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ABSTRACT

Endogenous Fluctuations in Unionised Economy with Productive Externalities*

We discuss the effects of unions on steady-state multiplicity and welfare, and on the occurrence of local indeterminacy, local bifurcations and (stochastic and deterministic) endogenous fluctuations driven by self-fulfilling expectations. We consider an overlapping generations economy with capital externalities and we focus on underemployment equilibria. We find that for wide regions in the parameter space, including an arbitrarily small degree of externalities and a Cobb-Douglas technology, unions increase steady-state employment and welfare, and local indeterminacy (sunspots) emerges. Our results also show that the role of unions in shaping local dynamics depends on technology (externalities and factor's substitutability). Indeed, if capital externalities take intermediate values, local indeterminacy requires implausibly low degrees of factor substitutability when unions are sufficiently strong. We also find that Hopf bifurcations and local deterministic endogenous fluctuations generically occur, with an elasticity of factor substitution the closest to one the lower the unions' bargaining power. Moreover, for lower levels of the elasticity of factor substitution transcritical bifurcations generically occur, implying the existence of two close steady states, one being a saddle and the other being indeterminate. With a CES technology, however, multiplicity of steady states is only possible in the presence of unions.

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1 Introduction

The aim of this paper is twofold. On one hand we dispute the conventional idea that unions, being a source of economic inefficiency, necessarily harm welfare. On the other hand, we analyze the role of unions on the emergence of multiple steady states, local indeterminacy, bifurcations and endogenous fluctuations. Indeed, the present study extends the existing studies on endogenous fluctuations in overlapping generations economies with capital accumulation to the case where the labour market is not perfectly competitive, but rather is unionized.

Within an OG framework the pioneer work on endogenous fluctuations for economies with capital accumulation is Reichlin (1986). Assuming perfect competition and constant returns to scale, he showed that endogenous fluctuations only emerged for implausible low values of the elasticity of substitution between capital and labour. More recently, Lloyd-Braga (1995) showed the compatibility between endogenous fluctuations and higher values of the elasticity of factor substitution when the model is amended to incorporate increasing returns internal to the firm.[1] However, the same is possible when increasing returns to scale are due to capital externalities, as in Cazzavillan (2001), where it is shown that local indeterminacy, and thereby sunspots, exist for large elasticities of inputs substitution and arbitrarily small capital externalities, provided labour supply is sufficiently elastic. The case of an infinite labour supply together with increasing returns (externalities) is considered in Coimbra (1999). He showed that the emergence of endogenous cycles is possible with both an elasticity of inputs substitution and an amount of (small) externalities in consonance with empirical evidence.

In this paper we pursue this latter analysis, our innovation concerning the labour market structure. We consider the existence of an (efficient) bargaining, between unions and firms, over wages and employment, under non binding contracts. The case of a perfectly competitive economy is, in our model, recovered as the case where unions' bargaining power is nil. Our aim is to analyze, in a simple dynamic environment, how the degree of unions' bargaining power and the relevant properties of technology (degree of externalities and substitution between factors in production) interact in shaping local dynamics. In order to do that we only impose minimal assumptions on technology, covering a wide set of different types of production functions.

[1]Cazzavillan et al. (1998) got similar results in a Woodford's (1986) type of model.

However, in order to keep the model simple, we restrict our analysis to representative household preferences that are additively separable between a Cobb-Douglas index, over future and current consumption (possibly absent), and leisure (or employment), being linear in the latter.[2] These features imply the existence of an endogenous reservation wage (which depends on forward looking expectations of interest rates) and an infinitely elastic labour supply.[3] Hence, at equilibrium, full employment may not be obtained. Indeed, even under perfect competition in the labour market, as in d'Aspremont et al. (1995),[4] underemployment may emerge and we restrict our analysis to this type of equilibria, along which employment may exhibit endogenous fluctuations. We also discuss our theoretical results in terms of their empirical plausibility, comparing the values of the relevant parameters under which the results are obtained with those usually found in empirical works.

Our main results may be summarized as follows. For arbitrarily small externalities, local *indeterminacy* occurs for a wide range of values for the elasticity of factor substitution, containing those considered plausible and including the standard case of a Cobb-Douglas technology. For a given low level of externalities it is possible that, for some values of the elasticity of substitution considered plausible, indeterminacy does not exist under perfect competition while it occurs when unions are sufficiently strong. On the contrary, for a given middle but still reasonable degree of externalities, the values of the elasticity of factor substitution required for indeterminacy when unions are sufficiently strong may become too low to be empirically plausible. Also local *Hopf* bifurcations, and thereby local deterministic endogenous fluctuations, generically occur with an elasticity of factor substitution the closest to 1 the lower the unions' bargaining power. Indeed, when externalities are small Hopf bifurcations occur with a value for the elasticity of factor substitution higher than one, and this value is increasing in the degree of unions' bargaining power. However, when externalities take intermediate values,

[2]An advantage of our specification is that it nests the more standard cases studied in the literature, where current consumption and unions bargaining power are absent, as for instance in Cazzavillan (2001).

[3]Note that the case of an infinitely elastic labour supply has been considered in several contributions. See for instance, besides Hansen (1985), the recent works of Farmer and Guo (1994), Benhabib and Farmer (1994), Barinci and Chéron (2001).

[4]d'Aspremont et al (1995) studied the effects of increasing returns on endogenous fluctuations in a OG model with underemployment, but they did not consider capital accumulation.

Hopf bifurcations occur with a value of the elasticity of factor substitution below one, and this value is decreasing in the degree of unions bargaining power. These results suggest that what really matters for the dynamics is not the degree of union power *per se* but the combination of the degree of union power and the existing type of technology, described by the degree of externalities and the elasticity of factor substitution. Local *Transcritical* bifurcations generically occur when the elasticity of factors substitution is lower than 1/2. In this case, even when the steady state under analysis is a saddle there is, close to it, another steady state that is locally indeterminate. However, in the absence of current consumption and with a CES technology, multiplicity of steady states requires the presence of unions.

Finally we find that unions increase steady state employment and welfare for plausible values of the parameters. Note that, given our second best environment, this outcome was indeed possible. However, externalities do not seem to be the force driving this result, since unions improve steady state welfare even for arbitrarily small externalities.[5]

The rest of the paper is organized as follows. In section 2 we describe the model and obtain the equilibrium dynamic equations. In section 3 we do the steady state analysis and discuss the steady state effects of union power. In section 4 we present the local dynamics and bifurcation analysis. Finally, in section 5, we present some concluding remarks.

2 The Model

We consider an overlapping generations model of two-period lived agents, with constant population and perfect foresight. In each period, $t = 1, 2...\infty$, a continuum of identical individuals (households) of mass N is born. Indi-

[5]Indeed, the fundamental welfare theorems may not apply to perfectly competitive equilibria without externalities in OG economies. Also, Devereux and Lockwood (1991), using an OG model without externalities, showed that unions could have a positive effect on steady-state capital accumulation and welfare. However, their model differs substantially from the current one. They consider a specific Cobb Douglas production function and also assume that employment is constant at the full employment level. Moreover, the focus of their analysis is quite different from ours. They are mainly interested on the effects of moving from binding to non-binding wage contracts on steady state capital. Here, we focus instead on the effects of unions on steady state employment and welfare. Note that both can increase without an increase in steady state capital. Also, they stick to steady state analysis, while we also study local dynamics.

viduals may consume in youth and in old age but can only work when young. Young individuals can either work a fixed amount of hours for a firm, receiving a wage w_t, or not work at all.[6] In each period there is a single output, exchanged under perfect competition, which is either consumed or added to the capital stock. Output is the numeraire and is produced by m identical firms[7], each using labour and capital. Capital services are rented each period in a perfectly competitive market. However, wages and employment are determined through a bargaining process (efficient bargaining) between unions and firms.

2.1 Households

We assume that preferences of a representative household born at $t \geq 1$ are described by the following utility function:

$$c_{1t}^{\beta} c_{2t+1}^{(1-\beta)} - a d_t, \tag{1}$$

where c_{1t} and c_{2t+1} are respectively consumption while young and old, $a > 0$ is the (constant) desutility of work and $0 \leq \beta < 1$. Each individual either decides to work a fixed amount of hours when young ($d_t = 1$), receiving a wage w_t, or not work at all ($d_t = 0$). The wage income received by a young worker in period t is spent in current consumption, c_{1t}, or saved for future consumption, c_{2t+1}, through the purchase of productive capital, k_{t+1}^h, available for production at the outset of period $t+1$. We assume that capital fully depreciates after a period of use in production. When old, individuals rent to firms the capital goods, purchased while young, at the (perfectly foreseen) expected rate $r_{t+1.}$ The problem solved by a representative individual born at $t \geq 1$ is then the maximisation of (1) subject to the following constraints:[8]

$$c_{1t} + k_{t+1}^h = w_t d_t \tag{2}$$

$$c_{2t+1} = r_{t+1} k_{t+1}^h. \tag{3}$$

[6] Although we are here assuming indivisible labour, our equilibrium equations would be the same if we had instead considered that households could choose the number of hours of work, as long as the marginal desutility of labour is constant.

[7] We assume that in each period firms's ownership is exogenously distributed among the old generation. Therefore, we do not consider the existence of a stock market, so that, as in standard OG models with capital, capital is the unique asset.

[8] In our model, as we shall see in section 2.2, there are no pure profits. Therefore, there are no dividends to be paid. Moreover, we assume that the generation born old at the beginning of this economy ($t = 1$) owns all the existing productive capital and spends all the rentals in consumption.

Then, for an employed young individual ($d_t = 1$) we have that:

$$c_{1t} = \beta w_t \tag{4}$$
$$c_{2t+1} = (1-\beta) w_t r_{t+1} \tag{5}$$

and:

$$k^h_{t+1} = (1-\beta) w_t. \tag{6}$$

Substituting (4) and (5) in (1) we can write the indirect utility function of each employed individual born at t as:

$$V_t = V(w_t, r_{t+1}) = \beta^\beta (1-\beta)^{(1-\beta)} r_{t+1}^{(1-\beta)} w_t - a. \tag{7}$$

If unemployed the utility of an individual born at t is zero, since $d_t = c_{1t} = c_{2t+1} = 0$. Hence, a young individual will only be willing to work if $V_t \geq 0$. This means that he/she will decide to work only when $w_t \geq \overline{w}_t$, where $\overline{w}_t$ is the real reservation wage given by:[9]

$$\overline{w}_t = a \left[\beta^\beta (1-\beta)^{(1-\beta)}\right]^{-1} r_{t+1}^{(\beta-1)} \equiv a\delta_{(\beta)} r_{t+1}^{(\beta-1)}. \tag{8}$$

2.2 Unions and firms

2.2.1 Unions

Unions are firm-specific, i.e. we have one union per firm. Unions represent all youngsters. We assume that at the beginning of each period t the young are matched exogenously and uniformly with unions[10] so that each of the m identical unions represents $n \equiv N/m$ young individuals. We assume that unions are utilitarian. See Oswald (1982). This means that each union maximizes the sum of all of its members utilities, whether employed or not. Using (7) and (8), we have that each union maximizes the following function:

$$\Omega_t = \beta^\beta (1-\beta)^{(1-\beta)} r_{t+1}^{(1-\beta)} (w_t - \overline{w}_t) l_t \tag{9}$$

where l_t is employment at the respective firm.[11]

[9] Note that for $\beta = 0$ we obtain $\overline{w}_t = a/r_{t+1}$, as we have that $\lim_{\beta \to 0} \delta(\beta) = 1$.

[10] Workers cannot move between firms or unions.

[11] Note that the union treats r_{t+1} and thereby $\overline{w}_t$, see (8), as given.

2.2.2 Firms

The private production function of a representative firm exhibits constant returns to scale, but there are increasing returns at the social level. We use externalities to combine a social technology that displays increasing returns with a competitive behaviour in the output market.[12] To simplify we shall only consider multiplicative capital externalities.[13] Accordingly, we consider the following production function:

$$\overline{k}_t^{\upsilon} AF(k_t, l_t) = \overline{k}_t^{\upsilon} Al_t f(x_t) \tag{10}$$

where $x \equiv k/l$ is the capital labour ratio, $A > 0$ is a scaling factor, $\overline{k}$ is the average capital stock and $\upsilon > 0$ is the elasticity of capital externalities.[14] Note that social increasing returns to scale are given by $1 + \upsilon > 1$. In (10) the second expression follows from the constant returns to scale assumption. We further assume that:

Assumption 1 *$f(x)$ is a real, continuous function for $x \equiv k/l \geq 0$, positively valued and differentiable as many times as needed for $x > 0$, with* $f'(x) > 0$ and $f''(x) < 0$. *We shall denote the elasticity of $f(x)$ by $\theta_{(x)}$, where* $0 < \theta_{(x)} \equiv \frac{f(x)x}{f(x)} < 1$; *and the elasticity of substitution between capital and labour by $\sigma_{(x)} > 0$, where $\frac{1}{\sigma_{(x)}} \equiv -\frac{f(x)f''(x)x}{f'(x)\left[f(x)-f'(x)x\right]}$.*

Each period producers maximize profits: $\pi_t \equiv \overline{k}_t^{\upsilon} Al_t f(x_t) - w_t l_t - r_t k_t$, where r_t is the rental cost of capital. Each small individual producer (m big) takes $\overline{k}_t$ as given when solving his optimizing problem. Producers, when maximizing profits, solve a two-stage problem. First, at the beginning of period t producers, given r_t, rent capital on the economy-wide capital market. After this wages and employment are determined through a bargain between

[12]This device has been widely used in macro dynamic models. See, for instance, Farmer and Guo (1994), Benhabib and Farmer (1994), Cazzavillan et al. (1998) and Cazzavillan (2001).

[13]This assumption is compatible with the standard learning-by-doing argument (see Arrow (1962), Romer (1986)) where the capital stock only is assumed to originate social increasing returns. See also Cazzavillan (2001) where a similar assumption is considered.

[14]Note that our results would be the same if we had considered a more general specification for capital externalities e.g. $\Psi(\bar{k})$. In this case υ would have be given by $\Psi'(\bar{k})\bar{k}/\Psi(\bar{k})$.

unions and firms on the distribution of the surplus over and above capital rental costs. Finally production takes place. The income generated in production, after the payment of the rental cost of capital, is then distributed among workers and the owners of the firms.

A relevant issue in the modelisation of this problem concerns the degree of commitment allowed for the wage-employment contract. We assume that workers cannot sign binding wage contracts, so that the wage and employment are determined after the capital stock decision has been made. Therefore to ensure time consistency of the equilibrium we solve the problem backwards, starting with the wage-employment bargain.

2.2.3 Wage, employment and capital decisions

We model the bargaining process using the generalized Nash bargaining solution.[15] This means that first we have to specify the fallback payoffs of each party, in the event that no agreement is reached. For the union the fallback utility is zero, i.e. the level of utility of an unemployed worked. For the firm, as we have considered non-binding wage contracts, the fall back profit is minus the rental cost of capital $(-r_t k_t)$.[16] Hence, the wage and employment at the typical firm solve the following problem:

$$\underset{(w_t,l_t)\in\Re^2_{++}}{Max} \quad \pi_t^{*^{\alpha}} \, \Omega_t^{(1-\alpha)} \tag{11}$$

where $0 < \alpha \leq 1$ measures the firm's bargaining power and where $\pi_t^* \equiv \pi_t - (-r_t k_t) = \overline{k}_t^{v} A l_t f(x_t) - w_t l_t$.

From the first order conditions of problem (11) we obtain:

$$\overline{w}_t = \overline{k}_t^{v} A \left[f(x_t) - f^{'}(x_t) x_t \right] \tag{12}$$

$$w_t = \overline{k}_t^{v} A \left[f(x_t) - \alpha f^{'}(x_t) x_t \right]. \tag{13}$$

[15]Note that this solution can been seen as a limit solution of a non-cooperative bargaining game. See Sutton (1986).

[16]With binding contracts the fall back profit would be zero as w and l could have been determined before installing the capital stock. Also, as the reader can check, if workers could sign binding contracts the solution, in our case, would be equivalent to the competitive outcome.

We assume that $l_t < n \equiv N/m, \forall_t$. This means that there is always some degree of underemployment in our economy.

The firm then chooses $k_t > 0$ to maximize profits that, using (13), can be written as:

$$\pi_t = \alpha \overline{k}_t^v A f'(x_t) k_t - r_t k_t \tag{14}$$

where x_t satisfies (12). This yields the first order condition:[17]

$$\alpha \overline{k}_t^v A f'(x_t) = r_t. \tag{15}$$

Substituting now (15) in (14) we can see that profits are zero, i.e. there are no pure profits in equilibrium.[18] Therefore, there are no dividends to be paid.

When the firm has all the power in the bargaining process ($\alpha = 1$) we obtain the competitive outcome: i.e., the rental cost of capital is identical to the marginal productivity of capital (see (15)) and the wage equals both the marginal productivity of labour and the reservation wage (see (12) and (13)).

From (12) we see that employment is determined by the equality between the marginal productivity of labour (MPL) and the reservation wage, for all α. This means that the contract curve is vertical,[19] so that, given a reservation wage, union power would have no effect on the level of employment for a fixed level of the capital stock. See figure 1 where $w(\alpha)$ represents the contract curve. However, as we shall see, union power influences the level of employment through its general equilibrium effects on the dynamics of capital accumulation. The latter is, in our OG structure, driven by the wage bill, whose value is influenced by the bargaining power of unions. Indeed, from (12) and (13), we have that unions are able to set wages above the reservation wage.[20] This mark-up depends positively on unions' bargaining

[17] Note that, given (12), the derivative of $\alpha \overline{k}_t^v A f'(x_t)$ with respect to k_t is zero.

[18] The assumption of private constant returns to scale is crucial for this result. Indeed, with decreasing returns to scale profits would be positive, as in Devereux and Lockwood (1991).

[19] Note that this result follows from the linearity of Ω in w.

[20] Note that for $\alpha < 1$ unemployment is involuntary at the individual level, since in this case all individuals are willing to work at the existing wage. However, underemployment is also present when the labour market is perfectly competitive, $\alpha = 1$. Whether in this case unemployment is voluntary or involuntary is more a question of semantic. Indeed, as we shall see in section 3 below, even in this case an increase in employment may be desirable from a social welfare point of view.

power and on the ratio between the marginal contributions of capital and labour for output, i.e. $w_t/\overline{w}_t = 1 + (1-\alpha)\frac{\theta_{(x_t)}}{1-\theta_{(x_t)}}$.[21]

(insert Figure 1 here)

2.3 Equilibrium

Equilibrium in the capital market requires that $mk_{t+1} = k^h_{t+1}ml_t$, i.e. in any period the capital stock available for production in the economy is predetermined by the savings of the previous young generation. Moreover, since all firms are identical, in equilibrium $k_t = \overline{k}_t$. Therefore, using (6) and (13), at equilibrium we have:

$$k_{t+1} = (1-\beta)l_t k^v_t A\left[f(x_t) - \alpha f'(x_t)x_t\right] \tag{16}$$

Also using (8), (12) and (15) we obtain:

$$k^v_{t+1}Af'(x_{t+1}) = \frac{1}{\alpha}\left\{\frac{k^v_t A\left[f(x_t) - f'(x_t)x_t\right]}{a\delta_{(\beta)}}\right\}^{\frac{1}{\beta-1}}. \tag{17}$$

Equations (16) and (17) govern the equilibrium dynamics of the model.[22] These equations implicitly define a two dimensional dynamic system, $(k_{t+1}, l_{t+1}) = Z(k_t, l_t)$, with one predetermined variable, k, whose value in any period is fixed by past savings, and one non predetermined variable, l, whose value is influenced by expectations of future interest rates (see (8) and figure 1).[23]

Definition 1 *An intertemporal equilibrium with perfect foresight is a sequence* $(k_t,\, l_t) \in \Re^2_{++}$, $t = 1, 2, ...\infty$, *such that (16) and (17), where* $x \equiv k/l$, *are satisfied.*

Moreover we shall assume that $l_t \in (0, n \equiv N/m)$ which will be verified for non explosive trajectories as long as N is big enough.

[21] Note that the capital share of output is given by $\alpha\theta_{(x)}$and the labour share of output is $1 - \alpha\theta_{(x)}$.

[22] Note that the output market also clears by Walras law.

[23] At period t, given a fixed level of the predetermined variable k_t , an increase (decrease) in $\overline{w}_t$, due to a decrease (increase) in expectations of r_{t+1}, shifts to the left (right) the vertical contract curve and decreases (increases) the level of employment.

3 Steady State Analysis

In this section we study the existence, uniqueness or multiplicity of the stationary states for our dynamic system. Our analysis will closely follow Cazzavillan et al. (1998) and Aloi et al. (2000).

3.1 Existence

A steady state solution (k, l) for all t is a solution of (16)-(17), with $x \equiv k/l$, such that:

$$x = (1-\beta)k^{\upsilon}A\left[f(x) - \alpha f^{'}(x)x\right] \tag{18}$$

$$a\delta_{(\beta)} = \left[f(x) - f^{'}(x)x\right](k^{\upsilon}A)^{2-\beta}\left[\alpha f^{'}(x)\right]^{1-\beta} \tag{19}$$

In what follows we ensure the existence of an interior steady state $(k^1, l^1) \in \Re^2_{++}$ by choosing appropriate values of the scaling parameters $A > 0$ and $a > 0$. Fixing (k, l) at $(k^1, l^1) \in \Re^2_{++}$ there is, under Assumption 1, a unique solution $A > 0$ to (18). Given this solution for A there is also a unique solution $a > 0$ to (19). Therefore:

Proposition 1 Existence of the steady state

Under Assumption 1, given constant values for $0 \leq \beta < 1$, $0 < \alpha \leq 1$ *and* $\upsilon > 0$, $(k, l) = (k^1, l^1) \in \Re^2_{++}$, *with* $x^1 \equiv k^1/l^1$, *is a stationary solution of the dynamic system (16)-(17) if and only if* $(A, a) \in \Re^2_{++}$ *are the unique solution of (18)-(19), i.e.,* $A = \frac{x^1}{(1-\beta)[f(x^1)-\alpha f'(x^1)x^1](k^1)^{\upsilon}}$ *and* $a = \frac{\left[f(x^1)-f^{'}(x^1)x^1\right]\left(k^1\right)^{(2-\beta)\upsilon}\left[\alpha f^{'}(x^1)\right]^{1-\beta}}{\delta_{(\beta)}}\left[\frac{x^1}{(1-\beta)[f(x^1)-\alpha f'(x^1)x^1](k^1)^{\upsilon}}\right]^{(2-\beta)}$.

3.2 Uniqueness versus multiplicity

Given fixed positive values for A and a as defined in Proposition 1, and fixed values for $0 \leq \beta < 1$, $0 < \alpha \leq 1$ and $\upsilon > 0$ we now study uniqueness and multiplicity of the (interior) steady states. In view of (18)-(19), an interior stationary solution (k, l) with $x = k/l$ must satisfy:

$$G(x) \equiv \alpha f^{'}(x)\left(\frac{x^{(2-\beta)}\left[f(x) - f^{'}(x)x\right]}{a\delta_{(\beta)}(1-\beta)^{(2-\beta)}\left[f(x) - \alpha f^{'}(x)x\right]^{(2-\beta)}} \right)^{\frac{1}{1-\beta}} = 1 \qquad (20)$$

where $G(x)$ is a continuous positive function for $x > 0$. To each value of x satisfying (20), the corresponding steady state solution (k, l) is uniquely determined through (18) and (19) when $\upsilon > 0$.[24] Hence, studying the existence of multiple steady states involves studying the number of solutions in x for (20). In particular if $G(x)$ is monotonic (i.e. if either $G^{'}(x) > 0$ for all x or $G^{'}(x) < 0$ for all x) there exists at most one steady-state, i. e. the one defined in Proposition 1. Also if $G'(x)$ changes its sign only once, then $G(x)$ is single peaked (or caved), crossing the value 1 at most twice. In this case, two steady state solutions would exist. Finally, if $G(x)$ is constant then the existence of a steady state under Proposition 1 requires that $G(x) \equiv G(x^1) = 1$ for all values of x and there is a continuum of steady states.

To check whether $G(x)$ is monotonic or not, we now analyze the sign of $G^{'}(x)$. Differentiating $G(x)$ we obtain:

$$H(x) \equiv \frac{G^{'}(x)x}{G(x)} = \frac{(1-\alpha\theta_{(x)}) + (2-\beta)(1-\theta_{(x)})(\sigma_{(x)} - 1)}{(1-\beta)\sigma_{(x)}(1-\alpha\theta_{(x)})} \qquad (21)$$

where $\theta_{(x)} \in (0, 1)$ and $\sigma_{(x)} \in (0, +\infty)$ are defined in Assumption 1. Since $x > 0$ and $G(x) > 0$, studying the sign of $G^{'}(x)$ is equivalent to studying the sign of the function $H(x)$. The following proposition presents the main results.

Proposition 2 Uniqueness of the steady state

[24]Indeed, if $\upsilon > 0$, to each value x^* satisfying (20) corresponds a unique steady state (k^*, l^*) where $k^* = \left[\frac{x^*}{f(x^*) - \alpha f'(x^*)x^*}\frac{1}{A(1-\beta)}\right]^{\frac{1}{\upsilon}}$ by (18) and $l^* = x^* k^*$. However, if $\upsilon = 0$ (no externalities), equation (18) only has as argument the variable x. In this case a value of x^* satisfying (20) would be a steady state of the system if and only if: $A(1-\beta) = \frac{x^*}{[f(x^*) - \alpha f'(x^*)x^*]}$. Moreover, in this case, a steady state (k^*, l^*) would not be locally unique. For each x^* there would be a continuum of steady states $(k, l) = l.(x^*, 1)$ with $0 < l < n$. Therefore, in the no externalities case our steady state welfare analysis of next section becomes meaningless.

Let us define the following critical value $\sigma_1 \equiv \frac{(2-\beta)(1-\theta_{(x)})-\left(1-\alpha\theta_{(x)}\right)}{(2-\beta)(1-\theta_{(x)})}$. *Under Proposition 1, there is a unique steady state if one of the following conditions is satisfied:*

(i) $\sigma_{(x)} > \sigma_1$ *, i.e.* $H(x) > 0$ *for all* $x > 0$.

(ii) $0 < \sigma_{(x)} < \sigma_1$*, i.e.* $H(x) < 0$ *for all* $x > 0$.

Note that $\sigma_1 \leq \frac{1}{2}$, since $\theta_{(x)} \in (0,1)$ and $\beta \in [0,1)$. Hence, if σ is constant (CES technology) the cases where $\sigma > \frac{1}{2}$ fall into configuration *(i)* and, therefore, the steady state is unique. Moreover, if $\alpha = 1$ (the competitive case) uniqueness of the steady state is also generically obtained in the CES case, since either *(i)* or *(ii)* are generically satisfied. These results mean that for a CES technology, multiple steady states may only arise in the presence of unions, i.e. when $\alpha < 1$, and if $\sigma < \frac{1}{2}$.

We address now the issue of multiplicity versus uniqueness of the steady state by analyzing the circumstances under which the conditions stated in Proposition 2 are violated. We focus on the cases where $H(x)$ changes sign at most only once. In this case $G(x)$ is either single-caved or single-peaked, so that there are at most two steady states.

Proposition 3 Multiplicity of steady states

Let $B(x) \equiv \sigma_{(x)}(2-\beta)\alpha(1-\theta_{(x)}) - \left(1-\alpha\theta_{(x)}\right)[2-(\alpha+\beta)]$*, then under Assumption 1, there are at most two steady states if either:*

(i) $B(x)$ *is increasing in* x *or;*

(ii) $B(x)$ *is decreasing in* x.

Under these conditions $H(x)$ changes sign at most once. Under condition *(i)* $H(x)$ is an increasing function. If $H(0) < 0$ and $H(+\infty) > 0$, then $H(x)$ crosses the value zero exactly once (from below), and the $G(x)$ function is single-caved. Under case *(ii)* $H(x)$ is an decreasing function. Hence if $H(0) > 0$ and $H(+\infty) < 0$, then $H(x)$ crosses the value zero exactly once (from above), and the $G(x)$ function is single-peaked. In these cases, if the assumptions of Proposition 1 are satisfied and $H(x)$ does not vanish at $x = x^1$, there are exactly two steady states whenever (i) or (ii) of Proposition 3 hold, provided that the appropriate boundary conditions are satisfied (namely that $G(x) - 1$ has the same sign for x close to zero and x close to $+\infty$).[25]

[25] In the nongeneric case where $H(x^1) = 0$ there is exactly one steady state. The function

In Appendix A.1 we show that, for $\beta = 0$, when technology is of the CES type with $\sigma < 1/2$ there are generically two steady states when $\alpha < 1$. As we have seen, this would not be possible in the perfectly competitive situation where for CES technologies the steady state is unique.

3.3 Steady state effects of union power

In this section, assuming that a steady state exists (Proposition 1), we analyze the effects of union power on steady state welfare. We are mainly interested in understanding if an increase in unions's power (a decrease in α) has a clear positive effect on economic performance at a steady state. This will be the case if unions are able to increase steady state employment and the representative worker's utility. In this case, if consumption of the old also increases, steady state social welfare would be higher in the presence of unions.[26]

A change in α changes the stationary solution (k, l) of (18)-(19) and, therefore, it influences employment and steady state agents' utility. Consider fixed values for the parameters $\upsilon > 0$, $A > 0$, $a > 0$, and $0 \leq \beta < 1$. Assume that (k^*, l^*) is a steady state solution associated with a given α^* satisfying (18) and (19). Then, as seen before, the steady state (k^*, l^*) is generically locally unique. Moreover, given Assumption 1, $k^*(\alpha)$ and $l^*(\alpha)$ are C^r functions for α close to α^*. Also, by continuity, if α slightly changes from α^*, the system (18)-(19) still has a unique steady state solution close to (k^*, l^*). Therefore, using differential analysis we can study how k^*, l^* and x^* change when α is slightly decreased from α^*.

$G(x)$ would have a maximum (if single peaked) or minimum (if caved) value at x^1. See Cazzavillan et al. (1998) where it is explained why, under these conditions, uniqueness of the steady state does not persist. Later on in the paper, when local dynamics and bifurcations around the steady state are studied, it can be checked that the case just described corresponds to the appearance of a transcritical bifurcation.

[26]Here, as in Azariadis (1993), we might want to consider a social welfare function of the type $SWF = b_0 U(c_2) + b_1 [U(c_1,c_2) - a]\, l + b_2 (n - l) U_2$, where b_0, b_1, b_2 are non negative weights and U_2 is the utility level of those unemployed (zero at the market equilibrium). This means that social welfare is a weighted average of the utilities of all types of households that may exist in a period t where a steady state solution is observed. Remark that the objective function of a representative union is a special case of the SWF presented above, with $b_0 = 0$ and $b_1 = b_2$. However, unions being myopic will never act as a social planner with identical weights. Indeed unions treat r as given and do not internalize production externalities.

The following proposition summarizes the main results.

Proposition 4 *Consider that Assumption 1 is verified and take fixed values for the parameters* $\upsilon > 0$, $A > 0$, $a > 0$, *and* $0 \leq \beta < 1$. *For a given* $\alpha \equiv \alpha^*$ *let* (k^*, l^*) *be a steady state solution of (18)-(19),* $\theta \equiv \theta_{(x^*)}$ *and* $\sigma \equiv \sigma_{(x^*)}$ *being the corresponding elasticity of* $f(x)$ *and elasticity of substitution between capital and labour. Then, a slight decrease in* α *(increase in union power) from* α^*, *increases simultaneously* l^* *and the corresponding representative worker utility if and only if one of the following conditions is met:*

(i) either $0 < \upsilon < \upsilon_1$ *and* $\sigma > \max\{\sigma_2, \sigma_3\}$

(ii) or $0 < \upsilon < \upsilon_1$ *and* $\sigma < \min\{\sigma_1, \sigma_2\}$

(iii) or $\upsilon > \upsilon_1$ *and* $\sigma < \sigma_1$,

where:

$\upsilon_1 \equiv \frac{(1-\beta)(1-\theta)}{1-\beta+\alpha\theta}$, $\quad \sigma_1 \equiv \frac{(2-\beta)(1-\theta)-(1-\alpha\theta)}{(2-\beta)(1-\theta)}$,

$\sigma_2 \equiv \frac{\alpha\theta^2}{(1-\beta)(1-\theta)-(1-\beta+\alpha\theta)\upsilon}$ *and* $\sigma_3 \equiv \frac{1-\beta}{1-\beta+\alpha}$.

Proof. See Appendix A.2.[27] ■

In Appendix A.3 we also show that, following a slight decrease in α, the steady state consumption of the old, c_2, will increase if and only if one of the following conditions holds: *i)* either $\beta < (1+\alpha)\theta$ and $\sigma > \max\{\sigma_1, \sigma_4\}$ *ii)* or $\beta < (1+\alpha)\theta$ and $\sigma < \min\{\sigma_1, \sigma_4\}$ or *iii)* or $\beta > (1+\alpha)\theta$ and $\sigma < \sigma_1$, where $\sigma_4 \equiv \frac{\theta}{\theta(1+\alpha)-\beta}$.

These results imply that an economy with unions may have, contrary to conventional wisdom, a better economic performance than an economy with a perfectly competitive labour market. Another and more important issue is whether this can happen for plausible values of the parameters. Empirical studies point to values for $\sigma \in [0.4, 2]$. See Hamermesh (1993). Moreover, as θ is the capital share of output in perfectly competitive economies, a reasonable range for θ would be $\theta \in [0.25, 0.4]$. Empirical estimations for

[27] The results of this proposition are stated in terms of conditions whose fulfillment may depend on the steady state under evaluation. When there are multiple steady states it may happen that in one steady state the requirements of proposition 4 are satisfied, while in the other they are not. However when the steady state is unique proposition 4 is fully informative and, as we have seen, this happens for instance for a CES technology with $\sigma > \sigma_1$, a case compatible with configuration (i) of Proposition 4.

υ usually point to values higher than zero but small.[28] We shall therefore consider that υ is bounded above by some value that should not be high, let us say $0 < \upsilon < \bar{\upsilon}$, with $\bar{\upsilon} \in [0.3, 0.6]$. With these ranges in mind we can see that our results may well be obtained under reasonable parameter values.[29] In figure 2 we represent σ_1, σ_2, σ_3 and σ_4 as functions of υ in the space (υ, σ). We considered that $\alpha = 1$, $\theta < 1/2$ and to simplify that $\beta = 0$. The shaded areas correspond to configurations of values (υ, σ) such that, departing from a perfectly competitive labour market, a slight increase in unions' bargaining power leads to an improvement in employment, welfare of a representative worker and consumption of the old generation.

(insert Figure 2 here)

The fact that an increase in unions' bargaining power may be beneficial should not surprise us. Indeed, in a second best environment, as ours, an additional distortion (unions) may be welfare improving. Note however that capital externalities may not be very important in engendering our results, since they are also obtained for arbitrarily small υ. See Proposition 4 and Figure 2.

Finally, note that, following a slight decrease in α, the capital stock may decrease. See Appendix A.2. This will happen for $\alpha = 1$, $\beta = 0$ and $\theta < 1/2$ when $\sigma_5 < \sigma < \sigma_1$ where $\sigma_5 \equiv \theta^2/(1-\theta)$. Inspection of Figure 2, shows that the intersection of the set defined by $\sigma_5 < \sigma < \sigma_1$ and the shaded area is a non empty set. Therefore it is possible to have an increase of employment and an improvement in welfare at the steady state with a decrease of capital (and of capital per worker). This is a typical phenomenon in OG models when there is dynamic inefficiency.

4 Local dynamics and bifurcation analysis

In this section, assuming that a steady state (k, l) exists (Proposition 1), we study how the local dynamic properties of a steady state are influenced by some relevant 'parameters' of the model. As in Cazzavillan (2001), and

[28] See for instance Caballero and Lyons (1992) or the more recent papers of Burnside (1996) and Basu and Fernald (1995).

[29] For example if $\upsilon = 0.01$, $\theta = 0.34$ and $\beta = 0.5$, we have that departing from a perfectly competitive economy ($\alpha = 1$) a slight increase in unions power will be beneficial if $\sigma > 0.36$ and also if $\sigma < 0.33$.

in order to simplify the analysis, we assume in this section that $\beta = 0$, i.e. that the young generation does not consume and fully devotes its labour income to savings and future consumption. We use the standard procedure of considering the linearized dynamics of our two-dimensional system (16) and (17) around a steady state:[30]

$$\begin{pmatrix} k_{t+1} - k \\ l_{t+1} - l \end{pmatrix} = \begin{pmatrix} J_{kk} & J_{kl} \\ J_{lk} & J_{ll} \end{pmatrix} \begin{pmatrix} k_t - k \\ l_t - l \end{pmatrix}.$$

The matrix J is the Jacobian matrix of our system (16)-(17) evaluated at the steady state with:

$$J_{kk} = \upsilon + \frac{\theta\left[\sigma\left(1-\alpha\right)+\alpha\left(1-\theta\right)\right]}{\sigma\left(1-\alpha\theta\right)} \tag{22}$$

$$J_{kl} = \frac{k}{l}\left\{1+\upsilon-J_{kk}\right\} \tag{23}$$

$$J_{lk} = -\frac{l}{k}\left\{\frac{J_{kk}\left[\sigma\upsilon-\left(1-\theta\right)\right]+\left(\sigma\upsilon+\theta\right)}{\left(1-\theta\right)}\right\} \tag{24}$$

$$J_{ll} = -J_{kl}\frac{l}{k}\frac{\sigma\upsilon-\left(1-\theta\right)}{\left(1-\theta\right)}+\frac{\theta}{\left(1-\theta\right)} \tag{25}$$

where σ and θ are both evaluated at the considered steady state solution. Moreover, recall that $\sigma \geq 0,\ 0 < \theta < 1,\ \upsilon > 0$, and $0 \leq (1-\alpha) < 1$.

The trace and the determinant of J are then given by the following expressions:

$$T = T_1 + T_2\sigma \tag{26}$$

$$D = D_1 + D_2\sigma \tag{27}$$

[30] This procedure is valid for non linear systems as long as the Jacobian matrix is invertible and has no eigenvalue on the unit circle (Hartman-Grobman Theorem, see for instance Grandmont (1988) pg 28, Theorem B.4.1 or Azariadis (1993), pg 59, Theorem 6.1).

where:

$$D_2 = -T_2 \equiv \frac{\upsilon}{1-\alpha\theta} > 0 \tag{28}$$

$$T_1 \equiv \frac{\upsilon\,(1-\theta) + (1-\alpha\theta)}{(1-\theta)\,(1-\alpha\theta)} > 0 \tag{29}$$

$$D_1 \equiv \frac{[\upsilon(1-\alpha) + 1 - \alpha\theta]\,\theta}{(1-\theta)\,(1-\alpha\theta)} > 0 \tag{30}$$

The trace and the determinant of J correspond respectively to the sum and product of its eigenvalues, i.e. the roots of the associated characteristic polynomial $Q(\lambda) \equiv \lambda^2 - \lambda T + D$. Therefore, the eigenvalues defining the local stability properties also depend on the values of the parameters σ, θ, υ, and α, which we shall take as characterizing our economies. In order to study how these parameters influence the local dynamic properties we use a geometrical method as in Grandmont et al. (1998). The basic idea of the method is to analyse in the space (T, D) how the trace and determinant, and accordingly the local eigenvalues of the system, change as the relevant parameters are made to vary.

Consider fixed values for all parameters, except σ (*the bifurcation parameter).* From (26) and (27), the locus $(T_{(\sigma)}, D_{(\sigma)})$, obtained as σ continuously change, is defined through the following expression:

$$D = \Delta(T) \equiv \frac{\upsilon + 1 + \theta}{1-\theta} - T \tag{31}$$

The expression Δ is linear, defining a Δ line in the space (T, D) with a negative slope of -1. Given that $\sigma \geq 0$, only part of the Δ line is relevant to our analysis. That is the part beginning at (T_1, D_1) when $\sigma = 0$ (see (29) and (30)) and ending at $(-\infty, +\infty)$ for $\sigma = +\infty$. This *half-line* Δ, represented in the space (T, D), points upwards to the left. See an example in figure 3.

(insert Figure 3 here)

To proceed with the geometrical method it is convenient to define in the plane (T, D) the following lines. The *line* AC $(D = T - 1)$ where a local eigenvalue is equal to 1; the *line* AB $(D = -T - 1)$, where one eigenvalue is equal to -1; and the *segment* BC $(D = 1$ and $|T| < 2)$ where two eigenvalues are complex conjugates of modulus 1. See figure 3. These lines divide the plane (T, D) into three different regions: *sink*, *source* and *saddle*[31], according to the values assumed by the local eigenvalues of the system and the respective dynamic properties near the steady state.

Using our half-line Δ, as in figure 3, we can identify whether the steady state is a sink, a saddle or a source, depending on the values taken by σ. Note that, given the existence of a non predetermined variable l in our system, local indeterminacy arises when the steady state is a sink.[32]

When the half-line Δ crosses the boundaries defined by lines AC, AB or the interior of segment BC a local bifurcation generically occurs.[33] A local *Hopf* bifurcation generically occurs when, by slightly changing a parameter of the model, a pair of complex conjugate eigenvalues cross the unit circle so that the values of T and D cross the interior of the segment BC (i.e. the bifurcation parameter crosses a certain value σ_H). A *flip* bifurcation occurs when the values of T and D cross the AB line. However this will not be possible in our model, since the half-line Δ is parallel to the line AB. Finally, when the values of T and D cross the AC line, and if Propositions 1 and 3 are satisfied, a *transcritical* bifurcation generically occurs.[34] Later, on subsection 4.2, we shall discuss the relevance of indeterminacy and bifurcations for the emergence of endogenous fluctuations.

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[31]The system is locally stable a sink (the two local eigenvalues have modulus less than 1) if and only if the point (T, D) lies in the interior of the triangle ABC. The stationary state is a source when both eigenvalues have modulus higher than one. It is a saddle in all other cases. For more details see Azariadis (1993).

[32]Local indeterminacy occurs when the number of eigenvalues with modulus strictly lower than 1 is larger than the number of predetermined variables. Here, this means that for a given initial condition of the predetermined variable k_t, close to the steady state, there is a continuum of values for l_t defining equilibrium paths that converge to the steady state.

[33]A local bifurcation occurs in nonlinear systems when, by slightly changing one parameter of the model, there is a qualitative change in the dynamic properties of the equilibrium trajectories nearby the steady state, i.e., when a local eigenvalue crosses the unit circle. For a more technical approach the reader may wish to consult Grandmont (1988) or Hale and Koçak (1991).

[34]When T and D cross the line AC, an eigenvalue crosses the value 1. Transcritical or other types of bifurcations may occur (pitchfork or saddle-node). However, since we have assumed the existence of at least one steady state (Proposition 1) and at most of two (Proposition 3) these other bifurcations are ruled out.

4.1 The role of unions bargaining power

Back to our dynamic system we shall now discuss how the half-line Δ moves in the space (T, D) as α varies from 0 to 1. This amounts to analyze how the origin $(T_{1(\theta,\alpha,\upsilon)}, D_{1(\theta,\alpha,\upsilon)})$ of the half-line Δ changes with α, since its slope, as we have seen, is constant (equal to -1). From (31) we can see that the expression for the Δ line does not depend on α. This means that as α changes the values taken by the determinant and the trace lay over the same Δ line in the space (T, D). However, the relevant range of this line (for $\sigma \in (0, +\infty)$) changes with α because its initial point $(T_{1(\theta,\alpha,\upsilon)}, D_{1(\theta,\alpha,\upsilon)})$ smoothly varies with α. As α decreases the initial point move along the line Δ towards the left. Indeed both $T_{1(\theta,\alpha,\upsilon)}$ and $D_{1(\theta,\alpha,\upsilon)}$ are always positive and $T_{1(\theta,\alpha,\upsilon)}$ is increasing in α, while $D_{1(\theta,\alpha,\upsilon)}$ decreases with α.

From now on, and for an exposition device let us fix relevant ranges for our parameters θ and υ:

Aassumption 2 $0 < \theta < \frac{1}{2}$ *and* $0 < \upsilon < 2(1 - 2\theta)$.[35]

Using (29) and (30) it is easy to check that, under Assumption 2, this initial point lies to the right of the line AC, i.e., $D_{1(\alpha)} < T_{1(\alpha)} - 1$, for all values of $0 < \alpha \leq 1$. See an example in figure 3. Therefore, as σ increases from 0 to $+\infty$ the half-line Δ must cross the line AC for some positive value of σ and a transcritical bifurcation occurs. This will happen when σ crosses the critical value σ_T:

$$\sigma_T \equiv \frac{1 - 2\theta + \alpha\theta}{2(1 - \theta)} \tag{32}$$

Moreover, using (31) we have that, under Assumption 2, the value taken by the trace of J, when its determinant is 1, is always lower than 2. This means that the Δ line always intersects the segment BC in its interior, and

[35] This assumption simplifies considerably our analysis, as it will become apparent below. Note however that it does not impose 'strange' values on θ or υ.

Hopf bifurcations generically occur[36] when σ crosses the critical value σ_H:

$$\sigma_H \equiv \frac{1 - 2\theta - \theta\upsilon - \alpha\theta\,(1 - 2\theta - \upsilon)}{\upsilon\,(1-\theta)}. \tag{33}$$

The following proposition summarizes the results on local stability and bifurcation analysis that can now be easily established.

Proposition 5 *Given fixed values for* $0 < \alpha \leq 1$, υ *and* θ *satisfying Assumption 2, and the definition of the critical values* σ_T *and* σ_H *given respectively in (32) and (33), the following generically holds:*

The steady state is a saddle for $0 < \sigma < \sigma_T$*; a transcritical bifurcation occurs for* $\sigma = \sigma_T$ *and the steady state becomes a sink for* $\sigma_T < \sigma < \sigma_H$*; a Hopf bifurcation occurs for* $\sigma = \sigma_H$ *and the steady state becomes a source for* $\sigma > \sigma_H$.

The above findings on local stability and bifurcations are depicted in figures 4a and 4b. There, for given values of $\upsilon > 0$ and θ satisfying Assumption 2, we have plotted σ_T and σ_H as functions of α. Both σ_T and σ_H are always positive and $\sigma_H > \sigma_T$. Note that σ_T is increasing with α, with a maximum value $\sigma_T = 0.5$ for $\alpha = 1$. However σ_H is decreasing or increasing in α depending on wether $\upsilon < 1 - 2\theta$ or $\upsilon > 1 - 2\theta$. The first case is represented in figure 4a and the second in figure 4b. It can be checked that $\sigma_H > 1$ in the first case and that $\sigma_H < 1$ in the second. Note that the curve representing σ_T is the same in both plots since σ_T does not depend on υ.

(insert Figure 4 here)

4.2 Discussion of the results

In this subsection we interpret and discuss the results obtained. Proposition 5 implies that endogenous fluctuations (deterministic and/or stochastic) may emerge in our model. For $\sigma_T < \sigma < \sigma_H$ the steady state is a

[36]Notice that local Hopf bifurcations could not occur if there were no externalities, i.e., if $\nu = 0$. Indeed, in this case, the dynamics for x (a non predetermined variable) are completely described by a one dimensional dynamic equation (17). Since Hopf bifurcations can only occur in two dimensional dynamical systems, they would be excluded. The same conclusions are obtained for the case $\upsilon = 0$, if we consider the dynamics for k and l, which are given by the two dimensional dynamic system (16) and (17). In this case we have that $D = T - 1$, so that the respective Δ line could never cross the line BC in its interior.

sink. In this case there is locally indeterminacy and, as shown for instance in Grandmont et al.(1998), there are infinitely many stochastic endogenous fluctuations (sunspots) arbitrarily near the steady state. If the system were linear this would be the only possibility for the emergence of endogenous fluctuations. However, in non linear systems like ours, deterministic and stochastic endogenous fluctuations may also emerge due to the occurrence of local bifurcations. When a *transcritical* bifurcation occurs there are, at least for σ close to σ_T, two steady states exchanging stability properties, in our case between a *sink* and a *saddle*. Hence, even when the steady state under analysis is a *saddle* there are sunspots around the other one (*a sink*). Also, as σ is increased and crosses the value σ_H the steady-state that was a *sink* becomes a *source* through the occurrence of a *Hopf* bifurcation. This, at least for some values of σ near σ_H generates the appearance of an invariant closed curve surrounding the steady state in the state space, over which we observe *deterministic periodic or quasiperiodic dynamics*.[37] When the *Hopf* bifurcation is supercritical[38] this cyclic behaviour appears for values of σ such that the steady state is a *source*.[39] In this case, as shown in Grandmont el al. (1998), there are also infinitely many stochastic equilibria staying in a compact neighborhood of the steady state that contains in its interior the invariant close curve.[40]

[37]As noted before in footnote 36, Hopf bifurcations cannot occur if $v = 0$. Therefore local deterministic endogenous fluctuations may only appear in the presence of externalities, which here implies the existence of technologies with social increasing returns to scale. This result is in accordance with Aloi et al. (2000).

[38]Bifurcations are supercritical or subcritical depending on certain conditions, in terms of the values of higher order derivatives of the dynamic system, which usually are difficult to obtain since they involve complex analytical computations.

[39]When the steady state is a source the economy may face adjustment problems if there is an exogenous shock. Trajectories may become explosive or explosiveness may become contained due to the existence of an attracting invariant closed curve (supercritical Hopf). In any case, employment may reach its full employment level and, when this happens, dynamics change. Note that under full employment l_t is fixed, $l_t = n$, and therefore the dynamic system is unidimensional. Under perfect competition it would be given by (16) with $\alpha = 1$. In this case, it is well known that, when externalities are strong enough, endogenous growth may emerge. See Jones and Manuelli (1992). However, endogenous growth may also emerge in the presence of unions even without externalities. See Dos Santos Ferreira and Lloyd-Braga (2002). Nevertheless, in our case, taking into account the possibility of full employment would obviously lead to a much more complex behaviour, with the possibility of dynamics regime switching, which is beyond the scope of this paper.

[40]Note that even when the local steady state under analysis is determinate (saddle or source) endogenous fluctuations emerge in our nonlinear model if σ is close enough to the

Moreover, Proposition 5 and figure 4 show that the above findings apply no matter the value of α. This could imply that the role of unions in shaping local dynamics is not very important. However, the combinations of σ and α required for the emergence of bifurcations and indeterminacy are quite different, depending on whether the degree of capital externalities is low ($\upsilon < 1-2\theta$) as in figure 4a, or takes intermediate values ($1-2\theta < \upsilon < 2(1-2\theta)$) as in figure 4b. To further highlight this issue we summarize in figure 5 the local stability properties and the occurrence of bifurcations in the (υ,σ) plane, considering two cases for α: $\alpha = 1$, the perfect competition case and $\alpha < 1$, the unions case. From (33), it is easy to see that the $\sigma_{H(\upsilon)}$ curve is decreasing in υ, becoming steeper when α decreases and always crossing the point $(1-2\theta, 1)$. Moreover, from (32) we see that the curve $\sigma_{T(\upsilon)}$ is an horizontal line that shifts downwards when α decreases.

(insert Figure 5 here)

By inspection of the figures above we see that: *i)* for intermediate values of υ ($1-2\theta < \upsilon < 2(1-2\theta)$), unions only influence the local dynamics of the system when $\sigma < 1$; also, in this case, the value of $\sigma < 1$ compatible with *Hopf* bifurcations becomes lower when α decreases; *ii)* on the contrary, for lower values of υ ($\upsilon < 1-2\theta$), and assuming (reasonably) that $\sigma > 1/2$, unions' bargaining power is only relevant for the dynamics when $\sigma > 1$, and, in this case, the value of $\sigma > 1$ compatible with *Hopf* bifurcations becomes higher when α decreases. This suggests that what really matters for the dynamics is not the degree of unions bargaining power *per se* but the existing combination of the degree of union power and the "parameters" characterizing technology, namely the degree of capital externalities and the elasticity of substitution between inputs.

Moreover, we also have that: *iii)* the occurrence of local *Hopf* bifurcations for values of σ around 1 is possible no matter the value of α, only requiring a value for υ sufficiently close to $1-2\theta$;[41] *iv)* no matter the value of υ, the steady state can only be a saddle for low values of σ, namely in an interval $(0, \sigma_{T(\alpha)} \leq 1/2)$ that shrinks as α decreases; and *v)* indeterminacy may arise with an arbitrarily small degree of externalities.

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sink boundaries (σ_T and σ_H).

[41]However, the case of a Cobb Douglas technology with $\nu = 1-2\theta$ is non generic in our model. Indeed in this case no Hopf bifurcations occur since the dynamic system becomes log linear. This happens because we assumed full depreciation of capital.

Indeed, as in Cazzavillan (2001), for (arbitrarily) small capital externalities indeterminacy may arise for a very wide range of values for σ, containing those values usually considered empirically plausible. More precisely, indeterminacy is obtained for $\sigma \in \left(\sigma_{T(\alpha)}, \sigma_{H(\alpha,\upsilon)}\right)$ where $\sigma_{T(\alpha)} < 1/2$ and $\sigma_{H(\alpha,\upsilon)} > 1$, the latter converging to $+\infty$ as υ tends to zero. This result is obtained independently of the value of α. Note however that, as referred above, for a given low value of υ, $0 < \upsilon < 1-2\theta$, $\sigma_{H(\alpha,\upsilon)}$ is increasing in $1-\alpha$ (i.e., the unions bargaining power), while $\sigma_{T(\alpha)}$ is decreasing in $1-\alpha$, so that the range of values for σ compatible with indeterminacy enlarges as the degree of unions bargaining power increases. Then, it may be the case that for some empirically plausible values of σ indeterminacy does not arise under perfect competition, while it occurs when there are sufficiently strong unions.[42] However, with intermediate capital externalities, $1 - 2\theta < \upsilon < 2\left(1 - 2\theta\right)$, things become different. As before, indeterminacy occurs when $\sigma \in \left(\sigma_{T(\alpha)}, \sigma_{H(\alpha,\upsilon)}\right)$. But, in this case, $\sigma_{H(\alpha,\upsilon)} < 1$ so that indeterminacy may only emerge for values of σ below one. Also, in this case, $\sigma_{H(\alpha,\upsilon)}$ is a decreasing function of $1-\alpha$, so that the range of values for σ compatible with indeterminacy depart from those considered plausible as unions' bargaining power increases. See figure 4b. In this case, if unions are strong enough the emergence of indeterminacy may require values for the elasticity of factor substitution much too low to be compatible with empirical estimations.[43] Therefore, in the presence of sufficiently strong unions indeterminacy of the steady state may become implausible if capital externalities take intermediate, but still reasonable, values.

5 Concluding Remarks

In this paper we analyze the contribution of unions to the emergence of indeterminacy, bifurcations and endogenous fluctuations, and discuss the effects of unions on steady-state multiplicity and welfare. We consider an overlapping generations economy with increasing returns to scale due to positive capital externalities and focus on underemployment equilibria. As we have seen, unions may increase steady state employment and welfare for wide and

[42] Take for example the case of $\upsilon = 0.12$ and $\theta = 0.4$. Then indeterminacy arises for $\alpha = 1$ if $\sigma \in (0.18; 1.67)$, for $\alpha = 0.5$ if $\sigma \in (0.12; 1.89)$; for $\alpha = 0.1$ if $\sigma \in (0.07; 2.07)$.

[43] Take for example the case of $\upsilon = 0.3$ and $\theta = 0.4$. Then indeterminacy arises for $\alpha = 1$ if $\sigma \in (0.18; 0.67)$, for $\alpha = 0.5$ if $\sigma \in (0.12; 0.56)$; for $\alpha = 0.1$ if $\sigma \in (0.07; 0.47)$.

plausible regions in the parameter space. Moreover this outcome, as well as local indeterminacy, is possible with both arbitrarily small externalities and values for the elasticity of factors substitution around 1. With respect to local dynamics and the emergence of endogenous fluctuations, our results show that what really matters is not union power *per se* but the existing combination of union power and "*technology*", i.e., the degree of externalities and factor substitutability in production. Indeed, if externalities are small, it is possible, for plausible values of elasticity of factors substitution, that local indeterminacy only arises when unions are sufficiently strong. In this case unions would be responsible for the existence of stochastic endogenous fluctuations (sunspots) arbitrarily near the steady state. On the contrary, if capital externalities take intermediate but still reasonable values, indeterminacy may only occur for implausible low substitutability between factors if unions are sufficiently strong. We have also seen that *Hopf* bifurcations, and thereby local deterministic endogenous fluctuations, occur in our model. Their occurrence requires a value for the elasticity of substitution the closer to 1 the lower the bargaining power of unions, i.e., the closer the labour market structure is to the perfectly competitive one. Transcritical bifurcations also generically occur when the elasticity of factor substitution is lower than 1/2. In this case even when the steady state under analysis is a saddle there is, close to it, another steady state that is locally indeterminate. However, for a CES technology, multiplicity of steady states is ruled out when the labour market is perfectly competitive, while two steady states may exist as soon as unions are present.

Some extensions of our analysis and further developments of the model may be desirable. Wages are usually higher in the presence of unions. In our OG economy savings and capital accumulation are obtained out of wages while capital rentals are used for consumption. Thereby it is possible that the effects of unions here obtained are linked to the saving structure considered. The consideration of a Woodford (1986) type of model where, on the contrary, due to financial constraints, capital rentals are invested in capital accumulation and wages are used for consumption, may bring some different results. We also leave for future research other interesting extensions of this study, like the consideration of other settings for the labour market and the analysis of the effects of certain stylized economic policy rules. However, this work has shown that unions may have important effects on economic performance, not only from a positive point of view but also from a normative point of view. Therefore we believe that further research on this subject is much welcome.

A Appendix

A.1 Multiplicity of steady states in the CES case

Consider a CES technology where $f(x) = \left[\delta x^{\frac{\sigma-1}{\sigma}} + 1 - \delta\right]^{\frac{\sigma}{\sigma-1}}$ with $0 < \delta < 1$. Then for $\beta = 0$ we see, using (20), that in this case $G(x)$ can be written as:

$$G(x) = \frac{\delta(1-\delta)\alpha x^{\frac{2\sigma-1}{\sigma}}}{\delta x^{\frac{\sigma-1}{\sigma}}(1-\alpha) + 1 - \delta}.$$

Hence $\lim_{x\to\infty} G(x) = 0$ if $\sigma < 1/2$ and $\lim_{x\to 0} G(x) = 0$ so that $G(x) - 1$ has the same sign for x close to zero and x close to $+\infty$. Note also that in this case, for $\beta = 0$, we have that $B(x) = 2\sigma\alpha(1-\theta_{(x)}) - (1-\alpha\theta_{(x)})(2-\alpha)$ with $\frac{\partial B}{\partial \theta} = \alpha(2-\alpha-2\sigma) > 0$ when $\sigma < 1/2$, since $0 < \alpha < 1$. As $\theta_{(x)} = \delta x^{\frac{\sigma-1}{\sigma}}/(\delta x^{\frac{\sigma-1}{\sigma}} + 1 - \delta)$ is a decreasing function of x, we have that $B(x)$ is a decreasing function of x when $\sigma < 1/2$ so that configuration *(ii)* of Proposition 3 applies. Moreover, $H(0) = \frac{1}{\sigma} > 0$ and $H(+\infty) = \frac{2\sigma-1}{\sigma} < 0$ when $\sigma < 1/2$ so that $G(x)$ is single peaked and there are exactly two steady states.

A.2 Proof of Proposition 4

Proof. We start by analyzing the effects of α on x and k. Differentiating, respectively, (20) and (18) we have that:

$$\varepsilon_{x,\alpha} = -\frac{(1-\beta+\alpha\theta)}{(1-\beta)(1-\alpha\theta)}\frac{1}{H(x)} \tag{34}$$

$$\upsilon\varepsilon_{k,\alpha} = \frac{\alpha\theta}{(1-\alpha\theta)} + \frac{(1-\theta)(\sigma-\alpha\theta)}{\sigma(1-\alpha\theta)}\varepsilon_{x,\alpha} \tag{35}$$

For steady state employment, as we have that $\varepsilon_{l,\alpha} = \varepsilon_{k,\alpha} - \varepsilon_{x,\alpha}$, using (35) we obtain:

$$\upsilon\varepsilon_{l,\alpha} = \frac{\alpha\theta}{(1-\alpha\theta)} + \frac{(1-\theta)(\sigma-\alpha\theta) - \upsilon\sigma(1-\alpha\theta)}{\sigma(1-\alpha\theta)}\varepsilon_{x,\alpha.} \tag{36}$$

Substituting now (34) in (36), and after some simple algebraic computations, it is easy to see that an increase in unions' bargaining power will lead to an increase in employment, i.e. $\varepsilon_{l,\alpha} < 0$, when one of the following conditions is satisfied:

(1) $0 < \upsilon < \upsilon_1$, $\sigma < \sigma_2$ and $\sigma < \sigma_1$.

(2) $0 < \upsilon < \upsilon_1$, $\sigma > \sigma_2$ and $\sigma > \sigma_1$.

(3) $\upsilon > \upsilon_1$ and $\sigma < \sigma_1$.

We analyze now how the steady state utility of the representative employed individual varies with union power. Substituting (8), (12), and (13) in (7) we obtain:

$$a + V(w, r) = a\frac{\left[f(x) - \alpha f'(x)x\right]}{[f(x) - f'(x)x]} = V^*(x). \tag{37}$$

Differentiating the previous expression we have that

$$\varepsilon_{V^*,\alpha} = \frac{\alpha\theta}{1-\alpha\theta}\left\{\frac{(1-\alpha)}{\alpha}\frac{(\sigma-1)}{\sigma}\varepsilon_{x,\alpha} - 1\right\}. \tag{38}$$

Therefore, since a is constant, V is decreasing in α, i.e. increasing in union power, when $\varepsilon_{V^*,\alpha} < 0$. Using (34), we obtain after some simple algebraic computations that this happens when either $\sigma > \sigma_3$ or $\sigma < \sigma_1$. Once $\sigma_3 > \sigma_1$, we have that $\varepsilon_{V^*,\alpha} < 0$ if and only if one of the following conditions are verified:

(4) $\sigma > \sigma_3$.

(5) $\sigma < \sigma_1$.

Combining now the previous results Proposition 4 follows.

For completeness let us finally obtain the conditions under which capital per worker, x, and capital, k, increase or decrease following an increase in unions' bargaining power. Using (34) it is easy to see that $\varepsilon_{x,\alpha} < 0 \Leftrightarrow \sigma > \sigma_1$ and $\varepsilon_{x,\alpha} > 0 \Leftrightarrow \sigma < \sigma_1$ Substituting (34) in (35) we can check that $\varepsilon_{k,\alpha} < 0$ when one of the following conditions are verified. Either $\sigma > \sigma_1$ and $\sigma > \sigma_5$,or $\sigma < \sigma_1$, and $\sigma < \sigma_5$, where:

$$\sigma_5 = \frac{\theta^2\alpha}{(1-\beta)(1-\theta)}. \tag{39}$$

Also $\varepsilon_{k,\alpha} > 0 \Leftrightarrow \min\{\sigma_1, \sigma_5\} < \sigma < \max\{\sigma_1, \sigma_5\}$. ■

A.3 Effects of union power on steady state consumption of the old

Using (5) and (6) we have that at the steady state $c_2 = k^h r$. As equilibrium in the capital market requires that $mk_{t+1} = k^h_{t+1} ml_t$, at the steady state we obtain $k^h = x$, so that we can rewrite $c_2 = xr$. Differentiating this last expression we obtain:

$$\varepsilon_{c_2,\alpha} = \frac{-\alpha\theta}{(1-\beta)(1-\alpha\theta)} + \left(1 - \frac{[\theta(1-\alpha\theta) + (1-\theta)(\sigma-\alpha\theta)]}{(1-\beta)(1-\alpha\theta)\sigma}\right)\varepsilon_{x,\alpha} \quad (40)$$

Substituting now (34) in (40) we have that $\varepsilon_{c_2,\alpha} < 0$ when one of the following conditions holds: *i)* if $\beta < (1+\alpha)\theta$ then either $\sigma > \max\{\sigma_1, \sigma_4\}$ or $\sigma < \min\{\sigma_1, \sigma_4\}$; *ii)* if $\beta > (1+\alpha)\theta$ and $\sigma < \sigma_1$, where $\sigma_4 \equiv \frac{\theta}{\theta(1+\alpha)-\beta}$.

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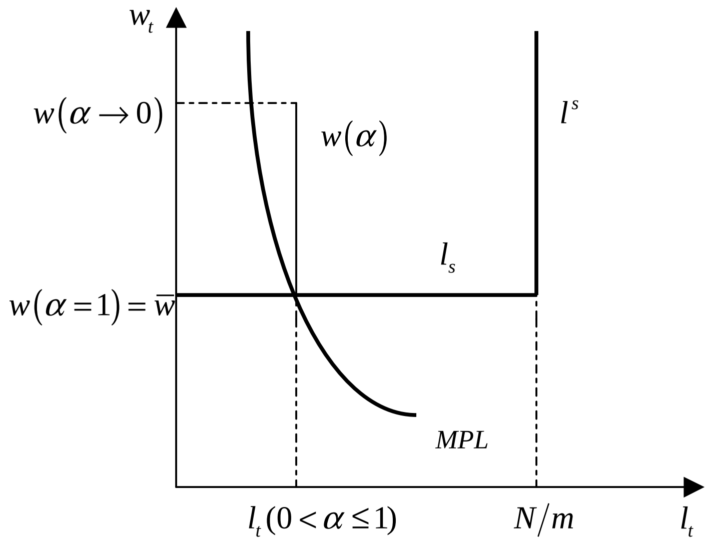
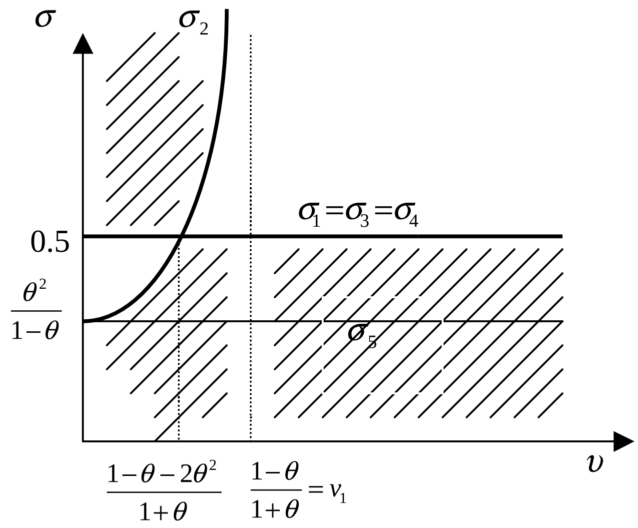


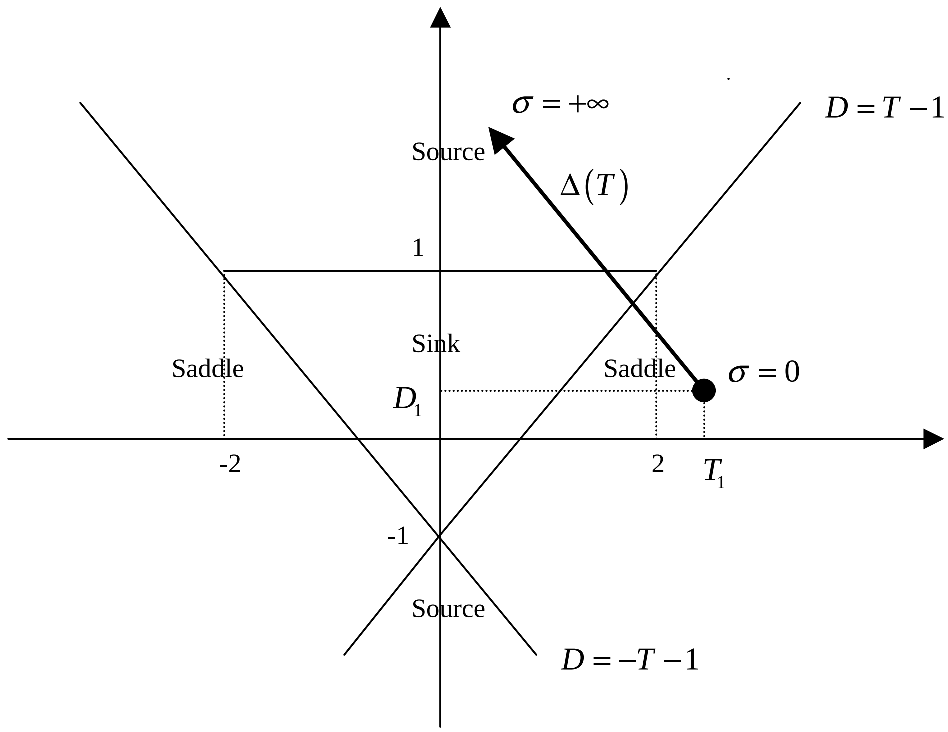
w_t
$w(\alpha \to 0)$
$w(\alpha)$
l^s
l_s
$w(\alpha = 1) = \bar{w}$
MPL
$l_t(0 < \alpha \leq 1)$
N/m
l_t

FIGURE 1



σ
σ_2
$\sigma_1 = \sigma_3 = \sigma_4$
0.5
$\frac{\theta^2}{1-\theta}$
σ_5
υ
$\frac{1-\theta-2\theta^2}{1+\theta}$
$\frac{1-\theta}{1+\theta} = v_1$

FIGURE 2



$\sigma = +\infty$
$D = T - 1$
Source
$\Delta(T)$
1
Sink
Saddle
Saddle
$\sigma = 0$
D_1
-2
2
T_1
-1
Source
$D = -T - 1$

FIGURE 3

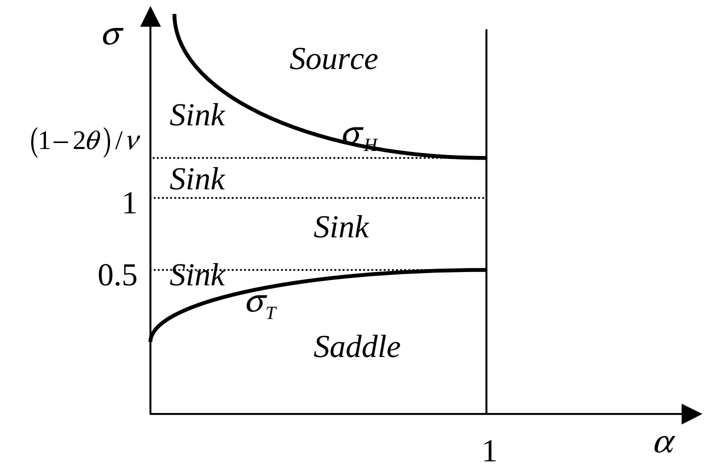


FIGURE 4a: $0 < \upsilon < (1-2\theta)$

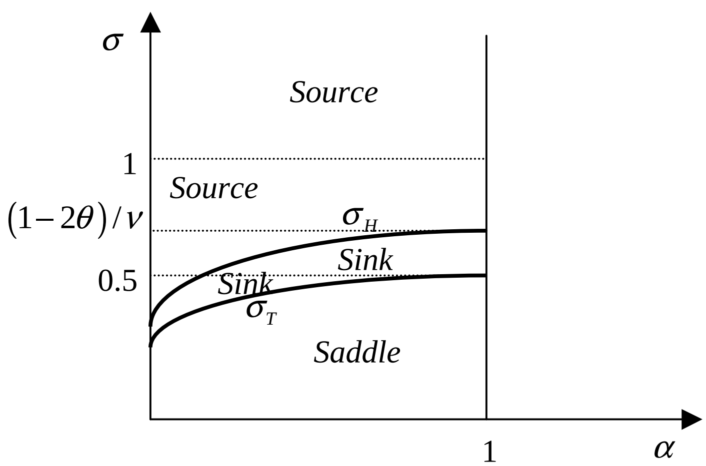
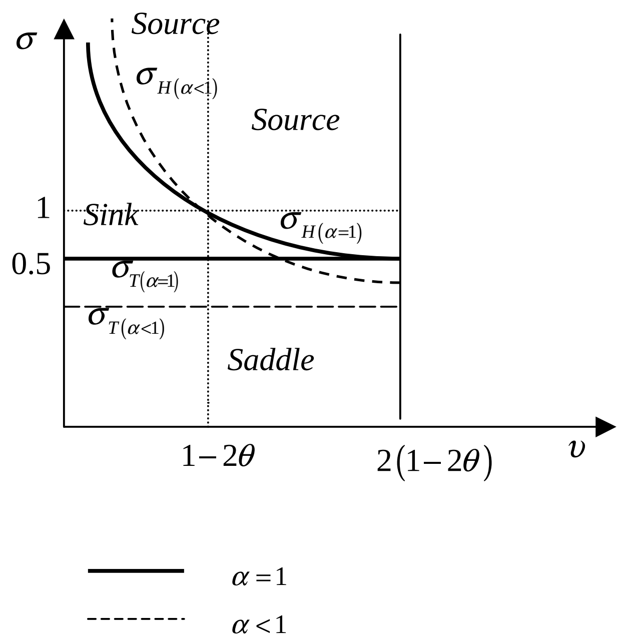


FIGURE 4b: $1-2\theta < \upsilon < 2(1-2\theta)$



σ
Source
σ_H(α<1)
Source
1
Sink
σ_H(α=1)
0.5
σ_T(α=1)
σ_T(α<1)
Saddle
1−2θ
2(1−2θ)
υ
α = 1
α < 1

FIGURE 5