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ABSTRACT

Spatial Agglomeration Dynamics*

This paper develops a model of economic growth and activity locating endogenously on a 3-dimensional featureless global geography. The same economic forces influence simultaneously growth, convergence, and spatial agglomeration and clustering. Economic activity is not concentrated on discrete isolated points but instead a dynamically-fluctuating, smooth spatial distribution. Spatial inequality is a Cass-Koopmans saddlepath, and the global distribution of economic activity converges towards egalitarian growth. Equality is stable but spatial inequality is needed to attain it.

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Keywords: cluster, continuous space, convergence, distribution dynamics, globalization, growth, knowledge, saddlepath dynamics, spatial inequality, spatial spillovers

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Spatial agglomeration dynamics

by

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Income inequality across geography is as profound as it is across people. In spatial inequality, agglomeration and clustering constitute the observations to be formalized and explained. Understanding their evolution draws on ideas in economic development, growth, and economic geography.

When spatial inequality analyses are motivated by contrasting, say, New York City and Yuma, Arizona the set of economic forces a researcher identifies distinguishes prototypes of the two locations one has in mind. New York might have economic activity showing high increasing returns; Yuma, by contrast, might produce goods with significant transport costs. The reasoning—canonical in economic geography—then addresses why New York enjoys an income level higher than Yuma's, i.e., it explains income inequality.

That analysis is silent, however, on a number of interesting questions. Why are the clusters potentially only in Yuma and New York, not anywhere in between or beyond? How many clusters should endogenously emerge—if N locations are a priori possible, does Yuma/New York reasoning predict $N/2$ high-income agglomerations, or just 1? If $N/2$, are they interspersed in between low-income points, or do they collect all together at one end of the physical geography? (And what if N isn't given?) Does it matter that from Yuma, New York is 2100 miles northeast, or would the same reasoning work for comparing with San Francisco—how do spatial relations matter? Put differently, where is geography in this model of economic geography?

This paper describes research in spatial dynamics that address these and related questions (Paul Krugman and Anthony Venables (1997), Danny Quah (2000, 2001)).[1] I illustrate the ideas in a dynamic perfect-foresight equilibrium model that integrates growth, geography, and distribution in an explicit geographical space, namely a 3-dimensional

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[1] The mathematical tools here might appear unfamiliar but are firmly classical: their core goes back at least to Ulf Grenander and Gabor Szegö (1958) and Alan Turing (1952).

globe. The model determines the number of spatial modes (agglomerations, clusters) in economic activity on this globe, the locations of these agglomerations relative to one another, and their dynamics along convergent paths to balanced growth steady state. (Multiple spatial modes can imply the kind of twin-peaked income distributions described in Quah (1997).)

The model is neoclassical with the driver of economic growth being technology, or knowledge accumulation.[2] In the model, new knowledge is generated exogenously. Transportation costs are zero so that knowledge potentially disseminates freely across space. However, in any given location and at any given time, the effective use of knowledge depends on past choices made there on the use of knowledge, and on current and past choices made in surrounding locations. The model displays imperfect productivity spillovers across space and time, and determines spatial and dynamic fluctuations jointly.

Uniformity—an egalitarian spatial income distribution—is always an equilibrium, and characterizes balanced-growth steady state. However, spatial agglomerations or clusters appear in perfect foresight Cass-Koopmans saddlepath transitions: such inequality dynamics are *necessary* for convergence to balanced-growth steady state.

Close in spirit to this paper—despite the differences in model, methods, and conclusions—Eeckhout and Jovanovic (2001) study knowledge spillovers in production where permanent inequality resolves a tension between catching up and free riding. In Dilip Mookherjee and Debraj Ray (2002), equality is unstable. Here, equality is stable, but spatial inequality is needed to achieve it. Kiminori Matsuyama (2002) studies the stability and general structure of discrete equilibria in complementarity games, in a way related to the concerns expressed above on the unsatisfactory nature of two-point (or, more generally, discrete) equilibrium outcomes.

1 The Model

Let z denote a representative location on a geography $\mathbb{G}$, and normalize $\int_{\mathbb{G}} dz$ to 1. (Section 2.2 below specializes $\mathbb{G}$ to the surface of a 3-dimensional globe, but the discussion until then is general. If it helps

[2] James Feyrer (2001) shows it is TFP, rather than capital or labor, that accounts for the twinpeakedness of the cross-country income distribution given in Quah (1997).

intuition, the reader can, without loss, visualize what happens between now and then using that special case.)

The technology level in use at time t in location z is $A_t(z) \geq 0$. Write the spatial profile or distribution as $A_t = \{A_t(z) \mid z \in \mathbb{G}\}$. In this model, technology and (accumulated) knowledge are synonymous, so that the average state of knowledge worldwide is $\overline{A}_t = \int_{\mathbb{G}} A_t(z)\, dz$. Uniformity has $\overline{A}_t = A_t(z)\ \forall z$. Spatial agglomerations or clusters are modes in the spatial distribution A_t, inducing in turn modes in the spatial distribution of incomes (5) below.

As much A as demanded is supplied: Think of this as D in R&D, where R perpetually runs ahead of D (i.e., R grows by rate at least g given in (8) below), and is financed by nondistortionary taxation with output made costlessly available to everyone.

Each location is its own infinitesimally-small nation. It discounts the future at constant rate $\rho > 0$ and produces gross output

$$F(A_t(z) \mid A_t) = W_z(A_t) \times A_t(z) \tag{1}$$

with

$$W_z(A_t) = \int_{\mathbb{G}} K(z, z')A_t(z')\, dz' \tag{2}$$

where for all z the weighting function $K(z, \cdot)$ is a probability density or probability kernel on $\mathbb{G}$, so that $K(z, z') \geq 0$ for all z' and $\int_{\mathbb{G}} K(z, z')\, dz' = 1$. By (1)–(2) output is linear in the state of knowledge $A_t(z)$, with coefficient (marginal product) equal to a weighted average of the current levels of A in the appropriate neighborhood of z.

Weighting function K is time-invariant; allowing it to evolve adds no additional insight. In section 2.2 we restrict K further, in line with the related specialization of $\mathbb{G}$. Until then, however, the discussion requires no further assumptions on the pair $(\mathbb{G}, K)$.

Robert Lucas (1988) described how for economic growth knowledge is necessarily at once global, not Chinese, or Korean, or US. This model maintains that. But how effective global knowledge A is at location z depends both on one's current state of knowledge and on one's neighbors'. Assume further that training or retooling costs need to be expended before knowledge can be used in production. These training costs at z are quadratic in A's rate of change:

$$C(\dot{A}_t(z)) = \frac{1}{2}\zeta \times \dot{A}_t(z)^2, \quad \zeta > 0. \tag{3}$$

While A's effectiveness in (1) is history- and geography-dependent, from (3) the cost of changing it is neither, and is the same everywhere. Transforming global knowledge to local use can also be interpreted as changing general, codifiable knowledge to specific, tacit knowledge.

Coefficient ζ parameterizes retooling costs: The larger is ζ, the more sluggish will be changes in A. Assume that

$$4\zeta^{-1} < \rho^2, \tag{4}$$

i.e., relative to how much the future is discounted, retooling costs generate sufficiently high sluggishness.

Income, net of retooling costs, is then

$$y_t(z) = F(A_s(z) \mid A_s) - C(\dot{A}_s(z)). \tag{5}$$

Economy z at time t solves

$$\sup_{\{A_s(z):s\geq t\}} \int_{s\geq t} y_s(z)e^{-\rho s}\, ds \tag{6}$$
$$\text{s. t.} \quad \text{conditions (1)–(5) and} \quad \begin{cases} A_t(z) \\ \{A_s(z') : s \geq t,\ z' \neq z\}\,. \end{cases}$$

An *equilibrium* is a collection of mutually consistent timepaths $\{A_s(z) : s \geq t\}$, one for each z, solving (6), or alternatively, a time path of profiles $\{A_s : s \geq t\}$ such that each z-section $A(z)$ solves (6) and follows what others expect of z. The equilibrium is rational expectations Nash in the strategy space comprising timepaths $\{A_s(z) : s \geq t\}$ since expectations are realized in equilibrium and (6) requires that each location select a feasible timepath taking as given the choices made in all other locations.

The model is one not only of a set of given locations choosing alternative patterns of development but, upon reinterpretation, also a model of location choice, i.e., of a planner deciding where to place resources, subject to feasibility constraints.

Each z's decision on the timepath for its $A(z)$ gives, for appropriate weighting kernel K, an optimizing version of the "gradual catchup" specifications in, e.g., Andrew Bernard and Charles I. Jones (1996) and Gavin Cameron, James Proudman, and Stephen Redding (1998). But rather than focus on the dynamics of any single z, the concern here is the simultaneous evolution of all z's in joint equilibrium. In this, it shares the same goals as Lucas (2000), who considers yet different mechanics for A spillover across countries.

2 Equilibrium Dynamics

At each z the program (6) has necessary first-order condition the Euler equation

$$W_z(A_t) + \zeta \frac{d\dot{A}_t(z)}{dt} - \rho\zeta\dot{A}_t(z) = 0,$$

which implies the decision rule

$$\dot{A}_t(z) = \zeta^{-1} \times \int_0^\infty W_z(A_{t+s}) e^{-\rho s}\, ds \tag{7}$$

where I have solved stable roots backwards and unstable roots forwards (e.g., Thomas Sargent (1987, Ch. 9)).

2.1 Balanced Growth Steady State

Uniformity is an equilibrium with $\overline{A}$ growing at proportional rate

$$g = \frac{\rho}{2} - \frac{(\rho^2 - 4\zeta^{-1})^{1/2}}{2} \in (0, \rho). \tag{8}$$

If, as in Section 2.2 below, $\mathbb{G}$ is the surface of 3-dimensional globe, then the balanced growth steady state equilibrium given in (8) can be visualized easily: Across geography the level of knowledge is a globe concentric with $\mathbb{G}$, growing proportionally outwards. So too then the spatial distribution of incomes. Nations everywhere have identical and growing incomes. Convergence, equality, and globalization are total.

Balanced growth steady state (8) is a uniform equilibrium, however, even without this restriction on $\mathbb{G}$. To see this, note that at uniformity with $\overline{A}$ growing at rate $g < \rho$, the right side of (7) becomes

$$\begin{aligned}\zeta^{-1} \int_0^\infty \overline{A}_{t+s} e^{-\rho s}\, ds &= \zeta^{-1}\overline{A}_t \int_0^\infty e^{-(\rho-g)s}\, ds \\ &= [(\rho - g)\zeta]^{-1}\overline{A}_t.\end{aligned}$$

Location z therefore has

$$\dot{A}_t(z)/\overline{A}_t = [(\rho - g)\zeta]^{-1},$$

so that in uniform equilibrium with $\dot{A}_t(z) = \overline{A}_t$, this becomes

$$g = [(\rho - g)\zeta]^{-1}.$$

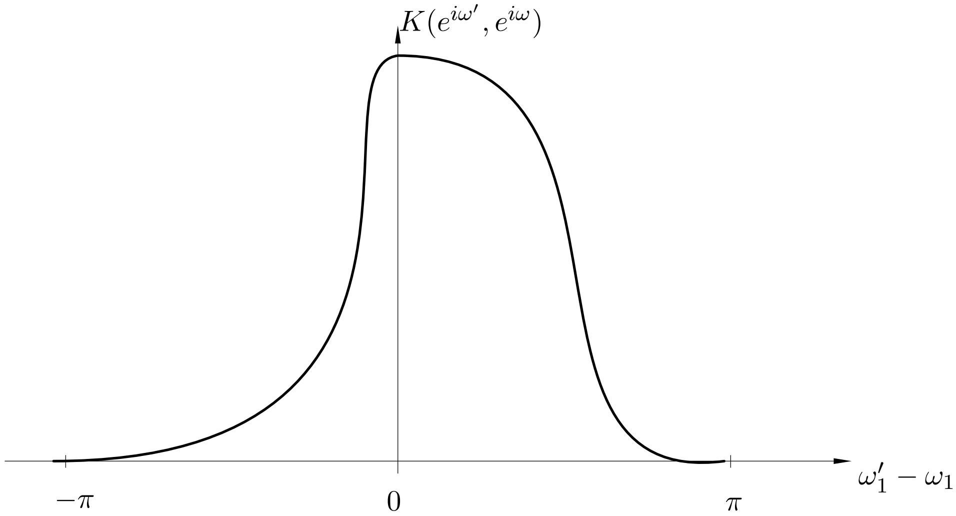

Fig. 1: Weighting kernel Spillovers across geography can be asymmetric and unimodal. Section through K shown has $\omega' = (\omega_1', \omega_2')$ and $\omega = (\omega_1, \omega_2)$, with ω_2' and ω_2 fixed.

This quadratic in g has one root given by (8); the other root exceeds $\rho/2$, implying infinite value to (6).

In (8), $dg/d\rho < 0$ and $dg/d\zeta < 0$. The intuition is straightforward. The more myopic are decision makers, the lower is the steady-state growth rate. Similarly, the higher are retooling costs, the slower does the world grow.

The analysis thus far has been blind to any special structure in $\mathbb{G}$ and K. Only in transition dynamics will that matter. We turn now to this.

2.2 Transition to Steady State

Specialize $\mathbb{G}$ to the surface of a 3-dimensional globe. Using polar coordinates the representative location is $z = (z_1, z_2) = e^{-i\omega} = (e^{-i\omega_1}, e^{-i\omega_2})$, with $i = \sqrt{-1}$ and $\omega \in [-\pi, \pi] \times [-\pi, \pi]$.

Next, require K be nondegenerate (i.e., not constant and have, for all z, $K(z, \cdot)$ place positive weight outside a small open neighborhood of z), continuously differentiable, and radially homogeneous (i.e., $K(e^{i\omega'}, e^{i\omega})$ depend only on $\omega' - \omega$). Radial homogeneity differs from symmetry, which would require instead dependence on $|\omega' - \omega|$.

Fig. 1 shows a section through a possible K. By radial homogeneity K is graphed as a function of only $\omega' - \omega$. Spillover weighting can increase in that separation and be asymmetric about the origin. From both these properties, it differs from the usual decay due to physical distance. A unimodal K is not ruled out, and indeed will suffice to generate multiple modes in spatial outcomes below.

The restrictions on K allow considerable flexibility. Technological catchup, when a follower economy's behavior depends on its separation from A in the leader economy, is intrincially aspatial—it doesn't matter how far the US is from the UK or India for its ideas and engineering blueprints to propagate there. Thus, that K can degrade with physical distance but need not is consistent with most specifications of technological catchup. Knowledge work, when collaboration needs either synchronicity or sequencing, makes timezone separation more important than physical distance: K can then vary along latitudes but be flat along longitudes (e.g., Quah (2000)). Latitudinal variation in this example might even lead to bumps in K where 8-hour timezone separations, say, might be more productive than 4-hour ones. While K matters importantly for equilibrium transition paths, we will see below it is *not* the only factor explaining spatial dynamics.

Outside of steady state, equilibrium transition behavior can be quite intricate. To rule out extraordinary but nonetheless uninteresting outcomes, assume equilibrium is Markov, i.e., at each location z there is a time-invariant mapping M_z such that (7) becomes $\dot{A}_t(z) = M_z(A_t)$. The present discounted value on the right of (7) can be calculated as depending only on the current knowledge profile A_t. Detrending both sides around the equilibrium growth path $\overline{A}_0 e^{gt}$ and then stacking into a spatial profile gives

$$\dot{\widetilde{A}}_t = \widetilde{M}(\widetilde{A}_t), \tag{9}$$

an ordinary differential equation in the space of bounded positive functions on $\mathbb{G}$. The dynamical system (9) has state vector that is infinite-dimensioned.

To obtain intuition for what follows, recall Cass/Koopmans-type dynamics when the transition equation (9) is finite and linear, with steady state $\widetilde{A}_t = \overline{\widetilde{A}}$, i.e.,

$$\dot{\widetilde{A}}_t = \widetilde{M} \times \left(\widetilde{A}_t - \overline{\widetilde{A}} \right). \tag{9$'$}$$

Suppose the finite matrix $\widetilde{M}$ is diagonalizable

$$\widetilde{M} = \boldsymbol{\Phi} \boldsymbol{V} \boldsymbol{\Phi}^{-1}, \quad \boldsymbol{V} = \mathrm{diag}\{\nu_1, \nu_2, \ldots\}.$$

Stable or unstable dynamics hinge on the sign of the real parts of eigenvalues ν_j by

$$(\mathbf{\Phi}^{-1}\dot{\widetilde{A}}_t)_j = \nu_j \times \left((\mathbf{\Phi}^{-1}\widetilde{A}_t)_j - (\mathbf{\Phi}^{-1}\overline{\widetilde{A}})_j\right). \tag{10}$$

The convergent manifold is the collection of initial states $\widetilde{A}_0$ such that (9′) takes the system to steady state $\overline{\widetilde{A}}$. Suppose $\boldsymbol{V}$ collects all ν_j with negative real parts in its leading entries. From (10) the convergent manifold has representation

$$\tilde{q}: \quad \widetilde{A}_0 - \overline{\widetilde{A}} = \mathbf{\Phi} \times \begin{pmatrix} \tilde{q} \\ \underline{0} \end{pmatrix}, \tag{11}$$

zeroing out components that multiply into unstable eigenvalues.

In (9) for the current model, the convergent manifold—a subset of the collection of spatial functions on $\mathbb{G}$ integrating to zero—is well defined. That manifold is the collection of initial states such that (9) converges to zero. However, $\widetilde{M}$ is an infinite-dimensioned nonlinear mapping. Any linearization into an equation such as (9′) will have the eigenvector-eigenvalue decomposition evolving in time.

Recognize, however, that by its construction from a radially symmetric K the operator $\widetilde{M}$ is Toeplitz (see, e.g., Turing (1952) and Grenander and Szegö (1958))—if it were a matrix its rows would be simply rotational shifts of one another. Alternatively, every fixed diagonal section in $\widetilde{M}$ is constant. Then $\widetilde{M}$ has a spectrum (counterpart to the set of eigenvalues) that is discrete, and comprised of Fourier transforms of $\widetilde{M}$'s horizontal sections, while its eigenfunctions (counterparts to eigenvectors) comprise only complex exponentials $e^{-i\omega j}$ for integer j.

In parallel with (10) the spectrum determines the dynamics of detrended profiles $\widetilde{A}$ about the steady-state growth path. In parallel with (11) the eigenfunctions determine the convergent manifold: Here, every element of the convergent manifold is a linear combination of complex exponentials, i.e., 2-dimensional waves of varying frequencies on $\mathbb{G}$. Moreover, when retooling costs ζ are neither too large nor too small, the nullifying of spectral elements required in (11) makes the convergent manifold a strict subset of the full span of the eigenfunctions.[3] Multiple modes then necessarily appear in the spatial distributions that comprise the convergent manifold—see Fig. 2.

[3] Quah (2000, 2001) provides explicit technical details in this reasoning.

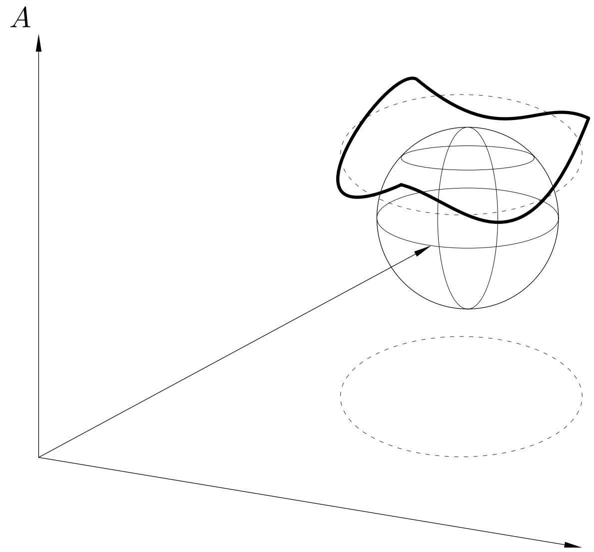

Fig. 2: Spatial agglomeration on 3-dimensional globe Figure shows a horizontal slice through the local perturbations that converge back to steady state only with clusters in the spatial distribution

Along the equilibrium path, where the spatial agglomerations locate relative to one another in $\mathbb{G}$—the bumps in Fig. 2—will be determined by the ω's that remain active in (11). Those ω's are, in turn, functions of all the parameters of the model (K, ζ, ρ). Economic activity across space has a profile that depends on both those active ω's and the corresponding spectra. The dynamics of the spatial distribution of economic activity can, in turn, be read off (10).

3 Conclusions

When spatial inequality is studied in a model with discrete locations fixed, many interesting questions cannot be addressed. This paper develops a model of economic growth and activity that permits a richer analysis needed for that discussion.

The spatial neoclassical growth model in this paper has knowledge accumulation as the engine of growth. Equilibrium in the model is rational expectations and Nash. Knowledge spillovers across geography and optimal knowledge accumulation decisions determine in equilibrium the distribution of knowledge used across space and over time. The resulting pattern of economic activity is not concentrated on discrete isolated points but is instead a dynamically-fluctuating, smooth spatial distribution. Spatial inequality is a Cass-Koopmans saddlepath in the space of spatial distributions, and the global distribution of economic activity converges towards egalitarian growth. Equality is stable but spatial inequality is needed to attain it.

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