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ABSTRACT

Monetary Policy with Uncertain Central Bank Preferences*

This Paper considers monetary policy when the weight policy makers put on output loss relative to inflation is their private information. I show that in the first period of a two-period term, all policy makers but the least inflation averse inflate less – but respond more to shocks – than if there were no private information. Moderately inflation-averse policy makers may reduce their inflation most. A tendency toward increased conservatism in their second period increases inflation in the first. The model is extended to T-period terms, $T < \infty$. It is shown that inflation depends solely on the policy maker's time left in office and not how long he has served or what he has already done. With unchanging preferences and no discounting, inflation is lower the longer he has left.

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1. Introduction

An important factor in monetary policy is how averse policy makers are to inflation relative to their dislike of output loss. Their preferences are their private information and this has implications for their behavior. The intent of this paper is to analyze the effect of unobservable preferences on policy makers' incentives to inflate.

In the basic model, policy makers serve two periods and each period they minimize a loss function which is decreasing in output and increasing in squared inflation. Output is increasing in unexpected inflation. Stochastic shocks, realized after the public's expectations are formed but before monetary policy is made, provide a role for activist monetary policy. Policy makers vary by the weight they put on output loss, relative to inflation. This weight is a policy maker's private information and there is a continuum of policy maker types.[1]

The policy maker takes current expected inflation as given when he chooses current inflation. The public has rational expectations. If it knew the policy maker's preferences, then on average it would predict inflation correctly in equilibrium. The policy maker does not take into account the rise in equilibrium current expected inflation associated with a higher current inflation choice. As a result, inflation would be too high. This is the familiar time-inconsistency problem.

If, however, the public believed the policy maker to be more inflation averse than he actually is, then its expectation of inflation would be too low and on average unexpected inflation would be strictly positive. Thus, output would be higher than it would be if preferences were guessed correctly. Thus, policy makers have an incentive to increase the public's perception of their inflation aversion. The public realizes this and knows the inflation the policy maker would

[1]The model builds on Vicker's (1986) framework where two types of policy makers inhabit a two-period model without output shocks.

pick, given its type. I show there exists a unique separating perfect Bayesian equilibrium, where the policy maker's type is revealed by his inflation choice.[2]

I show that all but the least conservative policy maker (that is, the one who puts the most weight on output) inflate less during their first period in office than they would with known preferences. This is because at their within-period optimal inflation, current welfare is insensitive to small changes in inflation. But, a small decrease in inflation raises the public's perception of their inflation aversion and increases welfare in their second period in office. The least conservative policy maker does not inflate less. His type is revealed in equilibrium and inflationary expectations are as bad as they can be. Thus, he has no incentive to signal and chooses his within-period optimal inflation. This result is similar to Vicker's (1986).

Two novel results are derived in the basic model. First, policy makers with intermediate preferences may lower their inflation the most as a result of their private information. The very conservative place little weight on output and, thus, put little weight on future expected inflation and have relatively little incentive to signal. The less conservative care more about future expected inflation. However, because the least conservative policy maker does not lower his inflation, relative to what he would do with known preferences, slightly less conservative policy makers do not have to reduce their inflation by much to distinguish themselves.

Second, unknown preferences make policy makers less inflationary in their first period in office, but *more* responsive to shocks than they otherwise would be. A shock increases within-period optimal inflation and the increase is larger the less conservative the policy maker. Thus, for all but the least conservative policy maker, inflation increases by more than within-period

[2]Other signaling models include Spence (1973) and Milgrom and Roberts (1986), both of which have two types of agents with private information. Milgrom and Roberts (1982) and Rogoff and Sibert (1988) consider a continuum of types.

optimal inflation does because more conservative policy makers do not have to lower their inflation, relative to that of less conservative policy makers, by as much to distinguish themselves. The tradeoff between central bank conservatism and activism may be less severe than Rogoff (1985) suggests.

In addition to the separating equilibrium just described, where the policy maker's type is revealed, the model can have a continuum of pooling equilibria. These equilibria are supported by beliefs that seem implausible and a further equilibrium refinement is used to rule them out.

The model is extended to consider changing preferences. I find the novel result that if policy makers tend to become more conservative while in office, this causes lower inflation in their second period and *higher* inflation in the first. Inflation rises in the first period because a lower weight is expected to be placed on future output, relative to inflation, thus lowering the gain from future unexpected inflation and the incentive to signal.

The model is also extended to $T < 4$ periods and two new results are derived. Inflation depends solely on the time left in office. If the probability of a preference change is zero and the discount factor is one, inflation is lower the longer the policy maker has left in office. Thus, longer terms lead to lower inflation on average.

This paper is related to Backus and Driffill's (1985) paper on uncertain central bank preferences. There, two types of policy makers inhabit a finite-horizon Barro-Gordon (1983) world. One type is strategic and maximizes social welfare; the other is mechanistic, always choosing zero inflation. A policy maker's type is his private information. The strategic type may attempt to masquerade as a mechanistic type early in his term in office to increase the public's belief that he might be mechanistic. This lowers future inflationary expectations, providing a more favorable output-inflation tradeoff later in his term.

Another related paper is Cukierman and Meltzer (1986). (Faust and Svensson (forthcoming) is a recent extension.) In their infinite-horizon model, the central bank has private information about its time-varying preferences and chooses money growth. Because the central bank chooses money growth, rather than an interest rate, Cukierman and Meltzer argue that the central bank's action (planned money growth) is imperfectly observable. This turns what would otherwise be a signaling problem, similar to the one here, into a statistical inference problem. This results in striking differences between their model and the one here. Unlike the model here, there are no pooling equilibria; the separating equilibrium may be unique, but is not revealing; the outcome depends upon the public's beliefs about the distribution of preferences.[3]

The policy maker in this model unilaterally chooses inflation. This setup is relevant for countries where policy is made by a monetary policy committee with both instrument and target independence and which is dominated by its chairman.[4] The United States may be an example. It is also relevant for countries where the government delegates instrument, but not target, independence to the central bank. In this case, the policy maker should be interpreted as the government or government official who chooses the target, not the central bank. An example might be the United Kingdom, where the Chancellor selects the target.

In Section 2, the basic model is presented. Section 3 discusses the model's implications for the tradeoff between policy maker conservatism and activism. Section 4 allows for uncertainty, in the form of random changes in the policy maker's preferences. Section 5 extends the basic model to $T < 4$ periods. Section 6 concludes.

2. The Model

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[3]Interestingly, as the variance of the shocks goes to zero and planned money growth becomes observable, the model does not become similar to the one here. See Carlsson and Dasgupta (1997) and Matthews and Mirman (1983) for a discussion of noisy signalling models.

[4]See Sibert (2001) for a model with an independent monetary policy committee made up of multiple policy makers serving overlapping terms.

A policy maker's term in office lasts two periods and his welfare loss in each period is increasing in inflation and decreasing in output. Inflation is disliked because it leads to shoe-leather and menu costs; it makes the domestic currency an inconvenient unit of account; it may redistribute income in a way that is perceived as unfair; and, in the presence of staggered nominal price contracts it distorts relative prices. Either nominal wage contracting and rational expectations, as in the Barro and Gordon (1983) model, or a Lucas (1973) expectations view of aggregate supply ensure output is increasing in unanticipated inflation.

The weight a policy maker puts on output loss relative to inflation is represented by the parameter χ and is his private information. I suppose that there is a continuum of possible preferences and it is common knowledge that the distribution of χ is given by the continuous density function f which is positive on $[\underline{\chi},\bar{\chi}] \subset \mathbb{R}_+$.

The two-period loss function of a policy maker with preference parameter χ is

$$\pi_1^2/2 - \chi(\pi_1 - \pi_1^e)\mu_1 + \beta E[\pi_2^2/2 - \chi(\pi_2 - \pi_2^e)\mu_2],\ 0 < \beta < 1, \quad \textbf{(1)}$$

where π_t is period-t inflation, π_t^e is the public's expectation of period-t inflation, and μ_t is an *i.i.d.* mean one shock with support $[\underline{\mu},\bar{\mu}] \subset \mathbb{R}_+$, $t = 1,2$. The discount factor, β, is common across policy makers and the public. The expectations operator, E, is conditional on information dated $t = 1$.

The variable μ_t can be viewed as a shock to the expectations-augmented Phillips curve. A high μ_t creates a more favorable tradeoff between output and unexpected inflation. In the Lucas (1973) framework, an example of a high μ_t would be a relatively small period-t variance of aggregate demand. In the Barro-Gordon framework with a Cobb-Douglas production function, the shock can be viewed as resulting from a relatively high labor share of output.

The within-period timing of events is as follows. First, expectations of inflation are formed. Second, the shock is realized and observed by both the public and the policy maker. Third, the policy maker chooses inflation. Thus, the public observes both first-period inflation and the shock before forming its expectation of second-period inflation.

The within-period-t welfare loss of the policy maker with preference parameter χ, $\pi_t^2/2 - \chi(\pi_t - \pi_t^e)\mu_t$, is minimized by setting $\pi_t = \chi\mu_t$. In period two there is no future to consider; hence, the policy maker's dominant strategy is $\pi_2 = \chi\mu_2$. Substituting this into equation (1) and ignoring terms that do not include the policy maker's decision variable allows the policy maker's first-period loss function to be written as

$$\pi_1^2/2 - \chi\pi_1\mu_1 + \beta\chi E(\pi_2^e\mu_2). \qquad \textbf{(2)}$$

The public has rational expectations; hence, it believes $\pi_2 = \chi\mu_2$. I initially focus on separating equilibria, where the policy maker's first-period inflation reveals its type. The public conjectures that the first-period inflation rule is $\pi_1 = \pi^*(\chi)$. Although not written out, the function π^* also depends on the realization of μ_1. Separability implies π^*: $[\underline{\chi},\overline{\chi}] \to \mathbb{R}$ is one-to-one. Thus, upon observing π_1 and μ_1, the public infers the policy maker's type is $\pi^{*-1}(\pi_1)$. Thus, $\pi_2^e = \pi^{*-1}(\pi_1)$ and $E(\pi_2^e\mu_2) = \pi^{*-1}(\pi_1)$. Substituting this into equation (2) yields

$$\pi_1^2/2 - \chi\pi_1\mu_1 + \beta\chi\pi^{*-1}(\pi_1). \qquad \textbf{(3)}$$

I conjecture (and later verify) that the equilibrium function π^* is twice differentiable on $[\underline{\chi},\overline{\chi}]$. Then first- and second-order conditions for a solution to the policy maker's problem are

$$\pi_1 - \chi\mu_1 + \beta\chi\pi^{*-1\prime}(\pi_1) = 0 \qquad \textbf{(4)}$$

$$1 + \beta\chi\pi^{(-1)''}(\pi_1) > 0, \tag{5}$$

for $\chi \in [\underline{\chi}, \overline{\chi})$, respectively.

Rational expectations requires that conjectures are correct so that $\pi^*(\chi) = \pi(\chi) = \pi_1$, the solution to equation (4) for $\chi \in [\underline{\chi}, \overline{\chi})$. Substituting this, $\pi^{-1\prime}(\pi_1) = 1/\pi'(\chi)$ and $\pi^{-1\prime\prime}(\pi_1) = -\pi''(\chi)/\pi'(\chi)^3$ into equations (4) and (5) yields

$$\beta\chi/\pi'(\chi) = \chi\mu_1 - \pi(\chi) \tag{6}$$

$$1 - \beta\chi\pi''(\chi)/\pi'(\chi)^3 > 0. \tag{7}$$

An equilibrium satisfying equations (6) and (7) is a perfect Bayesian equilibrium. The policy maker is optimizing while taking into account the effect of its action on the public's beliefs. The public's beliefs are consistent with Bayes' rule and are formed using its correct conjecture about the equilibrium strategies and its observation of the policy maker's action.

I make the counterfactual assumption here that inflation is precisely controlled. I could assume that planned inflation is $\pi_t^p = i_t + \delta_t$ and actual inflation is $\pi_t = \pi_t^p + \varepsilon_t$, where i_t is the central bank's observable instrument – typically an overnight interbank interest rate – and δ_t and ε_t are mean zero errors that are uncorrelated with other variables and each other. Ignoring terms unimportant to the optimization problem, the loss function (3) would be replaced by the expected loss function $i_1^2/2 - \chi i_1\mu_1 + \beta\chi i^{*(-1)}(i_1)$, where i^* is the public's conjecture of the first-period interest rate rule. The analysis is unchanged, except the variable i replaces the variable π. It might also be that measured inflation equals true inflation plus an error. This is also unimportant here as the public does observe the interest rate and will invert that to find the policy maker's type.

From inspection, it appears that the differential equation (6) may have solutions where $\pi(\chi) > \chi\mu_1$ and $\pi'(\chi) < 0$ and solutions where $\pi(\chi) < \chi\mu_1$ and $\pi'(\chi) > 0$. The second-order condition allows the former case to be ruled out. Inflation is strictly positive and a strictly increasing function of the policy maker's type, χ.

Proposition 1. *A solution to equations* (6) *and* (7) *must have* $\pi(\chi) > 0$, $\pi'(\chi) > 0$ *and* $\pi(\chi) < \chi\mu_1$ *for* $\chi \in [\chi, \bar{\chi})$.

Proof. Differentiating (6) and substituting the result into (7) yields $\pi'(\chi) > \beta/\mu_1 > 0$. This and (6) imply $0 < \pi(\chi) < \chi\mu_1$.

From Proposition 1, it is seen that in a separating equilibrium all policy makers with $\chi \in [\chi, \bar{\chi})$ must inflate less than their within-period optimum. To see this, suppose a policy maker were to choose its within-period optimum of $\chi\mu_1$ and the public were to infer that a bank that chooses $\chi\mu_1$ is type χ. Then a marginal decrease in inflation would have no effect on the first-period payoff since the derivative of first-period welfare with respect to first-period inflation is zero at $\chi\mu_1$. However, because π^* is strictly increasing, a decrease in inflation would yield a strictly positive benefit in the form of a public belief that χ is lower than it actually is.

This result is similar to the one in Vickers (1986). Using a version of Cukierman and Meltzer (1986) with a simpler stochastic structure, Cukierman (1992, ch. 9) also finds that policy makers inflate less than they would if there were symmetric information. The result contrasts with Backus and Driffill (1985), where the presence of mechanistic types gives strategic types an incentive to build a reputation for inflationary toughness by masquerading as a mechanistic type. Here, all agents are strategic and the more inflation averse reduce their inflation to distinguish themselves from the less inflation averse.[5]

The right-hand side of equation (6) gives the marginal cost of signaling. This is the marginal cost of setting inflation below its within-period optimum, $\chi\mu_1$. The left-hand side of equation (6) gives the marginal incentive to signal. This is the increase in welfare brought about by a decline in next-period's inflationary expectations. It is seen that the more weight that the policy maker puts on the future, the greater is his incentive to signal.

This signaling game has the unusual feature that the existence of symmetric information can improve welfare. With perfect information, the type-χ policy maker would like to be able commit to inflating less than $\chi\mu_1$ so that expected first-period inflation would be lower. But, he cannot do this; once expectations are in place, choosing $\chi\mu_1$ is optimal. The first-best outcome cannot be attained. This is the familiar time-inconsistency problem. With private information, a policy maker with $\chi < \bar{\chi}$ has an incentive to signal to the public that he is not inflation loving by lowering his inflation below $\chi\mu_1$. Thus, the threat that the public might update its beliefs in a damaging way serves as a commitment device.

Equation (6) is a differential equation with no boundary condition. Riley (1979) argues that the most reasonable separating equilibrium in a signaling model is the one where the sender of the most undesirable signal ($\bar{\chi}$) maximizes his within-period welfare. Here, this is because if $\chi = \bar{\chi}$, a separating equilibrium must have π_2^e ' $\bar{\chi}$. As inflationary expectations could be no worse, a type-$\bar{\chi}$ policy maker should pick his within-period optimum. It is typical in signaling models to claim the "Riley" outcome is the reasonable one. (See, for example, Milgrom and

[5]Cukierman (1992, ch. 16) supposes policy makers differ in their ability to commit to announced inflation targets. Private information about types and the existence of policy makers who cannot commit cause those who can to inflate more. This is to accommodate inflationary expectations which assume the existence of policy makers who cannot commit.

Roberts (1982) and Vickers (1986).) Thus, the boundary condition for equation (6) is[6]

$$\pi(\bar{\chi}) = \bar{\chi}\mu_1. \tag{8}$$

Some restrictions must be placed on the parameters to ensure a unique equilibrium exists. I assume

$$2\beta > \bar{\mu}^2 \tag{A1}$$

$$b(\underline{\mu})\tan^{-1}(b(\underline{\mu})) > \ln(\bar{\chi}/\underline{\chi}), \textit{ where } b(\mu) := \mu/\sqrt{4\beta - \mu^2}. \tag{A2}$$

Assumption (A1) is a sufficient (but not necessary) condition for the uniqueness of the solution to equation (6) (with the boundary condition (8)). Assumption (A2) is a sufficient condition for its existence. The second assumption requires that $\underline{\chi}$ not be too small relative to $\bar{\chi}$. Although the form is unconventional, it is easy to interpret this restriction.

A completely "conservative" policy maker ($\underline{\chi} = 0$) places no weight on output, and hence, does not care about next period's expected inflation. Such a policy maker has no incentive to convince the public he is conservative. A separating equilibrium must have $\underline{\chi}$ sufficiently greater than zero that the policy maker cares enough about expected inflation to have an incentive to signal his type. Thus, we need a lower bound on the admissible range of preference parameters.

Proposition 2. *A unique separating equilibrium exists*.

Proof. The differential equation (6) is homogeneous; it can be integrated (although not solved for explicitly) and solutions satisfy

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[6]Using the results of Ramey (1996), only equilibria satisfying (8) satisfy the D1 Criterion.

$$\ln\chi = \int \frac{(\mu_1 - v)dv}{v^2 - \mu_1 v + \beta} + c, \; where \; v := \pi/\chi, \tag{9}$$

where c is any constant.

Solving the integral in equation (9) yields

$$\ln(\pi^2 - \mu_1\chi\pi + \beta\chi^2) = 2b(\mu_1)\tan^{-1}(b(\mu_1)(2\pi - \mu_1\chi)/(\mu_1\chi)) + 2c. \tag{10}$$

Using the boundary condition to eliminate c yields

$$\begin{aligned} L(\pi) &:= \ln[(\pi^2 - \mu_1\pi\chi + \beta\chi^2)/(\beta\bar{\chi}^2)] = \\ R(\pi) &:= 2b(\mu_1)[\tan^{-1}(b(\mu_1)(2\pi - \mu_1\chi)/(\mu_1\chi)) - \tan^{-1}(b(\mu_1))]. \end{aligned} \tag{11}$$

I now prove a unique solution to (11) exists. The functions L and R are continuous on $[0,\mu_1\chi]$ with $L(0) = L(\mu_1\chi) = 2\ln(\chi/\bar{\chi}) < 0 = R(\mu_1\chi)$. Assumption (A2) ensures $L(0) > R(0)$. Thus a solution $\pi(\chi)$ exists on $(0,\chi\mu_1)$. The implicit function theorem ensures it is continuously differentiable in μ_1 as well as χ. If more than one solution exists, at least three must exist. Suppose this is true. $L'(\pi) < (>)\ 0$ for $\pi < (>)\ \mu_1\chi/2$ and $R'(\pi) > 0$. Either there is one solution on $(0,\mu_1\chi/2)$ and beginning from below R, L intersects R at least twice on $A := (\mu_1\chi/2,\mu_1\chi)$ or there are no solutions on $(0,\mu_1\chi/2)$ and beginning from above R, L intersects R at least three times on A. Thus, beginning from below R, L intersects R at least twice on A. By (11) and (A1), $L'' > 0$ on A and $R'' < 0$ on A. Thus, this is impossible. The properties of L and R are depicted in Fig.1.

The equilibrium must have inflation increasing in χ at an increasing rate.

Proposition 3. $\pi''(\chi) > 0$.

Proof. By (6), $\pi'(\chi) = \beta\chi/[\chi\mu_1 - \pi(\chi)]$. Differentiating this yields $\pi''(\chi) > 0$ iff $\chi\pi'(\chi) > \pi(\chi)$. Substituting (6) back in yields that this is true iff $h(\pi) := \pi^2 - \chi\mu_1\pi + \beta\chi^2 \geq \min_{\pi} h(\pi) = \chi^2(\beta - \mu_1^2/$

4) > 0. Assumption (A1) ensures this is true.

The intuition is that the higher is χ, the more the policy maker cares about future output and the greater incentive he has to distinguish himself from nearby types. An implication of this is that intermediate types may signal the most, and hence have the biggest gap between actual inflation and their within-period optimum. Low-χ types care little about future expected inflation and have little incentive to signal. The highest-χ type chooses his within-period optimum; hence, high-χ types do not need to lower inflation by much to signal they are different from neighboring types. The main features of inflation as a function of χ are depicted in Fig. 2.

A striking feature of the equilibrium is that the policy maker's action perfectly reveals his type and thus the public's prior beliefs, captured by the density function *f*, do not affect the outcome. This contrasts with Briault, Haldane and King's (1997) result that a reduction in preference uncertainty improves matters and with Faust and Svensson's (forthcoming) result that an increased ability to deduce the policy makers preferences is generally beneficial.

Clearly, $\pi^* = \pi$ is twice differentiable on $[\underline{\chi}, \overline{\chi})$, as previously conjectured. Having proved existence, one can calculate the second derivative from (6). The methodology employed, however, ignored the possibility that non-differentiable separating equilibria might exist. Mailath (1987) shows that for a wide class of signaling games, separating equilibria must be differentiable. The objective function (3) and the boundary condition (8) satisfy Mailath's regularity conditions (conditions (1) - (7) on page 1352). Hence, the equilibrium satisfying equations (6) - (8) is the only separable equilibrium.

In addition to perfect Bayesian separating equilibria, perfect Bayesian pooling equilibria may exist as they do in Vickers (1986) and other signaling models. To see this, let $\hat{\chi}$ be the (unconditional) expected value of χ. Let $\overline{\pi}(\chi)$ $(\underline{\pi}(\chi))$ be the highest (lowest) value of π_1 such that type χ is indifferent between choosing $\overline{\pi}(\chi)$ $(\underline{\pi}(\chi))$ and being believed to be type $\hat{\chi}$ and choosing his within-period optimum $\chi\mu_1$ and being thought type $\bar{\chi}$.

If the policy maker is thought to be type $\hat{\chi}$, then $\pi_2^e = \hat{\chi}$; if he is thought to be type $\bar{\chi}$, then $\pi_2^e = \bar{\chi}$. By equation (2), $\bar{\pi}(\chi) = \chi\mu_1 + \sqrt{2\beta\chi(\bar{\chi} - \hat{\chi})}$ and $\underline{\pi}(\chi) = \chi\mu_1 - \sqrt{2\beta\chi(\bar{\chi} - \hat{\chi})}$. Let $P := [\underline{\pi}(\bar{\chi}), \bar{\pi}(\underline{\chi})]$. P is nonempty if $\mu_1 \leq \sqrt{2\beta(\bar{\chi} - \hat{\chi})}/(\sqrt{\bar{\chi}} - \sqrt{\hat{\chi}})$. In this case, let π^p be any element of P. Then there exists a pooling equilibrium where $\pi(\chi) = \pi^p$, for every χ, and the policy maker is believed to be type $\hat{\chi}$ if $\pi_1 = \pi^p$ and type $\bar{\chi}$ otherwise. The equilibrium seems odd, however. It is difficult to justify why the public would infer $\chi = \bar{\chi}$ upon seeing $\pi < \pi^p$, but the perfect Bayesian equilibrium concept does not place any restrictions on out-of-equilibrium beliefs other than that they support the equilibrium.

Additional refinements can be used to rule out pooling equilibria. Ramey (1996) shows that for a wide class of signaling games with a continuum of types, which includes this one, equilibria in which more than one type choose the same action violate Criterion D1. Intuitively, in this model a perfect Bayesian equilibrium is a D1 equilibrium if the following is true. Suppose the public observes an out-of-equilibrium inflation rate π^* and that there is a type-χ policy maker who is indifferent between his equilibrium strategy and the resulting expected inflation and choosing π^* and having expected inflation of π_2^{e*}. The public must attach zero probability to the policy maker being type χ if there is another type, χ', who strictly prefers π^* and having expected inflation of π_2^{e*} to his equilibrium strategy and the resulting expected inflation.

The results so far are obtained in a model where welfare loss is linear in output, rather than quadratic in the deviation of output from its optimal level. This specification is also used by Backus and Driffill (1985), Vickers (1986) and Cukierman and Meltzer (1986) and is necessary for tractability. It ensures the policy maker has a dominant strategy in his second period in office and allows first-period expected inflation to be taken as a constant.

The alternative quadratic specification has the property that policy makers dislike positive deviations of output from the optimal level as much as negative ones. However, it also has the attractive feature, missing here, that policy makers dislike volatility of output.

3. Appointing Conservative Policy makers

Suppose that society has preference parameter $\chi^{(}$. The social (command) optimum would be achieved if the policy maker set inflation equal to $\chi^{*}\mu_t$ - χ^{*} in period *t*. In this case expected inflation equals zero and the policy maker responds optimally to shocks. The expected loss to society equals - $\chi^{*2}\sigma^2/2$, where σ^2 is the variance of the shock. Unfortunately, in a world where his preferences are known, a type-χ^{*} policy maker cannot commit himself to doing this. Rogoff (1985) suggests another solution: appoint a policy maker who is more conservative than society.[7] If the policy maker has preference parameter $\chi^{h,} < \chi^{*}$, then he will set inflation equal to $\chi^{h}\mu_t$ and the expected social welfare loss will be $\chi^{h^2}(\sigma^2 \ \% \ 1)/2 \ \& \ \chi^{(}\chi^{h}\sigma^2$. The conservative policy maker is strictly preferable to one with society's preferences if and only if $\chi^{h} > [(\sigma^2 - 1)/(\sigma^2 + 1)]\chi^{*}$. The optimal choice of χ^{h} is $\sigma^2\chi^{*}/(1 + \sigma^2)$ 0 $(0,\chi)$ and the associated loss is - $\chi^{*2}\sigma^4/[2(\sigma^2 + 1)] >$ $\& \ \chi^{(2}\sigma^2/2$. It is optimal to appoint a policy maker who is more conservative than society, but not so conservative as to place no weight on output. The optimal loss is not attained; lower inflation is achieved at the cost of a sub-optimal response to the shock.

The model here suggests the problem is less dire than that suggested by Rogoff. Suppose that society can appoint policy makers who are more conservative than society, but does not have exact control over how conservative the policy makers are. If policy makers are appointed for two-period terms, the results of the Section 2 suggest they will inflate less than they would if their preference parameter is known during their first period in office. The next proposition shows that they will be *more* responsive to shocks than if they had known preferences.

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[7]Rogoff (1985) assumes that the social welfare loss function is quadratic in deviations of output from its optimal level and that output shocks are additive.

To see this, I first consider a simple two-type example. One type -- a *dove* -- has the same preference parameter as society, χ^*. The other, a *hawk*, is more conservative, with $\chi = \chi^h$, where $\chi^* > \chi^h > 0$. In this case any separating perfect Bayesian equilibrium must have the hawk successfully distinguishing himself from the dove in the first period and inflating no less than he has to to do this. Applying the Riley criterion, the dove sets first-period inflation equal to his within-period optimum, $\chi^*\mu_1$. Thus, the hawk chooses inflation of π_1^h such that the dove is indifferent between choosing $\chi^*\mu_1$, and being thought a dove and choosing π_1^h and being thought a hawk. This and equation (2) yields $\pi_1^h = \chi^*\mu_1 - \sqrt{2\beta\chi^{(}(\chi^{(} \ \& \ \chi^h)} < \chi^h\mu_1$.[8]

Thus, in equilibrium the hawk inflates less than he otherwise would, but reacts to shocks in the same fashion as the policy maker with society's preferences. The reason is clear, equilibrium requires that the dove be indifferent between defecting and not defecting. Hence inflation must be some amount below his within-period optimum.[9]

During the first period of his term, any conservative policy maker is preferred to the one who minimizes society's welfare loss function. Thus, over the two periods a wider range of conservative policy makers are preferred by society than if preferences were known.

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[8]The existence of an equilibrium requires the parameters to be such that the inequality holds for every realisation of the shock.

[9]I have assumed a multiplicative shock that can be interpreted as a shock to the Phillips curve. That the expectations-augmented Phillips curve has a random slope seems a reasonable assumption. (It is also made by Dixit and Lambertini (1999) and Dixit and Jensen (2000)) However, additive shocks exist as well and coupled with quadratic preferences provide a *stabilisation* role for the policy maker. It is this stabilisation role which is typically discussed in the literature and it is not the same as the activist role modelled here. However, if a perfect Bayesian separating equilibrium exists for the case of a quadratic loss function with a linear output shock and two policy maker types, it is qualitatively similar to the one here. The hawk's inflation is a constant amount less than the dove's; hence the hawk stabilises as a dove. Details are available from the author.

For the continuum-of-types case, a similar result holds.[10] It is depicted in Fig. 3.

Proposition 4. $\partial\pi(\chi)/\partial\mu_1 > \chi,\ \chi \in [\underline{\chi},\overline{\chi})$.

Proof. Suppose $\exists\chi' \in [\underline{\chi},\overline{\chi})$ such that $\partial\pi(\chi')/\partial\mu_1 \leq \chi'$. By the implicit function theorem (applied to (11)), $\partial\pi(\chi)/\partial\mu_1$ is continuous in χ on $[\underline{\chi},\overline{\chi}]$ and equals $\overline{\chi}$ at $\overline{\chi}$. By (6), $\partial\pi'(\chi)/\partial\mu_1 = \beta\chi'[\partial\pi(\chi)/\partial\mu_1 - \chi]/[\chi\mu_1 - \pi(\chi)]^2 \leq 0$ at χ'. Thus, there must exist a $\chi'' \in (\chi',\overline{\chi})$ such that $\partial\pi(\chi'')/\partial\mu_1 < \chi''$ and $\partial\pi'(\chi'')/\partial\mu_1 > 0$. This is a contradiction.

The intuition is as follows. A rise in μ_1 causes policy makers' within-period optimal inflation to rise at rate χ. Thus, the marginal incentive to inflate increases more for less conservative policy makers. As a result, type χ ($\chi < \overline{\chi}$) can increase his inflation at a rate greater than χ because it is not necessary to lower his inflation, relative to less inflation-averse types' inflation, by as much to separate himself.

4. Changing Central Bank Preferences

In this section I allow preferences to change between periods one and two. This change may reflect an anticipated change in circumstances rather than a fundamental change in preferences, for example anticipation of a future job, pressure from the government, the press, the electorate, or a special interest group.

Suppose that in period one, the preference parameter χ is drawn from a distribution on $[\underline{\chi},\overline{\chi}]$, as in the previous section. At the start of period two, the preference parameter χ', is drawn from a distribution on $[\underline{\chi}',\overline{\chi}']$, which may depend on χ. Then the policy maker's expected second-period preference parameter is $E(\chi'|\chi)$ and $\pi_2^e = E(\chi'|\pi^{*\,-1}(\pi_1))$, where π^* is the public's conjecture of the inflation rule. As before I focus on the separating equilibrium so that π^* is one-to-one and is conjectured to be twice differentiable. Ignoring terms not depending on π_1, the

[10] This result differs from the prediction of Cukierman (1992)'s variant of the Cukierman and Meltzer (1986) model. There, the central bank is always less activist with private information than with perfect information.

policy maker minimizes $\pi_1^2/2 - \chi\mu_1\pi_1 + \beta E(\chi'|\chi)E(\chi'|\pi_1^{-1}(\pi_1))$.

The first- and second-order conditions and consistency of conjectures imply

$$\pi_1 - \chi\mu_1 + \frac{\beta E(\chi'|\chi)}{\pi'(\chi)}\frac{\partial E(\chi'|\chi)}{\partial\chi} = 0. \tag{12}$$

$$1 + \beta E(\chi'|\chi)[\partial^2 E(\chi'|\chi)/\partial\chi^2/\pi'(\chi)^2 - \pi''(\chi)\partial E(\chi'|\chi)/\partial\chi/\pi'(\chi)^3] > 0 \tag{13}$$

The last term in equation (12) is the marginal incentive to signal. Uncertain second-period preferences have two effects on incentives. First, the higher is the expected value of χ', the more important it is to the policy maker to raise output in period two. Thus, the more incentive he has to distinguish himself from less conservative types in period one. This tends to lower current inflation. Second, if causing the public to believe χ is lower causes the public to lower its estimate of χ', this also tends to lower current inflation.

What happens if the policy maker tends to become more conservative? Surprisingly, this results in not just lower inflation in the future, but higher inflation today. Suppose that $\chi' = \chi\eta$, where η is a random variable independent of μ_2 and χ and with mean $\hat{\eta} < 1$. An interpretation is that central banks tend to socialize policy makers who are central bankers, causing them to become more conservative over time. Alternatively, policy makers might anticipate a future job in the financial sector. A $\hat{\eta} > 1$ might arise if the policy maker believes he may want to appeal to a government or interest group wanting higher inflation.

The policy maker's expected future preference is $\hat{\eta}\chi$ and $\pi_2^e = \hat{\eta}\pi^{*-1}(\pi_1)$, where π^* is the conjectured inflation rule. Thus, the policy maker's first-period loss function is $\pi_1^2/2 - \chi\mu_1\pi_1 + \beta\hat{\eta}^2\chi\pi^{-1}(\pi_1)$. After insisting that conjectures are correct, the first-order condition is

$$\pi_1(\chi) - \chi\mu\varepsilon_1 + \beta'\chi/\pi'(\chi) = 0, \textit{ where } \beta' := \beta\hat{\eta}. \tag{14}$$

Increased future conservatism is equivalent to a fall in $\hat{\eta}$ and causes a fall in β'. The variable β' plays the role of β in equation (6), so the properties of the equilibrium are the same as in Section 2. A lower β' raises current inflation. This yields the following result.

Proposition 5. *If policy makers tend to become more conservative during their tenure in office, then first-period inflation is higher than if preferences were constant.*

An expected future fall in the preference parameter that is proportional to today's has two effects. First, it lowers the weight put on future output, lowering the policy maker's desire to create future inflationary surprises. Second, it decreases the benefit of signaling because a lower perceived current preference parameter implies a proportionately lower parameter is expected next period. Both effects tend to increase current inflation. Thus, if policy makers tend to become more conservative while in office, they might not inflate more in the second period than the first.

5. A *T*-Period Model

In this section I consider a model lasting T, $2 < T < \infty$, periods where policy makers' preferences can change over time. With probability $\rho \in (0,1)$, preferences change and the new value is drawn from the same distribution. I assume $\mu_t = 1$, $t = 1,\ldots,T$.

In a perfect Bayesian equilibrium, strategies must yield a perfect Bayesian equilibrium in the "continuation game" beginning in each period, given the public's posterior beliefs. Thus, I first solve for the two-period game beginning in $T - 1$, and then solve by backwards recursion for the continuation game beginning in $T - s$, $s = 2,\ldots,T - 1$. As in Section 2, I consider separating equilibria where the public conjectures that period-T-s inflation is given by the twice differentiable function $\pi_{T-s} = \pi^*_{T-s}(\chi)$, $s = 1,\ldots,T-1$.

In each $t < T$, if its current preference parameter is χ, the central bank's expectation of its future preference parameter is $(1 - \rho)\chi + \rho\hat{\chi}$, where $\hat{\chi}$ is the expected value of χ. The public believes that the central bank will choose its within-period optimum at T. Hence, $\pi^e_T = (1 - \rho)\pi^{*-1}_{T-1}(\pi_{T-1}) + \rho\hat{\chi}$. It believes the central bank will follow the rule π^*_{T-s+1} at $T - s + 1$. With probability 1 - ρ, this will be evaluated at the period T - s preference parameter and with probability ρ at a new preference parameter. Hence, $\pi^e_{T-s+1} = (1 - \rho)\pi^*_{T-s+1}(\pi^{*-1}_{T-s}(\pi_{T-s})) + \rho\int\pi^*_{T-s+1}(\chi')f(\chi')d\chi'$. Thus, ignoring terms not affecting the optimization problem, the central bank minimizes $\pi^2_{T-1}/2 - \chi\pi_{T-1} + z\pi^{*-1}_{T-1}(\pi_{T-1})$ in period T - 1 and $\pi^2_{T-s}/2 - \chi\pi_{T-s} + z\pi^*_{T-s+1}(\pi^{*-1}_{T-s}(\pi_{T-s}))$ in period T - s, s = 2,...,T-1. where $z = z(\chi) := \beta(1 - \rho)[(1 - \rho)\chi + \rho\hat{\chi}]$.

The first- and second-order conditions and consistency of conjectures yield

$$\begin{aligned} &\pi_{T-1}(\chi) - \chi + z/\pi'_{T-1}(\chi) = 0 \\ &\pi_{T-s}(\chi) - \chi + z\pi'_{T-s+1}(\chi)/\pi'_{T-s}(\chi) = 0, \; s = 2,...,T-1 \end{aligned} \quad (15)$$

$$\begin{aligned} &1 - z\pi''_{T-1}(\chi)/\pi'_{T-1}(\chi)^3 > 0 \\ &1 + z\pi''_{T-s+1}(\chi)/\pi'_{T-s}(\chi)^2 - z\pi'_{T-s+1}(\chi)\pi''_{T-s}(\chi)/\pi'_{T-s}(\chi)^3 > 0, \; s = 2,...,T-1, \end{aligned} \quad (16)$$

As in Section 2, the boundary condition $\pi_t(\bar{\chi}) = \bar{\chi}$ holds for every t.

Proposition 6. *A perfect Bayesian separating equilibrium has* $\pi_t < \chi$, $t < T$, $\chi \in [\underline{\chi}, \bar{\chi})$.

Proof. Differentiating (15) and substituting the result into (16) yields

$$\pi'_{T-1} > \beta(1 - \rho)^2 > 0, \; \pi'_{T-s} > \beta(1 - \rho)^2\pi'_{T-s+1} > 0, \; s = 2,...,T-1. \quad (17)$$

By (15), this requires that $\pi_{t-s} < \chi$, s = 2,...T-1.

In each period but T, all policy makers but the least inflation averse inflate less than they would without private information. Because the equilibrium in each continuation game does not depend on the public's initial beliefs about the distribution of χ, inflation depends solely on the time a policy maker has left in office and not on how long he has already served or what he has previously done. This contrasts with the existing literature. In Backus and Driffill (1985), separation may not exist in early periods and strategic types may choose their within-period optimum before the last period.

It is not clear whether inflation rises or falls with the amount of time left in office. To analyze this, I focus on the limiting case where $\rho = 0$ and $\beta = 1$.[11]

***Proposition** 7. If* $\beta = 1$, *then in the limit as* ρ *goes to zero,* $\pi_t(\chi) < \pi_{t+1}(\chi),\ t < T,\ \chi \in [\underline{\chi}, \overline{\chi})$.

Proof. By (17), $\pi'_{T-s}(\chi) > \pi'_{T-s+1}(\chi),\ t < T,\ \chi \in [\underline{\chi}, \overline{\chi})$. This and the boundary condition yield the result.

The equilibrium inflation rule specifies lower inflation for any χ, the longer the policy maker has left in office. To see why, consider case of $t = 3$. By lowering his inflation, a policy maker is thought to be more conservative. Expected inflation in period three falls one-for-one with the public's belief about the policy maker's type. In period two, expected inflation falls more rapidly than the public's belief about the policy maker's type if χ is not too small. This gives high-χ policy makers more incentive to lower inflation in period one than in period two. More conservative policy makers must then inflate less in period one to distinguish themselves.

6. Conclusion

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[11] If $\rho = 0$, first-period inflation reveals the central bank's type. If this is χ, then with probability one the public expects to observe $\pi_2(\chi)$ in period two. However, if it observes the out-of-equilibrium event $\pi_2(\chi')$ where $\chi' \neq \chi$, it updates its beliefs; the policy maker is then believed to be type χ'. Vincent (1998) presents a T-period signaling model with two types of monopolists. Analysing the case of $\rho = 0$, he notes it is not uncommon for perfect Bayesian (and sequential) equilibria to require increasing supports off the equilibrium path. With an arbitrarily small possibility of a preference change, this issue does not arise.

This paper analyzes a model where the monetary policy maker's loss function is increasing in inflation and decreasing in output and where output is increasing in unanticipated inflation. The existence of a shock, realized after expectations are formed, provides an activist role for monetary policy.

If the weight the policy maker puts on output, relative to inflation, is known then equilibrium inflation is too high to minimize the equilibrium value of the policy maker's loss function. If his preferences are his private information and there are a continuum of potential policy maker types, then all policy makers but the least inflation averse inflate less and react more strongly to stochastic shocks than they would without private information. Intermediate types may reduce their inflation by most.

If policy makers tend to become more conservative while in office, it is shown that they will inflate less in their final term and more in their first term than if preferences were constant. If policy makers serve $T < 4$ periods terms it is shown that inflation depends solely on their time left in office, and not on how long they have served or what they have done previously. If preferences do not change and policy makers do not discount the future while in office, then they inflate less the longer they have left in office.

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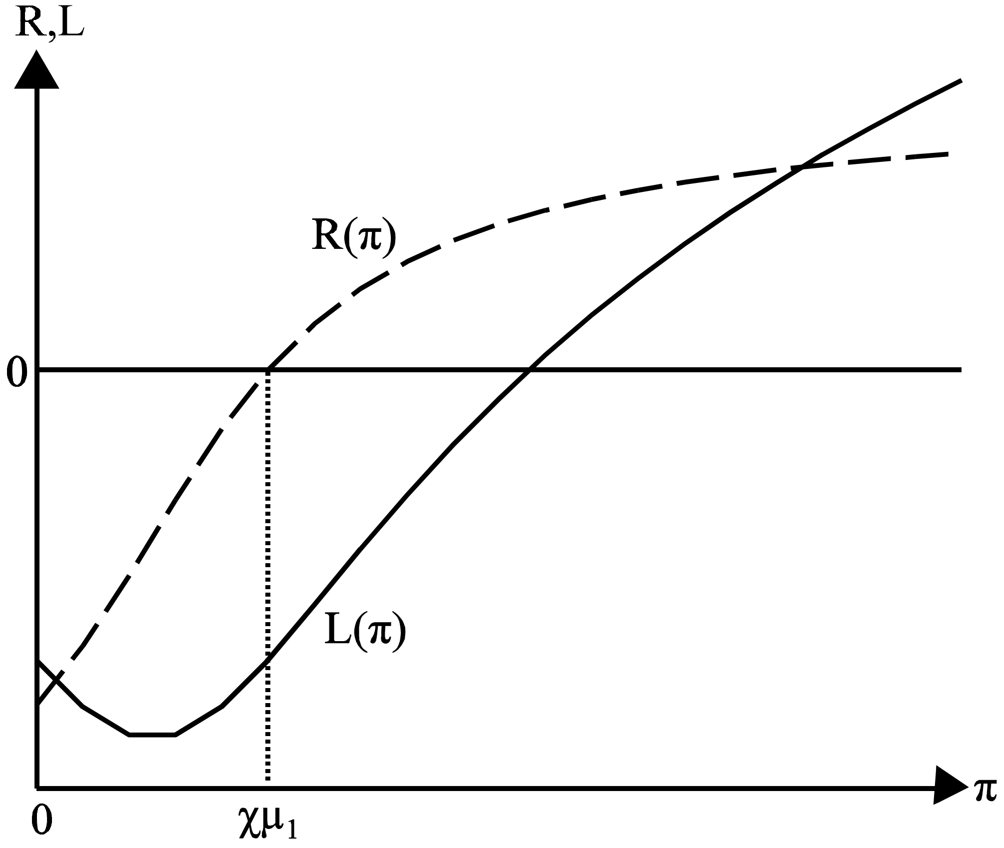
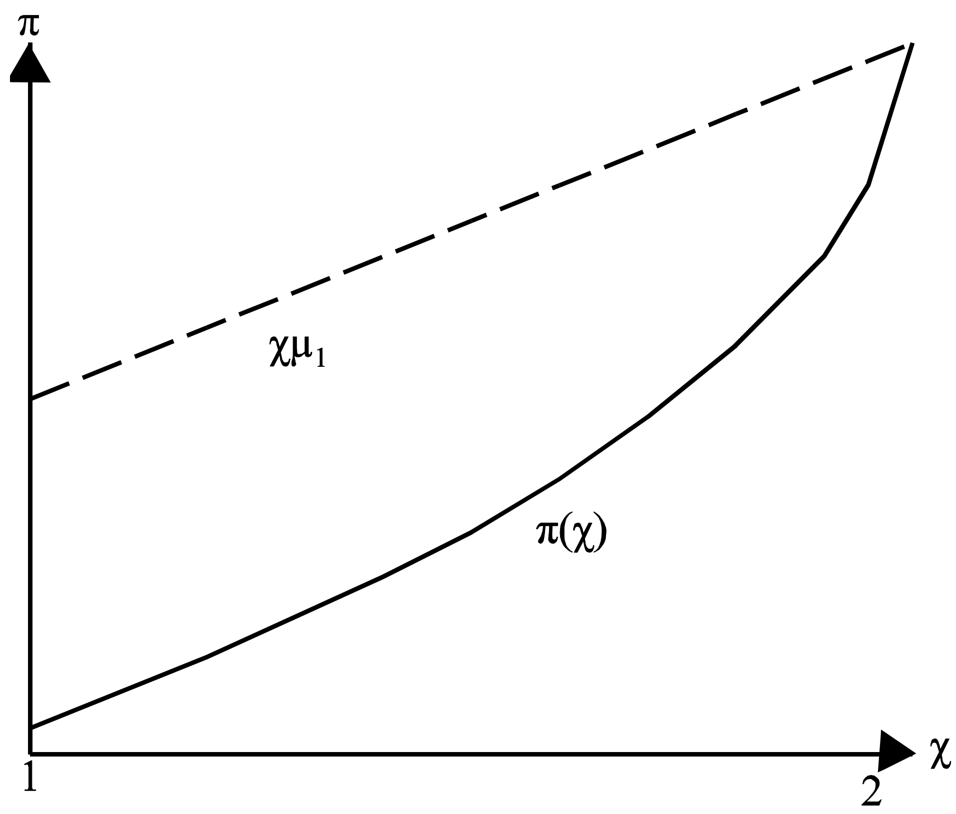
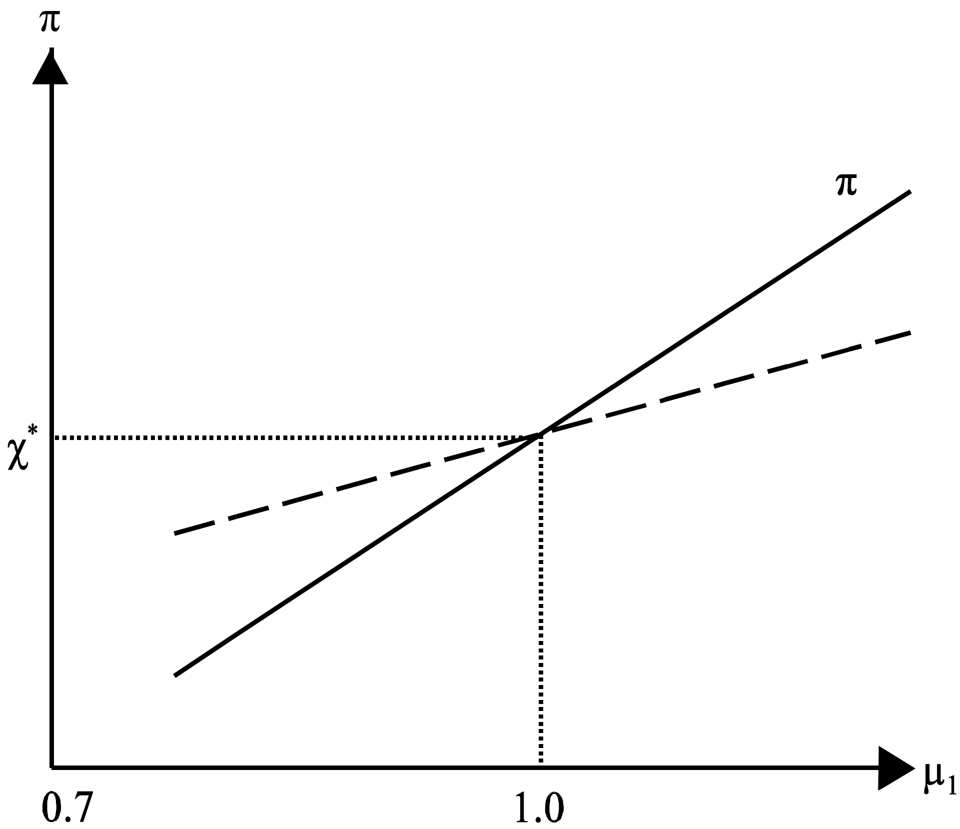


Fig. 1. Equation (11)



Drawn for $\beta = .8$, $\mu_1 = 1$, $\chi = 1$ and $\bar{\chi} = 2$. The distance between $\chi\mu_1$ and $\pi(\chi)$ is maximized at about 1.15.

Fig. 2. Inflation as a Function of the Policy Maker's Type



The dashed line represents the choice of the central banker with χ ' $\chi^($ when there is no private information.

Drawn for $\chi = 1, \overline{\chi} = 1.2, \beta = 1$.

Fig. 3. Inflation as a Function of the Shock