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ABSTRACT

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In this Paper we present a simple model where asset returns are functions of multiple investment growth rates. The model is tested for its ability to price the 25 Fama-French portfolios using the Generalized Methods of Moments (GMM) methodology, as well as Fama-MacBeth cross-sectional regressions. Comparisons on the basis of several metrics with other models, such as the CAPM, the Fama-French (1993) model and Cochrane's (1996) model, reveal that it consistently outperforms the CAPM and Cochrane's model. It also outperforms the Fama-French model in several tests. Our model can explain a significantly larger proportion of the cross-sectional variation in the 25 Fama-French portfolios than the Fama-French model does. Specification tests in the context of GMM and the Fama-MacBeth regressions show that in the presence of the investment growth factors included in our model, the size and book-to-market characteristics lose their ability to explain asset returns. Our model is successful in pricing book-to-market- and size-sorted portfolios, although it includes exclusively macroeconomic variables as factors.

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1. Introduction

Portfolio-based models have dominated the field of asset pricing in the 20th century. The Capital Asset Pricing Model (CAPM) of Sharpe (1964) and Lintner (1965) has been the model on which most of finance theory and practice was built. Unfortunately, tests of the CAPM by Fama and French (1992) revealed that the model is no longer able to explain the cross-section of asset returns. To bridge the gap that the demise of CAPM has created, Fama and French (1992, 1993) proposed an alternative empirical model whose factors are also portfolio returns. This model includes apart from the market portfolio, a factor related to the book-to-market (B/M) of stocks (HML) and a factor related to size (SMB). Fama and French (1992, 1993, 1995, 1996) and Davis, Fama and French (2000) show that the model performs well in explaining a cross-section of B/M and size portfolios. The Fama-French (FF) model has by now largely replaced the CAPM in all finance applications that require the use of an asset pricing model.

There are two outstanding issues with the Fama-French model. First, the HML and SMB factors are derived using empirical methods and therefore, it is not a priori clear that they are related to fundamental economic risk.[1] Second, as Cochrane (1996, and 2001) argues, asset pricing models that use portfolio returns as factors may be successful in describing asset returns, but they will never be able to explain them. The reason is that these models leave unanswered the question of what explains the return-based factors.

[1] Recently, Liew and Vassalou (2000) provide evidence that HML and SMB and related to future economic growth. Furthermore, Vassalou (2000) shows that much of the ability of HML and SMB to explain asset returns is due to news related to future Gross Domestic Product (GDP) growth. Both studies support a risk-based explanation for HML and SMB.

Ideally, one would want to explain asset returns using macroeconomic factors. A central paradigm in this literature is the Consumption CAPM (CCAPM) of Breeden (1979). Unfortunately, CCAPM has not been successful in explaining asset returns.[2] A more recent model that uses macroeconomic factors in pricing risky assets is the investment-based CAPM of Cochrane (1991, 1996). Cochrane (1996) shows that his model performs significantly better than the CCAPM and about as well as the CAPM and the Chen, Roll, and Ross (1986) model.

In this paper, we extend the work of Cochrane (1991, 1996). We derive a simple model where asset returns are linear functions of multiple investment growth rates. Cochrane (1996) shows that investment growth rates and investment returns can explain asset prices equally well. Cochrane's (1991) model is formulated in terms of investment returns. In contrast, our model provides an explicit link between investment returns and investment growth rates. Furthermore, because it allows for the existence of multiple firms, it derives an asset pricing relation where asset returns are linear functions of multiple rather than a single investment growth rate. Consistent with our model specification, the multiple investment growth rates are interpreted in our tests as sector investment growth rates.

As base assets for our tests, we use the 25 Fama-French portfolios. The reason we choose to test our model on those portfolios has to do with the fact that most of the challenges in empirical asset pricing in the 1990s are related to the difficulty of pricing those size- and B/M-sorted portfolios.

[2] Tests of the CCAPM include those of Breeden, Gibbons, and Litzenberger (1989), Campbell (1996), Cochrane (1996), and Lettau and Ludvingson (2001), among others.

The model performs surprising well in explaining the Fama-French portfolios. Comparisons with the CAPM, the FF model and Cochrane's model, show that it consistently outperforms the CAPM and Cochrane's model. It also outperforms the FF model in several tests.

We test the competing models using the Generalized Methods of Moments (GMM) approach of Hansen (1992), as well as Fama and MacBeth (1973) regressions. Within the GMM framework, we perform a number of tests to examine the robustness and comparative performance of our model. In particular, we compute Hansen's (1982) J-test on the overidentifying restrictions of the competing models. We also compute the Newey-West (1987) ΔJ test to examine whether the factors of the models account for all the priced information in HML and SMB. To compare the performance of the models, we use the Hansen and Jagannathan (1997) distance measure. We also test whether the parameters of the models are stable over time by employing Andrews' (1993) supLM test. Finally, we test whether our model, as well as the competing models, are robust to changes in the set of base assets used for their estimations using Cochrane's (1996) approach of scaled returns. Our model performs well with respect to all of the above metrics. The results from the Fama-MacBeth regressions show that the proposed model is also robust to alternative testing methodologies. Specification tests within the Fama-MacBeth framework reaffirm that the investment growth factors of our model absorb all the priced information in the B/M and size characteristics.

The factors in our model are five sector investment growth rates which we compute by summing the residential fixed investment, non-residential fixed investment and changes in private inventories of these sectors. Our tests also include the empirical examination of a reduced-form version of the model which includes only three investment growth rates. It turns out that the three factors which can best predict future GDP growth can also explain asset returns about as well as the five factors together.

The structure of the paper is as follows. Section 2 presents the model. Section 3 discusses the GMM and Fama-MacBeth testing approaches, as well as the various tests performed within their frameworks. Section 4 describes data. Section 5 provides the empirical results from the alternative estimations of the competing models. We conclude with a summary of our findings in Section 6.

2. The Model

The set-up of our model is as follows. A firm maximizes its present value subject to technological constraints. The firm's present value is the expected value of all the future net cash flows paid to the owner of the firm, i.e. the dividend payments. We adopt the time-to-build physical capital assumption. To simplify the model, we assume that capital stock is fully depreciated each period, as in Balvers, Cosimano and McDonald (1990). There is one period lag between investment and the time the investment becomes productive. In other words, investment I_t at time t only becomes productive at time $t+1$. Since the previous period's capital fully depreciates, I_t equals capital stock at time $t+1$. Let K_t be the capital of the firm at time t, and denote by y_t the output at time t. Then we have:

$$y_t = f(K_t), \text{ and} \tag{1}$$

$$K_{t+1} = I_t \tag{2}$$

Let φ be the adjustment cost function, in the sense that in order to make an investment I_t at time t, $I_t\left(1+\varphi\left(\frac{I_t}{K_t}\right)\right)$ units of capital stock will be used.

The firms' objective is to maximize its present value,

$$\max_{\{i_t\}} E_t \sum_{j=0}^{\infty} m_{t,t+j}\left(f(K_{t+j}) - I_{t+j} - I_{t+j}\varphi\left(\frac{I_{t+j}}{K_{t+j}}\right)\right) \tag{3}$$

where $m_{t,t+j}$ is the stochastic discount factor at the time t, which is used to discount cash-flows from $t+j$ to t.

Substitute $m_{t,t+j} = m_{t,t+1}m_{t+1,t+j}$ into equation (3) to get the first order condition

$$1 = E_t\left[m_{t,t+1} \frac{f'(K_{t+1}) + \left(\frac{I_{t+1}}{K_{t+1}}\right)^2 \varphi'\left(\frac{I_{t+1}}{K_{t+1}}\right)}{1+\varphi\left(\frac{I_t}{K_t}\right) + \frac{I_t}{K_t}\varphi'\left(\frac{I_t}{K_t}\right)}\right] \tag{4}$$

And denote

$$R_{t+1}{}^{i} = \frac{f'(K_{t+1}) + \left(\frac{I_{t+1}}{K_{t+1}}\right)^2 \varphi'\left(\frac{I_{t+1}}{K_{t+1}}\right)}{1+\varphi\left(\frac{I_t}{K_t}\right) + \frac{I_t}{K_t}\varphi'\left(\frac{I_t}{K_t}\right)} \tag{5}$$

Note that $R_{t+1}{}^{i}$ is the investment return from time t to time $t+1$. The denominator is the cost of investing one extra capital good at time t. The numerator is the payoff of this extra unit of capital at time $t+1$. In other words, the pricing kernel should price the investment return if it can price all assets correctly.

There are N firms in the economy. Each firm is a price taker in this partial equilibrium model and therefore, its investment decisions do not affect the stochastic discount factor.

Capital stock is the only production input for all firms. Output is produced by a stochastic AK production function. In this production function, the output is linear in capital, K, and A is the technology or marginal productivity. Shocks can be viewed as the marginal productivity of capital. Firms are heterogeneous in the sense that each firm is subject to a different shock.

Firm n's production function is:

$$f_n\left(K_{n,t}\right) = A_{n,t} K_{n,t}\,, \tag{6}$$

where $K_{n,t}$ and $A_{n,t}$ are respectively the capital and the marginal productivity of firm n at time t.

The timing of the production process is as follows. At the beginning of time t, the firm uses it's capital stock K_t to produce. During the production process, shocks are revealed and output y_t is materialized. The firm observes the output and makes decisions as to how much to invest, I_t, and how much to pay out as dividend D_t. As mentioned previously, investment I_t becomes productive capital in the next period, and since capital is fully depreciated in each period, it becomes equal to K_{t+1}. After the shocks at time $t+1$ are revealed, y_{t+1} is produced. This process continues in the following periods.

We assume that there are L common shocks in the economy and that the marginal productivity of each firm is a linear combination of these shocks (as in Brock 1982). That is:

$$A_{n,t} = A_n^0 + A_n^1 \tilde{\delta}_{1,t} + \ldots + A_n^L \tilde{\delta}_{L,t} . \tag{7}$$

We also assume that each shock follows an AR(1) process:

$$\tilde{\delta}_{l,t} = \mu_l + \beta_l \tilde{\delta}_{l,t-1} + \tilde{\varepsilon}_{l,t} . \tag{8}$$

$\tilde{\varepsilon}_{1,t}$, $\tilde{\varepsilon}_{2,t}$, …, $\tilde{\varepsilon}_{L,t}$ are noises at time t, which may be correlated. You may think of each firm as a linear combination of L different technologies. The shock to the *l*th technology is $\tilde{\delta}_{l,t}$.

Since the coefficients A_n^0, A_n^1, …, A_n^L do not depend on t, after the production is revealed, every firm can observe the components of its marginal productivity, i.e., every firm can observe $\tilde{\delta}_{1,t}$, $\tilde{\delta}_{2,t}$, …, $\tilde{\delta}_{L,t}$ after they are realized at time t. The assumption that A_n^0, A_n^1, …, A_n^L are constant over time means that the way shocks effect a firm's productivity is not time-varying.

At time t, each firm maximizes its market value:

$$\max_{\{I_{n,t+j}\}_{j=0}^{\infty}} D_{n,t} + E_t\left[\sum_{j=1}^{\infty} m_{t,t+j} D_{n,t+j}\right]$$
$$\text{s.t.} \begin{cases} D_{n,t+j} = A_{n,t+j}K_{n,t+j} - I_{n,t+j} - I_{n,t+j}\varphi\left(\dfrac{I_{n,t+j}}{K_{n,t+j}}\right) \\ K_{n,t+j+1} = I_{n,t+j} \end{cases} \qquad j = 0, 1, 2, 3, \ldots, \tag{9}$$

where φ is the adjustment cost function which satisfies:

$$\varphi(0) = 0, \varphi' > 0, \text{and } \varphi'' > 0 . \tag{10}$$

Denote the maximized market value of firm n at time t by $S^{*}{}_{n,t}$.

Note that $\tilde{\delta}_{l,t}$ follows a Markov process. Because the production process for firm n after time t depends only on $\tilde{\delta}_{1,t}$, $\tilde{\delta}_{2,t}$, …, $\tilde{\delta}_{L,t}$ and $K_{n,t}$, the optimal decision rule of the firm must be a function of $\tilde{\delta}_{1,t}$, $\tilde{\delta}_{2,t}$, …, $\tilde{\delta}_{L,t}$ and $K_{n,t}$. Therefore, $S^{*}{}_{n,t}$ is also a function of $\tilde{\delta}_{1,t}$, $\tilde{\delta}_{2,t}$, …, $\tilde{\delta}_{L,t}$ and $K_{n,t}$:

$$S^{*}{}_{n,t} = S^{*}\left(\tilde{\delta}_{1,t}, \tilde{\delta}_{2,t}, \ldots, \tilde{\delta}_{L,t}, K_{n,t}\right). \tag{11}$$

Under some regularity conditions, we may assume that there exists a unique continuous function $I^{*} = G_n{}^{*}\left(\delta_1, \delta_2, ..., \delta_L, K\right)$ which gives firm n its optimal market value.

Theorem 1. Function $G_n{}^{*}$ is homogenous of degree 1 in K**.**

(See proof in Appendix 1.)

By Theorem 1, $\dfrac{G_n{}^{*}\left(\delta_1, \delta_2, ..., \delta_L, K\right)}{K}$ is only a function of δ_1, δ_2, …, and δ_L. Using Taylor's Formula, we get $\dfrac{G_n{}^{*}\left(\delta_1, \delta_2, ..., \delta_L, K\right)}{K} \approx B_n^0 + B_n^1\delta_1 + B_n^2\delta_2 + ... + B_n^L\delta_L$.

The investment growth rate $i^{*}{}_{n,t} = \dfrac{I*_{n,t}}{I^{*}{}_{n,t-1}} = \dfrac{I^{*}{}_{n,t}}{K^{*}{}_{n,t}} = \dfrac{G_n{}^{*}\left(\tilde{\delta}_{1,t}, \tilde{\delta}_{2,t}, \ldots, \tilde{\delta}_{L,t}, K^{*}{}_{n,t}\right)}{K^{*}{}_{n,t}}$. This means that $i^{*}{}_{n,t}$ is an approximately linear combination of $\tilde{\delta}_{1,t}, \tilde{\delta}_{2,t}, \ldots,$ and $\tilde{\delta}_{L,t}$:

$$i^{*}{}_{n,t} \approx B_n^0 + B_n^1\tilde{\delta}_{1,t} + B_n^2\tilde{\delta}_{2,t} + ... + B_n^L\tilde{\delta}_{L,t}. \tag{12}$$

By now, we have established a relationship between investment growth rate and production shocks. Recall that the investment return for firm n is :

$$R_{n,t+1}{}^{i} = \frac{f'(K_{n,t+1}) + \left(\frac{I_{n,t+1}}{K_{n,t+1}}\right)^2 \varphi'\left(\frac{I_{n,t+1}}{K_{n,t+1}}\right)}{1+\varphi\left(\frac{I_{n,t}}{K_{n,t}}\right) + \frac{I_{n,t}}{K_{n,t}}\varphi'\left(\frac{I_{n,t}}{K_{n,t}}\right)} = \frac{A_{n,t+1} + (i_{n,t+1})^2 \varphi'(i_{n,t+1})}{1+\varphi(i_{n,t}) + i_{n,t}\varphi'(i_{n,t})} \tag{13}$$

Since the investment growth rate i_{t+1} of a firm can be written as an approximate linear function of the L common shocks $\tilde{\delta}_{1,t}, \tilde{\delta}_{2,t}, \ldots,$ and $\tilde{\delta}_{L,t}$ and $A_{n,t+1}$ is also a linear combination of the L common shocks, the investment return for each firm can be written as a linear function of the L common production shocks.

$$R^{i}{}_{n,t+1} \approx c_{0,t} + c_{1,t}\tilde{\delta}_{1,t+1} + c_{2,t}\tilde{\delta}_{2,t+1} + \ldots + c_{L,t}\tilde{\delta}_{L,t+1} \tag{14}$$

Suppose there are L artificial firms and the lth artificial firm makes use of only the lth technology available in the economy. That is, the lth artificial firm's marginal productivity is only affected by the lth common shock $\tilde{\delta}_{l,t}$. In that case, the investment growth rate for the lth artificial firm $\bar{i}_{l,t}$ depends only on shock $\tilde{\delta}_{l,t}$. We use $\bar{i}$ to denote the investment growth rate of the artificial firm. Moreover $\bar{i}_{l,t}$ can be written as a linear function of the shocks $\tilde{\delta}_{l,t}$ as in equation (12).

Since the L common shocks span the space of the investment return, and the L investment growth rates $\bar{i}_{l,t}$, $l = 1,2,\ldots L$ from the L artificial firms are just linear transformations of $\tilde{\delta}_{1,t}, \tilde{\delta}_{2,t}, \ldots,$ and $\tilde{\delta}_{L,t}$, it follows that $\bar{i}_{1,t}, \bar{i}_{2,t}, \ldots \bar{i}_{L,t}$ also span the space of investment returns. Thus, each firm's investment return can be written as a linear combination of the investment growth rates of these L artificial firms.

$$R^{i}{}_{n,t+1} \approx d_{0,t} + d_{1,t}\bar{i}_{1,t+1} + d_{2,t}\bar{i}_{2,t+1} + \ldots + d_{L,t}\bar{i}_{L,t+1} \tag{15}$$

The above specification links the investment returns with the investment growth rates of different technologies. For different firms, the source of risk is that the output of the firm depends not only on the capital invested by the firm, but also on some random shocks. Different firms are influenced by these shocks differently. Suppose one firm is in farming whereas another firm sells financial products. In years in which there is some natural disaster like drought, the first firm will be greatly affected by the weather, while the second one will be affected much less, if any.

In the absence of arbitrage, there exists a pricing kernel which is a linear combination of the investment growth rates. In the sections to follow, we test the hypothesis that the pricing kernel is a linear combination of investment growth factors and that it can price equity returns correctly.

$$m = \sum_{l} b_l \bar{i}_l \tag{16}$$

where $\bar{i}_l$ is the investment growth rate for lth technology.

3. Estimation Methodology

The model is tested using both the Generalized Methods of Moments (GMM) approach of Hansen (1992) and the Fama and MacBeth (1973) procedure. The two testing approaches have slightly different merits and therefore the use of both methodologies in the current application is warranted.

The GMM is a one-step procedure and therefore it is a priori preferable to two-stage estimation approaches such as the Fama-MacBeth. A number of tests can be performed within the GMM framework which help evaluate the robustness of a model as well as its relative performance vis-à-vis other competing models. However, the small-sample properties of GMM deteriorate rapidly as the ratio of assets used in the tests to time-series observations increases. The problem is more severe here because investment growth data are only available on a quarterly basis. As a result, only a small number of assets can be used to test the proposed model. In this study, we select 12 of the 25 Fama-French portfolios to perform GMM. Details about the base assets used in the GMM estimations are provided in Section 4.

Since much of the debate on asset pricing in the nineties has been about the ability of alternative models to price the 25 portfolios of Fama and French, (1993) it is important to examine how well a newly proposed model can explain all of those assets. To this end, we use the Fama-MacBeth methodology. Apart from the ability of the Fama-MacBeth procedure to accommodate a large number of assets, it also allows the computation of the cross-sectional R-square which is an intuitively appealing metric of the ability of a model to explain the cross-section of asset returns. The main drawback of the Fama-MacBeth procedure is that it is a two-stage estimation, where betas are estimated in the first stage, and risk premiums in the second. As a result, it suffers from the well-known errors-in-variables problem. To alleviate this shortcoming, we use Shanken's (1992) adjustment of the standard errors.

In the sections below we describe the two testing procedures, as well as the various tests we conduct within their framework.

3.1. Description of Optimal GMM and Various Tests Performed

In the absence of arbitrage opportunities in the economy, there exists a stochastic discount factor (pricing kernel) m such that

$$E(r \cdot m) = p\,, \tag{17}$$

where r is a $n\times1$ vector of asset returns, and p is a $n\times1$ vector of assets' prices. If r is the simple return of a portfolio, then its price p is equal to 1. However, if the asset return is expressed in excess of the risk-free rate, its price will be zero.

The pricing kernel of a linear factor model can be written as a linear combination of the factors:

$$m = b_0 + f' \cdot b_1\,, \tag{18}$$

where f is a $k\times1$ vector of factors, b_0 is a scalar, and b_1 is a $k\times1$ vector of coefficients.

Let β be a $n\times k$ matrix of assets' loadings on the factors:

$$\beta = \mathrm{cov}(r,\ f')\,\mathrm{var}(f)^{-1}\,, \tag{19}$$

Then equations (17) and (18) imply that

$$E(r) = r^0 p + \beta'\lambda\,, \tag{20}$$

where $r^0 = \dfrac{1}{E(m)} = \dfrac{1}{b_0 + E(f')b_1}$, and $\lambda = -r^0\,\mathrm{cov}(f, f')b_1$.

To test whether a factor j is priced, we test the null hypothesis $H_0:\ \lambda_j = 0$. When the factors are not orthogonal, and therefore their covariance matrix is not diagonal, testing the above hypothesis is not the same as testing whether a factor *j* helps price the assets. This second hypothesis can be tested by examining whether $b_j = 0$, (see Cochrane, 1996).

We use the optimal GMM of Hansen (1992) to estimate the coefficients in the pricing kernel, the prices of risk, and perform over-identification tests on the models. In other words, we choose b_0 and b_1 to minimize the objective function:

$$J_T = g_T' \cdot W \cdot g_T \tag{21}$$

Hansen (1982) shows that the optimal weighting matrix (in the sense that the variance of estimates is the smallest) is the inverse of the variance matrix of g_T. In practice, we start the procedure with W being the identity matrix. We then estimate the variance matrix $\hat{S}$, use $\hat{S}^{-1}$ to be the new weighting matrix, and iterate this process. A reason we favor the use of the optimal weighting matrix over the identity is related to our choice to perform tests using the Fama-MacBeth procedure in addition to GMM. Cochrane (2001) shows that the Fama-MacBeth procedure is essentially the same as a GMM that uses the identity matrix. Since we aim to examine whether the two alternative testing methodologies provide variant results, there is little scope in performing GMM using the identity matrix.

When the optimal weighting matrix is used, the test on the over-identifying restrictions of the model is given by:

$$T \cdot J_T \sim \chi^2(\#moments - \#\,parameters).$$

We explicitly examine whether the investment growth factors can absorb all of the priced information in the Fama-French factors HML and SMB. To test this hypothesis, we use Newey and West's (1987) ΔJ test. The test follows a χ^2 distribution. To perform the test, we first use GMM to estimate a model that includes the investment growth factors along with HML and SMB. We will call this specification the "unrestricted model". We then use the weighting matrix of this "unrestricted model" to estimate a model that includes only the investment growth factors. This is the "restricted model". The difference in the J functions from the two estimations is chi-square distributed:

$$TJ(restricted) - TJ(unrestricted) \sim \chi^2(\# of\ restriction)$$

The proposed model and its reduced form are also compared to other well-established models in the literature. These are the CAPM, the FF model, and Cochrane's model. To compare the various models examined, we use the Hansen and Jagannathan (1997) (HJ) distance measure.

The definition of the HJ distance is as follows. Denote the coefficients in the pricing kernel by b. Recall that

$$E_t(m_{t+1}r_{t+1}) = p\ , \tag{22}$$

In the above equation, p refers to the true price. Denote $\tilde{p}(b)$ as the prices that depend on the parameters of the pricing kernel so that $\tilde{p}(b) = E_t\left[m_{t+1}(b) \cdot r_{t+1}\right]$. The HJ distance is then equal to:

$$Dist(b) = \sqrt{\left(\tilde{p}(b) - p\right)' E\left[rr'\right]^{-1}\left(\tilde{p}(b) - p\right)} \tag{23}$$

To express it in the more standard GMM notation, we use $g(b) = \tilde{p}(b) - p$ as the moment condition. It then becomes

$$Dist(b) = \sqrt{g(b)' E[rr']^{-1} g(b)} \tag{24}$$

The HJ distance is also the least-square distance between the given pricing kernel and the closest point in the set of the pricing kernels that can price assets correctly.

Let's denote the sample price error as $g_T(b) = \frac{1}{T}\sum_{t=1}^{T} m_t(b) \cdot r_t - p$, and $W_T = \frac{1}{T}\sum_{t=1}^{T} r_t r_t'$.

The sample HJ distance is thus defined as:

$$Dist_T(b) = \sqrt{\min_b g_T(b)' W_T^{-1} g_T(b)} \tag{25}$$

To estimate the HJ distance we use the familiar GMM procedure with one difference. The weighting matrix is the inverse of the covariance matrix of the second moments of asset returns, instead of the optimal or identity matrix. The optimal weighting matrix cannot be used in this case because it is model-specific, a feature that makes it unsuitable for model comparisons. In contrast, the inverse of the covariance matrix of asset returns is invariant across models, and therefore, the HJ distance provides a uniform measure for comparing different models. Moreover, Jagannathan and Wang (1996) derive the asymptotic distribution of the HJ distance, which turns out to be a weighted sum of *n-k* i.i.d. random variables of $\chi^2(1)$ distribution. To get the p-value for the HJ distance, we simulate the weighted sum of $n-k$ $\chi^2(1)$ random variables 100,000 times.

In our discussion of the empirical results of the competing models we also present graphs of the pricing errors they generate, along with their two-standard-error band. The pricing error is given by $g_T(\hat{b}) = p - \frac{1}{T}\sum_{t=1}^{T} m_t(\hat{b}) \cdot r_t$, where $m_t(\hat{b}) = \hat{b}_0 + \hat{b}_1' f_t$. Hansen (1982) derives the asymptotic variance of the pricing error as being equal to:

$$\mathrm{var}\left(g_T(b)\right) = \frac{1}{T}\left(I - D_T\left(D_T' W_T D_T\right)^{-1} D' W_T\right) S_T \left(I - D_T\left(D_T' W_T D_T\right)^{-1} D' W_T\right)' \qquad (28)$$

We also perform robustness tests for the competing models following Cochrane's (1996) approach of scaled returns. In particular, we multiply both sides of equation (17) by a conditioning variable. The resulting scaled returns can be interpreted as managed portfolios where the fund manager adjusts his investment strategy according to the signal he receives from the conditioning variable. As alternative conditioning variables we use the dividend yield, the default premium and the variable cay proposed by Lettau and Ludvingson (2000). Lettau and Ludvingson show that cay is a powerful predictor of asset returns. The dividend yield and the default premium are also well-known in the literature for their ability to predict asset returns.

Finally, we examine the stability of the estimated parameters of the models using Andrews (1993) supLM test. Suppose there is a change point at time $T\pi$. Using GMM, we can estimate the parameters for the sample between 0 and $T\pi$, and the sample between $T\pi$ and T. We can impose the restriction that the parameters of the two samples are equal by also estimating the parameters for the whole sample period. To test whether this restriction holds, we can apply standard Wald, LR (Likelihood Ratio) or LM (Lagrange Multiplier) tests. The LM test is especially easy to perform, because it only uses the restricted estimate, which is just the whole sample estimation that we already got from our previous GMM. To test whether there is a structural change in the time period between $T\pi_1$ and $T\pi_2$, Andrews suggests to use the $\sup_{\pi \in [\pi_1, \pi_2]} LM(\pi)$ statistic. Unfortunately we cannot test whether there is a change point in the whole sample, because $\sup_{\pi \in [\pi_1, \pi_2]} LM(\pi)$ will go to infinity if the interval does not have a positive distance at both endpoints (see Andrews 1993). For that reason, we choose the interval of π_1 = 15% and π_2 = 85% . This is the interval recommended by Andrews (1993) when the change point is unknown.

3.2. Fama-MacBeth Cross-sectional Regression Methodology

An alternative and highly popular method to estimate the prices of risk is the Fama-MacBeth regression methodology. With 25 assets, the covariance matrix will have $\frac{25 \cdot 26}{2} = 325$ free parameters to estimate. In this case, the large-sample property of the optimal GMM estimates becomes somewhat problematic, whereas the Fama-MacBeth regressions can provide useful insights in the pricing of these assets.

The return process is given by,

$$R_t^{\,i} = a_i + \beta_{i,a} f_t^{\,a} + \beta_{i,b} f_t^{\,b} + ... + \varepsilon_t^{\,i}, \quad t = 1,2,...T \tag{29}$$

where f_t are defined factors. Equation (29) implies that the expected return of a risky asset is a linear combination of the asset's loadings on the risk factors. Specifically,

$$E(R^i) = \alpha + \beta_{i,a}\lambda^a + \beta_{i,b}\lambda^b + ... \quad i = 1,2,....N \tag{30}$$

Equation (30) can be tested by running a cross-sectional regression of average returns on betas.

Since the factor loadings β are estimated in the first stage and these βs are used as independent variables in the second stage, there is an errors-in-variables problem. As mentioned earlier, to remedy this problem, we use Shanken's (1992) method to adjust the standard errors. In the tables, we report t-values calculated both from unadjusted and adjusted standard errors.

To test whether the factors in the competing models absorb all the priced information in the size and B/M factors of the FF model, we use the Jagannathan and Wang (1998) approach. They show that useless factors cannot make firm characteristics insignificant in the second stage of cross-sectional regressions. We perform such specification tests by including the average portfolio size and book-to-market ratio in the second stage estimations.

4. Data

The base assets for our tests are the 25 portfolios constructed by Fama and French and obtained from Kenneth French's website.[3] These portfolios are formed from the intersection of five size and five B/M portfolios. The portfolios are rebalanced every end of June, using end-of-June market capitalization and six-month prior B/M information.

[3] We are thankful to Kenneth French for making the data available. Details about the portfolio construction can be obtained from http://web.mit.edu/kfrench/www/index.html

The time period we use is from 1952:1 to 1998:12, which corresponds to the time period for which investment data are also available.

The returns on the 25 portfolios are monthly. Since investment growth rates are only available on a quarterly basis, we compute quarterly returns by compounding the three monthly returns of each quarter. We denote the 25 portfolios as 11, 12, 13, …., 55, where the first digit indicates the portfolio's size group and the second digit indicates portfolio's B/M ratio group, with 1 being the lowest and 5 the highest.

Although we use all the 25 Fama-French portfolios in the Fama-MacBeth tests, we avoid doing that in the GMM tests, because of concerns with the small-sample properties of the estimator mentioned earlier. In an effort to reduce the number of assets and the small-sample problems associated with them, we choose for our GMM tests 12 out of the 25 FF portfolios. The 12 portfolios include three portfolios from the three smallest size quintiles as well as three portfolios from the largest size quintile. In particular, they include the portfolios 11, 13, 15, 21, 23, 25, 31, 33, 35, 51, 53, and 55. Our selection emphasizes the assets in the three smallest size quintiles because as Hodrick and Zhang (2000) confirm, the B/M effect on the small firms is the most difficult to price. In addition to the 12 portfolios, the GMM tests include the risk-free rate.

In the ΔJ tests of the GMM estimations, we use data on HML and SMB. Analogous specification tests performed as part of the Fama-MacBeth procedure use the size and B/M ratios of the 25 portfolios. In addition, the tests of CAPM and the Fama-French model include the return on the market portfolio. Once again, these data are obtained from Kenneth French's website. Size is the portfolio's average size. Furthermore, the B/M ratio of a portfolio is the sum of the book value of firms in the portfolio divided by the sum of their market value. Size is a monthly series whereas B/M is available on an annual basis. The return on the market portfolio is the value-weighted return of all stocks in the CRSP database.

The three-month T-bill rate, RF, is obtained from CRSP. In our estimations, we use the last observation of the previous quarter as the safe rate for the next quarter.

Investment data are obtained from the Federal Reserve Board Statistical Releases. The sample period is 1952Q1 to 1998Q4 with a total of 188 observations. Gross private domestic investment is comprised of non-residential fixed investment, residential fixed investment, and changes in private inventories. It is also divided into five sectors according to the data source. We denote nonresidential investment as NONRES, residential investment as RES, and inventory changes as CHGINV. The five sectors are the following: Household and nonprofit organizations (HHOLDS), non-farm non-financial corporate business (NFINCO), non-farm non-corporate business (NONCOR) farm business (FARM), and financial corporations (FINAN).

Our tests of the proposed model and its reduced form concentrate on the pricing of the investment growth rates of the five sectors. As mentioned earlier, the use of the five-sector investment growth rates is consistent with the derivation of the model we present in Section 2. Recall that the pricing kernel implied by our model is a linear function of the investment growth rates of the L artificial firms. The marginal productivity of these artificial firms is affected only by the lth common shock, since each artificial firm uses only the lth technology in the economy. This justifies the use of sector investment growth rates in our empirical tests. To calculate the gross private domestic investment within each sector, we sum up the residential fixed investment, non-residential fixed investment and changes in private inventories of these sectors.

To select the investment growth factors that are included in the reduced-form model, we explore the relation of the investment growth rates with future Gross Domestic Product (GDP) rate, (see Section 5.1.) Quarterly seasonally-adjusted GDP data are obtained from the Federal Reserve Board Statistical Releases.

To scale returns for our robustness tests we use the dividend yield, the default premium and the variable cay. Dividends are imputed from the value-weighted CRSP return, by including and excluding dividends and then annualizing by summing up the previous 12-month observations. The dividend yield is defined as the annualized dividend level divided by the price level, as in Hodrick (1992). Once the monthly dividend yield series is computed, we use the end-of-quarter observation to construct the quarterly series. The default premium is calculated as the difference between Moody's Seasoned yields on Aaa and Baa corporate bonds. The data source is the Federal Reserve Economic Data. Finally, cay is the consumption-wealth ratio calculated by Lettau and Ludvingson (2000).[4]

We present summary statistics of our investment growth factors and the decomposition of gross private domestic investment in Table 1. It is worth noting the large standard deviations of CHGINV and FARM in Panel A. CHGINV is greatly affected by business cycles. Investment in FARM business also has a very volatile

[4] The data series is available from Martin Lettau's website at http://www.ny.frb.org/rmaghome/economist/lettau/lettau.html

component in inventory changes which is responsible for its large standard deviation. In addition, FARM is a counter-cyclical component of GDP growth rate as can be seen from its negative correlation with the GDP growth rate. The remaining investment growth variables are contemporaneously positively correlated with GDP growth. NFINCO has the highest correlation with the GDP growth and it is equal to 0.675. Furthermore, NFINCO constitutes the largest component of gross private domestic investment with a share of 51.24%. The second largest component is HHOLDS with 26.71% .

5. Estimation Results

5.1. The relation between the investment growth rates and future GDP Growth

From the derivation of our model, it can be seen that the output of a firm at time $t+1$ is a function of the investment at time t. In the aggregate level, this implies a strong relationship between GDP growth rate at time $t+1$ and investment growth rate at time t. We exploit this idea to see which investment growth rates have the most significant relationship with future GDP growth.

Part of the purpose of this study is to throw more light in the information content of SMB and HML. Liew and Vassalou (2000) show that HML and SMB can predict future GDP growth and their ability to do so is largely independent of that of the market portfolio. Furthermore, Vassalou (2000) shows that much of the priced information in HML and SMB is due to news related to future GDP growth. Since GDP growth is an aggregate variable, it is interesting to examine which components of GDP growth SMB and HML are most closely related to. In this study, we test whether components of investment growth rates can account for all the priced information in HML and SMB.

To identify those sector investment growth rates that are most closely related to future GDP growth, we use regression analysis. The results are presented in Table 2. HHOLDS significantly predicts one-quarter ahead future GDP growth. The slope coefficient has a t-value of 5.86 and the R-square of the regression is 16%. FARM can also predict GDP growth one quarter ahead, but the R-square is less than 1%. Finally, NFINCO can predict future GDP growth and the R-square of the regression is 2%.

In Figure 1, we plot the GDP growth rate series and the investment growth variables with the shaded area denoting the NBER recession periods. HHOLDS and NFINCO decline significantly during recession times. In contrast, FINAN does not seem to be influenced by recessions very much and NONCOR has a weaker relation with variations in the business cycles than HHOLDS and NFINCO do. FARM has peaks in periods of recession due to the significant increase in the inventory component.

The results in Table 2 showed that the slope coefficients from regressions of future GDP growth on HHOLDS, NFINCO and FARM individually, produce statistically significant coefficients. In addition, Figure 1 suggests that the same investment growth rates react more to variations in the business cycles than FINAN and NONCOR. For the above reasons, we also test whether a model that includes only HHOLDS, NFINCO, and FARM as factors is able to explain the asset returns and the value premium of the 25 Fama-French portfolios.

5.2. GMM Estimation Results

Non-scaled returns

We first examine the performance of the model with the five investment growth factors. The results are presented in Panel A of Table 3. The model performs reasonably well. The J-test cannot reject the model. The associated p-value is 0.656, which implies that the model can price the 13 assets correctly. However only NONCOR and NFINCO have statistically significant coefficients. As a result, the Wald test cannot reject the hypothesis that the *b* coefficients of the model are jointly zero. The HJ-distance is 0.383. Assuming an annual standard deviation of 20%, as in Campbell and Cochrane (2000), the maximum annualized pricing error is 7.7%. Furthermore, the p-value of the HJ-distance is 0.368, which means that we cannot reject the hypothesis that the HJ-distance is zero. The ΔJ statistic has a p-value of 0.210 implying that after we include HML and SMB in the pricing kernel, the change in the objective function is not statistically significant. In other words, it appears that in the presence of the five investment growth factors in the pricing kernel, HML, SMB lose their ability to explain the prices of assets. Finally, the results of the supLM test imply that the parameters of the model are stable over time, and there are no indications of potential structural breaks.

Panel B of Table 3 reports the results on the three-investment-growth asset pricing model, with HHOLDS, NFINCO, and FARM being the factors. Both HHOLDS, and NFINCO have significant coefficients in the pricing kernel and the model passes the Wald test that its *b* coefficients are jointly significant at the 10% level. However, the J-test provides some evidence against the model. The p-value of the test is 0.025, implying that the model is rejected at the 5% level of significance. The HJ-distance of the model is 0.462, which translates into a maximum annualized pricing error of 9.2%. This is larger than the one found or the five-factor investment growth model of Panel A. Furthermore, the p-value of the HJ-distance is 0.078 suggesting that it is statistically significant at the 10% level. The ΔJ test reveals that HML and SMB lose once again their ability to explain asset prices when the three investment growth factors are present in the pricing kernel. In addition, the model exhibits parameter stability as shown by the supLM test.

To compare the performance of the proposed models, we also examine three other models which are well-established in the literature. These are the Capital Asset Pricing Model (CAPM), the Fama and French (1993) (FF) model and Cochrane's (1996) investment-based asset pricing model. The results are presented in Panels C to E of Table 3.

In the CAPM, the market factor is significantly priced. However, there is strong evidence against the model since the J-statistic is 40.856 with a p-value of 0.000. It also fails to pass the HJ test. The supLM statistic is 12.330. Furthermore, we reject the hypothesis of parameter stability at the 5% level of significance.

The Fama-French model performs better than the CAPM and Cochrane's model. The market factor and HML are statistically significant and have significant prices of risk. The three factors are also jointly significant, as shown by the p-value of the Wald test. However, the model does not pass the J-test on the overidentifying restrictions. The p-value is 0.008. The HJ-distance is 0.397, which translates into a maximum annual pricing error of 7.9%. The p-value for the HJ-distance is 0.003, and therefore the hypothesis that it is zero is rejected again. The supLM test shows that the parameters of the model are stable over time.

In testing Cochrane's model, we use the residential (RES) and nonresidential (NONRES) investment growth rates rather than the investment returns. As mentioned earlier, Cochrane (1996) shows that the performance of his model is similar when investment growth rates are used instead of investment returns.

The NONRES investment growth rate has both a statistically significant coefficient and risk premium. We reject the hypothesis that the factors of the model are jointly zero. Similarly to the other benchmark models, Cochrane's model does not pass the J-test on the overidentifying restrictions. The HJ-distance is equal to 0.528 and has a p-value of zero. It corresponds to a maximum annualized pricing error of 10.6%. This is larger than those of the other models examined. Furthermore, when HML and SMB are added to the pricing kernel, the ΔJ-test has a p-value of 0.063. This means that we can reject, at the 10% level of significance, the hypothesis that HML and SMB do not add explanatory power to the model. Finally, the supLM test detects some degree of parameter instability. It is statistically significant at the 10% level. Our findings on Cochrane's model are consistent with those presented in Hodrick and Zhang (2000).

We plot the pricing errors of all the competing models in Figure 2. All models have difficulty in explaining the returns of small growth portfolios. In the case of CAPM, the B/M effect is quite significant for the 4 B/M groups, with some pricing errors being outside the two-standard-deviation band. The Fama-French model exhibits a far less distinct pattern in the pricing errors than the other models examined. Note that the two proposed models, as well as Cochrane's model, have much larger standard errors than the

CAPM or the FF model. This results from the fact that they include exclusively macro-variables as factors in their pricing kernels. The two proposed models generate smaller pricing errors in general than Cochrane's model. That is also evident from the HJ-distances of the three models. In particular, the pricing errors of the proposed models are smaller for the first two small size quintiles than they are for Cochrane's model. This is again consistent with the results of the ΔJ tests in Table 3 which show that the factors in the proposed models absorb all the priced information in HML and SMB, whereas the factors in Cochrane's model do not.

Robustness tests: Scaled Returns

An important robustness test for any asset pricing model is to examine how well the model performs when it is asked to price different sets of assets. To that end, we use Cochrane's (1996) approach and scale returns by conditioning variables. The scaled returns can be interpreted as returns on managed portfolios, where the fund manager adjusts his investment strategy according to the information provided by the conditioning variable. The conditioning variables considered are the dividend yield, the default premium and cay. All conditioning variables are lagged by one quarter.

Table 4 presents results of the models when returns are scaled by the dividend yield. Note that the two proposed models perform better with scaled returns by dividend than they do with unscaled returns, in the sense that the maximum annualized pricing errors they generate are now smaller. The five-factor model has a maximum pricing error of

5.96%, whereas the three-factor model has a maximum pricing error of 7.8%. The remaining tests of the two models provide similar evidence to that of Table 3.

The results for the benchmark models are also similar to those of Table 3 for unscaled returns. Again, we notice that the HJ distances for the benchmark models are smaller than those generated by the same models for unscaled returns. The FF model still performs somewhat worse than the five-factor model, in terms of this metric. However, Cochrane's model has now an HJ-distance which is slightly lower than that of the CAPM, whereas the opposite was true in the case of unscaled returns.

When returns are scaled by the default premium, the evidence for the models changes to some extent. The results are reported in Table 5. The HJ-distance for the FF model is now smaller than those of the two proposed models. Furthermore, there is some evidence against the three-factor investment growth model, as shown by the J-test which has a p-value of 0.018. However, both of the proposed models pass the ΔJ-test. The HJ distance for Cochrane's model is smaller than that of the CAPM, but larger than those of the two proposed models. Furthermore, Cochrane's model is the only one which now exhibits parameter instability, as revealed by the supLM test.

Table 6 reports the results from testing the models when returns are scaled by cay. These results are very similar to those of unscaled returns. The FF model has an HJ distance which is larger than that of the five factor investment growth model, but smaller than the HJ distance of the three factor investment growth model. Cochrane's model continues to have an HJ distance which is larger than that of the CAPM, and both models generate maximum pricing errors which are larger than those of all the other models. There is again some evidence against the three-factor investment growth model as shown by the J-test. Both of the proposed models pass the ΔJ test. In other words, no matter whether we use unscaled returns or returns scaled by any of the three conditioning variables, the ΔJ-test shows that in the presence of the proposed investment growth factors, HML and SMB lose their ability to explain the prices of the base assets. Finally, the supLM test reveals evidence of parameter instability for the CAPM and Cochrane's model at the 5% and 10% level of significance respectively.

Tables 4 to 6 show that the results on the proposed models are robust to changes in the base assets.

5.3. Fama-MacBeth Cross-sectional Regressions

Recall that the Fama-MacBeth tests use all the 25 Fama-French portfolios, instead of only 12 of them.

The results are reported in Table 7. We denote the market excess return by EMKT. CAPM can explain about 4% of the cross-sectional variation in returns, and the premium on the market portfolio is not statistically significant. This is consistent with the poor performance of CAPM previously documented in Fama and French (1992), Jagannathan and Wang (1996), and Lettau and Ludvigson (2000). Similarly to Lettau and Ludvigson (2000), we find that the market risk premium is negative.

In the FF model, only the risk premium on HML is statistically significant. This is again consistent with the evidence presented in Lettau and Ludvingston (2000).[5] The FF model can explain 63% of the cross-sectional variation in returns.

Cochrane's model can explain 50% of the cross-sectional return variation. The results imply that the NONRES investment growth factor is both economically and statistically more significant for explaining asset returns than the RES investment growth. This is consistent with our findings from the GMM estimations.

In contrast to the above benchmark models, the five-factor investment growth model has a cross-sectional adjusted R-square of 84%. The risk premium on NFINCO is statistically significant even after applying Shanken's correction to the standard errors.[6] Recall that NFINCO is the largest component in gross private domestic investment.

The three-factor investment growth model can still explain 73 percent of the total variation. This is larger than the cross-sectional variation explained by the FF model and the other two benchmark models.

The ability of the two proposed models to explain the cross-section of the 25 portfolios can also be seen from the pricing errors they generate. Figure 3 plots the

[5] Note that Lettau and Ludvingston's tests cover the period from 1963Q3 to 1998Q3, whereas ours cover the period from 1952Q2 to 1998Q4.

[6] Note that Shanken's (1992) adjustment significantly decreases the t-values of the risk premiums in the two proposed models and Cochrane's model, where only macroeconomic factors are used. This is not the case for the risk premiums estimated as part of the CAPM and the FF models, where the adjustment has a minimal impact on the value of the t-statistic. Shanken's (1992) adjustment factor for the estimated covariance matrix is $(1+c)$, where $c = \Gamma'\Sigma^{-1}{}_F\Gamma$. Γ is the estimated premium (omitting the constant term), and Σ is the covariance matrix of the factors. As can be seen from Table 1, all the investment growth factors have high standard deviations. Thus, the adjustments for the risk premiums of these factors will be high, leading to big reductions in the unadjusted t-values. Note that the standard deviations of the

realized versus the fitted returns of the 25 portfolios for all competing models. Overall, the two proposed models perform better than the benchmark models. In particular, it is interesting to note that the five- and three-factor investment growth models are better able to price small growth stocks than the FF model.

5.4. Specification Tests

In this section we explicitly test the ability of the size and B/M firm characteristics to explain the cross-section of asset returns when they are added as factors in the competing models.

Table 8A reports the results from estimating the models including the size characteristic. As it is revealed from the t-values, the size coefficient is not statistically significant for either of the proposed models. It is not significant in Cochrane's model either. These results imply that both the proposed models and Cochrane's model can adequately account for the explanatory power of size. As expected, the addition of size in the CAPM results in a slope coefficient which is highly statistically significant. The large effect that size has in explaining the cross-section of asset returns can also be seen from the dramatic increase in the adjusted R-square of the CAPM model. It is now 57%, compared to only 4% in the absence of the size factor.

In Table 8B we report the results from the specification tests that include the B/M characteristic. Once again, the slope coefficient of the B/M is not statistically significant in either of the two proposed models. It is, however, significant at the 10% level in

quarterly FF factors EMKT, SMB and HML are 0.079, 0.053, 0.049 respectively. It can be seen from Table 1 that they are generally smaller than those of the investment growth rates.

Cochrane’s model. It appears that the investment growth factors used in the proposed models are better able to account for the B/M characteristic than the residential and nonresidential investment factors of Cochrane’s model. As expected, B/M has a large impact on the ability of the CAPM to explain the cross-section. The slope coefficient is highly statistically significant and the adjusted R-square is 57%.

Note that the results from the specification tests of this section are consistent with the evidence provided from the ΔJ tests discussed earlier. Both types of tests reveal that the proposed investment growth factors contain all the priced information of HML and SMB, rendering the inclusion of these two factors pointless. Furthermore, the evidence of the two types of tests is also consistent for the CAPM and for Cochrane’s model. Recall that in the case of CAPM the p-value of the ΔJ test is always zero. In contrast, in the case of Cochrane's model, we can generally reject the hypothesis that HML and SMB are not important for explaining asset prices at the 10% level.

The performance of the proposed investment growth models in the Fama-MacBeth cross-sectional tests compliments our findings from the GMM estimations. Therefore, our proposed models are robust to alternative testing methodologies, and their performance is not influenced by the number of assets used in these alternative testing approaches.

6. Conclusions

This paper develops a simple investment growth asset pricing model which allows for multiple investment growth factors to affect asset returns. We test the model using five sector investment growth rates as factors, as well as a reduced form of it which uses only three investment growth rates. Our tests follow the GMM methodology and our findings are supplemented with results from Fama-MacBeth cross-sectional regressions.

The performance of the proposed models is compared to those of the CAPM, the FF model and Cochrane's investment-based asset pricing model. Our results show that the proposed models always outperform Cochrane's model, as well as the FF model in several cases. Furthermore, the proposed models account for all the priced information in the size and B/M factors proposed by Fama and French (1992, 1993). The parameters of the proposed models are stable over time and the maximum annualized pricing errors that they generate are often similar to those of the FF model. In terms of their ability to explain the cross-section of asset returns in the Fama-MacBeth regressions, even our three-factor investment growth model outperforms the FF model, by generating a significantly higher cross-sectional adjusted R-square.

The proposed models include exclusively macroeconomic variables as factors, and therefore, their ability to outperform models that use asset return-based factors is important.

Appendix 1.

First we show that S^* is homogeneous of degree 1 in K, that is: $S^*(\delta_1, \delta_2, ..., \delta_L, \mathrm{c} \cdot K) = \mathrm{c} \cdot S^*(\delta_1, \delta_2, ..., \delta_L, K)$.

Suppose there are two firms with identical marginal productivity. Firm 1 has initial capital $c \cdot K$, whereas firm 2 has initial capital K. We need to prove that $S_1^* = c \cdot S_2^*$.

We define $\tilde{G}(\delta_1, \delta_2, ..., \delta_L, K) = c \cdot G^*\left(\delta_1, \delta_2, ..., \delta_L, \frac{K}{c}\right)$. We omit the subscript n from the function G, because the claim in Theorem 1 holds for every firm. Let $\tilde{S}_1$ be the stock valuation of firm 1 when it applies $\tilde{G}$ as the decision rule, and assume that the firm 2 is using the optimal strategy. Then for firm 1, next period's capital will be

$$\tilde{F}(\delta_1, \delta_2, ..., \delta_L, \mathrm{c} \cdot K) = c \cdot F^*(\delta_1, \delta_2, ..., \delta_L, K).$$

This is exactly c times the firm 2's capital in the next period. By induction, we can see that firm's 1 capital is always c times firm's 2 capital at any time. This also implies that the investment made by firm 1(using strategy $\tilde{G}$) is always c times the investment made by firm 2 (using strategy G^*). Hence, the dividend $D = AK - I - \varphi\left(\frac{I}{K}\right)$ for firm 1 is also c times the dividend of firm 2 at any time. This means $\tilde{S}_1 = c \cdot S_2^*$. Therefore, we get:.

$$S_1^* \geq \tilde{S}_1 = c \cdot S_2^*. \tag{A.1}$$

This results from the fact that firm's 2 capital is $\frac{1}{c}$ times firm's 1 capital. By the same token, we get:

$$S_2^* \geq \frac{1}{c} S_1^* . \tag{A.2}$$

It follows from (A.1) and (A.2) that $S_1^* = c \cdot S_2^*$.

By (A.1), this implies that $S_1^* = \tilde{S}_1$. Thus, $\tilde{G}$ is the optimal decision rule for firm 1. Because G^* is continuous, $\tilde{G}$ is also continuous. Since the continuous decision rule is unique, $G^*(\delta_1, \delta_2, ..., \delta_L, K) = c \cdot G^*\left(\delta_1, \delta_2, ..., \delta_L, \frac{K}{c}\right)$ must hold, and this proves the conclusion in Theorem 1.

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Table 1: Summary Statistics of the Data

Panel A

Variable	mean	std	ρ_1	% of GPI
GDP	0.018	0.011	0.392	
NONRES	0.020	0.026	0.369	67.60%
RES	0.018	0.048	0.535	28.74%
CHGINV	0.282	6.299	-0.023	3.66%
HHOLDS	0.018	0.044	0.464	26.71%
NFINCO	0.022	0.083	-0.035	51.24%
NONCOR	0.019	0.078	-0.114	14.10%
FARM	-0.349	6.218	0.023	3.50%
FINAN	0.038	0.095	-0.099	4.45%

Panel B

	GDP	NONRES	RES	CHGINV		
GDP	1.000					
NONRES	0.566	1.000				
RES	0.410	0.160	1.000			
CHGINV	0.061	-0.026	-0.025	1.000		
	GDP	HHOLDS	NFINCO	NONCOR	FARM	FINAN
GDP	1.000					
HHOLDS	0.407	1.000				
NFINCO	0.675	0.189	1.000			
NONCOR	0.326	0.286	0.306	1.000		
FARM	-0.058	-0.121	-0.101	-0.069	1.000	
FINAN	0.175	0.002	-0.035	-0.149	0.019	1.000

Note: Panel A of Table 1 provides the means, standard deviations (std), first-order autocorrelations (ρ_1), and the percentage of Gross Private Investment (%GPI) that each investment growth rate accounts for. Panel B reports the correlation coefficients of the investment growth rates with the Gross Domestic Product (GDP) growth rate. The time period is from 1952Q2 to 1998Q4.

Table 2: The Ability of the Investment Growth Rates to Predict GDP Growth One-Quarter Ahead.

	Coefficient	t-value	R^2
Constant	1.588	19.692	0.167
HHOLDS	9.883	5.863	0.163
Constant	1.718	21.129	0.023
NFINCO	1.934	1.871	0.018
Constant	1.735	22.430	0.011
NONCOR	1.460	1.186	0.006
Constant	1.756	22.787	0.007
FARM	-0.014	-8.574	0.002
Constant	1.738	21.273	0.003
FINAN	0.587	0.829	-0.003

Note: Table 2 presents regressions of the one-quarter ahead GDP growth rate on the investment growth rates. We denote the investment growth rate of the household and nonprofit sector by HHOLDS, the non-financial, non-farm sector by NFINCO, the non-farm non-corporate sector by NONCOR, the farming sector by FARM, and the financial sector by FINAN. Note that the slope coefficients are scaled by 100. T-values are corrected for White (1980) heteroskedasticity. The second row of the R-square column refers to the adjusted R-square. The time period is from 1952Q2 to1998Q4.

Table 3: GMM Estimation of Competing Models: Unscaled Returns

Panel A: Five-Factor Investment Growth Model

	Constant	HHOLDS	NFINCO	NONCOR	FARM	FINAN
Coefficient	0.658	-16.135	13.708	19.400	0.133	0.802
t-value	1.931	-1.648	1.668	1.966	0.852	0.123
Premium		0.007	-0.104	-0.121	-4.254	-0.010
t-value		0.345	-1.892	-2.092	-0.710	-0.185
Tests:	*J*	ΔJ	*Wald(b)*	*HJ Dist*	*sup LM*	
statistic	5.036	3.022		0.383	15.601	
p-value	0.656	0.210	0.143	0.452		

Panel B: Three-Factor Investment Growth Model

	Constant	HHOLDS	NFINCO	FARM	
Coefficient	1.068	-16.474	11.464	0.079	
t-value	6.747	-2.294	2.009	0.881	
Premium		0.026	-0.057	-3.067	
t-value		2.015	-1.678	-0.869	
Tests:	*J*	ΔJ	*Wald(b)*	*HJ Dist*	*sup LM*
statistic	19.031	3.971		0.462	7.323
p-value	0.025	0.137	0.079	0.088	

Panel C: CAPM

	Constant	EMKT			
Coefficient	1.068	-2.612			
t-value	21.459	-2.465			
Premium		0.017			
t-value		2.465			
Tests:	*J*	ΔJ	*Wald(b)*	*HJ Dist*	*sup LM*
statistic	40.856	21.358		0.508	12.330**
p-value	0.000	0.000	0.014	0.000	

Panel D: The Fama-French Model

	Constant	EMKT	SMB	HML	
Coefficient	1.229	-5.139	1.916	-7.536	
t-value	16.065	-3.841	1.279	-4.571	
Premium		0.020	0.002	0.012	
t-value		2.652	0.597	3.193	
Tests:	*J*	ΔJ	*Wald(b)*	*HJ Dist*	*sup LM*
statistic	22.261			0.397	10.807
p-value	0.008		0.000	0.003	

Panel E: Cochrane's Model

	Constant	NONRES	RES		
Coefficient	0.354	35.904	-3.055		
t-value	1.030	2.444	-0.554		
Premium		-0.020	0.001		
t-value		-2.250	0.040		
Tests:	*J*	ΔJ	*Wald(b)*	*HJ Dist*	*sup LM*
statistic	29.947	5.536		0.528	14.094*
p-value	0.001	0.063	0.009	0.000	

Note: The GMM estimations of the models use 12 of the 25 Fama-French portfolios in addition to the risk-free rate. The stock portfolios are the following: 11, 13, 15, 21, 23, 25, 31, 33, 35, 51, 53, and 55, where the first digit denotes the size quintile while the second digit denotes the book-to-market quintile. The number 1 stands for small size (low B/M) and the number 5 for big size (high B/M). We denote the investment growth rate of the household and nonprofit sector by HHOLDS, the non-financial, non-farm sector by NFINCO, the non-farm non-corporate sector by NONCOR, the farming sector by FARM, and the financial sector by FINAN. In the tests of Cochrane's model, the residential investment growth rate is denoted by RES and the nonresidential by NONRES. HML is a zero-investment portfolio which is long on high book-to-market (B/M) stocks and short on low B/M stocks. Similarly, SMB is a zero-investment portfolio which is long on small capitalization stocks and short on big capitalization stocks. EMKT refers to the excess return on the stock market portfolio. The J-test is Hansen's (1982) test on the overidentifying restrictions of the model. The ΔJ test is the Newey-West (1987) chi-square difference test. It examines the increase in the J function of a model when HML and SMB are added in the pricing kernel. The Wald(b) test is a joint significance test of the *b* coefficients in the pricing kernel. The J, ΔJ , and Wald(b) tests are computed in GMM estimations that use the optimal weighting matrix. We denote by "HJ Dist" the Hansen-Jagannathan (1997) distance measure. It refers to the least-square distance between the given pricing kernel and the closest point in the set of pricing kernels that price the assets correctly. The p-value of the measure is obtained from 100,000 simulations. The supLM test refers to Andrews (1993) stability test. It examines whether the parameters of the model are stable during the sample period. We indicate that a model does not pass the stability test at the 10%, 5%, and 1% level of significance with one, two, and three asterisks respectively. The critical values for the supLM test are obtained from Andrews (1993). The HJ distance and the supLM tests are computed using the Hansen and Jagannathan weighting matrix of second moments of asset returns. The estimation period is from 1953Q1 to 1998Q4.

Table 4: GMM Estimations of the Competing Models: Scaled Returns by Dividend Yield

Panel A: Five-Factor Investment Growth Model

	Constant	HHOLDS	NFINCO	NONCOR	FARM	FINAN
Coefficient	0.541	-16.514	11.077	20.723	0.142	3.040
t-value	1.523	-1.929	1.757	1.903	0.977	0.542
Premium		0.009	-0.096	-0.132	-5.107	-0.010
t-value		0.492	-2.068	-1.950	-0.855	-0.188
Tests:	*J*	ΔJ	*Wald(b)*	*HJ Dist*	*sup LM*	
Statistic	3.909	4.975		0.298	14.637	
p-value	0.790	0.083	0.134	0.748		

Panel B: Three-Factor Investment Growth Model

	Constant	HHOLDS	NFINCO	FARM	
Coefficient	1.048	-16.989	13.249	0.099	
t-value	6.844	-2.343	2.586	0.935	
Premium		0.026	-0.066	-3.720	
t-value		2.024	-2.225	-0.905	
Tests:	*J*	ΔJ	*Wald(b)*	*HJ Dist*	*sup LM*
statistic	11.909	3.986		0.392	10.064
p-value	0.218	0.136	0.037	0.279	

Panel C: CAPM

	Constant	EMKT			
Coefficient	1.091	-3.271			
t-value	20.347	-3.047			
Premium		0.021			
t-value		3.047			
Tests:	*J*	ΔJ	*Wald(b)*	*HJ Dist*	*sup LM*
statistic	39.483	17.052		0.471	12.259**
p-value	0.000	0.000	0.002	0.000	

Panel D: The Fama-French Model

	Constant	EMKT	SMB	HML	
Coefficient	1.233	-5.230	1.194	-7.121	
t-value	16.494	-3.992	0.772	-4.124	
Premium		0.022	0.005	0.011	
t-value		3.053	1.180	2.675	
Tests:	*J*	ΔJ	*Wald(b)*	*HJ Dist*	*sup LM*
statistic	21.299			0.367	8.126
p-value	0.011		0.000	0.008	

Panel E: Cochrane's Model

	Constant	NONRES	RES		
Coefficient	0.242	40.075	-1.178		
t-value	0.865	3.410	-0.235		
Premium		-0.023	-0.005		
t-value		-3.211	-0.363		
Tests:	*J*	ΔJ	*Wald(b)*	*HJ Dist*	*sup LM*
statistic	21.595	5.533		0.457	12.938*
p-value	0.017	0.063	0.001	0.023	

Note: Same comments as in Table 3 apply.

**Table 5: GMM Estimations of the Competing Models:
Scaled Returns by the Default Premium**

Panel A: Five-Factor Investment Growth Model

	Constant	HHOLDS	NFINCO	NONCOR	FARM	FINAN
Coefficient	0.538	-12.450	6.594	18.563	0.138	3.996
t-value	1.484	-1.842	1.038	2.111	1.092	0.602
Premium		0.006	-0.067	-0.116	-5.303	-0.022
t-value		0.369	-1.414	-2.018	-0.986	-0.357
Tests:	*J*	ΔJ	*Wald(b)*	*HJ Dist*	*sup LM*	
statistic	9.203	4.259		0.404	11.680	
p-value	0.238	0.119	0.108	0.028		

Panel B: Three-Factor Investment Growth Model

	Constant	HHOLDS	NFINCO	FARM	
Coefficient	1.028	-12.189	8.905	0.063	
t-value	8.684	-2.560	2.349	1.015	
Premium		0.019	-0.045	-2.447	
t-value		2.140	-1.936	-0.993	
Tests:	*J*	ΔJ	*Wald(b)*	*HJ Dist*	*sup LM*
statistic	19.995	4.581		0.423	5.744
p-value	0.018	0.101	0.023	0.016	

Panel C: CAPM

	Constant	EMKT			
Coefficient	1.098	-3.406			
t-value	20.594	-3.234			
Premium		0.022			
t-value		3.234			
Tests:	*J*	ΔJ	*Wald(b)*	*HJ Dist*	*sup LM*
statistic	36.784	17.203		0.454	4.535
p-value	0.000	0.000	0.001	0.000	

Panel D: The Fama-French Model

	Constant	EMKT	SMB	HML	
Coefficient	1.250	-5.803	1.699	-7.101	
t-value	16.034	-4.168	1.054	-4.147	
Premium		0.025	0.004	0.010	
t-value		3.320	1.053	2.576	
Tests:	*J*	ΔJ	*Wald(b)*	*HJ Dist*	*sup LM*
statistic	20.169			0.343	12.437
p-value	0.017		0.000	0.014	

Panel E: Cochrane's Model

	Constant	NONRES	RES		
Coefficient	0.328	36.071	-1.967		
t-value	1.203	3.224	-0.440		
Premium		-0.021	-0.002		
t-value		-2.976	-0.173		
Tests:	*J*	ΔJ	*Wald(b)*	*HJ Dist*	*sup LM*
statistic	21.202	6.336		0.436	14.215**
p-value	0.020	0.042	0.000	0.015	

Note: Same comments as in Table 3 apply.

Table 6: GMM Estimations of the Competing Models: Scaled Returns by cay

Panel A: Five-Factor Investment Growth Model

	Constant	HHOLDS	NFINCO	NONCOR	FARM	FINAN
Coefficient	0.658	-16.272	13.445	19.464	0.133	0.880
t-value	1.728	-1.688	1.656	1.975	0.863	0.134
Premium		0.007	-0.103	-0.122	-4.292	0.009
t-value		0.369	-1.884	-2.086	-0.724	0.171
Tests:	*J*	ΔJ	*Wald(b)*	*HJ Dist*	*sup LM*	
statistic	5.069	2.355		0.381	15.586	
p-value	0.652	0.308	0.136	0.467		

Panel B: Three-Factor Investment Growth Model

	Constant	HHOLDS	NFINCO	FARM	
Coefficient	1.073	-16.867	11.571	0.081	
t-value	6.735	-2.344	2.024	0.884	
Premium		0.027	-0.057	-3.116	
t-value		2.068	-1.686	-0.875	
Tests:	*J*	ΔJ	*Wald(b)*	*HJ Dist*	*sup LM*
statistic	18.548	3.940		0.460	7.276
p-value	0.029	0.139	0.072	0.089	

Panel C: CAPM

	Constant	EMKT			
Coefficient	1.071	-2.688			
t-value	21.307	-2.532			
Premium		0.018			
t-value		2.532			
Tests:	*J*	ΔJ	*Wald(b)*	*HJ Dist*	*sup LM*
statistic	40.878	21.522		0.507	12.505**
p-value	0.000	0.000	0.011	0.000	

Panel D: The Fama-French Model

	Constant	EMKT	SMB	HML	
Coefficient	1.231	-5.200	1.931	-7.545	
t-value	16.090	-3.898	1.290	-4.588	
Premium		0.020	0.002	0.012	
t-value		2.704	0.615	3.182	
Tests:	*J*	ΔJ	*Wald(b)*	*HJ Dist*	*sup LM*
statistic	22.134			0.394	10.756
p-value	0.008		0.000	0.003	

Panel E: Cochrane's Model

	Constant	NONRES	RES		
Coefficient	0.350	36.351	-3.319		
t-value	1.030	2.504	-0.603		
Premium		-0.021	0.001		
t-value		-2.300	0.078		
Tests:	*J*	ΔJ	*Wald(b)*	*HJ Dist*	*sup LM*
statistic	29.725	5.414		0.526	14.049*
p-value	0.001	0.067	0.007	0.000	

Note: Same comments as in Table 3 apply.

Table 7: Tests of the Competing Models using Fama-MacBeth Regressions

							R^2	Joing Sig
	Constant	HHOLDS	NFINCO	NONCOR	FARM	FINAN	Adj R^2	Adj Joint Sig
Premium	0.041	-0.009	-0.094	-2.727	1.164	-0.015		
t-value	(6.045)	(-0.893)	(-4.030)	(-1.179)	(2.083)	(-1.309)	0.873	0.000
t-value(adj)	(3.543)	(-0.523)	(-2.362)	(-0.691)	(1.221)	(-0.767)	0.839	0.057

					R^2	Joing Sig
	Constant	HHOLDS	NFINCO	FARM	Adj R^2	Adj Joint Sig
Premium	0.044	-0.007	-0.120	1.300		
t-value	(6.231)	(-0.677)	(-3.196)	(1.434)	0.808	0.000
t-value(adj)	(3.282)	(-0.357)	(-1.684)	(0.755)	0.781	0.072

			R^2	Joing Sig
	Constant	EMKT	Adj R^2	Adj Joint Sig
Premium	0.049	-0.010		
t-value	(4.985)	(-0.863)	0.078	0.388
t-value(adj)	(4.946)	(-0.856)	0.038	0.392

					R^2	Joing Sig
	Constant	EMKT	SMB	HML	Adj R^2	Adj Joint Sig
Premium	0.041	-0.007	0.002	0.012		
t-value	(3.238)	(-0.510)	(0.484)	(3.338)	0.679	0.008
t-value(adj)	(3.135)	(-0.494)	(0.468)	(3.231)	0.633	0.012

				R^2	Joing Sig
	Constant	NONRES	RES	Adj R^2	Adj Joint Sig
Premium	0.031	-0.074	0.007		
t-value	(5.131)	(-3.142)	(0.466)	0.545	0.002
t-value(adj)	(2.690)	(-1.647)	(0.244)	0.504	0.176

Note: The Fama-MacBeth regression tests use all of the 25 Fama-French portfolios. The premiums are estimated in the second-stage cross-sectional regressions and they are the coefficients on the betas of the factors listed on the column headings. We denote the investment growth rate of the household and nonprofit sector by HHOLDS, the non-financial, non-farm sector by NFINCO, the non-farm non-corporate sector by NONCOR, the farming sector by FARM, and the financial sector by FINAN. In the tests of Cochrane's model, the residential investment growth rate is denoted by RES and the nonresidential by NONRES. HML is a zero-investment portfolio which is long on high book-to-market (B/M) stocks and short on low B/M stocks. Similarly, SMB is a zero-investment portfolio which is long on small capitalization stocks and short on big capitalization stocks. EMKT refers to the excess return on the value-weighted stock market portfolio. We report two t-values for each parameter. The first one is calculated using the uncorrected Fama-MacBeth standard errors. The second one is calculated using Shanken's (1992) adjusted standard errors. The unadjusted cross-sectional R-square is denoted by R^2, and the adjusted for degrees of freedom R-square by Adj. R^2 . The last column of the table reports p-values from chi-square tests on the joint significance of the betas of each model. The first p-value is computed using the uncorrected variance-covariance matrix, while the second one uses Shanken's (1992) correction. The estimation period is 1952Q2 to 1998Q4.

Table 8A: Fama-MacBeth Regressions that Include the Size Characteristic

	Constant	HHOLDS	NFINCO	NONCOR	FARM	FINAN	SIZE	R^2 Adj R^2
Premium	0.060	-0.003	-0.085	-4.240	0.722	-0.023	-0.002	
t-value	(4.487)	(-0.287)	(-3.608)	(-2.283)	(1.312)	(-1.828)	(-1.367)	0.899
t-value(adj)	(2.682)	(-0.171)	(-2.156)	(-1.364)	(0.784)	(-1.092)	(-0.817)	0.858

	Constant	HHOLDS	NFINCO	FARM	SIZE	R^2 Adj R^2
Premium	0.038	-0.009	-0.113	1.658	0.001	
t-value	(3.234)	(-0.967)	(-4.039)	(2.207)	(0.390)	0.816
t-value(adj)	(1.708)	(-0.511)	(-2.133)	(1.166)	(0.206)	0.768

	Constant	EMKT	SIZE	R^2 Adj R^2
Premium	0.103	-0.039	-0.004	
t-value	(5.742)	(-3.148)	(-3.126)	0.625
t-value(adj)	(5.159)	(-2.828)	(-2.808)	0.572

	Constant	EMKT	SMB	HML	SIZE	R^2 Adj R^2
Premium	0.078	0.028	-0.030	0.007	-0.01	
t-value	(4.787)	(1.965)	(-3.638)	(1.765)	(-4.694)	0.824
t-value(adj)	(3.529)	(1.449)	(-2.682)	(1.301)	(-3.460)	0.778

	Constant	NONRES	RES	SIZE	R^2 Adj R^2
Premium	0.036	-0.076	0.000	-0.001	
t-value	(2.131)	(-3.598)	(0.009)	(-0.379)	0.549
t-value(adj)	(1.107)	(-1.870)	(0.005)	(-0.197)	0.459

Note: Size is the log of the portfolio size. The same comments as in Table 7 apply.

Table 8B: Fama-MacBeth Regressions that Include the Book-to-Market Characteristic

	Constant	HHOLDS	NFINCO	NONCOR	FARM	FINAN	B/M	R^2 Adj R^2
Premium	0.037	-0.012	-0.091	-1.025	1.086	-0.021	0.003	
t-value	(5.327)	(-1.212)	(-3.898)	(-0.533)	(1.931)	(-1.853)	(1.265)	0.886
t-value(adj)	(3.257)	(-0.741)	(-2.383)	(-0.326)	(1.181)	(-1.133)	(0.774)	0.839

	Constant	HHOLDS	NFINCO	FARM	B/M	R^2 Adj R^2
Premium	0.035	-0.013	-0.104	0.699	0.004	
t-value	(5.448)	(-1.439)	(-3.247)	(0.662)	(1.496)	0.869
t-value(adj)	(3.198)	(-0.845)	(-1.906)	(0.389)	(0.878)	0.834

	Constant	EMKT	B/M	R^2 Adj R^2
Premium	0.028	0.001	0.009	
t-value	(2.350)	(0.113)	(3.285)	0.627
t-value(adj)	(2.350)	(0.113)	(3.284)	0.574

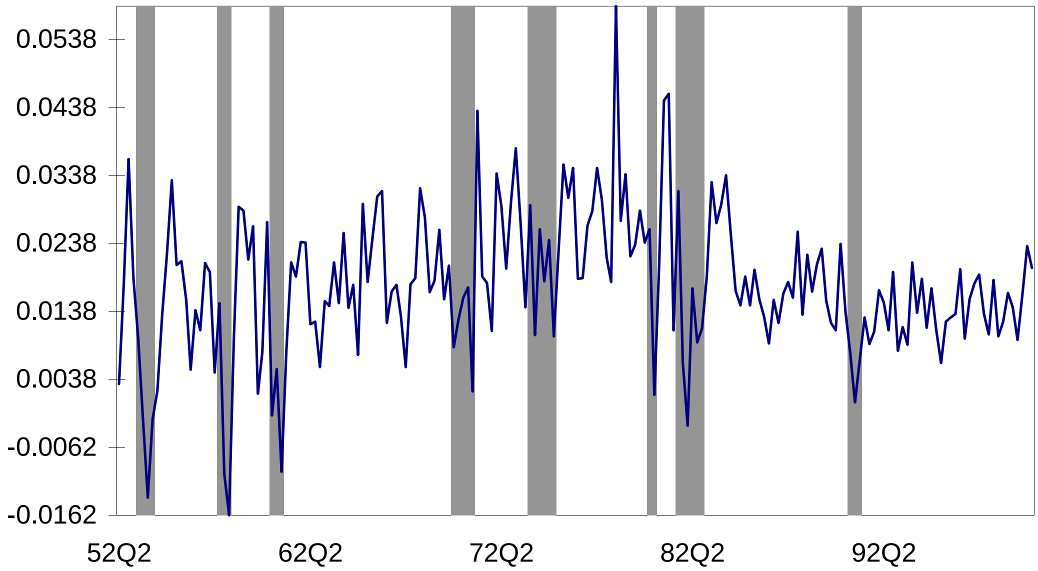
	Constant	EMKT	SMB	HML	B/M	R^2 Adj R^2
Premium	0.049	-0.018	0.002	0.006	0.005	
t-value	(3.776)	(-1.221)	(0.405)	(1.252)	(2.033)	0.697
t-value(adj)	(3.651)	(-1.180)	(0.392)	(1.211)	(1.966)	0.618

	Constant	NONRES	RES	B/M	R^2 Adj R^2
Premium	0.029	-0.033	0.003	0.006	
t-value	(4.635)	(-2.390)	(0.211)	(2.221)	0.676
t-value(adj)	(3.734)	(-1.925)	(0.170)	(1.789)	0.611

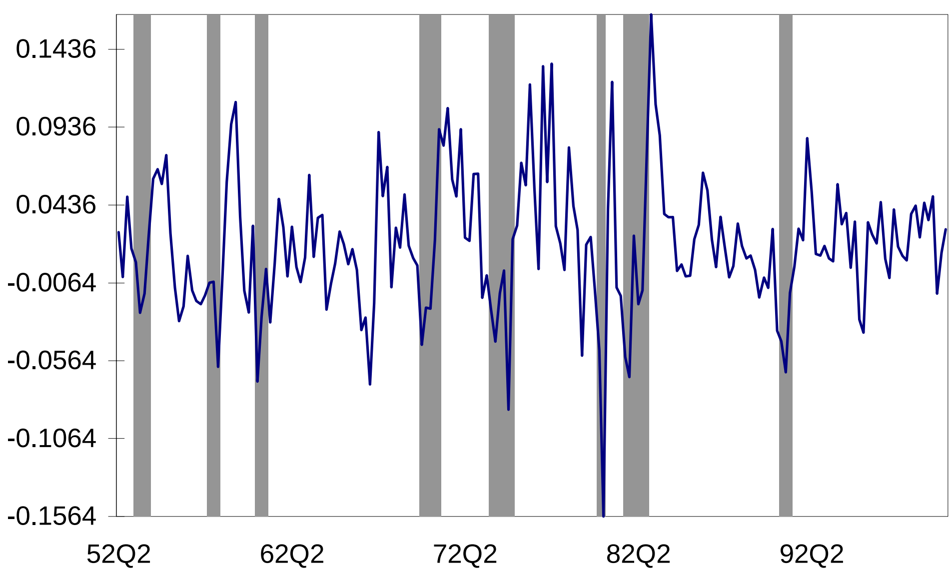
Note: B/M stands for book-to-market and is defined as the log of a portfolio's book-to-market ratio. Same comments as in Table 7 apply.

Figure 1: GDP and Investment Growth Rates

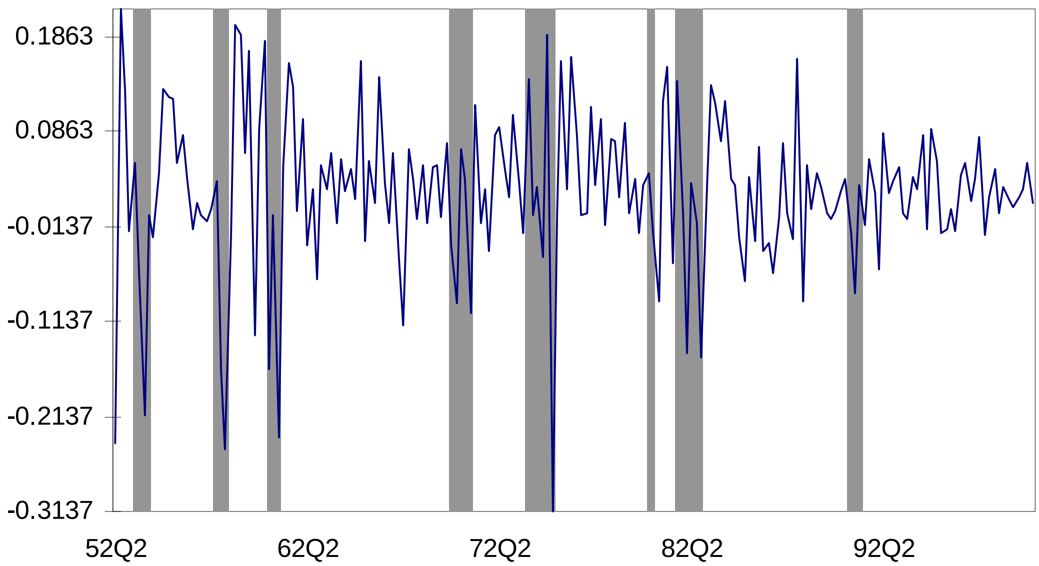
Panel A: GDP Growth Rate



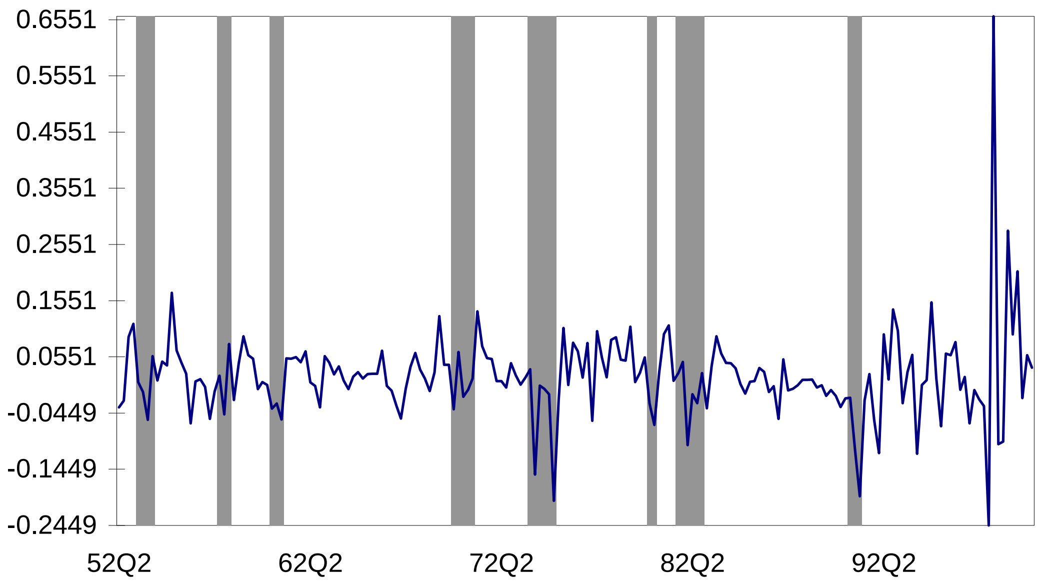
Panel B: Household and NonProfit (HHOLDS) Sector Investment Growth Rate



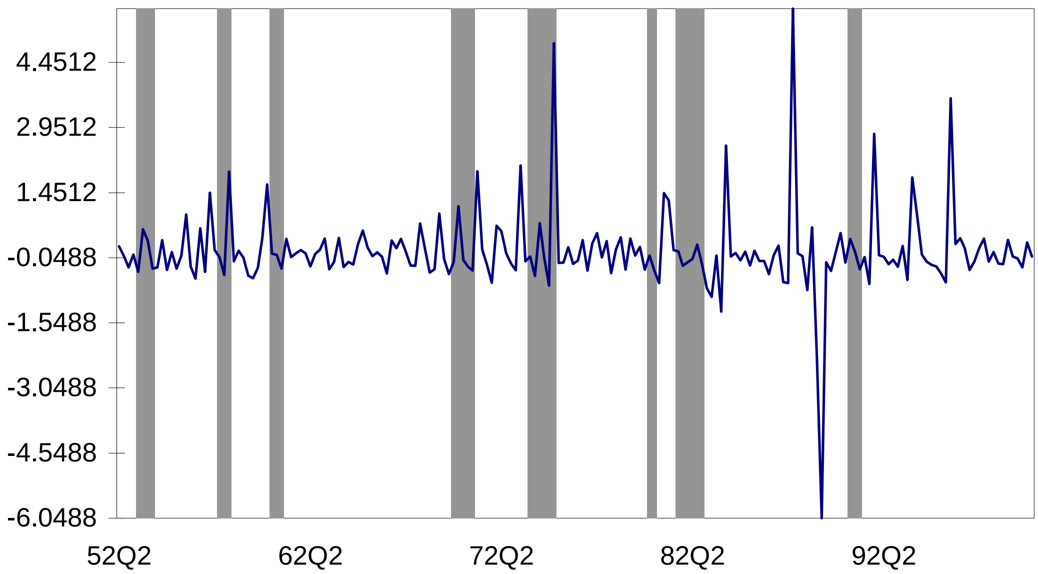
Panel C: Non-Financial Non-Corporate (NFINCO) Sector Investment Growth rate



Panel D: Non-Farm, Non-Corporate (NONCOR) Sector Investment Growth Rate



Panel E: Farm (FARM) Sector Investment Growth Rate



Panel F: Financial (FINAN) Sector Investment Growth Rate

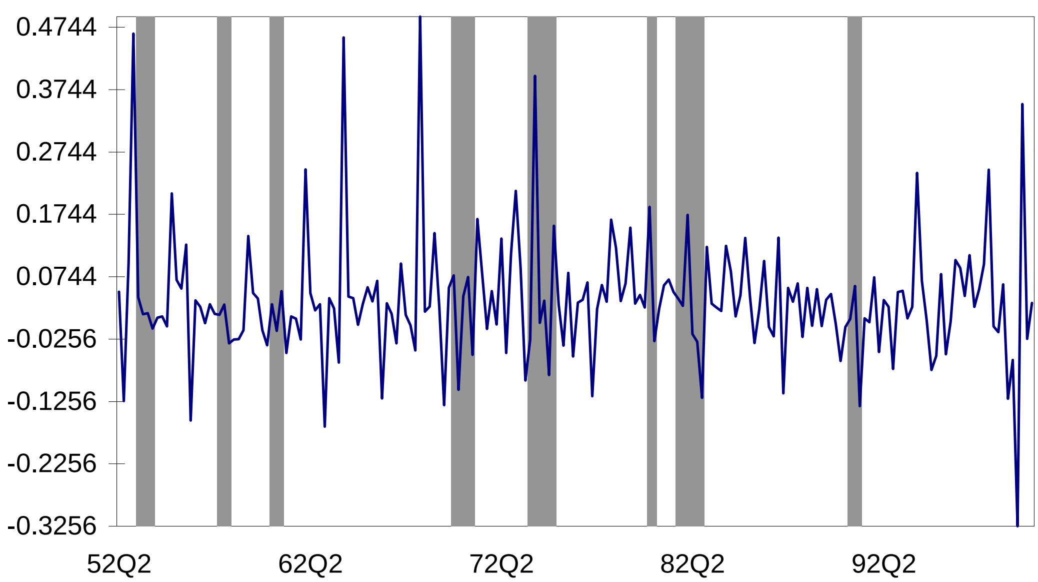
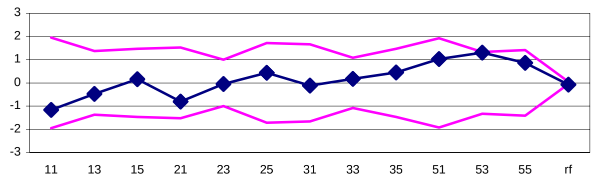
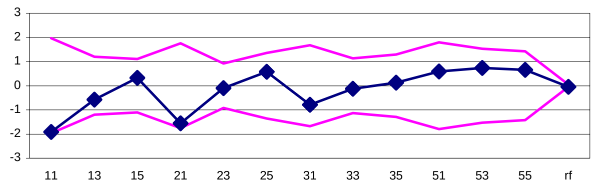


Figure 2: Pricing Errors of Unscaled Returns From GMM Estimations

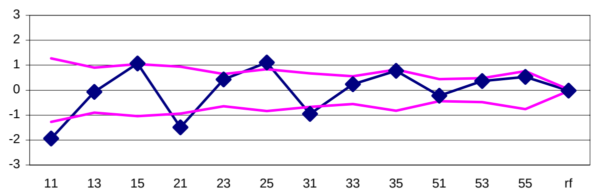
Five-Factor Investment Growth Model



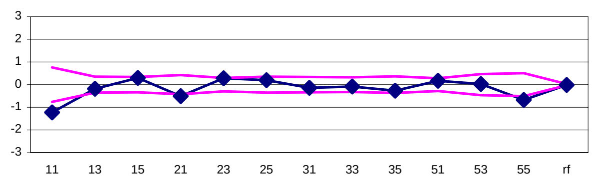
Three-Factor Investment Growth Model



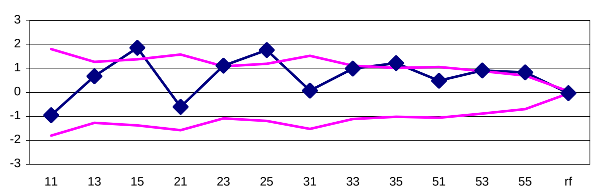
CAPM



The Fama-French Model



Cochrane's Model



Note: The labels on the x-axis indicate the portfolio number. The first digit refers to the size quintile and the second digit to the B/M quintile. The last asset is the risk-free (rf) rate. The scale on y-axis is in percentage points. The two lines denote the two standard-error band.

Figure 3: Realized vs Fitted Returns from the Fama-MacBeth Regressions: 25 Portfolios