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ABSTRACT

Economic Growth in an Interdependent World Economy*

We outline six facts that should be explained by an international growth model: 1) conditional convergence; 2) cross-country dispersion of growth rates; 3) cross-country dispersion of *per capita* income levels; 4) cross-country dispersion of savings rates; 5) within-country correlation of savings and investment and 6) cross-country equality of real rates of interest. We argue that the neoclassical model performs poorly in several dimensions and we provide an alternative two-sector endogenous growth model based on the work of Lucas and Romer that can account for all of our stylized facts. Our model accounts for the observation that poor countries grow faster than rich ones (fact number 1) as a consequence of the transitional dynamics of the ratio of physical to human capital. We show that opening capital mobility across countries does not necessarily equate the physical to human capital ratios across countries despite the resultant equalization of factor prices.

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1 Introduction

Over the last decade and a half there has been a resurgence of interest in the causes of economic growth. Theoretical work was fueled by the construction of a large panel data set, by Alan Heston and Robert Summers, and empirical work using the Heston-Summers data has provided a number of stylized facts that theoretical models of growth have sought to reproduce. These include:

1. Conditional convergence: (after controlling for government policies and education levels, poorer countries grow faster, on average, than richer countries).

2. There is a wide dispersion in cross-country growth rates.

3. Per capita income levels and per capita capital stocks differ across countries.

4. Savings and investment rates differ across countries for long periods of time.

5. National savings and domestic investment are highly correlated at both high and low frequencies.

6. Real interest rates are approximately equal across countries with similar institutional characteristics.

Facts 1, 2, and 3 are frequently cited by authors of work on growth (see Barro and Sala-i-Martin [2]), Facts 4 and 5 originate in the paper by Feldstein and Horioka [6] and evidence for Fact 6 is found in studies by Frankel [7], Harberger [8], Obstfeld [15] and Popper [16].[1]

Much of the theoretical work on international growth comparisons treats the world as a collection of closed economies, each evolving as a neoclassical growth model of the kind studied by Robert

[1] The studies by Frankel [7], Obstfeld [15] and Popper [16] look at the offshore-onshore return differentials on assets of the same currency and conclude that these differentials (both long term and short term) – which are indicators of departure from perfect capital mobility – were quite small. Harberger's [8] work is a summary of a slightly older literature which directly examined sectoral and aggregate rates of return on capital and concluded that the *ex post* differentials across countries were quite small.

Solow [20] and Trevor Swan [21]. The Solow-Swan model predicts that, after controlling for variables that influence the steady state, poor countries should grow faster than rich ones. This is because low initial income per capita is accounted for by a lack of physical capital and countries with the same fundamentals will asymptotically approach the same level of GDP per person. This is Fact 1 in our collection of stylized facts. The observation that growth rates are widely dispersed in the data (Fact 2) follows from the assumption that the world consists of a collection of economies that have not attained their balanced growth paths.

In the Heston-Summers data set, not all countries are converging to the same level of per capita income. This fact is consistent with the neoclassical model if one allows preferences or government policies to differ across countries since these variables influence savings rates. Investment equals savings because the economies are closed by assumption. Since long run per-capita incomes depend on investment rates, the closed economy model accounts for different per-capita income levels (Fact 3), different savings and investment rates, (Fact 4) and correlation of saving and investment (Fact 5) as a direct result of the closed economy characterization. Fact 6, (similar rates of return) however, is problematic when viewed in conjunction with Facts 3 (different per capita incomes) and 4 (different savings and investment rates). If countries do not converge in per capita income levels, there is no reason for them to have similar rates of return.

The convergence prediction of the neoclassical model has attracted considerable attention in the literature. Barro and Sala-i-Martin [2] have argued that after controlling for differences in rates of investment in human and physical capital, differences in government policies, and differences in a range of political and social variables, there is evidence for convergence in income levels across countries, across US states and across Japanese prefectures. Since the Barro-Sala-i-Martin regressions control for explanatory variables other than initial GDP per capita, the tendency of poor countries or regions to grow faster than rich ones is referred to as conditional convergence.

The evidence for conditional convergence is often cited as lending support to the neoclassical model. However, the predicted rate of income convergence in the data is extremely slow relative to that predicted by the model.[2] To address this issue, Mankiw, Romer and Weil [12] argue that one should reinterpret capital to include both physical and human capital. Using an adjusted measure, they claim that the elasticity of capital in production is of the order of 0.8 rather than the conventional measure of 0.3 that is based on income share data. This higher level of the capital elasticity enables these authors to reconcile the rate of convergence predicted by a closed economy neoclassical growth model with the observed rate of convergence in data.

Once one allows for trade in factors of production however, the Mankiw-Romer-Weil reinterpretation no longer helps since the neoclassical model with open capital markets predicts that convergence of per capita income levels should be instantaneous. In the absence of international capital mobility the rate of return should be higher in poorer countries since, relative poverty is accounted for by a lack of capital. Allowing for capital mobility across countries, one should observe instantaneous capital flows from richer to poorer countries as investors take advantage of arbitrage opportunities. This prediction is not backed up in the data. Nor does it help to invoke political risk or institutional factors since capital flows are low even amongst countries with different savings and investment rates, but similar characteristics and open capital markets such as those of the OECD. This evidence is particularly puzzling in the light of the fact that rates of return are approximately equalized internationally but per capita incomes are very different. It raises an obvious question: what is the mechanism that equates rates of return?[3]

Barro, Mankiw and Sala-i-Martin [1] address this issue by assuming that there are restrictions on borrowing and lending to finance human capital acquisition. This characterization of capital markets slows down the predicted speed of convergence but the model continues to predict large capital flows from rich to poor countries during the transition to the steady state. Moreover, in the

[2] King and Rebelo [9] have shown that this problem is due to the extremely high real interest rates that arise at low per capita capital and output levels when there are diminishing returns to capital.

[3] Jaume Ventura [22] has pointed out that the two-sector neoclassical model can rationalize the fact that rates of returns are equalized across countries since trade in factors and trade in goods are substitutes within the cone of diversification. But the two-sector neoclassical model still has difficulty with the fact that per capita incomes differ substantially across countries since trade in commodities should also equalize per capita incomes.

data, savings and investment rates differ markedly and there is no tendency for them to converge. In optimizing versions of the neoclassical model it is typical to assume that a country is represented by an agent with a rate of time preference that is common across countries. A more sophisticated version of the neoclassical model allows for an endogenous rate of time preference that depends on wealth. In this version, patient countries save more and become wealthier, but increasing wealth makes them more impatient thereby equalizing rates of time preference endogenously. Since saving behavior is governed by the rate of time preference these models predict that savings rates should be equalized over time. Even if one accepts the Barro, Mankiw, Sala-i-Martin argument that imperfect capital markets can account for a slow transition, it is still difficult to account for different savings rates across countries within an optimizing framework.

Our paper takes off from the endogenous growth literature initiated by Lucas [11] and Romer [18], [19]. We show that an endogenous growth model of a two-country-world with two factors, physical and human capital, can explain all of the facts cited above. Our key insight is that in the AK model of endogenous growth, (for an exposition see Rebelo [17]), rates of return are equated by technology and there is no motive for different countries to trade in the asset markets. We develop a two-sector version of an AK model in which the rate of interest is technologically determined. Like the simple AK model, our two-sector version leaves no room for intertemporal trade but unlike the one-sector version our model is consistent with the empirical evidence for conditional convergence. Other things equal, it predicts that under the closed economy assumption, a poor country will grow faster than a rich country as a consequence of the transitional dynamics of the ratio of physical to human capital.[4] Importantly, however, the model only predicts conditional convergence and *not* absolute convergence in income levels across countries.

A key contribution of our work is to show that in the two sector AK model, opening up capital flows across countries *does not* necessarily lead to an instantaneous disappearance of the imbalance

[4]Mulligan and Sala-i-Martin [14] recognize that the two-sector closed economy model generates transitional dynamics that imply conditional convergence in the closed economy but they do not pursue the implications of their analysis for international trade.

between physical and human capital. Importantly, this holds true even though relative prices, and hence, real rates of return on physical capital, get equalized instantaneously. Hence, the conditional convergence prediction of the closed economy model carries over to the open economy version as well. Intuitively, within the cone of diversification of the two sector AK model, a change in relative prices can be accommodated solely through changes in the sectoral factor intensities and sectoral factor allocations rather than through a cross-country movement of physical capital. Crucially for our interpretation of the data, our model also permits different rates of time preferences across countries and, hence, it can explain different long run savings, investment, and growth rates.

Section 2 discusses the data and section 3 lays out the model. In Section 4 we present a closed economy version before proceeding to a two country world in Sections 7 and 8. Section 9 concludes with some final remarks.

2 Some Evidence on Saving, Investment and Growth

In figure 1 we plot savings and investment rates for Japan, the United States and Egypt. The data is taken from the Heston-Summers data set and for each country we have plotted national saving and national investment as percentages of GDP. We chose these three countries as examples of a high saving country, a medium saving country and a low saving country, however, the broad features of the saving-investment facts are similar across all countries in the world. Savings and investment differ widely across countries at both low and high frequencies but within countries they are highly correlated.

[Insert Figure 1 about here]

Much of the work in the theory of international economics begins with the assumption that the world can be modeled as a collection of countries each characterized by a representative individual. These representative individuals are assumed to have the same rate of time preference in order to guarantee that the theoretical model has a well defined steady state wealth distribution. There is a refinement of these models, based on recursive preferences, in which agents are assumed to have utility functions that are intertemporally non-separable. In these models agents have a rate of time preference that is endogenous and that is a function of steady state wealth. Neither the separable nor the non-separable versions of standard models can easily account for the fact, illustrated in Figure 1, that savings and investment rates differ across countries for long periods of time.

In separable models, the rate of time preference determines the savings rate. In models with recursive preferences, equilibrium is achieved by a convergence of rates of time preference, and therefore of savings rates. This convergence is achieved through the assumption that as individuals become wealthier this increase in wealth endogenously lowers their discount rates. In these models, one should expect to see savings rates either equal or converging over time. The data displays no such tendency and we believe that this failure of the standard model is a strong reason to reconsider the implications of the endogenous growth model.

Figure 2 depicts the time series plots of the continental real per capita GDP levels between 1970 and 1990. While this picture masks income mobility across countries, it nevertheless provides a rather stark picture of the absence of any clear convergence in income levels over time across the continents. The corresponding evidence at the cross-country level is, as is well known, similar.[5]

[Insert Figure 2 about here]

3 The Model

We construct a two-country model. Each country is inhabited by a representative consumer with rate of time preference ρ and instantaneous utility function $U(C)$, where C is the flow rate of consumption. We use the notation ρ' and C' to denote the rate of time preference and consumption in country 2.

[5] Chari, Kehoe and McGrattan [4] provides an excellent description of the distribution of relative incomes across countries and the evolution of this distribution over time.

3.1 Technology

Our economy contains three commodities, K^W, H, and H'. K^W represents the world stock of a unique physical commodity that can be consumed or used in production in either country. H and H' refer to the stocks of human capital in countries 1 and 2. We assume that H and H' are distinct and that human capital cannot be traded internationally. There are four technologies that we describe below.

$$Y = F\left(K_Y, H_Y\right) \tag{1}$$

$$Y' = F\left(K'_Y, H'_Y\right). \tag{2}$$

Technologies (1) and (2) are identical increasing, concave, constant returns-to-scale production functions that describe the production of physical commodities. K_Y and H_Y represent the inputs of capital and human capital used to produce the physical commodity in country 1. Similarly K'_Y and H'_Y are inputs of physical and human capital in country 2.

To produce human capital we assume the following increasing, concave, constant returns-to-scale production functions.

$$\dot{H} = I = G\left(K_I, H_I\right) \tag{3}$$

$$\dot{H}' = I' = G\left(K'_I, H'_I\right). \tag{4}$$

The symbols I and I' mean investment in human capital in countries 1 and 2, and K_I and H_I (K'_I and H'_I) are inputs of physical capital and human capital to the sectors that produce human capital. Throughout the paper a dot over a variable denotes its time derivative.

Adding up constraints give the following restrictions on the alternative uses of resources,

$$K^W = K + K' \tag{5}$$

$$H = H_Y + H_I \tag{6}$$

$$H' = H'_Y + H'_I \tag{7}$$

$$K = K_Y + K_I \tag{8}$$

$$K' = K'_Y + K'_I. \tag{9}$$

Equation (5) is the aggregate constraint on world capital, and K (K') denotes the stock of capital in country 1 (2). Equations (6) and (7) are the constraints on the uses of human capital in each country and equations (8) and (9) are constraints on the uses of physical capital.

4 The Closed Economy Case

We begin by studying a world composed of two autarchic economies and without loss of generality, we analyze the equilibrium in country 1. Although we analyze the planning problem, the planning optimum for our economy can be decentralized with competitive markets since it is a relatively standard version of a convex economy with a finite number of agents.

The social planner maximizes the lifetime welfare of the representative agent,

$$Max \int_{t=0}^{\infty} e^{-\rho t} U(C)\, dt, \tag{10}$$

subject to the constraints

$$\dot{K} = F(K_Y, H_Y) - C \tag{11}$$

$$\dot{H} = G(K_I, H_I), \tag{12}$$

and the adding up constraints, equations (6) and (8). We assume throughout the paper that instantaneous utility is logarithmic,

$$U(C) = \log C.$$

Although we believe that our main results generalize to isoelastic utilities, in some cases this generalization is likely to be at considerable cost in terms of expositional complexity.

4.1 Static Production Plans

To solve the planner's problem we define a Hamiltonian, H that we maximize by choosing the control variables K_I, $K_Y, H_Y, H_I,$ and C. Let Λ and M be the costate variables associated with K

and H. The first order conditions to this problem are given by,

$$\frac{1}{C} = \Lambda \tag{13}$$

$$\Lambda F_K(K_Y, H_Y) = MG_K(K_I, H_I) \tag{14}$$

$$\Lambda F_H(K_Y, H_Y) = MG_H(K_I, H_I) \tag{15}$$

$$\dot{\Lambda} = [\rho - F_K(K_Y, H_Y)]\,\Lambda \tag{16}$$

$$\dot{M} = [\rho - G_H(K_I, H_I)]\,M, \tag{17}$$

plus the transversality conditions,

$$\lim_{T\to\infty} e^{-\rho T}\Lambda K = 0 \tag{18}$$

$$\lim_{T\to\infty} e^{-\rho T} MH = 0, \tag{19}$$

for K and H.

Equations (14) and (15) can be combined to generate conditions for an efficient production plan,

$$\frac{F_K(K_Y, H_Y)}{G_K(K_Y, H_Y)} = \frac{M}{\Lambda} = \frac{F_H(K_Y, H_Y)}{G_H(K_Y, H_Y)}. \tag{20}$$

Although we do not permit trade, at this point we will explicitly introduce notation for country 2 to facilitate the exposition later in our presentation. Let the variables ψ and ψ' be the ratios of the shadow value of human capital production to physical output in the two countries;

$$\psi = \frac{M}{\Lambda}, \quad \text{and} \quad \psi' = \frac{M'}{\Lambda'}. \tag{21}$$

In a decentralized economy these variables would be equated to the relative prices of human capital to output in the two countries. The following definitions of physical to human capital ratios in each industry enable us to simplify notation by making use of the constant returns-to-scale assumption.

$$k_I = \frac{K_I}{H_I}, \;\; k_Y = \frac{K_Y}{H_Y}, \;\; k_I' = \frac{K_I'}{H_I'}, \;\; k_Y' = \frac{K_Y'}{H_Y'}. \tag{22}$$

Using these definitions, writing $f_K\left(k\right)$ for $f_K\left(K/H, 1\right)$, and exploiting constant returns-to-scale, the first order conditions (14), (15) and (20) can be written more compactly;

$$f_K\left(k_Y\right) = \psi g_K\left(k_I\right) \tag{23}$$

$$f_H(k_Y) = \psi g_H(k_I) \tag{24}$$

$$\frac{f_K(k_Y)}{g_K(k_I)} = \psi = \frac{f_H(k_Y)}{g_H(k_I)}. \tag{25}$$

So far, we have allowed for very general technologies. At this point we introduce two additional assumptions that allow us to prove uniqueness of the steady state equilibrium.

Axiom 1 (No Factor Intensity Reversals) *One of the following two conditions holds. Either*

$$-\frac{f(x) f_{KK}(x)}{f_K(x)^2} > -\frac{g(x) g_{KK}(x)}{g_K(x)^2}, \qquad \textit{for all } x > 0, \tag{26}$$

or:

$$-\frac{g(x) g_{KK}(x)}{g_K(x)^2} > -\frac{f(x) f_{KK}(x)}{f_K(x)^2}, \qquad \textit{for all } x > 0. \tag{27}$$

Axiom 2 (Interiority) *The elasticities of the reduced form production functions are strictly between zero and one,*

$$0 < \frac{f_K(x)\, x}{f(x)} < 1, \quad \textit{and} \quad 0 < \frac{g_K(x)\, x}{g(x)} < 1. \tag{28}$$

Appendix A shows how to solve equations (23) and (24) in two unknowns for k_Y and k_I as functions of the relative price ψ. We represent these solutions with the notation,

$$k_Y = k_Y(\psi), \quad k_I = k_I(\psi), \tag{29}$$

from which it follows that the social planner's static production plan can be summarized by a single variable, ψ. If condition (26) holds then in equilibrium, $k_I > k_Y$. In this case, we say that the human capital industry is more intensive in its use of physical capital. We will often abbreviate this and say that the human capital industry is more capital intensive. If condition (27) holds then $k_Y > k_I$ and in this case we say that the physical capital industry is more capital intensive. The force of Axiom (1) is to allow us to concentrate on economies with a unique steady state equilibrium. Axiom (2) implies that both factors are essential and allows us to avoid dealing with corner conditions in some of the proofs.

There are several implications that follow from an analysis of the first order conditions. In the decentralized version of our model, ψ would represent the relative price of a unit of newly produced human capital in terms of a unit of the physical commodity and one could represent this price as the slope of a production possibilities frontier. Once one knows ψ, the ratio of physical to human capital is determined in each industry, as is the relative output of each of the two produced commodities. More important for our interpretation of data is the observation that if ψ happens to equal ψ', there will be no reason for trade in capital between the two countries, even if the capital markets are open.

4.2 Dynamic Plans

In this section we introduce notation that lets us describe a balanced growth path. We define two new variables

$$\lambda \equiv \Lambda H, \; k = K/H, \tag{30}$$

and we use the notation $f(k_Y)$, $g(k_I)$ and $f_K(k_Y)$, $g_H(k_I)$ to represent the production functions F and G and the marginal products F_K and G_H in intensive form. In Appendix B we show that the optimal plan can be described by the following system of three differential equations in these transformed variables. We assume, but do not prove, the existence of a balanced growth path. Existence requires a set of restrictions on the technologies and preferences that are analogous to the Inada conditions for the one-sector model. Given existence, uniqueness follows from the additional assumption of no factor intensity reversals.

$$
\begin{aligned}
\frac{\dot{\psi}}{\psi} &= f_K\left(k_Y(\psi)\right) - g_H\left(k_I(\psi)\right) && (31)\\
\frac{\dot{\lambda}}{\lambda} &= \rho - f_K\left(k_Y(\psi)\right) + \left(\frac{k_Y\left(\psi\right) - k}{k_Y\left(\psi\right) - k_I\left(\psi\right)}\right) g\left(k_I(\psi)\right) && (32)\\
\frac{\dot{k}}{k} &= \left(\frac{k - k_I(\psi)}{k_Y(\psi) - k_I(\psi)}\right) \frac{f\left(k_Y(\psi)\right)}{k} - \frac{1}{\lambda k} && (33)\\
&\quad - \left(\frac{k_Y(\psi) - k}{k_Y(\psi) - k_I(\psi)}\right) g\left(k_I(\psi)\right) \\
\lim_{T\to\infty} e^{-\rho T}\lambda k &= 0 && (34)\\
\lim_{T\to\infty} e^{-\rho T}\psi\lambda &= 0 && (35)\\
k\left(0\right) &= k_0. && (36)
\end{aligned}
$$

In the following section we study the dynamics of the closed economy model in the presence of two alternative assumptions about capital intensities.

5 Solving the Model

Equations (31), (32) and (33) constitute a three variable system of autonomous non-linear differential equations with boundary conditions (34), (35) and (36). In the following sections we let $\{k^*, \psi^*, \lambda^*\}$ represent the balanced growth path and we analyze the properties of dynamic equilibria that are close to this path. We will show that equilibria are locally determinate. This requires that there is a one dimensional manifold that converges to the balanced growth path for any initial value of k in the neighborhood of k^*.

5.1 Simplifying the Model

We begin by drawing on a well known result from utility theory. Individuals with Cobb-Douglas preferences will choose to consume a constant fraction of their wealth. To exploit this insight, consider the following definition of wealth,

$$W = K + \psi H. \tag{37}$$

We refer to W as the wealth of the household since in a market economy ψ would represent the relative price of human to physical capital and W would be equal to the market value of assets. Using this definition we are able to reduce the dimension of the equations that define equilibrium by solving explicitly for λ as a function of k and ψ. By rearranging the equation $1/C = 1/\rho\,(K + \psi H)$, using the definitions of λ and k, one can show that along the stable manifold the following condition holds,

$$\lambda = \frac{1}{\rho\,(k + \psi)}. \tag{38}$$

This result is proved in Appendix C. The intuition for why it holds follows from the observation that, a representative consumer with logarithmic preferences will choose to set $C = \rho W$. In neo-classical growth models the expression for wealth involves the discounted value of future income. In the endogenous growth model there is no future labor income and the appropriate concept of wealth is the market value of human plus non-human capital.

5.2 Transition Dynamics

In this section, we describe the path that a closed economy would follow to attain its balanced growth path. We will show that equilibria are very different depending on whether the human capital or physical capital industry is more intensive in its use of physical capital. In the former case, when $k_I > k_Y$, the model predicts that relative prices and rental rates for physical and human capital will jump to their steady state values. This version of the model behaves a lot like the one-sector AK model in which the rental rate is always equal to the constant A.

In the case when $k_Y > k_I$ there are non-trivial relative price dynamics along the adjustment path. In this latter case, we derive an important result that characterizes the adjustment dynamics. We will show that the relative price of human capital ψ and the ratio of physical to human capital k move in the same direction. In the subsequent section of the paper we will use this result to show that the model has similar predictions for conditional convergence to those of the Solow-Swan model.

We begin by exploiting equation (38) to rewrite the dynamical system in the following way,

$$\dot{\psi} = \psi\phi(\psi, k), \tag{39}$$

$$\dot{k} = \gamma(\psi, k), \tag{40}$$

where the functions ϕ and γ are defined as follows,

$$\phi(\psi, k) \equiv f_K(k_Y(\psi)) - g_H(k_I(\psi)) \tag{41}$$

$$\gamma(\psi, k) \equiv \left(\frac{k - k_I(\psi)}{k_Y(\psi) - k_I(\psi)}\right) f(k_Y(\psi)) \tag{42}$$

$$-\rho(k + \psi) - \left(\frac{k_Y(\psi) - k}{k_Y(\psi) - k_I(\psi)}\right) kg(k_I(\psi)).$$

To reduce the system in this way we have exploited the assumption of logarithmic utility to find an explicit solution for the multiplier λ in terms of the remaining variables k and ψ. To establish the properties of equilibria in the neighborhood of the balanced growth path we will linearize this system around the point $\{k^*, \psi^*\}$.

Let the Jacobian of the system, evaluated around the balanced growth path, be given by the matrix

$$J = \begin{bmatrix} \phi_\psi & 0 \\ \gamma_\psi & \gamma_k \end{bmatrix}.$$

In Appendix C, we derive explicit expressions for ϕ_ψ, γ_ψ, and γ_k and we establish that

$$k_Y - k_I < 0, \implies \phi_\psi > 0, \quad \gamma_k < 0 \tag{43}$$

$$k_Y - k_I > 0, \implies \phi_\psi < 0, \quad \gamma_k > 0, \quad \gamma_\psi < 0. \tag{44}$$

Since the matrix J is lower triangular its roots are equal to its diagonal elements. The above expressions imply that these roots switch sign depending on whether physical capital or human capital is more capital intensive.

Equations (39) and (40) describe the dynamics of the optimal path. The initial ratio of physical to human capital is predetermined and the relative price ψ is a jump variable that adjusts to set the system on its stable manifold. Figures 3 and 4 illustrate how the dynamics behave for the two cases, $k_I > k_Y$ and $k_Y > k_I$. When the human capital sector is more intensive in its use of physical capital, the relative price of human capital, ψ, jumps to its steady state value ψ^* and the physical to human capital ratio approaches its steady state value k^* asymptotically. In the case that the final goods sector is more physical capital intensive, the stable manifold is upward sloping. If k begins below k^* then k and ψ both increase over time asymptotically approaching their steady state values. If k begins above k^*, they both decrease over time, approaching $\{k^*, \psi^*\}$ from above.

[Insert Figure 3 about here]

[Insert Figure 4 about here]

Locally, the saddle path of the system is found from the decomposition

$$J = Q\Lambda Q^{-1}$$

where Λ is the diagonal matrix of eigenvalues

$$\Lambda = \begin{bmatrix} \phi_\psi & 0 \\ 0 & \gamma_k \end{bmatrix}$$

and the matrices Q and Q^{-1} are stacked matrices of eigenvectors. The eigenvalue decomposition leads to the expressions

$$Q = \begin{bmatrix} 1 & 0 \\ \frac{\gamma_\psi}{\phi_\psi - \gamma_k} & 1 \end{bmatrix}, \qquad Q^{-1} = \begin{bmatrix} 1 & 0 \\ -\frac{\gamma_\psi}{\phi_\psi - \gamma_k} & 1 \end{bmatrix},$$

and the stable manifold is found by setting the row of

$$Q^{-1} \begin{bmatrix} d\psi \\ dk \end{bmatrix}$$

associated with the positive eigenvalue of J, equal to zero, where $d\psi$ and dk are deviations from the steady state.

To establish the properties of figures 3 and 4 one must distinguish two cases. In the case when $k_Y < k_I$ Appendix C establishes that $\phi_\psi > 0$ and so the first row of Q^{-1} must be set equal to zero. In this case the relative price of physical to human capital must jump straight to its steady state value and, since $\gamma_k < 0$, the ratio of human to non-human capital will converge to its steady state asymptotically. An econometrician observing this economy would see no change in relative prices or rental rates of physical and human capital along the transition path.

In the case when $k_Y > k_I$, γ_k is positive and in this case one defines the stable manifold by setting the second row of Q^{-1} equal to zero. In this case,

$$dk = \frac{\gamma_\psi}{\phi_\psi - \gamma_k} d\psi,$$

where $\frac{\gamma_\psi}{\phi_\psi - \gamma_k} > 0$, since, when $k_Y > k_I$ we have established that $\gamma_\psi < 0$, $\phi_\psi < 0$ and $\gamma_k > 0$. In the following section we will exploit these results to examine the implications of the two-sector endogenous growth model for the empirical work on conditional convergence.[6]

6 Conditional Convergence

In this section we discuss the implications of our analysis for the debate on conditional convergence. Since it is the convergence debate that has led many economists to reject the endogenous growth model in favor of the neoclassical growth model we regard our findings on conditional convergence to be an important insight of our work.

6.1 GDP in the Endogenous Growth Model

To discuss the issue of convergence we need a definition of GDP. In many economies the production of human capital is organized by the state and there are likely to be distortions in the market value of human capital. Since we have no clear idea in which direction these distortions might operate, we have chosen to use a market based definition of GDP. We define it to be the value of output that

[6] These results on the dynamic properties of the model in the closed economy case are similar to the results obtained by Bond et al [3] who analyzed a similar model. However, they did not pursue the cross-country growth and conditional convergence implications of their results. Instead, their focus was on the long run dynamic effects of changes in rates of time preference and factor taxes.

we would be attained if the social planning optimum were decentralized in competitive markets:

$$Z = F(K_Y, H_Y) + \psi G(K_I, H_I), \tag{45}$$

where $F(K_Y, H_Y)$ is the output of the physical goods sector, $G(K_I, H_I)$ is the output of the human capital sector and ψ is the relative price of human capital. From the national income accounting identity we can also write this expression as

$$Z = rK + \omega H,$$

or more compactly,

$$Z = H(rk + \omega), \tag{46}$$

where $r = f_K(k_Y)$ and $\omega = f_H(k_Y)$ are the rental rates for physical and human capital. The terms r and ω can both be considered functions of ψ since k_Y and k_I are themselves functions of ψ.

Using this definition we now show that GDP will grow more slowly during the transition to balanced growth if the economy starts out with too much physical capital. To see this, we differentiate equation (46) to arrive at an expression for γ_Z, the growth rate of GDP in terms of γ_H, the growth rate of human capital and the growth rates of k and ψ,

$$\gamma_Z \equiv \gamma_H + \left(\frac{rk}{rk+\omega}\right)\left(\frac{\dot{k}}{k} + \varepsilon_r \frac{\dot{\psi}}{\psi}\right) + \left(\frac{\omega}{rk+\omega}\right)\varepsilon_\omega \frac{\dot{\psi}}{\psi}. \tag{47}$$

In this expression, ε_r and ε_ω are the elasticities of r and ω with respect to ψ. Collecting terms and using the fact that $\omega\varepsilon_\omega = -rk_Y\varepsilon_r$ we can write this expression as[7]

$$\gamma_Z \equiv \gamma_H + a_1 \frac{\dot{k}}{k} + a_2 \frac{\dot{\psi}}{\psi}, \tag{48}$$

where $a_1 = \left(\frac{rk}{rk+\omega}\right)$ and $a_2 = \left(\frac{k-k_Y}{rk+\omega}\right) r\varepsilon_r$. Notice that $a_1 > 0$ and $a_2 > 0$ in the case when $k_I < k < k_Y$.

6.2 GDP Growth and Conditional Convergence

Equation (47) has important implications for the growth regressions described by Barro and Sala-i-Martin [2]. Along a balanced growth path the GDP growth rate is equal to the human capital growth rate. But a country that begins with too much physical capital will grow more slowly during the transition than in the steady state as it reduces its excess physical capital. To see why this is so, consider the two cases of $k_I \lesseqqgtr k_Y$ separately.

Consider first the case of a world in which the production of human capital is more capital intensive, $k_I > k_Y$. Recall that in this case, $\frac{\dot{\psi}}{\psi} = 0$, and for a country that starts with too much physical capital, $\frac{\dot{k}}{k} < 0$ along the transition path (see Figure 3). It follows from equation (48) that we would observe slower GDP growth during the transition path than in the steady state.

Now consider the case in which the physical capital industry is more capital intensive. In this case it is no longer true that relative prices are constant during the transition path, but we have established in the previous section that ψ and k move in the same direction (see Figure 4). Since, when $k_Y > k_I$, the coefficient $a_2 = \left(\frac{k-k_Y}{rk+\omega}\right) r\varepsilon_r$ in equation (48) is positive, it follows that once

[7] Recall that $\omega = f_H(k_Y(\psi))$ and $r = f_K(k_Y(\psi))$. Define

$$\varepsilon_\omega = \frac{k_Y f_{HK}}{f_H}\frac{k_{Y\psi}\psi}{k_Y}, \qquad \varepsilon_r = \frac{k_Y f_{KK}}{f_K}\frac{k_{Y\psi}\psi}{k_Y}$$

and note that from the homogeneity of f

$$\frac{k_Y f_{HK}}{f_H}\frac{k_{Y\psi}\psi}{k_Y} = -\frac{k_Y f_{KK}}{f_K}\frac{k_{Y\psi}\psi}{k_Y}\frac{k_Y f_K}{f_H}$$

or

$$\varepsilon_\omega = -\varepsilon_r k_Y \frac{r}{\omega}.$$

again an economy that is initially rich in physical capital will grow more slowly along the transition path than in the steady state.

6.3 Implications for Growth Regressions

Barro and Sala-i-Martin regress average growth rates over fifteen years on initial income, initial educational attainment (a variable that picks up the initial level of human capital) and a set of other variables that includes measures of average investment in both physical and human capital. Their main finding is that countries that start out rich grow more slowly than countries that start out poor and they claim that this is evidence in favor of the Solow model and against endogenous growth models. However, our model offers an alternative interpretation of these results. It suggests that controlling for initial human capital, rich countries are those with an initially high ratio of physical to human capital. Once one controls for investment in human capital over the period (a variable that determines γ_H) our model predicts exactly what Barro and Sala-i-Martin find in the data. Rich countries should grow more slowly than poor ones. Crucially, our model generates this prediction despite exhibiting endogenous growth.

We have shown that by regressing the average growth rate on initial GDP per person, controlling for human capital, the Solow-Swan model and two-sector endogenous growth model both predict that rich countries will grow more slowly than poor ones. But this does not imply that the two models are observationally equivalent. The Solow-Swan model predicts that countries with the same investment rates will converge to the same level of GDP per capita. The endogenous growth model predicts instead that they will converge to the same growth rates. One might hope that the differences in these predictions might be exploited to better differentiate the two theories.

7 The Interdependent World Economy

We now turn to an open economy two-country version of our model. We assume that the world is composed of two representative agent economies that trade freely in the final good and, hence, in physical capital. However, human capital is non-traded and country specific. The structure of the two economies is unchanged from that presented above. Our chief interest is in determining the impact of openness on the equilibrium dynamics of the model. In particular, we want to determine the ability – or lack thereof – of the interdependent economy version to reproduce the stylized facts cited in the introduction.

7.1 The Social Planning Problem

We start by asking how a benevolent social planner would allocate consumption and organize production in the world economy. The social planner solves the problem,

$$Max \int_{t=0}^{\infty} e^{-\rho t} \left[bU(C) + e^{-(\rho' - \rho)t} (1-b) U\left(C'\right) \right], \tag{49}$$

subject to the constraints:

$$\dot{K}^W = F(K_Y, H_Y) + F\left(K'_Y, H'_Y\right) - C - C' \tag{50}$$

$$\dot{H} = G(K_I, H_I) \tag{51}$$

$$\dot{H}' = G\left(K'_I, H'_I\right). \tag{52}$$

In addition the social planner respects the adding up constraints (5–9) and the initial conditions

$$K^W(0) = \bar{K}^W,\ H(0) = \bar{H},\ H'(0) = \bar{H}'.$$

The parameter b is the welfare weight that the social planner attributes to country 1 and we allow for the possibility that the social planner uses different discount rates, ρ and ρ', for the two countries. This is an important feature of our model since, in order to explain the data, we must permit savings rates to differ across countries.

As before, let Λ, M and M' be the costate variables associated with W, H and H' and retain the assumption that U is logarithmic. The social planner chooses the controls C and C' to satisfy the following first order conditions:

$$\Lambda = \frac{b}{C} = \frac{1-b}{C'} e^{-(\rho' - \rho)t}. \tag{53}$$

7.2 Static Production Plans

The analysis of efficient production plans in the open economy is identical to the closed economy case studied in section 4.1 and it leads to the first order conditions in intensive form

$$f_K(k_Y) = \psi' g_K(k_I') \tag{54}$$

$$f_K(k_Y') = \psi' g_K(k_I') \tag{55}$$

$$f_H(k_Y) = \psi g_H(k_I) \tag{56}$$

$$f_H(k_Y') = \psi' g_H(k_I') \tag{57}$$

$$\psi g_K(k_I) = \psi' g_K(k_I'), \tag{58}$$

where the variables k_Y $(k_Y')$and k_I (k_I') , are the sectoral capital labor ratios as defined in Equation (22). Equations (54) – (58) imply that the social planner's production plan can be summarized by a single variable, ψ and that the optimal plan will choose

$$k_Y(\psi) = k_Y'(\psi') \tag{59}$$

$$k_I(\psi) = k_I'(\psi') \tag{60}$$

$$\psi = \psi'. \tag{61}$$

The functions $k_Y(\psi)$ and $k_I(\psi)$ are identical to those derived in the closed economy case.

Equation (61) implies that, if one were to decentralize the planning solution, the relative price of human capital to physical capital would be equalized across countries even though human capital is non-traded. This is a restatement of the well known result that in a two-factor two-good model, trade in goods and trade in factors are substitutes. Allowing for trade in the final good is sufficient to equate the relative price of the non-traded good as well.

7.3 Dynamic Production Plans

In the following analysis we show that the dynamics of an optimal production plan is described by a dynamical system in the four variables k^w, λ, ψ and θ. ψ is the variable defined previously and k^w, λ, and θ are defined as:

$$k^w = \frac{K^W}{H + H'} \tag{62}$$

$$\lambda = \Lambda\left(H + H'\right) \tag{63}$$

$$\theta = \frac{H}{H + H'}. \tag{64}$$

We use k^w to refer to the world ratio of physical to human capital. λ is the shadow price of consumption, ψ is the relative price of physical and human capital and θ gives the world distribution of the stocks of H and H'. It is important to note that we have redefined the variable λ in the two country context: previously it was equal to ΛH.

In Appendix D we show that an optimal plan for the two-sector, two-country economy is characterized by the following system of four differential equations in the variables k^w, λ, ψ and θ.

$$\begin{aligned}
\frac{\dot{k}^w}{\dot{k}^w} &= \left(\frac{k^w - k_I[\psi]}{k_Y[\psi] - k_I[\psi]}\right)\frac{f(k_Y[\psi])}{k^w} - \frac{s(t)}{\lambda k^w} \\
&\quad - \left(\frac{k_Y[\psi] - k^w}{k_Y[\psi] - k_I[\psi]}\right) g(k_I[\psi]) && (65)\\
\frac{\dot{\lambda}}{\lambda} &= \rho - f_k(k_Y[\psi]) + \left(\frac{k_Y[\psi] - k^w}{k_Y[\psi] - k_I[\psi]}\right) g(k_I[\psi]) && (66)\\
\frac{\dot{\psi}}{\psi} &= f_k(k_Y[\psi]) - g_H(k_I[\psi]) && (67)\\
\frac{\dot{\theta}}{\theta} &= \frac{(1-\theta)}{\theta}\left(q' - \left[\frac{k^w - k_I[\psi]}{k_Y[\psi] - k_I[\psi]}\right]\right) g(k_I[\psi]) && (68)\\
\lim_{T\to\infty} e^{-\rho T}\lambda k^w &= 0 && (69)\\
\lim_{T\to\infty} e^{-\rho T}\psi\lambda &= 0 && (70)\\
\theta(0) &= \theta_0 && (71)\\
k^w(0) &= k_0^w. && (72)
\end{aligned}$$

The function $s(t)$ that appears in Equation (65) is given by the expression

$$s(t) \equiv b + (1-b)\, e^{-(\rho'-\rho)t}$$

and is equal to 1 for the case when rates of time preference are equal in the two countries. In this special case, the equations of motion are autonomous and the economy converges to a balanced growth path. When $\rho < \rho'$, Equation (65) is no longer autonomous. In this case the share of the more patient individual in world consumption asymptotically approaches 1 and the equations of the world economy converge to those of the more patient of the two closed economies.

We use the notation $q \equiv \frac{H_Y}{H}$ and $q' \equiv \frac{H'_Y}{H'}$ to refer to the shares of human capital in each country that are allocated to the production of physical commodities. Using this notation, the growth rates of human capital in each country are related to q and q' by the following expressions:

$$\begin{aligned} \frac{\dot{H}}{H} &= (1-q)\, g(k_I[\psi]) \\ \frac{\dot{H}'}{\dot{H}'} &= \left(1-q'\right) g(k_I[\psi])\,. \end{aligned}$$

In any equilibrium in which countries 1 and 2 grow at the same rate we must have $\dot{\theta} = 0$ and from equation (68) that

$$q = q' = \tilde{q} = \frac{k^w - k_Y[\psi]}{k_Y[\psi] - k_I[\psi]}, \tag{73}$$

where

$$\tilde{q} \equiv q\theta + q'(1-\theta)$$

is the average world allocation of human capital to the physical goods sector. In the following section we discuss the determination of the time path of q, q' and $\tilde{q}$ in an optimal plan.

8 Interpreting the Equilibrium of the Two-Country Model

In this section we consider the economic implications of the solution to the planners problem that we derived in section 7.3. We begin by asking how two-countries would respond if they were opened up to trade.

8.1 Opening the World to Trade

Consider a world of two closed economies in which each economy has the same technological opportunities but representative agents in the two economies have different rates of time preference. Suppose that each economy in isolation has attained its balanced growth path and that the economies are now opened to trade. Since the economies have identical technologies all relative prices in the two economies will be the same. However, the level of GDP per capita may differ since it depends on initial conditions. It follows that, in the endogenous growth model, there need be no trade even if capital markets are open and GDP per capita *is different*.

Now consider what happens if two countries that have not attained balanced growth are opened to trade. Suppose first, that $k_I > k_Y$. This is the case when the human capital industry is more capital intensive and we have shown that for this case the rental rates of human and physical capital and the relative price of human and physical and human capital are all independent of the state. There are no relative price dynamics, even in the transition, and two countries that are opened to trade have no incentive to borrow and lend even if agents have different rates of time preference and different stocks of physical and human capital.

Now suppose that $k_Y > k_I$. In this case, if the two economies have not attained their balanced growth paths, then there will be incentives to borrow and lend since the rental rates of physical capital will be different at the date that trade is opened. In terms of equations (65–68), we interpret the opening of the world to trade as the solution to a new planning problem that gives rise to equations (65–68) in place of two sets of closed economy equations like those discussed in section 4. If the world is in equilibrium before trade is opened, and if the opening of the economy to trade was unanticipated, then the welfare weight b will reflect the relative wealth of the two economies at the date at which the opening occurs. Opening the world to trade resets the planner's ratio of K^W to $H + H'$. Rates of return will be equated and the economy will follow a different path from the point that trade is initiated. But although the opening of the economy to trade will alter relative prices, it need not alter the growth rates of the two economies. The following section elaborates further on this point.

8.2 The Location of Production

In this section we discuss the location of world production. Since physical goods can be produced in either location, and since technologies are homogenous of degree one in reproducible inputs, there is an important indeterminacy in the nature of the solution to the social planning optimum. The social planner cares how much of each commodity is allocated to each country, but he does not care about the physical location of production. This characteristic of the solution arises solely from the linearity of the technology set.

The irrelevance of the physical location of production is reflected in the fact that equations (65), (66) and (67) are independent of θ. The variable θ is the share of world human capital located in country 1 and the evolution of this variable is governed by Equation (68). It depends on q', the share of human capital in country 2 that is allocated to the production of physical commodities. But the time path of q' is indeterminate, reflecting the fact that the technologies available to the social planner are linearly homogenous in reproducible inputs. If one were to assume that human capital grows at the same rate in the two countries then q' would be determined from equation (73). But there is no reason to impose this solution. It would correspond to a decision by the social planner to maintain the initial value of θ. If the two countries begin with different levels of human capital then it is consistent with an optimal plan to choose to maintain that situation for ever. An observer of such an equilibrium would see the two countries growing at the same rate, with a permanently higher level of output per capita in the country that began with more human capital.

There are many other welfare equivalent solutions to this optimization problem since the social planner can transfer physical resources from one country to the other without affecting the total quantity of physical output produced in the world. As physical resources are transferred from country 1 to country 2, human capital in the receiving country must be transferred to the industry that is more capital intensive to maintain the physical to human capital ratios at their stationary levels. For the same reason, the social planner will shift production in the opposite direction in the country from which physical capital has been transferred. The linear homogeneity of production implies that there is a continuum of ways of allocating resources across the two countries to maintain the same physical output. This argument is independent of the way that resources are allocated to consumption and it is this fact that implies that the alternative production plans are welfare equivalent.

This has important implications for the cross-country movement of physical capital that is induced by the opening of capital markets. In particular, opening the economies to trade immediately equates the relative price ψ, and hence, the sectoral physical to human capital ratios across countries. This effect, in general, also changes the relative price in each country vis-a-vis their autarkic prices. Recall that the allocation of human capital to the final goods sector in the home country ($H_Y/H = q$) is given by $q = \frac{k - k_I(\psi)}{k_Y(\psi) - k_I(\psi)}$. Thus a change in ψ must be accompanied by changes in either q or k or by some combination of the two. Within the cone of diversification, it is feasible to keep k unchanged and accommodate a change in the relative price ψ through a change in the sectoral allocation of human capital q alone. This feature characterizes any two sector–two factor model where factors are mobile across sectors.[8] In the neoclassical model the precise distribution of the world capital stock, and hence k and k', in the free trade equilibrium gets tied down by another condition, namely, diminishing returns to the accumulable factors. In our two sector AK model, however, there are constant returns to the accumulable factors which makes k and k', and hence, the direction of the cross border capital movement, indeterminate.[9] Thus, the cross-country equalization of factor prices does not necessarily lead to an equalization of the physical to human capital ratios across countries.

[8] A typical example of this is the well known Heckscher-Ohlin model. In the static version of that model with fixed capital and labor, all changes in relative prices are accommodated through sectoral factor reallocations.

[9] Note that since human capital is a stock which cannot be traded, any *discrete* change in the physical to human capital ratios k and k' at a given point in time can only occur through cross-country movements of physical capital.

8.3 Implications for Convergence

The fact that location is irrelevant has additional implications for the convergence debate. Consider a world that has attained its balanced growth path so that k^*, λ^*, and ψ^* are at their steady state levels. An important implication of this assumption is the world growth rate, the world ratio of physical to human capital and all rental rates and relative prices are constant over time.[10] Now suppose that the social planner arbitrarily decides to raise q', thereby causing θ to increase over time. The variable θ is equal to $H/(H+H')$. It follows, if θ is to increase, that human capital must grow faster in country 1 than in country 2.

The implications of raising q' for trade flows will depend on which industry is more capital intensive. If $k_Y > k_I$ then physical resources will flow to country 2 to enable the capital intensive physical goods industry to operate more intensively. As long as the social planner maintains q' above its steady state level, one will observe a trade deficit in country 2 as goods flow from the faster growing country 1 to the slower growing country 2. If the opposite capital intensity condition prevails, one will see the high growth country importing rather than exporting goods. In this case $k_I > k_Y$. Since the production of physical goods is less capital intensive, country 2 will need less physical capital rather than more.

The implications of the world model for the convergence regressions are ambiguous. The fact that there are many possible equilibria suggests that care should be taken in interpreting these results and that one should seek richer implications of endogenous and exogenous growth theories that exploit their differential implications for the time series properties of variables in panel data sets. Growth rates could switch at any time as a result of a movement of resources across countries as the social planner switches from one solution to another. Crucially, our model predicts that these switches in growth rates should be linked to trade flows but not, necessarily, to initial conditions. However, as we have shown, even the sign of the trade flows associated with the growth switches

[10] One would hope that in a stochastic verision of the model developed in this paper one might replace the time invariance of relative prices with the weaker implication that relative prices follow a stationary stochastic process that is independent of the distributions of growth rates across countries.

may be reversed if one were to take account the difference in factor intensities between consumption and investment goods industries.

8.4 Implications for Savings and Investment

In the introduction we drew attention to the inability of the neoclassical model to account for the low frequency pattern of savings and investment rates both across time and across countries. This is one of the strongest arguments in favor of the endogenous growth model. We have shown that despite perfect capital mobility across countries there need not be significant cross-country capital flows. In our model there is no incentive for agents to shift capital abroad since there are no rate of return differentials across countries. Hence, the model can explain the Feldstein-Horioka finding that cross-country long-term capital flows are small. We are also able to explain different savings rates and different growth rates that may persist over time by allowing for differences in rates of time preference.

9 Conclusion

In the introduction to this paper, we drew attention to six stylized facts. We argued that these facts present difficulties for the neoclassical growth model when one recognizes that modern capital markets are relatively open. We have made the case that a relatively standard endogenous growth model can account for all of these facts.

We believe that most economists would cite the evidence on conditional convergence as evidence against the endogenous growth model. But we have shown that the two-sector endogenous growth model can account for this evidence as a result of the adjustment dynamics of the stocks of capital to human capital. In addition, we have argued that the two-sector model can account for a wide variety of additional evidence that presents significant difficulty for the neoclassical model. This additional evidence includes the Feldstein-Horioka observation that national savings and domestic investment are highly correlated at high and low frequencies, the fact that real interest rates are approximately equalized across countries, and the wide dispersions in living standards across countries.

The key results of the paper follow from the basic insight that in a two-sector AK model the location of production is irrelevant. The linear homogeneity of the production technology in accumulable inputs makes the social planner indifferent between different geographic allocations of production. One limitation of the paper is that these alternative production allocations across countries are all welfare equivalent. In related work (see Farmer and Lahiri [5]) we have shown that in a two-country version of the endogenous growth model analyzed in Lucas [11], the indeterminacy of cross-country production plans survives but where the alternative equilibria can be Pareto ranked due to the presence of production externalities.

A key criticism of the endogenous growth model has been that it predicts a very tight one–to–one link between the investment rate and the growth rate. Jones [10] tested this prediction for the OECD countries using post World War II data and concluded that there was no evidence to support this prediction. However, McGrattan [13] looks at a much longer time series (starting from 1870) and finds a strong positive relationship between the rates of investment and growth rate for the OECD countries. Moreover, she finds that this positive investment–growth relationship also emerges in the post-war data in a much larger sample of countries than just the OECD. Lastly, McGrattan also demonstrates that slight modifications in the standard AK model allow it to rationalize the short-run deviations of the growth rate from the investment rate.

Understanding the determinants of growth remains perhaps the most important challenge facing economists and policymakers across the world. Between the two key competing visions of the growth process – the neoclassical and the endogenous growth models – the profession appears to have leaned lately toward the neoclassical view. This may have been due to a combination of factors: the comfort of the profession with the traditional diminishing returns to capital specification, and a lot of evidence over the last decade suggesting the presence of conditional convergence. Our main goal in this paper has been to highlight the fact that neither of these factors necessarily conflict with the endogenous growth model. Moreover, the endogenous growth version performs better than the neoclassical model in more realistic open economy specifications of the world. A lot more empirical work needs to be done in order to choose between these two competing visions of the world. But perhaps it is time to give the AK model another look.

10 Appendix A: Results from Static Efficiency

In Appendix A we establish some properties that follow from the static first order conditions for an efficient production plan.

10.1 Deriving a Function $h(k_I)$ Relating k_Y and k_I

We begin by establishing that, in equilibrium, there a exists a monotonic increasing function that relates k_Y to k_I. From the conditions for productive efficiency, equations (23), (24) and (25), we know that:

$$f_K(k_Y) = \psi g_K(k_I), \tag{74}$$

and

$$f_H(k_Y) = \psi g_H(k_I). \tag{75}$$

Using the facts that $f_H = f - k_Y f_K$ and $g_H = g - k_I g_K$ we can eliminate ψ from these equations to yield the following implicit equation linking k_I and k_Y :

$$\frac{f(k_Y)}{f_K(k_Y)} - k_Y = \frac{g(k_I)}{g_K(k_I)} - k_I. \tag{76}$$

Now define the functions $a(k_Y)$, $b(k_I)$;

$$a(k_Y) \equiv \frac{f(k_Y)}{f_K(k_Y)} - k_Y, \tag{77}$$

and,

$$b(k_I) \equiv \frac{g(k_I)}{g_K(k_I)} - k_I. \tag{78}$$

The derivatives of $a(k_Y)$ and $b(k_I)$, given by the expressions

$$a_K(k_Y) = -\frac{f(k_Y) f_{KK}(k_Y)}{f_K(k_Y)^2}, \quad b_K(k_I) = -\frac{g(k_I) g_{KK}(k_I)}{g_K(k_I)^2},$$

are strictly positive since f_{KK} and g_{KK} are negative from the assumption that f and g are strictly concave. From the fact that $f(0) = g(0) = 0$, it follows that if f_K and g_K are non-zero then $a(0) =$

$b(0) = 0$. If $f_K = 0$ (or $g_K = 0$) then one establishes that $f(0)/f_K(0) = 0$ $(g(0)/g_K(0) = 0)$ using L'Hospitâl's rule plus the strict concavity assumption. Using assumption 2, one can show that $\lim_{x\to\infty} a(x) = \lim_{x\to\infty} b(x) = \infty$. It follows that $a(x)$ and $b(x)$ are continuous functions mapping the positive real line into itself and the inverse function $a^{-1}(x) : [0,\infty] \to [0,\infty]$ is well defined. This reasoning implies that there exists a function $k_Y : [0,\infty] \to [0,\infty]$ defined as $k_Y = h(k_I) \equiv a^{-1}(b(k_I))$ with strictly positive derivative given by the expression,

$$h_{K_I} = \frac{g(k_I)\, g_{KK}(k_I)}{g_K(k_I)^2} \frac{f_K(h(k_I))^2}{f(h(k_I))\, f_{KK}(h(k_I))}. \tag{79}$$

10.2 Capital Intensities

We now use the definition of $h(k_I)$ to prove the assertion in the text that there are no factor intensity reversals in our economy. The function $h(k_I)$ is implicitly defined by the equations $a(k_Y) = t$, $b(k_I) = t$ for non-negative t. Further, $a(0) = b(0) = 0$. Under assumption (26), $a_K > b_K$. The graphs of $a(x)$ and $b(x)$ begin at the origin and $a(x)$ is always steeper than $b(x)$. It follows that if $a(k_Y) = b(k_I)$ then $k_Y < k_I$. Under assumption (27), $a_K < b_K$. Similar reasoning implies that in this case $b(x)$ is steeper than $a(x)$ and hence if $a(k_Y) = b(k_I)$ then $k_Y > k_I$.

10.3 The Relationship Between k_Y, k_I and ψ.

Define the function $\psi(k_I) : [0,\infty] \to [\psi_{\min}, \psi_{\max}]$ as

$$\psi = \frac{f_K(h(k_I))}{g_K(k_I)},$$

where

$$\psi_{\min} = \min\left\{\frac{f_K(h(0))}{g_K(0)}, \lim_{x\to\infty} \frac{f_K(h(x))}{g_K(x)}\right\},$$

and

$$\psi_{\max} = \max\left\{\frac{f_K(h(0))}{g_K(0)}, \lim_{x\to\infty} \frac{f_K(h(x))}{g_K(x)}\right\}.$$

We show below that this function is increasing if $k_Y > k_I$ and decreasing if $k_Y < k_I$. In the former case

$$\psi_{\min} = \frac{f_K(h(0))}{g_K(0)}, \quad \text{and} \quad \psi_{\max} = \lim_{x\to\infty} \frac{f_K(h(x))}{g_K(x)},$$

and in the latter, these definitions are reversed.

One can interpret $-\psi_{\min}$ as the slope of the production possibilities frontier as all capital is transferred to the production of physical capital. Similarly, $-\psi_{\max}$ is the slope of the production possibilities frontier as all capital is transferred to the production of human capital. The derivative of the function $\psi(k_I)$ is given by

$$\psi_{k_I} = \frac{g_K f_{kk}}{g_K^2} \frac{\partial k_Y}{\partial k_I} - \frac{f_K g_{KK}}{g_K^2}.$$

Rearranging terms, using (79) gives

$$\psi_{k_I} = -\frac{f_K g_{KK}}{g_K^2} \frac{f_K}{f} (k_Y - k_I), \tag{80}$$

which is negative if $k_Y < k_I$ (whenever (26) holds) and positive if $k_Y > k_I$ (when (27) holds).

From the inverse function theorem we can find a function $k_I : [\psi_{\min}, \psi_{\max}] \to [0, \infty]$ which we write as

$$k_I = k_I(\psi).$$

The derivatives of the function k_I is, from the inverse function theorem, equal to $1/\psi_K$ which is increasing when $k_Y > k_I$ and decreasing when $k_Y < k_I$. Similarly one can define a function $k_Y : [\psi_{\min}, \psi_{\max}] \to [0, \infty]$ as $k_Y = h(k_I(\psi))$. Since h is an increasing function k_Y is increasing in ψ when $k_Y > k_I$ and decreasing when $k_Y < k_I$.

11 Appendix B: Equilibria in the Closed Economy

In Appendix B we derive the equations that define dynamic equilibria in the closed economy model. We also establish several results concerning the properties of dynamic equilibria.

11.1 Deriving the Equations that Define Equilibria.

We begin by defining $\frac{H_Y}{H} = q$,and $\frac{H_I}{H} = 1 - q$. From the adding up constraint for the uses of physical capital we obtain $k = qk_Y + (1-q)k_I$, from which it follows that,

$$q = \frac{k - k_I}{k_Y - k_I}, \qquad 1 - q = \frac{k_Y - k}{k_Y - k_I}. \tag{81}$$

We will make use of these expressions in the following derivations.

We now turn to the derivation of equations (31), (32) and (33). From the definition $\psi \equiv \frac{M}{\Lambda}$, we have $\frac{\dot{\psi}}{\psi} = \frac{\dot{M}}{M} - \frac{\dot{\Lambda}}{\Lambda}$. Using equations (16) and (17) it follows directly that

$$\frac{\dot{\psi}}{\psi} = f_K\left(k_Y\right) - g_H\left(k_I\right), \tag{82}$$

which is equation (31) in the text.

From the definition $\lambda = \Lambda H$ we have $\frac{\dot{\lambda}}{\lambda} = \frac{\dot{\Lambda}}{\Lambda} + \frac{\dot{H}}{H}$. Using equations (12) and (16) one can write $\frac{\dot{\lambda}}{\lambda} = \rho - f_K\left(k_Y\right) + \frac{H_I}{H} g\left(k_I\right)$, and since $\frac{H_I}{H} = 1 - q = \frac{k_Y - k}{k_Y - k_I}$ (from (81)) one has

$$\frac{\dot{\lambda}}{\lambda} = \rho - f_K\left(k_Y\right) + \left(\frac{k_Y - k}{k_Y - k_I}\right) g\left(k_I\right),$$

which is equation (32) in the text.

From the definition $k = \frac{K}{H}$ it follows that $\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \frac{\dot{H}}{H}$. Dividing equation (11) by K and (12) by H one then has $\frac{\dot{k}}{k} = q\frac{f(k_Y)}{k} - \frac{C}{K} - (1-q)\, g\left(k_I\right)$ and using (81) and (13) one has

$$\frac{\dot{k}}{k} = \left(\frac{k - k_I}{k_Y - k_I}\right) \frac{f\left(k_Y\right)}{k} - \frac{1}{\lambda k} - \left(\frac{k_Y - k}{k_Y - k_I}\right) g\left(k_I\right),$$

which is equation (33) in the text. The transversality condition (34) for the transformed system follows from (18) by recognizing that $\Lambda K = \lambda k$ and (35) follows from combining (18) and (19).

12 Appendix C: The Dynamics of Equilibrium Paths

In this section we collect together a number of results related to the dynamics of transition paths to steady state.

12.1 Reducing the Dimension of the System

In this section we establish that

$$C = \rho\left(K + \psi H\right). \tag{83}$$

Differentiating (83) gives

$$\frac{\dot{C}}{C} = \frac{K}{K + \psi H}\frac{\dot{K}}{K} + \frac{\psi H}{K + \psi H}\left(\frac{\dot{\psi}}{\psi} + \frac{\dot{H}}{H}\right). \tag{84}$$

We now find expressions for the left side and right side of (84) and show that these expressions are equal when (83) holds.

We begin with the left side, which using equations (13) and (16) we write as follows.

$$LHS = \frac{\dot{C}}{C} = -\frac{\dot{\Lambda}}{\Lambda} = f_K - \rho. \tag{85}$$

We now turn to an expression for the right side. Using equations (31), (11) and (12) it follows that:

$$\begin{aligned} RHS &= \frac{K}{K+\psi H}\frac{\dot{K}}{K} + \frac{\psi H}{K+\psi H}\left(\frac{\dot{\psi}}{\psi} + \frac{\dot{H}}{H}\right) \\ &= \frac{1}{K+\psi H}\left(F - C + \psi H\left(f_K - g_H\right) + \psi G\right). \end{aligned}$$

Now expand the terms F and G using the facts that

$$F = f_K K_Y + f_H H_Y, \qquad G = g_K K_I + g_H H_I, \qquad H = H_I + H_Y$$

to give

$$RHS = \frac{1}{K+\psi H}\left(f_K K_Y + f_H H_Y - C + \psi H f_K - \psi H_Y g_H + \psi g_K K_I\right).$$

Since the first order conditions for productive efficiency imply

$$f_H = \psi g_H, \quad f_K = \psi g_K$$

this further reduces to

$$RHS = \frac{1}{K+\psi H}\left(f_K K_Y - C + \psi f_K H + f_K K_I\right)$$

or

$$= \frac{f_K\left(K+\psi H\right) - C}{K+\psi H}.$$

Hence, if $C = \rho\left(K+\psi H\right)$ then

$$RHS = f_K - \rho$$

which is the expression given above in (85) for the left side of equation (84). This establishes that (83) holds along the stable manifold.

12.2 The Elements of the Jacobian

In the text we defined the Jacobian of (39–40) to have elements.

$$J = \begin{bmatrix} \phi_\psi & 0 \\ \gamma_\psi & \gamma_k \end{bmatrix}$$

The following subsections derive expressions for each of these elements and derive values for their signs as functions of the sign of $k_Y - k_I$.

12.2.1 Deriving an Expression for ϕ_ψ

From equation (41)

$$\phi(\psi, k) \equiv f_K(k_Y(\psi)) - g_H(k_I(\psi)). \tag{86}$$

The derivative of ϕ w.r.t. ψ is given by

$$\frac{\partial \phi}{\partial \psi} = f_{KK}\frac{\partial k_Y}{\partial \psi} + k_I g_{KK}\frac{\partial k_I}{\partial \psi}.$$

This function has opposite sign to that of $\frac{\partial k_Y}{\partial \psi}$ and, since we assume no factor intensity reversals, if $k_I > k_Y$ then

$$sgn\left(\frac{\partial \phi}{\partial \psi}\right) > 0,$$

and if $k_Y > k_I$, then

$$sgn\left(\frac{\partial \phi}{\partial \psi}\right) < 0.$$

12.2.2 Deriving an Expression for γ_k

From equation (42) it follows that

$$\gamma_k = \frac{f}{D} - \rho - \frac{k_Y g}{D} + 2\frac{kg}{D}.$$

Suppose first that $D = k_Y - k_I > 0$. Then from equation (40) it follows that, when $\dot{k} = 0$,

$$\frac{f}{D} - \rho - \frac{k_Y g}{D} = \left(\frac{k_I f}{kD} - \frac{kg}{D}\right) + \frac{\rho\psi}{k} > 0.$$

Substituting this back into the expression for γ_k yields $\gamma_k > 0$ as claimed in the text.

Now suppose instead that $D < 0$. In this case it follows from equation (32) that,

$$\rho = f_K - \frac{k_Y g}{D} + \frac{kg}{D},$$

from whence it follows that

$$\gamma_k = \frac{f}{D} - f_K + \frac{kg}{D} < 0,$$

as claimed in the text.

12.2.3 Deriving an Expression for γ_ψ

Differentiating equation (42) with respect to ψ leads to the expression,

$$\gamma_\psi = f q_\psi + q f_K k_{Y\psi} - \rho - (1-q)\, k g_K k_{I\psi} + g k q_\psi, \tag{87}$$

where $q = \frac{k - k_I}{k_Y - k_I}$, $q_\psi = \frac{\partial q}{\partial \psi}$, $k_{Y\psi} \equiv \frac{\partial k_Y}{\partial \psi}$, and $k_{I\psi} \equiv \frac{\partial k_I}{\partial \psi}$. Since γ_ψ does not influence the dynamics in the case when $k_I > k_Y$, we restrict out attention to finding the sign of γ_ψ in the case, $k_Y > k_I$. First note that by using the chain rule and rearranging terms one can find the following expression for q_ψ,

$$q_\psi = -\frac{q k_{Y\psi}}{D} - \frac{(1-q)\, k_{I\psi}}{D} < 0, \tag{88}$$

where the sign follows from the assumption $D = k_Y - k_I > 0$ and the fact that k_Y and k_I are increasing when $D > 0$ (see equation (80). It follows that the only positive term in the expression (87) is

$$q f_K k_{Y\psi},$$

which, by multiplying and dividing by D, we write as

$$q f_K k_{Y\psi} = \frac{q f_K k_Y k_{Y\psi}}{D} - \frac{q f_K k_I k_{Y\psi}}{D}. \tag{89}$$

Using equation (88) we can rewrite the first term of (87) as

$$f q_\psi = -\frac{f q k_{Y\psi}}{D} - \frac{f\,(1-q)\, k_{I\psi}}{D},$$

or since $f = k_Y f_K + f_H$

$$fq_\psi = -\frac{qf_K k_Y k_{Y\psi}}{D} - \frac{f_H q k_{Y\psi}}{D} - \frac{f(1-q)k_{I\psi}}{D}. \tag{90}$$

Combining (89) and (90) establishes that

$$\begin{aligned} fq_\psi + qf_K k_{Y\psi} &= -\frac{qf_K k_Y k_{Y\psi}}{D} - \frac{qf_H k_{Y\psi}}{D} \\ &\quad -\frac{f(1-q)k_{I\psi}}{D} + \frac{qf_K k_Y k_{Y\psi}}{D} - \frac{qf_K k_I k_{Y\psi}}{D} \\ &= -\frac{qf_H k_{Y\psi}}{D} - \frac{f(1-q)k_{I\psi}}{D} - \frac{qf_K k_I k_{Y\psi}}{D} < 0. \end{aligned}$$

Since the other terms in expression (87) are all negative it follows that $\gamma_\psi < 0$ when $k_Y > k_I$.

13 Appendix D: Equilibria in the Open Economy

In this Appendix we derive the equations of motion that describe equilibria in the world economy.

13.1 Preliminary Definitions

Define the employment shares in each country $q \equiv \frac{H_Y}{H}$, $q' \equiv \frac{H'_Y}{H'}$. From the adding up constraints, (5)–(9) we can write these shares as follows,

$$q = \frac{k - k_I}{k_Y - k_I}, \qquad q' = \frac{k' - k'_I}{k'_Y - k'_I} \tag{91}$$

$$1 - q = \frac{k_Y - k}{k_Y - k_I}, \qquad 1 - q' = \frac{k'_Y - k'}{k'_Y - k'_I}. \tag{92}$$

Now let $\theta \equiv \frac{H}{H+H'}$ be the share of human capital in country 1 and let $k^w \equiv \frac{K^W}{H+H'}$ be the ratio of world capital to world human capital. Using these definitions write the adding up constraint for physical capital

$$K^W = K_Y + K_I + K'_Y + K'_I,$$

as follows:

$$k^w = \theta q k_Y + \theta(1-q)k_I + (1-\theta)q'k'_Y + (1-\theta)(1-q')k'_I. \tag{93}$$

Static efficiency requires

$$k_Y = k'_Y, \text{ and } k_I = k'_I,$$

from which it follows that

$$k^w = \left[q\theta + q'(1-\theta)\right] k_Y + \left[(1-q)\theta + \left(1-q'\right)(1-\theta)\right] k_I. \tag{94}$$

A further simplification follows by defining $\tilde{q} \equiv q\theta + q'(1-\theta)$, to be the relative intensity with which the social planner runs the physical and human capital sectors. $\tilde{q}$ is related to the state variables k^w and ψ by the expression

$$\tilde{q} = \frac{k^w - k_I[\psi]}{k_Y[\psi] - k_I[\psi]}, \quad 1-\tilde{q} = \frac{k_Y[\psi] - k^w}{k_Y[\psi] - k_I[\psi]}. \tag{95}$$

One can use this definition to simplify Expression (94) as follows

$$k^w = \tilde{q} k_Y + (1-\tilde{q}) k_I. \tag{96}$$

Next we use the definitions of q, q', and $\tilde{q}$ to derive the equations of motion of the international economy. We begin with the evolution of human capital in each country.

13.2 Human Capital Accumulation

Growth of human capital in each country is governed by Equations (51) and (52) which we write in intensive form:

$$\frac{\dot{H}}{H} = (1-q)\, g\left(k_I[\psi]\right), \tag{97}$$

$$\frac{\dot{H}'}{H'} = \left(1-q'\right) g\left(k'_I[\psi]\right). \tag{98}$$

Since $k_I = k'_I$, from the static conditions for an optimum, the growth rate in each sector will depend only on the relative magnitudes of q and q'.

13.3 Physical Capital Accumulation

Using Definitions (91) and (92) and exploiting the linear homogeneity of the production function one can rewrite the capital accumulation equation, (50), in intensive form:

$$\frac{\dot{K}^W}{K^W} = \frac{q\theta(H+H')}{K^W} f(k_Y) + \frac{q'(1-\theta)(H+H')}{K^W} f\left(k'_Y\right) - \frac{C}{K^W} - \frac{C'}{K^W}, \tag{99}$$

Noting that $k^w = K^W/(H+H')$ using facts that $k_I = k_I'$, $k_Y = k_Y'$ and $\tilde{q} \equiv q\theta + q'(1-\theta)$, k^w must evolve according to

$$\frac{\dot{k}^w}{k^w} = \frac{\tilde{q} f(k_Y[\psi])}{k^w} - \left(\frac{c+c'}{k^w}\right) - (1-\tilde{q})\, g(k_I[\psi]) . \tag{100}$$

Equation (53) and definitions of c, c' and λ imply that

$$c + c' = \frac{s(t)}{\lambda},$$

where

$$s(t) \equiv b + (1-b)\, e^{-(\rho'-\rho)t}.$$

Using this fact, plus the expression for $\tilde{q}$ given by Equation (95) leads to the expression we seek:

$$\frac{\dot{k}^w}{k^w} = \left(\frac{k^w - k_I[\psi]}{k_Y[\psi] - k_I[\psi]}\right)\frac{f(k_Y[\psi])}{k^w} - \frac{s(t)}{\lambda k^w} - \left(\frac{k_Y[\psi] - k^w}{k_Y[\psi] - k_I[\psi]}\right) g(k_I[\psi]) . \tag{101}$$

which is equation (65) in the text.

13.4 Relative Prices

The costate variables, for an optimal plan, must satisfy the conditions:

$$\frac{\dot{\Lambda}}{\Lambda} = \rho - f_k(k_Y), \tag{102}$$

$$\frac{\dot{M}}{M} = \rho - g_H(k_I), \tag{103}$$

$$\frac{\dot{M}'}{M} = \rho - g_H(k_I'), \tag{104}$$

and transversality requires that

$$\lim_{t\to\infty} e^{-\rho t}\Lambda K^W = \lim_{t\to\infty} e^{-\rho t} MH = \lim_{t\to\infty} e^{-\rho t} M'H' = 0. \tag{105}$$

Combining Equations (102)–(104), using the definition of ψ:

$$\frac{\dot{\psi}}{\psi} = \left[f_k(k_Y[\psi]) - g_H(k_I[\psi])\right] . \tag{106}$$

which is equation (67) in the text.

13.5 The Shadow Price of Consumption

Recall that $\lambda = \Lambda(H + H')$. It follows from Equations (102), (97) and (98) that the equation of motion for λ is given by:

$$\frac{\dot{\lambda}}{\lambda} = \rho - f_k(k_Y[\psi]) + (1 - \tilde{q})\, g(k_I[\psi]).$$

From the definition of $\tilde{q}$, this gives

$$\frac{\dot{\lambda}}{\lambda} = \rho - f_k(k_Y[\psi]) + \left(\frac{k_Y[\psi] - k^w}{k_Y[\psi] - k_I[\psi]}\right) g(k_I[\psi]), \tag{107}$$

which is equation (66) in the text.

13.6 The Evolution of θ

From the definition of θ:

$$\frac{\dot{\theta}}{\theta} = \frac{\dot{H}}{H} - \theta\frac{\dot{H}}{H} - (1 - \theta)\frac{\dot{H}'}{H'}.$$

Using equations (97) and (98) and the fact that $g(k_I) = g(k_I')$ we can rewrite this expression as:

$$\frac{\dot{\theta}}{\theta} = (1 - \theta)\left(q' - q\right) g(k_I[\psi]). \tag{108}$$

Using expression (95)

$$q' - q = \frac{q' - \tilde{q}}{\theta} = \frac{1}{\theta}\left(q' - \left[\frac{k^w - k_I[\psi]}{k_Y[\psi] - k_I[\psi]}\right]\right),$$

from whence it follows that

$$\frac{\dot{\theta}}{\theta} = \frac{(1 - \theta)}{\theta}\left(q' - \left[\frac{k^w - k_I[\psi]}{k_Y[\psi] - k_I[\psi]}\right]\right) g(k_I[\psi]), \tag{109}$$

which is equation (68) in the text.

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Figure 1: Savings and Investment as Percentages of GDP

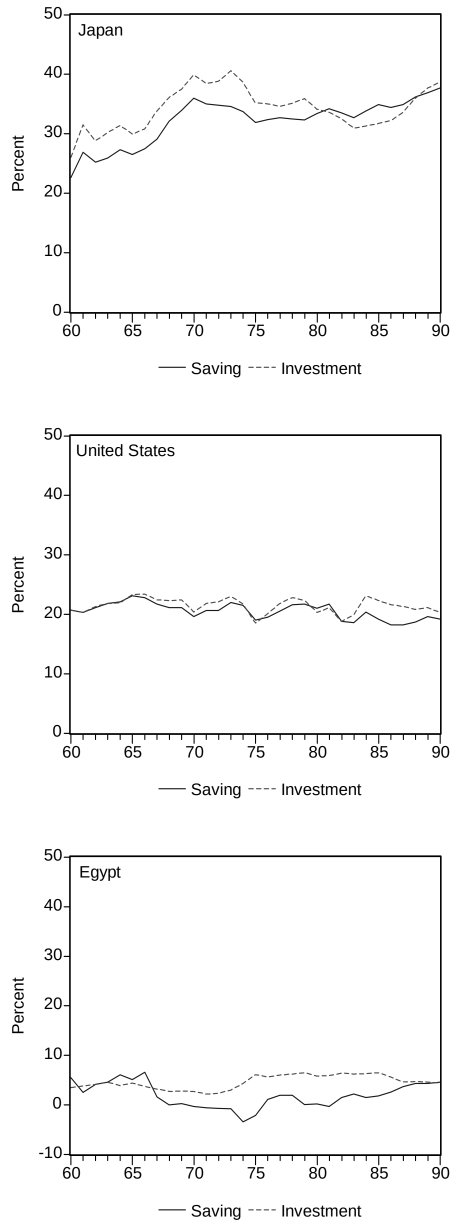

Figure 2: Continental per capita real GDP

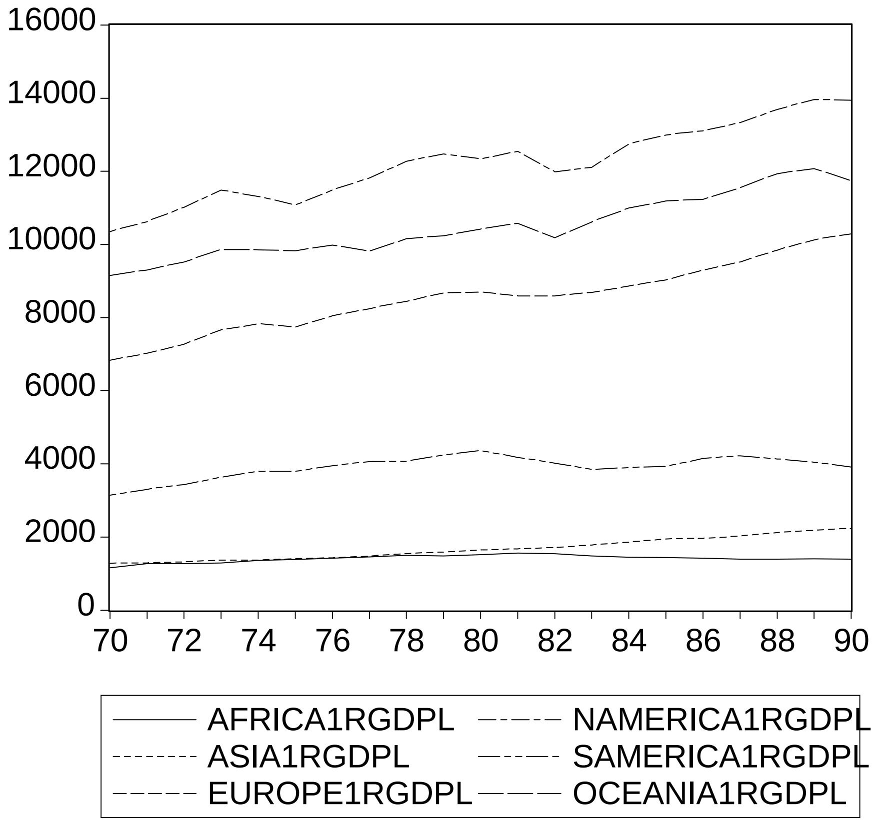

Figure 3: The Stable Manifold when Human Capital is more Capital Intensive

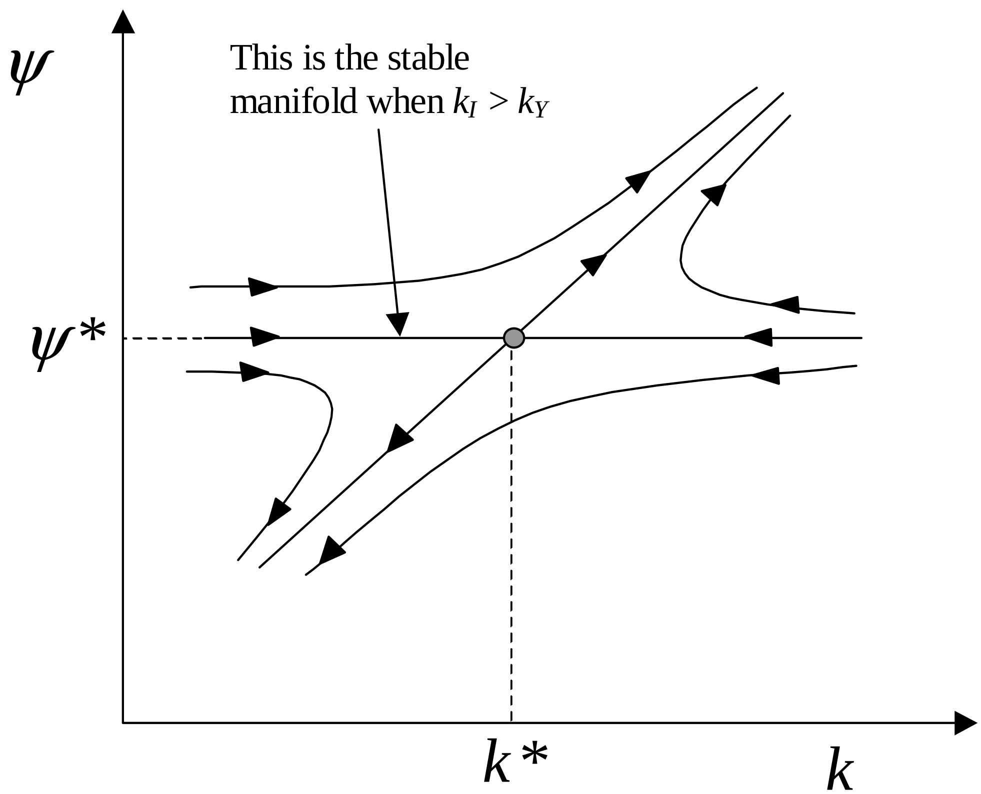

Figure 4: The Stable Manifold when Physical Capital is more Capital Intensive