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TECHNOLOGY DISSEMINATION AND ECONOMIC GROWTH: SOME LESSONS FOR THE NEW ECONOMY

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ABSTRACT

Technology Dissemination and Economic Growth: Some Lessons for the New Economy*

This Paper attempts to draw lessons for the New Economy from what economists know about technology dissemination and economic growth. It argues that what is most notable about the New Economy is that it is knowledge-driven, not just in the sense that knowledge now assumes increasing importance in production, thereby raising productivity. Instead, it is that consumption occurs increasingly in goods that are like knowledge — computer software, video entertainment, gene sequences, Internet-delivered goods and services — where material physicality matters little. That knowledge is aspatial and nonrival is key. Understanding the effective exchange and dissemination of such knowledge-products will matter more than resolving the so-called productivity paradox.

JEL Classification: N10, N15, O33 and O57
Keywords: aspatial, demand, endogenous growth, endogenous technology, human capital, industrial revolution, infinitely expansible, neoclassical growth, nonrival, productivity paradox, weightless economy

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NON-TECHNICAL SUMMARY

Because the New Economy is so intertwined with Information and Communications Technology, we are primed to think of New Economy developments as nothing more than technology-driven, productivity-improving changes on the supply side. We then want New Economy developments to do what all technical progress has historically done. And we emerge disappointed when we find productivity has not sky-rocketed, inflation has not forever disappeared, business downturns have not permanently vanished, and financial markets have not remained stratospheric. This Paper argues that the most profound changes in the New Economy are not productivity or supply-side improvements, but instead consumption or demand-side changes. The Paper summarizes the case for the importance of technical progress in economic growth, argues why the New Economy differs, and draws lessons from economic history to highlight potential pitfalls and dangers as the New Economy continues to evolve. A technical appendix studies the role of human capital in economic growth, clarifying when human capital influences levels of income but not growth rates and when human capital influences growth rates. The discussion emphasizes the distinction between human capital used for improving technology and human capital used in producing goods and services. Both matter and each separately can influence growth in an economy. The key lesson for the New Economy is that endogenous growth results from the interaction of demand and supply characteristics, not just production-side developments.

Technology Dissemination and Economic Growth:
Some Lessons for the New Economy
by

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January 2002

1 Introduction

Pick up a newspaper today, and you have to realize how words and concepts that didn't even exist a decade ago—Internet browsers, desktop operating systems, Open Source Software, WAP delivery, the 3 billion letters of the human genome, political organization and mobilization by Internet chat rooms—now appear regularly in front page headlines. These headlines describe *news* items—not science fiction trends, not arcane academic technologies, not obscure scientific experiments.

Someone out there with a handle on the social zeitgeist has determined that these items—part of the New Economy—impact readers' lives. Evidently, they are right, for these ideas subsequently insinuate their way into hundreds of thousands of non-specialist but informed discussions. When did popular culture evolve to where relative merits of different Internet browsers can be quietly debated at dinner (sometimes not so quietly), or where personal affinity for different desktop operating systems can constitute a basis for liking or disliking someone [Stephenson, 1999]?

When you live in that world, it is puzzling when you meet people intent on proving to you that none of those things you think you

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see and experience is real. These people, many of them academic economists, seem to come from an alternate, orthogonal universe. They say the New Economy is as nothing compared to the truly great inventions of the past (surely a strawman hypothesis if ever one was needed). These skeptics show you charts and figures, bristling with numerical calculations, arguing that the changes you figured to be deep and fundamental apply, in reality, only to the miniscule group of people working in companies that manufacture computers.

Are academic economists undermining their own credibility and doing their profession a disservice, when they argue a case so ridiculously opposite to what others think is plain and obvious? Or, are they providing a needed reality check as rampant hyperbole takes over all else?

Either way, a tension has built up between two groups of observers on the New Economy. In this paper, I seek to describe how such a situation might have come about, and I want to suggest some possible ways to understand and resolve that tension.

1.1 Technologies and Consumers

Anyone who visits urban centers in the Far East and South East Asia notices immediately the extreme, in-your-face nature to modern technologies here. Advanced technological products are sold, incongruously, in grubby marketplaces. Sophisticated software and hardware change hands in crowded stores that seem better suited to trading fresh homegrown agricultural produce.

To be clear, it's not that the nature of the underlying technologies differs between here and the rest of the world. It's that modern Asia uses modern technology more visibly, forging a sharper, more direct link between that technology and ordinary consumers. Internet cafes were invented in Thailand, and proliferated widely in Asia early on. Next-generation wireless mobile applications in Japan have been among the most innovative worldwide, and are globally admired and imitated. Urban center road pricing and seaport management in Singapore have attained timesliced precision that are orders of magnitudes better than anywhere else in the world. In many East Asian states, the Internet is a critical source of information, shortcircuiting barriers in a way that nothing else can. Hong Kong has cash card transactions rates unmatched elsewhere. In city squares throughout the Far East, up-to-the-second, streaming information screams out in high-tech high definition at throngs of ordinary shoppers. Digital entertainment imaging and animation here are unparalleled: East Asia continues to make the best toys in the world, high-tech or otherwise.

This technology/final consumer linkage is, of course, not unique in the world. Nokia Corporation in Helsinki has gotten to be the world's leading mobile telecommunications company by focusing on exactly this, delivering leading-edge technology directly (and literally) into the hands of hundreds of millions of consumers worldwide.

But, if not unique, this linkage is not particularly commonplace either. Take that example of Finnish wireless banking, mobile telecommunications, and information dissemination applications. In the eyes of some, when compared to daily life in Helsinki, consumer usage of technology in Silicon Valley is akin to that of a relatively backward Third World country. Perhaps so too, when compared to Hong Kong and other parts of Asia.

1.2 Accumulating capital under Joseph Stalin

In 1994, Paul Krugman [1994] suggested that because Singapore appeared to have developed primarily by heavily accumulating physical capital, its high economic growth rate could not be sustainable—the same way that Joseph Stalin's program for economic growth, embodied in exhorting Soviet steel production to match the US, was ultimately bound to fail.

In this interpretation, Krugman used the economists' prediction that ongoing physical capital accumulation—other things equal—would eventually run into diminishing returns. Putting into operation big machines, steel factories, bridges and other physical infrastructure, and heavy machinery can contribute to growth only temporarily—and then only in a relatively minor way.

But if not physical capital, then what drives economic performance? Many economists now agree that technical progress and its close relative, technology dissemination, constitute the ultimate source of sustained economic growth. That is the position I take in this paper.

But if that view is held almost uniformly, its connection to the New Economy is not as obviously uncontroversial. Economists such as Robert Gordon [2000] have been delightedly skeptical on the contribution of the New Economy to economic performance. To caricature those views, the New Economy has been a scam, foisted on an unsuspecting public and naive, trend-chasing policy-makers by the New Economy's slick sales and public relations machine.

1.3 Shopping the Internet

Towards the end of 2000, I got to have breakfast with a successful multimillionaire Internet entrepreneur in London. I asked him if he thought, as some seemed to, that Internet developments amounted to a new Industrial Revolution. He replied, "We're just talking about selling more groceries through a big out-of-town shopping centre—how revolutionary is that?"

My entrepreneur acquaintance—for the record, not an Internet grocer—has a self-aware, tongue-in-cheek manner about him. His statement is pithy to an extreme on the New Economy. It displays the same focus on the technology/consumer linkage I described above. The statement is, in my view, spot on, mostly, but it is a little too flippant on what is new in the New Economy.

This paper attempts to show why the technology/consumer linkage is critical in the New Economy—against a background of what economists know about economic growth and technology, and about the importance of technology's dissemination over time and across economies. It is here where the New Economy is truly new (well, almost)—and where it diverges most sharply from conventional mechanisms relating technology and economic growth.

2 Technology in Economic Growth: Knowledge and Economic Performance

From early on, economists studying growth had found that capital accumulation accounted for only 13% of the improvement in economic welfare experienced over the first part of the 20th century [Solow, 1957]. The rest of economic progress—almost 90% of it—had to be attributed to technology, or total factor productivity (TFP). Recent empirical analyses, notably Feyrer [2001], document how yet other key features of patterns of cross-country development similarly hinge importantly on TFP.

Those early conclusions followed from the so-called neoclassical growth model (see, e.g., Solow [1956, 1957] or the Technical Appendix, Section 7, below). But the key policy implication that many took from this work was exactly opposite to what the research showed—at least as I am interpreting it. In the 1960s and 1970s, researchers and policy-makers read Solow's work on the neoclassical growth model to mean that physical capital accumulation was what mattered most for economic growth. The reason, perhaps, is that, on the theoretical side, neoclassical growth analysis focused on the economic incentives surrounding decisions to save and invest in physical capital; it was downplayed that empirical analysis showed instead technology or TFP accounting for a much greater effect on economic performance and growth.

(Some authors still now take TFP to be no more than a residual, whereupon many possibilities remain open for its interpretation and explanation—it might be political barriers, monopoly inefficiency, X-efficiency, political economy inefficiency, moral hazard, social capital, and so on. In this paper, I adopt principally the discipline of the neoclassical growth model, and identify TFP with only technology and possibly human capital, including the latter under technology more generally. The Technical Appendix below makes this more precise.)

Thus, the development community devoted energy to putting in place physical infrastructure for growth, while academic economists sought to re-calibrate models and re-define variables to reduce the measured contribution due to technology. As an example of these efforts, consider human capital—education and training—which improves labor quality and thus increases the effective quantity of labor. Accounting explicitly for human capital might then reduce the importance of technology in explaining economic growth.

By the time Paul Krugman [1994] articulated his justly-famous critique of Singaporean development policy, the weight of opinion had swung full circle back to an emphasis on technology—thanks to forceful arguments developed meanwhile in Lucas [1988] and Romer [1986, 1990, 1992]. Economies could not hope to sustain high growth through savings and capital accumulation alone. Thus, by the mid 1990s, conventional wisdom was that a high TFP contribution to economic growth indicated a successful economy, not one with mismeasured capital stock and labor input. The way to increase TFP growth was research and development (R&D)—raising the science and knowledge base of the economy. Economists' focus had shifted from the incentive to accumulate physical capital to incentives for knowledge accumulation and technical progress.

A simple formalization will help clarify the issues here as well as others below. Suppose that total output Y satisfies a production function:

$$Y = F(K, N, \tilde{A}), \tag{1}$$

with K denoting the capital stock, N the quantity of labor, and $\tilde{A}$ a first, preliminary index of technology.

To deal with potential mismeasurement in technology and to highlight the role of human capital, suppose that $\tilde{A}$ has two components, h human capital per worker, and A technology proper. Because human capital is embodied in workers, h is specific to an economy—assuming for the discussion here that workers can be identified as belonging to particular economies. By contrast, A is disembodied and global. An alternative characterization might be that A describes codifiable knowledge, while h describes tacit knowledge.

Denoting quantities in different economies using subscripts, we assume that

$$\tilde{A}_j = (h_j, A) \tag{2}$$

applied to (1) gives either

$$Y_j = F(K_j, N_j \times h_j, N_j \times A) \tag{3}$$

or

$$Y_j = F(K_j, N_j \times h_j \times A). \tag{4}$$

The technical appendix shows that in one important class of models (section 7.3) standard assumptions surrounding (3) and (4) imply equilibria where *levels* of per capita incomes or labor productivity, Y/N, can be influenced by decisions on human capital. Growth rates in labor productivity, however, remain equal to the growth rate of technology A and thus invariant to decisions and policies on human capital.

In a different class of models (section 7.4) growth rates *are* influenced by human capital accumulation decisions. A key feature of such models is that growth arises from interaction between demand and supply-side characteristics, not just production-side developments.

The technical appendix clarifies the structural features distinguishing these two class of models. Notably, however, the models in sections 7.3–7.4 take human capital to be used only in producing goods and services. Then, advances in human capital can increase labor productivity, even taking the state of technology as given. Such models should be distinguished from those in, say, Romer [1990] where human capital is an input into R&D and thus technical progress, which thereby evolves endogenously. Human capital can therefore play dual but conceptually distinct roles in economic growth.

Working out the relative contributions to growth of technology and human capital, although not always distinct, matters. In the decomposition (2), technology A is the accumulation of a kind of knowledge resembling a global public good. Human capital h, however, is different. One part of knowledge that matters for growth is codifiable; the other, tacit.

3 Dissemination and Catchup? A Persistent and Growing Divide

While A has always been viewed as an important engine of economic growth—and the evidence and discussion of the previous section reconfirm this—recognizing the peculiar nature of the incentives for A's creation and dissemination raises a number of subtle issues.

A first natural inclination is to view knowledge—ideas, blueprints, designs, recipes—simply as a global public good. Two observations argue for this.

First, knowledge is *non-rival* or infinitely expansible [David, 1993, Romer, 1990]: However costly it might be to create the first instance of a blueprint or an idea, subsequent copies have marginal cost zero. The owner of an idea never loses possession of it, even after giving away the idea to others.

This observation differs from ideas being intangible: Haircuts are intangible, but obviously not infinitely expansible.

Second, knowledge disrespects physical geography and other barriers, both natural and artificial. Knowledge is aspatial; ideas and recipes can be transported arbitrary distances without degradation. (As before, the intangibility of haircuts but their extreme location-specificity makes clear why intangibility alone cannot be the defining characteristic for knowledge.) The acceptability of different ideas might of course differ across locations, depending on the users of those ideas—but that varies not strictly with geographical or national barriers, nor monotonically in physical distance.

An extreme view following from the two observations—first, that codifiable A accounts for most of economic growth and second, that codifiable A is non-rival and has global reach—is that the world should be roughly egalitarian, with all economies having approximately the same income levels. Or, if not, then at least income gaps between countries should be gradually narrowing.

But the opposite is happening. While the whole world is getting richer, the gap between poorest and richest is growing. Average per capita income (real, purchasing power parity adjusted) has grown at 2.25% per year since 1960. At the same time, however, the income ratio between the world's 90th-percentile and 10th-percentile economies grew from 12.3 in the first half of the 1960s to 20.5 in the second half of the 1980s [Quah, 1997, 2001a]. Moreover, distinct income clusters—one at the high end of the income range, another at the low end—appear to be emerging. The cross-economy income distribution has dynamics that are difficult to reconcile with a naive view of knowledge dissemination.

If, to explain these observations, we allow the possibility that A, the driver of growth, might differ across countries, then technology dissemination—how A_j in economy j helps improve $A_{j'}$ in economy j'—becomes paramount for economic growth.

Dissemination mechanisms have been studied [e.g., Barro and Sala-i-Martin, 1997, Cameron, Proudman, and Redding, 1998, Coe and Helpman, 1995, Eaton and Kortum, 1999, Grossman and Helpman, 1991], typically assuming that knowledge and technology are embodied in intermediate inputs, and that property rights permit monopoly operation by the owners of items of knowledge. However, in all these, that A is non-rival and aspatial is never explicitly considered. But it is those peculiar properties—nonrivalry and aspatiality—that allow greatest parallel between developments in the New Economy and what economists might know about technology dissemination.

Parente and Prescott [2000] have posed questions that come closest to the ones stated above. They too focus on A, and its apparent inability to disseminate globally. They conclude that it is vested interests within a potentially A-receiving country that represent significant barriers to A's dissemination. By contrast Quah [2001a] suggested that those obstacles emerge from an equilibrium interaction between A-transmitting and A-receiving economies. In section 5 below I consider the possibility that it is high aversion to change and newness, and low expertise among potential users of A that prevent A's dissemination. This possibility had also been considered previously in Quah [2001b] and Quah [2001c].

4 The New Economy: Puzzles and Paradoxes

If we understand the New Economy to be no more than what has emerged from the proliferation of information and communications technology (ICT), then the New Economy ought to contain no great surprises. ICT is just the most recent manifestation of an ongoing sequence of technical progress. It should then also contribute to economic performance the same way technical progress has always done.

4.1 Why might the New Economy be new?

Two observations suggest potential differences. First, for many, ICT is a General Purpose Technology (GPT), bearing the power to influence profoundly all sectors of an economy simultaneously [Helpman, 1998]. Unlike technical advances in, say, pencil sharpeners, ICT's productivity improvements can ripple strongly through the entire economy, affecting everything from mergers and acquisitions in corporate finance, to factory-floor rewiring of inventory management mechanisms.

Second, ICT products themselves behave like knowledge [Quah, 2001c], in the sense described in Section 3 above. Whether or not we consider, say, a Britney Spears MP3 file downloadable off the Internet as a piece of scientific knowledge—and I suspect most people would not—the fact remains, such an item has all the relevant economic properties of knowledge: infinite expansibility and disrespect of geography. Thus, models of the spread of knowledge, like those described earlier, can shed useful light on the forces driving the creation and dissemination of ICT products. This view suggests something markedly new in the New Economy—a change in the nature of goods and services to become themselves more like knowledge.

This transformation importantly distinguishes modern technical progress from earlier: The economy is now more knowledge-based, not just from knowledge being used more intensively in production, but from consumers' having increasingly direct contact with goods and services that behave like knowledge.

4.2 Puzzles and paradoxes?

I now describe some puzzles relating technology, economic growth, and the New Economy. I will suggest below that interpreting the New Economy in the terms I have just described helps resolve some, although not all, of these puzzles.

To overview, paradoxes in the knowledge-driven, technology-laden economy are of three basic kinds:

1. What used to be just the Solow productivity paradox [Solow, 1987]—"you see computers everywhere except in the productivity numbers"—extends more generally to science and technology. Put simply, a skeptic of the benefits of computers must, on the basis of productivity evidence, be similarly skeptical of science and technology's impact on economic growth.
2. It is not just that science and technology or ICT seem unrelated to economic performance, the correlation is sometimes negative. When output growth has increased, human capital deployment in science and technology appears to have fallen.
3. Although it is by most measures the world's leading technology economy, the US imports more ICT than it exports. And its TFP dynamics haven't changed as much as have TFP dynamics in other economies.

4.3 Solow productivity paradoxes

Fig. 1 contrasts rapidly expanding information technology (IT) investment with insignificant labor productivity improvement in the US between the mid-1960s and the early 1990s [Kraemer and Dedrick, 2001]. In 1973, annual growth in IT spending rose to 17% from an average of -0.2% over the preceding eight years. It then averaged 15.7% for the twenty-two years afterwards. Productivity growth averaged 2.3% for the first period, and then an anemic 0.9% subsequently. Thus, a potentially key addition to technological base of the US economy appears, in reality, to have contributed not at all to US productivity growth.

Fig. 2 shows, however, that the puzzle is more profound than the Solow paradox alone. From 1950 through 1988, the fraction of the US labor force employed as scientists and engineers in R&D increased four-fold, from 0.1% to 0.4% [Jones, 1995]. The increase in this series is much smoother: As much increase occurred after 1972 as before. Yet, as we earlier saw from Fig. 1, labor productivity growth *fell* sharply. (For completeness, Fig. 2 also graphs TFP growth, which relates much the same story as labor productivity growth.) The smooth secular rise in science and technology inputs engendered nothing remotely similar in incomes or productivity.

I conclude that whatever mechanism relates technology inputs—scientists and engineers; information technology—with measured productivity improvements, it is little understood. That mechanism is no more transparent for prosaic and uncontroversial inputs such as scientists and R&D engineers than it is for ICT.

The puzzle only deepens turning to more recent evidence on the US economy. Over 1995–1999, growth in nonfarm business sector productivity rose to an annual rate of 2.9%, more than double its average over the previous two decades [U. S. Department of Commerce, 1999]. Was this the long-awaited resolution of the Solow productivity paradox? If so, yet a different paradox emerges. Over this time, human capital indicators for science and technology in the US declined almost uniformly. Figures from the National Science Foundation (`http://caspar.nsf.gov/`) show that while between 1987 and 1997 the total number of bachelor's degrees increased by 18%, that for computer science *fell* by 36%, for mathematics and statistics by 23%, for engineering 16%, and for physical sciences, 1%. Burrelli [2001] reports that US science and engineering graduate enrollment fell in every single year since 1993, turning around only in 1999. Just as US productivity growth was starting to increase, measurable science and engineering inputs for generating new technology were doing exactly the opposite.

The preceding observations suggest, in my view, a number of complications in the stylization that science and engineering constitute direct inputs into technical progress in turn, driving economic growth. If there is a productivity paradox for ICT and the New Economy, then a yet larger one holds for science and technology more broadly.

4.4 International puzzles

Most studies have thus far focused on the US, but cross-country evidence raises yet further puzzles. Is the US the world's leading New Economy? In 1997 the share of ICT in total business employment was the same, 3.9% [OECD, 2000], for both the US and the European Union (EU). However, comparing the two blocs, the US is clearly well ahead on both value added and R&D expenditure. In the US, the share of ICT value added in the business sector was 8.7%, while the share of ICT R&D expenditure was 38.0%. The EU, by contrast, had ICT value added of only 6.4%, and R&D expenditure in ICT 23.6%.

That the EU numbers are averages across nation states, however, disguises wide diversity across different economies. Thus, a number of EU member states as well as other OECD economies show up *ahead* of the US in New Economy/ICT indicators [OECD, 2000, Tables 1–3, pp. 32–34]. Compared to the US, ICT share in total business employment is higher in Sweden (6.3%), Finland (5.6%), the UK (4.8%), and Ireland (4.6%). Similarly, Korea (10.7%), Sweden (9.3%), the UK (8.4%), and Finland (8.3%) each have ICT shares of value added that exceed the US's. The share of ICT R&D expenditure is 51% in Finland and 48% in Ireland. Moreover, in 1998 the US imported USD 35.9 billion more ICT than it exported [OECD, 2000, Table 4, p. 35]. By contrast, Japan (USD 54.3 billion), Korea (USD 13.6 billion), Ireland (USD 5.8 billion), Finland (USD 3.6 billion), and Sweden (USD 2.8 billion) all showed ICT trade surpluses.[1]

Finally, if the New Economy and ICT are supposed to have af-

[1] I have not been able to get more disaggregated statistics on the *kinds* of ICT products that are aggregated in the statistics above. Perhaps intra-industry trade and product differentiation might be insightful for thinking about these numbers. If so, however, it also suggests that an aggregate, macro emphasis on ICT and productivity is misleading for assessing economic performance.

fected TFP's dynamics in the US economy, they appear to have done so *less* than in economies like Finland, Ireland, and Sweden. Vanhoudt and Onorante [2001] document that for the US the contribution of TFP to economic growth has remained approximately constant at 71%–72% throughout both the 1970s and the 1990s. By contrast, Finland saw an increase in TFP contribution to its growth performance from 60% to 85%; Ireland, from 63% to in essence 100%; and Sweden, from 51% to 72%.

No single piece of empirical evidence here is overwhelming by itself, but the range of them suggests to me a couple of surprising possibilities. First, it is economies like Finland, Ireland, Sweden, Korea, and Japan that, in different dimensions, are more New Economy than the US—the first three of these, most consistently so. Second, to the extent that the US has been a successful New Economy and has powered ahead on the technology supply side, it is its ICT consumption, the demand side, that has grown even more.

4.5 What does the New Economy have to be?

This discussion brings us full circle to my Introduction, that the consumption or demand side of the New Economy deserves greater attention than it has thus far attracted.

By contrast, productivity-focused New Economy analyses are numerous and varied, and include the influential and provocative study of Gordon [2000]. In that work, the author identifies the New Economy as the acceleration in the rate of price declines of computers and related technologies since 1995. He compares New Economy developments to what he calls "Five Great Inventions" from the past, identified as product clusters surrounding (1) electricity; (2) the internal combustion engine; (3) chemical technologies (notably molecule-rearranging technologies, incorporating developments in petroleum, plastics, and pharmaceuticals); (4) pre-World War 2 entertainment, communications, and information (including the telegraph, telephone, and television); and (5) running water, indoor plumbing, and urban sanitation infrastructure. In Gordon's analysis, these clusters of technological developments drove the immense productivity improvements of the Second Industrial Revolution, 1860—1910. In Gordon's definition, the New Economy pales by comparison.

There is no question that Gordon's list of Great Inventions includes critically important technical developments. But comparing mere price reductions—if that is all the New Economy is—in inventions already extant (computers, telecommunications) to the items in the list hardly seems a balanced beginning to assess their relative importance. Moreover, the past always looks good—the further back the past, the better. The further-back past has been around longer than the only-recent past, and so has had greater opportunity to influence the world around us.

As an extreme, consider that at the end of 1999 a group of leading thinkers were asked what they considered the critical inventions of the millennium. Freeman Dyson, the renowned theoretical physicist, extended the choice to cover two millennia, and nominated dried grass:

> "The most important invention of the last two thousand years was hay. In the classical world of Greece and Rome and in all earlier times, there was no hay. Civilization could exist only in warm climates where horses could stay alive through the winter by grazing. Without grass in winter you could not have horses, and without horses you could not have urban civilization. Some time during the so-called dark ages, some unknown genius invented hay, forests were turned into meadows, hay was reaped and stored, and civilization moved north over the Alps. So hay gave birth to Vienna and Paris and London and Berlin, and later to Moscow and New York."
>
> (Freeman Dyson, 1999)

Very prosaic, minor changes can have profound effects, if they stay around long enough.

Gordon's list focuses on how the supply side of the economy has changed. Even (4) from his list is of interest, in his analysis, because it made the world smaller ("in a sense more profound than the Internet" [Gordon, 2000]), and really should include the postal system and public libraries leading, in turn, to literacy and reading.

In the analysis I develop here, by contrast, the New Economy is not only or even primarily a change in cost conditions on the supply side, then affecting the rest of the economy that uses that technology. Instead, it is the change in the nature of goods and services to become increasingly like knowledge. To draw out again the underlying theme, this is not just to say those goods and services are science and technology-intensive, but instead that their physical properties in consumption are the same as those of knowledge.

Such goods and services are becoming more important in two respects: first, as a fraction of total consumption; and second, in their increasingly direct contact with a growing number of consumers. To be concrete then, I include in this New Economy definition:

1. information and communications technology, including the Internet;
2. intellectual assets;
3. electronic libraries and databases;
4. biotechnology, i.e., carbon-based libraries and databases.

The common, distinctive features of these categories are, as earlier indicated: they represent goods and services with the same properties as knowledge; they are increasingly important in value added, and they represent goods and services with whom a growing number of final consumers are coming into direct contact. Quah [2001c] has called such goods *knowledge-products*. (This is partly to distinguish the issues here from those typically studied in, say, the "economics of information." The economic impact of a word-processing package, process-controller software, gene sequence libraries, database usage, or indeed the Open Source Software movement can be fruitfully considered without necessarily bringing in ideas such as moral hazard, adverse selection, or contract theory—the usual "economics of information" concerns.)

Categories (1)–(4) in my definition are, of course, not mutually exclusive. Intellectual assets (2) include both patentable ideas and computer software, with the latter obviously included in ICT (1) as well. But by intellectual assets, I refer also to software in its most general form, i.e., not just computer software, but also video and other digital entertainment, and recorded music. Finally, I prefer the term "intellectual assets" because it does not presume a social institution—such as patents and copyrights—to shape patterns of use, the way that, say, the term "intellectual property" does.

Viewing the New Economy as changes only on the supply or productivity side can give only part of the picture. This simplification is sometimes useful. Here it misleads. It generates an unhealthy obsession with attempting to measure the New Economy's productivity impacts. But even were that focus justified, shifting attention to the demand or consumption side helps raise other important and subtle new issues.

5 Knowledge in Consumption and Economic Growth

When the New Economy is identified with its potential supply-side impact, the critical links are threefold. First, the New Economy emphasizes knowledge, and knowledge raises productivity. Second, improved information allows tighter control of distribution channels, and with better-informed plans, inventory holdings can be reduced. Third, delivery lags have shortened so that productive factor inputs—capital and labor—can be reallocated faster and with less frictional wastage.

In the stylization from Section 2 and running through most of the discussion of Sections 3 and 4, knowledge and the New Economy are represented by A in the production function

$$Y = F(K, N, A) \tag{5}$$

(now ignoring the distinction between A and $\tilde{A}$ from Section 2). In the conventional analysis, controversy surrounds the quantitative dimension to this relation: Just how much does the New Economy affect A; what is the multiplier on A for Y?

What I have tried to argue above is that the New Economy is most usefully viewed as moving A from the production function (5) to be an argument in agents' preferences. The New Economy is a set of structural changes in the economy that have ended up inserting into utility functions objects that have the characteristics of A. Succinctly, if U represents a utility function, and C the consumption of other, standard commodities, then the New Economy is

$$U = U(C, A). \tag{6}$$

Quah [2001c] has studied a model where learning to use new A is costly in time, and therefore A affects consumers' budget constraint. The indirect utility function is then a reduced-form representation with exactly the features of (6).

That A disrespects geography and is infinitely expansible has profound implications for the behavior of consumers as well as producers. For one, transportation costs and end-user location can no longer satisfactorily explain what we see in patterns of economic geography [Fujita, Krugman, and Venables, 1999, Quah, 2000, 2001b]. For another, demand-side characteristics assume increased importance in determining market outcomes [Quah, 2001c].

To see this second point, consider two possibilities. First, suppose societies have established institutions—intellectual property rights (IPRs) like patents, say—that prevent driving the market price of knowledge products to zero marginal cost. Social institutions do this by making copying illegal for all but the IPR holder. The IPR holder then operates as a monopolist, delivering a quantity and charging a price determined entirely by the demand curve. Cost considerations determine profits, but not price or quantity—it is demand alone that determines market outcomes.

Second, suppose the opposite, i.e., that IPR institutions do not exist. Knowledge-products then are not protected by IPRs, but have incentive mechanisms for their creation and dissemination separated—as might happen, say, under systems of patronage or procurement [David, 1993]. Then infinite expansibility of the knowledge-product results in the supply side supplying as much as the demand side will bear, in a way divorced from the structure of costs in creation. Again, then, the ultimate determinant of market outcomes is the demand side.

These observations suggest the seemingly paradoxical conclusion that the most serious obstacle impeding progress in the New Economy might be consumer-side reluctance to participate in it. The advanced technologies around us might well turn out to be unproductive, not because of any defect inherent to them, but instead simply because we users have *chosen* not to use those technologies to best effect.

Statistical evidence in Jalava and Pohjola [2002] suggests two conclusions that bear on this hypothesis. First, in the US in the 1990s, ICT use provided benefits exceeding those from ICT production. Second, in Finland the contribution of ICT use to output growth has more than doubled in the 1990s.

Evidence of a different nature also sheds light on this demand-side hypothesis. Quah [2001c] describes a historical example where demand-side considerations mattered critically for technical progress. China at the end of the Sung dynasty in the 14th century was neither chockful of dot-com entrepreneurs nor brimming with Internet infrastructure. However, it did stand on the brink of an industrial revolution, four centuries before the Industrial Revolution of late 18th-century Western Europe.[2]

China produced more iron per capita in the 14th century than did Europe in the early 18th. Blast furnace and pig/wrought iron technologies were more advanced in China in 200 BCE than European ones in the 1500s. In China, iron's price relative to grain fell, within

[2] The analysis in Quah [2001c] had been originally motivated by my reading of Jones [1988] and Mokyr [1990]. Since those, Landes [1998] has further re-ignited controversy over the historical facts; see, e.g., Pomeranz [2000]. What matter for my discussion are not precise details on how much exactly China might have been ahead of Europe, when—within a 5-century span of time—catchup from one to the other occurred, or if the reversal was sudden or gradual. No one disputes that 14th-century China was technologically advanced nor that afterwards China lost significant technologies that it had earlier had. It is these that I draw on for the current discussion.

a century, to a third of its level at the end of the first millennium—a technological improvement not achieved in the West until the 18th century. Paper, gunpowder, water-powered spinning machines, block printing, and durable porcelain moveable type were all available in China between 400 to 1000 years earlier than in Europe. China's invention of the compass in 960 and ship construction using watertight bouyancy chambers made the Chinese the world's most technologically formidable sailors, by as much as five centuries ahead of those in the West.

China's lead over Europe along this wide range of technical fronts has long suggested to some that China should have seen an industrial revolution 400 years before Europe. Detractors from this view do, of course, have a point: Perhaps China wasn't ahead in every single dimension of technological prowess. But fretting over specific details on, for instance, whether the Chinese used gunpowder mostly for fireworks rather than warfare, or whether their understanding of technology was more bluesky science rather than engineering oriented (or indeed vice versa), seems niggardly—academic even—in light of the impressively broad array of demonstrated technical competencies in China. Yet, despite this, the subsequent five centuries saw dismal Chinese economic decline, rather than sweeping economic progress. Why?

One reasonable conjecture, it seems to me, is that China's failure to exploit its technical base was a failure of demand. In 14th-century China, technological knowledge was tightly controlled. Scholars and bureaucrats kept technical secrets to themselves; it was said that the Emperor "owned" time itself. The bureaucrats believed that disseminating knowledge about technology subverted the power structure and undermined their position. That might well have been so. But, as a result, no large customer base for technology developed, and technological development languished after its early and promising start.

Eighteenth-century European entrepreneurs, in contrast, were eager to use high-technology products such as the spinning jenny and the steam engine. Strong demand encouraged yet further technical progress. In 1781, to encourage sharper engineering effort, Matthew

Boulton wrote James Watt that "The people in London, Manchester, and Birmingham are steam-mill mad" [Pool, 1997, p. 126].

Great excitement across broad swathes of society fired the economic imagination and drove technology into immediate application, as described in equation (6). Europe took the lead; China languished.

I do not know if these demand-side considerations explain the paradoxes in section 4 above. But they suggest to me that perhaps we might have been looking in the wrong place all along for evidence on the New Economy.

6 Conclusion

Because the New Economy is so intertwined with Information and Communications Technology, we are primed to think of New Economy developments as nothing more than technology-driven, productivity-improving changes on the supply side. We then want New Economy developments to do what all technical progress has historically done. And we emerge disappointed when we find productivity has not skyrocketed, inflation has not forever disappeared, business downturns have not permanently vanished, and financial markets have not remained stratospheric.

This paper argued that the most profound changes in the New Economy are not productivity or supply-side improvements, but instead consumption or demand side changes. The paper summarized the case for the importance of technical progress in economic growth, argued why the New Economy differs and described how it is truly new, and drawn lessons from economic history to highlight potential pitfalls and dangers as the New Economy continues to evolve.

The technical appendix studied the role of human capital in economic growth, clarifying when human capital affects income levels but not growth rates and when it does so growth rates. It emphasized the distinction between human capital used for improving technology and human capital used in producing goods and services. Both matter and each separately can influence economic growth. The key finding is that endogenous growth results from the interaction of demand and supply features, contrasting sharply with economic growth emerging solely from production-side characteristics.

Policy implications from this analysis are twofold. The first involves measurement; the second, longer-term concerns. We might be looking in the wrong place—supply-side developments—for evidence on the impact of the New Economy. Demand-side changes—the behavior of consumers—might be where we need to document more carefully the New Economy. This is not to suggest a naive Keynesian-type conclusion that only the demand-side is important. Both supply and demand matter—in growth as in all other economic outcomes.

This altered emphasis in the ultimate source of economic growth leads in turn to the second, longer-term implication. If the profound changes are to be on the part of consumers, and those changes take a while to filter through to steady-state equilibrium growth, perhaps we should simply stay the course, have faith in the New Economy, and not obsess about measuring productivity changes in the short term. Skilled, discerning consumers and increased levels of broad-based education—for encouraging improved uses of technology, for raising labor productivity, for pushing back the frontiers of science and technology—are what will drive economic growth, one way or another.

7 Technical Appendix

This appendix studies the role of human capital in growth. It considers two classes of models: First, where human capital choices influence levels but not growth rates; second, where human capital choices influence steady-state growth rates. (To isolate the direct role of human capital, this Appendix does not consider the case where technology is influenced by inputs of human capital [e.g., Romer, 1990].)

We will see that, in general, it is not the details on the mechanism for accumulating human capital that matter for distinguishing the two different effects. Instead, it is the a priori assumption on how human capital enters the production function. Recall production function

(1),

$$Y = F(K, N, \tilde{A}),$$

and assume that $\tilde{A}$ comprises two components (h, A), where h is per worker human capital and A is technology proper.

In the first class of models—where human capital affects income levels but not growth rates—the total stock of human capital is a separate capital input, paralleling physical capital

$$Y = F(K, N, \tilde{A}) = F(K, H, NA), \qquad \text{with } H = hN. \qquad \text{(PF0)}$$

The second class of models has human capital attached explicitly to workers [e.g., Lucas, 1988, Rebelo, 1991, Uzawa, 1965]:

$$Y = F(K, N, \tilde{A}) = F(K, hNA) = F(K, HA). \qquad \text{(PF1)}$$

Human capital then augments labor the same way as does technology, and—we will see below—affects growth rates in steady state.[3]

Section 7.3 below will treat the first class of models, while section 7.4 the second.[4] Assume throughout that F, whether in (PF0) or (PF1), is constant returns to scale or homogeneous degree 1 (HD1).

The core of the material below is sufficiently well known that it appears in a number of textbooks [e.g., Barro and Sala-i-Martin, 1995]. However, the organization and emphases differ. Most important, this Appendix explicitly includes in the analysis technical

[3] To emphasize, in (PF0) the aggregate human capital stock H appears as factor input, additional to and separate from labor N. Such a production function is used in, e.g., Mankiw, Romer, and Weil [1992], where it takes the specific form $K^{\alpha}H^{\beta}(NA)^{1-\alpha-\beta}$, with $\alpha, \beta > 0$ and $\alpha + \beta < 1$.

[4] A third class of models—e.g., Jones [1998, Ch. 3] or Romer [2001, 3.8]—specifies production function (PF1) as in the second class of growth models, but then bounds the amount of human capital per worker that can be accumulated. The results then are the same as in levels-but-not-growth models, so this Appendix will incorporate them in section 7.3 below.

change, population growth, and different depreciation rates on human and physical capital. This is more than just book-keeping, as without them one is unable to examine the interaction between, say, technical change and human capital accumulation. Thus, we will see in section 7.4 below that with ongoing technical progress, when human capital contributes to growth its reduced-form relationship with income and physical capital shows a diminishing significance—even though were human capital absent, growth would fall. Put differently, even when human capital matters, an empirical researcher will discover no stable cointegrating relationship of it with physical capital and income.

Next, under the same conditions, we will see that, unlike physical capital, human capital must become progressively costlier to accumulate. As technology advances, incrementing the typical worker's stock of human capital will, in equilibrium , demand ever-greater resources. Thus the analysis in section 7.4 captures the intuition that technologically-advanced economies require substantial, costly training, even if measured human capital shows no large corresponding increases resulting from that training.

Turning from substantive to expositional considerations, the analysis including all the above additional possibilities and using general functional forms is conceptually easier than when applying just, say, Cobb-Douglas functions.[5] Without being any more complicated, the

[5] Using general functional forms—assuming, say, no more than constant returns to scale—clears up any lingering doubts about a possible knife-edge nature to the conclusions. And it prevents the usual explosive cascade of exponents in α's and $(1-\alpha)$'s in the exposition where descriptions such as "the net marginal product of physical capital" then become ambiguously aliased into a whole range of other possible interpretations. As just one example, equation (5.13) on p. 180 in Barro and Sala-i-Martin [1995] uses ν to mean two logically different things—one a Lagrange multiplier, the other an allocation share. Later on, just before equation (5.18) the authors use a "significant amount of algebra" (omitted) to obtain a critical result. Of course, their accurate and powerful economic intuition gets them

development in section 7.4 includes as convenient special cases a number of well-known models of growth with human capital.

Although all the material below is technically more difficult than that in the text, sections 7.1–7.3 remain relatively less formal and rigorous. Section 7.4, on the other hand, requires greater precision in the statements, and so uses a much more formal (definition/theorem/proof) presentation.

7.1 General setup

As far as possible I use the following notational convention: Uppercase letters denote economy-wide quantities; lowercase, their per capita or per worker versions. the Roman alphabet denotes observable economic timeseries; Greek, parameters or coefficients. The more complicated is the symbol (tildes, underscores) the less easily is what it denotes found in national income accounts. Necessarily, however, there will be some exceptions: the state of technology, A, cannot be directly measured, but the symbol is so much used in the literature, calling it something else would only confuse.

Assume

$$\dot{N}/N = \nu \geq 0, \ N(0) > 0, \text{ and} \tag{7}$$

$$\dot{A}/A = \xi \geq 0, \ A(0) > 0, \tag{8}$$

i.e., the labor force and technology evolve at constant proportional growth rates. Endogenous population and technology models alter (7) and (8), respectively, setting out mechanisms and incentives for determining $\dot{N}/N$ and $\dot{A}/A$. This Technical Appendix focuses on human capital, however, and so we will retain (7) and (8).

Let the labor force equal the population, and define per worker output and capital:

$$y \stackrel{\text{def}}{=} Y/N \quad \text{and} \quad k \stackrel{\text{def}}{=} K/N,$$

to the correct answer in any case. My exposition below, conversely, never uses any significant amount of algebraic manipulation.

and their technology-adjusted versions:

$$\tilde{y} \stackrel{\text{def}}{=} Y/NA \quad \text{and} \quad \tilde{k} \stackrel{\text{def}}{=} K/NA. \tag{9}$$

In this formulation y is simultaneously also per capita income as well as average labor productivity. Following the same convention also define H to denote total human capital $H \stackrel{\text{def}}{=} h \times N$, and the technology-adjusted version

$$\tilde{h} = H/NA = h/A. \tag{10}$$

(This last definition will turn out to be useful only in section 7.3 below.) Aggregate physical and human capital depreciate at instantaneous flow rates δ_K and δ_H respectively.

To fix ideas, section 7.2 establishes the Solow neoclassical growth model in our notation. Section 7.3 extends this to where human capital affects levels but not growth rates. To clarify the connection to the Solow model, the discussion here follows Mankiw, Romer, and Weil [1992] in assuming ad hoc accumulation in physical and human capital. This is not crucial though: An optimizing Cass-Koopmans analysis obtains the same results. What matters is assuming the production function (PF0) rather than (PF1).

Section 7.4 turns to an optimizing framework, and shows how switching between production functions (PF0) and (PF1) allows human capital to affect growth rates.

7.2 Neoclassical growth

Following Solow [1956], let physical capital K evolve as:

$$\dot{K} = \tau_K Y - \delta_K K, \quad K(0) > 0, \ \tau_K \in (0, 1), \ \text{and } \delta_K > 0, \tag{11}$$

with $\dot{K}$ denoting K's time derivative and τ_K the savings or investment rate. It will be useful to define the deepening constant

$$\zeta_K \stackrel{\text{def}}{=} (\nu + \xi) + \delta_K > 0.$$

In this first model take h to be constant. Specialize production function (1) to the constant returns to scale function

$$Y = F(K, NA). \tag{12}$$

A *balanced-growth steady state* (BGSS) is a collection of timepaths

$$\{\, y(t),\ k(t):\ t \,\}$$

such that $\dot{y}/y$ and k/y are constant in time. An *equilibrium* is a collection of time paths

$$\{\, y(t),\ k(t):\ t \in [0, \infty) \,\}$$

satisfying equations (11)–(12). A *BGSS equilibrium* is a BGSS satisfying equations (11)–(12).

To understand the properties of equilibrium, divide (12) throughout by NA to obtain

$$\tilde{y} = F(\tilde{k}, 1) \stackrel{\text{def}}{=} f(\tilde{k}).$$

Using (7)–(9) in equation (11) then gives

$$\dot{\tilde{k}}/\tilde{k} = \tau_K \times f(\tilde{k})\tilde{k}^{-1} - \zeta_K, \quad \tilde{k}(0) > 0. \tag{13}$$

Under standard economic assumptions on $f = F(\cdot, 1)$ the differential equation (13) implies that $\tilde{k}$ converges from any initial point $\tilde{k}(0)$ to the unique solution of

$$f(\tilde{k})\tilde{k}^{-1} = \zeta_K \times \tau_K^{-1}.$$

Thus in equilibrium at BGSS, capital per worker

$$k = K/N = \tilde{k}A$$

grows at the constant rate $\dot{A}/A = \xi$. Output per worker

$$y = Y/N = \frac{F(K, NA)}{N} = f(\tilde{k})A$$

converges similarly to a unique time path that grows in BGSS at the same constant, exogenously-given rate ξ.

Summarizing, in this model with h constant, in BGSS the growth rate of per capita income equals that for technology.

7.3 Two Models of Growth with Human Capital: Levels but not Growth Rates

This section studies two different models for human capital in economic growth. In the first h human capital per worker increases without bound; in the second h remains finite in steady state. Both models, however, predict that choices on human capital influence only the level of output per worker. Steady-state growth rates will remain fixed at that for technology, $\dot{A}/A = \xi$, as in the model above.

First, [following Mankiw, Romer, and Weil, 1992] suppose production function (1) now takes the form of equation (PF0)

$$Y = F(K, H, NA),$$

with constant returns to scale in all three arguments.

Parallel with physical capital accumulation (11) let H evolve as:

$$\dot{H} = \tau_H Y - \delta_H H, \quad H(0) > 0,\ 0 < \tau_K + \tau_H < 1,\ \text{and}\ \delta_H > 0, \quad (14)$$

with τ_H the rate of investment in human capital. Human capital increases from resources spent on it—schooling, for example—and depreciates at a constant proportional rate. Investment on human capital is a constant fraction of income. Equation (14) allows $h = H/N$ to increase without bound. Indeed, in the equilibrium described below, h will diverge to infinity.

A *balanced-growth steady state* (BGSS) is a collection of time paths

$$\{\, y(t),\ k(t),\ h(t) :\ t \,\}$$

such that $\dot{y}/y$, k/y, and h/y are constant in time. An *equilibrium* is a collection of time paths

$$\{\, y(t),\ k(t),\ h(t) :\ t \in [0, \infty) \,\}$$

satisfying equations (PF0), (11), and (14). A *BGSS equilibrium* is a BGSS satisfying equations (PF0), (11), and (14).

To see the properties of equilibrium, rewrite (PF0) in technology-adjusted per capita form:

$$\tilde{y} = F(\tilde{k}, \tilde{h}, 1) \stackrel{\text{def}}{=} f(\tilde{k}, \tilde{h}).$$

As with the definition of ζ_K, let

$$\zeta_H \stackrel{\text{def}}{=} (\nu + \xi) + \delta_H > 0.$$

Then just as we obtained (13) for the neoclassical growth model, we have

$$\dot{\tilde{k}}/\tilde{k} = \tau_K \times f(\tilde{k}, \tilde{h})\tilde{k}^{-1} - \zeta_K \quad \text{and} \tag{15}$$

$$\dot{\tilde{h}}/\tilde{h} = \tau_H \times f(\tilde{k}, \tilde{h})\tilde{h}^{-1} - \zeta_H. \tag{16}$$

The pair of equations (15)–(16) implies a steady state in $(\tilde{k}, \tilde{h})$ satisfying

$$f(\tilde{k}, \tilde{h})\tilde{k}^{-1} = \zeta_K \times \tau_K^{-1} \quad \text{and} \quad f(\tilde{k}, \tilde{h})\tilde{h}^{-1} = \zeta_H \times \tau_H^{-1}. \tag{17}$$

Because F is HD1, function f will not be. Equation (17) then has a full-rank Jacobean and thus determines a unique pair $(\tilde{k}, \tilde{h})$. From (15)–(16) the vector $(\tilde{k}, \tilde{h})$ globally converges to the unique solution of (17). (Note that were f HD1, then the Jacobean of (17) would be singular. Then, if a solution existed, equation (17) would determine not $(\tilde{k}, \tilde{h})$ separately, but only their ratio.)

A useful interpretation of this result derives from recognizing that the left side of equations (17) are the average products of physical and human capital respectively, holding fixed technology-augmented labor NA. When F is HD1 those average products decline to zero even when the other capital input rises proportionally. Although no explicit optimization informs the accumulation decision, the hypothesized savings functions imply slowing accumulation, (15) and (16), with declining average products. Therefore, $\tilde{k}$ and $\tilde{h}$ do not grow indefinitely but instead converge to unique, finite values.

From the dynamics of $(\tilde{k}, \tilde{h})$ per capita income $y = Y/N$ converges too to a unique steady-state path that grows at rate $\dot{A}/A = \xi$. This is exactly as in the neoclassical growth model in section 7.2. The level of the steady-state path in y varies: For instance, it increases in steady-state $\tilde{h}$, which could be caused by, among other possibilities, a higher investment rate τ_H on human capital. However, to repeat, the growth rate of per capita income remains entirely unaffected, equalling ξ always.

The second model—following Jones [1998, Ch. 3] or Romer [2001, 3.8]—again leaves unaffected the key growth predictions of the neoclassical model. Suppose as before that h increases through investment, or through education in particular. However, while education can raise a worker's human capital with no diminishing returns, the amount of time that a worker can devote to education is bounded. Then even if all the worker's lifetime were spent on education, her human capital can, at most, reach some finite upper limit. Specifications that embody this implication include many typically used in labor economics. For instance,

$$h(s) = h_0 e^{\psi s}, \quad s \in [0, 1];\ h_0,\ \psi > 0,$$

with s denoting the fraction of time spent in schooling, implies a constant proportional effect for education

$$\frac{h'(s)}{h(s)} = \psi$$

(usually taken to equal 0.10 [e.g., Jones, 1998, Ch. 3]). But then even as s increases to its upper limit of 1, per worker human capital h approaches only at most $h_0 e^{\psi} < \infty$.

Use production function (PF1)

$$Y = F(K, NhA),$$

assumed to satisfy constant returns to scale, so that

$$\tilde{y} = F(\tilde{k}, h).$$

Denote the solution to a worker's optimization problem on education choice by the constant $\bar{s}$, so that the corresponding human capital level is

$$\bar{h} = h_0 e^{\psi \bar{s}} \in [h_0, h_0 e^{\psi}].$$

Then, using (PF1), (7), and (8), the physical capital accumulation equation (11) becomes

$$\dot{\tilde{k}}/\tilde{k} = \tau_K \times F(\tilde{k}, \bar{h})\tilde{k}^{-1} - \zeta_K. \tag{18}$$

But the behavior $\tilde{k}$ from (18) is exactly the same as that from (13), up to a shift factor in levels, induced by $\bar{h}$. Thus, again, $\tilde{k}$ converges from any initial point $\tilde{k}(0)$ to the unique solution of

$$F(\tilde{k}, \bar{h})\tilde{k}^{-1} = \zeta_K \times \tau_K^{-1}.$$

Under standard assumptions on F, the steady state level of $\tilde{k}$ is increasing in $\bar{h}$, and thus in $\bar{s}$. However, the steady growth rate of capital per worker $k = K/N$ is simply $\dot{A}/A = \xi$, independent of $\bar{s}$. Output per worker $y = Y/N$ inherits the same properties of global convergence and invariant steady-state growth rate. Thus, while levels of output per worker increase with education, growth rates are unchanged.

7.4 Growth with Human Capital

The models thus far have used arbitrary accumulation processes (11) and (14) and either production function (PF0) or production function (PF1) with bounded per worker human capital. In all cases per capita income growth occurred only from technical progress $\dot{A}/A = \xi$. This section adopts production function (PF1) and allows per worker human capital to grow without bound. For completeness, the discussion also takes an optimizing approach to accumulating physical and human capital, in place of the arbitrary (11) and (14). It is easy to see, however, that replacing (PF1) with (PF0) would restore the growth results of the previous section.

The analysis in this section includes, in a consistent notation, special cases such as the one-sector model in Barro and Sala-i-Martin [1995, 5.1] and the two-sector model in Rebelo [1991]—and therefore the Lucas model [Lucas, 1988] as well.

A social planner for the economy will solve a welfare optimization program that can then be decentralized with markets. Let C denote aggregate consumption so that, as above,

$$c = C/N \quad \text{and} \quad \tilde{c} = c/A$$

respectively define per capita and technology-intensive, per capita consumption.

Everyone in the economy is identical and infinitely-lived. The representative agent discounts the future at constant rate $\rho > 0$, and has instantaneous utility $U(c)$, where $U' > 0$, $U'' < 0$, and

$$U'(c) \to \infty \text{ as } c \to 0.$$

Social welfare is

$$\int_0^\infty e^{-\rho t} N(t) U(c(t))\, dt = N(0) \times \int_0^\infty e^{-(\rho-\nu)t} U(c(t))\, dt$$

Define

$$R(c) = \frac{-cU''(c)}{U'(c)} > 0.$$

If U has the CRRA form

$$U(c) = \frac{c^{1-\theta} - 1}{1 - \theta}, \qquad \theta > 0,$$

then $R(c) = \theta$ constant. However, to clarify the role that utility function U plays in the growth analysis, I will write R in general and assume it constant when necessary, rather than introduce a new parameter θ.

Assume the production functions (PF0) and (PF1) are everywhere continuously differentiable. Denote partial derivatives with respect to their j-th argument by F_j. As mnemonic, write $F_K = F_1$ and $F_H = F_2$, noting that in general $F_H \neq \partial F/\partial H$. For instance, in (PF1), $\partial F/\partial H$ equals $F_2 A = F_H A$. Since F is HD1, each F_j is HD0. The technology-adjusted per capita versions of (PF0) and (PF1) are, respectively,

$$\tilde{y} = F(\tilde{k}, \tilde{h}, 1) \stackrel{\text{def}}{=} f(\tilde{k}, \tilde{h}) \quad \text{and}$$
$$\tilde{y} = F(\tilde{k}, h) \stackrel{\text{def}}{=} f(\tilde{k}, h).$$

The function f corresponding to (PF0) is decreasing returns to scale. That for (PF1) has h rather than $\tilde{h}$ as argument, and retains the HD1 property—it is the same function as F, but I will write f to treat (PF0) and (PF1) simultaneously. I will also carry along the mnemonic f_K and f_H for partial derivatives in the first and second arguments; again, $f_K = \partial f/\partial\tilde{k} \neq \partial f/\partial K$. Second partial derivatives will, analogously, be denoted f_{KK} and so on. For now assume only that all first partial derivatives are non-negative; they might or might not satisfy Inada-type conditions. Because further assumptions on f vary with the model, I will restrict f as necessary below rather than here.

Denote by I_K aggregate investment devoted to changing physical capital, and by I_H that for changing human capital. Here, I_H excludes learning-by-doing but includes formal schooling and training—activities that draw resources away from consumption and physical capital investment. Assume that I_K, subject to being non-negative, can be costlessly transformed with consumption C, so both are measured in the same numeraire units. By contrast, private agents can trade I_H only at price q, not necessarily unity. The aggregate economy might, of course, face additional constraints on I_H—the two, I_K and I_H, might never be directly tradeable—but this q interpretation allows a consistent treatment of a range of different models. The usual per capita and technology-adjusted versions are:

$$\begin{aligned} i_K &\stackrel{\text{def}}{=} I_K/N \qquad & \tilde{i}_K &\stackrel{\text{def}}{=} i_K/A; \\ i_H &\stackrel{\text{def}}{=} I_H/N \qquad & \tilde{i}_H &\stackrel{\text{def}}{=} i_H/A. \end{aligned}$$

The national income identity is

$$Y = C + I_K + I_H \cdot q,$$

with technology-adjusted per capita version

$$\tilde{y} = \tilde{c} + \tilde{i}_K + \tilde{i}_H \cdot q.$$

Since $\tilde{y} = f(\tilde{k}, h)$, when q is positive this equation describes the tension between consumption and accumulating physical capital on the one hand and accumulating human capital on the other. Models where H increases through, say, learning by doing significantly depart from such a tension.

Physical capital accumulation follows:

$$\dot{K} = I_K - \delta_K K \implies \dot{\tilde{k}} = \tilde{\imath}_K - \zeta_K \tilde{k}. \tag{19}$$

How H depends on I_H will vary, depending on what is being studied in a particular model, and won't necessarily be exactly as the relation above between $\dot{K}$ and I_K.

Definition 7.1 *A balanced-growth steady state (BGSS) is a collection of time paths*

$$\{\, y(t),\ c(t),\ k(t),\ h(t),\ q(t) :\ t \,\}$$

such that $\dot{y}/y$, $\dot{h}/h$, c/y, k/y, *and* q *are invariant in time.*

The definition implies $\dot{c}/c = \dot{k}/k = \dot{y}/y$. However, the relation between h and y is left unspecified: this will matter below. Write $g \overset{\text{def}}{=} \dot{y}/y$ for the growth rate of per capita income or, equivalently, worker productivity in BGSS.

Without pretending to replace an equilibrium analysis, we can already conjecture at the formal results to come. If F is either (PF0) or (PF1) with h bounded, then BGSS has:

$$\dot{y}/y = \dot{c}/c = \dot{k}/k = \xi = \dot{A}/A.$$

When F is (PF0), then we also have in BGSS $\dot{h}/h = \xi$ so that h/y is invariant. Growth comes only from technical progress: No other outcome is possible with f displaying decreasing returns to scale.[6]

If, however, F is (PF1) then BGSS potentially has

$$\dot{y}/y = \dot{c}/c = \dot{k}/k = \dot{h}/h + \xi,$$

[6] This overstates somewhat. Even with F given by (PF0), BGSS with $\dot{y}/y > \xi$ might be possible if h/y grows without bound. However, for consumption to remain bounded from below given the national income identity, h accumulation must then become progressively less resource-demanding. This seems implausible.

so that the economy's growth rate g exceeds both $\dot{h}/h$ and $\dot{A}/A$. We of course need a model still to determine g in equilibrium, but regardless of g's value, with $\xi = \dot{A}/A > 0$, the above already implies that in BGSS:

1. The ratios of human capital to income and to physical capital, h/y and h/k (or equivalently H/Y and H/K), converge to zero;

2. Human capital must become increasingly costly to produce from I_H.

Thus, even with human capital mattering critically for growth, it will trend neither with income nor capital: In this model the failure to find a stable cointegrating relationship between human capital and income is evidence *for* rather than against the importance of human capital in growth.

To understand the second implication, suppose it failed and instead a counterpart to equation (19) held:

$$\dot{\tilde{h}} = \tilde{i}_H - (\nu + \xi + \delta_H)\tilde{h} \iff \dot{h} = i_H - (\nu + \delta_H)h,$$

or

$$\tilde{i}_H = \left(\dot{\tilde{h}}/\tilde{h} + [\nu + \xi + \delta_H]\right)\tilde{h}.$$

Since f is HD1, BGSS has

$$\tilde{y} = f(\tilde{k}, h) \implies \dot{h}/h = \dot{\tilde{y}}/\tilde{y} = \dot{\tilde{k}}/\tilde{k} = g - \xi,$$

so that

$$\dot{\tilde{h}}/\tilde{h} = \dot{h}/h - \xi = g - 2\xi.$$

But then in BGSS the right side of the national income identity

$$\tilde{y} = \tilde{c} + \left(\dot{\tilde{k}}/\tilde{k} + \zeta_K\right)\tilde{k} + \left(\dot{h}/h + [\nu + \xi + \delta_H]\right)\tilde{h} \cdot q$$

cannot grow at $g - \xi$, the growth rate of the left side.

Instead, what is needed is something like

$$\dot{h} = i_H/A - (\nu + \delta_H)h. \tag{20}$$

In words, the contribution of i_H to $\dot{h}$ becomes progressively difficult as A rises.[7]

From the discussion we have:

Proposition 7.2 *If production F is* (PF0) *then BGSS has*

$$\dot{h}/h = \dot{y}/y = \xi.$$

If, however, production F is (PF1) *then BGSS has*

$$\dot{h}/h = \dot{y}/y - \xi.$$

The specification above specializes to several well-known cases. With (PF0), setting $q = 1$ and $\dot{H} = I_H - \delta_H H$, and requiring

$$\tilde{c} + \tilde{i}_K + \tilde{i}_H \leq f(\tilde{k}, \tilde{h})$$

recovers an optimizing version of the model in Mankiw, Romer, and Weil [1992]. Specifying (PF1) and bounding $\tilde{h}$ gives the model in Jones [1998, Ch. 3] and Romer [2001, 3.8].

Using (PF1) and fixing $q = 1$ gives the one-sector growth model in Barro and Sala-i-Martin [1995, 5.1]. Freeing up q but requiring that for some HD1 (sub) production functions $\mathcal{F}$, $\mathcal{G}$ and allocation shares $s_K, s_H \in [0, 1]$ we have:

$$\begin{aligned} F(K, HA) &= \mathcal{F}\big(s_K K, s_H HA\big) \\ &\qquad\qquad + q \cdot \mathcal{G}\big([1 - s_K]K, [1 - s_H]HA\big) \\ C + I_K &\leq \mathcal{F}\big(s_K K, s_H HA\big) \\ I_H &\leq \mathcal{G}\big([1 - s_K]K, [1 - s_H]HA\big) \end{aligned}$$

gives the model in Rebelo [1991]. As before call the partial derivatives $\mathcal{F}_K$, $\mathcal{F}_H$, and so on. Then, restricting further $\mathcal{G}_K = 0$ gives the Lucas model. Since this case bears specific interest, the discussion below will take care to account for it with $s_K = 1$ at the corner optimum.

Hereafter, consider the following:

[7] Alternatively, the definition of BGSS in Defn. 7.1 to require invariant q can be modified appropriately.

Definition 7.3 *Assume production is given by* (PF1) *and human capital accumulation by* (20)*. Suppose the economy solves the social welfare optimization program:*

$$\sup_{\{\tilde{c},\tilde{i}_K,\tilde{i}_H,q;s_K,s_H\}} \int_0^\infty U(\tilde{c}(t)A(t))e^{-(\rho-\nu)t}\,dt \tag{21}$$

$$\text{s.t.} \quad \tilde{c},\ \tilde{i}_K,\ \tilde{i}_H,\ q \geq 0 \text{ and } 0 \leq s_K,\ s_H \leq 1$$

$$\dot{\tilde{k}} = \tilde{i}_K - \zeta_K \tilde{k} \tag{22}$$

$$\dot{h} = \tilde{i}_H - (\nu + \delta_H)h \tag{23}$$

$$\tilde{c} + \tilde{i}_K + q\tilde{i}_H \leq f(\tilde{k}, h) = \tilde{y} \tag{24}$$

$$f(\tilde{k}, h) = \mathcal{F}\big(s_K\tilde{k}, s_H h\big) + q \cdot \mathcal{G}\big([1-s_K]\tilde{k}, [1-s_H]h\big) \tag{25}$$

and either

$$\tilde{i}_H \leq \mathcal{G}\big([1-s_K]\tilde{k}, [1-s_H]h\big) \tag{26a}$$

or

$$q = 1. \tag{26b}$$

A BGSS equilibrium is a BGSS together with pair (s_K, s_H) *invariant in time solving* (21)–(26).

When (26a) holds, (24) and (25) imply

$$\tilde{c} + \tilde{i}_K \leq \mathcal{F}\big(s_K\tilde{k}, s_H h\big),$$

i.e., the technology for producing I_H differs from that for producing $C+I_K$. Call $C+I_K$ *goods*, so that $\mathcal{F}$ and $\mathcal{G}$ describe goods production and human capital production respectively.

To analyze equilibrium define for non-negative Lagrange multipliers

$$(m_K, m_H, m_C, m_Y, m_I, m_q)$$

the Hamiltonian:

$$
\begin{aligned}
\mathcal{H} = e^{-(\rho-\nu)t}\Big[& U(\tilde{c}A) \\
& + (\tilde{i}_K - \zeta_K \tilde{k})m_K + (\tilde{i}_H - (\nu + \delta_H)h)m_H \\
& - \left(\tilde{c} + \tilde{i}_K + q\tilde{i}_H - f(\tilde{k}, h)\right) m_C \\
& - \Big(f(\tilde{k}, h) - \mathcal{F}\big(s_K\tilde{k}, s_H h\big) \\
& \qquad - q \cdot \mathcal{G}\big([1 - s_K]\tilde{k}, [1 - s_H]h\big)\Big) m_Y \\
& - \left(\tilde{i}_H - \mathcal{G}\big([1 - s_K]\tilde{k}, [1 - s_H]h\big)\right) m_I \\
& - (1 - q)m_q\Big].
\end{aligned}
$$

The first-order conditions at an optimum are:

$$\frac{\partial \mathcal{H}}{\partial \tilde{c}} = 0 \implies AU' - m_C = 0 \tag{27}$$

$$\frac{\partial \mathcal{H}}{\partial \tilde{i}_K} = 0 \implies m_K - m_C = 0 \tag{28}$$

$$\frac{\partial \mathcal{H}}{\partial \tilde{i}_H} = 0 \implies m_H - (q \cdot m_C + m_I) = 0 \tag{29}$$

$$\frac{\partial \mathcal{H}}{\partial s_K} \lesseqqgtr 0 \implies \mathcal{F}_K \cdot m_Y - (q \cdot m_Y + m_I)\mathcal{G}_K \lesseqqgtr 0 \tag{30}$$

$$\frac{\partial \mathcal{H}}{\partial s_H} \lesseqqgtr 0 \implies \mathcal{F}_H \cdot m_Y - (q \cdot m_Y + m_I)\mathcal{G}_H \lesseqqgtr 0 \tag{31}$$

$$\frac{\partial \mathcal{H}}{\partial q} = 0 \implies -\tilde{i}_H \cdot m_C + \mathcal{G} \cdot m_Y + m_q = 0 \tag{32}$$

and

$$
\begin{aligned}
& \frac{\partial \mathcal{H}}{\partial \tilde{k}} = -\frac{d}{dt}\left[e^{-(\rho-\nu)t} m_K(t)\right] \\
\implies & f_K \cdot m_C + (1 - s_K)\mathcal{G}_K \cdot m_I - \zeta_K \cdot m_K \\
& \qquad - (f_K - s_K \cdot \mathcal{F}_K - q[1 - s_K]\mathcal{G}_K)\, m_Y \\
& \qquad\qquad = [(\rho - \nu) - \dot{m}_K/m_K]\, m_K
\end{aligned}
\tag{33}
$$

with, finally,

$$
\begin{aligned}
&\frac{\partial \mathcal{H}}{\partial h} = -\frac{d}{dt}\left[e^{-(\rho-\nu)t} m_H(t)\right] \\
\Longrightarrow\ & f_H \cdot m_C + (1-s_H)\mathcal{G}_H \cdot m_I - (\nu+\delta_H)\cdot m_H \\
&\qquad - (f_H - s_H \cdot \mathcal{F}_H - q[1-s_H]\mathcal{G}_H)\, m_Y \\
&\qquad\qquad = [(\rho-\nu) - \dot{m_H}/m_H]\, m_H. \qquad (34)
\end{aligned}
$$

Conditions (30) and (31) work in the obvious way if it is optimal to set s_K or s_H to their boundary values at either 0 or 1. For instance, in the Lucas case, $\mathcal{G}_K = 0$ so that share s_K is optimally set to 1 whereupon (30) becomes the inequality $\mathcal{F}_K \cdot m_Y > (q \cdot m_Y + m_I)\mathcal{G}_K$. Related, when q is not restricted to 1, equation (32) fails and so provides no additional restriction in the solution. Finally, conditions (27)–(29) have been stated as equalities rather than more generally because all equilibria of interest below will have $\tilde{c}$, $\tilde{i}_K$, and $\tilde{i}_H$ positive.

In these first-order conditions the price q only ever appears together with the Lagrange multiplier m_I. When q is not restricted to 1 (as in (26b)), the pair (q, m_I) are then determined only jointly, not individually. This implies that the level of measured output y in (24)–(25) is indeterminate as well, although its growth rate might be uniquely tied down. We will see this in 7.4b. below. The economics is straightforward: When (26a) is activated the economy physically cannot instantaneously transform resources between goods and human capital. A range of possible prices q can then be consistent with the observed outcomes in goods and human capital production. Put another way, agents' decisions are optimally at a corner solution. Then, up to limits, the Lagrange multiplier m_I on (26a) moves appropriately to compensate for alternative settings of q. As the market price q varies, again up to limits, optimal decisions remain unaltered, with m_I transparently adjusting to maintain equilibrium. Being only a shadow value, m_I is invisible to GDP accounting, whereas q appears explicitly. Setting q to zero recovers what Barro and Sala-i-Martin [1995, Ch.5] call "narrow output"; setting q to its maximum value within the feasible range, "broad output".

7.4a. Identical technologies for human capital and goods

When $\tilde{i}_H$ is freely interchangeable with c and $\tilde{i}_K$, set $m_I = m_Y = 0$ and $m_q > 0$. Then conditions (30)–(31) are irrelevant and $q = 1$ so that first-order conditions (29), (32), (33), and (34) become respectively

$$\begin{aligned} m_H - m_C &= 0 \\ -\tilde{i}_H \cdot m_C + m_q &= 0 \\ f_K \cdot m_C - \zeta_K \cdot m_K &= [(\rho - \nu) - \dot{m_K}/m_K]\, m_K \\ f_H \cdot m_C - (\nu + \delta_H) \cdot m_H &= [(\rho - \nu) - \dot{m_H}/m_H]\, m_H. \end{aligned}$$

Calling m the common value $m_C = m_K = m_H$ and log-differentiating (27) with respect to time, the collection of first-order conditions collapses to:

$$\dot{m}/m = \rho + \delta_K + \xi - f_K = \rho + \delta_H - f_H \tag{35}$$

$$\dot{\tilde{c}}/\tilde{c} = [(1 - R(\tilde{c}A))\xi - \dot{m}/m]\, R(\tilde{c}A)^{-1}. \tag{36}$$

From the HD0 property of f_K and f_H, equation (35) implies

$$f_H(1, h/\tilde{k}) - f_K(1, h/\tilde{k}) = (\delta_H - \delta_K) - \xi \tag{37}$$

so that $h/\tilde{k}$ is constant in time,[8] depending only on δ_H, δ_K, ξ, and f.

Significantly, (37) holds everywhere in equilibrium, not only in BGSS. Thus, the model does not in general admit an equilibrium—BGSS or otherwise—with arbitrary initial conditions in K and H. At arbitrary initial levels of physical and human capital the implied marginal products need not line up as required in (37). In this model physical and human capital can change only gradually, and so cannot be instantaneously adjusted to meet marginal productivity conditions. But when (37) does hold at a particular value of $h/\tilde{k}$ then equation (35) gives $\dot{m}/m$, which in turn determines $\dot{\tilde{c}}/\tilde{c}$ through (36).

[8] When $\delta_H - \delta_K = \xi = 0$ and $f(\tilde{k}, h) = \tilde{k}^{\alpha} h^{1-\alpha}$, then (37) gives $h/\tilde{k} = (1-\alpha)^{-1}\alpha$. This special case is, however, neither more insightful nor easier to obtain than the general case considered in the text of this paper. More important, it is strictly misleading in hiding the dependence of equilibrium $h/\tilde{k}$ on model parameters.

That this gives the growth rate of the economy overall is shown in the following Proposition, which also summarizes the discussion thus far and provides further details:

Proposition 7.4 *Assume in Defn. 7.3 that $\tilde{\imath}_H$ is freely interchangeable with c and $\tilde{\imath}_K$. Suppose that $R(\tilde{c}A)$ is constant and f satisfies*

$$\begin{aligned} \forall\text{fixed } h\text{:}\quad & f_K(\tilde{k}, h) \to 0 \text{ as } \tilde{k} \to \infty, \\ & f_K(\tilde{k}, h) \to \infty \text{ as } \tilde{k} \to 0, \\ & f_{KK} < 0, \\ \forall\text{fixed } \tilde{k}\text{:}\quad & f_H(\tilde{k}, h) > (\delta_H - \delta_K) - \xi \text{ uniformly in } h \text{ on} \\ & \qquad\qquad\qquad \text{a neighborhood of zero,} \\ \text{and}\quad & f_{HH} \leq 0. \end{aligned}$$

Then, for any given initial value $\tilde{k}^* > 0$, BGSS equilibrium exists and is unique, with the ratio $h/\tilde{k}$ taking a value $\underline{h}^*$ constant in time and independent of $\tilde{k}^*$. The BGSS growth rate is

$$\begin{aligned} g &= [f_K(1, \underline{h}^*) - (\rho + \delta_K)]\, R^{-1} \\ &= [(f_H(1, \underline{h}^*) + \xi) - (\rho + \delta_H)]\, R^{-1}, \end{aligned} \tag{$\mathfrak{G}1$}$$

bounded from above by the average product of K in producing goods $(C + I_K)$ net of per capita depreciation. If $\xi > 0$ then the ratios of human capital to income and to physical capital converge to zero.

Proof *By the assumptions on f, the left side of equation (37), $f_H - f_K$, exceeds its right side at $h/\tilde{k} = 0$ and strictly declines monotonically without bound. Thus (37) admits a unique positive finite solution $\underline{h}^*$ in $h/\tilde{k}$. Using $\underline{h}^*$ in (35) and plugging the result into (36) gives the growth rate $\dot{\tilde{c}}/\tilde{c}$, varying with $h/\tilde{k}$ but not $\tilde{k}$ itself. The definition of BGSS then gives*

$$\begin{aligned} \dot{\tilde{y}}/\tilde{y} = \dot{\tilde{k}}/\tilde{k} = \dot{\tilde{c}}/\tilde{c} &= [(1 - R)\xi - \dot{m}/m]\, R \\ &= [(1 - R)\xi - (\rho + \delta_K + \xi - f_K(1, \underline{h}^*)]\, R \end{aligned}$$

Moreover, $h/\tilde{k}$ *constant implies also* $\dot{h}/h = \dot{\tilde{k}}/\tilde{k} = \dot{\tilde{c}}/\tilde{c}$. *Then*

$$\begin{aligned} g &= \dot{y}/y = \dot{\tilde{y}}/\tilde{y} + \xi = \dot{\tilde{c}}/\tilde{c} + \xi \\ &= [(1-R)\xi - \dot{m}/m]\, R^{-1} + \xi = [\xi - \dot{m}/m]R^{-1} \\ &= [f_K(1, \underline{h}^*) - (\rho + \delta_K)]\, R^{-1} \\ &= [(f_H(1, \underline{h}^*) + \xi) - (\rho + \delta_H)]\, R^{-1}, \end{aligned} \qquad \text{from (35)}$$

verifying ($\mathfrak{G}1$). *Since* $\dot{\tilde{k}}/\tilde{k} = g - \xi$ *in BGSS, we also have* $\tilde{k}(t) = \tilde{k}^* e^{(g-\xi)t}$. *To see this establishes an equilibrium, note that equations* (22)–(24) *imply:*

$$\tilde{y} = \tilde{c} + \left(\dot{\tilde{k}}/\tilde{k} + \zeta_K\right)\tilde{k} + \left(\dot{h}/h + (\nu + \delta_H)\right)h,$$

so that $(\tilde{k}, \underline{h}^*)$ *then determine the other endogenous variables:*

$$\begin{aligned} \tilde{i}_H &= \left(\dot{h}/h + [\nu + \delta_H]\right)\underline{h}^* \times \tilde{k} \\ \tilde{i}_K &= \left(\dot{\tilde{k}}/\tilde{k} + \zeta_K\right)\tilde{k} \\ \tilde{c} &= f(1, \underline{h}^*)\tilde{k} - \tilde{i}_H - \tilde{i}_K \\ \tilde{y} &= f(1, \underline{h}^*)\tilde{k} \\ m &= m_K = m_H = m_C = AU'(\tilde{c}A) \\ m_q &= \tilde{i}_H \times m \quad \text{and} \quad q = 1. \end{aligned}$$

Define $\underline{c} \stackrel{\text{def}}{=} \tilde{c}/\tilde{k}$. *In BGSS equilibrium* $\dot{\underline{c}}/\underline{c} = \dot{\tilde{c}}/\tilde{c} - \dot{\tilde{k}}/\tilde{k} = 0$ *so that, from* (22), (23), (24), *and* $\dot{\tilde{k}}/\tilde{k} = g - \xi$, *we have:*

$$\begin{aligned} \underline{c} &= f(1, \underline{h}^*) - \zeta_K - \left(\dot{h}/h + [\nu + \delta_H]\right)\underline{h}^* - (g - \xi) \\ &= \left\{ f(1, \underline{h}^*) - \left(\dot{h}/h + [\nu + \delta_H]\right)\underline{h}^* \right\} - (\nu + \delta_K) - g. \end{aligned}$$

Since $m < \infty$ *so that* (27) *gives* $\underline{c} > 0$, *the expression on the right must be positive. The term in braces is the average product of* K *in producing* $C + I_K$. *Net of per capita depreciation, i.e., taking away*

$\nu + \delta_K$, this average product must therefore exceed growth rate g. Finally, for $\xi > 0$,

$$\dot{h}/h = \dot{\tilde{y}}/\tilde{y} = \dot{y}/y - \xi < \dot{y}/y = \dot{k}/k$$
$$\Longrightarrow \; h/y, \; h/k \to 0 \text{ as } t \to \infty.$$

Q.E.D.

The hypotheses on f as stated in Prop. 7.4 might appear unusual, but are implied by the usual strict concavity and Inada conditions. The statement gives an explicit lower bound on f_H that might well be negative, whereupon the condition is redundant. I have chosen to give the hypotheses as above to allow for situations in the literature that violate standard assumptions but cause no difficulties otherwise. A prominent example would be where the technology for accumulating H is linear [e.g., Lucas, 1988].

BGSS equilibrium growth rate ($\mathfrak{G}1$) has interesting features that should be emphasized:

Proposition 7.5 *Under the hypotheses of Prop. 7.4 the steady-state growth rate g exceeds technology's growth rate ξ precisely when*

$$\begin{aligned} & f_K(1, \underline{h}^*) > R\xi + \rho + \delta_K \\ \Longleftrightarrow \; & f_H(1, \underline{h}^*) > (R-1)\xi + \rho + \delta_H. \end{aligned}$$

Proof *Immediate from ($\mathfrak{G}1$).* *Q.E.D.*

The economy's growth rate ($\mathfrak{G}1$) exceeds that of technology when the equilibrium steady state capital ratio $\underline{h}^*$ implies marginal products f_K and f_H sufficiently high. The threshold for these marginal products depends, notably, on both the production side $(\xi, \delta_K, \delta_H)$ and the consumer side (ρ, R). Moreover, when the threshold is exceeded, the equilibrium growth rate itself depends, again, on both production features $(f, \xi, \delta_K, \delta_H)$ *and* consumer characteristics (ρ, R). This contrasts with equilibrium growth rates in sections 7.2 and 7.3, that vary only with technology, i.e., just with ξ. In the longer term, it might be this—rather than convergence or divergence, scale effects, stochastic trends, or a range of others—that turns out to be the single most distinctive characterization of endogenous growth. Emphasize this—it will appear again below—as follows:

Corollary 7.6 (Endogenous Growth Meta) *Growth varies with not only supply-side properties but demand-side features as well.*

Finally, also worth observing is that here population growth ν has *no* influence on the per capita income growth rate g. This finding, however, is quite special and easily overturned, despite the relatively general specification of the model above.

7.4b. Different technologies for human capital and goods

The setup here makes straightforward extending the discussion to where human capital investment differs in essential ways from consumption and physical capital investment. This is the case considered in Lucas [1988], Rebelo [1991], and Uzawa [1965].

Numerous special cases are possible. To keep things manageable I rule out $s_K = 0$ and $s_H = 1$, i.e., where no K is used in $\mathcal{F}$ for producing goods and no H is used in $\mathcal{G}$ for generating human capital.[9] Taken together those possibilities represent the extreme version of what Barro and Sala-i-Martin [1995] call empirically irrelevant "reversed factor intensities." Ruling out $s_K = 0$ and $s_H = 1$ simply formalizes two properties: first, *some* physical capital is always necessary in goods production and second, it is not possible to produce new human capital without *some* human capital to begin. Indeed, human capital is most of what goes into producing yet more human capital. A leading case of interest, which implies the exclusion, is Lucas's, which assumes $\mathcal{G}_K = 0$ and $\mathcal{F}_H > 0$ everywhere, so that in equilibrium $s_K = 1$ and $s_H \in (0, 1)$.

[9] This exclusion will be used below in (33′) and (34′). Given the current setup, an interested reader can easily see the implications of relaxing the restriction.

Next, $s_H = 0$ can also be excluded. That boundary value would imply that human capital is not used in producing goods. But then it cannot be optimal to continue to produce any human capital at all in equilibrium, for human capital is neither consumed nor used in producing anything except itself. Thus, in the analysis to follow, the first-order condition (31) is strengthened to an equality.

Suppose that (26a) constrains $\tilde{\imath}_H$ to $\mathcal{G}$ while q is unrestricted so that $m_q = 0$. Then (32) gives $m_C = m_Y$. Equation (25) implies:

$$\begin{aligned} f_K &= s_K \cdot \mathcal{F}_K + q \times (1 - s_K)\mathcal{G}_K \\ f_H &= s_H \cdot \mathcal{F}_H + q \times (1 - s_H)\mathcal{G}_H. \end{aligned}$$

From these and (29), the FOC (33) becomes

$$\begin{aligned} s_K\mathcal{F}_K \cdot m_C + (1 - s_K)\mathcal{G}_K \cdot m_H - \zeta_K \cdot m_K& \\ = [(\rho - \nu) - \dot{m_K}/m_K]\, m_K.& \end{aligned}$$

If $s_K = 1$ then the left side becomes just $\mathcal{F}_K \cdot m_C - \zeta_K \cdot m_K$. If, conversely, $s_K \in (0, 1)$ then (30) holds with equality so that together with (29) it gives $\mathcal{F}_K \cdot m_C = \mathcal{G}_K \cdot m_H$ so that again the left side above $\mathcal{F}_K \cdot m_C - \zeta_K \cdot m_K$. Thus, ruling out $s_K = 0$, using $m_C = m_K$ from (28) gives for the above:

$$\mathcal{F}_K - \zeta_K = (\rho - \nu) - \dot{m_C}/m_C. \tag{33$'$}$$

Again, by the partial derivatives of (25), the FOC (34) becomes

$$\begin{aligned} s_H\mathcal{F}_H \cdot m_C + (1 - s_H)\mathcal{G}_H \cdot m_H - (\nu + \delta_H) \cdot m_H& \\ = [(\rho - \nu) - \dot{m_H}/m_H]\, m_H,& \end{aligned}$$

so that ruling out $s_H = 1$, analogous reasoning to that above gives

$$\mathcal{G}_H - (\nu + \delta_H) = (\rho - \nu) - \dot{m_H}/m_H. \tag{34$'$}$$

(Counterparts to (33′)–(34′) are easily obtained if exclusion restrictions $s_K \neq 0$ and $s_H \neq 1$ are reversed.)

Define

$$m \stackrel{\text{def}}{=} m_H/m_C, \qquad \underline{c} \stackrel{\text{def}}{=} \tilde{c}/\tilde{k}, \qquad \underline{h} \stackrel{\text{def}}{=} h/\tilde{k}.$$

Now collect three dynamic equations for the just-defined m, $\underline{c}$, and $\underline{h}$. First, combining (33′) and (34′) gives:

$$\begin{aligned} \dot{m}/m &= \dot{m}_H/m_H - \dot{m}_C/m_C \\ &= \delta_H - \delta_K - \xi + \mathcal{F}_K - \mathcal{G}_H, \end{aligned} \tag{38}$$

where, because $\mathcal{F}_K$ and $\mathcal{G}_H$ are each HD0, $s_K \neq 0$, and $s_H \neq 1$, we can evaluate $\mathcal{F}_K$ and $\mathcal{G}_H$ in (38) at

$$\left(1, \frac{s_H}{s_K}\underline{h}\right) \quad \text{and} \quad \left(\frac{1-s_K}{1-s_H}\underline{h}^{-1}, 1\right)$$

respectively. The reason for taking $\mathcal{F}_K$ and $\mathcal{G}_H$ at these points will become clear below.

Second, as earlier, log-differentiate (27) with respect to time to get

$$\dot{\tilde{c}}/\tilde{c} = \left[(1 - R(\tilde{c}A))\xi - \dot{m}_C/m_C\right] R(\tilde{c}A)^{-1}.$$

Combining this with $\dot{m}_C/m_C$ from (33′) and recognizing

$$\begin{aligned} \dot{\tilde{k}}/\tilde{k} &= \tilde{i}_K/\tilde{k} - \zeta_K = \mathcal{F}\big(s_K, s_H \cdot \underline{h}\big) - \underline{c} - \zeta_K \\ &= s_K \mathcal{F}\left(1, \frac{s_H}{s_K}\underline{h}\right) - \underline{c} - \zeta_K \end{aligned}$$

(where I have used $\mathcal{F}$ HD1) gives

$$\begin{aligned} \dot{\underline{c}}/\underline{c} &= \dot{\tilde{c}}/\tilde{c} - \dot{\tilde{k}}/\tilde{k} \\ &= (\nu + \delta_K) - (\rho + \delta_K)R(\tilde{c}A)^{-1} \\ &\qquad + \underline{c} + R(\tilde{c}A)^{-1}\mathcal{F}_K - s_K \cdot \mathcal{F} \end{aligned} \tag{39}$$

with both $\mathcal{F}_K$ and $\mathcal{F}$ evaluated at $(1, s_H \cdot s_K{}^{-1}\underline{h})$.

The term $s_K \cdot \mathcal{F}$ will play a key role in subsequent discussion. Since $\mathcal{F}\big(1, s_H \cdot s_K{}^{-1}\underline{h}\big)$ is the output-physical capital ratio in the $C + I_K$

sector (or physical capital's average product in producing goods), the product $s_K \cdot \mathcal{F}$ is the ratio of goods produced to the *economy-wide* quantity of physical capital, not just the quantity used in goods production. Call this the *goods-physical capital ratio*. Its counterpart

$$(1 - s_H) \times \mathcal{G}\left(\frac{1 - s_K}{1 - s_H}\underline{h}^{-1}, 1\right),$$

or the ratio of the flow of new human capital to the economy-wide stock of human capital, will be similarly useful in the analysis below.

Return now to the third of the dynamic equations. Using $\mathcal{G}$ HD1, we have

$$\begin{aligned} \dot{h}/h &= \mathcal{G}\big([1 - s_K]/\underline{h}, 1 - s_H\big) - (\nu + \delta_H) \\ &= (1 - s_H)\mathcal{G}\left(\frac{1 - s_K}{1 - s_H}\underline{h}^{-1}, 1\right) - (\nu + \delta_H) \end{aligned}$$

so that

$$\begin{aligned} \dot{\underline{h}}/\underline{h} &= \dot{h}/h - \dot{\tilde{k}}/\tilde{k} \\ &= \delta_K - \delta_H + \xi + \underline{c} + (1 - s_H) \cdot \mathcal{G} - s_K \cdot \mathcal{F}. \end{aligned} \tag{40}$$

Equation (40) combines together $\underline{c}$, $\mathcal{G}$, and $\mathcal{F}$ without using prices. This causes no problems, however, as by this point these terms are all simply numbers—they are ratios of the appropriate quantities.

Provided R is constant the three equations, (38)–(40), together with (30) and (31) rewritten (using $m_C = m_Y$ and equation (29)) as the pair:

$$\text{either of} \begin{cases} m = \mathcal{F}_K \cdot \mathcal{G}_K^{-1} & \text{or} \\ s_K = 1 \end{cases} \tag{41}$$

$$m = \mathcal{F}_H \cdot \mathcal{G}_H \tag{42}$$

all $\mathcal{F}, \mathcal{F}_K, \mathcal{F}_H$ evaluated at $(1, s_H s_K^{-1} \cdot \underline{h})$ and all $\mathcal{G}, \mathcal{G}_K, \mathcal{G}_H$ evaluated at $([1 - s_K][1 - s_H]^{-1} \cdot \underline{h}^{-1}, 1)$, give five conditions that jointly determine $(m, \underline{c}, \underline{h}, s_K, s_H)$. The reason is now apparent for the evaluation point given right after equation (38).

Growth behavior here parallels Prop. 7.4. However, the more involved nonlinear equations (38)–(41) make less transparent existence and uniqueness of the equilibrium, in contrast to the single equation (37) needed above. Special cases assuming explicit functional forms for $(\mathcal{F}, \mathcal{G})$—e.g, the Cobb-Douglas pair model in Barro and Sala-i-Martin [1995, 5.2] and Rebelo [1991] or the Cobb-Douglas/linear model in Lucas [1988]—can be studied from the algebra of (38)–(41) directly.[10] The proposition that follows therefore hypothesizes a unique solution to these equations, leaving unspecified the more primitive assumptions on $(\mathcal{F}, \mathcal{G})$ that would transform the hypothesis into a conclusion. Nevertheless, some work remains to confirm that this solution is a BGSS equilibrium.

Proposition 7.7 *Assume in Defn. 7.3 that $R(\tilde{c}A)$ is constant and that human capital accumulates through a production function $\mathcal{G}$ different from $\mathcal{F}$ (that for producing goods). Assume $(\mathcal{F}, \mathcal{G})$ implies that equations (41) and (42) together with the zeroes of equations (38)–(40) have a unique solution $(m^*, \underline{c}^*, \underline{h}^*, s_K^*, s_H^*)$, where $s_K \neq 0$ and $s_H \neq 1$. Then, for any given initial value $\tilde{k}^* > 0$, BGSS equilibrium exists and—except in (y, q)—is unique. It is characterized by a $(m^*, \underline{c}^*, \underline{h}^*, s_K^*, s_H^*)$ constant in time and independent of $\tilde{k}^*$, with the equilibrium nonuniqueness given as:*

$$q \in [0, m^*] \quad \text{and} \quad \tilde{y} = \mathcal{F} + q \cdot \mathcal{G} \in [\mathcal{F}, \mathcal{F} + m^* \cdot \mathcal{G}] \, .$$

The BGSS equilibrium growth rate is

$$g = [\mathcal{F}_K - (\rho + \delta_K)] \, R^{-1} = [(\mathcal{G}_H + \xi) - (\rho + \delta_H)] \, R^{-1}, \qquad (\mathfrak{G}2)$$

bounded from above by the goods-physical capital ratio net of per capita depreciation. If $\xi > 0$ then the ratios of human capital to income and to physical capital converge to zero.

[10] As an exercise, the interested reader is encouraged to plug in specific functional forms and confirm that the resulting solutions verify equilibria previously obtained in the literature. See also the discussion at the end of this section.

Proof *By the hypotheses,* (26a) *is satisfied with equality and* q *is determined endogenously in equilibrium, so that* (26b) *no longer holds. In BGSS equilibrium, HD1 in production function* (PF1)*, Prop. 7.2, and* (42) *give* m *constant and therefore* $\dot{m} = \dot{\underline{c}} = \dot{\underline{h}} = 0$. *Therefore, BGSS equilibrium has* (38)–(40) *become*

$$s_K \cdot \mathcal{F} - \underline{c} - (1 - s_H) \cdot \mathcal{G} = \xi + \delta_K - \delta_H \tag{43}$$

$$s_K \cdot \mathcal{F} - \underline{c} - R^{-1}\mathcal{F}_K = (\nu + \delta_K) - (\rho + \delta_K)R^{-1} \tag{44}$$

$$\mathcal{F}_K - \mathcal{G}_H = \xi + \delta_K - \delta_H. \tag{45}$$

By hypothesis, these together with (41)–(42) *admit a solution*

$$(m^*, \underline{c}^*, \underline{h}^*, s_K^*, s_H^*).$$

This allows us to evaluate:

$$\begin{aligned} \dot{m}_C/m_C &= (\rho - \nu) - (\mathcal{F}_K - \zeta_K) \\ &= (\rho - \nu) - (\mathcal{G}_H - (\nu + \delta_H)) \\ \dot{\tilde{c}}/\tilde{c} &= \left[(1 - R)\xi - \dot{\tilde{m_C}}/\tilde{m_C}\right] R^{-1}. \end{aligned}$$

By BGSS Defn. 7.1

$$\dot{\tilde{y}}/\tilde{y} = \dot{\tilde{k}}/\tilde{k} = \dot{\tilde{c}}/\tilde{c}$$

so that

$$\begin{aligned} g &= \dot{y}/y = \dot{\tilde{c}}/\tilde{c} + \xi \\ &= \left[\mathcal{F}_K - (\rho + \delta_K)\right] R^{-1} = \left[(\mathcal{G}_H + \xi) - (\rho + \delta_H)\right] R^{-1}, \end{aligned}$$

verifying ($\mathfrak{G}$2). *In BGSS, either Prop. 7.2 or* $\underline{h}^*$ *constancy gives* $\dot{h}/h = \dot{\tilde{k}}/\tilde{k} = g - \xi$. *From any initial* $\tilde{k}^*$ *we then have* $\tilde{k}(t) = \tilde{k}^* e^{(g-\xi)t}$. *To see this establishes an equilibrium, calculate*

$$\begin{aligned} \tilde{i}_H &= \left(\dot{h}/h + [\nu + \delta_H]\right) \underline{h}^* \times \tilde{k} \\ \tilde{i}_K &= \left(\dot{\tilde{k}}/\tilde{k} + \zeta_K\right) \tilde{k} \\ \tilde{c} &= \underline{c}^* \tilde{k} = \mathcal{F}\left(s_K, s_H \underline{h}^*\right) \tilde{k} - \tilde{i}_K \\ m_Y &= m_K = m_C = AU'(\tilde{c}A) \\ m_H &= m^* \times m_C. \end{aligned}$$

The solution $(m^*, \underline{c}^*, \underline{h}^*, s_K^*, s_H^*)$ *and an initial* $\tilde{k}^*$ *uniquely determine the endogenous variables above. However, not so for* $(m_I, q, \tilde{y})$ *individually. Instead, from* (24), (25), *and* (29), *we have*

$$
\begin{aligned}
m_I &= m_H - q \cdot m_C = (m^* - q) \cdot m_C \\
\tilde{y} &= \mathcal{F}\left(s_K^*, s_H^* \cdot \underline{h}^*\right) \tilde{k} + q \cdot \mathcal{G}\left([1 - s_K^*]/\underline{h}^*, 1 - s_H^*\right) \underline{h}^* \tilde{k}
\end{aligned}
$$

so that any constant $q \in [0, m^*]$ *implies an* m_I *such that*

$$0 \leq m_I \leq m^* m_C = m_H,$$

and a $y = A\tilde{y}$ *that together with the above constitutes a BGSS equilibrium. Next,* (39) *gives*

$$
\begin{aligned}
\underline{c} &= s_K \mathcal{F} - R^{-1} \mathcal{F}_K - \left[(\nu + \delta_K) - (\rho + \delta_K) R^{-1}\right] \\
&= s_K \mathcal{F} - (\nu + \delta_K) - \left[\mathcal{F}_K - (\rho + \delta_K)\right] R^{-1} \\
&= \left[s_K \mathcal{F} - (\nu + \delta_K)\right] - g.
\end{aligned}
$$

The term in brackets is the goods-physical capital ratio net of per capita depreciation. Since $m < \infty$ *so that* (27) *gives* $\underline{c} > 0$, *the expression on the right must be positive: The growth rate* g *is bounded from above by the net of per capita depreciation goods-physical capital ratio. Finally, for completeness, reproduce the previous argument that for* $\xi > 0$,

$$
\begin{aligned}
\dot{h}/h = \dot{\tilde{y}}/\tilde{y} = \dot{y}/y - \xi < \dot{y}/y = \dot{k}/k& \\
\implies h/y,\ h/k \to 0 \text{ as } t \to \infty.&
\end{aligned}
$$

Q.E.D.

Is there intuition for the indeterminacy in $(q, \tilde{y})$? Recall from (24)–(25) in Defn. 7.3 that q is a relative price. It serves two functions: First, q accounts for what is immediately added to national income by human capital accumulation. Second, q is a market signal to allocate resources between producing goods and producing human capital. When technologies $\mathcal{F}$ and $\mathcal{G}$ differ and restriction (26a) holds, the equilibrium production decision is a corner solution: goods and human capital cannot be transformed into each other—not just costlessly, but at all. The relative price that decentralizes this allocation decision is determined only up to an appropriate range. All prices within that range imply the same observed outcome in quantities; the slack is taken up by some shadow value, in this case, the Lagrange multiplier m_I. But then using q in national income accounts leads similarly to a range of possible values for GDP. When q is set to zero, GDP fails to include human capital accumulation and is then what Barro and Sala-i-Martin [1995, Ch. 5] call "narrow output". Conversely, at the maximum feasible equilibrium value for q, namely $m^* = \mathcal{F}_H \cdot \mathcal{G}_H^{-1}$ (corresponding to equation (5.16) in Barro and Sala-i-Martin [1995]), GDP evaluates to what Barro and Sala-i-Martin [1995, Ch. 5] call "broad output". The analysis above, however, suggests that *any* level of GDP between narrow and broad output is equally meaningful. All of them grow at the same rate in BGSS equilibrium; all of them imply an identical value to the program (21)–(26).

As earlier, the BGSS equilibrium growth rate has interesting features:

Proposition 7.8 *Under the hypotheses of Prop. 7.7 the steady-state growth rate g exceeds technology's growth rate ξ precisely when*

$$\begin{aligned} & \mathcal{F}_K(s_K^*, s_H^* \cdot \underline{h}^*) > R\xi + \rho + \delta_K \\ \iff\ & \mathcal{G}_H([1 - s_K^*]/\underline{h}^*, 1 - s_H^*) > (R-1)\xi + \rho + \delta_H. \end{aligned}$$

Proof *Immediate from* ($\mathfrak{G}2$). *Q.E.D.*

The equilibrium growth rate ($\mathfrak{G}2$) resembles ($\mathfrak{G}1$) in the earlier discussion. For the economy's growth rate to exceed that of technology, the marginal productivity of physical capital in goods production or, equivalently, the marginal productivity of human capital in generating new human capital must be sufficiently high. The critical threshold depends on both production $(\xi, \delta_K, \delta_H)$ and consumption (ρ, R) characteristics. When the threshold is exceeded, again, the equilibrium growth rate depends on both production features $(\mathcal{F}, \mathcal{G}, \xi, \delta_K, \delta_H)$ and consumer characteristics (ρ, R).

Prop. 7.7, as already discussed above, hypothesizes that $(\mathcal{F}, \mathcal{G})$ implies a unique solution to equations (38)–(42). A reasonable conjecture is that standard Inada-type conditions would deliver this. However, those curvature conditions would unnecessarily rule out, among others, the leading case with $\mathcal{G}$ linear [Lucas, 1988], and where the equilibrium can be studied explicitly. To see this, note that, in our notation, that model has

$$\begin{aligned}
&\mathcal{F}\left(s_K \cdot \tilde{k}, s_H \cdot h\right) = (s_K \cdot \tilde{k})^{\alpha}(s_H \cdot h)^{1-\alpha}, \quad \alpha \in (0,1) \\
&\mathcal{G}\left([1 - s_K] \cdot \tilde{k}, [1 - s_H] \cdot h\right) \\
&\qquad\qquad = \gamma \times [1 - s_H] \cdot h, \quad \gamma > \max\{0, -[\xi + \delta_K - \delta_H]\}.
\end{aligned}$$

Then the ratios and marginal products in Prop. 7.7 are

$$\begin{aligned}
\mathcal{F} &= \left(\frac{s_H}{s_K} \cdot \underline{h}\right)^{1-\alpha} \\
\mathcal{F}_K &= \alpha \times \left(\frac{s_H}{s_K} \cdot \underline{h}\right)^{1-\alpha} \\
\mathcal{F}_H &= (1 - \alpha) \times \left(\frac{s_H}{s_K} \cdot \underline{h}\right)^{-\alpha} \\
\mathcal{G} &= \mathcal{G}_H = \gamma \quad \text{and} \quad \mathcal{G}_K = 0.
\end{aligned}$$

By the last of these, $s_K^* = 1$ in equation (41). Using this in (45) determines $s_H^* \cdot \underline{h}^*$, since $\gamma > -[\xi + \delta_K - \delta_H]$ by hypothesis. In turn, equation (44) then gives $\underline{c}^*$, and equation (43), s_H^* and $\underline{h}^*$ separately. Finally, (42) gives m^*. The BGSS equilibrium growth rate is

$$g = \dot{h}/h + \xi = \gamma \times (1 - s_H^*) - (\nu + \delta_H) + \xi.$$

This depends on consumer characteristics through s_H^* being determined in (43)–(45).

References

Robert J. Barro and Xavier Sala-i-Martin. *Economic Growth.* McGraw-Hill, New York, 1995.

Robert J. Barro and Xavier Sala-i-Martin. Technology diffusion, convergence, and growth. *Journal of Economic Growth*, 2(1):1–25, March 1997.

Joan S. Burrelli. Graduate enrollment in science and engineering increases for the first time since 1993. *National Science Foundation Division of Science Resources Studies, Data Brief*, 11 January 2001.

Gavin Cameron, James Proudman, and Stephen Redding. Productivity convergence and international openness. In James Proudman and Stephen Redding, editors, *Openness and Growth*, chapter 6, pages 221–260. Bank of England, London, 1998.

David T. Coe and Elhanan Helpman. International R&D spillovers. *European Economic Review*, 39(5):859–887, May 1995.

Paul A. David. Intellectual property institutions and the panda's thumb: Patents, copyrights, and trade secrets in economic theory and history. In M. B. Wallerstein, M. E. Mogee, and R. A. Schoen, editors, *Global Dimensions of Intellectual Property Rights in Science and Technology*, chapter 2, pages 19–61. National Academy Press, Washington DC, 1993.

Jonathan Eaton and Samuel Kortum. International technology diffusion: Theory and measurement. *International Economic Review*, 40(3):537–570, August 1999.

James Feyrer. Convergence by parts. Working paper, Dartmouth College, Hanover, December 2001.

Masahisa Fujita, Paul Krugman, and Anthony Venables. *The Spatial Economy: Cities, Regions, and International Trade.* MIT Press, Cambridge, 1999.

Robert J. Gordon. Does the 'New Economy' measure up to the great inventions of the past? *Journal of Economic Perspectives*, 14(4): 49–74, Fall 2000.

Gene M. Grossman and Elhanan Helpman. *Innovation and Growth in the Global Economy*. MIT Press, Cambridge, 1991.

Elhanan Helpman, editor. *General Purpose Technologies and Economic Growth*. MIT Press, Cambridge, 1998.

Jukka Jalava and Matti Pohjola. Economic growth in the New Economy: Evidence from advanced economies. *Information Economics and Policy*, 14(2), June 2002.

Charles I. Jones. Time series tests of endogenous growth models. *Quarterly Journal of Economics*, 110:495–525, May 1995.

Charles I. Jones. *Introduction to Economic Growth*. W. W. Norton, New York, 1998.

Eric L. Jones. *Growth Recurring: Economic Change in World History*. Oxford University Press, Oxford, 1988.

Kenneth L. Kraemer and Jason Dedrick. Information technology and productivity: Results and policy implications of cross-country studies. In Matti Pohjola, editor, *Information Technology, Productivity, and Economic Growth*, UNU/WIDER and Sitra, chapter 12, pages 257–279. Oxford University Press, Oxford, 2001.

Paul Krugman. The myth of Asia's miracle. *Foreign Affairs*, 73(6): 62–78, November/December 1994.

David S. Landes. *The Wealth and Poverty of Nations*. Little, Brown and Company, London, 1998.

Robert E. Lucas. On the mechanics of economic development. *Journal of Monetary Economics*, 22(1):3–42, July 1988.

N. Gregory Mankiw, David Romer, and David N. Weil. A contribution to the empirics of economic growth. *Quarterly Journal of Economics*, 107(2):407–437, May 1992.

Joel Mokyr. *The Lever of Riches: Technological Creativity and Economic Progress*. Oxford University Press, Oxford, 1990.

OECD. *Measuring the ICT Sector*. OECD, Paris, 2000.

Stephen L. Parente and Edward C. Prescott. *Barriers to Riches*. Walras-Pareto Lectures. MIT Press, Cambridge, 2000.

Kenneth Pomeranz. *The Great Divergence: China, Europe, and the Making of the Modern World Economy*. Princeton University Press, Princeton, 2000.

Robert Pool. *Beyond Engineering: How Society Shapes Technology*. Oxford University Press, New York, 1997.

Danny Quah. Empirics for growth and distribution: Polarization, stratification, and convergence clubs. *Journal of Economic Growth*, 2(1):27–59, March 1997.

Danny Quah. Internet cluster emergence. *European Economic Review*, 44(4–6):1032–1044, May 2000.

Danny Quah. Cross-country growth comparison: Theory to empirics. In Jacques Dreze, editor, *Advances in Macroeconomic Theory*, volume 133 of *Proceedings of the Twelfth World Congress of the International Economic Association, Buenos Aires*, chapter 16, pages 332–351. Palgrave, London, 2001a.

Danny Quah. Demand-driven knowledge clusters in a weightless economy. Working paper, Economics Dept., LSE, April 2001b.

Danny Quah. The weightless economy in economic development. In Matti Pohjola, editor, *Information Technology, Productivity, and Economic Growth*, UNU/WIDER and Sitra, chapter 4, pages 72–96. Oxford University Press, Oxford, 2001c.

Sergio Rebelo. Long-run policy analysis and long-run growth. *Journal of Political Economy*, 99(3):500–521, June 1991.

David Romer. *Advanced Macroeconomics*. McGraw-Hill, New York, second edition, 2001.

Paul M. Romer. Increasing returns and long-run growth. *Journal of Political Economy*, 94(5):1002–1037, October 1986.

Paul M. Romer. Endogenous technological change. *Journal of Political Economy*, 98(5, part 2):S71–S102, October 1990.

Paul M. Romer. Two strategies for economic development: Using ideas and producing ideas. *Proceedings of the World Bank Annual Conference on Development Economics*, pages 63–91, March 1992.

Robert M. Solow. A contribution to the theory of economic growth. *Quarterly Journal of Economics*, 70(1):65–94, February 1956.

Robert M. Solow. Technical change and the aggregate production function. *Review of Economics and Statistics*, 39(3):312–320, August 1957.

Robert M. Solow. We'd better watch out. *New York Times Book Review Section*, page 36, 12 July 1987.

Neal Stephenson. *In the Beginning was the Command Line*. Avon Books, New York, 1999.

U. S. Department of Commerce. *The Emerging Digital Economy II*. U. S. Department of Commerce, 1999.

Hirofumi Uzawa. Optimal technical change in an aggregative model of economic growth. *International Economic Review*, 6:18–31, January 1965.

Patrick Vanhoudt and Luca Onorante. Measuring economic growth and the new economy. *EIB Papers*, 6(1):63–83, 2001.

Figure 1

Information
Technology

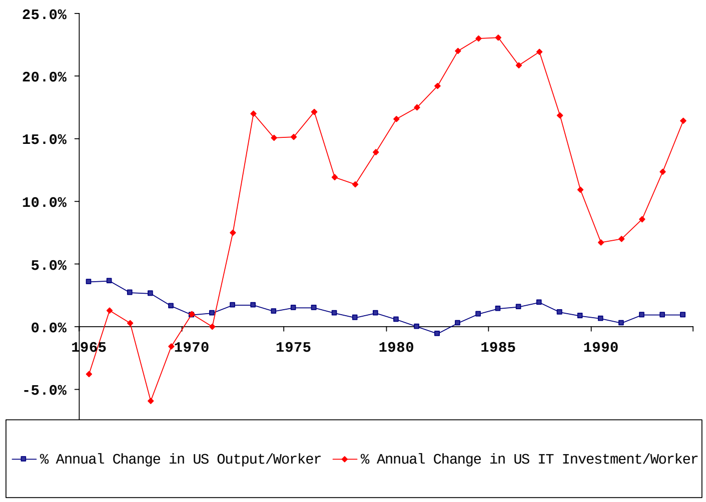

Figure 2

Scientists and engineers in R&D