analysis of the dynamics of sports competitions and understanding the nature of spectators' demand for drama.

[1] In recent years, contracts for players in some team sports have been loaded with incentive clauses, so that player compensation often takes on a strong piece rate flavor. Such incentive contracts are necessary because the rewards for winning in these sports usually accrue to the owner of the team, who in turn pays the players for their effort. The players in team sports are therefore motivated by their personal contracts with the owner, not directly by the reward structure of the game. The principal-agent problem in team production has been well explored in the literature. In contrast, this paper focuses on the optimal reward schemes of the sports game.

The starting point of the present paper is a dynamic version of Lazear and Rosen's (1981) tournament model. In their original static model, tournament participants exert efforts that determine "scores." Rank order schemes and others based on more informative relative performance measures such as score differences perform equally well. When designed optimally, all these schemes achieve the first best outcome, if participants are risk-neutral.

This conclusion is dramatically changed in a sports game with two halves where two contestants choose efforts at the beginning of each half. In a rank order scheme, contestants are rewarded according to whose total score is greater. In an alternative linear score difference scheme, contestants are also rewarded according to the final score difference, but the difference matters not just in terms of its sign but also linearly in terms of its magnitude. We show that the optimal linear score difference scheme dominates rank order schemes. The reason is simple. Under a rank order scheme, contestants keep up the efforts in the second half only when the game is still close at the end of the first half.[2] In contrast, a linear score difference scheme gives constant incentives for contestants to exert effort, independent of the stage of the game and of the score difference at the end of the first half. Under the standard assumption that contestants face convex effort costs, constant allocation of efforts across different states of the game reduces effort costs to the contestants. As a result, linear score difference schemes out-perform rank order schemes. Indeed, under reasonable assumptions, the optimal linear score difference scheme induces the first best efforts.

The result that linear score difference schemes dominate rank order schemes is puzzling given the prominence of rank order schemes in sports. It suggests that spectators in a sports event care about other characteristics of the sports game besides contestants' effort levels. We capture a unique feature in the demand for sports by assuming that spectators enjoy "suspense" in the game: Instead of caring about efforts per se, spectators derive greater utility from contestants' efforts when the game is closer and the outcome is still uncertain.

Performance of linear score difference schemes is independent of how much spectators value suspense. Under a linear score scheme, contestants continue to exert efforts in the

[2] Ehrenberg and Bognanno (1990) document this dynamics of efforts in golf tournaments.

second half to collect the rewards which are based on how large the final score difference is, even when the game has become lop-sided and spectators have lost interest. In contrast, when spectators demand suspense, a rank order scheme provides incentives for continuing efforts exactly when such efforts matter to spectators. We show that as the demand for suspense increases, the optimally designed rank order scheme increases the stake for the contestants. The more spectators demand suspense, the better rank order schemes perform relative to linear score difference schemes. When the demand for suspense is sufficiently high, in a sense to be properly defined, the optimal rank order scheme dominates the optimal linear score difference scheme. Indeed, the optimal rank order scheme dominates all incentive schemes that reward contestants on the basis of the final score difference.

The literature on optimal design of tournaments that began with Lazear and Rosen (1981) explains how tournaments work, but it has not shed much light on when these schemes should be used relative to other incentive mechanisms, particularly when the agents are risk neutral. Prendergast (1999, pp 36-37) reviews several reasons for using tournaments. None of the reasons explains why prizes do not depend on relative performance measures in sports events where these measures are readily available. Holmstrom (1982) casts doubt on the importance of rank order tournaments in labor contracts by demonstrating that relative performance schemes such as rank order tournaments have no intrinsic value if output measures of agents are uncorrelated. There are also some works on design of tournaments in a dynamic setting (Aron and Lazear, 1990; Cabral, 1999) but their focus is on risk taking rather than on effort choice.

In the principle-agent literature, Holmstrom and Milgrom (1987) have considered the problem of providing incentives to an agent who sequentially chooses efforts after observing the outcomes of previous efforts. The present paper can be viewed as an extension of the Holmstrom-Milgrom model of dynamic incentives to the case of multiple agents. While Holmstrom and Milgrom show that linear incentive contracts are optimal, we recover the optimality of rank order schemes in sports contests by introducing suspense in spectators' preference.

Our assumption that spectators enjoy suspense in sports events is consistent with the "uncertainty of outcome hypothesis" in the empirical sports literature, which states that spectators are willing to pay more for more uncertain games (Knowles, Sherony and Haupert, 1992). The uncertainty of outcome hypothesis may be explained by the standard tournament model of Lazear and Rosen, where contestants supply more effort when the game is closer. The assumption of demand for suspense in our model suggests that spectators of a rank order sports tournament lose interest when the game becomes lop-sided, both because they anticipate that contestants will slack off, and because they no longer care much about efforts by contestants.[3] A related finding in the empirical sports literature is that sports leagues try to keep some balance by minimizing the disparity between the strong and weak teams, known as "competitive balance" (Fort and Quirk, 1995). The idea that competitive balance helps provide effective effort incentives is well known since Lazear and Rosen (1981), and there has been some recent research about its implications to income distribution in sports (Palomino and Rigotti, 2000; Szymanski, 2001). While competitive balance is consistent with the public's demand for suspense, in that games in a more balanced sports league are less predictable and thus more desirable, our analysis focuses instead on the dynamics of efforts in the game.

The next section presents a simple model of dynamic sports game, with two contestants and two halves. Section 3 assumes that spectators care about efforts per se and characterizes the optimal rank order and linear score difference schemes. Section 4 defines spectators' demand for suspense and shows that as the demand for suspense increases, the relative performance of the optimal rank order scheme improves and eventually dominates linear score difference schemes. The last two sections of the paper discuss some extensions of our model and summarize our results.

2. The Model

There are two players in a sports game that consists of two halves. In each half $k = 1, 2$, the two players $j = A, B$ choose efforts μ_k^j simultaneously. The score difference δ_k in either

[3] A simple way of validating our assumption, and our explanation for the prominence of rank order schemes, is to test the uncertainty of outcome hypothesis after controlling for the contestants' efforts. We hope to make this the subject of future research.

half, defined as A's score minus B's score, is determined by the difference of $\mu_k^A - \mu_k^B$ and a random factor ϵ_k. Throughout the paper, we assume that ϵ_k is i.i.d. across the two halves, and has a differentiable, uni-modal density f that is symmetric around 0. Denote as F the corresponding distribution function of ϵ_k. We allow the support of ϵ_k to be finite or infinite.

The two players are risk-neutral, and do not discount. The two players simultaneously choose their effort at the beginning of each half to maximize their expected award less the sum of effort costs in the two halves. Players observe the first-half score difference δ_1 before choosing their efforts in the second half. We assume that the cost of effort, C, is the same in each half and the same for the two players. In addition to the standard assumption that C is increasing and convex, we make the following technical assumption.

ASSUMPTION 1: $0 \leq C''' \leq (C'')^2/C'$, *with at least one strict inequality.*

Assumption 1 imposes two global restrictions on the cost function: the third derivatives be non-negative but at most equal to the ratio of squared second derivatives to first derivatives. An increasing quadratic cost function, for example, satisfies the above assumption because $C''' = 0$ and $C' > 0$. The restriction of $C''' \leq (C'')^2/C'$ means that the first derivative of C is log-concave. The exponential cost function also satisfies these restrictions.

The incentive designer chooses a reward scheme to maximize spectators' utility minus the expected payoff to the players, subject to voluntary participation of players and equilibrium response by the players. Let $\underline{U}$ be the reservation utility of each player before entering the game. Spectators derive utility from efforts exerted by players during the game. We define P_k as the rate of spectator utility per unit of effort μ_k^j in half $k = 1, 2$, and we assume that this rate is the same for the two players. Players' efforts are observable to spectators, but not contractible. This ensures that the designer's objective function can involve efforts explicitly, but that the designer cannot condition rewards directly on efforts. Moreover, rewards can depend only on the final score difference, but not on the score difference δ_1 at the end of the first half. These contractual restrictions are reasonable for incentive design in sports context.

We will distinguish between the case where P_2 is constant and the case where it depends on the first-half score difference $P_2(\delta_1)$. When P_2 is constant, we will say that spectators care only about "excitement." One goal of the model is to capture the idea that spectators care also about "suspense" in addition to excitement. A simple way of modeling demand for suspense is by assuming that spectators care more about efforts when the game is closer. We will say that spectators care also about suspense when P_2 as a function of δ_1 is symmetric around and single-peaked at $\delta_1 = 0$. A constant $P_2(\delta_1)$ should be viewed as a polar case corresponding to no preference for suspense. The other polar case occurs when the function $P_2(\delta_1)$ is an indicator function with all the weight at $\delta_1 = 0$ (tied first half), which corresponds to an extreme preference for suspense. Throughout the paper, we make the following assumption:

Assumption 2: $P_1 = \int P_2(\delta_1) f(\delta_1) d\delta_1$.

In the absence of any presumption regarding how much spectators enjoy the excitement of the game in the first half versus in the second half, Assumption 2 is a natural starting point. In the case when spectators care only about the excitement of the game, Assumption 2 implies that $P_2 = P_1$. We use separate notation for P_1 and P_2 throughout the paper, to highlight the distinction between the case where P_2 is constant (section 3) and the case where P_2 depends on δ_1 (section 4). Assumption 2 is not needed for some of the analysis; its role will become clear later.

3. Excitement Only

In this section, we focus on the benchmark case where spectators care only about the excitement of the game (that is, P_2 is constant). We derive the optimal rank order scheme and the optimal linear score difference scheme, and compare the performance of these two incentive schemes.

A rank order scheme rewards players entirely on the basis of who wins and who loses the whole game, regardless of the score difference at the end. Such a scheme is represented by an "incentive prize" r, which is the difference between the winner's and the loser's

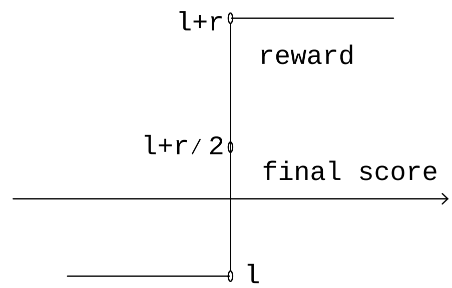

Figure 1. A rank order scheme

rewards, and a fixed transfer l, which can be either positive or negative. See Figure 1 for an illustration. To find the optimal rank order scheme, we use backward induction to characterize the equilibrium response to an arbitrary rank order scheme (r, l).

In the second half, given first-half score difference δ_1, the probability that A's wins the contest is $F(\delta_1 + \mu_2^A - \mu_2^B)$. Player A chooses μ_2^A to maximize

$$rF(\delta_1 + \mu_2^A - \mu_2^B) - C(\mu_2^A),$$

where B's effort μ_2^B is taken as given. The first-order condition for A is therefore

$$C'(\mu_2^A) = rf(\delta_1 + \mu_2^A - \mu_2^B).$$

Switching the roles of A and B, we get the first-order condition for B. Since the random variable ϵ_2 has symmetric density function f, the two players exert the same second-half effort μ_2 in equilibrium. The common first-order condition is then (the second-order condition is assumed to be satisfied):

$$C'(\mu_2) = rf(\delta_1).$$

Since μ_2 depends both on the first-half score difference δ_1, and on the choice of r, we write it as $\mu_2(\delta_1, r)$. For any r, $\mu_2(., r)$ is symmetric around 0 because f is.

The state of the game at the beginning of the second half is entirely summarized by δ_1, so we can write the continuation payoff of each player at the beginning of the second half as

$$v(\delta_1) = rF(\delta_1) - C(\mu_2(\delta_1, r)) + l.$$

Taking derivative and using the first-order condition for μ_2, we have

$$v'(\delta_1) = rf(\delta_1)\left(1 - \frac{\partial \mu_2}{\partial \delta_1}(\delta_1, r)\right).$$

Intuitively, a greater first-half effort by either side not only increases the chance of winning but also affects the cost of effort in the second half. In particular, a large and positive score difference δ_1 reduces the equilibrium second-half effort cost by discouraging the opponent from exerting effort in the second half. To see this, take derivative of the first-order condition in the second half,

$$\frac{\partial \mu_2}{\partial \delta_1}(\delta_1, r) = \frac{rf'(\delta_1)}{C''(\mu_2)}.$$

Since C is convex, under our assumption that f is single-peaked at $\delta_1 = 0$, the sign of $\partial\mu_2/\partial\delta_1$ is determined by f', the slope of the density function: $\partial\mu_2/\partial\delta_1$ is positive if $\delta_1 < 0$ and negative if $\delta_1 > 0$. This means that the second-half equilibrium effort increases if the score difference δ_1 gets closer to 0 and decreases if δ_1 drifts away from 0. Moreover, the continuation function v for a player increases faster with his effort when he is leading than when he is falling behind (that is, $v'(\delta_1) > v'(-\delta_1)$ for any $\delta_1 > 0$.)[4]

In the first half, player A chooses μ_1^A to maximize

$$\int v(\mu_1^A - \mu_1^B + \epsilon_1)f(\epsilon_1)d\epsilon_1 - C(\mu_1^A),$$

where $\mu_1^A - \mu_1^B + \epsilon_1$ represents the random score difference δ_1 at the end of the first half, and μ_1^B is taken as given. In the symmetric equilibrium, both players exert the same effort μ_1 in the first half, which satisfies the following first-order condition (the second-order condition is again assumed to be satisfied):

$$C'(\mu_1) = \int v'(\epsilon_1)f(\epsilon_1)d\epsilon_1.$$

From the symmetry of $\mu_2(., r)$ it follows that the first-order condition for the first half can be simplified as

$$C'(\mu_1) = \int rf^2(\epsilon_1)d\epsilon_1.$$

[4] Even though $\partial\mu_2/\partial\delta_1$ is positive for $\delta_1 < 0$, it is impossible that $v'(\delta_1)$ is negative for some large negative score difference δ_1, if the second-order condition with respect to μ_2 is satisfied. From the second-order condition, we have $\partial\mu_2/\partial\delta_1 \leq 1$ for any δ_1, and it follows that $v'(\delta_1) \geq 0$ for any δ_1.

Thus, the first-half effort is chosen as in a static game with a noise term equal to the sum of the noise in the two halves. Given that the two players exert the same effort μ_1 in the first half, the equilibrium score difference δ_1 is a random variable with the distribution function F. In what follows, we continue to write $\mu_2(\delta_1, r)$ instead of $\mu_2(\epsilon_1, r)$, to stress that μ_2 depends on δ_1, even though in equilibrium δ_1 is equal to ϵ_1.[5] Equilibrium dynamics of efforts are characterized in the next two results.

LEMMA 1. *In a rank order scheme, the second half involves greater efforts by both players than in the first half if the first-half score difference is small, and lower efforts if it is large.*

PROOF: Compare the first-order conditions $C'(\mu_1) = \int rf^2(\epsilon_1)d\epsilon_1$ and $C'(\mu_2) = rf(\delta_1)$. Since $\mu_2(\delta_1, r)$ is symmetric around and single-peaked at $\delta_1 = 0$, so is $C'(\mu_2(\delta_1, r))$. Since

$$C'(\mu_1) = \int C'(\mu_2(\delta_1, r))f(\delta_1)d\delta_1,$$

there exists some $d > 0$ such that $C'(\mu_1) > C'(\mu_2(\delta_1, r))$, and hence $\mu_1 > \mu_2(\delta_1, r)$, if and only if $\delta_1^2 < d^2$. *Q.E.D.*

Although δ_1 is completely random in equilibrium, μ_1 and μ_2 can be compared on average terms. The difference in expected levels of efforts in the two halves of a rank order scheme is summarized in the following result.

LEMMA 2. *In a rank order scheme, the second half is on average less exciting than the first half.*

PROOF: The first-order conditions for the two halves imply that,

$$C'(\mu_1) = \int C'(\mu_2(\delta_1, r))f(\delta_1)d\delta_1.$$

[5] Later we will show that under any linear score difference scheme, the first half score difference is also equal to the random noise. Thus, which scheme is used does not directly affect the score difference. Our main theorem that rank order schemes dominate linear score difference schemes when spectators care about suspense sufficiently does not result because rank order schemes induce closer scores. However, when the game is modeled with more than two periods, the score difference is no longer pure noise under a rank order scheme. See the discussion in section 5.

Under Assumption 1, $C''' \geq 0$, and so C' is convex.

$$C'(\mu_1) \geq C'\left(\int \mu_2(\delta_1, r))f(\delta_1)d\delta_1\right),$$

which implies that $\mu_1 \geq \int \mu_2(\delta_1, r)f(\delta_1)d\delta_1$. *Q.E.D.*

The incentive designer chooses the rank order scheme (r, l) to maximize "profits" per contestant:

$$\max_{r,l} \; P_1\mu_1 + \int P_2\mu_2(\delta_1, r)f(\delta_1)d\delta_1 - \left(l + \frac{1}{2}r\right)$$

subject to the participation constraint

$$l + \frac{1}{2}r - C(\mu_1) - \int C(\mu_2(\delta_1, r))f(\delta_1)d\delta_1 \geq \underline{U},$$

where μ_k, $k = 1, 2$, are equilibrium efforts defined above as functions of r. Since the two conditions depend only on the choice of r, the optimization problem is solved by first choosing an r, which determines μ_1 and $\mu_2(\delta_1, r)$, and then choosing l to bind the participation constraint. The necessary first-order condition with respect to r is[6]

$$\Big(P_1 - C'(\mu_1)\Big)\frac{d\mu_1}{dr} + \int \Big(P_2 - C'(\mu_2(\delta_1, r))\Big)\frac{\partial \mu_2}{\partial r}(\delta_1, r)f(\delta_1)d\delta_1 = 0.$$

Assumption 1 ensures that the second-order condition of the designer's problem is satisfied.[7]

Next, we consider the optimal linear score difference incentive scheme. A score difference scheme rewards players on the basis of the score difference at the end of the game. The simplest such scheme has two parameters: the fixed transfer t, which can be either

[6] Clearly, profits initially increase when r is close to zero, and eventually decrease when r is sufficiently high.

[7] To see this, write the objective function as the difference between revenue $i(r)$ and cost $k(r)$, where $i(r) = P_1\mu_1 + \int P_2\mu_2(\delta_1, r)f(\delta_1)d\delta_1$, and $k(r) = \underline{U} + C(\mu_1) + \int C(\mu_2(\delta_1, r))f(\delta_1)d\delta_1$. The first-order condition for the optimal r is $i'(r) = k'(r)$, and the second order condition is $i''(r) < k''(r)$. A sufficient condition for the second-order condition is that $i''(r) \leq 0$ and $k''(r) \geq 0$ for all r, with at least one strict inequality. Using the first-order conditions for the equilibrium efforts μ_1 and $\mu_2(\delta, r)$, we can show that under the assumption that $C''' \geq 0$, these efforts are weakly concave in r, and so $i''(r) \leq 0$. Similarly, the assumption that $C''' \leq (C'')^2/C'$ implies that the effort cost in each half as a function of r is weakly convex, and so $k''(r) \geq 0$.

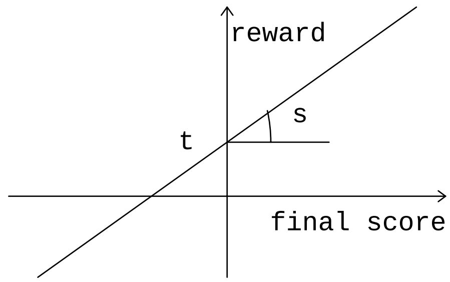

Figure 2. A linear score difference scheme

positive or negative, and a piece rate s. If the final score difference is δ_2, then A's reward is $t + s\delta_2$ and B's reward is $t - s\delta_2$. See Figure 2 for an illustration.

Lemma 3. *Under linear score difference schemes, the level of effort is the same for the two players and for the two halves, and independent of the first-half score difference.*

Proof: Again, we work backwards to find players' equilibrium response to a particular linear score difference scheme (t, s). Given the score difference δ_1 at the beginning of the second half, player A chooses μ_2^A to maximize

$$\int (t + s(\mu_2^A - \mu_2^B + \epsilon_2 + \delta_1)) f(\epsilon_2) d\epsilon_2 - C(\mu_2^A),$$

where μ_2^B is taken as given. Given symmetry, the common second-half effort μ_2 satisfies the first-order condition (the second-order condition is clearly satisfied):

$$C'(\mu_2) = s.$$

The equilibrium effort level μ_2 is a constant determined entirely by the piece rate s.

The continuation payoff of each player at the beginning of the second half as a function of δ_1 is

$$v(\delta_1) = t + s\delta_1 - C(\mu_2).$$

In the first half, anticipating this continuation payoff, player A chooses μ_1^A to maximize

$$\int v(\mu_1^A - \mu_1^B + \epsilon_1) f(\epsilon_1) d\epsilon_1 - C(\mu_1^A),$$

where $\mu_1^A - \mu_1^B + \epsilon_1$ represents the random score difference δ_1 at the end of the first half, and μ_1^B is taken as given. In equilibrium both players exert the same effort μ_1 in the first half, which satisfies the following first-order condition (the second-order condition is satisfied):

$$C'(\mu_1) = \int v'(\epsilon_1) f(\epsilon_1) d\epsilon_1.$$

Since $v' = s$, the first-order condition for the first half coincides with that for the second half. *Q.E.D.*

A constant level of effort μ, determined by $C'(\mu) = s$, is exerted by the two players throughout the game. Given this, the designer's profit maximization problem for the optimal linear score difference scheme is

$$\max_{t,s} \ (P_1 + P_2)\mu - t$$

subject to the participation constraint

$$t - 2C(\mu) \geq \underline{U}.$$

The optimal piece rate s is given by $\frac{1}{2}(P_1 + P_2)$, and the fixed transfer t binds the participation constraint.

By Lemma 1 and Lemma 2, a rank order scheme on average gets boring in the second half, but can become exciting when the game is close at the end of the first half. In contrast, Lemma 3 states that under a linear score difference scheme, players keep up the same level of effort regardless of whether the game is close or lopsided after the first half. How do the two schemes compare if both are chosen optimally?[8]

Proposition 1. *When spectators care only about the excitement of the game, rank order schemes are inferior to linear score difference schemes.*

[8] The proof remains valid if Assumption 2 is replaced by the weaker condition $P_1 \leq P_2$. By Lemma 2, if P_1 is sufficiently greater than P_2, then the result of Proposition 1 is reversed. We ignore this possibility as it seems unreasonable in sports tournaments.

PROOF: Let (r, l) be the optimal rank order scheme, and let μ_1 and $\mu_2(\delta_1, r)$ be the equilibrium efforts. Define $\mu = \frac{1}{2}(\mu_1 + \int \mu_2(\delta_1, r) f(\delta_1) d\delta_1)$. Then, since C is convex,

$$C(\mu_1) + \int C(\mu_2(\delta_1, r)) f(\delta_1) d\delta_1 \geq C(\mu_1) + C\left(\int \mu_2(\delta_1, r) f(\delta_1) d\delta_1\right) \geq 2C(\mu).$$

From Assumption 2,

$$P_1 \mu_1 + P_2 \int \mu_2(\delta_1, r) f(\delta_1) d\delta_1 = (P_1 + P_2)\mu.$$

Define $s = C'(\mu)$. Then a score difference scheme with s induces μ, with a lower effort cost to the players and the same revenue to the designer. Define $t = \underline{U} + 2C(\mu)$, then (t, s) generates greater profits than (r, l). *Q.E.D.*

The result that linear score difference schemes dominate rank order schemes is related to Holmstrom and Milgrom's (1987) theory of linear incentive contracts in a principal-agent model. They show that in a dynamic environment in which the agent can adjust his efforts according to commonly observed history of output, the principal can do no better than making the payment conditional only on some aggregated output measure.[9] In particular, the two-wage payment schemes discovered by Mirrlees (1974) to approximate the first best do not work well because the agent can game such schemes by conditioning his efforts on the output path. Proposition 1 can be viewed as an extension of Holmstrom-Milgrom's result to multiple agents. Rank order schemes correspond to the two-wage payment schemes of Mirrlees, while linear score difference schemes correspond to linear contracts of Holmstrom and Milgrom.

4. Excitement and Suspense

The inferiority of rank order schemes to score difference schemes is puzzling because most sports competitions adopt rank order schemes. The answer to this "puzzle" is suggested

[9] The result of Holmstrom and Milgrom relies on their assumption that there is no wealth effect in the agent's utility function. Also, in their model the principal can condition payment to the agent on the entire history of the output, while in our model the incentive designer is restricted to rewarding the players according to the final score difference.

by the unique feature in sports that spectators care about the dynamics of the game. We model this by assuming that spectators value player's efforts more when the game is closer. Formally, we assume that P_2 depends on δ_1. In particular, $P_2(\delta_1)$ is symmetric around and single-peaked at $\delta_1 = 0$ (tied first half). This modification of spectators' preference captures the idea that spectators enjoy both excitement and suspense. Spectators do not just care about excitement in terms of great efforts: a lop-sided game bores them even when the losing side keeps up the effort. On the other hand, spectators do not just care about suspense in terms of close games: they do not like it when the leading player slacks off even though it makes the game close. We show in this section that rank order schemes perform better than score difference schemes when spectators have a strong enough preference for suspense.

We capture the concept of increasing demand for suspense as follows. We say that the demand for suspense (with respect to the chance in the game, represented by f) is greater under $P_2(\delta_1)$ than under $\tilde{P}_2(\delta_1)$ if (i) $\int (P_2(\delta_1) - \tilde{P}_2(\delta_1)) f(\delta_1) d\delta_1 = 0$, and (ii) there exists $\alpha > 0$ such that $P_2(\delta_1) - \tilde{P}_2(\delta_1) > 0$ if and only if $\delta_1^2 < \alpha^2$. Intuitively, $P_2(\delta_1)$ is more "concentrated" (with respect to f) than $\tilde{P}_2(\delta_1)$, in the sense that the two functions have the same expectation under density f, but the value of P is larger for close games (middle values of δ_1) and smaller when a player has acquired a strong lead (more extreme values of δ_1 in either direction). Whenever necessary, we further simplify the analysis by considering functions P_2 indexed by a one-dimensional parameter a, such that, with a slight abuse of notation, the demand for suspense is greater under $P_2(\delta_1, a)$ than under $P_2(\delta_1, \tilde{a})$ when $a > \tilde{a}$. In this case, we will say that spectators' demand for suspense increases when a increases, and rewrite condition (ii) above as: (iii) there exists a function $\alpha(a)$ such that $\partial P_2(\delta_1, a)/\partial a > 0$ if and only if $\delta_1^2 < \alpha^2(a)$.[10]

Increasing demand for suspense does not change the design of linear difference scheme. Since the two players exert the same effort in the two halves regardless of the score dif-

[10] Note that under condition (i), a sufficient for condition (iii) is that $\partial P_2(\delta_1, a)/\partial a$ is symmetric around and single-peaked at $\delta_1 = 0$. However, neither symmetry nor single-peak of $\partial P_2(\delta_1, a)/\partial a$ is necessary for the definition of increasing demand for suspense. Also, condition (ii) implies condition (iii), but not vice versa: in a class of $P_2(\delta_1, a)$ ranked by a which satisfies condition (iii), there can be two functions $P_2(\delta_1, a')$ and $P_2(\delta_1, a'')$ that intersect with each other more than twice, and therefore violate condition (ii).

ference, the optimal piece rate s depends only on the expectation of $P_2(\delta_1)$, which does not change. The fixed transfer t that binds the player's participation constraint is also unchanged.

In contrast, intuition suggests that the optimal rank order scheme should change as spectators' demand for suspense increases. As $P_2(\delta_1)$ becomes more concentrated around $\delta_1 = 0$, the designer will want to make $\mu_2(\delta_1, r)$ also more concentrated in order to take advantage of the fact that spectators have a greater demand for suspense. How can this be achieved? From the equilibrium condition for second-half effort μ_2, we see that increasing r will increase the whole function of $\mu_2(\delta_1, r)$. But since the density function $f(\delta_1)$ is uni-modal, the increase in μ_2 will be more pronounced around $\delta_1 = 0$. Thus, as $P_2(\delta_1)$ becomes more concentrated around $\delta_1 = 0$, the designer will want to increase r. This intuition is confirmed in the following result.

LEMMA 4. *As demand for suspense increases, the incentive prize under the optimal rank order scheme increases and the optimal profits also increase.*

PROOF: From the equilibrium condition of second-half effort μ under rank order scheme, $C'(\mu_2) = rf(\delta_1)$, we find that

$$\frac{\partial \mu_2}{\partial r}(\delta_1, r) = \frac{f(\delta_1)}{C''(\mu_2(\delta_1, r))}.$$

Since both $f(\delta_1)$ and $\mu_2(\delta_1, r)$ are symmetric around $\delta_1 = 0$, $\partial \mu_2(\delta_1, r)/\partial r$ is also symmetric. Furthermore,

$$\frac{\partial^2 \mu_2}{\partial r \partial \delta_1}(\delta_1, r) = f'(\delta_1)\frac{(C'')^2 - C'''C'}{(C'')^3}.$$

Under Assumption 1, $\partial \mu_2(\delta_1, r)/\partial r$ is also single-peaked around $\delta_1 = 0$.

With P_2 as a function of δ_1 and indexed by a, the first-order condition with respect to r in the optimal design problem of rank order schemes becomes

$$(P_1 - C'(\mu_1))\frac{d\mu_1}{dr} + \int (P_2(\delta_1, a) - C'(\mu_2(\delta_1, r)))\frac{\partial \mu_2}{\partial r}(\delta_1, r) f(\delta_1) d\delta_1 = 0.$$

Taking derivatives of the above condition with respect to a, we find that, dr/da, the effect of increasing demand for suspense, has the same sign as

$$\int \frac{\partial P_2}{\partial a}(\delta_1, a)\frac{\partial \mu_2}{\partial r}(\delta_1, r) f(\delta_1) d\delta_1.$$

By condition (i) of increasing demand for suspense, $\int (\partial P_2(\delta_1, a)/\partial a) f(\delta_1) d\delta_1 = 0$. Then, for any constant α we can write the above integral as

$$\int \frac{\partial P_2}{\partial a}(\delta_1, a) \left(\frac{\partial \mu_2}{\partial r}(\delta_1, r) - \frac{\partial \mu_2}{\partial r}(\alpha, r) \right) f(\delta_1) d\delta_1.$$

By condition (iii) of the definition of increasing demand for suspense, we can choose $\alpha > 0$ such that $\partial P_2(\delta_1, a)/\partial a$ is positive for all $\delta_1 \in (-\alpha, \alpha)$, and negative for any $\delta_1 < -\alpha$ or $\delta_1 > \alpha$. We have shown that $\partial \mu_2(\delta_1, r)/\partial r$ is symmetric around and single-peaked at $\delta_1 = 0$. Then, the above integral is positive both for $\delta_1 < -\alpha$ and for $\delta_1 > \alpha$, because $\partial P_2(\delta_1, a)/\partial a < 0$ and $\partial \mu_2(\delta_1, r)/\partial r < \partial \mu_2(\alpha, r)/\partial r$. The integral from $-\alpha$ to α is also positive because $\partial P_2(\delta_1, a)/\partial a > 0$ and $\partial \mu_2(\delta_1, r)/\partial r > \partial \mu_2(\alpha, r)/\partial r$. It follows that $dr/da > 0$.

By the envelope theorem, the change in the value of the objective function under the optimal rank order scheme has the same sign as

$$\int \frac{\partial P_2}{\partial a}(\delta_1, a) \mu_2(\delta_1, r) f(\delta_1) d\delta_1.$$

We know that $\mu_2(\delta_1, r)$ is symmetric and single-peaked, just like $\partial \mu_2(\delta_1, r)/\partial r$. By a similar argument as above, the above integral is positive, and therefore the value of the objective function under the optimal rank order scheme increases. *Q.E.D.*

Similar comparative statics about the design and the profits of the optimal rank order scheme can be carried out with respect to the density function f of the noise in the game. A more concentrated f represents an environment of sports competition that is less susceptible to pure luck of players, and therefore more responsive to their efforts in the game. Comparative statics with respect to the role of chance is interesting, because characteristics of a sports game can be modified, and indeed they often have been in the history of the game, when changes occurred to the rule of the game, training technology for athletes, or equipment used in the game. Formally, we can define "diminishing role of chance" in the game as follows. Let the density function f of the noise be indexed by a one-dimensional parameter b, such that there exists a function $\beta(b)$ with $\partial f(\delta_1, b)/\partial b > 0$ if and only if $\delta_1^2 < \beta^2(b)$. This condition means that f becomes more concentrated for middle values of δ_1.[11] Intuitively, when f becomes more concentrated, the game is more likely to be closer given any effort levels of the two players, and the incentive designer should respond by increasing the incentive prize, in the same way as when P_2 becomes more concentrated. Indeed, the proof of Lemma 4 can be directly extended to show that with a diminishing role of chance (that is, as b increases), the incentive designer increases the incentive prize and the profits under the optimal scheme also increase.[12] Thus, diminishing role for chance has the same effects on the design and the profits of the optimal scheme as increasing demand for suspense.[13]

In our model, performance measurement errors decrease the likelihood that the score will stay close in the second half and reduce the utility of the spectators. In the standard principal-agent moral hazard literature, measurement errors increase the risk premium of the agent. In both cases, measurement errors decrease profits by hampering the working of incentive contracts.

To resolve the puzzle raised by Proposition 1, we still need to show that when spectators care enough about suspense in the game, the optimal rank order scheme eventually dominates the optimal linear score difference scheme. We establish this result indirectly by first characterizing conditions under which a rank order scheme achieves the first best outcome.

LEMMA 5. *There is a rank order scheme that achieves the first best efforts in both halves if and only if $f(\delta_1)/P_2(\delta_1)$ is constant for all δ_1.*

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[11] Note that since $f(\delta_1, b)$ is a density function, implicitly we have $\int(\partial f(\delta_1, b)/\partial b)d\delta_1 = 0$ for any b.

[12] The argument uses the following general result invoked by Lemma 4. Let f be any density, and suppose two functions p and q are symmetric and single-peaked with p more concentrated than q with respect to f. Then, for any symmetric g with the same peak, $\int p(x)g(x)f(x)dx > \int q(x)g(x)f(x)dx$.

[13] There is an important difference between the comparative statics with respect to P_2 and with respect to f. In the case of increasing demand for suspense, the design and the profits of the optimal linear score difference schemes are not affected, and therefore the relative advantage of rank order schemes emerges. In the case of diminishing role of chance, one can show that the optimal piece rate s in a linear score difference scheme is given by $\frac{1}{2}(P_1 + \int P_2(\delta_1)f(\delta_1, b)d\delta_1)$, which increases with b. Similarly, the effect of increasing b on the profits under the optimal linear score difference scheme has the same sign as $\mu(s)\int P_2(\delta_1)(\partial f(\delta_1, b)/\partial b)d\delta_1$ (where $\mu(s)$ is defined by $C'(\mu(s)) = s$), which can be shown to be positive. Thus, with diminishing role of chance, performance is improved under both the optimal rank order difference scheme and the optimal linear score difference scheme. The net effect on the comparison of the two schemes is generally ambiguous. Note that in the above comparative statics exercise with respect to f, Assumption 2 is no longer satisfied.

PROOF: The first best efforts maximize the difference between the revenue

$$P_1\mu_1 + \int P_2(\delta_1)\mu_2(\delta_1)f(\delta_1)d\delta_1$$

and the cost

$$\underline{U} + C(\mu_1) + \int C(\mu_2(\delta_1))f(\delta_1)d\delta_1.$$

The first-order conditions for the first best efforts μ_1^* and $\mu_2^*(\delta_1)$ are therefore

$$C'(\mu_1^*) = P_1,$$

and

$$C'(\mu_2^*(\delta_1)) = P_2(\delta_1).$$

Suppose that there exists a constant k such that $P_2(\delta_1) = kf(\delta_1)$ for all δ_1. Consider a rank order scheme with a prize $r = k$. The first-order condition for the second-half equilibrium level of effort coincides with the condition for the first best $\mu_2^*(\delta_1)$. Under Assumption 2, the first-order condition for the first-half equilibrium level of effort coincides with the condition for μ_1^*. The first best outcome is achieved.

To prove the reverse, suppose there exist δ_1 and δ_1' with $f(\delta_1)/P_2(\delta_1) \neq f(\delta_1')/P_2(\delta_1')$. Compare the first-order condition for the second-half equilibrium level of effort under a rank order scheme, and the condition for $\mu_2^*(\delta_1)$. There exists no prize r such that the first best second-half effort is achieved both at δ_1 and δ_1'. *Q.E.D.*

The role of Assumption 2 is now apparent. When the assumption is not satisfied, no reward schemes based on final score difference, including rank order schemes and linear score difference schemes, can implement the first best efforts. This follows because the first-order conditions for the first best first-half efforts and the second-half efforts cannot be satisfied at the same time. Essentially, the problem is that we assume that payoffs for the players depend only on the final score difference. Under such schemes the incentives for the first half and for the second half are directly linked. When Assumption 2 is not satisfied, such link becomes a binding restriction on what can be achieved under a reward scheme based on the final score difference.

The proof of Lemma 5 implies that the optimal linear score difference scheme induces the first best efforts if and only if P_2 is constant and equal to P_1. In this case, the optimal linear score difference scheme dominates any rank order schemes. This conclusion is unchanged if we assume that each player's score in either half is determined by his effort in that half plus some noise that is independent across players, and allow the designer to condition reward on overall individual scores instead of on the score difference. Thus, we have verified in our model Holmstrom's (1982) result that tournaments have no intrinsic value in providing incentives in a team production problem, if individual output can be measured and measurement errors are independent. This reasoning no longer applies if P_2 depends on δ_1, because the objective function of the designer directly involves comparisons of individual performance.

According to Lemma 5, when the noise in the game has the same distribution (after proper rescaling) as the spectators' preference for suspense, the first best efforts can be implemented by a rank order scheme. In this case, rank order schemes dominate linear score difference schemes. The next proposition shows that this is true as long as spectators' demand for suspense is sufficiently high. Consider the problem of designing the optimal rank order scheme, for a given preference function $P_2(\delta_1)$. For simplicity, we assume that the rescaled functions $P_2(\delta_1)/P_1$ and $f(\delta_1)/(\int f^2(x)dx)$ intersect exactly twice, at d and $-d$.[14] For the following result, we say that the demand for suspense is high relative to the chance in the game, if $P_2(\delta_1)/P_1 > f(\delta_1)/(\int f^2(x)dx)$ when $\delta_1^2 < d^2$ (i.e., if $P_2(\delta_1)$ is more concentrated than $f(\delta_1)$ after proper rescaling.)

PROPOSITION 2. *If spectators' demand for suspense is high relative to the chance in the game, then the optimal rank order scheme dominates linear score difference schemes.*

PROOF: Consider a class of spectator preference functions $P_2(\delta_1, a)$ indexed by a, given by

$$P_2(\delta_1, a) = \frac{P_1 f(\delta_1)}{\int f^2(x)dx} + a\left(P_2(\delta_1) - \frac{P_1 f(\delta_1)}{\int f^2(x)dx}\right).$$

[14] It can be seen from the proof below that the result of Proposition 2 goes through as long as we can find a class of preference functions $P_2(\delta_1, a)$ of increasing demand for suspense, such that for some $a_1 > a_2$ we have $P_2(\delta_1, a_1) = P_2(\delta_1)$ and $P_2(\delta_1, a_2) = P_1 f(\delta_1)/(\int f^2(x)dx)$.

By construction, $\int (\partial P_2(\delta_1, a)/\partial a) f(\delta_1) d\delta_1 = 0$, so condition (i) of increasing demand for suspense is satisfied. By assumption, $\partial P_2(\delta_1, a)/\partial a$ is positive if and only if $\delta_1^2 < d^2$ for all a, so condition (iii) of increasing demand for suspense is also satisfied. Since $P_2(\delta_1, 0)$ is proportional to $f(\delta_1)$, Lemma 5 implies that at $a = 0$, the optimal rank order scheme dominates linear score difference schemes. Lemma 4 then implies that the optimal rank order scheme continues to dominate linear score difference schemes when $a = 1$. But this is precisely what we need, because $P_2(\delta_1, 1) = P_2(\delta_1)$ by construction. *Q.E.D.*

Proposition 2 can be strengthened. The continuity of the profits under the optimal rank order scheme in a implies that the conclusion of Proposition 2 can hold even if spectators' demand for suspense is "a little" lower than the chance in the game. More precisely, since when $P_2(\delta_1)$ is proportional to $f(\delta_1)$, the optimal rank order scheme implements the first best efforts and therefore dominates linear score difference schemes, if the given $P_2(\delta_1)$ is just a little less concentrated than $f(\delta_1)$, the optimal rank order scheme still dominates linear score difference schemes. Thus, for a given environment of sports competition with a role of chance represented by $f(\delta_1)$, there is a range of spectators' demand for suspense under which the optimal rank order scheme dominates linear score difference schemes.

By Proposition 2, as long as spectators' demand for suspense is sufficiently high, rank order schemes out-perform linear score difference schemes. Starting from the case where $P_2(\delta_1)$ and $f(\delta_1)$ are proportional, rank order schemes dominate when $P_2(\delta_1)$ becomes more concentrated, or alternatively, when the random variable ϵ_2 has a wider distribution. In these situations, probability of outcome reversal is significant even for large first-half score differences. Since spectators do not care about effort in the second half when the game becomes lop-sided by the end of the first half, rank order schemes provide less incentives (and more appropriate incentives) for players than linear score difference schemes. Though dominating linear score difference schemes, rank order schemes do not induce the first best efforts. The noise distribution is too spread out relative to spectators' preference function, and even rank order schemes fail to limit players' incentives in the second half when the game becomes lop-sided at the end of the first half.

The following explicit example helps to understand the comparison between linear score difference schemes and rank order schemes. Let the cost function $C(\mu)$ be given by

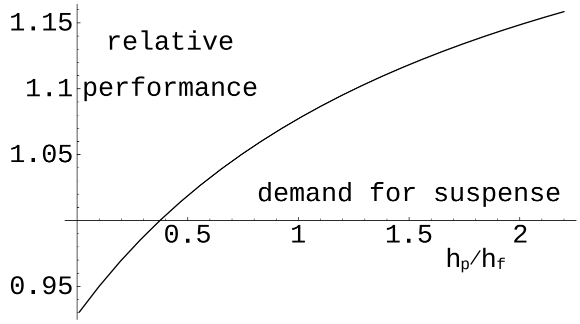

Figure 3. Relative performance: rank order and linear score difference

$\frac{1}{2}\mu^2$, and the reservation utility $\underline{U}$ be 0. Suppose that the noise term ϵ_k in each half k is normally distributed, with mean 0 and precision h_f. Let $P_1 = \int P_2(\delta_1) f(\delta_1) d\delta_1 = P$, and assume that rescaled P_2 is a normal density function, with mean 0 and precision h_p. This implies that

$$P_2(\delta_1) = P\sqrt{1+\frac{h_p}{h_f}}\exp\left(-\frac{1}{2}h_p\delta_1^2\right).$$

Using the analysis in section 3, we can show that the optimal linear difference scheme has a piece rate $s = P$, with profit per contestant P^2. On the other hand, the optimal rank order scheme has an incentive prize r given by

$$r = P(4\sqrt{3}-6)\sqrt{\frac{\pi}{h_f}}\left(1+\sqrt{1+\frac{1}{1+2h_f/h_p}}\right).$$

As shown more generally in Lemma 4, r increases with the precision h_p relative to h_f, that is, with greater demand for suspense relative to the role of chance. The profit per contestant under the optimal rank order scheme is given by

$$P^2\left(\sqrt{3}-\frac{3}{2}\right)\left(1+\sqrt{1+\frac{1}{1+2h_f/h_p}}\right)^2.$$

Figure 3 illustrates how the profit increases as demand for suspense increases: the optimal rank order scheme under-performs the optimal linear score difference scheme by about 7.7 percent when h_p/h_f is zero, catches up when h_p/h_f is roughly 0.37, out-performs by about 7.7 percent when $h_p = h_f$, and eventually out-performs by roughly 35 percent when h_p/h_f is sufficiently high.

5. Discussions

We have shown in Lemma 5 that if the density of the noise f is proportional to the preference P_2 for suspense, then a rank order scheme implements the first best efforts. An interesting question is what the optimal incentive scheme looks like when this proportionality condition is not satisfied. Answering this question extends our analysis beyond the comparison of rank order schemes and linear score difference schemes, and helps us understand the nature of the design problem in our setup. To begin, note that the two incentive schemes we considered (rank order and linear score difference) are both functions only of the final score difference δ_2. Whereas the reward function in a rank order scheme has only two values with a discontinuity at 0, the reward function in a linear score difference scheme is, by definition, linear. Thus, it is natural to consider nonlinear score difference schemes, with a reward function n that depends on the final score difference δ_2 in an arbitrary way. For analytical convenience, we restrict our attention to functions $n(\delta_2)$ that are differentiable, with symmetric derivatives n'. Then, following the same arguments as those in Lemma 5, we can show that a nonlinear score difference scheme $n(\delta_2)$ implements the first best efforts in both halves if and only if for all δ_1,

$$\int n'(\delta_1 + \epsilon_2) f(\epsilon_2) d\epsilon_2 = P_2(\delta_1).$$

If we think of the functions n', f and P_2 as density functions for independent random variables X, Y and Z, respectively, then the above condition means that n' is such that the random variable Z is given by $X - Y$.[15]

The above analysis suggests that under some special forms of the density function f and the preference function $P_2(\delta_1)$, explicit forms of the nonlinear scheme that induces the first best efforts can be found. For example, suppose that both f and P_2 (after proper rescaling) are normal, with mean 0 and precision h_f and h_p respectively. Then, the nonlinear scheme n achieves the first best if n' is proportional to the normal density

[15] Suppose that independent random variables X and Y have density functions f_x and f_y. Then, since X and Y are independent, the probability of the event $x - y \leq z$ is $\int \int_{x \leq z+y} f_x(x) dx f_y(y) dy$. Thus, the density function of the random variable $Z = X - Y$ is $\int f_x(z+y) f_y(y) dy$.

function with mean 0 and precision $(h_p^{-1} - h_f^{-1})^{-1}$, as long as $h_p < h_f$. For any fixed h_f, as h_p increases to h_f, the optimal nonlinear reward function n converges to a rank order scheme. In the limiting case of $h_p = h_f$, the rank order scheme achieves the first best efforts. This result serves as a special case of Proposition 2. If $h_p > h_f$, there is no reward function n that achieves the first best. In this case the designer wants to reduce the incentives for continuing second-half efforts to minimum when the first-half score difference is sufficiently large. Intuition suggests that the second best nonlinear scheme is a rank order scheme, but more assumptions on the noise distribution and the cost function may be necessary to validate the intuition.

In this paper we have assumed that the game is divided into two halves and players choose efforts simultaneously at the beginning of each half. Ideally one would like to study a model where efforts are continuously adjusted as the game proceeds. In such a model, equilibrium efforts under rank order schemes can be asymmetric between the two players as well as history-dependent, even if we maintain the assumption of symmetry among the players in terms of the effort cost function. To see this, consider a variation of our two-period model where there is handicap score δ_0 for one player in the beginning of the first half. Second-half efforts are characterized as before and continue to be symmetric. If the symmetry in the first-half efforts still holds, from the first order condition we must have

$$\int v'(\epsilon_1 + \delta_0) f(\epsilon_1) d\epsilon_1 = \int v'(\epsilon_1 - \delta_0) f(\epsilon_1) d\epsilon_1.$$

The above condition will not hold because v' is not symmetric. More precisely, using the equilibrium expression of v', and noting that the first part of v' is symmetric, we find that a necessary condition for equilibrium first-half efforts μ_1^A and μ_1^B is given by

$$C'(\mu_1^B) - C'(\mu_1^A) = 2r \int \frac{\partial \mu_2}{\partial \delta_1}(\epsilon_1 + \delta_0 + \mu_1^A - \mu_1^B) f(\epsilon_1 + \delta_0 + \mu_1^A - \mu_1^B) f(\epsilon_1) d\epsilon_1.$$

When $\delta_0 = 0$, the above condition is satisfied if $\mu_1^A = \mu_1^B$. If $\delta_0 \neq 0$, symmetric solutions cannot obtain. Indeed, one can show that if $\delta_0 > 0$ so that A is leading B, then either μ_1^A is strictly greater than μ_1^B, or μ_1^A is less than μ_1^B by at least the score difference δ_0.[16] The

[16] The proof of this statement uses the following result: If f is symmetric around and single-peaked at 0 and g satisfies $g(-x) = -g(x) > 0$ for any $x > 0$, then $\int g(\delta + x) f(x) dx < 0$ if and only if $\delta > 0$. To see this, suppose $\delta > 0$. Rewrite the integral as the sum of $\int_{x \geq \delta} g(x)(f(x-\delta) - f(x+\delta))dx$ and $\int_{0 \leq x < \delta} g(x)(f(x-\delta) - f(x+\delta))dx$. Both parts are negative.

first scenario is the more reasonable one: if A is leading B, then starting from the position that A and B are exerting the same efforts, the marginal benefit of asserting additional efforts is greater for A than for B, because keeping the score difference to the final period saves efforts for both A and B while reducing the score difference forces greater efforts in the final period from both players.[17] But stronger conditions on the density function f and the cost function C are needed to exclude the other possibility of $\mu^A < \mu^B - \delta_0$, and to characterize the effort dynamics.

The extension of our basic two-half model in the present paper to multiple periods does not invalidate the conclusion that spectators' demand for suspense is a necessary ingredient in explaining why rank order schemes are the dominant form of rewarding schemes in sports competitions. But characterizing the effort dynamics in a multiple-period model under different rewarding schemes is an interesting question in its own right. We plan to pursue it in future research.

6. Summary

This paper answers a fundamental question in the economics of sports: why are rank order schemes the dominant form of incentive mechanisms used in sports? Our answer is that spectators of a sports event care about efforts of contestants when there is suspense in the outcome, not just the efforts per se. This conclusion is reached by considering a dynamic version of tournaments first studied by Lazear and Rosen (1981). When spectators care only about efforts in the sports game, we found that linear score difference schemes dominate rank order schemes, in the sense that the former induces greater effort with lower expected reward. But when we incorporated the preference for suspense, which type of scheme is better depends on how much spectators demand suspense. The more spectators enjoy suspense, the better rank order schemes perform relative to linear score difference schemes. When spectators' demand for suspense is sufficiently high, the optimal rank order scheme dominates linear score difference schemes.

[17] This scenario is consistent with Dixit's (1987) result that in a static tournament the favored team has incentives to over-commit efforts in order to preempt the other side.

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