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## ABSTRACT

# Does Geographical Agglomeration Foster Economic Growth? And Who Gains and Loses From It?*

This Paper proposes a two-region model of endogenous growth, which is a natural combination of a core-periphery model *à la* Krugman and of a model of endogenous growth *à la* Grossman/Helpman/Romer. Specifically, we add to the core-periphery model an R&D sector that uses skilled labour to create new varieties for the modern sector, while forward-looking migration behaviour is introduced. The innovation activity in the R&D sector involves knowledge externalities among skilled workers. Our analysis suggests that the presence of such a sector reinforces the tendency toward agglomeration, and supports the idea that the additional growth spurred by agglomeration may lead to a Pareto-dominant outcome such that when the economy moves from dispersion to agglomeration, innovation follows a much faster pace. As a consequence, even those who stay put in the periphery are better off than under dispersion, provided that the growth effect triggered by the agglomeration is strong enough.

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# 1 Introduction

The main effect of interregional and international integration is likely to increase economic efficiency within the space-economy. However, as often argued in the general press, a deepening in market opening - globalization - might well be accompanied by the appearance of some core regions whose wealth is, in part, obtained at the expense of peripheral regions. So far, however, such a result has been obtained in the context of static models in which the total number of firms is constant (Krugman, 1991a; Fujita, Krugman and Venables, 1999). In other words, most of economic geography does not account for the possible impact of agglomeration on the rate of innovation, which in turn is likely to influence the geographical distribution of welfare levels. It is, therefore, fundamental to figure out what the core-periphery property becomes in a dynamic setting where the number of firms may grow over time.

More precisely, we want to know whether regional discrepancies widen or fall over time, and what are the main reasons for such a possible divergence or convergence? Since, by the time frame of human history, the current disparities between rich and poor regions are recent (Bairoch, 1993), it is important to understand how they may change over time. As regional discrepancies are often considered as socially undesirable, the issue is indeed critical from the policy standpoint. If one finds strong reasons for the existing disparities to persist or, worse, for growth and agglomeration to make those living in the periphery worse off, governments and international bodies should become more active in designing policies fostering a more equitable distribution of wealth across nations and regions.

Clearly, space and time are intrinsically mixed in the process of economic development but the study of their interaction is a hard task. Indeed, because either agglomeration or growth is a complex phenomenon by itself, one should expect any integrated analysis to face many conceptual and analytical hurdles. Not surprisingly, therefore, the field is still in its infancy and relevant contributions are not many. Yet, the task is not out of reach. Because both the "new" theories of growth and "new economic geography" share the same basic framework of monopolistic competition, there exists a solid foundation for cross-fertilization between the two fields. The existing contributions that have recently explored the mutual influences between growth and location exploit this formal analogy; see, e.g. Waltz (1996), Baldwin (1999), Martin and Ottaviano (1999, 2001), Baldwin, Martin and Ottaviano (2001). However, we still lack a framework of reference that would allow us to check (i) how the rate of growth is related to the degree of spatial concentration of activities and (ii) whether the spatial concentration of innovation may boost a sufficiently high rate of growth for those living the periphery to be better off than under dispersion, once *skilled people working for the R&D sector are mobile.*

It should be stressed here that the introduction of workers' migration into an endogenous growth model under perfect foresights raises unsuspected problems that are discussed below. Despite those difficulties, it is possible to derive some tentative conclusions that appear to be reasonable. To the best of our knowledge, today only three papers allow for labor mobility in a multiregional (or multi-city) endogenous growth model under perfect foresights, that is, Waltz (1996), Baldwin and Forslid (2000), and Black and Henderson (1999). Although they represent pioneering works towards a unified analysis of growth and location, their treatment of migration seems to leave a room for further considerations. The assumption of costless migration in Waltz (1996) tends to generate a bang-bang migration behavior, which is too far from reality. Our framework is close to that of Baldwin and Forslid (2000) but is more tractable analytically, thus permitting more concrete results. Finally, the endogenous growth model of an urban system by Black and Henderson (1999) represents a significant achievement; however, its implicit assumption of an optimally controlled migration process seems fairly restrictive.

In this paper, we propose a new and simple model of endogenous growth for a two-region economy, which represents a natural combination of a Krugman-type core-periphery model and of a Grossman-Helpman-Romer-type model of endogenous growth with horizontally differentiated products. Specifically, this means that we add to the core-periphery model a research and development sector that uses skilled labor to create new varieties for the modern sector so that the number of firms producing in this sector is variable, while forward-looking behavior and migration are formalized in the same spirit as in Ottaviano, Tabuchi and Thisse (2002). Our model may be viewed as an attempt at integrating several issues addressed in previous works. More precisely, it combines (i) *the demand effect generated by the migration of skilled workers*, as in core-periphery models and (ii) *the productivity effect generated by the existence of spillovers*, such as those studied in endogenous growth models. These two effects are in turn associated with the growth in the number of varieties, which gives rise to a second demand effect. Hence, our model provides a "general" framework of reference, which is amenable to a complete analytical solution.

For simplicity, we neglect the transitionary period (except in the stability analysis) and focus on a steady-state spatial equilibrium; this equilibrium is such that the spatial distribution of skilled workers is time-invariant while the total number of patents/varieties/firms grows at a constant rate, both being determined at the equilibrium outcome. One of the most stimulating results obtained in this paper is that the growth rate, measured by the variation in the number of varieties, changes with the spatial distribution of skilled workers. In other words, we show that *the growth of the global economy depends on the spatial distribution of the innovation sector across regions.*

Skilled workers being mobile, we make the complementary assumption that patents for new products can be transferred costlessly between regions, presumably because the technology required to produce the new varieties is available everywhere or because blueprints developed in a region requires no adjustments when used in another region (there is no need for the "tropicalization" of technology). Although somewhat restrictive, this assumption agrees with the observation made by Pollard (1982) regarding the transfer of technologies during the Industrial Revolution:

> "The steam-engines, spinning-mules, blast furnaces, or railway systems installed on the Continent were exactly like the British ones...It was not until several generations later...that alternative technologies ...were to be developed." (p.85).

In such a context, production is footloose and agglomeration economies turn out to be very strong: *the entire R&D activity always concentrates into a single region, which also accommodates a dominant share of the industrial sector*. Stated differently, the symmetric spatial configuration (in which each region contains half of each sector) is never a stable outcome here. Hence, we may conclude that the existence of a R&D sector appears to be a strong centripetal force at the multiregional level, which amplifies the circular causation that lies at the heart of the core-periphery model.

This result confirms the idea that *growth and agglomeration would go hand in hand*, as suggested by Hirschman (1958, p.183) long ago:

> "we may take it for granted that economic progress does not appear everywhere at the same time and that once it has appeared powerful forces make for a spatial concentration of economic growth around the initial starting points."

In particular, our positive analysis seems to give credit to the existence of a trade-off between growth and spatial equity, as suggested by many economic policy debates that take place in several industrialized countries. It also seems to confirm analyses undertaken by economic historians such as Pollard (1982, p. 325) for whom:

> "Given a sufficient effort from the centre, ...all these [backward] regions can be developed, but frequently only at the cost of slowing down the progress of the country as a whole."

However, our welfare analysis supports the idea that the additional growth spurred by agglomeration may lead to a Pareto-superior outcome. Specifically, when the economy moves from dispersion to agglomeration, innovation follows a faster pace. As a consequence, *even those who stay put in the periphery are better off than under dispersion provided that the growth effect triggered by the agglomeration is strong enough.* It should be stressed that this Pareto-optimal property does not require any transfer whatsoever: it is a pure outcome of market interaction. To be sure, the unskilled who live in the core of the economy enjoy a higher level of well-being than those in the periphery. Thus, what appears here is a situation in which everybody can be made better off because agglomeration generates more growth. However, the gap between those who live, respectively, in the core and in the periphery enlarges. Put differently, *the rich get richer and so may do the poor, but without ever catching up.* Hence, according to Rawls' principle, there would be no conflict between growth and equity since all the unskilled, even those residing in the periphery, can be made better off.

Yet, absolute discrepancies widen across individuals, due to the differential benefits generated within the core region.[1] In other words, *agglomeration gives rise to regressive effects in terms of income distribution.* Such enlarging welfare gaps may call for corrective policies, even though such policies might in turn hurt growth and, thus, individual welfare. Indeed, the reduction of regional disparities is a major concern in several parts of the world. For example, in the case of the European Union, Article 130a of the Amsterdam Treaty states that the "Community shall aim at reducing disparities between the levels of development of the various regions and the backwardness of the least favoured regions or islands, including rural areas", so that a clear social cohesion objective exists.

[1] However, relative discrepancies remain constant.

Note, finally, that, in our setting, *the regional income discrepancy reflects, at least to a large extent, the spatial distribution of skills*. The welfare gap between the core and the periphery arises because of the additional gains generated by a faster growth that the skilled are able to spur by being agglomerated. This in turn makes the unskilled residing in this region better off, even though their productivity is the same as the one of those living in the periphery. The redistributive issue is, therefore, less simple than what it might seem at first sight.

The remainder of the paper is organized as follows. The model is presented in Section 2. Our analysis is performed in Section 3; note that assuming that skilled workers are mobile requires a new and detailed analysis of the stability of the equilibrium paths (proofs are given in the appendix). Section 4 ends the paper.

# 2 A model of agglomeration and growth with mobility

The purpose of this section is to present a simple setting in which growth is driven by the increase in the number of varieties, while the skilled workers who create the blueprints, which are necessary for the production of the new varieties, are free to move across regions. Although changes in the populations of skilled and unskilled workers could be important, we make the simplifying assumption that each population keeps the same size over time.[2]

## 2.1 The model

In terms of labor requirement in each sector, our model is similar to the one presented in Ottaviano (2001) in which the fixed cost of a firm belonging to the modern sector is expressed in terms of skilled labor, while its marginal cost is expressed in terms of unskilled labor. The main difference resides in the presence of a R&D sector in which patents for new varieties are developed. In turn, the fixed cost of a firm producing a given variety is equal to the cost of acquiring the corresponding patent.

[2]It is worth noting that empirical works cast serious doubts on the idea that growth would be triggered by an increase in the proportion of skilled workers (Jones, 1995; Greenwood and Jovanovic, 1998).

The economy consists of two regions, $A$ and $B$, and three production sectors, namely the traditional sector ($\mathbb{T}$), the modern sector ($\mathbb{M}$), and the innovation sector ($\mathbb{R}$). There are two production factors, the low-skilled workers (L) and the high-skilled workers (H). Both the $\mathbb{T}$- and $\mathbb{M}$-sectors use unskilled workers, while the $\mathbb{R}$-sector uses skilled workers. Each unskilled is endowed with one unit of L-labor per unit of time and is immobile. Every region has the same number of unskilled ($L/2$) over time, where $L$ is a constant. Each skilled is endowed with one unit of H-labor and can move between regions at some positive cost (more below). *The total number of skilled in the economy is constant over time*; without loss of generality, this number is normalized to 1 so that $L$ may be interpreted as the relative size of the unskilled to the skilled. Given this interpretation, it is important to mention that the value of $L$ does not matter for our main results, thus suggesting that our assumption of fixed populations might be less critical than what it seems at first sight. Although the total number of skilled is constant over time, we show how growth is made possible through another variable, *the knowledge capital*, which rises together with the number of patent/varieties.

First, we describe consumers' preferences (the location and/or time argument is suppressed when no confusion arises from doing so). All workers have the same instantaneous utility function given by

$$u = Q^{\mu}\mathcal{T}^{1-\mu}/\mu^{\mu}(1-\mu)^{1-\mu} \qquad 0 < \mu < 1 \tag{1}$$

where $\mathcal{T}$ is the consumption of the homogeneous $\mathbb{T}$-good, while $Q$ stands for the index of the consumption of the $\mathbb{M}$-varieties given by

$$Q = \left[\int_0^M q(i)^{\rho}\, di\right]^{1/\rho} \qquad 0 < \rho < 1$$

In this expression, $M$ is the total mass of $\mathbb{M}$-varieties available in the global economy at time $t$, while $q(i)$ represents the consumption of variety $i \in [0, M]$.

The homogeneous $\mathbb{T}$-good is produced under constant returns and perfect competition. Furthermore, this good is costlessly shipped between the two regions, thus enabling us to normalize its price to one across time and location. Hence, if $\epsilon$ denotes the expenditure of a consumer at a given time $t$ while $p(i)$ is the price of variety $i$, then his demand functions are as follows:

$$\mathcal{T} = (1-\mu)\epsilon \tag{2}$$

$$q(i) = \mu\epsilon p(i)^{-\sigma} P^{\sigma-1} \qquad i \in [0, M] \tag{3}$$

where $\sigma \equiv 1/(1-\rho)$, and $P$ is the price index of $\mathbb{M}$-varieties given by

$$P \equiv \left[ \int_0^M p(i)^{-(\sigma-1)} di \right]^{-1/(\sigma-1)} \tag{4}$$

Introducing (2) and (3) into (1) yields the indirect utility function

$$v = \epsilon P^{-\mu}$$

We now describe the behavior of an arbitrary consumer $j$ in space and time. If this consumer chooses an expenditure path, $\epsilon_j(t)$ for $t \in [0, \infty)$ such that $\epsilon_j(t) \geq 0$, and a location path, $r_j(t)$ for $t \in [0, \infty)$ such that $r_j(t) \in \{A, B\}$, then his indirect utility at time $t$ is given by

$$v_j(t) = \epsilon_j(t)[P_{r_j(t)}(t)]^{-\mu} \tag{5}$$

where $P_{r_j(t)}(t)$ is the price index of the $\mathbb{M}$-good in region $r_j(t)$ at time $t$.[3] If $r_j(t_-) \neq r_j(t)$, then he relocates at time $t$ and we denote by $t_h$ ($h = 1, 2, ...$) the sequence of such moves.[4]

Moving between regions requires various psychological adjustments that negatively affect a migrant. Hence, a consumer who relocates at time $t$ bears a cost $C(t)$ expressed in terms of his lifetime utility. Following a standard approach in endogenous growth theory, the lifetime utility of consumer $j$ at time 0 is then defined by

$$U_j(0) = V_j(0) - \sum_h e^{-\gamma t_h} C(t_h) \tag{6}$$

where $\gamma > 0$ is the subjective discount rate common to all consumers, while

$$V_j(0) \equiv \int_0^\infty e^{-\gamma t} \ln[v_j(t)] dt \tag{7}$$

is the lifetime utility gross of migration costs (hence, preferences are intertemporally CES with unit elasticity of intertemporal substitution).

[3] If $j$ is a L-worker, then $r_j(t)$ is either $A$ or $B$ for all $t$.

[4] So a H-worker is allowed to move back and forth several times.

The intertemporal allocation of resources is governed by *a global and perfectly competitive capital market* in which bonds, bearing an interest rate equal to $\upsilon(t)$ at time $t$, are traded. The interest rate is common to both regions because the capital market is equally accessible to all consumers and firms, wherever they reside. We must now specify consumer $j$'s intertemporal budget constraint, that is, the present value of expenditure equals wealth. Let $w_{r_j(t)}(t)$ be the wage rate which this consumer receives when he resides in $r_j(t)$ at $t$. Then, the present value of wage income is given by

$$W_j(0) = \int_0^\infty e^{-\bar{\nu}(t)t} w_{r_j(t)}(t)dt \tag{8}$$

where $\overline{\nu}(t) \equiv (1/t)\int_0^t \nu(\tau)\, d\tau$ is the average interest rate between 0 and $t$; in (8), the term $\exp[-\overline{\nu}(t)]$ converts one unit of income at time $t$ to an equivalent unit at time 0. Using the budget flow constraint, Barro and Sala-i-Martin (1995, p.66) show that the consumer's intertemporal budget constraint may be written as follows:

$$\int_0^\infty \epsilon_j(t)e^{-\overline{\nu}(t)t}dt = a_j + W_j(0) \tag{9}$$

where $a_j$ is the value of the consumer's *initial assets*.

Consider any given location path $r_j(\cdot)$. Then, if $\epsilon_j(\cdot)$ stands for an expenditure path that maximizes (6) subject to (9), the first order condition implies that

$$\dot{\epsilon}_j(t)/\epsilon_j(t) = \nu(t) - \gamma \qquad t \geq 0 \tag{10}$$

where $\dot{\epsilon}_j(t) \equiv d\epsilon_j(t)/dt$. Since (10) must hold for every consumer, it is clear that the following relation must hold

$$\dot{E}(t)/E(t) = \nu(t) - \gamma \qquad t \geq 0 \tag{11}$$

where $E(t)$ stands for the total expenditure in the economy at time $t$.

We now turn to the production side of the economy. As noted above, the $\mathbb{T}$-sector operates under constant returns: one unit of $\mathbb{T}$-good is produced using one unit of L-labor. We assume that the expenditure share $(1-\mu)$ on the $\mathbb{T}$-good is sufficiently large for the $\mathbb{T}$-good to be always produced in both regions.[5] In this case, the wage rate of the unskilled is always equal to 1 in each region

[5] A sufficient condition for this to hold is that $1 - \mu > \rho/(1+\rho)$.

$$w_A^L = w_B^L = 1 \qquad t \geq 0 \tag{12}$$

since the price of the traditional good is 1 across regions.

In the $\mathbb{M}$-sector, the production of any variety, say $i$, requires the use of the patent specific to this variety, which has been developed in the $\mathbb{R}$-sector. Once a firm has acquired the patent at the market price (which corresponds to this firm's fixed cost), it can produce one unit of this variety by using one unit of L-labor. The transportation of this variety within the same region is costless. However, when it is moved from one region to the other, only a fraction $1/\Upsilon$ arrives at destination, where $\Upsilon > 1$. Hence, if variety $i$ is produced in region $r = A, B$ and sold at the mill price $p_r(i)$, then the price $p_{rs}(i)$ paid by a consumer located in region $s \neq r$ is

$$p_{rs}(i) = p_r(i)\Upsilon \tag{13}$$

Let $E_r$ be the total expenditure in region $r$ at the time in question and $P_r$ be the price index of the $\mathbb{M}$-good in this region. Then, using (3) and (13), the total demand for variety $i$ produced in region $r$ equals

$$q_r(i) = \mu E_r p_r(i)^{-\sigma} P_r^{\sigma-1} + \mu E_s [p_r(i)\Upsilon]^{-\sigma} P_s^{\sigma-1}\Upsilon \tag{14}$$

where $r, s = A, B$ and $r \neq s$, while the last $\Upsilon$ accounts for the melting of the variety during its transportation. The corresponding profit is

$$\pi_r(i) = [p_r(i) - 1]q_r(i)$$

which yields the equilibrium price common to all varieties produced in region $r$:

$$p_r^* = 1/\rho \tag{15}$$

For notational convenience, we set

$$\phi \equiv \Upsilon^{-(\sigma-1)}$$

Then, if $M_r$ denotes the number of $\mathbb{M}$-varieties produced in region $r$ at the time in question (which may differ from the number of patents developed in this region), substituting (15) into (4) yields

$$P_r = (1/\rho)(M_r + M_s\phi)^{-1/(\sigma-1)} \tag{16}$$

where $r, s = A$ or $B$ and $r \neq s$. Furthermore, substituting (15) and (16) into (14), we obtain the equilibrium output of any variety produced in region $r$:

$$q_r^* = \mu\rho \left( \frac{E_r}{M_r + \phi M_s} + \frac{\phi E_s}{\phi M_r + M_s} \right) \tag{17}$$

whereas the equilibrium profit is given by

$$\pi_r^* = q_r^* / (\sigma - 1) \tag{18}$$

since

$$\frac{1}{\rho} - 1 = \frac{1}{\sigma - 1}$$

We now study the labor market clearing conditions for the unskilled. If $L_r^M$ denotes the demand of the unskilled by the $\mathbb{M}$-sector in region $r$, then

$$L_r^M = M_r q_r^*$$

and, by (17),

$$L_A^M + L_B^M = \mu\rho(E_A + E_B)$$

or, setting $E \equiv E_A + E_B$,

$$L_A^M + L_B^M = \mu\rho E \tag{19}$$

Using (2), the total demand for the $\mathbb{T}$-good is $T = (1 - \mu)E$ so that the total demand of L-labor in the $\mathbb{T}$-sector is equal to

$$L^T = (1 - \mu)E \tag{20}$$

In equilibrium, we must have

$$L^T + L_A^M + L_B^M = L$$

so that (19) and (20) imply that, in equilibrium, the total expenditure

$$E^* = \frac{L}{1 - \mu(1 - \rho)} \tag{21}$$

is independent of time since $L$ is constant. Therefore, we may conclude from (11) that *the equilibrium interest rate is equal to the subjective discount rate over time*

$$\nu^*(t) = \gamma \qquad \text{for all } t \geq 0 \tag{22}$$

As a result, using (10), the expenditure of any specific consumer $j$ is also a constant, which is readily obtained from (9) and (22):

$$\epsilon_j = \gamma[a_j + W_j(0)] \tag{23}$$

## 2.2 The R&D sector

Turning to the innovation sector, the patents for the new varieties are produced by perfectly competitive labs that use skilled workers and benefit from technological spillovers. Following the literature on endogenous growth theory (Romer, 1990; Grossman and Helpman, 1991), *we assume that the productivity of a researcher increases with the total capital of past ideas and methods, while this capital has the nature of a (possibly local) public good.* To be specific, when the knowledge capital in region $r$ is $K_r$, the productivity of each skilled residing in $r$ is given by $K_r$. Recall that the mass of skilled in the economy is constant and normalized to one ($H_A + H_B = 1$). Hence, when the share of skilled in region $r$ is $\lambda_r$, the number of patents developed per unit of time in region $r$ is such that

$$n_r = K_r \lambda_r \tag{24}$$

Furthermore, it is assumed that the knowledge capital in each region is determined as the outcome of the interactions among *all* skilled workers because each one has something to learn from the others. However, the intensity of these interactions varies with the spatial distribution of skilled. More precisely, when worker $j$ has a personal knowledge given by $h(j)$ (e.g., his human capital or the number of papers he has read), the knowledge capital available in region $r$ is given by

$$K_r = \left[\int_0^{\lambda_r} h(j)^\beta dj + \eta \int_0^{1-\lambda_r} h(j)^\beta dj\right]^{1/\beta} \qquad 0 < \beta < 1 \tag{25}$$

where $\beta$ represents an inverse measure of skilled workers' complementarity in knowledge creation, while the parameter $\eta$ ($0 \leq \eta \leq 1$) expresses the intensity of knowledge spillovers between the two regions.

Finally, we assume that worker $j$'s personal knowledge rises with the number of existing patents (e.g., published papers) in the global economy. For simplicity, it is taken to be proportional to the stock of patents:

$$h(j) = \alpha M$$

Normalizing $\alpha$ to one without loss of generality, it then follows from (25) that

$$K_r = M[\lambda_r + \eta(1-\lambda_r)]^{1/\beta} \qquad 0 < \beta < 1 \tag{26}$$

When $\eta = 1$, we have $K_r = M$, which corresponds to the case in which there is no distance-decay effect in knowledge diffusion so that knowledge is a pure public good. By contrast, when $\eta = 0$, we have $K_r = Mk(\lambda_r)$, thus implying that knowledge is a local public good. In this way, the parameter $\eta$ is a measure of the "localness" of knowledge.

As will be seen below, it is not necessary to consider a specific functional form such as (26). For our analysis to hold, it is sufficient to assume that

$$K_r = Mk[\lambda_r + \eta(1-\lambda_r)] \tag{27}$$

where $k(\cdot)$ is a strictly convex and increasing function, such that

$$k(0) = 0 \text{ and } k(1) = 1$$

Expression (27) implies that both regions are in a relationship of symmetry in the sense that their own knowledge capital depends only upon the distribution of the skilled, and not upon their specific attributes. Substituting (27) into (24) yields

$$n_r = Mk[\lambda_r + \eta(1-\lambda_r)]\lambda_r \tag{28}$$

The length of patents is assumed to be infinite so that a firm that produces a particular variety enjoys a monopoly position forever. This yields the following equation of motion for the number of varieties (or, equivalently, of patents) in the economy:

$$\begin{aligned} \dot{M} &= n_A + n_B \\ &= M\left\{\lambda k[\lambda + \eta(1-\lambda)] + (1-\lambda)\, k(1-\lambda+\eta\lambda)\right\} \end{aligned}$$

where $\lambda \equiv \lambda_A$ and $1 - \lambda \equiv \lambda_B$. For notational simplicity, we set

$$k_A(\lambda) \equiv k[\lambda + \eta(1-\lambda)] \qquad k_B(\lambda) \equiv k(1 - \lambda + \eta\lambda)$$

and

$$g(\lambda) \equiv \lambda k_A(\lambda) + (1-\lambda) k_B(\lambda) \tag{29}$$

Consequently, the above equation of motion becomes

$$\dot{M} = g(\lambda)M \tag{30}$$

where $g(\lambda)$ is the *growth rate* of the number of patents/varieties in the global economy when the distribution of skilled is $\lambda$. It is readily verified that $g(\lambda)$ is symmetric about $1/2$ and such that

$$g(0) = g(1) = 1$$

while, for $\eta < 1$

$$g^{'}(\lambda) \gtreqless 0 \qquad \text{as } \lambda \gtreqless \frac{1}{2} \quad \text{and} \quad g^{''}(\lambda) > 0 \qquad \lambda \in (0,1)$$

This implies that, for any given $\eta < 1$, *the number of varieties grows at the highest rate when the innovation sector is agglomerated in one region, while it grows at the lowest rate when this sector is fully dispersed.* For any given function $k(\cdot)$, this rate depends only upon the spatial distribution of skilled. It is readily verified that for $\eta = 1$

$$g(\lambda) = 1 \qquad \lambda \in [0,1]$$

which corresponds to the normalization of the function $k(\cdot)$ made above, in which case the spatial distribution of the $\mathbb{R}$-sector does no longer matter. Furthermore, $g(\lambda)$ is shifted upward when $\eta$ rises and reaches its maximum value when $\eta = 1$. This means that *the existence of a distance-decay effect in the diffusion of knowledge slows down the pace of innovation.*

We now turn to the formation of wages for the skilled workers. In the production function of patents (28), each $\mathbb{R}$-firm located in region $r$ takes the knowledge capital $K_r$ as given. Hence, from such a firm' viewpoint, the marginal productivity of H-labor is equal to $K_r$. Since the equilibrium wage of a skilled residing in $r$, denoted by $w_r$, is given by the average productivity of H-labor in this region, (28) implies that the unit costs of a new patent is given by

$$w_r/Mk_r(\lambda)$$

Firms enter freely into the $\mathbb{R}$-sector. Hence, if $\Pi_r$ denotes the market price of a patent developed in region $r$, the zero-profit condition implies that

$$\Pi_r = w_r/Mk_r(\lambda)$$

so that

$$w_r^* = \Pi_r Mk_r(\lambda) \tag{31}$$

In addition, free entry in the $\mathbb{M}$-sector implies that $\Pi_r$ equals the asset value of an $\mathbb{M}$-firm that starts producing a new variety by using the corresponding patent. This value, however, cannot be determined without specifying the conditions that govern the interregional mobility of patents and, therefore, that of the $\mathbb{M}$-firms. These conditions are discussed in the two sections below.

We may now determine individual expenditure for each type of worker. We assume that all $\mathbb{M}$-firms at time zero are equally shared among the skilled workers.[6] Using (23), this implies that $a_L = 0$ while $W_j(0) = \int_0^\infty e^{-\gamma t} dt = 1/\gamma$ so that

$$\epsilon_j^* = 1 \qquad j \in L \tag{32}$$

On the other hand, for each skilled, we have

$$\epsilon_j = \gamma[a_H + W_j(0)] \qquad j \in H \tag{33}$$

where the initial endowment of a skilled is given by

$$a_H = M_A(0)\Pi_A(0) + M_B(0)\Pi_B(0) \tag{34}$$

while $W_j(0)$ is determined through (8) and (31) under the specific location path followed by the worker.

[6] Alternatevely, we could assume that all the $\mathbb{M}$-firms at time 0 are equally shared by both types of workers. Our results would remain essentially the same.

## 2.3 Migration behavior

Concerning migration, we assume that when moving from one region to the other, workers incurs a utility loss that depends on the rate of migration inasmuch as a migration typically imposes a negative externality on the others (Mussa 1978, Krugman 1991b). Specifically, we assume that the migration cost, $C(t)$, in terms of utility loss for a migrant at time $t$ is given by

$$C(t) = \left| \dot{\lambda}(t) \right| / \delta \tag{35}$$

where $\dot{\lambda}(t)$ represents the flow of skilled moving from one region to the other and $\delta > 0$ a positive constant. It is positive (negative) when skilled workers move from $B$ to $A$ (from $A$ to $B$).

Consider the following case that will be relevant for the stability analysis of a steady-state equilibrium at $\widetilde{\lambda} \in (0, 1]$. Without loss of generality, let the initial distribution of skilled be lower than $\widetilde{\lambda}$. Suppose that $T > 0$ exists such that a flow of skilled from $B$ to $A$ starts at 0 and stops at $T$. Hence we have

$$\begin{aligned} \dot{\lambda}(t) &> 0 \qquad t \in (0, T) \\ \lambda(t) &= \widetilde{\lambda} \qquad t \geq T \end{aligned} \tag{36}$$

In this case, all the skilled residing in region $B$ are identical except for their migration time. As a result, we can identify them on the basis of their migration time: for each $t \in [0, T)$, denote by $W(0; t)$ the lifetime wage of a skilled who migrates from $B$ to $A$ at time $t$, that is,

$$W(0; t) = \int_0^t e^{-\gamma s} w_B(s) ds + \int_t^\infty e^{-\gamma s} w_A(s) ds \tag{37}$$

Then, using (6) and (35), the lifetime utility of such a migrant is given by

$$U(0; t) = V(0; t) - e^{-\gamma t}\, \dot{\lambda}(t)/\delta \tag{38}$$

where $V(0; t)$ is the lifetime utility gross of migration costs. Using (5) and (7), $V(0; t)$ may be determined as follows:

$$V(0; t) = \frac{1}{\gamma} \ln \gamma + \frac{1}{\gamma} \ln[a_H + W(0; t)] \tag{39}$$

$$-\mu\left[\int_0^t e^{-\gamma s}\ln[P_B(s)]ds + \int_t^\infty e^{-\gamma s}\ln[P_A(s)]ds\right]$$

Furthermore, since in equilibrium the skilled residing in region $B$ do not want to delay their migration beyond $T$ (Fukao and Bénabou, 1993), it must be that

$$\lim_{t\to T} C(t) = 0$$

Taking the limit of (38), we therefore obtain

$$\begin{aligned} U(0;T) &= V(0;T) \\ &= \frac{1}{\gamma}\ln\gamma + \frac{1}{\gamma}\ln[a_H + W(0;T)] \\ &\quad -\mu\left[\int_0^T e^{-\gamma s}\ln[P_B(s)]ds + \int_T^\infty e^{-\gamma s}\ln[P_A(s)]ds\right] \end{aligned} \tag{40}$$

Since, in equilibrium, all migrants are indifferent about their migration time, it must be that $U(0;t) = U(0;T)$ for all $t \in (0,T)$. Therefore, using (38), (39) and (40), we get

$$\begin{aligned} \dot{\lambda}(t) &= \delta e^{\gamma t}[V(0;t) - V(0;T)] \\ &= \frac{\delta}{\gamma}e^{\gamma t}\ln\left[\frac{a_H + W(0;t)}{a_H + W(0;T)}\right] - \delta\mu e^{\gamma t}\int_t^T e^{-\gamma s}\ln\left[\frac{P_A(s)}{P_B(s)}\right]ds \end{aligned} \tag{41}$$

for any $t \in (0,T)$. This expression describes the equilibrium migration dynamics of skilled workers under the expectation (36), while $\delta$ is the speed of adjustment in workers' migration.

# 3 Agglomeration and growth when production is footloose

## 3.1 The market outcome

Consider the case in which firms are footloose in that they are free to produce any new variety anywhere. In other words, a firm producing a specific variety can freely choose its location at each time $t$, regardless of the region where the patent was developed. Therefore, at any given time, if both $M_A$ and $M_B$ are positive, firms' profits at that time must be identical across regions. This in turn implies by (18) that $q_A^* = q_B^*$. Hence, using (17) as well as $E_A + E_B \equiv E^*$ and $M_A + M_B \equiv M$, we obtain

$$M_A = \frac{E_A - \phi E_B}{(1-\phi)E^*} M \qquad M_B = \frac{E_B - \phi E_A}{(1-\phi)E^*} M \tag{42}$$

so that

$$M_A > 0 \quad \text{and} \quad M_B > 0 \qquad \text{iff} \quad \phi < E_A/E_B < 1/\phi \tag{43}$$

Substituting (42) into (16) and (17) respectively leads to

$$P_r = (1/\rho)[(1+\phi)(E_r/E^*)M]^{-1/(\sigma-1)} \tag{44}$$

and

$$q_A^* = q_B^* = \mu\rho E^*/M \tag{45}$$

In a similar way, it can be shown that

$$M_A = M \quad \text{and} \quad M_B = 0 \qquad \text{iff} \quad E_A/E_B \geq 1/\phi \tag{46}$$

$$P_A = (1/\rho)M^{-1/(\sigma-1)} \qquad P_B = (1/\rho)(\phi M)^{-1/(\sigma-1)} \tag{47}$$

$$q_A^* = \mu\rho E^*/M \geq q_B^* = \mu\rho[\phi E_A + E_B/\phi]/M \tag{48}$$

Likewise, we have

$$M_A = 0 \quad \text{and} \quad M_B = M \qquad \text{iff} \quad E_A/E_B \leq \phi \tag{49}$$

$$P_A = (1/\rho)(\phi M)^{-1/(\sigma-1)} \quad \text{and} \quad P_B = (1/\rho)M^{-1/(\sigma-1)} \tag{50}$$

$$q_A^* = \mu\rho[E_A/\phi + \phi E_B] \leq q_B^* = \mu\rho E^*/M \tag{51}$$

In the three cases, the equilibrium profit common to all $\mathbb{M}$-firms is given by

$$\pi^* \equiv \max\{\pi_A^*, \pi_B^*\} = \frac{\mu E^*}{\sigma M} \tag{52}$$

in which we have used (18), (45), (48) and (51).

### 3.1.1 The ss-growth path when $\lambda$ is fixed

To start with, we choose any $\lambda \in [0, 1]$ and study the *steady-state growth path (*in short, the ss-growth path*)* under that specific $\lambda$. Given (30), the number of patents (which is equal to the number of $\mathbb{M}$-firms) at time $t$ is such that

$$M(t) = M_0 e^{g(\lambda)t} \tag{53}$$

where $M_0$ is the initial number of varieties. Using (52), the asset value of a firm at time $t$ is as follows:

$$\begin{aligned} \Pi(t) &\equiv \int_t^\infty e^{-\gamma(\tau-t)} \pi^*(\tau) d\tau, \\ &= \int_t^\infty e^{-\gamma(\tau-t)} \frac{\mu E^*}{\sigma M(\tau)} d\tau \end{aligned} \tag{54}$$

which is also identical to the equilibrium price of any new patent developed at that time (recall that the place where a patent is developed is immaterial). Hence, the asset value of all firms in the modern sector at time $t$ is such that

$$M(t)\,\Pi(t) = \frac{\mu E^*}{\sigma} \int_t^\infty e^{-\gamma(\tau-t)} \frac{M(t)}{M(\tau)} d\tau$$

Since $M(t)/M(\tau) = \exp[-g(\lambda)(\tau - t)]$ by (30), we obtain

$$M(t)\,\Pi(t) = \frac{\mu E^*}{\sigma[\gamma + g(\lambda)]} \equiv a^*(\lambda) \tag{55}$$

which is constant over time. Substituting $a^*(\lambda)$ for $M\Pi_r$ in (31) therefore yields the equilibrium wage rate of the skilled in each region:

$$w_A(\lambda) = a^*(\lambda)\, k[\lambda + \eta(1-\lambda)] \equiv a^*(\lambda)\, k_A(\lambda) \tag{56}$$

$$w_B(\lambda) = a^*(\lambda)\, k(1 - \lambda + \eta\lambda) \equiv a^*(\lambda) k_B(\lambda) \tag{57}$$

Since, given (34), $a_H = a^*(\lambda)$ and $W_j(0) = w_r(\lambda)/\gamma$ for each skilled living in region $r$, (33) implies that the total expenditure of all workers in region $r$ at any time equals

$$\begin{aligned} E_r(\lambda) &= \frac{L}{2} + \lambda_r \gamma [a^*(\lambda) + w_r(\lambda)/\gamma] \\ &= \frac{L}{2} + \lambda_r a^*(\lambda)[\gamma + k_r(\lambda)] \end{aligned} \tag{58}$$

using (56) and (57), which leads to

$$\frac{E_A(\lambda)}{E_B(\lambda)} = \frac{L/2 + \lambda a^*(\lambda)[\gamma + k_A(\lambda)]}{L/2 + (1-\lambda)a^*(\lambda)[\gamma + k_B(\lambda)]} \tag{59}$$

It is then readily verified that

$$\frac{E_A(1)}{E_B(1)} = \frac{\sigma+\mu}{\sigma-\mu} \qquad \frac{E_A(1/2)}{E_B(1/2)} = 1 \qquad \frac{E_A(0)}{E_B(0)} = \frac{\sigma-\mu}{\sigma+\mu} \tag{60}$$

while[7]

$$\frac{d[E_A(\lambda)/E_B(\lambda)]}{d\lambda} > 0 \qquad \lambda \in (0,1) \tag{61}$$

Regarding the ss-growth path under the chosen value of $\lambda$, there are two different configurations that depend on the value of $\phi \equiv \Upsilon^{-(\sigma-1)}$. First, when the transport cost of the $\mathbb{M}$-good is such that

$$\Upsilon^{\sigma-1} \equiv 1/\phi \geq \frac{\sigma+\mu}{\sigma-\mu} \tag{62}$$

then we have the situation depicted in Figure 1.

Figure 1: The expenditure ratio under high transport costs

Given (60) and (61), (62) implies that

$$\phi < \frac{E_A(\lambda)}{E_B(\lambda)} < 1/\phi \qquad \lambda \in [0,1] \tag{63}$$

Hence, it follows from (43) that, along the ss-growth path associated with our chosen value of $\lambda$, the $\mathbb{M}$-good is always produced in both regions. This should not come as a surprise since we consider a situation in which the transport cost of this good is high.

Second, when the transport cost of the $\mathbb{M}$-good is such that

$$\Upsilon^{\sigma-1} \equiv 1/\phi \leq \frac{\sigma+\mu}{\sigma-\mu}$$

[7] Since $E_A(\lambda) + E_B(\lambda) = E^*$ is constant, it is sufficient to show that $E_A(\lambda)$ increases with $\lambda$, a property that follows immediately.

then we have the situation described in Figure 2.

Figure 2: The expenditure ratio under low transport costs

We see that, for $\lambda$ sufficiently close to 1/2 (that is, when $\lambda$ is between $\lambda'$ and $\lambda''$), (63) holds so that the $\mathbb{M}$-good is produced in both regions. However, when $\lambda$ is larger than $\lambda''$ or smaller than $\lambda'$, we have

$$\frac{E_A(\lambda)}{E_B(\lambda)} \geq 1/\phi \equiv \Upsilon^{\sigma-1} \quad \text{or} \quad \frac{E_A(\lambda)}{E_B(\lambda)} \leq \phi \equiv \Upsilon^{-(\sigma-1)}$$

Accordingly, by (46) and (49), *the* $\mathbb{M}$*-good is entirely produced in the region that has the greater share of the* $\mathbb{R}$*-sector*. Again, this is not very surprising since we consider the case of low transport cost for the $\mathbb{M}$-good. The location of the $\mathbb{M}$-sector is then driven by the home market effect generated by the larger share of skilled workers.

### 3.1.2 The ss-growth path when migration is allowed

So far, we have examined the growth path under a fixed distribution of skilled workers between the two regions. Using the results above, we are now equipped to study the steady-state growth when these workers are free to move but choose not to do so. For that, we must compare the lifetime utility levels of the skilled in the two regions associated with the growth path under any fixed $\lambda$ and determine the values of $\lambda$ for which this is an equilibrium.

In (6), we may neglect the last term because no migration arises in a steady-state equilibrium. Then, for the chosen value of $\lambda$, $V_r(0;\lambda)$ stands for the lifetime utility of a skilled worker in region $r$, while $v_r(t;\lambda)$ is the corresponding instantaneous utility at time $t$. This means that

$$V_r(0;\lambda) = \int_0^\infty e^{-\gamma t} \ln[v_r(t;\lambda)]dt \tag{64}$$

so that

$$V_A(0;\lambda) - V_B(0;\lambda) = \int_0^\infty e^{-\gamma t} \ln\left[\frac{v_A(t;\lambda)}{v_B(t;\lambda)}\right] dt \tag{65}$$

Using (7), (33), (56) and (57), the expenditure of each skilled worker in region $r$ is

$$\epsilon_r = a^*(\lambda)[\gamma + k_r(\lambda)]$$

Applying (5), we get

$$v_r\,(t;\lambda) = a^*(\lambda)[\gamma + k_r\,(\lambda)][P_r(t)]^{-\mu} \tag{66}$$

which leads to

$$\frac{v_A\,(t;\lambda)}{v_B\,(t;\lambda)} = \frac{\gamma + k_A\,(\lambda)}{\gamma + k_B(\lambda)}\left[\frac{P_A\,(t)}{P_B\,(t)}\right]^{-\mu} \tag{67}$$

Hence, using (44), (47) and (50) respectively, we obtain

$$\left[\frac{P_A\,(t)}{P_B\,(t)}\right]^{-\mu} = \left[\frac{E_A(\lambda)}{E_B(\lambda)}\right]^{\mu/(\sigma-1)} \qquad \phi < E_A/E_B < 1/\phi \tag{68}$$

$$\left[\frac{P_A\,(t)}{P_B\,(t)}\right]^{-\mu} = \phi^{-\mu/(\sigma-1)} \qquad E_A/E_B \geq 1/\phi \tag{69}$$

$$\left[\frac{P_A\,(t)}{P_B\,(t)}\right]^{-\mu} = \phi^{\mu/(\sigma-1)} \qquad E_A/E_B \leq \phi \tag{70}$$

where the expenditure ratio $E_A\,(\lambda)\,/E_B\,(\lambda)$ is given by (59). Among other things, these expressions imply that the ratio (67) is the same over time when $\lambda$ is fixed.

Setting

$$\Phi(\lambda) \equiv \frac{v_A(t;\lambda)}{v_B(t;\lambda)}$$

we have

$$V_A(0;\lambda) - V_B(0;\lambda) = \frac{1}{\gamma}\ln\Phi(\lambda) \tag{71}$$

and, hence,

$$V_A(0;\lambda) \gtreqless V_B(0;\lambda) \quad \text{as} \quad \Phi(\lambda) \gtreqless 1 \tag{72}$$

It is then readily verified that

$$\Phi(1/2) = 1$$

so that

$$V_A(0; 1/2) = V_B(0; 1/2) \qquad (73)$$

thus implying that full dispersion is always a steady-state equilibrium.

Furthermore, (61) means that $E_A(\lambda)/E_B(\lambda)$ increases with $\lambda$. Similarly, for $\eta < 1$, $k_A(\lambda)$ is increasing while $k_B(\lambda)$ is decreasing in $\lambda$ while, for $\eta = 1$, $k_A(\lambda) = k_B(\lambda) = 1$ for all $\lambda$. Thus, for any given $\eta \in [0, 1]$, it follows from (67) as well as from (68)-(70) that

$$\frac{d\Phi(\lambda)}{d\lambda} \geq 0 \qquad \lambda \in (0, 1) \qquad (74)$$

so that we may conclude by using (71) that

$$\frac{d[V_A(0;\lambda) - V_B(0;\lambda)]}{d\lambda} \geq 0 \qquad \lambda \in (0, 1) \qquad (75)$$

Observe also that, since $\phi < E_A(\lambda)/E_B(\lambda) < 1/\phi$ holds in a neighborhood of $\lambda = 1/2$, (74) and (75) hold with a strict inequality in that neighborhood. Therefore, we may conclude that

$$V_A(0;\lambda) \gtreqless V_B(0;\lambda) \qquad \text{as } \lambda \gtreqless 1/2$$

These inequalities imply that the economy can be in a steady-state equilibrium under three different values of $\lambda$ only, i.e., $\lambda = 1$, $\lambda = 0$ and $\lambda = 1/2$.

They also suggest that the equilibrium $\lambda = 1/2$ is unstable, while the equilibria $\lambda = 1$ and $\lambda = 0$ are stable. Note, however, that the self-fulfilling nature of the migration process makes stability more difficult to define. Indeed, our model may yield several perfect-foresight solutions under the same initial distribution of skilled workers, $\lambda_0$. Consequently, for a given ss-growth path under $\widetilde{\lambda}$ $(= 0, 1/2, 1)$, there may exist a neighborhood $\Lambda$ of $\widetilde{\lambda}$ such that for each $\lambda_0 \in \Lambda$ an equilibrium path based on a certain expectation converges to this ss-growth path, while another equilibrium path based on another expectation diverges from the same ss-growth path. In this case, is the ss-growth path stable or unstable? A natural way to escape from such a difficulty is to impose a priori some reasonable restriction on the expectations which must be satisfied when an equilibrium path converges to the ss-growth path in question. Since there is perfect foresight, this is equivalent to imposing a restriction on the equilibrium path itself. More precisely, we introduce the following restriction:

Let $\widetilde{\lambda} \in [0,1]$ and $\lambda_0 \in [0,1]$ such that $\lambda_0 \neq \widetilde{\lambda}$. If $\{\lambda(t)\}_{t=0}^{\infty}$ is an equilibrium path satisfying the initial condition $\lambda(0) = \lambda_0$, this path satisfies the *monotonic convergence hypothesis under* $\widetilde{\lambda}$ (mc-hypothesis) when there exists $0 < T \leq \infty$ such that

$$\text{when } \lambda_0 < \widetilde{\lambda} \quad \begin{array}{ll} \dot{\lambda}(t) > 0 & \text{for } t \in (0,T) \\ \lambda(t) = \widetilde{\lambda} & \text{for } t \geq T \end{array} \tag{76}$$

$$\text{when } \lambda_0 > \widetilde{\lambda} \quad \begin{array}{ll} \dot{\lambda}(t) < 0 & \text{for } t \in (0,T) \\ \lambda(t) = \widetilde{\lambda} & \text{for } t \geq T \end{array} \tag{77}$$

The ss-growth path under $\widetilde{\lambda}$ is said to be *stable* if there exists a neighborhood $\Lambda$ of $\widetilde{\lambda}$ such that, for any $\lambda_0 \in \Lambda$ with $\lambda_0 \neq \widetilde{\lambda}$, there exists an equilibrium path which satisfies the mc-hypothesis under $\widetilde{\lambda}$.[8] The ss-growth path under $\widetilde{\lambda}$ is said to be *unstable* when there is no such a neighborhood of $\widetilde{\lambda}$. Observe that conditions (76) and (77) are satisfied when the economy moves on a 'stable arm' leading to $\widetilde{\lambda}$; the same holds when the economy moves on the outer part of a 'spiral' leading to $\widetilde{\lambda}$. We show in the Appendix that $\widetilde{\lambda} = 1/2$ is unstable (Propositions A.1) while $\widetilde{\lambda} = 0, 1$ are stable (Proposition A.2) under the mc-hypothesis.

It remains to consider the distribution of the $\mathbb{M}$-sector in the case $\lambda = 1$. The $\mathbb{R}$-sector is entirely agglomerated in region $A$. However, this is not necessarily true for the location of the $\mathbb{M}$-sector since patents are costlessly mobile. We show below that two different patterns may emerge according to the values of the transport costs of the $\mathbb{M}$-good.

As shown by Figure 1, when the transport cost of the $\mathbb{M}$-good is so high that relation (62) holds, this good is always produced in the two regions. In particular, using (42) and the first relation in (60), we see that

$$0 < \frac{M_B(1)}{M_A(1)} = \frac{\sigma - \mu - \phi(\sigma + \mu)}{\sigma + \mu - \phi(\sigma - \mu)} < 1 \qquad \text{iff } \Upsilon^{\sigma-1} \equiv 1/\phi > \frac{\sigma + \mu}{\sigma - \mu} \tag{78}$$

[8] A neighborhood $\Lambda$ of $\widetilde{\lambda}$ is defined within the subspace $[0,1]$.

We call this spatial configuration a core-periphery pattern of type 1: *the core region contains the entire* $\mathbb{R}$*-sector and the larger share of the* $\mathbb{M}$*-production sector* (but not all of it).

As $\Upsilon^{\sigma-1} \equiv 1/\phi$ decreases towards $(\sigma+\mu)/(\sigma-\mu)$, the ratio $M_B/M_A$ in (78) decreases continuously towards zero. Eventually we reach the situation in which

$$M_A = M \quad \text{and} \quad M_B = 0 \qquad \text{iff } \Upsilon^{\sigma-1} \equiv 1/\phi \leq \frac{\sigma+\mu}{\sigma-\mu} \tag{79}$$

In this spatial configuration, called a core-periphery pattern of type 2, *the core region contains entirely the* $\mathbb{R}$*- and* $\mathbb{M}$*-sectors.*

Consequently, we may conclude as follows.

**Proposition 1** *When patents are freely mobile, the stable spatial configuration exhibits*

*(i) a dominant agglomeration involving entirely the innovation sector and a large fraction of the modern sector in the same region when*

$$\Upsilon^{\sigma-1} > \frac{\sigma+\mu}{\sigma-\mu} \tag{80}$$

*(ii) a global agglomeration involving entirely the innovation and the modern sectors in the same region when*

$$\Upsilon^{\sigma-1} \leq \frac{\sigma+\mu}{\sigma-\mu} \tag{81}$$

*As the transport cost parameter* $\Upsilon$ *decreases towards* 1*, the transition from one pattern to the other occurs smoothly.*

In either type-1 or type-2 core-periphery structure, the whole $\mathbb{R}$-sector is agglomerated in the core region. Since the origin of the patents does not matter, the R&D firms are able to take full advantage of being agglomerated. Hence, when patents are footloose, the symmetric spatial configuration (in which each region contains half of each $\mathbb{M}$- and $\mathbb{R}$-sector) is never a stable outcome. Such a strong tendency towards concentration of economic activity seems to confirm Pollar (1982, p.116) for whom:

> "once the process starts, it will grow cumulatively, success begetting success, while the regions which lost out in the first round are increasingly sucked dry by the winners and left further and further behind."

Yet, as discussed below, the situation may not be so simple.

## 3.2 Should we mind the gap?

The analysis above suggests that the pace of growth is faster when agglomeration arises. It is, therefore, tempting to conclude that there is a conflict between growth and spatial equity in that the peripheral region would be a loser when growth is boosted by the agglomeration of mobile activities. This would be so in a zero-sum game but ours is not. Quite the contrary. As we will see, there might be only gainers in our game, although some regions would gain more than others. This is because *global growth may be strong enough for everybody, including the unskilled who live in the peripheral region, to be better off.*

In order to study some of the main aspects of the trade-off between growth and equity, we assume that the economy is initially on a ss-growth path involving dispersion ($\lambda = 1/2$). From the spatial equity standpoint, this is the best possible outcome since both types of workers reach respectively the same utility level regardless of the region in which they live. Although this outcome is unstable, one could imagine to enforce it by controlling the mobility of the skilled.

Assume now that the economy is left unrestricted so that any small perturbation will lead the economy toward a core-periphery structure in which all skilled are agglomerated in, say, region $A$ so that $\lambda = 1$. We also assume that, in (41), the speed of adjustment ($\delta$) is sufficiently large for the transition period to be short and, hence, the comparison of the two patterns to be meaningful. There are three groups of individuals to consider: the unskilled residing in $A$ and $B$, respectively, as well as the skilled.

Consider first the case of a core-periphery-structure of type 1, so that transport costs are high in the sense of (80). For the unskilled, we know that $w_r^L = \epsilon_r^L = 1$ for $r = A, B$ so that (5) becomes

$$v_r^L(t;\lambda) = [P_r(t)]^{-\mu}$$

Using (44), (53), (55), and (58), this implies

$$\frac{v_A^L(t;1)}{v_A^L(t;1/2)} = \left(\frac{\sigma+\mu}{\sigma}\right)^{\mu/(\sigma-1)} \exp\left\{\frac{\mu}{\sigma-1}\left[1-k\left(\frac{1+\eta}{2}\right)\right]t\right\}$$

which always exceeds one since $\mu > 0$. Hence, using (65) for the L-workers

$$V_A^L(0;1) - V_A^L(0;1/2) = \frac{\mu}{\gamma(\sigma-1)}\left[\frac{1-k\left(\frac{1+\eta}{2}\right)}{\gamma} + \ln\left(\frac{\sigma+\mu}{\sigma}\right)\right] > 0 \quad (82)$$

so that *the unskilled residing in the core region always prefer agglomeration to dispersion*. Regarding the unskilled living in the periphery, we obtain

$$\frac{v_B^L(t;1)}{v_B^L(t;1/2)} = \left(\frac{\sigma-\mu}{\sigma}\right)^{\mu/(\sigma-1)} \exp\left\{\frac{\mu}{\sigma-1}\left[1-k\left(\frac{1+\eta}{2}\right)\right]t\right\}$$

so that

$$V_B^L(0;1) - V_B^L(0;1/2) = \frac{\mu}{\gamma(\sigma-1)}\left[\frac{1-k\left(\frac{1+\eta}{2}\right)}{\gamma} - \ln\left(\frac{\sigma}{\sigma-\mu}\right)\right] \tag{83}$$

The first term inside the bracketed expression stands for the *growth effect* associated with the agglomeration of the $\mathbb{R}$-sector. More precisely, given that $g(1) = k(1) = 1$ and $g(1/2) = k[(1+\eta)/2]$, the numerator of the first term represents the increase in the growth rate of varieties in the economy due to the $\mathbb{R}$-sector agglomeration into the core region; thus the first term represents the life-time impact of agglomeration on consumers' welfare. It is strictly positive if and only if $\eta < 1$. The second term represents the disadvantage of being located in the peripheral region, which is measured by the relative increase in the price index of the $\mathbb{M}$-goods in region $B$. Given (83), the unskilled living in the periphery prefer agglomeration to dispersion if and only if

$$\frac{1-k\left(\frac{1+\eta}{2}\right)}{\gamma} > \ln\left(\frac{\sigma}{\sigma-\mu}\right) \tag{84}$$

namely when the extra growth boosted by agglomerating the R&D sector in one region is sufficiently large. This is more likely, the lower the discount rate ($\gamma$), the weaker the spillover effect ($\eta$), and the larger the size of the modern sector ($\mu$); on the other hand, more product differentiation ($\sigma$ falls) enhances the locational disadvantage of the periphery. Thus, *the unskilled residing in the lagging region prefer a core-periphery structure to a dispersed one when the former leads to a sufficiently high rate of growth in the global economy.* In this case, however, there is a welfare gap between the unskilled located in the core and the periphery. Stated differently, growth generates inequalities within the unskilled who are treated differently according to the region in which they live. Specifically, when $\lambda = 1$, we have

$$\frac{v_A^L(t;1)}{v_B^L(t;1)} = \left(\frac{\sigma+\mu}{\sigma-\mu}\right)^{\mu/(\sigma-1)}$$

so that the welfare gap is

$$V_A^L(0;1) - V_B^L(0;1) = \frac{\mu}{\gamma(\sigma-1)} \ln\left(\frac{\sigma+\mu}{\sigma-\mu}\right) > 0 \tag{85}$$

It remains to consider the skilled. Using (66), we obtain

$$\frac{v_A^H(t;1)}{v_A^H(t;1/2)} = \frac{v_A^L(t;1)}{v_A^L(t;1/2)}$$

Thus, when they are agglomerated, the well-being of the skilled increases by the same proportion as the unskilled residing in the core. Indeed,

$$V_A^H(0;1) - V_A^H(0;1/2) = \frac{\mu}{\gamma(\sigma-1)} \left[\frac{1-k\left(\frac{1+\eta}{2}\right)}{\gamma} + \ln\left(\frac{\sigma+\mu}{\sigma}\right)\right] > 0$$

so that *the skilled always prefer the agglomerated pattern.*

Consider now a core-periphery-structure of type 2, thus implying that transport costs are low (see (81)). Repeating the same argument as in the foregoing, we obtain the following inequalities:

$$V_A^L(0;1) - V_A^L(0;1/2) = \frac{\mu}{\gamma(\sigma-1)} \left[\frac{1-k\left(\frac{1+\eta}{2}\right)}{\gamma} + \ln\left(\frac{2}{1+\Upsilon^{-(\sigma-1)}}\right)\right]$$

$$V_B^L(0;1) - V_B^L(0;1/2) = \frac{\mu}{\gamma(\sigma-1)} \left[\frac{1-k\left(\frac{1+\eta}{2}\right)}{\gamma} - \ln\left(\frac{1+\Upsilon^{\sigma-1}}{2}\right)\right] \tag{85a}$$

In this case, the unskilled living in $B$ prefer the core-periphery structure if and only if

$$\frac{1-k\left(\frac{1+\eta}{2}\right)}{\gamma} > \ln\left(\frac{1+\Upsilon^{\sigma-1}}{2}\right) \tag{86}$$

Hence, the lower the transport costs, the more likely the unskilled in the periphery are better off under agglomeration.

Finally, under the core-periphery structure, the welfare gap within the unskilled population is given by

$$V_A^L(0;1) - V_B^L(0;1) = \frac{\mu}{\gamma} \ln \Upsilon > 0 \tag{87}$$

Summarizing all the results above, we may conclude as follows.

**Proposition 2** *Assume that patents are freely mobile. Then, the welfare levels of the three group of workers (the skilled, the unskilled in region A and the unskilled in region B) under the core-periphery growth path Pareto-dominates the symmetric growth path if and only if the additional growth boosted by agglomerating the R&D sector in one region is sufficiently large:*

$$\frac{1-k\left(\frac{1+\eta}{2}\right)}{\gamma} > \ln\left(\frac{\sigma}{\sigma-\mu}\right) \quad \textit{when } \Upsilon^{\sigma-1} > \frac{\sigma+\mu}{\sigma-\mu}$$

*or*

$$\frac{1-k\left(\frac{1+\eta}{2}\right)}{\gamma} > \ln\left(\frac{1+\Upsilon^{\sigma-1}}{2}\right) \quad \textit{when } \Upsilon^{\sigma-1} \leq \frac{\sigma+\mu}{\sigma-\mu}$$

As expected, at the border case where

$$\Upsilon^{\sigma-1} = \frac{\sigma+\mu}{\sigma-\mu}$$

(84) and (86) (as well as (85) and (87)) are identical.

Finally, note that (86) has an interesting implication: when the spillover effect is global ($\eta = 1$), the unskilled residing in region $B$ are always worse off in the core-periphery structure than under dispersion. This is because there is no growth effect triggered by agglomeration while the adverse effect of the price-index on the unskilled in region $B$ is still there. Consequently, footloose knowledge (if any) would have some unexpected effects: it would affect negatively the extra growth boosted by agglomeration without affecting the emergence of a core-periphery structure that makes those in the periphery worse off.

# 4 Concluding remarks

The results obtained in this paper seem to support Myrdal's (1957, p.26) claim:

> "The main idea I want to convey is that the play of the forces in the market normally tends to increase, rather than to decrease, the inequalities between regions."

When transport costs are sufficiently low, both the modern and innovation sectors concentrate within the same region while the other region specializes in the production of the traditional good. This is so even though the number of firms operating in the modern sector keeps rising over time. In fact, our analysis strongly supports the idea that *agglomeration and growth reinforce each other*, confirming and enlarging results obtained in a different context by Martin and Ottaviano (2001). An interesting implication of our analysis is that *policies fostering dispersion are likely to hurt global economic growth.*

However, the increase of regional disparities does not necessarily imply the impoverishment of the peripheral regions. This may happen, however, when agglomeration does not succeed to boost enough growth. In this case, the transfer of more economic activities into the core region does hurt those who keep living in the periphery. In the opposite case, *it is not so clear that agglomeration, growth and equity do conflict:* even the people residing in the periphery are better off in the core-periphery structure than under dispersion.[9] There is a conflict only when a fairly narrow interpretation of justice, i.e. egalitarianism, is considered since the unskilled living in the core region are better off than those in the periphery. At this stage of the debate, we do not have much to say: the answer depends on societal values. But whatever the answer, it is our contention that understanding multiregional growth is crucial for improving our knowledge of how modern economies do or may develop.

Finally, we hope to investigate the case in which patents developed in one region are not transferable to the other, presumably because there are social and cultural barriers to the adoption of new technologies. For example, it is well known that there are problems which hamper the effective implementation of blueprints in a foreign region due to the tacit knowledge they require and which are hard to transfer abroad (Sachs, 2000).

# APPENDIX

When firms are free to produce any variety in any region, we have seen in Section 3.1 that the economy can follow a steady-state growth path under

[9]In a spatial competition context, Combes and Linnemer (2000) obtain a somewhat similar result: all consumers may be better off under asymmetric equilibrium firms' locations than under symmetric locations. This is because price competition may be fiercer in the former case than in the latter.

three different values of $\lambda$, i.e., 0, 1/2 and 1. In this appendix, we study the stability of each of these ss-growth paths and show that the ss-growth path under $\widetilde{\lambda} = 1/2$ is unstable, while it is stable under $\widetilde{\lambda} = 0, 1$.

1. To start with, consider the case where $\widetilde{\lambda} = 1/2$. Since the two regions are symmetric, it is sufficient to focus on the values of $\lambda_0$ lower than 1/2. The mc-hypothesis then reduces to (76), which is itself equivalent to (36) in 11.2.3. In this case, the equilibrium migration dynamics of skilled workers is given by (41). In order to evaluate (41), we need several preliminary results.

First, recall that the asset value of a firm in the modern sector at time $t$ is given by (54). Using (30), we obtain:

$$M(t)/M(\tau) = e^{-\int_t^\tau g[\lambda(s)]ds}$$

Hence, at each $t \geq 0$

$$\begin{aligned} a(t) &\equiv M(t)\Pi(t)A.1 \\ &= \frac{\mu E^*}{\sigma} \int_t^\infty e^{-\int_t^\tau [\gamma + g(\lambda(s))]ds} d\tau \end{aligned} \tag{88}$$

implying under (76) that

$$a(T) = \frac{\mu E^*}{\sigma} \frac{1}{\gamma + g(\widetilde{\lambda})} \tag{A.2}$$

It follows from (A.1) that $a(t)$ is independent of $M(0) = M_0$. As a consequence, $M_0$ has no influence on the equilibrium values of our variables except $M(t)$.

Next, using (31), the wage rate of skilled workers in region $r$ at time $t \geq 0$ is given by

$$w_r^*(t) = a(t)k_r[\lambda(t)] \tag{A.3}$$

Under (76), substituting (A.3) into (37) yields for $t \leq T$

$$W(0;t) = W(0;T) + \int_t^T e^{-\gamma\tau} a(\tau) \{k_A[\lambda(\tau)] - k_B[\lambda(\tau)]\}]d\tau \tag{A.4}$$

where

$$W(0;T) = \int_0^T e^{-\gamma\tau} a(\tau) k_B[\lambda(\tau)] d\tau + \frac{a(T) k_A(\widetilde{\lambda})}{\gamma} e^{-\gamma T}$$

By definition, $W(0;t)$ represents the life-time wage of a skilled who migrates from $B$ to $A$ at time $t \leq T$. By contrast, the life-time wage of a skilled who stays in region $r$ forever is given by

$$\begin{aligned} W_r(0) &= \int_0^\infty e^{-\gamma t} w_r(t) dt A.5 \\ &= \int_0^\infty e^{-\gamma t} a(t) k_r(\lambda(t)) dt \qquad r = A, B \end{aligned} \tag{89}$$

Turning now to the aggregate regional expenditure, $E_r(t)$, we use (32) and (33), and set $a_H = a(0)$. Then, under (76), the aggregate expenditure in region $A$ at time $t \leq T$ can be obtained as follows:

$$E_A(t) = \frac{L}{2} + \lambda(0)\gamma\left[a(0) + W_A(0)\right] + \int_0^t \dot{\lambda}(\tau)\gamma\left[a(0) + W(0;\tau)\right] d\tau \tag{A.6}$$

where the first two terms represent respectively the expenditure of the unskilled and that of the skilled who stay in region $A$ forever, while the last term stands for the expenditure by the skilled who have moved from $B$ to $A$ by the time $t$. Since $E_A(t) + E_B(t) = E^*$, we have

$$E_B(t) = E^* - E_A(t) \tag{A.7}$$

It turns out, however, that another expression of $E_B(t)$ is often more useful. To obtain it, observe that under (76), it must be that

$$E_B(T) = \frac{L}{2} + (1 - \widetilde{\lambda})\gamma\left[a(0) + W_B(0)\right] \tag{A.8}$$

while differentiating (A.6) and (A.7) with respect to $t$ leads to

$$\dot{E}_B(t) = -\dot{E}_A(t) = -\dot{\lambda}(t)\gamma\left[a(0) + W(0;t)\right]$$

Hence, for each $t \leq T$, we get

$$E_B(t) = E_B(T) - \int_t^T \dot{E}_B(\tau) d\tau$$

$$
\begin{aligned}
&= \frac{L}{2} + \left(1 - \widetilde{\lambda}\right) \gamma \left[a(0) + W_B(0)\right] \\
&\quad + \int_t^T \dot{\lambda}(\tau)\gamma \left[a(0) + W(0;\tau)\right] d\tau
\end{aligned}
$$

Putting (A.6) and (A.9) together yields

$$
\begin{aligned}
E_A(t) - E_B(t) &= \lambda_0 \gamma \left[a(0) + W_A(0)\right] - (1 - \widetilde{\lambda})\gamma \left[a(0) + W_B(0)\right] \\
&\quad + \int_0^t \dot{\lambda}(\tau)\gamma \left[a(0) + W(0;\tau)\right] d\tau \\
&\quad - \int_t^T \dot{\lambda}(\tau)\gamma \left[a(0) + W(0;\tau)\right] d\tau \qquad t \leq T
\end{aligned}
$$

We are now ready to establish the following result.

**Proposition A.1** *Assume that patents are freely mobile. Then, the ss-growth path under* $\widetilde{\lambda} = 1/2$ *is unstable.*

**Proof**. Under (76) and $\widetilde{\lambda} = 1/2$, we have

$$
\lambda(t) < 1/2 \quad \text{for } t < T \qquad \lambda(t) = 1/2 \quad \text{for } t \geq T
$$

implying that

$$
k_A[\lambda(t)] \equiv k\left\{\lambda(t) + \eta[1 - \lambda(t)]\right\} \leq k\left[1 - \lambda(t) + \eta\lambda(t)\right] \equiv k_B(\lambda(t)) \quad \text{(A.11)}
$$

for $t \geq 0$ since $k(\cdot)$ is increasing and $\eta \leq 1$. Furthermore, $a(t) > 0$ for $t \geq 0$ by (A.1). Hence, (A.4) implies that

$$
W(0;t) \leq W(0;T) \qquad t \leq T
$$

while $a_H = a(0) > 0$, implying that

$$
\frac{a_H + W(0;t)}{a_H + W(0;T)} = \frac{a(0) + W(0;t)}{a(0) + W(0;T)} \leq 1 \qquad t \leq T \qquad \text{(A.12)}
$$

Next, (A.5) and (A.11) together imply that $W_A(0) \leq W_B(0)$. Thus, setting $\widetilde{\lambda} = 1/2$ in (A.10), for $t < T$ we obtain

$$
E_A(t) - E_B(t) < (\lambda_0 - \frac{1}{2})\gamma \left[a(0) + W_B(0)\right] + \int_0^T \dot{\lambda}(\tau)\gamma \left[a(0) + W(0;\tau)\right] d\tau
$$

Furthermore, for $\tau < T$, it follows from (37) and (A.5) that

$$\begin{aligned} W_B(0) - W(0;\tau) &= \int_\tau^\infty e^{-\gamma s}\left[w_B(s) - w_A(s)\right] ds \\ &= \int_\tau^\infty e^{-\gamma s} a(s)\left[k_B(\lambda(s)) - k_A(\lambda(s))\right] ds \end{aligned}$$

which is nonnegative by (A.11). Hence, given that $\lambda(T) = 1/2$, we get

$$\begin{aligned} E_A(t) - E_B(t) &< \left(\lambda_0 - \frac{1}{2}\right)\gamma\left[a(0) + W_B(0)\right] + \left(\int_0^T \dot{\lambda}(\tau) d\tau\right)\gamma\left[a(0) + W_B(0)\right] \\ &= (\lambda_0 - \frac{1}{2})\gamma\left[a(0) + W_B(0)\right] + (\frac{1}{2} - \lambda_0)\gamma\left[a(0) + W_B(0)\right] = 0 \end{aligned}$$

or

$$E_A(t) < E_B(t) \qquad t < T$$

Therefore, using (43)-(44) or (46)-(47), we obtain

$$\frac{P_B(t)}{P_A(t)} = \max\left\{\left[\frac{E_A(t)}{E_B(t)}\right]^{1/(\sigma-1)}, \phi^{1/(\sigma-1)}\right\} < 1 \qquad \text{(A.13)}$$

Inequalities (A.12) and (A.13) imply that the right hand side of (41) is negative for $t < T$, thus contradicting (76). Consequently, for any given $\lambda_0 < 1/2$, there is no equilibrium path that satisfies the mc-hypothesis under $\widetilde{\lambda} = 1/2$. In other words, that the ss-growth path under $\widetilde{\lambda} = 1/2$ is unstable. Q.E.D.

2. Showing the stability of the ss-growth path under $\widetilde{\lambda} = 1$ (or $\widetilde{\lambda} = 0$) is more involved because we must prove the existence of a neighborhood $\Lambda$ of $\widetilde{\lambda} = 1$ such that, for any $\lambda_0 \in \Lambda$, there is an equilibrium path leading to $\widetilde{\lambda} = 1$. We show this through several steps.

First, given $\lambda_0 \in [1/2, 1)$, we assume the existence of an equilibrium path that satisfies (76) under $\widetilde{\lambda} = 1$ and examines its properties. Observe that under the hypothesis (76) and $\widetilde{\lambda} = 1$, for any given $\lambda_0 \geq 1/2$ we have

$$1/2 < \lambda(t) < 1 \quad \text{for } t \in (0,T) \qquad \lambda(t) = 1 \quad \text{for } t \geq T \qquad \text{(A.14)}$$

implying that

$$k_A[\lambda(t)] \geq k_B[\lambda(t)] \qquad t \geq 0$$

It then follows from (A.4) that

$$W_A(0) \equiv W(0;0) \geq W(0;t) \geq W(0;T) \qquad t \leq T \tag{A.15}$$

which means

$$\frac{a_H + W(0;t)}{a_H + W(0;T)} = \frac{a(0) + W(0;t)}{a(0) + W(0;T)} \geq 1 \qquad t \leq T \tag{A.16}$$

Furthermore, setting $\widetilde{\lambda} = 1$ in (A.10) and using (A.15) yield

$$\begin{aligned} E_A(t) - E_B(t) &> \lambda_0 \gamma \left[a(0) + W_A(0)\right] - \int_0^T \dot{\lambda}(\tau) \gamma \left[a(0) + W(0;\tau)\right] d\tau \\ &\geq \lambda_0 \gamma \left[a(0) + W_A(0)\right] - (\int_0^T \dot{\lambda}(\tau) d\tau) \gamma \left[a(0) + W_A(0)\right] \qquad t \in (0, T] \end{aligned}$$

Since

$$\int_0^T \dot{\lambda}(\tau) d\tau = 1 - \lambda_0$$

it follows that

$$E_A(t) - E_B(t) > (2\lambda_0 - 1)\gamma \left[a(0) + W_A(0)\right] \qquad t \in (0, T] \tag{A.17}$$

implying that $E_A(t) > E_B(t)$ when $\lambda_0 \geq 1/2$. Hence, using (43) and (46), we obtain

$$\frac{P_B(t)}{P_A(t)} = \min \left\{ \left[\frac{E_A(t)}{E_B(t)}\right]^{1/(\sigma-1)}, (1/\phi)^{1/(\sigma-1)} \right\} > 1 \qquad t \in (0, T] \tag{A.18}$$

Substituting each equality in (A.16) and (A.18) into (41), let us define for $t \in [0, T]$ that

$$\begin{aligned} \Delta V(t) &\equiv e^{\gamma t} \left[V(0;t) - V(0;T)\right] A.19 \qquad (90) \\ &= \frac{1}{\gamma} e^{\gamma t} \ln \left[\frac{a(0) + W(0;t)}{a(0) + W(0;T)}\right] \\ &\quad + \frac{\mu}{\sigma - 1} e^{\gamma t} \int_t^T e^{-\gamma \tau} \ln \left[\min \left\{\frac{E_A(\tau)}{E_B(\tau)}, \frac{1}{\phi}\right\}\right] d\tau \end{aligned}$$

Then, given any $\lambda_0 \in [1/2, 1)$, it follows from (A.16), (A.18) and (A.19) that

$$\Delta V(t) > 0 \quad \text{for } t \in [0, T) \qquad \text{and} \qquad V(T) = 0 \tag{A.20}$$

implying that

$$\dot{\lambda}(t) = \delta \Delta V(t) > 0 \qquad t \in [0, T) \tag{A.21}$$

which is consistent with (76). Since $\lambda_0 \in [1/2, 1)$ by assumption and since

$$\dot{\lambda}(0) \equiv \lim_{t \to 0} \dot{\lambda}(t) > 0$$

by (A.20), it follows that $\dot{\lambda}(0) > 0$ even when $\lambda_0 = 1/2$, thus showing that expectations do matter.

Next, we show that starting with any $\lambda_0 \in [1/2, 1)$, the equilibrium path does reach $\widetilde{\lambda} = 1$ in a finite time. To do so, first observe by (A.1) and (A.14) that

$$\frac{\mu E^*}{\sigma} \frac{1}{\gamma + 1} \leq a(t) \leq \frac{\mu E^*}{\sigma} \frac{1}{\gamma + g(1/2)} \qquad \text{for } t \leq T \tag{A.22}$$

while setting $\widetilde{\lambda} = 1$ in (A.2) yields

$$a(T) = \frac{\mu E^*}{\sigma} \frac{1}{\gamma + 1} \tag{A.23}$$

In turn, we use (A.5) and (A.14) to obtain

$$W_A(0) \geq \frac{\mu E^*}{\sigma} \frac{1}{\gamma + 1} \frac{k\left[\frac{1}{2}(1 + \eta)\right]}{\gamma} \tag{A.24}$$

It also follows from (A.17) that

$$E_B(t) \leq E^*/2 \qquad t \leq T \tag{A.25}$$

Using (A.22), (A.24) and (A.25) yields that

$$\begin{aligned} \frac{E_A(t)}{E_B(t)} &> 1 + \frac{(2\lambda_0 - 1)\,\gamma\,[a(0) + W_A(0)]}{E_B(t)} \\ &\geq 1 + 2(2\lambda_0 - 1)\frac{\mu E^*}{\sigma} \frac{\gamma + k\left[\frac{1}{2}(1 + \eta)\right]}{\gamma + 1} \qquad t \leq T \end{aligned}$$

and hence we have by (A.16) and (A.19)

$$\begin{aligned} \Delta V(t) &> \frac{\mu}{\sigma - 1} e^{\gamma t} \int_t^T e^{-\gamma s} \ln \left[ \min \left\{ 1 + 2\,(2\lambda_0 - 1)\,\frac{\mu E^*}{\sigma} \frac{\gamma + k\left(\frac{1+\eta}{2}\right)}{\gamma + 1}, \frac{1}{\phi} \right\} \right] ds \\ &= \frac{1 - e^{-\gamma(T - t)}}{\gamma} J(\lambda_0) \qquad t < T \end{aligned}$$

where

$$J(\lambda_0) \equiv \frac{\mu}{\sigma - 1} \ln \left[ \min \left\{ 1 + 2(2\lambda_0 - 1) \frac{\mu E^*}{\sigma} \frac{\gamma + k \left( \frac{1+\eta}{2} \right)}{\gamma + 1}, \frac{1}{\phi} \right\} \right]$$

which is positive for $\lambda_0 > 1/2$ and increasing in $\lambda_{0.}$ Hence,

$$\dot{\lambda}(t) = \delta \Delta V(t) > \frac{\delta J(\lambda_0)}{\gamma} \left[ 1 - e^{-\gamma(T-t)} \right]$$

Integrating both sides from $t = 0$ to $T$ and setting $\lambda(0) = \lambda_0$ and $\lambda(T) = 1$, we get

$$1 - \lambda_0 > \frac{\delta J(\lambda_0)}{\gamma^2} \left[ \gamma T - \left( 1 - e^{-\gamma T} \right) \right]$$

or

$$\frac{\gamma^2}{\delta} \frac{1 - \lambda_0}{J(\lambda_0)} > \gamma T - \left( 1 - e^{-\gamma T} \right)$$

Let $T_{\sup}(\lambda_0)$ be the solution to the equation:

$$\frac{\gamma^2}{\delta} \frac{1 - \lambda_0}{J(\lambda_0)} = \gamma T - \left( 1 - e^{-\gamma T} \right) \tag{A.26}$$

Then, it is readily verified that, for each $\lambda_0 \in (1/2, 1)$, there exists a single solution $T_{\sup}(\lambda_0)$, which is positive, continuous and decreasing on $(1/2, 1)$, while

$$\lim_{\lambda_0 \to 1} T_{\sup}(\lambda_0) = 0$$

By construction, the value of $T$ associated with the equilibrium path starting with $\lambda_0$ is less than $T_{\sup}(\lambda_0)$. Hence, we may conclude as follows.

**Lemma A.1** Let $\widetilde{\lambda} = 1$ and assume that (76) holds. Then, there is a function $T_{\sup}(\lambda_0)$ defined on $(1/2, 1)$, which is positive, continuous, decreasing and such that the equilibrium path starting with $\lambda_0 \in (1/2, 1)$ at time 0 reaches $\widetilde{\lambda} = 1$ before $T_{\sup}(\lambda_0)$, where

$$\lim_{\lambda_0 \to 1} T_{\sup}(\lambda_0) = 0$$

Since $J(1/2) = 0$, the function $T_{\sup}(\lambda_0)$ defined as the solution to (A.26) has the property:

$$\lim_{\lambda_0 \to 1/2} T_{\sup}(\lambda_0) = \infty$$

However, it can be shown that for $\lambda_0 = 1/2$, the actual time to reach $\widetilde{\lambda} = 1$ is finite. This is because $\Delta V(0) > 0$ by (A.20) even when $\lambda_0 = 1/2$, while $\Delta V(t)$ is continuous on $[0, T]$. Therefore, along the equilibrium path starting with $\lambda_0 = 1/2$, (A.21) implies that $\lambda(t) > 1/2$ for any small $t > 0$. Then, as in Lemma A.1, we can show that the time required for the path to move from $\lambda(t) > 1/2$ to $\widetilde{\lambda} = 1$ is finite, implying that the total time is finite too.

Using the results above, our remaining task is to show the existence of a neighborhood $\Lambda$ of $\widetilde{\lambda} = 1/2$ such that, for any $\lambda_0 \in \Lambda$, there is an equilibrium path leading to $\widetilde{\lambda} = 1$. To do so, it is convenient to express the dynamics of such an equilibrium path by means of differential equations.

Let

$$\epsilon(t) \equiv \gamma \left[a(0) + W(0;t)\right]$$

Then, if

$$(\lambda(t), \Delta V(t), a(t), \epsilon(t), E_A(t))_{t=0}^{T}$$

is the equilibrium path which starts with the initial distribution $\lambda_0$ at time 0 and reaches $\widetilde{\lambda} = 1$ at time $T$, its dynamics can be obtained by using (A.1), (A.4), (A.6), (A.19) and (A.21) as follows: for $t \in (0; T)$

$$\dot{\lambda} = \delta \Delta V$$

$$\dot{\Delta V} = \gamma \Delta V - \frac{a}{\epsilon}\left[k_A(\lambda) - k_B(\lambda)\right] - \frac{\mu}{\sigma - 1} \ln \left[\min\left\{\frac{E_A}{E^* - E_A}, \frac{1}{\phi}\right\}\right]$$

$$\dot{a} = \left[\gamma + g(\lambda)\right] a - \frac{\mu E^*}{\sigma}$$

$$\dot{\epsilon} = -\gamma e^{-\gamma t} a \left[k_A(\lambda) - k_B(\lambda)\right]$$

$$\dot{E}_A = \delta \Delta V \epsilon$$

where the associated terminal conditions can be obtained by using (76), (A.7), (A.8), (A.19) and (A.23) as follows:

$$\lambda(0) = \lambda_0 \qquad \lambda(T) = 1 \tag{A.27}$$

$$V(T) = 0$$

$$a(T) = \frac{\mu E^*}{\sigma}\frac{1}{1+\gamma}$$

$$E_A(T) = E^* - \frac{L}{2}$$

$$\begin{aligned} \epsilon(T) &= \gamma\left[a(0) + W(0;T)\right] A.28 \\ &= \gamma\left[a(0) + \int_0^T e^{-\gamma\tau} a(\tau) k_B(\lambda(\tau)) d\tau + \frac{\mu E^*}{\gamma\sigma}\frac{e^{-\gamma T}}{\gamma+1}\right] \end{aligned} \tag{91}$$

The set of terminal conditions derived above is unusual in two respects. First, $T$ is an unknown, while $\lambda$ is specified at both endpoints (see (A.27)). Second, (A.28) is a complex condition involving integrals. Thus, it is not straightforward to show the existence of an equilibrium path starting with each $\lambda_0 \in \Lambda$, where $\Lambda$ is a neighborhood of $\widetilde{\lambda} = 1$. Therefore, we take a slightly different approach to reach the desired result. That is, given that most terminal conditions are specified at $t = T$, we move backward from $t = T$ to $t = 0$ by introducing a new time variable:

$$s \equiv T - t$$

Furthermore, instead of specifying $\lambda_0$, we specify $T$ and then obtain the associated $\lambda_0$. That is, using the new variable $s$, we may rewrite the dynamics as follows: for $s \in (0, T)$ we have

$$\dot{\lambda} = -\delta\Delta V \tag{A.29}$$

$$\dot{\Delta V} = -\gamma\Delta V + \frac{a}{\epsilon}\left[k_A(\lambda) - k_B(\lambda)\right] + \frac{\mu}{\sigma - 1}\ln\left[\min\left\{\frac{E_A}{E^* - E_A}, \frac{1}{\phi}\right\}\right]$$

$$\dot{a} = -\left[\gamma + g(\lambda)\right] a + \frac{\mu E^*}{\sigma}$$

$$\dot{\varepsilon} = \gamma e^{-\gamma(T-s)} a\left[k_A(\lambda) - k_B(\lambda)\right]$$

$$\dot{E_A} = -\delta\Delta V\varepsilon$$

where

$$\lambda(0) = 1$$

$$\Delta V(0) = 0$$

$$a(0) = \frac{\mu E^*}{\sigma}\frac{1}{1+\gamma}$$

$$E_A(0) = E^* - \frac{L}{2} \tag{A.30}$$

$$\epsilon(0) = \gamma\left\{a(T) + \int_0^T e^{-\gamma\tau}a(\tau)k_B[\lambda(\tau)]d\tau + \frac{\mu E^*}{\gamma\sigma}\frac{e^{-\gamma T}}{\gamma+1}\right\} \tag{A.31}$$

We may then proceed as follows (see Fujita and Thisse, 2001, for more details). Since (A.31) is a complex condition, we replace it with

$$\epsilon(0) = \epsilon_0 \tag{A.32}$$

where $\epsilon_0$ is a parameter to be chosen appropriately. It can then be shown that, for each $T > 0$ sufficiently small, there exists a closed interval, $I_\varepsilon(T)$, in the positive part of $\mathbb{R}$ such that, for each $\varepsilon_0 \in I_\varepsilon(T)$, the system (A.29) to (A.30) and (A.32) has a unique solution, denoted

$$\lambda[(s;T,\epsilon_0), \Delta V(s;T,\epsilon_0), a(s;T,\epsilon_0), \epsilon(s;T,\epsilon_0), E_A(s;T,\epsilon_0)]_{s=0}^T$$

Let $\varepsilon(0;T,\varepsilon_0)$ be the associated value of the right side of (A.31):

$$\epsilon\left(0;T,\epsilon_0\right) \equiv \gamma\left[a\left(T;T,\epsilon_0\right) + \int_0^T e^{-\gamma\tau}a(\tau;T,\epsilon_0)k_B\left(\lambda\left(\tau;T,\epsilon_0\right)\right)d\tau + \frac{\mu E^*}{\gamma\sigma}\frac{e^{-\gamma T}}{\gamma+1}\right]$$

Then, it can be shown that the equation,

$$\epsilon\left(0;T,\epsilon_0\right) = \epsilon_0$$

has a unique solution, denoted $\epsilon_0(T)$, which yields the associated value of $\lambda$ at $s = T$, denoted

$$\lambda_0(T) \equiv \lambda\left[T;T,\epsilon_0(T)\right]$$

Finally, by showing that $\lambda_0(T)$ is a continuous function on the interval $(0, \hat{T}]$ and is such that

$$\lim_{T\to 0}\lambda_0(T) = 0$$

we obtain the desired neighborhood of $\widetilde{\lambda} = 1$, $[\lambda_0(\hat{T}), 1)$. This is sufficient to establish the stability of the ss-growth path under $\widetilde{\lambda} = 1$. We may then conclude as follows.

**Proposition A.2** *Assume that patents are freely mobile. Then, the ss-growth path under* $\widetilde{\lambda} = 1$ *is stable.*

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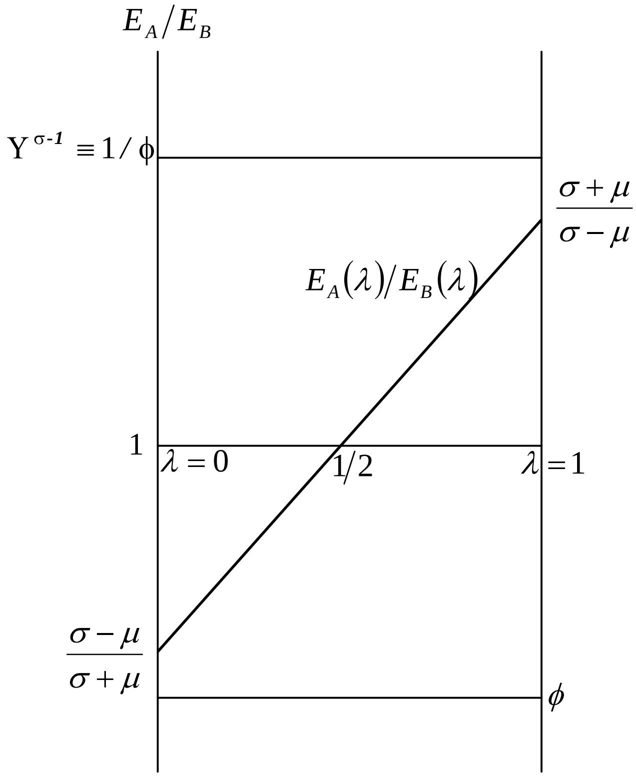


Figure 1. The expenditure ratio under high transport costs

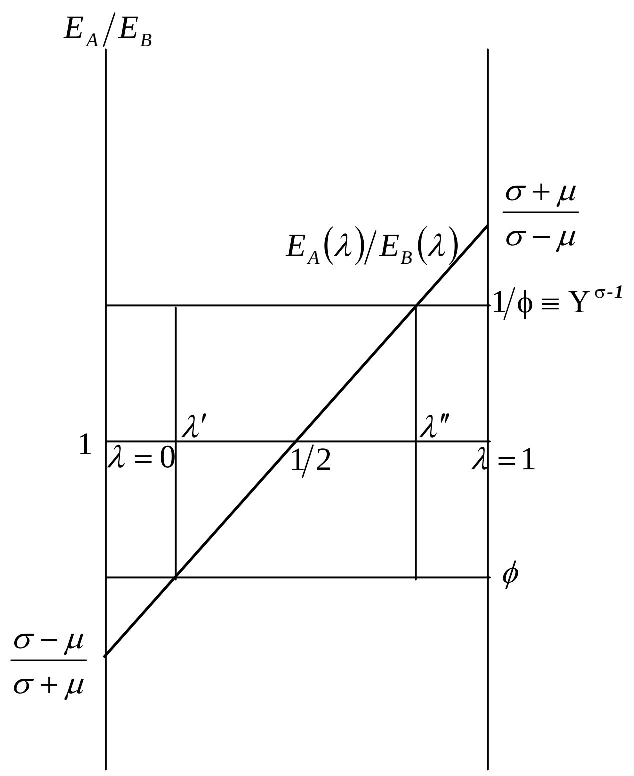


Figure 2. The expenditure ratio under low transport costs