DISCUSSION PAPER SERIES

No. 3233

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FINANCIAL ECONOMICS

Centre for Economic Policy Research

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ISSN 0265-8003

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Discussion Paper No. 3233
February 2002

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CEPR Discussion Paper No. 3233

February 2002

ABSTRACT

Coordination Failures and the Lender of Last Resort: Was Bagehot Right After All?*

The classical doctrine of the Lender of Last Resort, elaborated by Thornton (1802) and Bagehot (1873), asserts that the Central Bank should lend to 'illiquid but solvent' banks under certain conditions. Several authors have argued that this view is now obsolete: when interbank markets are efficient, a solvent bank cannot be illiquid. This Paper provides a possible theoretical foundation for rescuing Bagehot's view. Our theory does not rely on the multiplicity of equilibria that arises in classical models of bank runs. We build a model of banks' liquidity crises that possesses a unique Bayesian equilibrium. In this equilibrium, there is a positive probability that a solvent bank cannot find liquidity assistance in the market. We derive policy implications about banking regulation (solvency and liquidity ratios) and interventions of the Lender of Last Resort as well as on the disclosure policy of the Central Bank.

JEL Classification: G28
Keywords: central bank policy, global games, interbank market, liquidity ratio, orderly failure resolution, prompt corrective action, prudential regulation, solvency ratio, supermodular games and transparency

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* We are grateful to Sudipto Bhattacharya, Ramon Caminal, Xavier Freixas, Martin Hellwig, John Moore, Bruno Parigi, Hyun Shin, Elu von Thadden, and Jean Tirole for helpful discussions and comments. This Paper is produced as part of a CEPR research network on 'The Industrial Organisation of Banking and Financial Markets in Europe', funded by the European Commission under the Training and Mobility of Researchers Programme (Contract No: ERBFMRX-CT98-0222)

Submitted 06 February 2002

1 Introduction

There have been several recent controversies about the need for a Lender of Last Resort (LLR) both within national banking systems (Central Bank) and at an international level (IMF).[3] The concept of a LLR was elaborated in the XIX century by the governor of the bank of England, Thornton and by the editor of *The Economist*, Bagehot. An essential point of the "classical" doctrine associated to Bagehot asserts that the LLR role is to lend to "solvent but illiquid" banks under certain conditions.[4]

Banking crises have been recurrent in most financial systems. The LLR facility and deposit insurance were instituted precisely to provide stability to the banking system and avoid the consequences for the real sector. Indeed, financial distress may cause important damage to the economy as the example of the Great Depression makes clear (Bernanke (1983) and Bernanke and Gertler (1989)). Traditional banking panics were eliminated with the LLR facility and deposit insurance by the end of the XIX century in Europe, after the crisis of the 1930s in the US and also mostly in emerging economies, which have suffered numerous crises until today.[5] Modern liquidity crises associated to securitized money or capital markets have also required the intervention of the LLR. Indeed, the Federal Reserve intervened in the crises provoked by the failure of Penn Central in the US commercial paper market in 1970, by the stock market crash of October 1987 and by Russia's default in 1997 and subsequent collapse of LTCM (in the latter case a "lifeboat" was arranged by the New York Fed). For example, in October 1987 the Federal Reserve

[3] See for instance Calomiris (1998a,b), Kaufman (1990), Fisher (1998), Mishkin (1998), and Goodhart and Huang (1999a,b).

[4] The LLR should lend freely against good collateral, valued at pre-crisis levels, and at a penalty rate. Bagehot (1873), also presented for instance in Humphrey (1975) and Freixas et al. (1999).

[5] See Gorton (1988) for US evidence and Lindgren et al (1996) for evidence on IMF member countries.

supplied liquidity to banks with the discount window.[6]

The function of the LLR of providing emergency liquidity assistance has been criticized for provoking moral hazard on the banks' side. Perhaps more importantly, Goodfriend and King (1988) (see also Bordo (1990), Kaufman (1991) and Schwartz (1992)) remark that Bagehot's doctrine was elaborated at a time where financial markets were underdeveloped. They argue that, while central banks interventions on aggregate liquidity (monetary policy) are still warranted, individual interventions (banking policy) are not anymore: "with sophisticated interbank markets, banking policy has become redundant".

Open market operations can provide sufficient liquidity which is then allocated by the interbank market. The discount window is not needed. In other words, Goodfriend and King argue that when financial markets are efficient, a solvent institution cannot be illiquid. Banks can finance their assets with interbank funds, negotiable certificates of deposit (CDs) and repurchase agreements (repos). Well informed participants in this interbank market will make out liquidity from solvency problems. This view has consequences also for the debate about the need of an international LLR. Indeed, Chari and Kehoe (1998) claim, for example, that such an international LLR is not needed because the joint action of the Federal Reserve, the European Central Bank and the Bank of Japan can take care of any international liquidity problem.

Those developments have led qualified observers to dismiss bank panics as a phenomenon of the past and express confidence on the efficiency of financial markets, in particular the interbank market, to resolve liquidity problems of financial intermediaries. This is based on the view that participants in the interbank market are the most well informed agents to ascertain the solvency of an institution with liquidity problems.[7]

[6] See Folkerts-Landau and Garber (1992).

[7] For example, Tommaso Padoa-Schioppa, member of the Executive Committee of the European Cen-

The main objective of this article is to provide a theoretical foundation for Bagehot's doctrine in a model that fits the modern context of sophisticated and presumably efficient financial markets. We are thinking of a short time horizon (say 2 days) that corresponds to liquidity crises. We shift emphasis from maturity transformation and liquidity insurance of small depositors to the "modern" form of bank runs where large well-informed investors refuse to renew their credit (CDs for example) on the interbank market. The decision not to renew credit may arise as a result on an event (failure of Penn Central, October 1987 crash or LTCM failure) which puts in doubt the repayment capacity of an intermediary or a number of intermediaries. The Central Bank may then decide to provide liquidity to those troubled institutions. The question arises about whether such intervention is warranted. At the same time it is debated whether central banks should disclose the information they have on potential crisis situations (or the predictions of their internal forecasting models) and what degree of transparency should a Central Bank's announcements have.[8] We also hope to shed some light on the issue of transparency and optimal disclosure of information by the Central Bank.

Since Bryant (1980) and Diamond and Dybvig (1983), banking theory has insisted on the fragility of banks due to possible coordination failures between depositors (bank runs). However it is hard to base any policy recommendation on their model, since it systematically possesses multiple equilibria. Furthermore, a run equilibrium needs to be justified

tral Bank in charge of banking supervision, has gone as far as saying that classical bank runs may occur only in textbooks, precisely because measures like deposit insurance and capital adequacy requirements have been put in place. Furthermore, despite recognizing that "rapid outflows of uninsured interbank liabilities" are less unlikely, Padoa-Schioppa states that "However, since interbank counterparties are much better informed than depositors, this event would typically require the market to have a strong suspicion that the bank is actually insolvent. If such a suspicion were to be unfounded and not generalised, the width and depth of today's interbank market is such that other institutions would probably replace (possibly with the encouragement of the public authorities as described above) those which withdraw their funds" (Padoa-Schioppa (1999)).

[8] See, for example, Tarkka and Mayes (2000).

with the presence of sunspots that coordinate the behavior of investors. Indeed, otherwise no one would deposit in a bank that will be subject to run. This view of banking instability has been disputed by Gorton (1985) and others who argue that crises are related to fundamentals and not to self-fulfilling panics. In this view, crises are triggered by bad news about the returns to be obtained by the bank. Gorton (1988) studies panics in the National Banking Era in the US and concludes that crises were predictable by indicators of the business cycle.[9] There is an ongoing empirical debate about whether crises are predictable and their relation to fundamentals.[10]

Our approach is inspired by Postlewaite and Vives (1987), who display an incomplete information model with a unique Bayesian equilibrium with a positive probability of bank runs, and the model is adapted from the "global game" analysis of Carlsson and Van Damme (1993) and Morris and Shin (1998).[11] This approach builds a bridge between the "panic" and "fundamentals" view of crises by linking the probability of occurrence of a crisis to the fundamentals. A crucial property of the model is that, when the private information of investors is precise enough, the game among them has a unique equilibrium. Moreover, at this unique equilibrium there is an intermediate interval of values of the bank's assets for which, in the absence of intervention by the Central Bank, the bank is solvent but can fail by the fact that a too large proportion of depositors withdraw their money. In other words, in this intermediate range for the fundamentals there is the potential for a coordination failure. Furthermore, the range in which such coordination failure occurs diminishes with the ex ante strength of fundamentals.

Given that this equilibrium is unique and based on the fundamentals of the bank, we are

[9] The phenomenon has been theorized in the literature on information-based bank runs such as Chari and Jagannathan (1988), Jacklin and Bhattacharya (1988) and Allen and Gale (1998).

[10] See also Kaminsky et al (1999) and Radelet and Sachs (1998) for perspectives on international crisis.

[11] See also Heinemann and Illing (2000) and Corsetti et al (2000).

able to provide some policy recommendations on how to avoid such failure. More specifically, we discuss the articulation between ex-ante regulation of solvency and liquidity ratios and ex-post provision of emergency liquidity assistance. It is found that liquidity and solvency regulation can solve the coordination problem but typically the cost is too high in terms of foregone returns. This means that prudential measures have to be complemented with emergency discount window loans.

We also introduce a public signal and discuss the optimal disclosure policy of the Central Bank. Indeed, the Central Bank typically has information about banks that the market does not have (and, conversely, market participants have also information complementary to the Central Bank knowledge).[12] The model allows for the information structures of the Central Bank and investors to be non-nested. Our discussion has a bearing on the slippery issue of the optimal degree of transparency of Central Bank announcements. Indeed, Alan Greenspan has become famous for his oblique way of saying things, fostering an industry of "Greenspanology" or interpretation of his statements. Our model may rationalize oblique statements by central bankers that seem to add noise to a basic message. Indeed, we will show that precisely because the Central Bank may be in a unique position to provide information that becomes common knowledge it has the capacity to destabilize expectations in the market (which in our context means to move the interbank market to a regime of multiple equilibria). By fudging the disclosure of information the Central Bank makes sure that somewhat different interpretations of the release will be made preventing destabilization. The potential damaging effects of public information is a theme also developed in Morris and Shin (2001).

Finally, we endogenize the short-term debt structure as a way to discipline bank managers

[12] See Peek et al (1999), De Young et al (1998), and Berger et al (1998).

because of a moral hazard problem. The framework allows us to discuss early closure policies of banks and the interaction of the LLR, prompt corrective action and orderly resolution of failures. We can study then the adequacy of Bagehot's doctrine in a richer environment.

The rest of the article is organized as follows:

- Section 2 presents the model.
- Section 3 discusses runs and solvency.
- Section 4 characterizes the equilibrium of the game between investors.
- Section 5 studies the properties of this equilibrium and the effect of prudential regulation on coordination failure.
- Section 6 discusses the LLR policy implications of our model and the relations with Bagehot's doctrine.
- Section 7 introduces a public signal and discusses transparency.
- Section 8 sketches how to endogenize the liability structure and its welfare implications with attention to crisis resolution.
- Concluding remarks end the paper.

2 The Model

Consider a market with three dates: $\tau = 0, 1, 2$. At date $\tau = 0$ the bank possesses own funds E, and collects uninsured wholesale deposits (CDs for example) for some amount D_0, normalized to 1. These funds are used in part to finance some investment I in risky assets (loans), the rest being held in cash reserves M. Under normal circumstances, the returns RI on these assets are collected at date $\tau = 2$, the CDs are repaid, and the stockholders of the bank get the difference (when it is positive). However, early withdrawals may occur at an interim date $\tau = 1$, following the observation of private signals on the future realization of R. If the proportion x of these withdrawals exceeds the cash reserves M of the bank, the bank is forced to sell some of its assets. To summarize our notation, the bank's balance sheet at $\tau = 0$ is represented as follows:

I	$D_0 = 1$
M	E

where:

- D_0 (=1) is the volume of uninsured wholesale deposits, normally repaid at $\tau = 2$ but that can also be withdrawn at $\tau = 1$. The nominal value of deposits upon withdrawal is $D \geq 1$ independently of the withdrawal date. So, early withdrawal entails no cost for the depositors themselves (when the bank is not liquidated prematurely). We assume that the withdrawal decision is delegated to fund managers who typically prefer to renew the deposits (i.e. not to withdraw early) but are penalized by the depositors if the bank fails. Suppose that fund managers obtain a benefit $B > 0$ if they get the money back or if they withdraw and the bank fails. They get nothing otherwise. However, to withdraw involves a cost $C > 0$ for the managers (for example because their reputation suffers if they have to recognize that they have made a bad investment). The net expected benefit of withdrawing is $B - C > 0$ while the one of not withdrawing is $(1 - P)B$, where P is the probability that the bank fails. Accordingly, fund managers adopt the following behavioral rule: withdraw if and only if they anticipate $P > \gamma = C/B$, where $\gamma \in (0, 1)$.

- E represents the value of equity (or more generally long term debt; it may also include insured deposits[13]).
- I denotes the volume of investment in risky assets, which have a random return R at $\tau = 2$.
- Finally, M is the amount of cash reserves (money) held by the bank.

At $\tau = 1$, uninsured fund manager i privately observes a signal $s_i = R + \varepsilon_i$, where the $\varepsilon_i s$ are i.i.d. and also independent of R. As a result, a proportion x of them decides to "withdraw" (i.e. not to renew their CDs). By assumption there is no other source of financing for the bank (except maybe the Central Bank, see below) so if $x > \frac{M}{D}$, the bank is forced to sell a volume y of assets:[14] if the needed volume of sales y is greater than the total of available assets I the bank fails at $\tau = 1$. If not, the bank continues until date 2. Failure occurs at $\tau = 2$ whenever

$$R(I - y) < (1 - x)D. \tag{1}$$

Our modeling tries to capture in the simplest possible way the main institutional features of modern interbank markets. In our model, banks essentially finance themselves by two complementary sources: equity (or long term debt) and uninsured short term deposits (or CDs), which are uncollateralized and involve fixed repayments. However, in case of a liquidity shortage at date 1, banks also have the possibility to sell some of their assets (or equivalently borrow against collateral) on the repo market. This secondary market for bank assets is assumed to be informationally efficient, in the sense that the secondary

[13]If they are fully insured, these deposits have no reason to be withdrawn early and can thus be assimilated to stable resources.

[14]These sales are typically accompanied with a repurchase agreement or repo. They are thus equivalent to a collateralized loan.

price aggregates the decentralized information of investors about the quality of the bank's assets.[15] Therefore we assume that the resale value of the bank's assets depends on R. However bank owners cannot obtain the full value of these assets but only a fraction of this value $\frac{1}{1+\lambda}$, with $\lambda > 0$. Accordingly the volume of sales needed to face withdrawals x is given by:

$$y = (1+\lambda)\frac{[xD - M]_+}{R}$$

where $(xD - M)_+ = \max(0, xD - M)$.

The parameter λ measures the cost of "fire sales" in the secondary market for bank assets. It is crucial for our analysis[16], and can be explained by different types of considerations: limited commitment of future cash flows (as in Hart and Moore (1994) or Diamond and Rajan (1997)), moral hazard (as in Holmstrom and Tirole (1997)) or adverse selection (as in Flannery (1996)). We have chosen to stress the last explanation, because it gives a simple justification for the superiority of the Central Bank over financial markets in the provision of liquidity to banks in trouble. Suppose indeed that the risky assets of the bank consist of a continuum of infinitesimal loans indexed by $j \in [0,1]$ of returns Rv_j where the $v_j s$ are i.i.d. and uniformly distributed on the interval $[\frac{1}{1+\lambda}, \frac{1+2\lambda}{1+\lambda}]$. Suppose also that individual investors are all infinitesimal (so that they can only buy one of the loans) and cannot observe the $v_j s$ (which are privately observed by the bank). Each individual investor is therefore afraid to get the lowest quality loan, thus the maximum price he is ready to pay is $\frac{R}{1+\lambda}$. The superiority of the Central Bank resides in its large financial capacity, and thus its ability to eliminate the adverse selection problem by buying the

[15] We can imagine for instance that the bank organizes an auction among investors for the sale of its assets. The investors bid optimally given their private signals s_i. Since we assume that there is a large number of such depositors and that their signals are independent, the law of large numbers implies that the equilibrium price p of this auction is a deterministic function of R .

[16] For a similar assumption in a model of an international lender of last resort, see Goodhart and Huang (1999b).

entire portfolio (or a representative sample) at a unit price of R. The parameter λ can also be interpreted as a liquidity premium, i.e. the interest margin that the market requires for lending on a short notice.[17] Therefore our model can be thought as either applying to the financial distress of an individual bank (a bank is close to insolvency when R is small) or to a generalized banking crisis (a liquidity shortage implying a large λ).

We do not assume any direct inefficiency of interbank markets since operations on these markets do not involve any physical liquidation of bank assets. However, we will show that when a bank is close to insolvency (R small) or when there is a liquidity shortage (λ large) the interbank markets do not suffice to prevent early closure of the bank. Early closure involves the physical liquidation of assets and this is costly. We model this liquidation cost (not to be confused with the fire sales premium λ) as proportional to the future returns on the bank's portfolio. If the bank is liquidated at $\tau = 1$, the (per unit) liquidation value of its assets is νR, with $\nu \ll \frac{1}{1+\lambda}$.[18]

3 Runs and solvency

We focus in this section on some features of banks' liquidity crises that cannot be properly taken into account within the classical Bryant-Diamond-Dybvig (BDD) framework. In doing so we take the banks' liability structure (and in particular the fact that an important fraction of these liabilities can be withdrawn on demand) as exogenous. A possible way to endogenize the bank's liability structure is to introduce a disciplining role for liquid deposits. In Section 8 we explore such an extension.

[17] See Allen and Gale (1998) for a model where costly liquidation (asset sales) arises due to the presence of liquidity constrained speculators in the resale market.

[18] We could carry out our analysis assuming a physical liquidation cost at $\tau = 1$, identifying ν and $\frac{1}{1+\lambda}$. However, this simplification would come at the cost of not modelling properly the interbank market.

We adopt explicitly the short time horizon (say 2 days) that corresponds to liquidity crises. This means that we shift the emphasis from maturity transformation and liquidity insurance of small depositors to the "modern" form of bank runs, i.e. large investors refusing to renew their CDs on the interbank market.

A second element that differentiates our model from BDD is that our bank is not a mutual bank, but a corporation that acts in the best interest of its stockholders. This allows us to discuss the role of equity and the articulation between solvency requirements and provision of emergency liquidity assistance. However a proper modeling of the role of equityholders remains to be done.

As a consequence of these assumptions, the relation between x, the proportion of early withdrawals, and the failure of the bank is different from that in BDD. To see this, let us recapitulate the different cases:

- $xD \leq M$: there is no liquidation at $\tau = 1$. In this case there is failure at $\tau = 2$ if and only if

  $$RI + M < D \Leftrightarrow R < R_s = \frac{D - M}{I} = 1 - \frac{1 + E - D}{I}.$$

  R_s can be interpreted as the solvency threshold of the bank. It is a decreasing function of the solvency ratio $\frac{E}{I}$.

- $M < xD \leq M + \frac{RI}{1+\lambda}$: there is partial liquidation at $\tau = 1$. Failure occurs at $\tau = 2$ if and only if

  $$RI - (1 + \lambda)(xD - M) < (1 - x)D \Leftrightarrow R < R_s + \lambda \frac{xD - M}{I} = R_s \left[1 + \lambda \frac{xD - M}{D - M}\right].$$

This formula illustrates how, because of the premium λ , solvent banks can fail when the proportion x of early withdrawals is too big[19]. Notice however an important difference with BDD: when the bank is "supersolvent" ($R > (1+\lambda)R_s$) it can never fail, even if everybody withdraws ($x = 1$).

- Finally, when $xD > M + \frac{RI}{1+\lambda}$, the bank is closed at $\tau = 1$ (early closure). This

The failure thresholds are summarized in Figure 1 below:

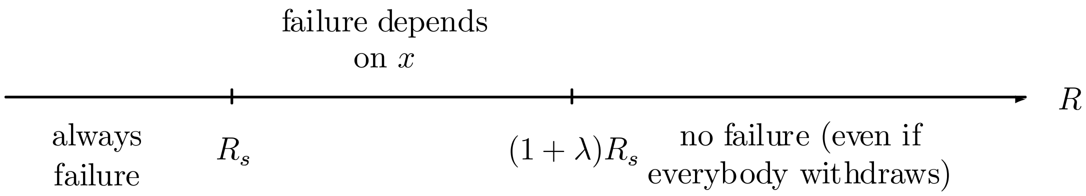


Figure 1

Several comments are in order:

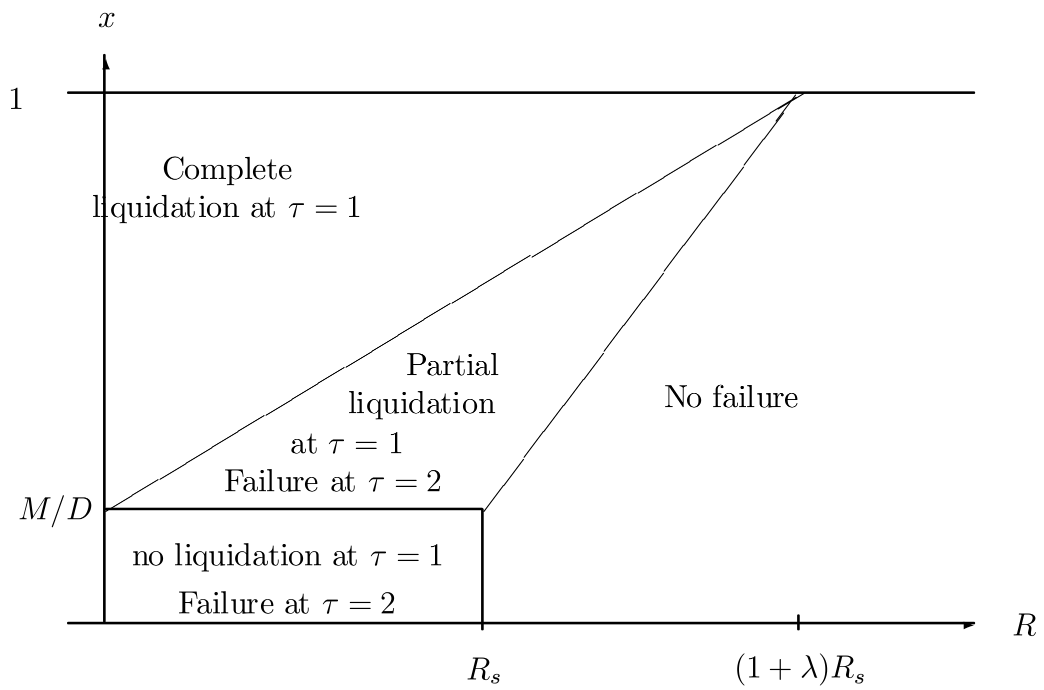
- In our model, early closure is never ex post efficient because to physically liquidate assets is costly. However, as discussed in Section 8, early closure may be ex ante efficient to discipline bank managers and induce them to exert effort.

- The perfect information benchmark of our model (where R is common knowledge at $\tau = 1$) has different properties than in BDD: the multiplicity of equilibria only arises in the median range $R_s \leq R \leq (1+\lambda)R_s$. When $R_s > R$ everybody runs ($x = 1$), when $R > (1+\lambda)R_s$ nobody runs ($x = 0$) and only in the intermediate region both

[19] Note that we can interpret that to obtain resources $xD - M > 0$ we need to liquidate a fraction of the portfolio $\mu = \frac{xD-M}{RI}(1+\lambda)$ and therefore at $\tau = 2$ we have left $R(1-\mu)I = RI - (1+\lambda)(xD - M)$.

equilibria coexist.[20] As we will see, and following the ideas introduced by Carlsson and Van Damme (1993), this pattern is crucial for being able to selecting a unique equilibrium through the introduction of private noisy signals (when noise is not too important, as in Morris and Shin (1998)). Goldstein and Pauzner (2000) adapt the same methodology to the BDD model, in which the perfect information game always has two equilibria, even for very large R. Accordingly, they have to make an extra assumption, namely that "there exists an external lender who would be willing to buy any amount of the investment... if she knew for sure that the long-run return was excessively high" (Goldstein and Pauzner (2000), p.11), in order to obtain a unique equilibrium in the presence of private signals with small noise.[21]

The different regimes of the bank, as a function of R and x, are represented in Figure 2.



[20]When $R_s > R$ fund managers get $B - C > 0$ by withdrawing and nothing by waiting. When $R > (1+\lambda)R_s$fund managers by withdrawing get $B - C$ and by waiting B. Note that if depositors made directly the investment decisions the equilibria would be the same provided that there is a small cost of withdrawal.

[21]See also Morris and Shin (2000).

Figure 2

The critical value of R below which the bank is closed early is given by:

$$R_{ec}(x) = (1+\lambda)\frac{(xD-M)_+}{I}.$$

The critical value of R below which the bank fails is given by:

$$R_f(x) = R_s + \lambda\frac{(xD-M)_+}{I}. \tag{2}$$

The parameters R_s, M and I are not independent. Since we want to study the impact of prudential regulation on the need for Central Bank intervention, we will focus on R_s (a decreasing function of the solvency ratio E/I) and $m = \frac{M}{D}$ (the liquidity ratio). Replacing I by its value $\frac{D-M}{R_s}$, we obtain:

$$R_{ec}(x) = R_s(1+\lambda)\frac{(x-m)_+}{1-m}, \text{ and}$$

$$R_f(x) = R_s(1+\lambda\frac{(x-m)_+}{1-m}).$$

4 Equilibrium of the investors' game

In order to simplify the presentation, we concentrate on "threshold" strategies, in which each fund manager decides to withdraw if and only if his signal is below some threshold t.[22] As we will see later this is without loss of generality. For a given R, a fund manager

[22] It is assumed that the decision on whether to witdraw is taken before the secondary market is organized and fund managers have the opportunity to learn about R from the secondary price. (On this issue see Atkeson's comments on Morris and Shin (2000).)

withdraws with probability

$$\Pr[R+\varepsilon<t]=G(t-R),$$

where G is the c.d.f. of the random variable ε. Given our assumptions, this probability also equals the proportion of withdrawals $x(R,t)$.

A fund manager withdraws if and only if the probability of failure of the bank (conditional on the signal s received by the manager and the threshold t used by other managers) is large enough. That is, $P(s,t)>\gamma$, where

$$\begin{aligned} P(s,t) &= \Pr[\text{failure}|s,t] \\ &= \Pr[R<R_f(x(R,t))|s]. \end{aligned}$$

Before we analyze the equilibrium of the investor's game let us look at the region of the plane (t,R) where failure occurs. For this, transform Figure 2 by replacing x by $x(R,t)=G(t-R)$. We obtain Figure 3 below.

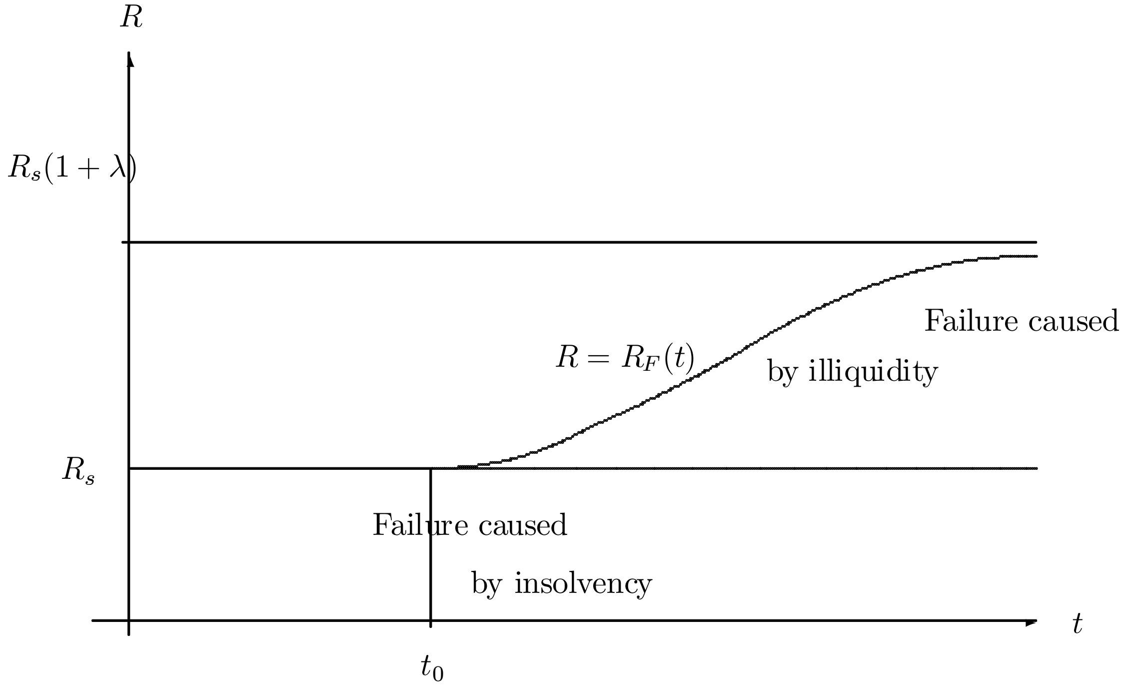


Figure 3

Notice that $R_F(t)$, the critical R that triggers failure is equal to the solvency threshold R_s when t is low and fund managers are confident about the strength of fundamentals:

$$R_F(t) = R_s \quad \text{if} \quad t \leq t_0 = R_s + G^{-1}(m).$$

However, for $t > t_0$, $R_F(t)$ is an increasing function of t and is defined implicitly by

$$R = R_s(1 + \lambda[\frac{G(t-R) - m}{1-m}]).$$

Let us denote by $G(.|s)$ the c.d.f. of R conditional on signal s :

$$G(r|s) = \Pr[R < r|s].$$

Then given the definition of $R_F(t)$

$$P(s,t) = \Pr[R < R_F(t)|s] = G(R_F(t)|s) \tag{3}$$

It is natural to assume that $G(r|s)$ is decreasing in s: the higher s, the lower the probability that R lies below any given threshold r. Then it is immediate that P is decreasing in s and nondecreasing in t: $\frac{\partial P}{\partial s} < 0$ and $\frac{\partial P}{\partial t} \geq 0$. This means that the depositors' game is one of strategic complementarities. Indeed, given that other fund managers use the strategy with threshold t the best response of a manager is to use a strategy with threshold $\overline{s}$: withdraw if and only if $P(s,t) > \gamma$ or equivalently if and only if $s < \overline{s}$ where $P(\overline{s},t) = \gamma$. Let $\overline{s} = S(t)$. Now we have that $S' = -\frac{\partial P/\partial t}{\partial P/\partial s} \geq 0$, a higher threshold t by others induce a manager to use also a higher threshold.

The strategic complementarity property holds for general strategies. For a fund manager all that matters is the conditional probability of failure for a given signal and this depends only on the aggregate withdrawals. Recall that the differential payoff to a fund manager for withdrawing over not withdrawing is given by $PB - C$ where $C/B = \gamma$. A strategy for a fund manager is a function $a(s) \in \{\text{not withdraw, withdraw}\}$. If more managers withdraw then the probability of failure conditional on receiving signal s increases. This just means that the payoff to a fund manager displays increasing differences with respect to the actions of others. The depositor's game is a supermodular game and there will exist a largest and a smallest equilibrium. In fact, the game is symmetric (that is, exchangeable against permutations of the players) and therefore the largest and smallest equilibria are symmetric.[23] At the largest equilibrium every fund manager withdraws in the largest number of occasions, at the smallest equilibrium every fund manager withdraws in the smallest number of occasions. The largest (smallest) equilibrium can be identified then with the highest (lowest) threshold strategy $\overline{t}(\underline{t})$.[24] These extremal equilibria bound the set of rationalizable outcomes. That is, strategies outside this set can be eliminated by iterated deletion of dominated strategies.[25] We will make assumptions so that $\overline{t} = \underline{t}$ and equilibrium will be unique.

The threshold $t = t^*$ corresponds to a (symmetric) Bayesian Nash equilibrium if and only if $P(t^*, t^*) = \gamma$. Indeed, suppose that funds managers use the threshold strategy t^*. Then for $s = t^*$, $P = \gamma$ and since P is decreasing in s for $s < t^*$ we have that $P(s, t^*) > \gamma$ and the manager withdraws. Conversely, if t^* is a (symmetric) equilibrium then for $s = t^*$

[23] See Remark 15, p.34 in Vives (1999). See also Chapter 2 in the same reference for an exposition of the theory of supermodular games.

[24] The extremal equilibria can be found with the usual algorithm in a supermodular game (Vives (1990)), starting at the extremal points of the strategy sets of players and iterating using the best responses. For example, to obtain $\overline{t}$ let all investors withdraw for any signal received (that is, start from $\overline{t}_0 = +\infty$ and $x = 1$) and applying iteratively the best response $S(\cdot)$ of a player obtain a decreasing sequence $\overline{t}_k$ that converges to $\overline{t}$.Note that $S(+\infty) = \overline{t}_1 < +\infty$ where $\overline{t}_1$ is the unique solution to $P(t, +\infty) = G(R_s(1+\lambda)|t) = \gamma$ given that G is (strictly) decreasing in t.

[25] See Morris and Shin (2000) for an explicit demonstration of the outcome of iterative elimination of dominated strategies in a similar model.

there is no withdrawal and therefore $P(t^*,t^*) \leq \gamma$. If $P(t^*,t^*) < \gamma$ then by continuity for s close but less than t^* we would have $P(s,t^*) < \gamma$, a contradiction. It is clear then that the largest and the smallest solutions to $P(t^*,t^*) = \gamma$ correspond respectively to the largest and smallest equilibrium.

An equilibrium can also be characterized by a couple of equations in two unknowns (a withdrawal threshold $t^*$and a failure threshold R^*):

$$G(R^*|t^*) = \gamma, \text{ and} \tag{4}$$

$$R^* = R_s(1 + \lambda[\frac{G(t^* - R^*) - m}{1 - m}]_+). \tag{5}$$

Equation (4) states that conditionally on observing a signal $s = t^*$, the probability that $R < R^*$ is γ. Equation (5) states that, given a withdrawal threshold t^*, R^* is the critical return (i.e. the one below which failure occurs). Equation (5) implies that R^* belongs to $[R_s, (1+\lambda)R_s]$. Notice that early closure occurs whenever $D.x(t^*, R) > M + \frac{IR}{1+\lambda}$,where $x(t^*, R) = G(t^* - R^*)$. This happens if and only if R is smaller than some threshold $R_{EC}(t^*)$. Clearly, $R_{EC}(t^*)$ is always smaller than the failure threshold R^* since early closure implies failure, while the converse is not true (see Figure 2).

In order to simplify the analysis of this system we are going to make distributional assumptions on returns and signals. More specifically, we will assume that the distributions of R and ϵ are normal, with respective means $\overline{R}$ and 0, and respective precisions (i.e. inverse variances) α and β. Denoting by Φ the c.d.f. of a standard normal distribution the equilibrium is characterized then by a pair (t^*, R^*) such that:

$$\Phi\left(\sqrt{\alpha+\beta}R^* - \frac{\alpha\bar{R} + \beta t^*}{\sqrt{\alpha+\beta}}\right) = \gamma, \tag{6}$$

and

$$R^* = R_s\left(1 + \lambda\left[\frac{\Phi(\sqrt{\beta}(t^* - R^*)) - m}{1-m}\right]_+\right). \tag{7}$$

We now can now state our first result

Proposition 1 *When β (the precision of the private signal of investors) is large enough relative to α (prior precision), there is a unique t^* such that $P(t^*, t^*) = \gamma$. We conclude that the investor's game has a unique (Bayesian) equilibrium. In equilibrium, fund managers use a strategy with threshold t^*.*

Proof of Proposition 1: We show that $\varphi(s) \stackrel{\text{def}}{=} P(s, s)$ is decreasing for $\beta \geq \beta_0 \stackrel{\text{def}}{=} \frac{1}{2\pi}\left(\frac{\lambda\alpha}{I}\right)^2$ with $I = \frac{D-M}{R_s}$. Under our assumptions R conditional on signal realization s follows a normal distribution $N(\frac{\alpha\bar{R}+\beta s}{\alpha+\beta}, \frac{1}{\alpha+\beta})$. Denoting by Φ the c.d.f. of a standard normal distribution, it follows that

$$\begin{aligned}
\varphi(s) &= P(s, s) = \Pr[R < R_F(s)|s] \\
&= \Phi\left[\sqrt{\alpha+\beta}R_F(s) - \frac{\alpha\bar{R}+\beta s}{\sqrt{\alpha+\beta}}\right]. \qquad (8)
\end{aligned}$$

This function is clearly decreasing for $s < t_0$ since, in this region, we have $R_F(s) \equiv R_s$. Now if $s > t_0$, $R_F(s)$ is increasing and its inverse is

$$t_F(R) = R + \frac{1}{\sqrt{\beta}}\Phi^{-1}\left(\frac{I}{\lambda}(R - R_s) + m\right).$$

The derivative of t_F is

$$t'_F(R) = 1 + \frac{1}{\sqrt{\beta}}\frac{I}{\lambda}\left[\Phi'\left(\Phi^{-1}\left(\frac{I}{\lambda}(R - R_s) + m\right)\right)\right]^{-1}.$$

Since Φ' is bounded above by $\frac{1}{\sqrt{2\pi}}$, t'_F is bounded below:

$$t'_F(R) \geq 1 + \sqrt{\frac{2\pi}{\beta}}\frac{I}{\lambda}.$$

Thus

$$R'_F(s) \leq \left[1 + \sqrt{\frac{2\pi}{\beta}}\frac{I}{\lambda}\right]^{-1}.$$

Given formula (8), $\varphi(s)$ will be decreasing provided that

$$\sqrt{\alpha+\beta}\left(1+\sqrt{\frac{2\pi}{\beta}}\frac{I}{\lambda}\right)^{-1} \le \frac{\beta}{\sqrt{\alpha+\beta}},$$

which, after simplification, gives: $\beta \ge \frac{1}{2\pi}\left(\frac{\lambda\alpha}{I}\right)^2$. If this condition is satisfied, there is at most one equilibrium. Existence is easily shown. When s is small $R_F(s) = R_s$ and equation (6) implies that $\lim_{s\to-\infty}\varphi(s) = 1$. On the other hand, when $s \to +\infty$, $R_F(s) \to (1+\lambda)R_s$ and $\varphi(s) \to 0$. ■

The limit equilibrium when β tends to infinity is easily characterized. From equation (6) we have that $\lim_{\beta\to+\infty}\sqrt{\beta}(R^* - t^*) = \Phi^{-1}(\gamma)$. Given that $\Phi\{-z\} = 1 - \Phi\{z\}$ we obtain that in the limit $t^* = R^* = R_s(1 + \frac{\lambda}{1-m}[\max\{1-\gamma-m, 0\}])$. The critical cutoff R^* is decreasing with γ and ranges from R_s for $\gamma \ge 1-m$ to $(1+\lambda)R_s$ for $\gamma = 0$. It is also nonincreasing in m. As we establish in the next section, these features of the limit equilibrium are also valid for $\beta \ge \beta_0$.

It is worth noting also that with a diffuse prior, $\alpha = 0$, the equilibrium is unique for any private precision of investors (indeed, we have that $\beta_0 = 0$). From (6) and (7) we obtain immediately that $R^* = R_s(1 + \frac{\lambda}{1-m}[\max\{1-\gamma-m, 0\}])$ and $t^* = R^* - \frac{\Phi^{-1}(\gamma)}{\sqrt{\beta}}$. Both the cases $\beta \to +\infty$ and $\alpha = 0$ have in common that each investor faces the maximal uncertainty about the behavior of other investors at the switching point $s_i = t^*$. Indeed, it can be easily checked that in either case the distribution of the proportion $x(R, t^*) = \Phi(\sqrt{\beta}(t^* - R))$ of investors withdrawing is uniformly distributed over $[0,1]$ conditional on $s_i = t^*$. This contrasts with the certainty case with multiple equilibria when $R \in (R_s, (1+\lambda)R_s)$ where, for example, in a run equilibrium an investor thinks that with probability one all other investors will withdraw. It is precisely the need to entertain a wider range of behavior of other investors in the incomplete information game that pins down a unique equilibrium as in Carlsson and Van Damme (1993) or Postlewiate and

Vives (1987).

5 Coordination failure and prudential regulation

For β large enough, we have just seen that there exists a unique equilibrium whereby investors adopt a threshold t^* characterized by

$$\Phi\left(\sqrt{\alpha+\beta}R_F(t^*) - \frac{\alpha\bar{R}+\beta t^*}{\sqrt{\alpha+\beta}}\right) = \gamma,$$

or

$$R_F(t^*) = \frac{1}{\sqrt{\alpha+\beta}}\left(\Phi^{-1}(\gamma) + \frac{\alpha\bar{R}+\beta t^*}{\sqrt{\alpha+\beta}}\right). \tag{9}$$

For this equilibrium threshold, the failure of the bank will occur if and only if:

$$R < R_F(t^*) = R^*.$$

This means that the bank fails if and only if fundamentals are weak, $R < R^*$. When $R^* > R_s$ we have an intermediate interval of fundamentals $R \in [R_s, R^*)$ where there is coordination failure: the bank is solvent but illiquid. The occurrence of coordination failure can be controlled by the level of the liquidity ratio m as the following proposition shows.

Proposition 2 *There is a critical liquidity ratio of the bank $\overline{m}$ such that for $m \geq \overline{m}$ we have that $R^* = R_s$, which means that only insolvent banks fail (there is no coordination failure. Conversely, for $m < \overline{m}$ we have that $R^* > R_s$. This means that for $R \in [R_s, R^*)$ the bank is solvent but illiquid (there is coordination failure).*

Proof of Proposition 2: For $t^* \leq t_0 = R_s + \frac{1}{\sqrt{\beta}}\Phi^{-1}(m)$, the equilibrium occurs for $R^* = R_s$. By replacing in formula (6) this gives:

$$(\alpha+\beta)R_s \leq \sqrt{\alpha+\beta}\Phi^{-1}(\gamma) + \alpha\bar{R} + \beta R_s + \sqrt{\beta}\Phi^{-1}(m),$$

which is equivalent to:

$$\Phi^{-1}(m) \geq \frac{\alpha}{\sqrt{\beta}}(R_s - \bar{R}) - \sqrt{1 + \frac{\alpha}{\beta}}\Phi^{-1}(\gamma). \tag{10}$$

Therefore, the coordination failure disappears when $m \geq \overline{m}$, where

$$\overline{m} = \Phi\left(\frac{\alpha}{\sqrt{\beta}}(R_s - \bar{R}) - \sqrt{1 + \frac{\alpha}{\beta}}\Phi^{-1}(\gamma)\right) .\blacksquare$$

Notice that, since R_s is a decreasing function of $\frac{E}{I}$, the critical liquidity ratio $\overline{m}$ decreases when the solvency ratio $\frac{E}{I}$ increases.[26]

The equilibrium threshold return R^* is determined (when (10) is not satisfied) by the solution to:

$$\phi(R) = \alpha(R - \bar{R}) - \sqrt{\beta}\Phi^{-1}\left(\frac{1-m}{\lambda R_s}(R - R_s) + m\right) - \sqrt{\alpha + \beta}\Phi^{-1}(\gamma) = 0. \tag{11}$$

When $\beta \geq \beta_0$, $\phi'(R) < 0$ and the comparative statics properties of the equilibrium threshold R^* are straightforward. Indeed, we have that $\partial\phi/\partial m < 0$, $\partial\phi/\partial R_s > 0$, $\partial\phi/\partial\lambda > 0, \partial\phi/\partial\gamma < 0$ and $\partial\phi/\partial\overline{R} < 0$. The following proposition states the results.

Proposition 3 Comparative statics *of R^* (and thus of the probability of failure):*

- *R^* is a decreasing function of the liquidity ratio m and the solvency (E/I) of the bank, of the critical withdrawal probability γ and of the expected return on the bank's assets $\bar{R}$.*
- *R^* is an increasing function of the fire sales premium λ.*

[26] More generally, it is easy to see that in our model, the regulator can control the probabilities of illiquidity ($\Pr(R < R^*)$) and insolvency ($\Pr(R < R_s)$) of the bank by imposing minimum liquidity and solvency ratios.

We have thus that stronger fundamentals, as indicated by a higher prior mean $\bar{R}$ also imply a lower likelihood of failure. In contrast, a higher fire sales premium λ increases the incidence of failure. Indeed, for a higher λ a larger portion of the portfolio must be liquidated to meet the requirements of withdrawals. We have also that R^* is decreasing with the critical withdrawal probability γ and as $\gamma \to 0$, $R^* \to (1+\lambda)R_s$.

The effect of an increase in the precision of the prior α is potentially ambiguous. This is so because $\partial\phi/\partial\alpha = R - \bar{R} - \frac{\Phi^{-1}(\gamma)}{2\sqrt{\alpha+\beta}}$, whose sign depends on whether $R^* \lesseqgtr \bar{R}$ and $\gamma \lesseqgtr 1/2$ (recall that $\Phi^{-1}(\gamma) \lesseqgtr 0$ as $\gamma \lesseqgtr 1/2$). We should expect that the cost of withdrawal C is small in relation to the continuation benefit for the fund managers B. Therefore, we can always assume that $\gamma = C/B$ will be less than $1/2$. This means that when the prior fundamentals are bad ($\bar{R}$ low) we will have $R^* > \bar{R}$ and $\partial\phi/\partial\alpha > 0$. In consequence increasing α will increase R^*. Indeed, to have more precise prior information about a bad outcome worsens the coordination problem. When the prior fundamentals are good ($\bar{R}$ high) and $R^* < \bar{R}$ the outcome is ambiguous unless $R^* << \bar{R}$, in which case $\partial\phi/\partial\alpha < 0$. Then a more precise prior information about a very good outcome alleviates the coordination problem.

A similar analysis applies to changes in the precision of private information of investors β. The reason is that the sign of $\{\partial\phi/\partial\beta\}$ depends on the sign of $\Phi^{-1}\left(\frac{1-m}{\lambda R_s}(R-R_s)+m\right)$and of $\Phi^{-1}(\gamma)$ and we may have $\frac{1-m}{\lambda R_s}(R-R_s)+m \lesseqgtr 1/2$ and/or $\gamma \lesseqgtr 1/2$. For example, for β large enough it can be seen that sign $\{\partial\phi/\partial\beta\}$ = sign $\Phi^{-1}(\gamma)$[27] Then an improved precision of private signals decreases (increases) R^* and the failure rate, if the relative cost of withdrawal for the fund managers is small, $\gamma < 1/2$ (large, $\gamma > 1/2$). If we think as before that the reasonable case is to have $\gamma < 1/2$ then an improvement in the private

[27] For β large we have that, for $R = R^*$, sign $\{\partial\phi/\partial\beta\}$ = sign $\left\{\frac{\Phi^{-1}(\gamma)}{2}\left(\frac{1}{\sqrt{\beta}} - \frac{1}{\sqrt{\alpha+\beta}}\right)\right\}$ = sign $\Phi^{-1}(\gamma)$.

precision of investors (when it is already high) makes failure less likely.

6 Coordination failure and LLR Policy

The main contribution of our paper has been to show the theoretical possibility of a solvent bank being illiquid, due to a coordination failure on the interbank market. We are now going to explore the lender of last resort policy of the central bank and present a scenario where it is possible to give a theoretical justification to Bagehot's doctrine.

We start by considering that the central bank tries to minimize its involvement under the constraint that the coordination failure disappears. We analyse a more elaborate objective in Section 8. We have shown in Section 5 that a high enough liquidity ratio m eliminates the coordination failure altogether by inducing $R^* = R_s$. This is so for $m \geq \overline{m}$ $= \Phi\left(\frac{\alpha}{\sqrt{\beta}}(R_s - \bar{R}) - \sqrt{1+\frac{\alpha}{\beta}}\Phi^{-1}(\gamma)\right)$.

However, it is likely that imposing $m \geq \overline{m}$ might be too costly in terms of foregone returns (recall that $I + M = 1 + E$, where I is the investment in the risky asset). Therefore, we look at a lender of last resort policy that eliminates the coordination failure when $m < \overline{m}$.

Suppose the Central Bank announces it will lend at rate $r \in (0, \lambda)$, and without limits, but only to solvent banks. The Central Bank is not allowed to subsidize banks and is assumed to observe R. The knowledge of R may come from the supervisory knowledge of the Central Bank or perhaps by observing the amount of withdrawals of the bank. Then the optimal strategy of a (solvent) commercial bank will be to borrow exactly the liquidity it needs, i.e. $D(x - m)_+$. Whenever $x - m > 0$, failure will occur at date 2 if and only if:

$$\frac{RI}{D} < (1 - x) + (1 + r)(x - m).$$

Given that $\frac{D}{I} = \frac{R_s}{1-m}$, we obtain that failure at $t = 2$ will occur if and only if:

$$R < R_s(1 + r\frac{(x-m)_+}{1-m}).$$

This is exactly analogous to our previous formula giving the critical return of the bank, only that the interest rate r replaces the liquidation premium λ. As a result, the LOLR policy will be fully effective (yielding $R^* = R_s$) only when r is arbitrarily close to zero.

The described LLR policy corresponds then to Bagehot's doctrine of helping (without limits) illiquid but solvent banks with loans at a penalty rate . Note also that whenever the Central Bank helps with a very low rate the collateral of the bank is evaluated under "normal circumstances". That is, when there is no coordination failure. Consider as an example the limit case of β tending to infinity. The equilibrium with no Central Bank help is then $t^* = R^* = R_s(1 + \frac{\lambda}{1-m}[\max\{1-\gamma-m, 0\}])$. Suppose that $1-\gamma > m$ so that $R^* > R_s$. We have that withdrawals are $x = 0$ for $R > R^*, x = 1-\gamma$ for $R = R^*$, and $x = 1$ for $R < R^*$. Whenever $R > R_s$ the Central Bank will help avoiding failure and evaluating the collateral as if $x = 0$. This effectively changes the failure point to $R^* = R_s$.

However, in some circumstances the central bank may not be able to infer R exactly because of noise (be it in the supervisory process or in the observation of withdrawals). The central bank will only obtain an imperfect signal of R. In this case the central bank will not be able to distinguish perfectly between illiquid and insolvent banks (as in Goodhart and Huang (1999a)) so that whatever the lending policy chosen by the central bank, taxpayers' money may be involved with some probability. This situation is realistic given the difficulty in distinguishing between solvency and liquidity problems.[28]

[28] We may even think that the Central Bank can not help ex post once withdrawals have materialized but that it receives a noisy signal s_{CB} about R at the same time that investors. Indeed, in many countries, the Central Bank has also a supervisory role, and thus can be expected to estimate R with a good precision. The central bank then can act preventively and inject liquidity into the bank contingent

We will not pursue this avenue more but concentrate in the next section on the effects of public information.

7 Public Information and Transparency

Suppose now that fund managers have available also a public signal $v = R + \eta$, where $\eta \sim N\left(0, \frac{1}{\beta_p}\right)$ is independent from R and the error terms ε_i of the private signals. This public signal may come, for example, from an announcement made by the Central Bank.

In this case we may think that $\beta_p \gg \beta$. That is, private signals are not useless given the public signal v but the precision of the latter may be much higher. Despite this the collective information of investors reveals R and therefore dominates the public signal.

The information set of investor i now consists of his private signal s_i and the public signal v. The conditional distribution of R given v is $N(\widehat{R}, \frac{1}{\widehat{\alpha}})$ where : $\widehat{R} = \frac{\alpha\bar{R}+\beta_p v}{\alpha+\beta_p}$ and $\widehat{\alpha} = \alpha + \beta_p$.

Let us examine the new form of equilibrium conditions:

- The second equation (equation (7)) is unchanged, given that the conditional distribution of signals given R does not depend on v.

- However, the first equation now depends on v,

$$\Pr[R < R^* | s = t^*, v] = \gamma.$$

on the signal received $L(s_{CB})$. In this case also the risk exists that an insolvent bank ends up being helped. The game of the fund managers changes because the liquidity injection modifies the failure region.

Because of normality, this can be written :

$$\Phi\left[\frac{\alpha(R^* - \overline{R}) + \beta_p(R^* - v) + \beta(R^* - t^*)}{\sqrt{\alpha + \beta_p + \beta}}\right] = \gamma,$$

or

$$\Phi\left[\frac{\widehat{\alpha}(R^* - \widehat{R}) + \beta(R^* - t^*)}{\sqrt{\widehat{\alpha} + \beta}}\right] = \gamma.$$

Comparing with equation (6), we see that, as expected, the only impact of the public signal v is to replace the unconditional moments $\overline{R}$ and $\frac{1}{\alpha}$ of R by its conditional moments $\widehat{R}$ and $\frac{1}{\widehat{\alpha}}$. Indeed, the prior on R can be interpreted as the observation of $\overline{R}$ with precision α.

The condition for a unique equilibrium becomes therefore:

$$\beta \geq \frac{1}{2\pi}\left(\frac{\lambda R_s}{1-m}\right)^2 \widehat{\alpha}^2.$$

We observe that, as before, the uniqueness property is lost if public information is precise enough. When $\beta = 0$, corresponding to the case of common knowledge (public information only), multiplicity prevails. Uniqueness is also lost if we move along a ray of positive slope from the origin in the plane (β, β_p), $\beta_p = k\beta$ with $k > 0$. This corresponds to a situation where the precision β of private signals grows without bound but the ratio β_p/β remains strictly positive. This means that, asymptotically, some information is still brought by the public signal. Then for β large enough there are three equilibria. However, as in Morris and Shin, if we keep β_p constant then as β tends to infinity the uniqueness property holds. All these results are in line with a recent contribution by C. Hellwig (2000) who questions the robustness of the results of Morris and Shin.

Here we will not interpret the multiplicity arising from the presence of public information as a lack of robustness of the uniqueness result but rather from the perspective of the lessons that can be drawn for Central Bank policy in relation to transparency. Indeed, even if we were to think that public forecasts are always interpreted in an idiosyncratic way, the case could be made that the central bank may have the unique ability to make an announcement that becomes common knowledge. Should the central bank then announce his signal to the public?

The common wisdom is that a Central Bank has to be as transparent as possible. However, it is evident that this need not be the case in our model. Indeed, while in the initial game without a public signal we may well be in the uniqueness region, adding a precise enough public signal we will have three equilibria.

For example, in the case $\beta_p = k\beta$ with $k > 0$ it is easily checked that for γ in $(0, 1)$, if $R_s < v < (1 + \lambda)R_s$ for β large enough there are three equilibria. There is an interior equilibrium with threshold at the public signal (with x in $(0, 1)$ and $t^* = R^* = v$) and two "corner" equilibria. In one corner equilibrium everybody runs ($x = 1$, with $t^* > R^* > v, R^* = (1 + \lambda)R_s$, and in the other nobody runs ($x = 0$, with $t^* < R^* < v$, $R^* = R_s$).[29]

At the interior equilibrium we have a similar result than with no public information but run and no-run equilibria also exist. We may therefore end up in an "always run" situation when disclosing (or increasing the precision of) the public signal while the economy was sitting in the interior equilibrium without public disclosure. In other words, public disclosure of a precise enough signal may be destabilizing. This means that a Central

[29] An equilibrium pair (R, t) has to fulfill:
$\Phi\left[\frac{\alpha(R-\overline{R})+\beta_p(R-v)+\beta(R-t)}{\sqrt{\alpha+\beta_p+\beta}}\right] = \gamma$, and $R = R_s(1 + \lambda[\frac{\Phi(\sqrt{\beta}(t-R))-m}{1-m}]_+)$.

As $\beta \to \infty$ and given that $\gamma \in (0, 1)$ and $\beta_p = k\beta$, $k > 0$, we obtain from the first equation that $t = (k + 1)R - kv$. Assume that $R_s < v < (1 + \lambda)R_s$. Let $\Gamma = \lim_{\beta\to+\infty}\sqrt{\beta}(t - R)$. We have that if in the limit $t > (<)R$, $\Gamma = +\infty$ ($-\infty$). Note that $x = \Phi(\Gamma)$ when $\beta \to \infty$. At the interior equilibrium Γ remains bounded and $t = R = v$. At the run equilibrium $R = (1+\lambda)R_s$, $t = (k+1)(1+\lambda)R_s - kv, \Gamma = \infty$ (because $t = (k + 1)(1 + \lambda)R_s - kv > (1 + \lambda)R_s$ if an only if $(1 + \lambda)R_s > v$). At a no-run equilibrium $R = R_s$, $t = (k + 1)R_s - kv, \Gamma = -\infty$ (because $t = (k + 1)R_s - kv < R_s$ if an only if $R_s < v$).

Bank that wants to avoid entering in the "unstable" region may have to add noise to its signal if it is "too" precise.

Summarizing the discussion on transparency:

- If we take the view that extra information (on top of the prior with precision $\alpha > 0$) is interpreted in an idiosyncratic way then more transparency (entailing private signals of higher precision β) reduces the incidence of coordination failure for β large (under the assumption that $\gamma < 1/2$).
- If there is public information that becomes common knowledge, perhaps through Central Bank disclosure, then the public signal cannot have too high a precision β_p since otherwise multiple equilibria reappear. Furthermore, even if we remain in the uniqueness region increasing the precision of public information will aggravate the coordination failure when fundamentals are weak (low $E[R|v]$, and under the assumption that $\gamma < 1/2$).

8 Endogenizing the liability structure and crisis resolution

In this section we sketch a possible way to endogenize the short term debt contract assumed in our model according to which depositors can withdraw at $\tau = 1$ or otherwise wait until $\tau = 2$. We have seen that the ability of investors to withdraw at $\tau = 1$ creates a coordination problem. We argue here that this potentially inefficient debt structure may be the only way investors can discipline a bank manager subject to a moral hazard problem.

Suppose indeed that investment in risky assets requires the supervision of a bank manager and that the distribution of returns of the risky assets depends on the effort undertaken by the manager. For example, the manager can either exert or not exert effort, $e \in \{0,1\}$, and $R \sim N(\overline{R}_0, \alpha^{-1})$ when $e = 0$, and $R \sim N(\overline{R}, \alpha^{-1})$ when $e = 1$ with $\overline{R} > \overline{R}_0$. That is, exerting effort yields a return distribution that first order stochastically dominates the one obtained by not exerting effort. The bank manager incurs in a cost if he chooses $e = 1$; if he chooses $e = 0$ the cost is 0. The manager also receives a benefit from continuing the project until date 2. Assume for simplicity that the manager does not care about monetary incentives. The manager's effort cannot be observed so his willingness to undertake effort will depend on the relationship between his effort and the probability that the bank continues at date 1. Withdrawals may enforce then the early closure of the bank and provide incentives to the bank manager.[30]

In the banking contract, short term debt/demandable deposits can improve upon long term debt/nondemandable deposits. With long term debt incentives cannot be provided to the manager, because there is never liquidation, and therefore the manager does not exert effort. Furthermore, incentives cannot be provided either with renegotiable short term debt because early liquidation is ex post inefficient. Dispersed short term debt (i.e. uninsured deposits) is what is needed.

Let us assume that it is worthwhile to induce the manager to exert effort. This will be true for $\overline{R} - \overline{R}_0$ large enough and the (physical) cost of asset liquidation not too large. Recall that we model this liquidation cost as proportional to the future returns on the bank's portfolio. The banking contract will have short-term debt and will maximize the expected profits of the bank, choosing the investment in risky and safe assets and deposit payment, subject to the resource constraint, the individual rationality constraint of investors (zero

[30] This approach is based on Grossman and Hart (1982) and is followed in Gale and Vives (2001). See also Calomiris and Kahn (1991), Diamond and Rajan (1997) and Carletti (1999).

expected return), the incentive compatibility constraint of the bank manager,[31] and the closure rule associated with the (unique) equilibrium in the investors' game. In the absence of a LLR, this early closure rule is defined by the property: $D.x(t^*, R) > M + \frac{IR}{1+\lambda}$, which is satisfied if and only if R is smaller than some threshold $R_{EC}(t^*)$. As stated before, $R_{EC}(t^*) < R^*$ since early closure implies failure, while the converse is not true. Now, an interesting question is how the banking contract compares with the incentive efficient solution, which we now describe.

Given that the pooled signals of investors reveal R , we can define the incentive-efficient solution as the choice of investment in liquid and risky assets and probability of continuation at $t = 1$(as a function of R) which maximize expected surplus subject to the resource constraint and the incentive compatibility constraint of the bank manager.[32] Furthermore, given the monotonicity of the likelihood ratio $\frac{g(R\,|e=0)}{g(R\,|e=1)}$, the optimal region of continuation is of the cutoff form. More specifically,the optimal cutoff will be the smallest R, say R^o, that fulfills the incentive compatibility constraint of the bank manager. Since $R_{EC}(t^*)$ must also fulfil the incentive compatibility constraint of the bank manager, we will have that at the optimal banking contract with no LLR, $R_{EC}(t^*) \geq R^o$. In fact we will typically have a strict inequality, since there is no reason that the equilibrium threshold t^* satisfies $R_{EC}(t^*) = R^o$. This means that the market solution will lead to too many early closures of banks. Moreover, the market solution will involve inefficient hoarding of liquidity as compared with the incentive efficient solution.[33]

[31] More precisely, we assume as in the previous sections that the face value of the debt contract is the same in periods $t = 1, 2$ (equal to D) and we suppose also that investors in order to trust their money to fund managers need to be guaranteed a minimum expected return.

[32] We disregrad here the welfare of the bank manager and of the funds managers.

[33] It is worth to remark that at the incentive-efficient solution it is optimal not hold any reserves ($m^o = 0$). This should come as no surprise because we we assume that there is no cost of intervention. The incentive-efficient solution solves $\text{Max}_m \left\{(1 + E - Dm)(\overline{R} - (1 - \nu)E(R \mid R \leq R^o) + Dm\right\}$

where R^o is the minimal return cutoff that incentivates the bank manager. If $(\overline{R} - (1 - \nu)E(R \mid R < R^o) > 1$ we have that $m^o = 0$.

Therefore the role of the LLR can be viewed, in this context, as correcting these market inefficiencies while maintaining the incentives of bank managers. By announcing its commitment to provide liquidity assistance (without limits, and at a rate slightly above the market rate that would prevail under "normal circumstances") whenever R is greater than R^o, the LLR has the power to modify expectations of investors and in particular the withdrawal threshold t^*so that $R_{EC}(t^*) = R^o$. However R^o will typically be different from R_s,which leads to a view on the LLR that differs from Bagehot's doctrine and introduces interesting policy questions. Whenever $R^o > R_s$ there is a region (specifically, for R in (R_s, R^o)) where there should be early intervention (or prompt corrective action, to use the terminology of banking regulators). Indeed, in this region a solvent bank should be intervened to control moral hazard of the banker. However, it may well be that $R^o < R_s$. The reason is that R_s is determined by the promised payments to investors while R^o is just the minimum threshold that incentivates the banker to behave. In the range (R^o, R_{EC}) the bank should be helped and it may be insolvent. When $R^o < R_s$ an insolvent bank in the range $(R^o, \min\{R_s, R_{EC}\})$ should be helped. If the Central Bank cannot help then another institution (Deposit Insurance Fund, Regulatory Agency, Treasury) should come to the rescue.

When $R^o > R_s$ a Central Bank that can commit to a LLR policy can implement the closure threshold R^o. However, if the Central Bank cannot commit and instead optimizes ex post (be it because to build a reputation is not possible or because of weakness in the presence of lobbying), it will intervene too often. Some additional institutional arrangement is needed in the range (R_s, R^o) to implement prompt corrective action (i.e. early closure of banks that are still solvent).

When $R^o < R_s$ the Central Bank, being prohibited from subsidising banks, can only intervene when the bank is solvent. Therefore another institution (financed by taxation or insurance premiums) is needed to provide an "orderly resolution of failure" when R is in the range (R^o, R_s) .

In general we can say that a complementary regulatory institution to the Central Bank has to take charge whenever R is less than $\min\{R^o, R_s\}$.

In summary:

- With neither a LLR nor an interbank market, liquidation takes place whenever $x(R) > mD$, which limits inefficiently investment I.

- With an interbank market but no LLR (as advocated by Goodfriend and King) the closure threshold is $R_{EC}(t^*)$ and there is excessive failure whenever $R_{EC}(t^*) > R^o$.

- When $R^o > R_s$ with both a LLR facility and an interbank market, together with a policy of prompt corrective action in the range (R_s, R^o), the incentive-efficient solution can be implemented.

- When $R^o < R_s$, on top of a LLR facility and an interbank market, a different form of institution (financed by taxation or by insurance premiums) is needed to implement the incentive-efficient solution. The Central Bank helps whenever the bank is solvent and the other institution provides an "orderly resolution of failure" in the range (R^o, R_s).

9 Concluding remarks

In this paper we have provided a rationale for Bagehot's doctrine of helping illiquid but solvent banks in the context of modern interbank markets. Indeed, investors in the interbank market may face a coordination failure and intervention may be desirable. We have examined the impact of public intervention along the following four dimensions:

- solvency and liquidity requirements (at $\tau = 0$);
- Lender of Last Resort policy (at the interim date $\tau = 1$);
- transparency and public disclosure of central bank's information, and
- closure rules, which can consist of two types of policy: orderly resolution of failures or prompt corrective action.

The coordination failure can be avoided by appropriate solvency and liquidity requirements. However, the cost of doing so will typically be too large in terms of foregone returns and ex ante measures will only help partially. This means that prudential regulation needs to be complemented by a Lender of Last Resort policy. The paper shows how discount window loans can eliminate the coordination failure (or alleviate it if for incentive reasons some degree of coordination failure is optimal). When the Central Bank has access to a public signal it is shown that the effects of its disclosure depend on whether its signal becomes common knowledge or not. If it does then disclosure of a signal of high enough precision could be destabilizing. An oblique statement by a central banker may be optimal in that it either provides information without creating a common knowledge signal or, even if it does, it adds enough noise so that the information does not become destabilizing. In any case, increasing the precision of public information may aggravate the coordination failure whenever the fundamentals are weak.

Finally, the model also provides a frame to discuss the interaction of LLR policy, prompt corrective action and orderly resolution of failures. Indeed, the implementation of the incentive-efficient solution may require to complement the Bagehot's LLR facility with prompt corrective action (intervention of a solvent bank) or orderly failure resolution (help to an insolvent bank).

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