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ABSTRACT

Why are a Third of People Indian and Chinese? Trade, Industrialization and Demographic Transition*

This research argues that international trade has played a significant role in the timing of demographic transitions across countries and has thereby been a major determinant of the distribution of world population and a prime cause of sustained differences in population growth and income levels across countries. In industrial economies international trade enhanced the specialization in the production of skilled-intensive goods and stimulated technological progress. The rise in the demand for skilled labour induced an investment in the quality of the population, expediting the demographic transition, stimulating technological progress and further enhancing the comparative advantage of these industrial economies in the production of skilled-intensive goods. In non-industrial economies, in contrast, the specialization in the production of unskilled-intensive goods that was brought about by international trade reduced the demand for skilled labour and provided limited incentives to invest in population quality. The demographic transition was therefore delayed, increasing further the abundance of unskilled labour in these economies and enhancing their comparative disadvantage in the production of skilled-intensive goods. International trade has therefore widened the gap between the technological level as well as the skill abundance of industrial and non-industrial economies, enhancing the initial patterns of comparative advantage and generating sustained differences in income per capita across countries.

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1 Introduction

This research argues that international trade has played a major role in the timing of demographic transitions across countries and has thereby been a significant determinant of the distribution of world population and a prime cause of sustained differences in population growth and income levels across countries.

In the early stages of the industrial revolution international trade enhanced the specialization of industrial economies in the production of industrial, skilled-intensive, goods. The rise in the demand for skilled labor induced a gradual investment in the quality of the population, expediting a demographic transition, stimulating technological progress and further enhancing the comparative advantage of these industrial economies in the production of skilled intensive goods. In non-industrial economies, in contrast, international trade generated an incentive to specialize in the production of unskilled-intensive, non-industrial, goods. The absence of significant demand for human capital provided limited incentives to invest in the quality of the population and the gains from trade were utilized primarily for a further increase in the size of the population. The demographic transition in these non-industrial economies was significantly delayed, increasing further their relative abundance of unskilled labor, enhancing their comparative disadvantage in the production of skilled intensive goods and delaying their process of development.

The research suggests, therefore, that international trade has widened the gap between the technological level as well as the skill abundance of industrial and non-industrial economies. Trade has thereby affected persistently the distribution of population, skills, and technologies in the world economy and has generated sustained differences in income per capita across countries.

The transition of economies from an epoch of Malthusian stagnation to sustained economic growth, has been a subject of intensive research in the last few years (e.g., Oded Galor and David N. Weil (1999, 2000) and Robert Lucas (1999)).[1] The inconsistency of exogenous as well as endogenous neoclassical growth models with the evolution of economies throughout

[1] Recent growth models with endogenous fertility of the long transition from stagnation to growth also include Gary Hansen and Edward Prescot (2000), Charles Jones (2001), Tomas Kogel and Alexia Prskawetz (2001), Olivier Morand (2000), Nils Lagerlof (2000), among others.

most of human history has led recently to the development of unified growth models which encompass an epoch of Malthusian stagnation as well as the transition from Malthusian stagnation to sustained economic growth. In light of the central role that population growth has apparently played in the Malthusian world as well as in the take-off to sustained growth, these unified models are based on endogenous population growth.[2] In addition they incorporate the main Malthusian features.[3] Oded Galor and David N. Weil (1999, 2000) argue that the inherent positive interaction between population and technology during the Malthusian regime increased the rate of technological progress sufficiently so as to induce investment in human capital that led to further technological progress, a demographic transition, and sustained economic growth. Alternatively, Gary Hansen and Edward Prescott (2000) propose that the transition from stagnation to growth reflects a transition from a stagnating agricultural economy to a growing industrial economy, due to a persistent technological progress in a latent industrial technology.[4]

Several intriguing puzzles, however, have not received a satisfactory resolution: How does one account for the sudden take-off from stagnation to growth in some countries in the world and the persistent stagnation in others? Why have the differences in per capita incomes across countries increased so markedly in the last two centuries? Why has the link between income per capita and population growth been so dramatically reversed in some economies but not in others? Has the pace of transition to sustained economic growth in advanced economies adversely affected the process of development in less-developed economies?[5]

[2] The existing literature on the relation between population growth and output has tended to focus on only one of the regimes described above. The majority of the literature has been oriented toward the modern regime, trying to explain the negative relation between income and population growth either cross-sectionally or within a single country over time (e.g. Robert J. Barro and Gary S. Becker, 1989). Among the mechanisms highlighted in this literature are: (a) higher returns to child quality in developed economies induce a substitution of quality for quantity (Becker, Kevin M. Murphy, and Robert F. Tamura, 1990); (b) developed economies pay higher relative wages for women, thus raising the opportunity cost of children (Oded Galor and David N. Weil, 1996); (c) the net flow of transfers from parents to children grows (and possibly switches from negative to positive) as countries develop (John W. Caldwell, 1976); (d) higher fertility rates among unskilled workers increase the return to skills and an incentive to substitute quality for quantity (Momi Dahan and Daniel Tsiddon, 1998).

[3] Michael Kremer (1993) examines the historical co-evolution of population and technology in a Malthusian epoch, providing creative evidence for the importance of the scale effect in economic growth.

[4] A related interesting research by Marvin Goodfriend and John McDermott (1995) demonstrate that exogenous population growth increases population density and hence generates a greater scope for the division of labor inducing the development of markets and economic growth. See Kogel and Prskawetz (2001) as well.

[5] Recently, Baldwin et. al. (2001) and McDermott (2001) has linked trade to nidustrialization and divergence in economic performance.

The historical evidence described in the next section suggests that interaction between economies, via international trade, may be a key element in the resolution of these remaining puzzles. The asymmetric effect of international trade on the timing of the demographic transition in developed and less-developed economies, and its persistent effect therefore on the initial patterns of comparative advantage, may suggest that the rapid transition of the currently developed economies into a state of sustained economic growth may be the prime cause of the slow transition of less developed economies into a state of sustained economic growth. In particular the economic history of the UK and India is consistent with a significant role for international trade in the timing of the demographic transition and in the process of industrialization. During the nineteenth century the UK traded manufactured goods for primary products with India. As documented in Table 1, industrialization in India regressed over this century whereas industrialization in the UK accelerated. Furthermore, the UK experienced an impressive increase in the level of education throughout the 19th century and a demographic transition towards the end of the century, whereas the demographic transition in India has been delayed. The theory further suggests that the near abstention of China from international trade during this period, delayed its demographic transition, increased the level of its population and derailed its relative position in the world income distribution. As documented by David Landes (1998), the degree of industrialization in China, which was in the midst of an epoch of isolationism and discouragement of international trade, was declining in this period, despite being quite technologically advanced.

The paper develops a unified growth model that captures the prime forces that govern the evolution of economies from Malthusian stagnation to sustained economic growth. The model focuses on the asymmetric role that international trade may have played in expediting the transition to sustained economic growth in technologically advanced economies and in delaying the transition in technologically inferior economies. The theory suggests that sustained differences in income and population growth across counties may be attributed to the contrasting role that international trade had on industrial and non-industrial nations. Traditional trade theory suggests that factor endowments and production technologies affect the pattern of international trade, whereas international trade has no effect on factor endowments and production technologies and thus on the patterns of comparative advantage. In contrast, in the proposed theory the patterns of comparative advantage is endogenously determined based upon the effect of trade on factor endowments and the level of technology. The theory is therefore related to the important literature on the dynamics of comparative advantage, developed by Grossman and Helpman (1991) and Alwyn Young (1991), in which innovations affect the level of technology and the dynamics of comparative advantage. In contrast to this literature, however, the proposed theory focuses on the interaction between population dynamics and the dynamics of comparative advantage.

The model of this paper comprises four fundamental elements. The interaction between these four fundamental elements generates a dynamic pattern that is consistent with the observed evolution of the world economy from the epoch of Malthusian stagnation to the current era of sustained growth, along with widened differences in income per capita and population growth rates across economies. The first element consists of the main ingredients of a Malthusian world. The economy is characterized by a fixed factor of production, land, and a subsistence consumption constraint. In the proximity of the Malthusian equilibrium, if technological progress permits output per worker to exceed the subsistence level of consumption, population rises, the land-labor ratio falls, and in the absence of further technological progress wages fall back to the subsistence level. Similarly, if an adverse shock reduces income below subsistence, population falls and wages rise back to the subsistence level. Income per capita is therefore self-equilibrating and in the absence of a significant change in the production technology the economy remains in the vicinity of a Malthusian equilibrium.

The second element of the model links technological progress to the emergence of human capital accumulation that brings about the take-off from the Malthusian equilibrium and leads ultimately to a demographic transition and sustained economic growth. Technological progress increases the productivity of a latent advanced manufacturing production technology and makes it economically viable. The employment of this skilled-intensive technology raises the return to human capital, inducing parents to substitute child quality for child quantity. Household's choose the number of children and their quality in the face of a constraint on the total amount of resources that can be devoted to child-raising and labor market activities. Technological progress has therefore two contrasting effects on the evolution of population. First, it increases the return to human capital,[6] inducing parents to raise the quality of each child and reduce the number of children.[7] But, second, by raising parental income above the subsistence level, technological progress provides more resources for the quantity as well as quality of children. At low levels of technology when agents are still highly constrained by their subsistence constraint, the second effect dominates and overall fertility increases. Ultimately however the first effect dominates and technological progress induces a reduction in fertility rates, generating a demographic transition. The third element links the formation of human capital to an increase in the rate of technological progress. Following the well-documented and commonly employed hypothesis, human capital is assumed to have a positive effect on technological progress and therefore on economic growth.[8]

The fourth element links international trade and specialization in production to the return to human capital, the demographic transition, and the persistent patterns of comparative advantage across countries. Since technological progress benefits the manufacturing sector more than the agricultural sector, international trade induces the technologically advanced economy to specialize in the production of the skilled-intensive manufactured goods whereas the technologically inferior country specializes in the production of unskilled-intensive agricultural goods. International trade, therefore, increases the demand for human capital in the technologically advanced economy, increasing its human capital accumulation and expediting its demographic transition and technological progress. In contrast, trade reduces the demand for skilled labor in technologically inferior countries, increasing its population growth rate and reducing its rate of technological progress. International trade reinforces the initial patterns of comparative advantage over time and in the absence of intervention some less-developed economies may never develop a demand for human capital accumulation sufficient to reduce its rate of population

[6] Goldin and Katz (1998) provide evidence regarding technology-skill complementarity during the 20th century.

[7] Unlike Gary Becker (1981) in which a high level of income is inducing parents to switch to having fewer, higher quality children, the substitution of quality for quantity in this paper is in response to increase in the reward to human capital due to technological progress.

[8] This link between education and technological change was proposed by Richard R. Nelson and Edmund S. Phelps [1966]. For supportive evidence see Easterlin (1981) and Mark Doms, Timothy Dunne, and Kenneth R. Troske (1997). The model abstracts from the potential positive effect of the overall size of the population on the rate of technological progress. This appears like a plausible assumption for countries with sufficiently large populations, given the focus on the post-industrial revolution period. While Kremer (1993) provides some supporting historical evidence for the role of population in technological progress prior to 1500AD, Jones (1995) argues that in the 20th century it appears that there is no evidence for a scale effect.

growth and to reverse its initial patterns of comparative advantage. Thus international trade may have a persistent effect on the distribution of population and income per capita in the world economy.

2 Historical Evidence

This section provides some historical evidence that demonstrates that the fundamental hypothesis of this research is consistent with the experience of the UK, India and China since the 19th century.

2.1 Trade and Industrialization

It is widely accepted that the expansion of international trade played an important contributory role to the industrial revolution in the UK, although this role is not seen as 'vital' to its occurrence (Joel Mokyr, 1985 and Nicholas Crafts, 1985). For the UK economy as a whole total net exports grew from about 10% of national income in the 1780's to about 25% in the 1850's, and for certain industries exports were extremely important, especially in the cotton industry where 70% of output was exported in the 1870's (Mokyr, 1985). Thus while technological advances would have spawned the industrial revolution without an expansion of international trade, the growth in exports undoubtedly increased the pace of industrialization and the growth rate of income.

Trade with Asia was very significant for Britain. According to Paul Bairoch (1974) trade with Asia constituted over 20% of UK total exports throughout the nineteenth century. In contrast, trade with Asia was only 5% or less of French German or Italian exports. UK imports from Asia were also much more important for the UK than for Europe. Bairoch estimates 23.2% of UK imports were Asian in 1860 as compared with 12.1% for continental Europe.

For India however, international trade may well have played a vital role in the delay of the industrial revolution. As Chaudhuri (1983) argues "In India's case we know that external trade was of far greater importance for her in the past than it was later". This is especially the case in the period following the abolition of the East India Company's monopoly on Indian

Share of the Value of British Trade in Total Value of Indian Trade								
	1828-9	1839-40	1850-1	1860-1	1880-1	1900-1	1920-1	1940-1
Exports	48.2	57.1	44.6	43.1	41.6	29.8	22.1	34.7
Imports	65.0	75.7	72.1	84.8	82.9	65.6	60.9	22.9

Table 1. Share of the Value of British Trade in the Total Value of Indian Trade.
Source: K.N. Chaudhuri (1983)

external trade in 1813. Chaudhuri (1983) describes 1813-1850 as a period of a rapid expansion in the volume of exports and imports which gradually transformed India from being an exporter of manufactured products – largely textiles – into a supplier of primary commodities. Trade with the UK was fundamental in this process as Table 1 demonstrates.

Bairoch's (1974, 1982) analysis of international levels of industrialization and international trade supports the viewpoint that international trade was associated with a decrease in the per capita level of industrialization in India. As Table 2 suggests, the rapid industrialization in the UK in the nineteenth century was associated with a decline in the per capita level of industrialization in India. Furthermore, Bairoch (1974) found that whereas 'new technology' industries made up between 60 and 70 percent of the UK manufacturing industry in 1860 it made up less than 1 percent of manufacturing industries in the developing countries.[9] There is thus strong evidence that the expansion of international trade significantly delayed the industrialization of India.

China during the period was in the midst of a policy of self imposed isolation and was also as Table 2 shows experiencing decreases in its level of per capita industrialization. After

[9] This contrast with the experience of the non-UK European economies which produced more of the 'new technology' goods and which traded with themselves to a greater extent, (Bairoch, 1974).

Per Capita Industrialization Levels (Index normalized at UK 1900=100)					
	1800	1860	1913	1953	1980
UK	16	64	115	210	325
Europe	8	17	45	90	267
India*	6	3	2	5	16
China	6	4	3	5	24

Table 2. Table of per capita industrialization levels.
Source: Bairoch (1982)
***** India is measured using its boundaries in 1913 and is India, Pakistan and Bangladesh after 1928

a brief period of interest in foreign exploration at the start of the fifteenth century during which huge fleets were built, by the end of the fifteenth century things had turned around. As described by Landes (1998) "By 1500, anyone who built a ship of more than two masts was liable to the death penalty, and in 1525 coastal authorities were enjoined to destroy all ocean-going ships and arrest their owners. Finally in 1551 it became a crime to go to sea on a multimasted ship, even for trade". This era of isolationism was to last over four hundred years in which European technological advances left China far behind.

2.2 Industrialization and Demographic Transition

For the major part of human existence economies appear to have been in a state of Malthusian stagnation. Diminishing returns to labor along with a positive effect of the standard of living on the growth rate of population provided a self equilibrating role for the size of the population in a stationary economic environment. Changes in the technological environment or in the availability of land led to larger but not richer population. The growth rate of output per capita had been negligible over time and the standard of living had not differed greatly across countries. For instance, the average growth rate of GDP per capita in Western Europe in the years 0-1000 was nearly zero and only 0.14% in the years 1000-1820 (Angus Maddison, 1982). Similarly, the pattern of population growth over this era followed the Malthusian pattern. The average annual rate of population growth in Western Europe was 0% between the years 0 and 1000 and 0.1% in the years 1000-1820 (Madisson, 1982), and world population grew at an average pace of less than 0.1 percent per year from the year 1 to 1750 (Massimo Livi-Bacci, 1997), reflecting the slow pace of resource expansion and technological progress. Fluctuations in population and wages also reflected the structure of the Malthusian regime. Negative shocks to population, such as the Black Death, were reflected in higher real wages and faster population growth. Finally, differences in technology were reflected in population density but not in standards of living. Prior to 1800 differences in standard of living between countries were relatively small despite the existence of wide differences in technology (Richard Easterlin, 1981, Lucas, 1999, and Pritchett, 1997).

The emergence from Malthusian stagnation in Europe was initially very slow, (Madisson, 1982, 1995). During this slow transition, the Malthusian mechanism linking higher income to higher population growth continued to function, but the reduction in resources per capita caused by higher population was counteracted by technological progress, which allowed per capita income to keep rising. The average growth of output per capita over the period 1820-1870 rose to an annual rate of 1.0 percent along with an impressive increase in education.[10] During this time interval, the Malthusian mechanism linking higher income to higher population growth continued to function. Fertility rates increased in most of Western Europe until the second half of the nineteenth century, peaking in England and Wales in 1871 and in Germany in 1875. (Tim Dyson and Mike Murphy, 1985, and Ansley J. Coale and Roy Treadway, 1986).[11] Furthermore,

[10]See the next section, but for example, the average number of years of schooling in England and Wales rose from 2.3 for the cohort born between 1801 and 1805 to 5.2 for the cohort born 1852-56 and 9.1 for the cohort born 1897-1906. (Robert C. O. Matthews, Charles H. Feinstein, and John C. Odling-Smee, 1982).

[11]In addition, as living standards rose, mortality fell. Between the 1740s and the 1840s, life expectancy at

as the next section will show, the acceleration in technological progress increased the return to human capital and had ultimately triggered a demographic transition in which fertility rates declined rapidly paving the way to an era of sustained economic growth.[12] Furthermore, the level of resources invested in each child increased as well. Population growth fell and was one of the forces which brought about a sustained average annual increase in income per capita of 2.2 percent over the period 1929-1990.

Table 3 shows that in the UK, population growth increased rapidly during the industrial revolution before declining sharply in the twentieth century. Western Europe has a similar although less dramatic pattern. In contrast India and China have not until recently experienced a rapid increase in industrialization and has seen population growth increase with income growth rates in a Malthusian manner (see Landes (1998) and McEvedy and Jones (1978)).

2.3 Industrialization and Human Capital Accumulation

It is also widely accepted that industrialization causes an increase in the demand for human capital by the industrial sector. This is clearly evident in the economic history of England during the industrial revolution which is outlined below, but similar processes were occuring in all industrialized economies.

In the early stage of the Industrial Revolution in England skills and literacy requirements had been minimal. During this period, illiterate labor force could operate the existing technology, and economic growth was not impeded by educational retardation.[13] Workers developed skills primarily through on-the-job training, and child labor was highly valuable. As argued by Green, (1990, pp. 293-294), "Britain's early industrialization had occurred without direct

birth rose from 33 to 40 in England and from 25 to 40 in France (Livi-Bacci, 1997). Mortality reductions led to growth of the population both because more children reached breeding age and because each person lived for a larger number of years.

[12] The reduction in fertility was most rapid in Europe around the turn of the century. In England, for example, live births per 1000 women aged 15-44 fell from 153.6 in 1871-80 to 109.0 in 1901-10 (Wrigley, 1969). The exception was France, where fertility started to decline in the early 19th century.

[13] As argued by Mitch (1992 pp. 14-15), during the first stages of the Industrial Revolution, literacy was largely a cultural skill or a hierarchy symbol that had limited value in the labor market. For instance, in 1841 only 4.9% of male workers and only 2.2% of female workers were in occupations in which literacy was strictly required.

Population Growth Rates (Percentage Growth Per Year Over Previous 50 years)					
	1800	1850	1900	1950	2000
British Isles	0.94	1.1	0.81	0.50	0.31
Western Europe*	0.48	0.69	0.66	0.60	0.36
India**	0.16	0.38	0.46	0.86	2.24
China	0.68	0.55	0.18	0.43	1.56

Table 3. Table of Annualized growth rates of selected regions.
***** The population of Western Europe is the sum of Austria, Belgium, Czech Republic, Denmark, Finland, France, Germany, Hungary, Ireland, Italy, Luxembourg, Netherlands, Norway, Portugal, Slovakia, Spain, Sweden, Switzerland and UK.
****** The population of India is the sum of India, Pakistan and Bangladesh
Source: Figures calculated from McEvedy and Jones's (1978) population estimates except for 2000 population estimates which were from the United Nations Population Information Network's 's website at http://www.undp.org/popin/

state intervention and developed successfully, at least in its early stages, within a laissez-faire framework... the very success of Britain's early industrial expansion encouraged a complacency about the importance of scientific skills and theoretical knowledge which became a liability in a later period when empirical knowledge, inventiveness and rule of thumb methods were no longer adequate."

In the second phase of the industrial revolution skills became necessary for production.[14] In light of industrial competition from other countries, capitalists started to recognize the importance of technical education for the provision of skilled workers. Crafts and Thomas (1986) argue that "The source of Britain's industrial leadership in the nineteenth century was a favorable endowment of natural resources, combined with a stock of labor sufficient to exploit

[14]. "Job advertisements from the 1850s occasionally mentioned literacy as a desired characteristic for employment, even for occupations of modest status" (Mitch, 1993, p. 292).

these advantages; Britain's handicap in the later part of the century was a scarcity of the human capital which was an essential input to the technologically progressive product-cycle industries that dominated the Second Industrial Revolution".

This was recognized by the British government which in 1868 established the Parliamentary Select Committee on Scientific Education and so began a long process of parliamentary legislation and investigation concerning the relationship between science, industry, and education. The legislation had a positive effect on the level of education in Britain. School enrolment of 10-year olds increased from 40% in 1870 to 100% in 1900 and the percentage of males unable to sign their name on the marriage register fell from over 30% in 1840 to 1% in 1910 (West, 1985).

Education was not expanded to a similar degree in India in the 19th Century. As Aparna Basu (1974) notes about education in Bengal in 1918: "The populations of Bengal and the United Kingdom in 1917-18 were the same –45,000,000 - but [in] Bengal only one in ten of the population could read and write". The lack of broad based education in India can also be seen using the data of Barro and Lee (2000). Despite an expansion of education throughout the twentieth century Barro and Lee report that in 1960 72.2 percent of Indians aged 15 and above had "no schooling" compared with 2 percent in the UK.

3 An Autarkic Economy

This section analyzes the path of a closed economy from its Malthusian pre-industrial state through a transitional state of increased fertility, investment in human capital and economic growth to a modern state with high investment in human capital, low population growth, and sustained economic growth.

Consider an overlapping generations economy in which economic activity extends over infinite discrete time. In every period t two goods, a manufactured good, Y_t^m, and an agricultural good Y_t^a, may be produced using up to three factors of production, skilled labor, H_t, unskilled labor, L_t, and land, X. The supply of skilled and unskilled labor are endogenously determined and evolve over time, whereas the quantity of land is exogenously determined and remains constant over time.

3.1 Production

In each of the sectors in the economy production may take place with either an old technology or a new one. In early stages of development the new production technologies are latent and production is conducted using the old technologies. However, in the process of development the productivity of the new technologies grows faster than those of the old technologies and ultimately the new technologies become economically viable.[15] In the agricultural sector the introduction of the new technology represents the escape from the Malthusian trap, where wages in the agricultural sector are maintained despite an increase in population growth. In the manufacturing sector the introduction of the new technology reflects the industrial revolution and the associated increase in the demand for human capital.

3.1.1 Production of the Agricultural Good

The agricultural good can be produced by either an old technology or a new one. The output of the agricultural good produced with the old technology in period t, $Y_t^{a,o}$, is

$$Y_t^{a,0} = a_t^a (L_t^{a,0})^\gamma X^{1-\gamma}; \qquad 0 < \gamma < 1, \tag{1}$$

where $L_t^{a,o}$ is the amount of unskilled labor and X is the amount of land employed in period t in the production of agricultural good using the old technology, and a_t^a is the level of productivity of the old technology in period t. For simplicity the amount of land is normalized such that $X = 1$.

The output of the agricultural good produced with the new technology in period t, $Y_t^{a,N}$, is governed by a constant returns to scale production technology

$$Y_t^{a,N} = A_t^a L_t^{a,N}, \tag{2}$$

[15] See Hansen and Prescott (2000) and Kogel, and Prskawetz (2001) as well.

where $L_t^{a,N}$ is the amount of unskilled labor employed in the production of the agricultural good in period t using the new technology, and A_t^a is the level of productivity of the new technology in period t.

As will become apparent, in the early stages of development when the productivity of the new technology, A_t^a, is low relative to the productivity of the old technology, a_t^a, only the old technology will be employed. However in later stages of development, when A_t^a rises sufficiently relative to a_t^a, the new technology becomes economically viable.

3.1.2 Production of the Manufactured Good

The manufactured good can be produced by either an old technology or a new one. The output of the manufactured good produced with the old technology in period t, $Y_t^{m,0}$, is

$$Y_t^{m,0} = a_t^m L_t^{m,0}, \tag{3}$$

where $L_t^{m,0}$ is the amount of unskilled labor employed in period t in the production of the manufactured good using the old technology, and a_t^m is the level of productivity of the old manufacturing technology in period t.

The output of the manufactured good produced with the modern technology in period t, $Y_t^{m,N}$, is governed by a neoclassical constant returns to scale production function,

$$Y_t^{m,N} = A_t^m F(H_t^m, L_t^{m,N}) = A_t^m f(h_t^m) L_t^{m,N}, \tag{4}$$

where $h_t^m \equiv H_t^m / L_t^{m,N}$, A_t^m is the level of productivity of the new manufacturing technology in period t, and $L_t^{m,N}$ and H_t^m are the amounts of unskilled labor and skilled labor employed in the production of the manufacturing good in period t using the new technology.

As will become apparent, in early stages of development when the technological level A_t^m is low relative to a_t^m only the old manufacturing technology is economically viable. However in the process of development as A_t^m rises sufficiently relative to a_t^m, it becomes profitable for producers to employ the new manufacturing technology.

3.1.3 Factor Prices and Goods' Prices

Producers operate in perfectly competitive markets for final goods and for labor. In the absence of property rights to land, the return to land is zero and workers in the agricultural sector who use the old technology receive their average products[16].

The inverse demand for unskilled labor in the agricultural sector, given (1) and (2), is therefore

$$w_t^u = \begin{cases} p_t a_t^a (L_t^{a,0})^{\gamma-1} & \text{if} \quad Y_t^{a,0} > 0 \\ \\ p_t A_t^a & \text{if} \quad Y_t^{a,N} > 0, \end{cases} \tag{5}$$

where w_t^u is the wage of an unskilled labor in terms of the manufactured good, and p_t as the relative price of the agricultural good in terms of the manufactured good in period t.

The inverse demand for skilled and unskilled labor in the manufactured sector, given (3) and (4), is therefore

$$w_t^u = \begin{cases} a_t^m & \text{if} \quad Y_t^{m,0} > 0 \\ \\ A_t^m [f(h_t^m) - h_t^m f'(h_t^m)] \equiv A_t^m w^u(h_t^m) & \text{if} \quad Y_t^{m,N} > 0, \end{cases} \tag{6}$$

and

$$w_t^s = \; A_t^m f'(h_t^m) \equiv A_t^m w^s(h_t^m) \quad \text{if} \quad Y_t^{m,N} > 0. \tag{7}$$

Moreover,

$$\tfrac{w_t^s}{w_t^u} = \tfrac{f'(h_t^m)}{f(h_t^m) - h_t^m f'(h_t^m)} = \omega(h_t^m) \quad \text{if} \quad Y_t^{m,N} > 0, \tag{8}$$

where as follows from the neoclassical properties of the new manufacturing technology $f(h_t^m)$, $\omega'(h_t^m) < 0$.

[16] Since the fundamental mechanism explored in this paper focuses on the role of human capital accumulation and the demographic transition, rather than the role of capital and asset accumulation, in the process of development and in the emergence of sustained economic growth, this is a natural simplifying assumption. (See Galor-Weil (2000) and Jones (2000) as well). One could alternatively assume that the economy uses capital as a factor of production in agriculture and is small and open as in Galor and Weil (2000) or that land is collectively owned and the proceeds distributed lump sum across the population. Allowing for capital accumulation and property rights to land in a closed economy context would complicate the model to the point of intractability.

Since unskilled workers are costlessly mobile between the agricultural and the manufacturing sectors, the wages of unskilled labor in both sectors are equal if both goods are produced. As follows from (5) and (6), p_t, the relative price of the agricultural good in terms of the manufactured good in period t, is therefore

$$p_t = \begin{cases} \frac{a_t^m}{a_t^a (L_t^{a,0})^{\gamma-1}} & \text{if} \quad Y_t^{a,0} > 0 \text{ and } Y_t^{m,0} > 0 \\ \frac{a_t^m}{A_t^a} & \text{if} \quad Y_t^{a,N} > 0 \text{ and } Y_t^{m,0} > 0 \\ \frac{A_t^m w^u(h_t^m)}{a_t^a (L_t^{a,0})^{\gamma-1}} & \text{if} \quad Y_t^{a,0} > 0 \text{ and } Y_t^{m,N} > 0 \\ \frac{A_t^m w^u(h_t^m)}{A_t^a} & \text{if} \quad Y_t^{a,N} > 0 \text{ and } Y_t^{m,N} > 0. \end{cases} \tag{9}$$

3.2 Individuals: Fertility, Human Capital and Consumption

Individuals live for two periods. In their first period of life they consume a fraction of their parental unit time endowment; educated offspring require a larger fraction of parental time. In their second period of life they are endowed with one unit of time which they optimally allocate between child rearing and labor force participation.

3.2.1 Preferences and Budget Constraints

Individuals make optimal decisions over fertility, consumption and the training of their offspring (Becker (1976)). Individuals face a subsistence consumption constraint that they must consume a subsistence level of the agricultural good, $\widetilde{c}$.

Individual's preferences are defined over consumption and the potential aggregate income of their children[17]. The preferences of a member of generation t (i.e. an individual who is born in period $t-1$) are represented by the utility function,

$$u_t = (c_t^a)^\alpha (c_t^m)^\beta [w_{t+1}^s n_t^s + w_{t+1}^u n_t^u]^{1-\alpha-\beta}, \tag{10}$$

[17] A Stone-Geary utility function of the form: $u_t = (c_t^a - \widetilde{c})^\alpha (c_t^m)^\beta [w_{t+1}^s n_t^s + w_{t+1}^u n_t^u]^{1-\alpha-\beta}$ would generate identical qualitative results.

where c_t^a and c_t^m are the consumption of the agricultural good and the consumption of the manufactured good respectively. $\Sigma_{\{i=s,u\}} w_{t+1}^i n^i$ is the total potential income of the individual's offspring where n_t^s is the number of offspring trained to be skilled workers, n_t^u is the number of offspring trained to be unskilled workers, and w_{t+1}^s, and w_{t+1}^u are the wages paid to skilled and unskilled offspring in period $t+1$.

Individuals optimally allocate their time between labor force participation and child rearing. They further optimally choose both the number and quality of children and the amount of each good to consume. Denoting the time required to bring up a skilled offspring as, τ^s, and the time required to bring up unskilled offspring as, τ^u, where $\tau^s > \tau^u$, the budget constraint of a member i of generation t, $i = s, u$, is

$$p_t c_t^a + c_t^m + w_t^i (n_t^s \tau^s + n_t^u \tau^u) \leq w_t^i.$$

3.2.2 Optimization

A member i of generation t chooses $\{c_t^a, c_t^m, n_t^s, n_t^u\}$ so as to maximize the utility function.

$$\{c_t^a, c_t^m, n_t^s, n_t^u\} = \arg\max (c_t^a)^\alpha (c_t^m)^\beta [w_{t+1}^s n_t^s + w_{t+1}^u n_t^u]^{1-\alpha-\beta}$$

such that, for $i = s, u$, :

$$p_t c_t^a + c_t^m + w_t^i (n_t^s \tau^s + n_t^u \tau^u) \leq w_t^i;$$

$$c_t^a \geq \widetilde{c}.$$

The optimal decision depends on whether the subsistence consumption constraint is binding. If income was high enough, the constraint would not bind and the log-linearity of the utility function would imply that fixed shares of potential income are devoted to child rearing and consuming each of the two goods. However if the subsistence consumption constraint binds then a greater share of potential income must be devoted to agricultural consumption.

The consumption of the agricultural good, c_t^a, by a member i of generation t is

$$c_t^a = \begin{cases} \widetilde{c} & \text{if} \quad \alpha \frac{w_t^i}{p_t} < \widetilde{c} \\ \\ \alpha \frac{w_t^i}{p_t} & \text{if} \quad \alpha \frac{w_t^i}{p_t} \geq \widetilde{c}. \end{cases} \tag{11}$$

The consumption of the manufactured good, c_t^m, by a member i of generation t is therefore

$$c_t^m = \begin{cases} \frac{\beta}{1-\alpha}(w_t^i - p_t\widetilde{c}) & \text{if} \quad \alpha\frac{w_t^i}{p_t} < \widetilde{c} \\ \\ \beta w_t^i & \text{if} \quad \alpha\frac{w_t^i}{p_t} \geq \widetilde{c}. \end{cases} \tag{12}$$

Furthermore, the number of educated and uneducated offspring will be determined such that the aggregate time devoted by a member i of generation t to child rearing is

$$(n_t^s\tau^s + n_t^u\tau^u) = \begin{cases} \frac{1-\alpha-\beta}{1-\alpha}\frac{(w_t^i - p_t\widetilde{c})}{w_t^i} & \text{if} \quad \alpha\frac{w_t^i}{p_t} < \widetilde{c} \\ \\ (1-\alpha-\beta) & \text{if} \quad \alpha\frac{w_t^i}{p_t} \geq \widetilde{c}, \end{cases} \tag{13}$$

where,

$$\begin{array}{lll} n_t^u = 0 & \text{if} & w_{t+1}^s/w_{t+1}^u \geq \tau^s/\tau^u \\ n_t^s > 0 \text{ and } n_t^u > 0 & \textit{only if} & w_{t+1}^s/w_{t+1}^u = \tau^s/\tau^u \\ n_t^s = 0 & \text{if} & w_{t+1}^s/w_{t+1}^u < \tau^s/\tau^u. \end{array} \tag{14}$$

3.3 The Education Decision

This section demonstrates that in early stages of development, when the technological level is relatively low, individuals do not have an incentive to invest in the human capital of their offspring. However, as the level of technology improves in the process of development, the new manufacturing technology will ultimately become economically viable, human capital will be demanded and individuals will have an incentive to invest in the human capital of their offspring.

Lemma 1 *Consider the new manufacturing sector. There exists a unique ratio of skilled to unskilled labor, $(h^m)^*$, such that*

$$\frac{w_t^s}{w_t^u} = \omega((h^m)^*) = \frac{\tau^s}{\tau^u}.$$

where,

$$n_t^u = 0 \quad \textit{if} \quad h_t^m < (h^m)^*$$

$$n_t^s = 0 \quad \textit{if} \quad h_t^m > (h^m)^*.$$

Proof. As established in (8), $\omega'(h_t^m) < 0$. and uniqueness of $(h^m)^*$ follows. The remaining part is a corollary of (14). □

Hence, if $h_{t+1}^m < (h^m)^*$ then individuals would not have an incentive to raise unskilled offspring and the skilled to unskilled ratio will increase, whereas if $h_{t+1}^m > (h^m)^*$ then individuals would not have an incentive to raise skilled offspring and the skilled to unskilled ratio will decline till $h_{t+1}^m = (h^m)^*$.

Corollary 1

$$h_t^m = (h^m)^* \quad \text{if} \quad Y_t^{m,N} > 0,$$

and therefore

$$w_t^u = A_t^m w^u((h^m)^*) \quad \text{if} \quad Y_t^{m,N} > 0.$$

The new manufacturing technology will be able to employ unskilled workers if the wage paid in this sector $A_t^m w^u((h^m)^*)$ is at least as high as the wage paid to unskilled workers in the old manufacturing technology, a_t^m. Hence,

Lemma 2 *The new manufacturing technology is economically viable if*[18]

$$\frac{A_t^m}{a_t^m} \geq 1/[w^u((h^m)^*)].$$

Proof. $Y_t^{m,N} > 0$ if the marginal productivity of unskilled labor in the new manufacturing sector is at least as high as in the old manufacturing sector. Hence the lemma follows from (6) and Corollary 1. □

3.4 Aggregate Labor Allocation

Since preferences are such that both goods are consumed in every period, in autarky both goods must be produced in every period. Hence an equilibrium in the goods market requires that, in a given technological state, the demand for the agricultural and the industrial goods given by (11) and (12) equal the supply of the two goods given by (1)-(4).

[18] When $A^m w^u(h_m^*) = a^m$ then there is indeterminacy in the choice of how many skilled and unskilled offspring to produce. This indeterminacy can be resolved by assuming that *ceteris paribus* parents prefer educated children. The indeterminacy resolves itself after one period in any case as technology progresses.

Lemma 3 *If both goods are produced with the old technology*

(a) The employment of labor in the agricultural sector is

$$L_t^{a,0} = \begin{cases} [\frac{\widetilde{c}}{a_t^a} N_t]^{1/\gamma} & \text{if} \quad \alpha \frac{w_t^u}{p_t} < \widetilde{c} \\ \\ \alpha N_t & \text{if} \quad \alpha \frac{w_t^u}{p_t} \geq \widetilde{c}. \end{cases}$$

(b)The employment of labor in the manufacturing sector is

$$L_t^{m,0} = \begin{cases} \frac{\beta}{1-\alpha}(N_t - [\frac{\widetilde{c}}{a_t^a} N_t]^{1/\gamma}) & \text{if} \quad \alpha \frac{w_t^u}{p_t} < \widetilde{c} \\ \\ \beta N_t & \text{if} \quad \alpha \frac{w_t^u}{p_t} \geq \widetilde{c}. \end{cases}$$

(c) The aggregate time devoted to child rearing is

$$n_t^u \tau^u = \begin{cases} \frac{(1-\alpha-\beta)}{(1-\alpha)}(N_t - [\frac{\widetilde{c}}{a_t^a} N_t]^{1/\gamma}) & \text{if} \quad \alpha \frac{w_t^u}{p_t} < \widetilde{c} \\ \\ (1-\alpha-\beta) N_t & \text{if} \quad \alpha \frac{w_t^u}{p_t} \geq \widetilde{c}. \end{cases}$$

Proof. Follows from (11)-(13), (1) and (3), noting that $\frac{w_t^u}{p_t} = a_t^a (L_t^{a,0})^{\gamma-1}$. □

4 The Time Path of Macroeconomic Variables

4.1 Technological Progress

Suppose that the technological level in period $t+1$, λ_{t+1}, is affected positively by the skill intensity of the economy in period t, $h_t \equiv H_t/L_t$ and by the technological level in period t, λ_t.

$$\lambda_{t+1} = \phi(\lambda_t, h_t), \tag{15}$$

where $\phi(\lambda_t, h_t)$ is an increasing function and $\phi(\lambda_t, 0) > \lambda_t \ \ \forall \lambda_t$.

Suppose that the productivity levels in each sector are functions of the technological level in the economy as a whole. Namely, the productivity of the old and the new technologies in the agricultural sector, a, and the manufacturing sector, m, are

$$\begin{aligned} A_t^j &= A^j(\lambda_t) \\ & \qquad\qquad\qquad j = a, m \\ a_t^j &= a^j(\lambda_t) \end{aligned} \tag{16}$$

where, $dA^j/d\lambda > 0$ and $da^j/d\lambda > 0$, $j = a, m$.

The productivity parameters are restricted so as to assure that the process of technological progress is consistent with its historical patterns:

(a) The new manufacturing and agricultural technologies are not economically viable in period 0, i.e.,

$$\frac{A_0^m}{a_0^m} < 1/[w^u((h^m)^*)];$$
$$\frac{A_0^a}{a_0^a} < (L_0^{a,0})^{\gamma-1} = (\frac{\tilde{c}N_0}{a_0^a})^{\frac{\gamma-1}{\gamma}} \tag{A1}$$

where $N_0 > 0$ is the initial size of the adult population.[19]

(b) The advancement in the productivity of the manufacturing sector is higher than that in the agricultural sector, and the new technologies advance more rapidly that the old one, i.e.,

$$\frac{dA^m(\lambda_t)}{d\lambda_t} > \frac{dA^a(\lambda_t)}{d\lambda_t} > \frac{da^m(\lambda_t)}{d\lambda_t} > \frac{da^a(\lambda_t)}{d\lambda_t} > 0; \quad \lim_{\lambda_t \to \infty} \frac{Aj(\lambda_t)}{a^j(\lambda_t)} = \infty \quad j = a, m. \tag{A2}$$

Condition A2 ensures that a more technologically advanced economy has a comparative advantage in the industrial good[20].

Let $(t^m)^*$ be the time period in which the new manufacturing technology becomes economically viable, i.e., .

$$\frac{A_t^m}{a_t^m} \geq 1/[w^u((h^m)^*)] \quad \forall t \geq (t^m)^*,$$

and let $(t^a)^*$ be the time period in which the new agricultural technology becomes economically viable, i.e.,

$$\frac{A_t^a}{a_t^a} \geq ((L_t^{a,0})^*)^{\gamma-1} \qquad \forall t \geq (t^a)^*,$$

where $(L_t^{a,0})^*$ which is the level of employment in the old agricultural sector necessary for the old agricultural sector alone to satisfy the total demand for agricultural products at time t.

[19] The last equality follows from Lemma 3.

[20] Condition A2 also has the implication in this model that the relative price of agricultural goods is monotonically increasing over time, see equation (9) which may appear counter-factual. This problem can be rectified by assuming that the cost of acquiring skills is decreasing through time, (i.e. τ^s/τ^u is decreasing with λ.) In this case the relative price of agricultural goods may be decreasing through time in all economies, but decreasing more slowly in the advanced economy, thus giving it a comparative advantage in the manufactured good. However, in the context of the current model, the assumption of a fixed τ^s/τ^u is a reasonable simplifying assumption.

The existence of these time periods depends on the analysis of the equilibrium dynamics of the economy which is derived below. Thus the existence of $(t^a)^*$ and $(t^m)^*$ will be shown in the appendix. In order to simplify the exposition the new agricultural technology is assumed to become economically viable before the new manufacturing technology, i.e.,

$$(t^a)^* < (t^m)^*, \tag{A3}$$

4.2 Human Capital Accumulation

The ratio of skilled to unskilled labor employed in the economy at time $t+1$, h_{t+1}, depends on the level of λ_{t+1} and is thus not a dynamic relationship in itself, although agents' fertility and human capital accumulation decisions are by definition intertemporal. Parents' fertility decisions in the period t are based on their rational expectation of the relative wage rate of skilled and unskilled labor in the period $t+1$, w^u_{t+1}/w^s_{t+1}, which in turn depends on the level of technology in the period $t+1$, λ_{t+1}. Hence,

$$h_{t+1} = h(\lambda_{t+1}), \tag{17}$$

4.3 Population Dynamics

The level of adult population in period $t+1$, N_{t+1}, depends on three variables: the adult population in period t, N_t, the income level and income distribution of the adult population in period t, which are determined by λ_t, and the demand for skilled and unskilled workers in period $t+1$, which are determined by λ_{t+1}. Hence,

$$N_{t+1} = n(\lambda_{t+1}, \lambda_t, N_t). \tag{18}$$

5 The Dynamical System and the Evolution of the Economy

The dynamical system of the economy is given by equations (15) and (18), noting (17). This section shows how this dynamical system will evolve though qualitatively distinct stages as the time period crosses the thresholds, $(t^a)^*$ and $(t^m)^*$. As will become apparent the first stage is a pre industrial Malthusian stage and the final stage is a modern industrial stage. In between these two stages, growth gradually increases and the population growth rate first rises then falls.

5.1 The Malthusian Stage

When $t < (t^a)^*$ the new technology in both sectors is not economically viable. The economy will tend towards a state where agents are constrained in their choices by the subsistence consumption constraint. The share of the agricultural sector in production is thus higher than in subsequent stages and the budget share of manufactured goods is lower. As the new manufacturing technology is not economically viable, there is no demand for skilled labor and there is thus no human capital accumulation. The rate of technological progress is therefore slow since $\lambda_{t+1} = \phi(\lambda_t, 0)$. This accords with stylised facts for Europe before the industrial revolution, see Maddison (1982, 1995).

Population Dynamics in the Malthusian Stage

In the Malthusian stage since the new production technology in the manufacturing sector is not economically viable there is no demand for skilled labor. Parents therefore only rear unskilled children. The old agricultural production technology has a fixed factor of production - land - and so there are decreasing returns to scale to labor. Thus for a given level of technology, as population rises, the land-labor ratio falls, and wages fall. As stated in the following Lemma, under reasonable conditions this will be a stable process whereby population tends to a steady state level for a given level of technology.

Lemma 4 *In the Malthusian stage, if technology is stationary, the population will converge to a steady state level, if (i)* $(1-\alpha-\beta)/\tau^u > 1$, *(ii)* $\gamma > [(1-\alpha-\beta)-(1-\alpha)\tau^u]/(1-\alpha)\tau^u$, *and (iii)* $N_0 < (a_0^a/\widetilde{c})^{\frac{1}{1-\gamma}}$.

Proof. From Lemma 3, the population dynamic in the Malthusian stage can be written

$$N_{t+1} = \begin{cases} \frac{(1-\alpha-\beta)}{(1-\alpha)\tau^u}[1 - (\frac{\widetilde{c}}{a_t^a})^{\frac{1}{\gamma}} N_t^{\frac{1-\gamma}{\gamma}}] N_t & \text{if} \quad \alpha \frac{w_t^u}{p_t} < \widetilde{c} \\ \frac{(1-\alpha-\beta)}{\tau^u} N_t & \text{if} \quad \alpha \frac{w_t^u}{p_t} \geq \widetilde{c}, \end{cases} \tag{19}$$

Condition (i) ensures that when agents are unconstrained and are rearing only unskilled children, the population is rising. From the properties of the old agricultural production technology, this implies that the economy will eventually be in a state where its agents are constrained by the subsistence constraint.

The rate of change of the population level next period, N_{t+1}, with respect to the population level this period, N_t, is decreasing in N_t when agents are constrained by the subsistence constraint. This can be seen by differentiating the first expression of equation (19),

$$\frac{dN_{t+1}}{dN_t} = \frac{(1-\alpha-\beta)}{(1-\alpha)\tau^u}[1 - \frac{1}{\gamma}(\frac{\widetilde{c}}{a_t^a})^{\frac{1}{\gamma}} N_t^{\frac{1-\gamma}{\gamma}}]$$

At a steady state from equation (19), $(\frac{\widetilde{c}}{a_t^a})^{\frac{1}{\gamma}} N_t^{\frac{1-\gamma}{\gamma}} = 1 - \frac{(1-\alpha)\tau^u}{(1-\alpha-\beta)}$. This implies the unskilled wage, $a_t^a (L_t^{a,0})^{\gamma-1}$ is also constant and equal to $\widetilde{c}(1-\alpha-\beta)/[(1-\alpha-\beta)-(1-\alpha)\tau^u]$. Hence,

$$\frac{dN_{t+1}}{dN_t} \mid_{N_{t+1}=N_t} = \frac{(1-\alpha-\beta)}{(1-\alpha)\tau^u}[1 - \frac{1}{\gamma}[1 - \frac{(1-\alpha)\tau^u}{(1-\alpha-\beta)}]]$$

For the population process to be globally stable the slope must be greater than 0 and less than 1 at the steady state. Thus we require that

$$0 < \frac{dN_{t+1}}{dN_t} \mid_{N_{t+1}=N_t} < 1$$

which follows from condition (ii) after rearranging. Condition (iii) simply assumes that the initial level of population is not so large as to cause the initial average product of labor to be below the subsistence level. □

Technological progress has no effect on the real wage rate, $a_t^a (L_t^{a,0})^{\gamma-1}$, and just allows for a larger level of population. This is depicted in Figure 1. Technological progress initially causes output per worker to increase which in turn increases wages and fertility and so causes the population to rise. The average product of labor thus falls and in the absence of further technological progress, real wages fall back to the long run level of $\widetilde{c}(1-\alpha-\beta)/[(1-\alpha-\beta)-(1-\alpha)\tau^u]$.

This stage persists until the new agricultural technology becomes economically viable at $(t^a)^*$. Lemma 4 implies that if the demand for the agricultural good grows at least as fast after the new technologies become economically viable as they would have done if there were no new technologies, then the level of $a_t^a((L_t^{a,0})^*)^{\gamma-1}$ will be at or below $\widetilde{c}(1-\alpha-\beta)/[(1-\alpha-\beta)-(1-\alpha)\tau^u]$ and thus $(t^a)^*$ as defined above will exist. Since the new technology increases wages, increases the resources devoted to fertility and doesn't decrease the demand per capita for agricultural goods then this condition is likely to hold.[21]

5.2 The Population Expansion Stage

When $(t^a)^* < t < (t^m)^*$ the new technology in the manufacturing sector is not economically viable and so there is still no demand for skilled labor. Parents thus only rear unskilled offspring. The market equilibrium is very similar to that in the Malthusian stage except that in this stage the new agricultural production technology is economically viable and so the expression $a_t^a(L_t^{a,0})^{\gamma-1}$ is substituted by the expression A_t^a.

The important difference between this stage and its predecessor is that the Malthusian check on the economy is no longer present. In this stage increased population does not reduce the real wage and so from this point onwards the unskilled wage rises with the level of technology. From equations (11), (12) and (13) this implies that the budget share devoted to fertility and manufactured goods will increase. This implies that population and the rate of growth of

[21] For some parameter specifications the fall in fertility caused by the introduction of the new industrial technology reduces fertility so much and for so long that the population falls below the level it would have attained if the economy had continued on its Malthusian path without the new technologies. If this occurs then we cannot rule out the possibility that $a_t^a((L_t^{a,0})^*)^{\gamma-1}$ rises above A^a. We regard this as a highly unlikely and very counterfactual (From McEvedy and Jones's (1978) analysis, the population of the British Isles grew from 5 million in 1500 to 10 million in 1750. If growth continued at this rate then the current population of the British Isles would have been 20 million, much below its current actual level of approximately 60 million). We thus rule it out with the following assumption The population at the beginning of the modern industrial stage is greater than it would have without the existence of the new production technologies. This is a sufficient condition for the existence of a $(t^a)^*$.

population increases throughout this stage according to the equation

$$N_{t+1} = \frac{(1-\alpha-\beta)}{(1-\alpha)\tau^u}(1 - \frac{\widetilde{c}}{A_t^a})N_t$$

5.3 Industrialization and Demographic Transition

When $t > (t^m)^* > (t^a)^*$ the new manufacturing technology is economically viable and there is a demand for skilled labor. The ratio of skilled to unskilled labor employed in the economy at time $t+1$, h_{t+1}, is determined by the fertility decisions of the adult agents in period t, which are based on the expected relative wage rates of skilled and unskilled labor in period $t+1$, w_{t+1}^u/w_{t+1}^s, and also by the demand side of the economy.

We show below in Lemma 5 that there is a unique market clearing level of h_{t+1} for all values of λ_{t+1}. We present the initial case where λ_{t+1} is such that both skilled and unskilled workers will be constrained by the subsistence constraint. The analysis for the subsequent case where unskilled workers will be constrained by the subsistence constraint but skilled workers will not, follows trivially from this analysis.

Lemma 5 *When both sectors' new technology is economically viable and both skilled and unskilled workers will be constrained by the subsistence constraint in period* $t+1$, *there exists a unique market clearing level of* h_{t+1} *which will be rationally expected by agents making their period* t *fertility decisions*

Proof. See Appendix. □

Lemma 5 shows that there exists a unique relationship between h_{t+1} and λ_{t+1}. The following lemma shows that the level of h_{t+1} is non-decreasing in λ_{t+1}.

Lemma 6 *The ratio of skilled to unskilled labor employed in the economy at time* t, h_t, *is non-decreasing in the level of technology,* λ_t. *When some agents are constrained in their decisions by the subsistence constraint* h_t *is increasing in the level of* λ_t. *When no agent is constrained by the subsistence constraint* h_t *is a constant and is unaffected by the level of* λ_t.

Proof. This follows from the agents first order conditions, equations (11), (12) and (13). They imply that for all constrained agents the higher their wage the larger their budget share devoted to manufactured goods. Thus an increase in λ_t implies that the equilibrium ratio of the value of manufactured goods to agricultural goods produced must also increase. That is the following ratio must increase

$$\frac{Y_t^m}{p_t Y_t^a} = \frac{A_t^m f((h^m)^*) L_t^{m,N}}{p_t A_t^a L_t^a} = \frac{f((h^m)^*) L_t^{m,N}}{w^u((h^m)^*) L_t^a}$$

where L_t^a is the total employment of labor in the agricultural sector. This implies an increase in $L_t^{m,N}$. However an increase in $L_t^{m,N}$ also implies an increase in h_t since from the definition of h_t,

$$h_t \equiv \frac{H_t}{L_t} = \frac{(h^m)^* L_t^{m,N}}{L_t^a + L_t^{m,N}}$$

where the latter equality uses the fact that in equilibrium the ratio of H_t to $L_t^{m,N}$ always equals $(h^m)^*$. Thus when any agents are constrained an increase in λ_t implies an increase in h_t.

When agents aren't constrained equations (11), (12) and (13) show that the budget share of manufactured goods is unaffected by λ_t and so increases in λ_t have no effect on h_t. $\square$

In this stage there is thus a self-reinforcing relationship between technological progress and the human capital intensity of the economy which causes both the rate of technological progress and the level of human capital accumulation in the economy to rise. From equation (15) an increased level of h_t increases λ_{t+1} and from Lemma 6 an increase in λ_{t+1} increases h_{t+1}. This process creates two opposing forces on the population growth rate as the following section describes.

Population Dynamics in the Stage of Demographic Transition

The growing levels of technology and skill intensity apply two conflicting pressures on the rate of population growth. On the one hand they increase in the wage level which leads to an increase in the budget share of manufactured goods which in turn raises the demand for skilled workers and so tends to lower the fertility rate since $\tau^s > \tau^u$. However on the other hand increases in the wage level also allows more resources to be devoted to raising children which exerts a positive influence on the fertility rate. In corollary 2 below we show that we can restrict parameters in this model so that fertility has the inverted 'U' shaped relationship with income per capita that has been observed in many developed economies.

The equilibrium number of children produced in this stage is straightforward to derive given the unique equilibrium level of h_{t+1}, and assuming symmetry across agents. This is done in the following proposition where we will denote the number of skilled children produced by an agent i with wage, w_t^i for $i = u, s$, by $n_t^{i,s}$; the number of unskilled children produced by an agent i, by $n_t^{i,u}$; the total number of children produced by an agent i, by n_t^i, and the equilibrium supply of labor of skilled and unskilled agents as l_{t+1}^s and l_{t+1}^u, respectively. Again we present the initial case where λ_{t+1} will be such that both skilled and unskilled workers will be constrained by the subsistence constraint. The analysis for the subsequent case where unskilled workers will be constrained by the subsistence constraint but skilled workers will not, follows trivially from this analysis.

Lemma 7 *When both sectors' new technology is economically viable and the subsistence constraint is binding for skilled and unskilled workers in period $t+1$, then under symmetry, the fertility decisions of agents in period t are described by the following two equations,*
(i) The ratio of skilled to unskilled offspring of an agent i, $n_t^{i,s}/n_t^{i,u}$, is

$$n_t^{i,s}/n_t^{i,u} = h_{t+1}\frac{l_{t+1}^u}{l_{t+1}^s}.$$

(ii) The total number of offspring of an agent i, n_t^i, is given by the equation,

$$n_t^i = (1-\alpha-\beta)(1-\frac{p_{t+1}\widetilde{c}}{w_{t+1}^i})(1+h_{t+1}\frac{l_{t+1}^u}{l_{t+1}^s})/[(1-\alpha)(\tau^u + h_{t+1}\frac{l_{t+1}^u}{l^{t+1^s}}\tau^s)].$$

Proof. The level of h_{t+1} together with the assumption of symmetry determines the ratio of $n_t^{i,s}$ to $n_t^{i,u}$ since $h_{t+1} = n_t^{i,s}l_{t+1}^s/n_t^{i,u}l_{t+1}^u$. This directly implies condition (i) of the proposition. Thus $n_t^i = [1+h_{t+1}(l_{t+1}^u/l_{t+1}^s)]n_t^{i,u}$ and similarly $(n_t^{i,u}\tau^u + n_t^{i,s}\tau^s) = n_t^{i,u}[\tau^u + h_{t+1}(l_{t+1}^u/l_{t+1}^s)\tau^s]$. This together with the first order condition, equation(13) implies condition (ii). $\square$

This lemma demonstrates, as stated above, that there are two opposing forces on the population growth rate in this stage of development. On the one hand the increase in the wage level leads to an increase in the budget share of manufactured goods, which raises the demand for skilled workers. Since $\tau^s > \tau^u$, this exerts a negative influence on the fertility rate. On the other hand the increase in the wage level also allows more resources to be devoted to raising children which exerts a positive influence on the fertility rate. The following corollary shows that it is possible to restrict parameters so that fertility has an inverted 'U' shaped relationship with income per capita over time.

Corollary 2 *When both sectors' new technology is economically viable, the rate of population growth will eventually fall towards a lower level if* $[(1-\alpha-\beta)(1+\widetilde{h})-(\tau^u+\widetilde{h}\tau^s)]$ *is sufficiently small and positive, where* $\widetilde{h} = (h^m)^*/[1+\frac{\alpha}{\beta}\frac{f((h^m)^*)}{w^u((h^m)^*)}]$.

Proof. As λ_{t+1} increases real wages increase and eventually no agents will be bound by the subsistence constraint. In this case condition (ii) of Lemma7 becomes $n_t^i = (1-\alpha-\beta)(1+\widetilde{h})/(\tau^u+\widetilde{h}\tau^s)$. $\square$

5.4 The Modern Industrial Stage

When the level of technology rises sufficiently for the subsistence constraint not to bind for any agent, the economy reaches a state where the population growth rate and the skill intensity of the economy are constant. Since no agent is bound by the subsistence constraint, the budget share devoted to manufactured goods and the level of human capital accumulation will be higher than in the previous stages and from corollary 2 the fertility rate will be low. This implies that growth will also, *ceteris paribus,* be higher since from equation (15) $\lambda_{t+1} = \phi(\lambda_t, h_t)$ where $\phi(\lambda_t, h_t)$ is an increasing function of both arguments. This is described in the following proposition.

Proposition 1 *When both sectors' new technology is economically viable and neither skilled and unskilled workers are constrained by the subsistence constraint, the economy is in a state of balanced growth with a constant population growth rate and skill intensity.*

Proof. From the first order conditions, equations (11) and (12), the demand ratio $p_t c_t^a / c_t^m = \alpha/\beta$. Thus using the same analysis as in Lemma 5 the unique equilibrium level of h_{t+1} equals $\widetilde{h}, \forall t$ where $\widetilde{h}$ is the constant defined above in corollary 7. Also following the same analysis as in lemma 7 and using the first order condition equation (13) the total fertility of all agents is given by $n = (1-\alpha-\beta)(1+\widetilde{h})/(\tau^u + \widetilde{h}\tau^s)$. □

Corollary 3 *When both sectors' new technology is economically viable and neither skilled and unskilled workers are constrained by the subsistence constraint, the budget share devoted to manufactured goods and the level of human capital accumulation will be higher than in the previous stages and the fertility rate will be low.*

Proof. This follows from the first order conditions, equations (11), (12) and (13), lemma 6 and corollary 2. □

6 The Effect of International Trade

This section will show how international trade can accelerate a relatively technologically advanced economy's path through its transitional phase to a modern industrial economy and conversely prolong the time spent by a relatively less advanced economy in its transitional phase, perhaps indefinitely. We will examine the case where trade takes place after the new production technologies become viable. We will see that the world economy behaves very much like a standard Ricardian model of international trade and so there is large tendency for economies to specialize production in one of the two goods.

Consider two economies denoted A and B, which are identical in every respect except for their level of technology. Assume without loss of generality that economy B is more technologically advanced than economy A. From above we know that the autarkic relative price of the agricultural good in economy B, denoted, p^B, will be greater than that in economy A, p^A. Under international trade the equilibrium relative price, p^* must lie between p^A and p^B and at least one economy will be specialized in production. This is shown in the following proposition.

Proposition 2 *Under international trade at least one of economies A and B will be specialized in production.*

Proof. To see this note that from section 2.1, for each level of technology there is a unique level of p such that both goods are produced. In a closed economy this condition determines the autarkic level of p. If the level of p under international trade, denoted $p^{\star}$, is greater than its autarkic relative price then an economy will specialize in the agricultural good. Conversely if the level of p^* is less than its autarkic relative price then the economy will specialize in the industrial good. Defining the offer curve for the manufacturing good of economy i, denoted O^i_m, as the amount of Y^m produced minus the amount of Y^m consumed in economy i for a given relative price, then from above it is clear that O^i_m will be positive for levels of p^* below p^i and O^i_m will be negative for levels of p^* above p^i. This is depicted in Figure 2. International equilibrium is where $O^B_m + O^A_m = 0$ and is thus where the curves O^B_m and -O^A_m cross. The relative price under international trade, p^* lies between the two autarkic relative prices, p^A and p^B and thus from above at least one economy will be specialized in production. $\square$

Proposition 3 *International trade will initially decrease the rate of population growth of economy B and increases the rate of population growth of economy A.*

Proof. This follows from Proposition 2. From above we know that $p^A \leq p^* \leq p^B$ and thus that the production of the agricultural good in economy A must rise significantly under international trade and thus the level of h in the economy will fall which will cause the rate of population growth to rise. The opposite effect will occur in economy B where h will increase towards $(h^m)^*$ and will equal $(h^m)^*$ if $p^* < p^B$. $\square$

The initial effect of international trade will persist through time so that the initially relatively less advanced will become even less relatively advanced through time. This is shown in the following Proposition and Corollary.

Proposition 4 *The initial patterns of Comparative Advantage will persist through time.*

Proof. This follows from equation (15). The increased level of h will increase the rate of technological progress in economy B and the reduced level of h will decrease the rate of technological progress in economy A and so using (A2) maintains the initial pattern of comparative advantage. □

Corollary 4 *International trade will cause the initially relatively less technologically advanced economy to fall further technologically behind*

Proof. This follows directly from proposition 4 . □

Given this it follows that international trade will accelerate a relatively technologically advanced economy's path through its transitional phase to a modern industrial economy and conversely prolong the time spent by a relatively less advanced economy in its transitional phase, perhaps indefinitely. This is shown in the following proposition.

Proposition 5 *International trade will accelerate the demographic transition of economy B and will delay, perhaps indefinitely, the demographic transition of economy A.*

Proof. Following Propositions 2 and 4 international trade will cause economy A to specialize in the production of the agricultural good. This causes the level of h to be lower in economy A than in economy B and thus the rate of technological progress will be slower in economy A than economy B's. The total income of economy A relative to that of the world will be growing if its population growth rate is sufficiently high to compensate for its lower technological progress and vice versa. If the relative share of income of economy A in the world economy falls through time then economy A could completely specialize in agricultural production for ever and never demand skilled workers and so may never go through a demographic transition. Alternatively if the relative share of income of economy A in the world economy rises through time then at some point the output of the manufactured good from economy B will be insufficient to meet world demand and economy A will begin demanding skilled workers and eventually may go though a demographic transition. For economy B international trade unambiguously increases the rate of technological progress and so hastens the increase in demand for skilled labor and so accelerates the demographic transition □

7 Concluding Remarks

This research argues that sustained differences in population growth and income levels across countries as well as the current distribution of world population can be attributed, in part, to the contrasting effects that international trade played in the timing of the demographic transition in industrial and non-industrial countries. In industrial economies international trade enhanced the specialization in the production of skilled-intensive goods and stimulated technological progress. The rise in the demand for skilled labor induced an investment in the quality of the population, expediting the demographic transition, stimulating technological progress and further enhancing the comparative advantage of these industrial economies in the production of skilled intensive goods. In non-industrial economies, in contrast, the specialization in the production of unskilled-intensive goods that was brought about by international trade reduced the demand for skilled labor and provided limited incentives to invest in population quality. The demographic transition was therefore delayed, increasing further the abundance of unskilled labor in these economies and enhancing their comparative disadvantage in the production of skilled intensive goods. International trade has therefore widened the gap between the technological level as well as the skill abundance of industrial and non-industrial economies, enhancing the initial patterns of comparative advantage and generating sustained differences in income per capita across countries. The asymmetric effect of international trade on the timing of the demographic transition in developed and less-developed economies, and its persistent effect therefore on the initial patterns of comparative advantage, may suggest that the rapid transition of the currently developed economies into a state of sustained economic growth may be the prime cause of the slow transition of less developed economies into a state of sustained economic growth.

The economic history of the UK, India and China is consistent with the thesis that international trade played a significant role in the timing of the demographic transition and in the process of industrialization. Historical evidence suggests that during the nineteenth century the intensive trade relationship between India and the, technologically superior, UK lead to a regression in industrialization in India and acceleration in the industrialization in the UK. Whereas the UK experienced an impressive increase in the level of education throughout the 19th century and a demographic transition towards the end of the century, in India the demographic transition has been delayed and its comparative advantage in the production of labor-intensive goods has been enhanced. Furthermore, China could have taken advantage of superior technology by trading internationally. China's near abstention from international trade during this period, has apparently lead to a decline in China's degree of industrialization over this period, and has subsequently delayed its demographic transition and its relative position in the world income distribution.

The proposed model abstracts from several factors that are relevant for the effect of international trade on population growth and the process of development. The adverse effect of international trade on industrialization and thus on the timing of the demographic transition may be mitigated by the positive effect of trade on technological diffusion across countries and on the demand for manufactured goods associated with a rise in the level of income in the short run. If, however, the rate of technological diffusion depends upon the existing stock of factor endowments (e.g., Susanto Basu and David N. Weil, 1998, and Daron Acamoglu and Fabrizio Zilibotti, 2001) then the adverse effect of trade on factor endowment would be enhanced by the slower rate of technological diffusion. In either case, technological diffusion would not alter the patterns of comparative advantage among the initial trading partners. Moreover, cultural and institutional differences between countries in the determination of population growth, public provision of education, and in the process of technological change would be reflected in the timing of their demographic transition and their patterns of comparative advantage.

Appendix

In this appendix we place the proof of two lemmas: lemma 5 which derives the existence of a unique market clearing level of h_{t+1} during the stage of demographic transition and lemma A1 which proves the existence of $(t^a)^*$ and $(t^m)^*$.

Lemma 5 *When both sectors' new technology is economically viable and both skilled and unskilled workers will be constrained by the subsistence constraint in period* $t+1$, *there exists a unique market clearing level of* h_{t+1} *which will be rationally expected by agents making their period* t *fertility decisions*

Proof. λ_{t+1} is forecastable given period t information. Once λ_{t+1} is known then so are p_{t+1}, w^s_{t+1} and w^u_{t+1} - this follows from equations (6),(7) and (9). This implies, from equation (13), that the equilibrium supply of labor of skilled and unskilled agents, denoted l^s_{t+1} and l^u_{t+1}, respectively, are also known and equal to, $l^i_{t+1} = 1 - \frac{1-\alpha-\beta}{1-\alpha}(1 - \frac{p_{t+1}\widetilde{c}}{w^i_{t+1}})$ for $i = u, s$. Using these variables we can define, h_{t+1}, in the following way

$$h_{t+1} \equiv \frac{H_{t+1}}{L_{t+1}} = \frac{N^s_{t+1} l^s_{t+1}}{N^u_{t+1} l^u_{t+1}} \tag{20}$$

where N^u_{t+1} and N^s_{t+1} are the numbers of unskilled and skilled workers in the economy respectively.

The ratio of demand of agricultural goods to manufactured goods in the whole economy in period $t+1$ can be written using the first order conditions, equations (11) and (12) as,

$$\frac{p_{t+1}c^a_{t+1}}{c^m_{t+1}} = \frac{p_{t+1}\widetilde{c}[N^u_{t+1} + N^s_{t+1}]}{\frac{\beta}{1-\alpha}[(w^u_{t+1} - p_{t+1}\widetilde{c})N^u_{t+1} + (w^s_{t+1} - p_{t+1}\widetilde{c})N^s_{t+1}]}$$

Equation (20), implies that $N^s_{t+1} = N^u_{t+1} h_{t+1} \frac{l^u_{t+1}}{l^s_{t+1}}$. Using this and recognising that in equilibrium $w^s_{t+1} = \frac{\tau^s}{\tau^u} w^u_{t+1}$ and $w^u_{t+1} = p_{t+1} A^a_{t+1}$, we can rewrite the expression for the demand ratio as,

$$\frac{p_{t+1}c^a_{t+1}}{c^m_{t+1}} = \frac{\widetilde{c}(1 + \frac{l^u_{t+1}}{l^s_{t+1}} h_{t+1})}{\frac{\beta}{1-\alpha}[A^a_{t+1}(1 + \frac{\tau^s}{\tau^u}\frac{l^u_{t+1}}{l^s_{t+1}} h_{t+1}) - \widetilde{c}(1 + \frac{l^u_{t+1}}{l^s_{t+1}} h_{t+1})]} \equiv \frac{d + e h_{t+1}}{f + g h_{t+1}}$$

where d, e, f and g are positive constants. The latter equality follows from the fact that $A^a_{t+1} > \widetilde{c}$ and $\tau^s > \tau^u$.

The supply ratio of the economy will be given by the following expression

$$\frac{p_{t+1}Y^a_{t+1}}{Y^m_{t+1}} = \frac{p_{t+1}A^a_{t+1}L^a_{t+1}}{A^m_{t+1}f((h^m)^*)L^{m,N}_{t+1}} = \frac{w^u((h^m)^*)L^a_{t+1}}{f((h^m)^*)L^{m,N}_{t+1}}$$

where L^a_{t+1} and $L^{m,N}_{t+1}$ are the amount of hours of unskilled labor used in the agricultural and manufacturing sectors respectively.

In autarkic equilibrium the supply and demand ratios must be equal. This implies that

$$L^a_{t+1} = L^{m,N}_{t+1}\frac{f(h^{m*})}{w^u((h^m)^*)}\frac{(d+eh_{t+1})}{(f+gh_{t+1})}$$

Market clearing in the labor market implies $L_{t+1} = L^a_{t+1} + L^{m,N}_{t+1} = N^u_{t+1}l^u_{t+1}$ and as noted above $H_{t+1} = N^s_{t+1}l^s_{t+1} = (h^m)^*L^{m,N}_{t+1}$. Thus using equation (20) it follows that

$$h_{t+1} = \frac{H_{t+1}}{L_{t+1}} = \frac{L^{m,N}_{t+1}(h^m)^*}{L^{m,N}_{t+1}[1+\frac{f(h^{m*})}{w^u((h^m)^*)}\frac{(d+eh_{t+1})}{(f+gh_{t+1})}]}$$

which can be simplified to

$$h_{t+1} = \frac{(h^m)^*(f+gh_{t+1})}{(f+gh_{t+1}) + \frac{f(h^{m*})}{w^u((h^m)^*)}\widetilde{c}(1+\frac{l^u_{t+1}}{l^s_{t+1}}h_{t+1})} = \frac{D+Eh_{t+1}}{F+Gh_{t+1}}$$

where D, E, F and G are positive constants. This gives us a quadratic expression in h_{t+1} that has only one positive root which will be the unique equilibrium level of h_{t+1}. $\square$

Lemma A1 *Under A1, A2 and A4*

(a) there exists a time period $(t^m)^*$ in which the new manufacturing technology becomes economically viable, i.e.,

$$\frac{A^m_t}{a^m_t} \geq 1/[w^u((h^m)^*)] \qquad \forall t \geq (t^m)^*.$$

(b) there exists a time period $(t^a)^*$ in which the new agricultural technology becomes economically viable, i.e.,

$$\frac{A^a_t}{a^a_t} \geq (L^{a,0}_t)^{\gamma-1} \qquad \forall t \geq (t^a)^*.$$

Proof. (a) Follows from (A1), (A2) and Lemma 2 noting that $\phi(\lambda_t, 0) > \lambda_t \ \ \forall \lambda_t$. (b) Lemma 4 shows that under the old technology the unskilled wage, $a_t^a(L_0^{a,0})^{\gamma-1}$ tends to the constant level of $\widetilde{c}\frac{(1-\alpha-\beta)}{(1-\alpha-\beta)-(1-\alpha)\tau^u}$. However from (A2) A^a is rising through time. Therefore at some time period, t, $A_t^a > a_t^a(L_0^{a,0})^{\gamma-1}$. This time period is $(t^a)^*$. For time periods t where $(t^a)^* < t < (t^m)^*$ from equation (13) population will be higher than it would have been in the Malthusian regime. Therefore the shadow Malthusian unskilled wage[22] given by $a_t^a((L_t^{a,0})^*)^{\gamma-1}$, will be below the level $\widetilde{c}\frac{(1-\alpha-\beta)}{(1-\alpha-\beta)-(1-\alpha)\tau^u}$. In contrast A^a will have continued rising. Thus the inequality $\frac{A_t^a}{a_t^a} > ((L_t^{a,0})^*)^{\gamma-1}$ will still hold. For time periods t where $(t^m)^* < t < (t^{mis})^*$, where $(t^{mis})^*$ is the start of the modern industrial stage, the inequality $\frac{A_t^a}{a_t^a} > ((L_t^{a,0})^*)^{\gamma-1}$ will also still hold since we are assuming that the demand for agricultural goods is at least as high as it would have been without any new technologies, and thus the shadow Malthusian unskilled wage will be below the level $\widetilde{c}\frac{(1-\alpha-\beta)}{(1-\alpha-\beta)-(1-\alpha)\tau^u}$ whereas again A^a will have continued rising. Finally for time periods t where $t > (t^{mis})^*$, the agents first order conditions, equation (11), shows that the demand for agricultural goods will be growing by more than the growth rate of A^a, This is a greater rate than would be occurring under the Malthusian system. Thus again the shadow Malthusian unskilled wage will be below the level $\widetilde{c}\frac{(1-\alpha-\beta)}{(1-\alpha-\beta)-(1-\alpha)\tau^u}$ and so the inequality $\frac{A_t^a}{a_t^a} > ((L_t^{a,0})^*)^{\gamma-1}$ still holds.

[22] Where as defined above, the variable $(L_t^{a,0})^*$ is the level of employment in the old agricultural sector necessary for the old agricultural sector alone to satisfy the total demand for agricultural products at time t.

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Figure One

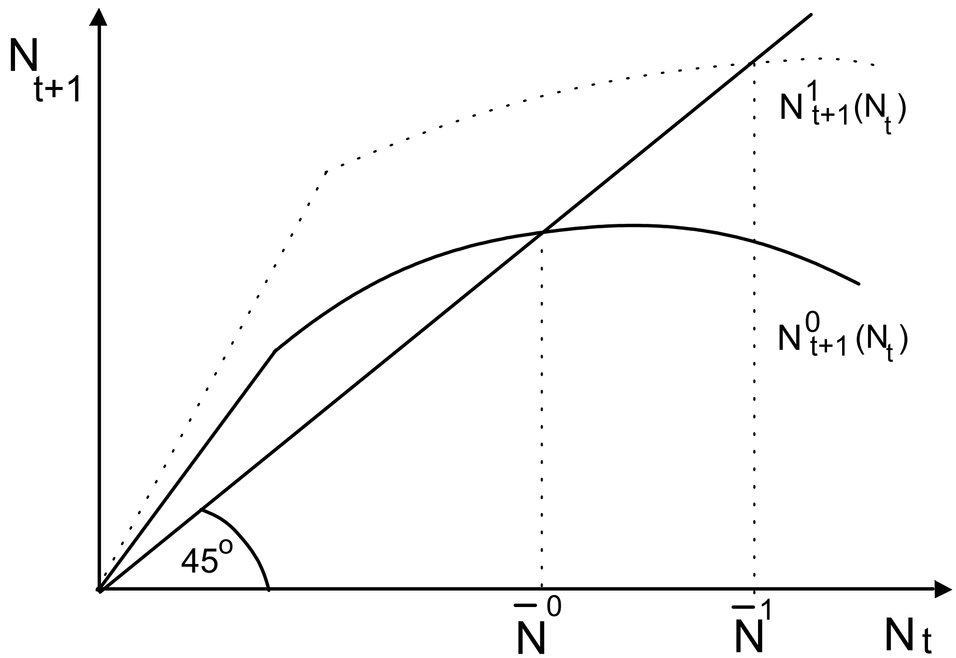

Figure 1. This figure depicts the population dynamics in the Malthusian Stage. The adult population next period, N_{t+1}, is drawn as a function of the adult population this period, N_t. For the initial level of technology this function is $N^0_{t+1}(N_t)$ and population tends to the steady state level $\overline{N}^0$. Technological progress shifts the relationship between N_{t+1} and N_t to $N^1_{t+1}(N_t)$ and so population tends to a higher steady state level $\overline{N}^1$. However although technological progress initially increases unskilled real wages, in the new steady state unskilled real wages return to their old steady state level.

Figure Two

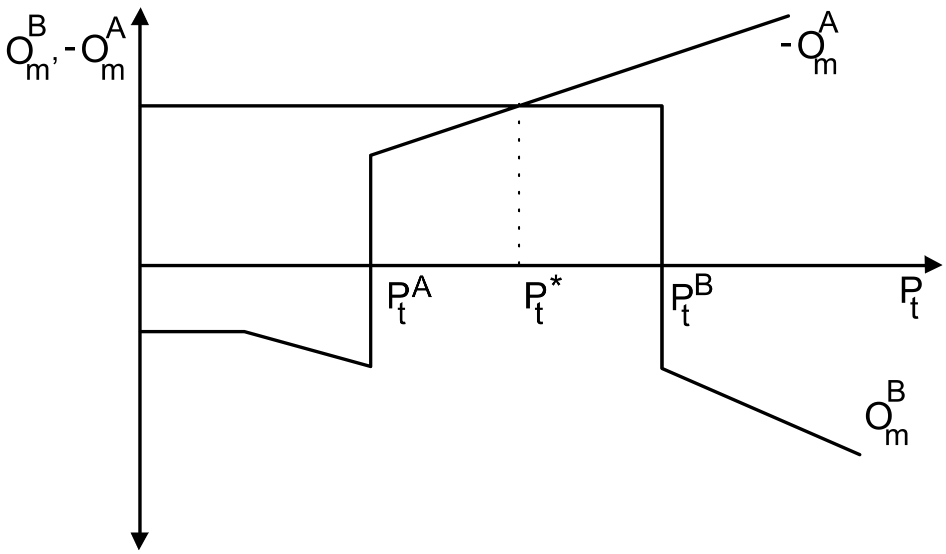

Figure 2. This figure depicts the international equilibrium. The offer curve of the relatively advanced economy, B, is denoted O_m^B while the offer curve of the less advanced economy, A, is denoted O_m^A. The offer curves are drawn for the case where economy B is not constrained by the subsistence constraint in autarky but economy A is. International equilibrium is where $O_m^B + O_m^A = 0$ and is thus where the curves O_m^B and -O_m^A cross. The relative price under international trade, p^* lies between the two autarkic relative prices, p^A and p^B and thus the figure depicts the case where both economies are specialized in production.