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FISCAL STABILIZATION POLICY IN A MONETARY UNION WITH INFLATION TARGETING

Torben M Andersen

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Torben M Andersen, University of Aarhus and CEPR

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Centre for Economic Policy Research
90–98 Goswell Rd, London EC1V 7RR, UK
Tel: (44 20) 7878 2900, Fax: (44 20) 7878 2999
Email: cepr@cepr.org, Website: www.cepr.org

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ABSTRACT

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JEL Classification: E52, E58, E61, E62 and E63
Keywords: EMU, policy cooperation, policy-mix and shocks

Torben M Andersen
Institute of Economics
Universitet Aarhus
Building 350
DK-8000 Aarhus C
DENMARK
Tel: (45 8) 942 1609
Fax: (45 8) 613 6334
Email: tandersen@econ.au.dk

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Fiscal Stabilization Policy in a Monetary Union with Inflation Targeting

Torben M. Andersen
Department of Economics, University of Aarhus
CEPR, EPRU and IZA

January 25, 2002

Abstract

The interaction betwen a common monetary policy targeting inflation and decentralized fiscal policies aiming at output stability is considered in a setting taking into account interdependencies between countries running via trade links and the common monetary policy. The setting is sufficiently general to capture various effects of fiscal policy, and to allow for both demand and supply shocks, which can be either aggregate or idiosyncratic. It is shown that a policy mix problem arises between the common monetary policy and the decentralized fiscal policy, which results in an inappropriate stabilization of shocks. In the case of aggregate shocks the inefficiency in responding to shocks is increasing in the number of member countries, while it is decreasing in the case of idiosyncratic shocks. Numerical illustrations show that the cost of non-cooperative fiscal policies can be large in the case of aggregate shocks, while they are small in the case of idiosyncratic shocks, provided that fiscal policy can be flexibly adjusted to cope with idiosyncratic shocks.

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1 Introduction

The European Monetary Union has a centralized monetary policy aiming at price stability, while fiscal policy remains decentralized at the level of member states. This raises a fundamental issue concerning policy coordination, namely, whether there is a tendency that the overall policy mix becomes inappropriate implying an inadequate stabilization of shocks. This possibility arises since single fiscal authorities to an insufficient degree take into account that their actions affect inflation and therefore in turn the monetary policy stance which affect all member states of the EMU[1]. This monetary policy externality may potentially

[1] In a discussion of fiscal policy, the ECB states "If inflationary or deflationary tendencies are reinforced, monetary policies might have to react more strongly than they would have had to without fiscal interventions", European Central Bank (2001, p 50).

cause a bias in the sense that fiscal policy tends to be too expansionary and monetary policy too contractionary.

However, fiscal policy also has other effects for trading partners. A standard interdependence runs via trade links since a domestic fiscal expansion may benefit trading partners through a demand spillover effect since increasing domestic demand tends to raise imports, as well as a cost spillover effect since higher domestic activity tends to lead to higher domestic wage/price increases which in turn makes it possible for trading partners to increase their market share. These two forms of externalities – termed trade externalities in the following – tend to run in the opposite direction of the monetary policy externality. Non-cooperative fiscal policy making tends to lead to insufficient expansionary fiscal policies, since single fiscal authorities do not take the positive trade externalities for trading partners into account (see e.g. Cooper (1995) or Hamada and Kawai (1997) for surveys).

What is the net effect of the monetary policy and the trade externalities? Do fiscal policies tend to be too contractionary or too expansionary, and is the policy mix affected in a significant way? Does the answer depend on the type of shock inducing business cycle fluctuations? To what extent are these externalities affected by the degree of decentralization of fiscal policy, that is, would these problems increase or decrease when the number of fiscal authorities increase (as would be an implication of an increase in the number of participants in the EMU)?

The present paper addresses these questions with outset in the way in which the macroeconomic policy framework is defined for member countries in the European Monetary Union according to the "Maastricht assignment" (Issing (2000)). This has two important parts. First, monetary policy is delegated to the European Central Bank, while fiscal policy remains a national issue. Second, the task for the ECB is explicitly formulated to have price stability as its mains objective. This is here interpreted in the way that the main objective of ECB is to target inflation (cf also ECB (1999)). The macroeconomic policy framework and the monetary policy objectives are thus given and taken for granted by national fiscal authorities.

An important question in a European context is how the interaction between centralized monetary policy and decentralized fiscal policy affect the scope for short-term stabilization policy, when the monetary policy has price stability as its primary objective, and national fiscal authorities use fiscal instruments to stabilize national output.

A first step[2] in addressing this type of questions is taken by Leitemo (2000) who considers how inflation targeting affects fiscal policy making in an open economy model with a trade-off between activity and inflation, a flexible exchange rate and fiscal policy affecting aggregate demand. If the fiscal authority

[2] Dixit (2000) and Dixit and Lambetini (2000b) also discuss the implications of decentralized fiscal policy in a monetary union, but focussing on situations where fiscal authorities do not take the monetary policy reaction into account, that is, monetary policy retains its discretionary power. This corresponds to a situation different from the "Maastricht assignment".

perceives the monetary policy rule (the Stackelberg case)[3] corresponding to the "Maastricht assignment" no policy mix problem arises, since the fiscal authorities realize that monetary policy pins down the output level, in which case an active fiscal policy contributes to stabilize interest rates and exchange rates[4]. Note that it is a model property that the inflation targeted depends only on output. The intuition for the above-mentioned results is thus that inflation targeting works as a constraint on fiscal policy, making it more difficult for the fiscal authorities to pursue an employment target in excess of the equilibrium level.

Decentralized fiscal policy is introduced in this framework by Sveen (2001)[5], by "splitting" the domestic economy up in two fiscal jurisdictions, and allowing for idiosyncratic shocks to output in these areas. When fiscal authorities are in a leader role vis-à-vis the monetary authority (i.e. monetary rule known = inflation targeting) the non-cooperative case tends to produce a stabilization bias[6]. The bias arises since the fiscal policy response to idiosyncratic shocks is too weak compared to the cooperative case – since the single fiscal authority does not take into account that the shock – via traditional international interdependencies – affects other countries causing a positive externality in stabilization policy.

This line of analysis is here extended to a setting which allows for a more general modelling of both the spillover effects between countries[7], and the monetary transmission mechanism as well as a variable number of member countries. In addition the set-up allows for both aggregate demand and supply shocks, which can be either idiosyncratic and aggregate within the monetary union. Specifically, this paper develops a model for a monetary union based on a standard framework in which inflation is determined by a Phillips-curve relating price/wage increases to domestic aggregate activity, fiscal policy and possibly various shocks (demand and supply). Activity is demand determined, and aggregate demand depends negatively on the real rate of interest, the terms of trade (costs externality) and positively on fiscal policy and aggregate income in the monetary union (demand externality) as well as possible shocks. The

[3] If the fiscal authority does not perceive the monetary rule but takes monetary policy as given, a policy mix problem arises in the presence of a conflict over output goals – monetary policy tends to be too contractionary and fiscal policy too expansionary – in this case an argument can be made for restricting flexibility in fiscal policy. This result is related to the findings in Dixit (2000) and Dixit and Lambertini (2000a,b).

[4] There is a so-called state-contingency in fiscal policy (see Svensson (1997))since the fiscal stance depends on the realization of the inflation measure targeted, provided the fiscal authorities are concerned about not only stabilization of output but also exchange rates and interest rates.

[5] There is still a passive "outside" country.

[6] There is a state contingent bias in the aggregate fiscal stance which is positively increasing in the inflation measure targeted. However, since expected inflation is zero, there is no systematic bias in the aggregate fiscal stance.

[7] One short-coming of the above-mentioned analyses is that the trade links between countries in a monetary union are either not present, or only modelled in a very simple way. Moreover, the monetary policy transmission runs only via the effect on the real exchange rate vis-à-vis third countries.

policy setting is one where the monetary authority uses its power to determine interest rates to target expected inflation, and the fiscal authorities knowing the monetary regime choose fiscal policy to minimize a loss function specified over variability in output variability and the fiscal policy instrument. There are a number of independent fiscal authorities choosing their fiscal policy non-cooperatively.

This setting captures the various international interdependencies mentioned above. One type – monetary policy externalities – arises from any action affecting country-specific inflation and therefore in turn aggregates inflation in the monetary union to which the monetary authority reacts. A second type arises via trading relations – trade externalities[8] – a domestic fiscal expansion would benefit trading partners both by increasing the level of export demand they are facing, and by improving their competitiveness (domestic inflation will increase relative to that of other countries). The latter link implies that fiscal authority will be concerned about the domestic inflationary pressure due to its consequences for trade, even though the monetary authority is targeting aggregate inflation in the monetary union

There are a number of surprising and interesting findings on the interaction between fiscal and monetary policy, when the latter targets inflation. Whether fiscal policy is expansionary or contractionary does not depend on how if affects aggregate demand, since this effect is neutralized by monetary policy to attain the inflation target. However, if fiscal policy has a direct (temporary) effect on the inflation process it has an expansionary effect on output if it reduces inflation, and oppositely a contractionary effect if it adds to inflation. For the same reason it turns out that a demand shock, which both tends to increase activity and the inflationary pressure in the economy, becomes contractionary in equilibrium and oppositely for a supply shock increasing equilibrium output and decreasing inflation. Aggregate shocks (demand and supply) create a policy mix problem since fiscal authorities try to stabilize such shocks via fiscal policy responses but with insufficient attention to the monetary policy externality. Non-cooperative policies tend thus to be excessively counter-cyclical to aggregate shocks. For idiosyncratic shocks the situation is opposite, since such shocks do not release the monetary policy externality, but single countries perceiving spillover effects tend to choose too moderate fiscal responses, implying that fiscal policy is insufficiently counter-cyclical relative to the cooperative case. The policy mix problem present in relation to aggregate shocks is larger, the larger the number of fiscal decision makers within the monetary union, while oppositely in the case of idiosyncratic shocks the inefficiencies caused by non-cooperative policy making decrease in the number of fiscal decision makers within the union.

It follows that a monetary union adhering to a target of price stability is not necessarily problematic for the response to idiosyncratic shocks, provided that the number of fiscal decision makers is sufficiently large (the number of participating countries in the union is sufficiently large). This result presumes

[8] In accordance with models with a more explicit microfoundation of product market interdependencies, see section 2 below.

that there are no limitations (like binding budget norms) on the ability of the fiscal authority to respond to idiosyncratic shocks.

The paper is organised as follows: The model structure is laid out in section 2, and section 3 considers fiscal policy decisions in case of non-cooperative and cooperative fiscal policies. Section 4 turns to a comparison of non-cooperative and cooperative policies, and also includes numerical illustrations on the quantitative importance of the policy mix problem between fiscal and monetary policy in a monetary union. Section 5 offers some concluding remarks, and discusses some open questions.

2 A model for a monetary union

Following the tradition in the recent literature on macroeconomic policy rules (see e.g. Svensson (1997), Dixit (2000), Dixit and Lambertini (2000a,b), Leitemo (2000), Sveen (2000) and Beetsma, Debrun and Klassen (2001)), the model structure builds on an aggregate supply curve (Phillips-curve relation), and an aggregate demand relation. In the present context it is important to be specific about the interactions between member countries of the monetary union, both in respect to inflation and output. To simplify, the currency union is considered to be a closed area to focus on the direct interdependencies between member countries of the currency area[9]. The currency area has n member countries indexed by i, which are all symmetric up to the realisation of shocks (see below). All variables – except the interest rate – are defined by the log value of the variable in question measured relative to its long-run value.

Wage and Price setting[10]
Nominal wage increases are assumed to be determined by an inflation augmented Phillips curve (see e.g. Ball (1999)) in which wage increases depend on current increases in country-specific consumer prices (π^c_{it}), output (y_{it}), fiscal policy captured by the fiscal indicator variable (g_{it}), and an innovation (u_{it})

$$\Delta W_{it+1} = \pi^c_t + \omega_y y_{it} + \omega_g g_{it} + \omega_u u_{it} \quad ; \quad \omega_y > 0$$

This relation can be interpreted as capturing the presence of nominal wage contracts, and the assumption that wage increases depend on current consumer price inflation π^c_t (rather than expected future inflation) is a simple way of modelling nominal inertia. At the cost of more complexity the same qualitative implications would arise under a structure with asynchronized wage contracts (see e.g. Taylor (1999)). The formulation allows for both the case where $\omega_g \gtrless 0$ since fiscal policy may have (temporarily) separate effects on wage (price) inflation depending on the particular instrument used. It is usually asserted that e.g. public expansions financed by value added and excise taxes add (temporarily)

[9] Since the consolidated trade share for EU is about 10% of GDP this assumption is a reasonable approximation.

[10] The model disregards forward-looking elements to simplify the analysis. However, under strict inflation targeting this has no major qualitative implications.

to the inflationary pressure in the economy ($\omega_g > 0$), but it is also possible that tax increases may lead to wage moderation ($\omega_g < 0$) [11]. Observe that fiscal policy also has an indirect effect on inflation via its effect on aggregate activity (see below). For the interpretation of ω_u see below.

Domestic firms set prices (P_{it}) based on a mark-up pricing rule implying that period t prices are given as

$$P_{it} = mW_{it} \quad ; \quad m \geq 1$$

where the mark-up m is assumed constant. It follows that the increase in domestic producer prices is given as

$$\pi_{it} = \Delta W_{it}$$

The consumer price index is defined over prices of domestically and foreignly produced commodities

$$P_{it}^c = \frac{1}{n} \sum_j P_{jt}$$

and formulated as rates of changes we have[12]

$$\pi_{it}^c = \pi_t \equiv \frac{1}{n} \sum_j \pi_{jt}$$

By substitution we find that the domestic producer price inflation can be written

$$\pi_{it+1} = \pi_t + \omega_y y_{it} + \omega_g g_{it} + \omega_u u_{it} \tag{1}$$

It is assumed that shocks have expected value zero, i.e.

$$E_t u_{it+j} = 0 \qquad \forall j > 0$$

and the variance is denoted σ_u^2. The aggregate value of shocks across the monetary union is given as[13]

$$u_t \equiv \frac{1}{n} \sum_j u_{jt}$$

[11] It is well-known that higher income taxes may lead to an increased labour supply and thus a downward pressure on the wage rate provided income effects are sufficiently strong, see e.g. Dixon (1987) and King and Baxter (1992). It can also arise via a production subsidy as analysed in e.g. Dixit and Lambertini (2000a).

[12] At the cost of more complexity it would be straigthforward to allow for different consumption baskets in the countries via e.g. a home bias for domestically produced goods. This would not, however, change anything qualitatively, cf below. Note that the expression for the consumer price inflation is consistent with the price index implied by e.g. CES preferences.

[13] One may think of $u_{it} = \epsilon_t + \varepsilon_{it}$, where ϵ is the aggregate part of the shock, and ε_i the country-specific or idiosyncratic part (where $\sum \varepsilon_j = 0$).

Aggregating over country-specific producer inflation we get

$$\pi_{t+1} = \pi_t + \omega_y y_t + \omega_g g_t + \omega_u u_t \tag{2}$$

It follows that there is no long-run trade-off between activity and inflation.

Aggregate demand

The microfoundations for the demand relation are not given explicitly, but can be derived in the context of a specialized production structure as in Obstfeld and Rogoff (1995) or through an allocation of production across countries depending on differences in comparative advantages and trade frictions (see e.g. Andersen (2001)). Likewise intertemporal substitution (via the real rate of interest) is introduced in a straightforward and traditional way.

The output relation for country i in period t is given as

$$y_{it} = -\delta_r(i_t - \pi^c_{it}) - \delta_\tau(\pi_{it} - \pi_t) + \delta_y y_t + \delta_g g_{it} + \delta_u u_{it} \quad ;\delta_r > 0, \delta_\tau > 0, \delta_y > 0, \delta_g > 0 \tag{3}$$

where the first term captures the negative effect of an increase in the real rate of interest on demand[14], the second term captures the role of competitiveness in the sense that a higher increase in domestic producer prices than in foreign producer prices shifts demand away from domestic producers (terms-of-trade effect), the third term is the level effect of foreign trade capturing how (net) exports depend on aggregate income in the monetary union ($y \equiv \frac{1}{n}\sum_j y_j$), the fourth term captures the demand effect of fiscal policy, and finally we have the effect of the contemporaneous innovation to the shock[15].

Note that different interpretations of the shocks are possible, if $\delta_u > 0$ and $\omega_u > 0$, the shock has an interpretation as a demand shock, and if $\delta_u > 0$ and $\omega_u < 0$, the shock has an interpretation as a supply shock.

The effect of domestic fiscal policy on domestic inflation is given by

$$\frac{\partial \pi_{it+1}}{\partial g_{it}} = \omega_g + \omega_y \delta_g$$

that is, there is both a direct effect (ω_g) and an indirect effect running via activity ($\omega_y \delta_g$). Although the direct effect is allowed to be either positive or negative, it is assumed throughout, that the net-effect on inflation of a fiscal expansion is positive (i.e. $\omega_y \delta_g + \omega_g > 0$).

It follows straightforward from (3) that aggregate output is given as

$$y_t = \frac{1}{1-\delta_y}\left[-\delta_r(i_t - \pi^c_t) + \delta_g g_t + \delta_u u_t\right] \tag{4}$$

[14] Note that since consumer price inflation is the same in all countries, the real rate of interest is also the same. However, allowing for different consumption bundles such that $\pi^c_{it} = \lambda\pi_{it} + (1-\lambda)\pi_t$ would imply that $-\delta_r(i_t - \pi^c_{it}) - \delta_\tau(\pi_{it} - \pi_t) = -\delta_r(i_t - \pi_t) - (\delta_\tau + \lambda\delta_\tau)(\pi_{it} - \pi_t)$, hence under a straightforward reinterpretation of the coefficients of the demand relation, the analysis applies for the case of different consumption bundles across countries.

[15] Note that the specification implies that only unanticipated shocks to the inflationary process has real effects.

and that

$$y_{it} - y_t = -\delta_\tau(\pi_{it} - \pi_t) + \delta_g(g_{it} - g_t) + \delta_u(u_{it} - u_t) \tag{5}$$

that is, output in a member country differs from the mean output in the monetary union if it either i) has a different producer price inflation (wage increases), ii) pursues a different fiscal policy, or iii) is affected by idiosyncratic shocks.

Monetary policy
The monetary authority is controlling the nominal interest rate (i) but since inflation is known it follows that the real rate of return ($r \equiv i - \pi$) is effectively determined. The monetary authority aims at strict inflation targeting (see Svensson (1997)), in the sense that expected inflation is kept at its target value (here assumed zero for convenience), i.e.

$$E_t \pi_{t+1} = 0$$

which by use of (2) is equivalent to keeping output at the level ensuring a zero inflation rate, i.e.

$$y_t = \frac{1}{\omega_y}\left[\pi_{t+1} - \pi_t - \omega_g g_t - \omega_u u_t\right]$$

Using the aggregate demand relation (4) we find this to be equivalent to

$$\pi_{t+1} = \pi_t + \frac{\omega_y}{1-\delta_y}\left(-\delta_r(i_t - \pi_t) + \delta_g g_t + \delta_u u_t\right) + \omega_g g_t + \omega_u u_t = 0$$

Note that since π_{t+1} is non-stochastic conditional on period t information it follows that $\pi_{t+1} = 0$. It is assumed that all past inflation rates are zero[16].

Accordingly, the inflation target is met if the interest rate is set according to the following rule,

$$(i_t - \pi_t) = \frac{1-\delta_y}{\omega_y \delta_r}\left[\pi_t + (\frac{\omega_y}{1-\delta_y}\delta_g + \omega_g)g_t + (\frac{\omega_y}{1-\delta_y}\delta_u + \omega_u)u_t\right] \tag{6}$$

The monetary reaction function entails a monetary tightening to increases in inflation, to a more expansionary fiscal policy[17] and to shocks raising inflation (negative supply shocks, or positive demand shocks). Conditional on the shock variable fiscal and monetary policy instruments are strategic complements, a higher value of the fiscal policy indicator g leads to a higher value of the monetary policy instrument ($i - \pi$). Using the monetary reaction function (6) the aggregate output level is given as

$$y_t = \frac{-1}{\omega_y}\left[\omega_g g_t + \omega_u u_t\right] \tag{7}$$

[16] The results hold qualitatively for any constant targetted inflation rate.

[17] Note that $\frac{1-\delta_y}{\omega_y \delta_r}\left[\frac{\omega_y}{1-\delta_y}\delta_g + \omega_g\right]$ is positive if $\omega_y \delta_g + \omega_g$ is positive as assumed.

This brings out that with inflation targeting the problem of stabilizing output becomes a question of choosing the aggregate fiscal stance such that it corrects for the direct inflation consequences of the aggregate shock. With a symmetric loss function (see below) there are thus no differences with respect to the ability to stabilize either demand or supply shocks, although the direction in which fiscal policy should be changed to stabilize output differs between the two cases.

The response of output to fiscal policy is interesting, and brings out important lessons on the interaction between fiscal and monetary policy under inflation targeting. Fiscal policy has a direct effect on activity (the $\delta_g$term), which in turn affects inflation. Since the monetary authority targets inflation, this effect is neutralized via the monetary policy reaction cf (6). For the direct inflation effect of fiscal policy (the ω_g term) the monetary policy reaction is different – if fiscal policy through this channel adds further to inflation ($\omega_g > 0$), monetary policy will have to be further tightened, and this works to contract output. Oppositely, if fiscal policy contributes to lowering inflation ($\omega_g < 0$), there is a monetary policy easening, and this boosts activity.

The shock variable is contractionary if it is a demand shock ($\omega_u > 0$), and expansionary if it is a supply shock ($\omega_u < 0$). The reason is also here to be found in the monetary response to the shock which in turn depends on how the shock affects inflation. Since a demand shock boosts inflation, a monetary contraction is necessary, and oppositely for a supply shock.

For later reference observe that output in country i can be written

$$\begin{aligned} y_{it} &= \frac{-(1-\delta_y)}{\omega_y}\left[\pi_t + (\frac{\omega_y}{1-\delta_y}\delta_g + \omega_g)g_t + (\frac{\omega_y}{1-\delta_y}\delta_u + \omega_u)u_t\right] \\ &\quad -\delta_\tau(\pi_{it} - \pi_t) + \delta_g g_{it} + \delta_y y_t + \delta_u u_{it} \end{aligned}$$

In the appendix it is shown that there exists a solution for period t equilibrium output in country i of the form

$$y_{it} = a_{it} + \sum_j b_{ij}u_{jt} + \sum_j c_{ij}g_{jt} \tag{8}$$

where

$$\begin{aligned} a_{it} &= -\delta_\tau \pi_{it} \\ b_{ii} &= -\frac{\omega_u}{\omega_y}\frac{1}{n} + (1-\frac{1}{n})\delta_u \gtrless 0 \\ b_{ij} &= -\frac{\omega_u}{\omega_y}\frac{1}{n} - \frac{1}{n}\delta_u \gtrless 0 \\ c_{ii} &= -\frac{\omega_g}{\omega_y}\frac{1}{n} + (1-\frac{1}{n})\delta_g \gtrless 0 \\ c_{ij} &= -\frac{\omega_g}{\omega_y}\frac{1}{n} - \frac{1}{n}\delta_g \gtrless 0 \end{aligned}$$

We have that a domestic shock (u_{it}) boosts domestic activity ($b_{ii} > 0$) if it is a supply shock ($\omega_u < 0$). It is ambiguous how a demand shock ($\omega_u > 0$) affects output due to two opposite effects, namely, a direct expansionary effect via more demand for domestically produced goods, and a contractionary effect running via the inflation channel (inducing a monetary contraction). Note that the larger the monetary union (n) the smaller the inflation effect and the larger the output effect. The reason being that the larger the number of member countries, the less effect do national (idiosyncratic) shocks have on monetary policy. Shocks have an externality for foreign output ($b_{ij}, i \neq j$) which is ambiguous in the case of a supply shock, but unambiguously negative in the case of a demand shock.

Inflation targeting has one important implication for the interdependencies between countries, since the usual trade interdependency (the income-trade multiplier captured by the parameter δ_y) is neutralized. Any change in aggregate output would induce a response in monetary policy, which neutralizes the effect so as to keep inflation at its target value. Accordingly, the income-trade multiplier is countered by monetary policy reactions. It follows that the demand spillover has no implications for equilibrium output under strict inflation targeting. Interestingly, the international linkage via aggregate demand (export/import) playing an important role in traditional analysis of international interdependencies in fiscal policy becomes of no importance in the present setting (coefficients are seen to be independent of δ_y).

For domestic fiscal policy we have that it is unambiguously expansionary if it lessens the inflationary pressure ($\omega_g < 0$) while the effect is ambiguous in the case where it strengthens the inflationary pressure ($\omega_g > 0$) . The reason is that higher inflation contributes to a monetary tightening, which in turn contracts output. Also here the net-effect depends critically on the size of the monetary union since we have

$$\frac{\partial c_{ii}}{\partial n} = \frac{1}{n^2}\left(\frac{\omega_g}{\omega_y} + \delta_g\right) > 0$$

that is, the more decentralized fiscal policy decisions are, the more policy makers perceive that a domestic fiscal expansion will boost domestic activity. The reason is that the larger the number of fiscal decision makers, the less a single fiscal policy maker perceives that its actions affect aggregate inflation within the monetary union, and the monetary policy reaction hereto. Note that c_{ii} is always negative for $n = 1$, while it becomes positive for a sufficiently large n, hence it follows that there exists a number of fiscal decision makers $\underline{n}$ such that $c_{ii} < 0$ for $n < \underline{n}$ and $c_{ii} > 0$ for $n > \underline{n}$. Finally, notice that the fiscal externality ($c_{ij}, i \neq j$) is unambiguously negative if fiscal policy boosts inflation, while it may be positive if it reduces the inflationary pressure.

3 Fiscal Policy

We can now turn to fiscal policy decisions under the assumption that each national fiscal authority chooses fiscal policy so as to minimize a loss function

given as

$$L_{it} = E_t \sum_{m=0}^{\infty} (1+\rho)^{-m} (\frac{1}{2}\alpha \left(y_{it+m}\right)^2 + \frac{1}{2}(g_{it+m})^2)$$

that is, the loss function penalizes variations in both output (around its steady state level) and public sector activity (fiscal policy instruments). Notice, that national fiscal policy makers are assumed not to be directly concerned about inflation, since the task of controlling inflation is delegated to a monetary authority targeting inflation. However, as will become apparent below, the policy maker is indirectly concerned about country specific inflation since it affects competitiveness and thus in turn activity, cf (8). The weight of output stabilization relative to instrument stability is $\alpha > 0$, and $\rho \geq 0$ is the subjective discount rate of the policy maker.

It is assumed that the loss function also represents the social welfare function, that is, potential conflicts between fiscal authorities and the electorate are assumed not to exist.

3.1 Non-cooperative policy making

In the non-cooperative case the fiscal authority in each country makes its decision taking the fiscal decisions of other member states as given (Cournot-Nash game), but recognizing the monetary policy reaction function since there is full information on the overall macroeconomic policy framework, cf the introduction. The fiscal policy reaction function can be written (see appendix) as depending on country-specific activity and (next period) inflation, i.e.

$$g_{it} = \kappa_y^{nc} y_{it} + \kappa_\pi^{nc} \pi_{it+1} \tag{9}$$

where

$$\begin{aligned} \kappa_y^{nc} &= -\alpha c_{ii} \gtrless 0 \\ \kappa_\pi^{nc} &= -\frac{\alpha}{1+\rho} \left(\delta_\tau\right)^2 \left(1 - \frac{1}{n}\right) \left(\delta_g \omega_y + \omega_g\right) < 0 \end{aligned}$$

It is seen that fiscal policy responds counter-cyclically to domestic activity provided that fiscal policy is expansionary ($c_{ii} > 0$), while it moves pro-cyclically if fiscal policy is contractionary ($c_{ii} < 0$). To interpret the policy response to an increase in the domestic inflationary pressure note that this effect arises via the cost spillover or competitiveness channel discussed above. An increase in future domestic inflation will lower future output by shifting demand away from domestic suppliers, since the policy maker is forward looking and concerned about output stability also in the future it follows that domestic inflation through this channel affects current fiscal policy. Finally observe, that the policy reaction function given above is not a closed form solution since both y_{it} and π_{it+1} are endogenous.

The difference between the fiscal policy stance in a given country and the overall fiscal stance within the monetary union can be written

$$g_{it} - g_t = \kappa_y^{nc}(y_{it} - y_t) + \kappa_\pi^{nc}\pi_{it+1}$$

Fiscal policy tends to be more contractionary in economies with a higher inflationary pressure, and with an output level above (see below) the average for $\kappa_y^{nc} < 0 \left(\kappa_y^{nc} > 0\right)$.

Aggregating over the fiscal reaction functions (9) we find that the aggregate fiscal stance is determined as[18]

$$g_t = \kappa_y^{nc} y_t$$

Implying that aggregate activity can be written

$$\begin{aligned} y_t &= -\frac{1}{\omega_y}\left[\omega_g \kappa_y^{nc} y_t + \omega_u u_t\right] \\ &= -\frac{\omega_u}{\omega_y + \omega_g \kappa_y^{nc}} u_t \end{aligned}$$

When[19] $\omega_y + \omega_g \kappa_y^{nc} > 0$, it follows that aggregate output in equilibrium moves counter-cyclically to demand shocks, and pro-cyclically to supply shocks, and vice versa. Note that there is an aggregate effect of fiscal policy only to the extent that fiscal policy has a direct impact on the wage/price formation process ($\omega_g \neq 0$).

3.2 Cooperative policy making

The cooperative solution is found under the assumption of a utilitarian criterion implying that fiscal policy for country i can be written (see appendix)

$$g_{it} = \kappa_i^c y_{it} + \kappa_a^c y_t + \kappa_\pi^c \pi_{it+1} \tag{10}$$

where

$$\begin{aligned} \kappa_i^c &= -\alpha\delta_g < 0 \\ \kappa_a^c &= -\alpha n c_{ji} \lesseqgtr 0 \\ \kappa_\pi^c &= -\frac{\alpha}{1+\rho}\left(\delta_\tau\right)^2\left(\omega_y \delta_g + \omega_g\right) < 0 \end{aligned}$$

[18] The so-called state-contingent bias (cf Svensson (1997), see also Sveen (2001)) according to which the aggregate fiscal stance depends on the realized value of the targeted inflation measure does not arise here since there are no independent cost-push shocks (inflation is non-stochastic). In e.g. Sveen (2001) such a shock is present and in the non-cooperative case this yields a state contingent bias in which the aggregate fiscal stance is increasing in the targeted rate of inflation. However, since expected inflation is zero there is no systematic bias in the aggregate fiscal stance.

[19] This holds if either $\omega_g < 0$ or for $\omega_g > 0$ when $n > \underline{n}$ ensuring that $c_{ii} > 0$.

Note some important differences to the non-cooperative policy reaction function (9). Domestic fiscal policy responds differently to local (κ_i^c) and aggregate activity (κ_a^c), where the former depends on the direct effect of fiscal policy on activity, and the latter on the international externality in fiscal policy (c_{ji}). The fiscal response is seen to be unambiguously counter-cyclical to local output variations, but pro-cyclical to aggregate output variations if the fiscal spillover is positive ($c_{ji} > 0$), and counter-cyclical if it is negative ($c_{ji} < 0$) . Finally, the policy coefficients are independent of the number of countries/fiscal decision makers (n).

In the aggregate we have (see appendix)

$$g_t^c = \kappa_y^c y_t \qquad ; \qquad \kappa_y^c \equiv \kappa_i^c + \kappa_a^c = \alpha \frac{\omega_g}{\omega_u}$$

where as expected κ_y^c is independent of n. Hence,

$$g_{it}^c - g_t^c = -\alpha(c_{ii} - c_{ji})(y_{it} - y_t)$$

Since $-\alpha(c_{ii} - c_{ji}) = -\alpha\delta_g < 0$, it follows that countries experiencing output levels above the average should contract fiscal policy relative to the average, and vice versa. Note this holds irrespective of the sign of ω_g.

4 Inefficiencies in non-cooperative fiscal policies

We can now turn to a comparison of cooperative and non-cooperative policy making. The cases of purely aggregate shocks and idiosyncratic shocks are considered in turn. The section ends by a numerical illustration of the results.

4.1 Aggregate shocks

In the case of aggregate shocks ($u_{it} = u_t$ for all i) we have that the difference in fiscal policy between the non-cooperative and the cooperative case depends on the policy parameter, where

$$\kappa_y^c - \kappa_y^{nc} = (1 - \frac{1}{n})\frac{\omega_g + \omega_y \delta_g}{\omega_y} > 0$$

Hence, we have that for the monetary union as a whole the cooperative fiscal policy stance is unambiguously more pro-cyclical than the non-cooperative fiscal policy stance - there is a policy-mix problem in the form of a counter-cyclical bias. The intuition is that each separate fiscal authority does not sufficiently take into account the inflation externality caused by fiscal expansions - this externality runs via both the direct effect fiscal policy has for price/wage formation

and the indirect effect from output to inflation. Note that this inefficiency is increasing in the number of fiscal decision makers (n),

$$\frac{\partial(\kappa_y^c - \kappa_y^{nc})}{\partial n} > 0$$

since the larger the monetary union the less effect does each separate fiscal authority perceive that its actions have on the aggregate policy stance within the monetary union. To put it differently, more fiscal decentralization reinforces the policy mix problem arising between a common monetary policy targeting inflation and decentralized fiscal policy. Note that this comparison only makes sense if there are aggregate shocks in the monetary union i.e. $u_t \neq 0$ (see below on idiosyncratic shocks).

Denote by L^{nc} the loss to fiscal policy makers in the non-cooperative case, and by L^c the loss in the cooperative case we have (see appendix)

$$\frac{L^{nc}}{L^c} = \frac{(\alpha + (\kappa_y^{nc})^2)}{(\alpha + (\kappa_y^c)^2)} \frac{(\omega_y + \kappa_y^c \omega_g)^2}{(\omega_y + \kappa_y^{nc} \omega_g)^2}$$

This ratio measures how much larger the loss is under non-cooperative policies relative to cooperative policies. There are two important properties of the relative loss ratio. First, it does not depend on whether the economy is exposed to demand or supply shocks (ω_u influences none of the coefficients in the expression). Second, it is also independent of trade externalities running via both income and relative prices (neither δ_y nor δ_τ affects the coefficients involved) - it follows that the consequences of non-cooperation wrt aggregate shocks is not changing with further product market integration.

4.2 Idiosyncratic shocks

In the case where the shocks are only country-specific or idiosyncratic (u=0) the situation is qualitatively different to the case of aggregate shocks (for technical details see appendix). In this case the non-cooperative policy function remains as given in (9) and in the cooperative case it can be written (note $y = 0$)

$$g_{it} = \kappa_i^c y_{it} + \kappa_\pi^c \pi_{it+1}$$

revealing that the cooperative response to a idiosyncratic shock is always counter-cyclical. Note that

$$\begin{array}{rcl} \kappa_i^c & < & \kappa_y^{nc} \\ 0 & > & \kappa_\pi^c > \kappa_\pi^{nc} \end{array}$$

Comparing the cooperative and the non-cooperative policy responses we have that

$$g_{it}^c - g_{it}^{nc} = -\alpha \left[\frac{1}{n} \left(\frac{\omega_g}{\omega_y} + \delta_g \right) \right] y_{it} - \frac{\alpha}{1+\rho} (\delta_\tau)^2 \frac{1}{n} (\delta_g \omega_y + \omega_g) \pi_{it+1} \qquad (11)$$

indicating that we have both a monetary policy and a cost spill over effect causing a difference between cooperative and non cooperative policy making. The monetary policy spill over effect – the first term on the RHS of (11) – arises because country-specific fiscal authorities perceive that their choice of fiscal policy will affect the monetary policy stance. Since a more expansionary fiscal policy tends to induce a more contractionary monetary policy, the single fiscal decision maker perceives that this effect reduces the potential of fiscal policy to stabilize domestic output. There is also a cost-spillover effect – the second term on the RHS of (11) - the fiscal authority perceives that an expansionary fiscal policy adds to inflation and therefore in turn induces future wage increases which harms competitiveness and thus future output. This lowers the incentive to pursue an expansionary fiscal policy to stabilize current output.

Considering variations in the number of member states it follows that

$$\frac{\partial(g_{it}^{c} - g_{it}^{nc})}{\partial n} < 0$$

that is, the difference between the non-cooperative and cooperative policy parameter is decreasing in the number of member states (n). This is so since both of the externalities referred to above are lowered. First, the effect on monetary policy of a single country fiscal policy is falling in n, and the same applies to the cost externality, since the larger the union the less influence will domestic producer inflation have on overall consumer price inflation and therefore on wage setting[20].

The relative loss ratio stating the loss to fiscal policy makers in the non-cooperative case relative to the loss under cooperative policies can be written (see appendix)

$$\frac{L^{nc}}{L^{c}} = \frac{\left[\alpha\left[\Phi_{\pi}^{nc2}\frac{(\Gamma_{y}^{nc}\Phi_{u}^{nc}+\Gamma_{u}^{nc})^{2}}{1-\Gamma_{y}^{nc2}\Phi_{\pi}^{nc2}} + \Phi_{u}^{nc2}\right] + \left[\varphi_{\pi}^{nc2}\frac{(\Gamma_{y}^{nc}\Phi_{u}^{nc}+\Gamma_{u}^{nc})^{2}}{1-\Gamma_{y}^{nc2}\Phi_{\pi}^{nc2}} + \varphi_{u}^{nc2}\right]\right]}{\left[\alpha\left[\Phi_{\pi}^{c2}\frac{(\Gamma_{y}^{c}\Phi_{u}^{c}+\Gamma_{u}^{c})^{2}}{1-\Gamma_{y}^{c2}\Phi_{\pi}^{c2}} + \Phi_{u}^{c2}\right] + \left[\varphi_{\pi}^{c2}\frac{(\Gamma_{y}^{c}\Phi_{u}^{c}+\Gamma_{u}^{c})^{2}}{1-\Gamma_{y}^{c2}\Phi_{\pi}^{c2}} + \varphi_{u}^{c2}\right]\right]}$$

The complexity of this relation makes it difficult to obtain transparent analytical results. It should, however, be noted that the elasticity of demand wrt to the terms of trade (δ_{τ}) does influence the relative loss ratio in the case of idiosyncratic shocks, and international integration may therefore affect the gains from fiscal policy cooperation, cf below.

4.3 Numerical illustrations

The numerical illustrations reported here rely on a number of parameter values the precise value of which can be debated given the simplified structure

[20] It may be argued that the fading away of this effect the larger the union depends critically on the specification of the consumer price index, in particular the symmetric weighting of all producer prices in the monetary union. In a formulation allowing for some home bias, it may be conjectured that this effect remains operative even with a large number of decision makers. However, the finding that it is decreasing in the number of decision makers should be robust.

of the model. A key parameter is the cyclical sensitivity of fiscal policy, and the parameter[21] has been calibrated such that the fiscal policy variable (g) in the non-cooperative case is counter-cyclical wrt to output fluctuations with a numerical elasticity of about 0.6. This implies that the fiscal variable is interpreted as the budget deficit, and the sensitivity corresponds roughly to the level of automatic stabilizers on average across EU-countries (see van der Noord (2000)). In the base cases it is assumed that fiscal policy has a direct positive effect on inflation (ω_g). The numerical exercise should not be interpreted as a calibration, but only as a numerical illustration providing a first quantitative idea on the strength of the mechanisms considered here.

Figures 1-3 illustrate[22] the loss under non-cooperative policy making relative to the loss under cooperative policy making. In the case $n = 1$ the non-cooperative and cooperative case are obviously identical. Observe that the relative loss does not say anything on the absolute level of the loss, that is, they do not answer the pertinent question of whether the gains from an active stabilization policy are large or small. Considering relative losses across the two regimes has the advantage that there is no need to take a stand on the variability of shocks. It is an implication of the results reported in e.g. figure 1 that if the gains from an active stabilization policy are large, then the costs of non-cooperative policies are also large, however, if the former are small, the latter may also be small. Finally, note that the loss function is not derived from an explicit welfare approach but gives the loss function of the fiscal authority assumed to represent the electorate.

In the case of aggregate shocks it is seen that the loss under non-cooperative policies can be a factor 2 or larger than in the cooperative case, cf figure 1. The relative loss is increasing in the number of fiscal authorities due to the policy mix externality outlined above. The relative loss flattens out when the number of fiscal authorities increases, hence, an increase in the number of fiscal authorities from 12 to say 20 implies that the loss in the non-cooperative case will increase from being approximately 2.3 times as large to being 2.4 as large. Enlargement of the EMU may thus reinforce policy-mix problem, but relatively little compared to the problems already present.[23]

In the case where shocks are completely idiosyncratic in the sense that they do not aggregate to something of common importance within the currency area we find that the relative loss due to non-cooperative fiscal policy making (evaluated for otherwise unchanged parameter values to the case of aggregate shocks) is much smaller, cf fig 2. The loss under non-cooperation is always less than 1.5 of that under cooperation. Moreover we find that the relative loss is decreasing in the number of fiscal authorities, but the relative loss function is relatively flat in n, that is, variations in n do not seem to matter much for the gains from

[21] The other parameter values are in the base case set at: $\alpha = 2$, $\omega_y = 0.5$, $\omega_g = 0.5$, $\omega_g = 0.2$, $\omega_u = 0.1$, $\delta_\tau = 0.2$, $\delta_y = 0.5$, $\delta_g = 0.3$, $\delta_u = 0.1$, $\rho = 0$.

[22] It is found that the critical number of countries ($\underline{n}$) yielding standard signs of fiscal multipliers is very small (2 or 3) for the cases considered here.

[23] The degree of fiscal decentralization within countries is disregarded here, and therefore assumed invariant to the number of fiscal decision makers in the union.

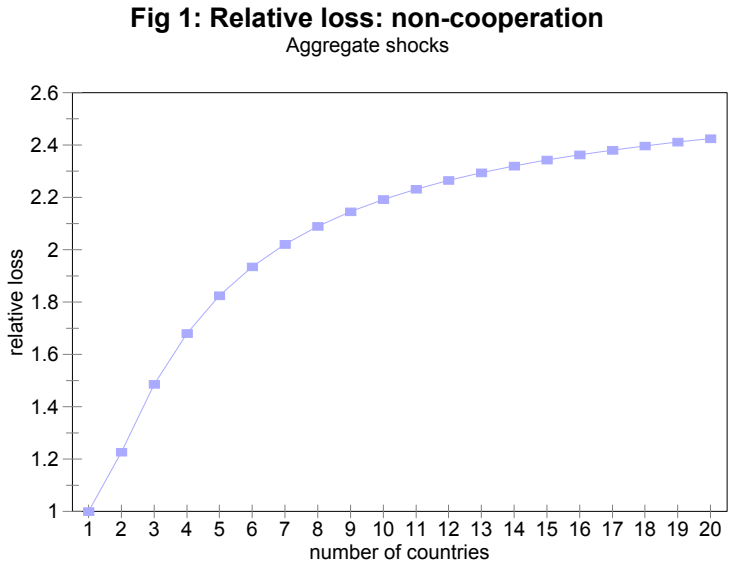

Figure 1:

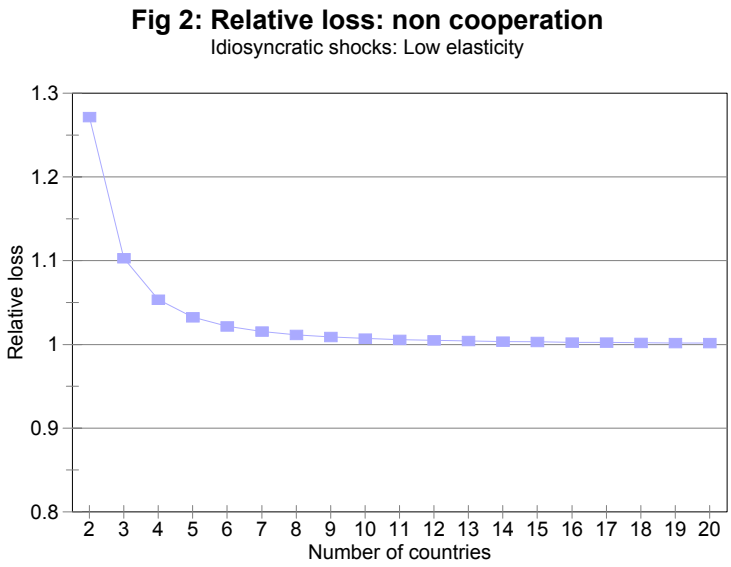

Figure 2:

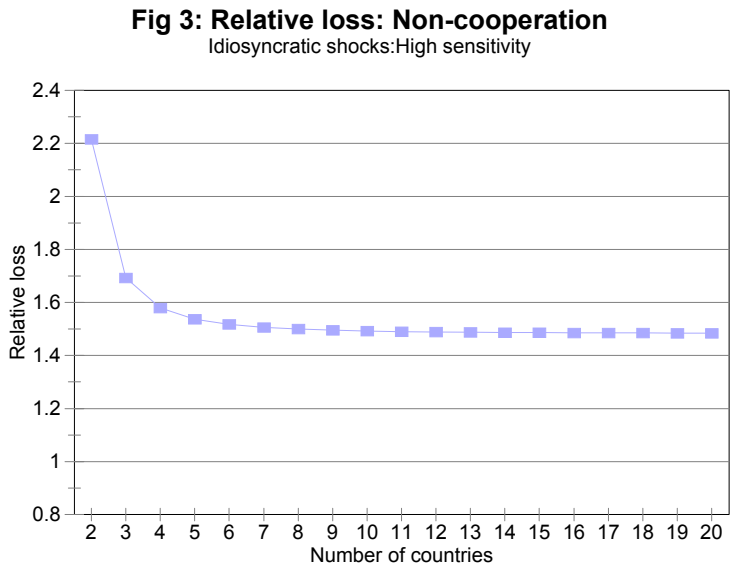

Figure 3:

cooperation.

As noted in the theoretical analysis, trade externalities do not influence the relative loss ratio in the case of aggregate shocks. However, for idiosyncratic shocks the sensitivity of demand to the terms of trade (δ_τ) plays a role. Figure 3[24] brings out that if the price elasticity is high implying, that the concern for the cost spillover is large, then the cost of non-cooperative policy making is somewhat larger, and a non-trivial cost remains even when the number of member countries increases.

Table 1 reports the relative loss for supply shocks for otherwise unchanged parameter to the base case underlying figure 1 and 2 in the case of demand shocks. It is seen that the qualitative conclusions are the same. Actually the relative loss for aggregate supply shocks is equivalent to that for aggregate demand shocks (cf also general proof in appendix). Note that the relative loss from non-cooperation are much smaller in the case of idiosyncratic supply shocks than for idiosyncratic demand shocks. The table also includes the calculations for the case where fiscal policy works to lower the inflationary pressure ($\omega_g < 0$). It is found that the difference in loss between the cooperative and non-cooperative case in general are much smaller. Still, the qualitative profile found above holds, that is, the relative loss is increasing in the number of countries for aggregate shocks and decreasing for idiosyncratic shocks.

[24] Changing δ_τ from 0.2. to 1.2.

Table 1: Relative loss for supply shocks, and negative inflation effect of fiscal policy

number of countries	$\omega_g = 0.2$		$\omega_g = -0.2$			
	supply shock		demand shock		supply shock	
	agg	idio	agg	idio	agg	idio
1	1	na	1	na	1	na
2	1.227	1.254	1.003	1.272	1.003	1.003
3	1.486	1.099	1.006	1.103	1.006	1.002
4	1.681	1.052	1.007	1.05	1.007	1.001
5	1.825	1.032	1.008	1.033	1.008	1.001
6	1.935	1.022	1.009	1.022	1.009	1
7	2.020	1.016	1.009	1.016	1.009	1
8	2.089	1.012	1.010	1.012	1.010	1
9	2.145	1.010	1.010	1.009	1.010	1
10	2.192	1.008	1.010	1.007	1.010	1
11	2.232	1.006	1.011	1.006	1.011	1
12	2.265	1.005	1.011	1.005	1.011	1
13	2.295	1.005	1.011	1.004	1.011	1
14	2.320	1.004	1.011	1.003	1.011	1
15	2.343	1.003	1.011	1.003	1.011	1
16	2.363	1.003	1.011	1.003	1.011	1
17	2.381	1.003	1.011	1.002	1.011	1
18	2.397	1.002	1.012	1.002	1.012	1
19	2.411	1.002	1.012	1.002	1.012	1
20	2.424	1.002	1.012	1.002	1.012	1

The numerical illustrations reported here suggest that the coordination problem can be of substantial importance for aggregate shocks while it is of little importance in the case of idiosyncratic shocks. Moreover, it shows that an increase in the number of fiscal decision makers tends to reinforce the policy mix problems in addressing aggregate shocks, but to decrease the problems related to the appropriate policy response to idiosyncratic shocks. It follows that a monetary union adhering to a target of price stability is not necessarily problematic for the response to idiosyncratic shocks, provided that the number of fiscal decision makers (the number of participating countries) in the union is sufficiently large. This result presumes that there are no limitations (like binding budget norms) on the ability of the fiscal authorities to respond to idiosyncratic shocks. This finding is interesting in the perspective of the traditional discussion of the need to maintain policy instruments to cushion idiosyncratic shocks in a monetary union (see e.g. Mundell (1961)). The present analysis confirms this, but also highlights that the issue of policy mix problems is related to aggregate shocks.

5 Concluding remarks

The interaction between fiscal and monetary policy within a monetary union has been considered in the case where monetary policy targets inflation. The framework used a simple version of standard macromodels, which is useful in bringing forth the basic mechanisms influencing policy interdependencies arising via trade interactions and the common monetary policy.

One interesting finding is that the coordination problem is larger in the case of aggregate than idiosyncratic shocks, where the latter does not seem to cause substantial problems in the numerical examples provided. There is thus a case to be made for further coordination of fiscal policy in the case of aggregate shocks. It is interesting to note that the European Commission in a recent communication on the policy coordination within the euro-area stressed that previous arguments have been based on the premises that interdependencies are small, and that they do not take appropriate consideration of the aggregate fiscal stance within the area (European Commission, 2001).

The present analysis shows that trade externalities under strict inflation targeting are not of any importance for the policy mix problem. The monetary interdependence is, however, and therefore there are gains from coordinating the aggregate fiscal stance relative to the monetary policy stance. While strict inflation targeting may be a too narrow interpretation of ECB policies, it does point out that the interdependencies between countries may change qualitatively when monetary policy is directed towards maintaining price stability, and an important question for future research would be to analyse whether this holds under various forms of flexible inflation targeting.

Since the policy mix problem arises for aggregate shocks, it may also be conjecture that more flexible inflation targeting where the monetary policy authority also pays attention to aggregate (within the monetary union) output stabilization would reduce the tension between fiscal and monetary policy. An interesting topic for future research would be to analyse whether the gains from fiscal policy coordination to aggregate shocks depends on the weight the monetary authority assigns to stabilization of aggregate output.

The analysis also shows that the policy mix problem depends critically on whether fiscal policy instruments tend to increase or decrease the inflationary pressure. This raises the question of whether policy mix problem could be reduced if short run stabilization policies are mainly done by instruments which can expand output at the same time as reducing the inflationary pressure. Temporary variations in taxes like value added tax and employment subsidies etc. seem to satisfy these criteria. It is an interesting question for further research to consider whether restrictions on the choice of fiscal instruments used for short-run stabilization can be a substitute for formal policy cooperation.

A key assumption for the present analysis has been that the monetary policy rule is known and taken into account by the fiscal authorities. The policy conclusions depend critically on this assumption. In the absence of monetary commitment in the sense that fiscal authorities take monetary policy actions for given when deciding on fiscal policy, there is a policy mix problem since fiscal discretion unravels the effects of monetary commitment and this effect is reinforced with fiscal policy cooperation. In this setting there are arguments for

restraining discretionary powers in fiscal policy (see e.g. Beetsma and Bovenberg (1998), Beetsma, Debrun and Klaassen (2001) and Dixit and Lambertini (2000a,b)). Oppositely, with monetary commitment implying that fiscal authorities plan under full knowledge of monetary policy reaction to fiscal policy, there is also a policy mix problem, but it is reduced with fiscal policy cooperation. In this case flexibility in fiscal policy is important, not least in respect to stabilization of idiosyncratic shocks. An important question for future research is how the monetary commitment depends on various lag structures in the response and effect time of both monetary and fiscal policy.

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APPENDIX

(I) Equilibrium output

Conjecture that equilibrium output in country i in period t can be written

$$y_{it} = a_{it} + \sum_j b_{ij} u_{jt} + \sum_j c_{ij} g_{jt} \tag{12}$$

We have by substitution in (3) that

$$\begin{aligned}
y_{it} &= \frac{-(1-\delta_y)}{\omega_y}\left[\pi_t + (\frac{\omega_y}{1-\delta_y}\delta_g + \omega_g)g_t + (\frac{\omega_y}{1-\delta_y}\delta_u + \omega_u)u_t\right] - \delta_\tau(\pi_{it} - \pi_t) + \delta_g g_{it} + \delta_y y_t + \delta_u u_{it} \\
&= \frac{-(1-\delta_y)}{\omega_y}\left[\pi_t + (\frac{\omega_y}{1-\delta_y}\delta_g + \omega_g)\frac{1}{n}\sum_k g_{kt} + (\frac{\omega_y}{1-\delta_y}\delta_u + \omega_u)\frac{1}{n}\sum_k u_{kt}\right] \\
&\quad -\delta_\tau(\pi_{i,t} - \pi_t) + \delta_g g_{it} + \delta_y \frac{1}{n}\left(\sum_k \left(a_{kt} + \sum_j b_{kj}u_{jt} + \sum_j c_{kj}g_{jt}\right)\right) + \delta_u u_{it}
\end{aligned}$$

Hence (12) is a solution (note $\pi_t = 0$) if

$$\begin{aligned}
a_{it} &= -\delta_\tau \pi_{it} + \delta_y \frac{1}{n}\sum_k a_{kt} \\
b_{i,j} &= \frac{-(1-\delta_y)}{\omega_y}(\frac{\omega_y}{1-\delta_y}\delta_u + \omega_u)\frac{1}{n} + \delta_y \frac{1}{n}\sum_k b_{kj} \qquad for \quad i \neq j \\
b_{ii} &= \frac{-(1-\delta_y)}{\omega_y}(\frac{\omega_y}{1-\delta_y}\delta_u + \omega_u)\frac{1}{n} + \delta_y \frac{1}{n}\sum_k b_{ki} + \delta_u \\
c_{ij} &= \frac{-(1-\delta_y)}{\omega_y}(\frac{\omega_y}{1-\delta_y}\delta_g + \omega_g)\frac{1}{n} + \delta_y \frac{1}{n}\sum_k c_{kj} \qquad for \quad i \neq j \\
c_{ii} &= \frac{-(1-\delta_y)}{\omega_y}(\frac{\omega_y}{1-\delta_y}\delta_g + \omega_g)\frac{1}{n} + \delta_y \frac{1}{n}\sum_k c_{ki} + \delta_g
\end{aligned}$$

It follows that

$$\begin{aligned}
\sum_j a_{jt} &= 0 \\
\sum_j b_{ij} &= \frac{-1}{\omega_y}(\frac{\omega_y}{1-\delta_y}\delta_u + \omega_u) + \frac{\delta_u}{1-\delta_y} \\
\sum_j c_{ij} &= \frac{-1}{\omega_y}(\frac{\omega_y}{1-\delta_y}\delta_g + \omega_g) + \frac{\delta_g}{1-\delta_y}
\end{aligned}$$

which implies

$$
\begin{aligned}
a_{it} &= -\delta_\tau \pi_{it} \\
b_{ii} &= -\frac{1}{n}\frac{1}{\omega_y}\left[\frac{\omega_y \delta_u}{1-\delta_y} + \omega_u\right] + \delta_u \left[\frac{1}{n}\frac{\delta_y}{1-\delta_y} + 1\right] \\
c_{ij} &= -\frac{1}{n}\frac{1}{\omega_y}\left[\frac{\omega_y \delta_g}{1-\delta_y} + \omega_g\right] + \delta_g \frac{1}{n}\frac{\delta_y}{1-\delta_y} \\
c_{ii} &= -\frac{1}{n}\frac{1}{\omega_y}\left[\frac{\omega_y \delta_g}{1-\delta_y} + \omega_g\right] + \delta_g \left[\frac{1}{n}\frac{\delta_y}{1-\delta_y} + 1\right]
\end{aligned}
$$

Note that the b- and c-coefficients are made up of an inflation component (the first term), and an activity component (the second term), we have

$$
\begin{aligned}
a_{it} &= -\delta_\tau \pi_{it} \\
b_{ii} &= -\frac{1}{n}\frac{\omega_u}{\omega_y} + \delta_u \left[1 - \frac{1}{n}\right] \gtrless 0 \\
b_{ij} &= -\frac{\omega_u}{\omega_y}\frac{1}{n} - \frac{1}{n}\delta_u \gtrless 0 \\
c_{ii} &= -\frac{\omega_g}{\omega_y}\frac{1}{n} + (1 - \frac{1}{n})\delta_g \gtrless 0 \\
c_{ij} &= -\frac{\omega_g}{\omega_y}\frac{1}{n} - \frac{1}{n}\delta_g < 0
\end{aligned}
$$

Note that

$$
\frac{\partial c_{ii}}{\partial n} > 0
$$

The more decentralized fiscal policy is the more the policy maker takes a fiscal expansion to boost activity.

We can write output as

$$
y_{it} = a_{it} + b_1 u_{it} + b_2 u_t + c_1 g_{it} + c_2 g_t
$$

where

$$
\begin{aligned}
b_1 &= b_{ii} - b_{ij} = \delta_u > 0 \\
b_2 &= -\frac{\omega_u}{\omega_y} - \delta_u \\
c_1 &= c_{ii} - c_{ij} = \delta_g \\
c_2 &= -\frac{\omega_g}{\omega_y} - \delta_g < 0
\end{aligned}
$$

For later reference note the inflation in country i producer prices can be written

$$
\begin{aligned}
\pi_{it+1} &= \pi_{it} - \left(1 - \frac{1}{n}\right)\frac{n}{n-1}\left(\pi_{it} - \pi_t\right) + \omega_y y_{it} + \omega_g g_{it} + \omega_u u_{it} \\
&= \omega_y y_{it} + \omega_g g_{it} + \omega_u u_{it}
\end{aligned}
$$

(II) Fiscal Policy: Non-cooperative
Loss function

$$L = E_t \sum_{m=0}^{\infty} (1+\rho)^{-m} (\frac{1}{2}\alpha \left(y_{it+m}\right)^2 + \frac{1}{2}(g_{it+m})^2)$$

The first order condition reads (non-cooperative)

$$\alpha c_{ii} y_{it} + g_{it} + \frac{\alpha}{1+\rho} E_t \sum_{m=1}^{\infty} (1+\rho)^{-m} \left[y_{it+m} \frac{\partial y_{it+m}}{\partial g_{it}} \right] = 0$$

where

$$E_t \left[y_{it+m} \frac{\partial y_{it+m}}{\partial g_{it}} \right] = E_t \left[y_{it+m} \left(\frac{\partial a_{it+m}}{\partial g_{it}} \right) \right]$$

We have

$$\begin{aligned} \frac{\partial a_{it+1}}{\partial g_{it}} &= -\delta_\tau \frac{\partial \pi_{it+1}}{\partial g_{it}} \\ &= -\delta_\tau \left(\omega_y c_{ii} + \omega_g\right) \end{aligned}$$

$$\frac{\partial a_{it+m}}{\partial g_{it}} = 0 \quad for \quad m > 1$$

Inserting we have

$$\begin{aligned} E_t \left[y_{it+1} \left(\frac{\partial a_{it+1}}{\partial g_{it}} \right) \right] &= -\delta_\tau \left(\omega_y c_{ii} + \omega_g\right) E_t \left[y_{it+1}\right] \\ &= -\delta_\tau \left(\omega_y c_{ii} + \omega_g\right) E_t [a_{it+1}] \\ &= \left(\delta_\tau\right)^2 \left(\omega_y c_{ii} + \omega_g\right) \pi_{it+1} \end{aligned}$$

Hence, the first order condition can be written

$$g_{it} = \kappa_y^{nc} y_{it} + \kappa_\pi^{nc} \pi_{it+1} \tag{13}$$

where

$$\begin{aligned} \kappa_y^{nc} &= -\alpha c_{ii} = -\alpha \left[-\frac{\omega_g}{\omega_y} \frac{1}{n} + (1 - \frac{1}{n}) \delta_g \right] \gtrless 0 \\ \kappa_\pi^{nc} &= -\frac{\alpha}{1+\rho} \left(\delta_\tau\right)^2 \left(\omega_y c_{ii} + \omega_g\right) \\ &= -\frac{\alpha}{1+\rho} \left(\delta_\tau\right)^2 \left(1 - \frac{1}{n}\right) \left(\delta_g \omega_y + \omega_g\right) \gtrless 0 \end{aligned}$$

In the following it is assumed (see text) that $\delta_g \omega_y + \omega_g > 0$ implying that $\kappa_\pi^{nc} > 0$ and $\kappa_y^{nc} > -\alpha \delta_g$.

Aggregate fiscal policy stance

$$\begin{aligned} g_t &= \kappa_y^{nc} y_t \\ &= \kappa_y^{nc} \left[-\frac{1}{\omega_y} \left[\omega_g g_t + \omega_u u_t \right] \right] \end{aligned}$$

$$g_t = \frac{-\kappa_y^{nc} \omega_u u_t}{\omega_y + \kappa_y^{nc} \omega_g}$$

i.e. (assuming $\omega_y + \kappa_y^{nc}\omega_g > 0$) the fiscal response is pro-cyclical to a supply shock ($\omega_u < 0$) and vice versa to a demand shock ($\omega_u < 0$).

For output we have

$$y_t = \frac{-\omega_u}{\omega_y + \kappa_y^{nc}\omega_g} u_t$$

$$\frac{\partial y_t}{\partial u_t} \gtreqless 0 \qquad for \qquad \omega_u \lesseqgtr 0$$

Supply shocks boost activity, and demand shocks contract activity.

Note that if all shocks are idiosyncratic, i.e. $u_t = 0$ it follows that $g_t = y_t = 0$.

III) Fiscal Policy: Cooperative

The first order condition reads (cooperative-utilitarian criterion)

$$\alpha \sum_j y_{jt} c_{ji} + g_{it} + \frac{\alpha}{1+\rho} E_t \sum_j \sum_{k=1}^{\infty} \left[y_{jt+k} \frac{\partial y_{jt+k}}{\partial g_{it}} \right] = 0 \tag{14}$$

Consider first the last term on the LHS. We have that

$$\begin{aligned} \frac{\partial y_{jt+k}}{\partial g_{it}} &= 0 \qquad for\ all\ j\ and\ k \geq 2 \\ \frac{\partial y_{jt+1}}{\partial g_{it}} &= -\delta_\pi \frac{\partial \pi_{jt+1}}{\partial g_{it}} = -\delta_\pi \omega_y c_{ji} \\ \frac{\partial y_{jt+k}}{\partial g_{it}} &= -\delta_\pi \frac{\partial \pi_{it+1}}{\partial g_{it}} = -\delta_\pi (\omega_y c_{ii} + \omega_g) \end{aligned}$$

Hence

$$\begin{aligned} E_t \sum_{k=1}^{\infty} \sum_j y_{jt+k} \frac{\partial y_{jt+k}}{\partial g_{it}} &= (\delta_\tau)^2 \left[[\omega_y c_{ji}] E_t \pi_{t+1} + (\omega_y \delta_g + \omega_g) E_t \pi_{it+1} \right] \\ &= (\delta_\tau)^2 (\omega_y \delta_g + \omega_g) E_t \pi_{it+1} \end{aligned}$$

since $E_t\pi_{t+1} = 0$. Turning to the first term on the LHS of (14) we have that

$$\begin{aligned}
\alpha \sum_j y_{jt} c_{ji} &= \alpha \left[(c_{ii} - c_{ji}) y_{it} + c_{ji} \sum_j y_{jt} \right] \\
&= \alpha (c_{ii} - c_{ji}) y_{it} + \alpha n c_{ji} y_t \\
&= \alpha \delta_g y_{it} + \alpha n c_{ji} y_t
\end{aligned}$$

which can be written

$$\begin{aligned}
g_{it} &= -\alpha \delta_g y_{it} - \alpha n c_{ji} y_t - \frac{\alpha}{1+\rho} (\delta_\tau)^2 (\omega_y \delta_g + \omega_g) E_t \pi_{it+1} \\
&= \kappa_i^c y_{it} + \kappa_a^c y_t + \kappa_\pi^c \pi_{it+1}
\end{aligned}$$

where

$$\begin{aligned}
\kappa_i^c &= -\alpha \delta_g < 0 \\
\kappa_a^c &= -\alpha n c_{ji} \lesseqgtr 0 \\
\kappa_\pi^c &= -\frac{\alpha}{1+\rho} (\delta_\tau)^2 (\omega_y \delta_g + \omega_g) < 0
\end{aligned}$$

In the aggregate we have

$$\begin{aligned}
g_t &= (\kappa_i^c + \kappa_a^c) y_t \\
&= \kappa_y^c y_t \qquad , \quad \kappa_y^c \equiv \alpha \frac{\omega_g}{\omega_y}
\end{aligned}$$

Finally, aggregate output can be written

$$y_t = \frac{-\omega_u}{\omega_y + \kappa_y^c \omega_g} u_t$$

IV) Comparing non-cooperative and cooperative policies

(i) Aggregate shocks:
Comparing the non-cooperative and the cooperative case we have

$$\kappa_y^c > \kappa_y^{nc}$$

Hence, provided that the net effect of fiscal policy on inflation is positive ($\omega_g + \omega_g \delta_g > 0$) we have that for the monetary union as a whole the cooperative fiscal policy stance is unambiguously more pro-cyclical than the non-cooperative fiscal policy stance. Note that this inefficiency is increasing in fiscal decentralisation, that is, more fiscal decentralization reinforces the policy mix problem arising between a common monetary policy targeting inflation and decentralized fiscal policy. Note this comparison only makes sense if there are aggregate shocks in the monetary union, i.e. $u_t \neq 0$ (see below on idiosyncratic shocks).

We have that

$$\begin{aligned} VAR(y^{nc}) &= \left[\frac{-\omega_u}{\omega_y + \kappa_y^{nc}\omega_g}\right]^2 VAR(u) \\ VAR(y^c) &= \left[\frac{-\omega_u}{\omega_y + \kappa_y^{c}\omega_g}\right]^2 VAR(u) \end{aligned}$$

Note that

$$VAR(g^{nc}) = (\kappa_y^{nc})^2 VAR(y^{nc})$$

$$VAR(g^c) = (\kappa_y^{c})^2 VAR(y^c)$$

The ex-ante loss under non-cooperative fiscal policy is therefore given as

$$\begin{aligned} L^{nc} &= \alpha VAR(y^{nc}) + (\kappa_y^{nc})^2 VAR(g^{nc}) \\ &= (\alpha + (\kappa_y^{nc})^2) VAR(y^{nc}) \\ &= (\alpha + (\kappa_y^{nc})^2) \left[\frac{-\omega_u}{\omega_y + \kappa_y^{nc}\omega_g}\right]^2 VAR(u) \end{aligned}$$

The relative loss under cooperation to non-cooperative fiscal decision making is therefore given as

$$\frac{L^c}{L^{nc}} = \frac{(\alpha + (\kappa_y^c)^2)}{(\alpha + (\kappa_y^{nc})^2)} \frac{(\omega_y + \kappa_y^{nc}\omega_g)^2}{(\omega_y + \kappa_y^{c}\omega_g)^2}$$

(ii) Idiosyncratic shocks

The above-mentioned case presumes that there are aggregate consequences of shocks, in the case of idiosyncratic shocks the situation may be very different. To consider this case assume that

$$u_t = 0$$

and hence $y_t = 0$.

$$g_{it} = \kappa_i^c y_{it} + \kappa_\pi^c \pi_{it+1}$$

it follows that the coop policy to idiosyncratic shocks is always counter-cyclical. Hence

$$\frac{\partial y_{it}}{\partial u_{it}} = \frac{\delta_u}{1 + \alpha\delta_g^2} < \delta_u$$

Comparing the cooperative and the non-cooperative policy responses we have that

$$g_{it}^{c} - g_{it}^{nc} = -\alpha \left[\frac{1}{n} \left(\frac{\omega_g}{\omega_y} + \delta_g \right) \right] y_{it} - \frac{\alpha}{1+\rho} (\delta_\tau)^2 \frac{1}{n} (\delta_g \omega_y + \omega_g) \pi_{it+1}$$

implying

$$\begin{aligned} \frac{\partial (g_{it}^{c} - g_{it}^{nc})}{\partial y_{it}} &= -\alpha \frac{1}{n} \left(\frac{\omega_g}{\omega_y} + \delta_g \right) < 0 \\ \frac{\partial (g_{it}^{c} - g_{it}^{nc})}{\partial \pi_{it}} &= \frac{\alpha}{1+\rho} (\delta_\tau)^2 \left(1 - \frac{1}{n} \right) (\delta_g \omega_y + \omega_g) < 0 \end{aligned}$$

that is the cooperative fiscal policy will be more counter-cyclical than the non-cooperative fiscal policy, and likewise the cooperative policy would respond more contractionary to domestic inflation than the non-cooperative policy.

Note that

$$\begin{aligned} -\alpha \frac{1}{n} \left(\frac{\omega_g}{\omega_y} + \delta_g \right) &\rightarrow 0 \text{ for } n \rightarrow \infty \\ \frac{\alpha}{1+\rho} (\delta_\tau)^2 \frac{1}{n} (\delta_g \omega_y + \omega_g) &\rightarrow 0 \text{ for } n \rightarrow \infty \end{aligned}$$

In the non-cooperative case we have

$$\begin{aligned} y_{it} &= -\delta_\tau \pi_{it} + \delta_g g_{it} + \delta_u u_{it} \\ \pi_{it+1} &= \omega_y y_{it} + \omega_g g_{it} + \omega_u u_{it} \\ g_{it} &= \kappa_y^{nc} y_{it} + \kappa_\pi^{nc} \pi_{it+1} \end{aligned}$$

Hence

$$\begin{aligned} \pi_{it+1} &= \omega_y y_{it} + \omega_g (\kappa_y^{nc} y_{it} + \kappa_\pi^{nc} \pi_{it+1}) + \omega_u u_{it} \\ &= \frac{1}{1 - \omega_g \kappa_\pi^{nc}} \left[(\omega_y + \omega_g \kappa_y^{nc}) y_{it} + \omega_u u_{it} \right] \\ &= \Gamma_y^{nc} y_{it} + \Gamma_u^{nc} u_{it} \end{aligned}$$

where

$$\begin{aligned} \Gamma_y^{nc} &= \frac{\omega_y + \omega_g \kappa_y^{nc}}{1 - \omega_g \kappa_\pi^{nc}} \\ \Gamma_u^{nc} &= \frac{\omega_u}{1 - \omega_g \kappa_\pi^{nc}} \end{aligned}$$

Using this we find

$$\begin{aligned} y_{it} &= -\delta_\tau \pi_{it} + \delta_g \left[\kappa_y^{nc} y_{it} + \kappa_\pi^{nc} \left[\Gamma_y^{nc} y_{it} + \Gamma_u^{nc} u_{it}\right]\right] + \delta_u u_{it} \\ &= \Phi_\pi^{nc} \pi_{it} + \Phi_u^{nc} u_{it} \end{aligned}$$

where

$$\begin{aligned} \Phi_\pi^{nc} &= \frac{-\delta_\tau}{1 - \delta_g(\kappa_y^{nc} + \kappa_\pi^{nc}\Gamma_y^{nc})} \\ \Phi_u^{nc} &= \frac{\delta_u + \delta_g \kappa_\pi^{nc}\Gamma_u^{nc}}{1 - \delta_g(\kappa_y^{nc} + \kappa_\pi^{nc}\Gamma_y^{nc})} \end{aligned}$$

Consequently, fiscal policy can be written

$$\begin{aligned} g_{it} &= \kappa_y^{nc} y_{it} + \kappa_\pi^{nc}(\Gamma_y^{nc} y_{it} + \Gamma_u^{nc} u_{it}) \\ &= (\kappa_y^{nc} + \kappa_\pi^{nc}\Gamma_y^{nc}) y_{it} + \kappa_\pi^{nc}\Gamma_u^{nc} u_{it} \\ &= (\kappa_y^{nc} + \kappa_\pi^{nc}\Gamma_y^{nc})(\Phi_\pi^{nc}\pi_{it} + \Phi_u^{nc} u_{it}) + \kappa_\pi^{nc}\Gamma_u^{nc} u_{it} \\ &= (\kappa_y^{nc} + \kappa_\pi^{nc}\Gamma_y^{nc})\Phi_\pi^{nc}\pi_{it} + ((\kappa_y^{nc} + \kappa_\pi^{nc}\Gamma_y^{nc})\Phi_u^{nc} + \kappa_\pi^{nc}\Gamma_u^{nc}) u_{it} \\ &= \varphi_\pi^{nc}\pi_{it} + \varphi_u^{nc} u_{it} \end{aligned}$$

where

$$\begin{aligned} \varphi_\pi^{nc} &= (\kappa_y^{nc} + \kappa_\pi^{nc}\Gamma_y^{nc})\Phi_\pi^{nc} \\ \varphi_u^{nc} &= ((\kappa_y^{nc} + \kappa_\pi^{nc}\Gamma_y^{nc})\Phi_u^{nc} + \kappa_\pi^{nc}\Gamma_u^{nc}) \end{aligned}$$

It follows that

$$\begin{aligned} \pi_{it+1} &= \Gamma_y^{nc} y_{it} + \Gamma_u^{nc} u_{it} = \Gamma_y^{nc}(\Phi_\pi^{nc}\pi_{it} + \Phi_u^{nc} u_{it}) + \Gamma_u^{nc} u_{it} \\ &= \Gamma_y^{nc}\Phi_\pi^{nc}\pi_{it} + (\Gamma_y^{nc}\Phi_u^{nc} + \Gamma_u^{nc}) u_{it} \end{aligned}$$

Hence

$$VAR(\pi) = \frac{(\Gamma_y^{nc}\Phi_u^{nc} + \Gamma_u^{nc})^2}{1 - \Gamma_y^{nc2}\Phi_\pi^{nc2}}$$

From which it follows that

$$\begin{aligned} VAR(y_i) &= \left[\Phi_\pi^{nc2}\frac{(\Gamma_y^{nc}\Phi_u^{nc} + \Gamma_u^{nc})^2}{1 - \Gamma_y^{nc2}\Phi_\pi^{nc2}} + \Phi_u^{nc2}\right] VAR(u) \\ VAR(g) &= \left[\varphi_\pi^{nc2}\frac{(\Gamma_y^{nc}\Phi_u^{nc} + \Gamma_u^{nc})^2}{1 - \Gamma_y^{nc2}\Phi_\pi^{nc2}} + \varphi_u^{nc2}\right] VAR(u_i) \end{aligned}$$

The expected loss under non-cooperative policies is therefore given as

$$\begin{aligned} L^{nc} &= \alpha VAR(y) + VAR(g) \\ &= \left[\alpha\left[\Phi_\pi^{nc2}\frac{(\Gamma_y^{nc}\Phi_u^{nc} + \Gamma_u^{nc})^2}{1 - \Gamma_y^{nc2}\Phi_\pi^{nc2}} + \Phi_u^{nc2}\right] + \left[\varphi_\pi^{nc2}\frac{(\Gamma_y^{nc}\Phi_u^{nc} + \Gamma_u^{nc})^2}{1 - \Gamma_y^{nc2}\Phi_\pi^{nc2}} + \varphi_u^{nc2}\right]\right] VAR(u) \end{aligned}$$

and the relative loss can therefore be written

$$\frac{L^{nc}}{L^{c}} = \frac{\left[\alpha\left[\Phi_{\pi}^{nc2}\frac{(\Gamma_{y}^{nc}\Phi_{u}^{nc}+\Gamma_{u}^{nc})^{2}}{1-\Gamma_{y}^{nc2}\Phi_{\pi}^{nc2}}+\Phi_{u}^{nc2}\right]+\left[\varphi_{\pi}^{nc2}\frac{(\Gamma_{y}^{nc}\Phi_{u}^{nc}+\Gamma_{u}^{nc})^{2}}{1-\Gamma_{y}^{nc2}\Phi_{\pi}^{nc2}}+\varphi_{u}^{nc2}\right]\right]}{\left[\alpha\left[\Phi_{\pi}^{c2}\frac{(\Gamma_{y}^{c}\Phi_{u}^{c}+\Gamma_{u}^{c})^{2}}{1-\Gamma_{y}^{c2}\Phi_{\pi}^{c2}}+\Phi_{u}^{c2}\right]+\left[\varphi_{\pi}^{c2}\frac{(\Gamma_{y}^{c}\Phi_{u}^{c}+\Gamma_{u}^{c})^{2}}{1-\Gamma_{y}^{c2}\Phi_{\pi}^{c2}}+\varphi_{u}^{c2}\right]\right]}$$