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ABSTRACT

Do We Need Multi-Country Models to Explain Exchange Rate, Interest Rate and Bond Return Dynamics?*

This Paper examines characterizations of the dynamics for first and second moments of the one-month interest rate, the 12-month excess bond return and exchange rates. The countries considered are the US, Germany, Japan and the UK. Our tests are based on the implications of multi-country versions of the Cox, Ingersoll and Ross (1985) class of term structure models. Multi-country models are in several cases better able to explain the dynamics of one-month interest rates and the 12-month excess bond returns than one-country models. Furthermore, in some cases, they can also explain the dynamics of the exchange rates better than two-country models. Multi-country models are particularly useful for explaining the second moment of the one-month US interest rate, the second moments of the 12-month excess bond returns in the US, Germany and Japan, as well as the first moment of the rate of appreciation of the Deutsche mark relative to the US dollar. In addition to results based on asymptotic distributions, we also provide inference using the small-sample distributions of test statistics.

JEL Classification: C61 and E40
Keywords: bond returns, exchange rates, interest rates and multi-country models

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1. Introduction

Several studies in the 1990's show that assets tend to be priced internationally (see Harvey (1991), Ferson and Harvey (1994), Dumas and Solnik (1995), and Vassalou (2000) among others). Bekaert and Harvey (1995) provide evidence of capital market integration. Finally, Fama and French (1992, 1993) show that the domestic CAPM can no longer explain the cross-section of asset returns, and Fama and French (1998) propose an alternative world two-factor model.

Although the above studies examine almost exclusively equity returns, one would expect that other asset classes, such as foreign exchange and fixed income, would also be priced internationally. Indeed, the world foreign exchange and fixed income markets are larger in terms of market capitalization than the global equity market. In addition, they are more liquid. If equity markets are becoming increasingly more integrated, so should be the markets of major currencies and fixed income securities. In globalized markets the dynamics of foreign exchange and fixed income assets should be affected mainly by international, rather than domestic, factors.

This paper examines the above issues. It tests whether the first and second moments of major exchange rates are affected by third-country factors. The countries considered are the US, Germany, Japan, and the UK. Furthermore, it tests whether multi-country models provide more information about the first and second moments of the one-month interest rates and the 12-month excess bond returns than simple first-order autoregressive (AR(1)) specifications do.

Our tests are based on the implications of the Nielsen and Saa-Requejo (1993) and Saa-Requejo (1993) models. These models are multi-country versions of the Cox, Ingersoll and Ross (1985) (CIR) affine model of the term structure. Two-country versions of the Nielsen and Saa-Requejo (1993) and Saa-Requejo (1993) models have received some attention lately in the papers of Backus, Foresi and Telmer (1995), Bansal (1997), and Brandt and Santa-Clara (1999).

The characterization of the exchange rate dynamics is important in the forward premium puzzle literature. This puzzle refers to the stylized fact that the slope coefficient in regressions of changes in an exchange rate on the nominal interest rate differential between the two currencies is significantly different from one (see Hodrick (1987), and Engel (1995) for surveys of the evidence). The forward rate can be a biased predictor of the future spot rate in large samples because rational investors require an exchange rate risk premium for holding foreign assets. Multi-country models of the term structure provide an alternative characterization of the exchange rate risk premium by making the market price of risk dependent on multiple interest rates (see Bansal, (1997)).

The dynamics of the short-term interest rate have useful implications for the term structure literature. Term structure models typically specify the dynamics of the factors with the result that the yields are functions of the factors and time to maturity. The short-term interest rate is a factor in the CIR class of models. Therefore, understanding its dynamics is necessary in order to further improve these models.

The paper contributes to our understanding of the evolution of first and second moments in interest-rates and exchange rates. It shows that in several cases multi-country models are better able to explain the dynamics of one-month interest rates, 12-month excess bond returns and exchange rates than one-country and two-country models respectively. This is particularly true for the second moment of the one-month US interest rate, the second moments of the 12-month excess bond returns and the first moment of the Deutsche mark – US dollar (DEM/USD) exchange rate.

Our inference is conducted using both asymptotic theory and Monte Carlo simulations. The results from the two approaches do not always coincide. Nevertheless, the evidence we present leads to the conclusion that at least in some cases, multi-country models are necessary in order to characterize the dynamics of interest rates and exchange rates.

The rest of the paper is organized as follows. Section 2 briefly presents the class of international affine models which constitutes the basis for our tests. Section 3 discusses our empirical methodology and describes the data. We present our statistical analysis on the dynamics of the one-month interest rate, the 12-month excess bond returns and the exchange rates in Section 4. Section 5 outlines our Monte Carlo experiments. Section 6 presents inference based on the small sample distributions of the statistics derived from the Monte Carlo experiments. We conclude in Section 7 with a summary of our results.

2. International Affine Models of the Term Structure of Interest Rates

In the international versions of the single-factor CIR model of Nielsen and Saá-Requejo (1993) and Saá-Requejo (1993), the one-dimensional state variables X and Y of the two countries i and j, have dynamics that are governed by the following stochastic differential equations:

$$dX(t) = (a_x - b_x X(t))dt + c_x \sqrt{X(t)} dZ_x(t) \quad \textbf{(1)}$$

and

$$dY(t) = (a_y - b_y Y(t))dt + c_y \sqrt{Y(t)} dZ_y(t) \quad \textbf{(2)}$$

where a_x, b_x, c_x, a_y, b_y, and c_y are positive constants such that $2a_x > c_x^2$ and $2a_y > c_y^2$. Z_y and Z_x denote uncorrelated Brownian motions. These conditions imply that X and Y are strictly positive and mean reverting. Furthermore, the instantaneous riskless interest rates of country j, r_j, and of country i, r_i, are linear functions of the two state variables X and Y. Thus, by changing variables, the dynamics of the short-term interest rates can be written as follows:

$$dr_i(t) = [á - g_i r_i(t) - ã_j r_j(t)]dt + \sqrt{h_j r_j(t) - ç_i r_i(t)}\,dZ_x + \sqrt{æ_i r_i(t) - æ_j r_j(t)}\,dZ_y \quad \textbf{(3)}$$

and

$$dr_j(t) = [\, b - p_i r_i(t) - p_j r_j(t)]dt + \sqrt{ê_j r_j(t) - ê_i r_i(t)}\,dZ_x + \sqrt{l_i r_i(t) - l_j r_j(t)}\,dZ_y \quad \textbf{(4)}$$

where $a, b, g_i, g_j, h_j, h_i, z_j, z_i, p_j, p, k_i, k_j, l_i,$ and l_j are functions of the structural parameters of the model. Note that the expressions under the square roots are equal to a positive constant times one of the two state variables, *X* or *Y*. Therefore, they are always positive.

In equilibrium, the exchange rate, *e*, expressed as currency *j* per unit of currency *i*, is a function of the state variables, and therefore, of the interest rates in the two countries. The resulting dynamics for the exchange rate are:

$$\frac{de_j(t)}{e_j(t)} = ì_j(t)dt + ó_{jx}(t)dZ_x(t) + ó_{jy}(t)dZ_y(t) \quad \textbf{(5)}$$

where $m_j(t) = f_j r_j(t) - f_i r_i(t)$ denotes the instantaneous expected rate of appreciation of currency *j* relative to the reference currency, and s_{jx} and s_{jy} are linear functions of the interest rates and reflect the time-varying sensitivities of the exchange rate to the sources of risk Z_x and Z_y respectively. The terms Z_x and Z_y describe uncertainty related to the state variables of the two economies[1]. Note that relation (5) implies that uncovered interest parity does not hold. The instantaneous expected rate of appreciation of currency *j* relative to the reference currency is given by the interest rate differential between the two countries plus a time-varying risk premium that is also a function of the interest rates. For simplicity, we will supress (t) in the notation of

[1] In Saá-Requejo (1993), the dynamics of the exchange rate are driven by an additional state variable which is the component of conditional heteroskedasticity of the exchange rate that is orthogonal to the information contained in the two term structures. However, in this paper, we will not focus on the effects this additional state variable has on the description of exchange rate dynamics.

the rest of the paper.

An n-country generalization of the model is straightforward. Assume that there are n state variables, one for each country. Assume again that each country's interest rate is a linear function of the set of state variables. Also, let the exchange rate dynamics of currency j relative to the reference currency be a function of the state. Then, we can write the exchange rate equations and the interest rate equations as functions of the interest rates. In particular, the rate of change of the exchange rate between the reference currency and currency j is

$$\frac{de_j}{e_j} = \mathbf{m}_j dt + \sum_{k=1}^{n} \mathbf{s}_{jk} dZ_k \qquad \textbf{(6)}$$

where

$$\mathbf{m}_j = \sum_{k=1}^{n} \mathbf{a}_{jk} r_k$$

with $\mathbf{a}_{jk}, k = 1,....,n$ being positive constants, and $\mathbf{s}_{jk}$ being instantaneous volatilities and linear functions of the interest rates.

We can reexpress the conditional mean of equation (6) as follows:

$$\mathbf{m}_j = \sum_{k=1}^{n} \mathbf{a}_{jk} r_k = \mathbf{b}_{ji} r_i + \sum_{k \neq i}^{n} \mathbf{b}_{jk} (r_i - r_k) \qquad \textbf{(7)}$$

and the conditional variance as

$$\mathbf{s}^2{}_j = c_{ji} r_i + \sum_{k \neq i}^{n} c_{jk} (r_i - r_k) \qquad \textbf{(8)}$$

Furthermore, the dynamics of the instantaneous interest rate of country j can be written as

$$dr_j = [\text{á}_j + \sum_{k=1}^{n} b_{jk} r_k] dt + \sum_{k=1}^{n} \sqrt{c_{jk} r_k} dZ_1 \qquad \textbf{(9)}$$

where a_j, b_{jk}, c_{jk} are constants.

Finally, the dynamics of the bond price will be given by

$$\frac{dP_i}{P_i} = m_{Pi} dt + \sum_{j=1}^{n} s_{Pij} dZ_j \qquad \textbf{(10)}$$

where $m_{Pi} = \sum_{j=1}^{n} f_{ij} r_j$ and $s_{Pij} = -h_{ij} \sqrt{\sum_{k=1}^{n} n_k r_k}$

Relations (7), (8), (9), and (10) will be the basis for our empirical tests which examine the dynamics of the first and second moments of the exchange rates, one-month interest rates, and excess bond returns in four developed countries. We concentrate our tests on the 12-month excess bond returns because this is the longest available maturity.

Our working hypothesis is that if multi-country models are useful for explaining the exchange rate, interest rate and bond return dynamics of the US, Germany, Japan, and the UK, they are also likely to be useful for explaining the exchange rate and interest rate dynamics of other countries. This should be particularly true for countries with smaller, emerging markets. However, we do not include emerging market interest rates, exchange rate and bond returns in our tests, because emerging markets are likely to be less integrated with developed markets than developed markets are among themselves. Bansal and Dahlquist (2000) provide evidence which suggests that differences across economies are related to per capita Gross National Product (GNP), average inflation rates, and inflation volatility. These findings set emerging markets apart from the developed markets, at least for the purposes of this study.

3. Empirical Methodology

In our empirical tests, we approximate the continuous-time stochastic processes in equations (7), (8), (9), and (10) by discrete-time Gaussian processes, and we proceed as if the data are generated by a discrete-time model based on continuously compounded rates of return. In this manner, we take advantage of the linearity in the expressions for the exchange rate and interest rate dynamics provided by the continuous-time theoretical models.

We denote the expectation at time t of the continuously compounded rate of appreciation of the reference currency relative to currency j by the discrete-time analog to equation (7):

$$E_t log(e_{t+1,j}) - log(e_{t,j}) = E_t g_{t+1,j} = p_j + \hat{a}_{ji} r_i + \sum_{k \neq i} \hat{a}_{jk}(r_i - r_k) \qquad \textbf{(11)}$$

where E_t denotes the expectation conditional on information at time t. The discrete-time analog to equation (8) can be written as follows:

$$Var_t(e_{t+1,j}) = p'_j + c_{ji} r_i + \sum_{k \neq i}^{n} c_{jk}(r_i - r_k) \qquad \textbf{(12)}$$

In addition, following equation (9), the discrete-time expected change in the short-term interest rate of country j, j=1,...,4 is given by:

$$E_t r_{t+1,j} - r_{t,j} = a_j + q_{ji} r_i + \sum_{k \neq i} q_{jk}(r_i - r_k) \qquad \textbf{(13)}$$

and the discrete-time conditional variance can be written as:

$$Var_t(r_{t+1,j}) - r_{t,j} = a'_j + c'_{ji} r_i + \sum_{k \neq i} c'_{jk}(r_i - r_k) \qquad \textbf{(14)}$$

Note the presence of a constant in equations (11), (12), (13), and (14). The discrete-time moments approximate their continuous-time counterparts as the length of the time interval in the discrete-time specification approaches zero. However, our empirical tests use data sampled at a monthly interval. Although the class of CIR models does not specify a constant in the exchange rate dynamics, we add one in our empirical specification, in case the appropriate discretization period is smaller than a month. A similar interpretation can be given to the constant term in the interest rate dynamics. It summarizes empirically the effect of the constant in the model specification of the interest rate dynamics plus any additional effects due to the simple discretization approach employed[2].

We estimate equations (10) and (11) by assuming rational expectations in which case

$$g_{t+1,j} = \acute{a}_j + \hat{a}_{ji} r_i + \sum_{k \neq i} \hat{a}_{jk} (r_i - r_k) + x_{t+1,j} \qquad \textbf{(15)}$$

and

$$(g_{t+1,j})^2 = \acute{a}'_j + \hat{a}'_{ji} r_i + \sum_{k \neq i} \hat{a}'_{jk} (r_i - r_k) + x'_{t+1,j} \qquad \textbf{(16)}$$

where $x_{t+1,j}$ and $x'_{t+1,j}$ are expectation errors. Similarly, under rational expectations, equations (12) and (13) become

$$\ddot{A} r_{t+1,j} = r_{t+1,j} - r_{t,j} = a_j + c_{ji} r_i + \sum_{k \neq i} c_{ji} (r_i - r_k) + \tilde{o}_{t+1,j} \qquad \textbf{(17)}$$

and

$$\ddot{A}(r_{t+1,j})^2 = a'_j + c'_i r_i + \sum_{k \neq i} c'_{ji} (r_i - r_k) + \tilde{o}'_{t+1,j} \qquad \textbf{(18)}$$

where again $u_{t+1,j}$ and $u'_{t+1,j}$ denote expectation errors.

In addition, we examine the implications of our analysis for the first and second moments of the 12-month excess bond returns in the four countries. We approximate the 12-month excess bond return by the one month change in the 12-month interest rate minus the one-month interest rate. The regression models estimated are of the form:

$$12R_{t+12,j} - 11R_{t+11,j} - r_{t,j} = d_j + k_{ji} r_i + \sum_{k \neq i} k_{ji} (r_i - r_k) + y_{t+1,j} \qquad \textbf{(19)}$$

and

$$(12R_{t+12,j} - 11R_{t+11,j} - r_{t,j})^2 = d'_j + k'_i r_i + \sum_{k \neq i} k'_{ji} (r_i - r_{ki}) + y'_{t+1,j} \qquad \textbf{(20)}$$

where R denotes the twelve-month interest rate and $y_{t+1,j}$ and $y'_{t+1,j}$ denote expectations errors.

Equations (15) to (20) are estimated using the Generalized Methods of Moments (GMM) of Hansen (1982). In constructing standard errors, we allow for conditional heteroskedasticity

[2] Saa-Requejo (1993) derives a class of discrete-time processes for the international affine models of the term structure. The discretization approach used here is consistent with his.

and the possibility of serial correlation up to two lags, following Newey-West (1987).

Our data source is the DRI/McGraw Hill database. The interest rate data are end-of-month quotes for one-month and 12-month eurocurrency deposits at the close of the London market. The exchange rates are also at the close of the London market. We calculate middle quotes using the bid and offered rates reported at DRI. The period spanned by our data is January 1981 to December 1997.

Summary statistics for the interest rates and exchange rates are provided in Table 1. All tables report results in percentage terms. The standard deviations indicate that exchange rates are more than ten times as variable as the one-month interest rates, but equally variable as the 12-month interest rates. The autocorrelations indicate that interest rates are highly serially correlated.

4. Empirical results

4.1 Exchange rates

Table 2 reports results from regressions of rates of currency appreciation on the interest rate factors specified in equation (14). The first panel provides the results for rates of appreciation relative to the US dollar, whereas the second and third panels examine the cross rates relative to the British pound and Japanese yen, respectively.

The coefficient on the one-month interest rate differential between the domestic interest rate and the interest rate of the reference currency is always negative and often statistically significant. This result is consistent with the majority of empirical evidence that rejects the uncovered interest parity hypothesis. Note, however, that the regressors in equation (15) are highly correlated with an average correlation of 0.62. This makes the interpretation of the individual coefficient estimates somewhat problematic. For that reason, we draw inferences based on hypothesis tests.

Table 2 also shows that the Deutsche mark falls in value relative to the dollar and yen in response to an increase in the difference between German and UK interest rates.

The chi-square tests reported in Table 2 examine the hypothesis of whether a two-country model is sufficient to describe the exchange rate dynamics. According to the asymptotic distributions of the chi-square statistic, the hypothesis is rejected at the 5% level for the DEM/USD and DEM/JPY exchange rates. The influence of the UK interest rate implies that a two-country specification is inappropriate. Furthermore, the two-country specification is rejected at the 10% level for the British pound - US dollar (GBP/USD) rate.

The results on the dynamics of the first moment of changes in exchange rates suggest that multi-country models, in some cases, contain more information than the traditional two-country frameworks. Furthermore, they show that multi-country models can provide a characterization of the exchange rate risk premium which has not been previously considered in the literature. The specification examined predicts between 1.9% and 6.4% of future changes in the exchange rates.

We have also tested equation (15) which refers to the dynamics of the second moments of the changes in exchange rates. The tests consider all the exchange rates defined by the four countries, including the cross-rates. The results, however, do not provide support to multi-country models. None of the third-country interest rates is able to provide significant information about the future variance of changes in any of the exchange rates examined. For that reason, and in order to conserve space, we do not report these results. Instead, we turn our attention to the first and second moments of interest rates.

4.2 Interest rates

4.2.1. First and second moments of the one-month interest rate.

The results from tests of equation (16) for the first moment of the four one-month interest rates are reported in Table 3. Once again, the contemporaneous correlations of the regressors do not allow us to give a clear-cut interpretation of the individual coefficient estimates. Nevertheless, the magnitudes of the coefficients and the t-statistics suggest that one-month interest rates, apart from that of the US, may react significantly to changes in the levels of one-month interest rates in other countries. The model explains between 2.1% and 4.5% of future changes in the four short-term interest rates.

Table 4 presents the results from hypothesis tests on the dynamics of one-month interest rates. In particular, we first test whether the one-month interest rate can be modeled in a single-country framework. In other words, we test whether the constant and the own lag of the domestic one-month interest rate are sufficient to explain future changes in the domestic one-month interest rate. This test follows a chi-square distribution with three degrees of freedom. The hypothesis is not rejected in the case of the US one-month interest rate. It is rejected, however, at the 1% in the case of Japan, and at the 10% level in the cases of Germany and the UK.

We also test whether a two-country model is sufficient to describe the dynamics of the first moment for the one-month interest rate using a chi-square test with two degrees of freedom. We test whether the second country is the US, Germany, Japan or the UK. Our results show that the two-country model is rejected for the German and Japanese one-month interest rates. Specifically, information about only the lagged German and US one-month interest rates is not sufficient to explain the dynamics of the German one-month interest rate. Furthermore, in the case of Japan, independently of whether we consider the US, Germany or the UK as the second country, the two-country model is rejected at the 5% level or less. These results again suggest that multi-country models are necessary to explain the evolution of the first moment in the four major one-month interest rates.

We also test whether a multi-country model can explain the dynamics of the second moments of the four one-month interest rates. The results are reported in Table 5. Again, due to multicollinearity issues, it is often difficult to interpret the individual coefficient estimates. It is notable, however, that the model explains between 4.8% in Germany and 31% in the US of the time-series variation in the second moment of the one-month interest rate.

Table 6 presents the results from the hypotheses tests. The hypothesis that a one-country model is sufficient to explain the dynamics of the second moment in the US one-month interest rate is strongly rejected at any conventional level of significance. The same hypothesis is also rejected at the 10% level for the UK one-month interest rate, whereas it is not rejected in the cases of Germany and Japan.

Furthermore, the hypothesis that a two-country model is sufficient in explaining the second moment is also strongly rejected in the case of the US, independently of whether the second country is Germany, Japan, or the UK. The same hypothesis is also rejected at the 5% level in the case of the UK when the second country is either Germany or Japan. It is not rejected, however, when the second country is the US. As expected, the hypothesis that a two-country model is correct is not rejected in the case of the German and Japanese one-month interest rates. This is because the one-country model was not rejected for those countries either.

The results on the second moments of the four one-month interest rates underline again the need to use, at least in some cases, multi-country models in order to describe the dynamics of the one-month interest rates.

4. 2.2. First and second moments of the 12-month excess bond returns

The tests on the dynamics for the first moments of excess bond returns are presented in Table 7. The results suggest that, only a small percentage of the time variation in the first moment of the 12-month excess bond returns can be explained by our model. The adjusted R-square varies between –0.3% for Germany and 2.7% for Japan. Furthermore, the hypotheses tests presented in Table 8 show that the one-country model is rejected only in Japan and only at the 10% level. Furthermore, the two-country model is rejected for Japan only when the second country is the US.

These results suggest that a multi-country model has limited ability to explain the dynamics of the first moment for the 12-month excess bond returns, at least in three of the four countries examined.

More interesting are the results on the second moments of excess bond returns presented in Table 9. A multi-country model can explain between 3.4% in the UK and 23.3% in the US of the time-variation in the second moment of the excess bond return.

Table 10 shows that the single-country model is rejected for the second moment of the US, German, and Japanese excess bond returns. Furthermore, the two-country model is rejected at the 5% level for the US, independently of whether the second country is Germany, Japan or the UK. The two-country model is also rejected in the case of Germany when the second country is either Japn or the UK. It is not rejected, however, when the second country is the US. Finally, the two-country model is also rejected in Japan when the second country is either the US or Germany. It is not rejected when the second country is the UK.

The results in Tables 9 and 10 reveal that a multi-country model can be particularly informative in describing the dynamics of the second moments of 12-month excess bond returns in Germany, Japan, and especially in the US.

5. Simulation model

Table 1 shows that interest rates are highly persistent. Therefore, as Bekaert, Hodrick, and Marshall (1997) argue, basing inference solely on the usual asymptotic distribution theory may not be reliable. Consequently, we also conduct inference using a simulated economy. The economy we simulate is not the only conceivable economy one can use to examine the small sample properties of the estimators. It is an economy, however, in which the null hypotheses are true by construction. The main drawback of this economy is that it is not consistent with the forward premium puzzle. We interpret the results of our simulations as setting a high hurdle for the null hypotheses of interest.

We calibrate this economy to match the persistence in interest rates and the volatilities of interest rates and exchange rates. We postulate that the interest rate of country j follows a single-factor CIR process:

$$r_{t+1,j} = (1-\varphi_j)\mu_j + \varphi_j r_{t,j} + \sigma_j r_{t,j}^{1/2}\varepsilon_{t+1,j} \qquad \textbf{(21)}$$

where $\varepsilon_{t+1,j}$ is N(0,1) and is correlated across countries. In economies that do not admit arbitrage, $E_t(M_{t+1,j} r_{t+1,j}) = 1$ where $M_{t+1,j}$ is country's j pricing kernel. We model the natural logarithm of the jth country's pricing kernel to be:

$$-m_{t+1,j} = \frac{1}{2}\theta_j^2 + (1+\frac{1}{2}\lambda_j^2)r_{t,j} + \lambda_j r_{t,j}^{1/2}\varepsilon_{t+1,j} + \theta_j \nu_{t+1,j} \qquad \textbf{(22)}$$

where $\nu_{t+1,j}$ is N(0,1) and independent of $\varepsilon_{t+1,j}$. The parameter λ_j controls the price of interest rate risk. In complete markets, the rate of appreciation of currency j relative to currency i is given by the difference in the logarithms of the two pricing kernels:

$$g_{t+1,j} = m_{t+1,i} - m_{t+1,j} \qquad \textbf{(23)}$$

The model parameters are the μ_j's, the ρ_j's, the σ_j's, the θ_j's, the λ_j's, and the correlations of the ε's. The following sample moments are used to recursively calibrate the parameters of the model: The sample mean of the interest rate process is used to estimate μ_j,

$$\mu_j = \hat{E}(r_{t,j}) \qquad \textbf{(24)}$$

where ^ indicates sample moment. The first-order autocorrelation coefficients of the interest rate processes identify the φ_j's,

$$\varphi_j = \frac{\hat{E}[(r_{t+1,j} - \mu_j)(r_{t,j} - \mu_j)]}{\hat{E}[r_{t,j} - \mu_j)^2]} \qquad \textbf{(25)}$$

Once μ_j and φ_j are known, the sample standard deviation can be written as follows:

$$\sigma_j = [1 - \varphi_j^2)\hat{E}[r_{t,j} - \mu_j)^2]/\mu_j]^{1/2} \qquad \textbf{(26)}$$

The covariance of the interest rates involves a first-order Taylor's series approximation. First, we write the interest rate process as the infinite sum of its innovations. Note that only contemporaneous terms will matter, and consider the innovation covariance:

$$E_{t-1}[(\sigma_i r_{t-1,i}^{1/2} \varepsilon_{t,i})(\sigma_j r_{t-1,j}^{1/2} \varepsilon_{t,j})] = \sigma_i \sigma_j (r_{t,i} r_{t,j})^{1/2} \rho_{ij} \qquad \textbf{(27)}$$

A first-order Taylor approximation gives

$$(r_{t,i} r_{t,j})^{1/2} = (\mu_i \mu_j)^{1/2} + \frac{1}{2}(\mu_i \mu_j)^{-1/2} \mu_j (r_{t,i} - \mu_j) + \frac{1}{2}(\mu_i \mu_j)^{-1/2} \mu_i (r_{t,j} - \mu_j) \qquad \textbf{(28)}$$

Substituting the first-order Taylor approximation into equation (19) and taking unconditional expectations results in

$$E[(\sigma_i r_{t-1,i}^{1/2}\varepsilon_{t,i})(\sigma_j r_{t-1,j}^{1/2}\varepsilon_{t,j})] = \sigma_i \sigma_j (\mu_i \mu_j)^{1/2} \rho_{ij} \quad \textbf{(29)}$$

By summing the infinite weighted sum of terms in equation (19), the covariance of the interest rates is well approximated as

$$Cov[r_{t,i}, r_{t,j}] \approx \frac{\sigma_i \sigma_j (\mu_i \mu_j)^{1/2}}{1-\varphi_i \varphi_j} \rho_{ij} \quad \textbf{(30)}$$

We therefore calibrate the covariance of the sample innovations to be equal to

$$\rho_{ij} = \frac{E(\varepsilon_{t+1,i}\varepsilon_{t+1,j})(1-\varphi_1\varphi_j)}{[(1-\varphi_j^2)(1-\varphi_j^2)]^{1/2}} \quad \textbf{(31)}$$

This leaves four θ_j's and four λ_j's to be determined. We limit the additional information used for determining these parameters to the three sample variances of the rates of appreciation of the dollar relative to currency j. In particular, we assume that each of the θ_j's equals a common θ, and we set θ^2 equal to ten percent of the average variance of the rates of appreciation of the three currencies relative to the dollar. Thus,

$$\theta^2 = 0.10 \sum_{j=1}^{3} V(g_{t,j})/3 \quad \textbf{(32)}$$

Given our sample period, θ is equal to 0.0108 or 1.08%. We also assume that the λ of the US is equal to the average of the λ's of the other three countries. Under these assumptions, we are able to solve for the remaining λ's by using the following equation for each j:

$$V(g_{t+1,j}) = \delta_1^2 V(r_{t,1}) + \lambda_1^2 \mu_1 + 2\theta^2 + \delta_j^2 V(r_{t,j}) + \lambda_j^2 \mu_j - \frac{2\delta_1\delta_j(\mu_1\mu_j)^{1/2}\rho_{1,j}}{(1-\varphi_1\varphi_j)} - 2\lambda_1\lambda_j(\mu_1\mu_j)^{1/2}\rho_{1,j} \quad \textbf{(33)}$$

where $\delta_j \equiv (1+\lambda_j^2/2)$, $j = 1,2,3,4$. Table 11 reports the parameter values used in the simulation model. The values are all reasonable. In particular, with $\lambda_j < 0$, the term structure is upward sloping on average.

6. Inference based on the small-sample distributions

Using the model described in Section 5, we perform 10,000 Monte Carlo experiments of length 204, and calculate p-values for the hypothesis tests of Section 4 using the small-sample distributions.

The simulated p-values for the hypothesis tests performed on the dynamics of exchange rates are reported in Table 12. Notice that our inference is now somewhat different. Using asymptotic distributions, Table 2 shows that the two-country model is rejected at the 5% level in the cases of the DEM/USD and DEM/JPY exchange rates, whereas it is rejected at the 10% level for the GBP/USD rate of appreciation. However, the p-values from the small-sample distributions in Table 12 reveal that the two-country model is rejected at the 10% level only in the case of the DEM/USD rate.

Table 13 presents the simulated p-values for the hypothesis tests performed on the dynamics of the first and second moments of the one-month interest rates. Once again, our inference differs from that presented in Table 4. We first test the hypothesis that a single-country model is sufficient to explain the interest rate dynamics of the four interest rates. This hypothesis is true in the simulated economies. Using the small sample distributions, the hypothesis is now rejected at the 1% level only in the case of the Japanese interest rate. Recall that based on the asymptotic distributions the hypothesis was also rejected at the 10% level for the German and UK interest rates.

Furthermore, we test whether a two-country model is sufficient to describe the dynamics of the first moment of the one-month interest rates. Again, this hypothesis is true in the context of our simulated model. The hypothesis is rejected at the 1% level for the Japanese interest rate when the second country is Germany, and at the 5% level when the second country is the US. It is not, however, rejected when the second country is the UK. These results are again different from those reported in Table 4, where the two-country model was rejected for the Japanese interest rate, independently of whether the US, Germany, or the UK was considered as the second country. The simulation results suggest that the Japanese interest rate reacts more strongly to changes in the UK interest rate than to changes in the US and German interest rates.

Note also that our inference based on small-sample distributions differs from that of Table 4 in two more ways. First, the two-country model is no longer rejected in the case of the German interest rate when the second country is the US. Second, the two-country model is not rejected in the case of the UK interest rate when the second country is Japan.

In the case of the second moments of the short-term interest rate, the hypothesis that a one-country model is correct is rejected in the US at the 1% level. Furthermore, the hypothesis that a two-country model is sufficient to describe the dynamics of the first moment of the one-month US interest rate is also rejected at the 1% level. The rejection holds independently of which country we consider as the second country. Furthermore, the hypothesis that a two-country model is correct is also rejected in the case of the UK one-month interest rate, when the second country is either Japan or the UK. Therefore, the results from the simulated p-values largely coincide with the results from the asymptotic ones.

Finally, Table 14 presents the simulated p-values for the dynamics of excess 12-month bond returns.

In the case of the first moment, the inference is similar to the one presented in Table 9 using the asymptotic distributions. The only exceptions are found in the case of Japan, where the one-country model is not rejected and the two-country model is no longer rejected even when the second country is the US.

In the case of the second moments of the 12-month excess bond returns, the inference based on the simulated p-values is quite different from that based on asymptotics. In particular, the one-country model is no longer rejected, whereas it was previously rejected for the US, Germany, and Japan. In addition, the two-country model is rejected in the case of the US, only when the second country is either Japan or the UK. The two-country model is also rejected in Germany when the second country is Japan.

The Monte Carlo experiments based on the simulated economy we consider produce in several cases results that contradict those obtained from the asymptotic distributions. These differences in the results may be due to one of two reasons. Either the asymptotic distributions are unreliable, or the simulated economy is not entirely appropriate for judging the small sample properties of the estimators. The latter may be the case because, as mentioned earlier, the simulated economy is not consistent with the forward premium puzzle. Therefore, the simulated p-values may need to be interpreted with caution.

7. Conclusions

In this paper we test whether multi-country models add to our understanding of the dynamics of exchange rates and short-term interest rates beyond what is already known from two- and single-country models respectively.

We examine the dynamics of the first and second moments of the one-month interest rate, the 12-month excess bond returns and the exchange rates defined by the US, Germany, Japan and the UK.

Our results reveal that in several cases, multi-country models are better positioned to explain these dynamics than more restrictive specifications. This is particularly true for the second moments of the one-month US interest rate, the second moments of the 12-month bond returns of the US, Germany and Japan, as well as the first moment of the DEM/USD exchange rate.

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Table 1: Summary statistics

All data are from the DRI/McGraw-Hill database. The one- and 12-month interest rates are end-of-month middle quotes for one-month and 12-month Eurocurrency deposits at the close of the London market. The exchange-rate data are also end-of-month middle quotes at the close of the London market. In all cases, we calculate middle quotes using the bid and offered rates provided by DRI. The period covered is from January 1981 to December 1997 or 204 observations. The table uses the symbols FC for foreign currency, USD for the US dollar, GBP for the British pound, and JPY for the Japanese yen.

Variable	Mean	Standard Deviation	Autocorrelations				
			r_1	r_2	r_3	r_6	r_{12}
One-month interest rates							
USA	0.636	0.282	0.959	0.923	0.895	0.775	0.592
Germany	0.522	0.206	0.966	0.931	0.902	0.794	0.549
Japan	0.390	0.203	0.975	0.952	0.933	0.870	0.725
UK	0.828	0.256	0.971	0.942	0.913	0.840	0.627
12-month interest rates							
USA	8.148	3.331	0.975	0.943	0.914	0.815	0.619
Germany	6.421	2.353	0.984	0.956	0.923	0.804	0.557
Japan	4.764	2.401	0.981	0.958	0.936	0.865	0.714
UK	10.057	2.819	0.978	0.948	0.919	0.834	0.617
Exchange rates relative to the US dollar (FC/USD)							
Germany	-0.081	3.367	0.060	0.065	0.009	-0.083	0.010
Japan	-0.224	3.443	0.095	0.045	0.054	-0.176	0.093
UK	0.181	3.432	0.098	0.019	-0.011	-0.134	0.032
Cross rates relative to the British Pound (FC/GBP)							
Germany	-0.262	2.532	0.177	-0.026	0.030	0.001	0.055
Japan	-0.405	3.388	0.165	0.038	-0.026	-0.164	0.006
Cross rate relative to the Japanese Yen (FC/JPY)							
Germany	0.144	2.839	0.071	0.018	-0.005	-0.127	-0.010

Table 2: First moments of changes in exchange rates

The dependent variable in these regressions is g_{t+1}. The letter r refers to interest rates whereas the subscript i refers to the home country. In the subscripts to r, Germany is denoted by GR, Japan by JP, United Kingdom by UK, and United States by US. The monthly data are expressed in percentage terms. Newey- West (1987) t-values corrected for heteroskedasticity and serial correlation up to lag two appear in parentheses under the coefficient estimates. The $\chi^2(2)$ test examines the hypothesis that a two-country model is sufficient. The second country is always that of the reference currency. The asymptotic p-value is given in parentheses below the value of the statistic. The *R*-squares are adjusted for degrees of freedom.

Home country	Constant (t-value)	$r_{t,i} - r_{t,US}$ (t-value)	$r_{t,i} - r_{t,GR}$ (t-value)	$r_{t,i} - r_{t,JP}$ (t-value)	$r_{t,i} - r_{t,UK}$ (t-value)	$r_{t,i}$ (t-value)	$\chi^2(2)$ (p-value)	$adj. R^2$
				Exchange rates relative to the US dollar				
Germany	0.580	-2.345		0.343	3.281	0.062	8.21	1.9%
	(0.65)	(-1.85)		(0.16)	(2.30)	(0.04)	(0.016)	
Japan	-1.209	-2.858	-1.956		-0.463	-0.447	1.93	4.1%
	(-1.20)	(-2.77)	(-1.38)		(-0.28)	(-0.37)	(0.380)	
UK	0.210	-2.702	-2.614	-2.391		2.820	5.63	5.1%
	(0.25)	(-2.10)	(-1.39)	(-1.16)		(2.69)	(0.060)	
				Cross rates relative to the British Pound				
Germany	0.370	0.357		2.734	-1.606	-2.758	2.38	
	(0.53)	(0.38)		(1.53)	(-1.18)	(-3.05)	(0.304)	2.3%
Japan	-1.419	-0.156	0.658		-5.350	-3.267	0.16	6.4%
	(-1.83)	(-0.13)	(0.38)		(-3.39)	(-2.90)	(0.925)	
				Cross rate relative to the Japanese Yen				
Germany	1.789	0.514		-5.381	3.745	0.509	8.47	
	(2.42)	(0.59)		(-3.05)	(2.65)	(0.48)	(0.014)	3.0%

Table 3: First moments of the one-month interest rates

The dependent variable in these regressions is Δr_{t+1} The letter r refers to interest rates and the subscript i to the home country. Germany is denoted by GR, Japan by JP, United Kingdom by UK, and United States by US. The monthly data are expressed in percentage terms. Newey-West (1987) t-values corrected for heteroskedasticity and serial correlation up to lag two appear in parentheses under the coefficient estimates. The R-squares are adjusted for degrees of freedom.

Home country	Constant (t-value)	$r_{t,i} - r_{t,US}$ (t-value)	$r_{t,i} - r_{t,GR}$ (t-value)	$r_{t,i} - r_{t,JP}$ (t-value)	$r_{t,i} - r_{t,UK}$ (t-value)	$r_{t,i}$ (t-value)	$adj. R^2$
USA	0.036		0.032	-0.025	0.019	-0.055	3.6%
	(1.82)		(1.03)	(-0.64)	(0.63)	(-1.80)	
Germany	-0.003	-0.008		-0.008	-0.025	-0.013	2.1%
	(-0.28)	(-0.34)		(-0.39)	(-1.53)	(-0.64)	
Japan	-0.015	-0.009	0.020		-0.042	-0.016	4.5%
	(-2.27)	(-1.46)	(1.49)		(-3.44)	(-1.73)	
UK	0.024	-0.036	0.032	-0.021		-0.025	2.5%
	(1.59)	(-1.54)	(1.17)	(-0.60)		(-1.08)	

Table 4: Hypotheses tests on the fist moments of the one-month interest rates

Two types of hypotheses are tested. The first hypothesis is that a one-country model is sufficient to explain the one-month interest rate dynamics in the four countries. This amounts to testing whether parameters other than the constant and the coefficient on the lagged domestic one-month interest rate are zero. This test is chi-square distributed with three degrees of freedom. The asymptotic p-value is reported under the value of the statistic. The second hypothesis tested is that a two-country model is correct. This tests the parameter restriction that the coefficients other than the constant, the lagged domestic one-month interest rate and the lagged one-month interest rate differential between the domestic and the second country's interest rate are zero. We test whether the second country is the USA (US), Germany (GR), Japan (JP), or the United Kingdom (UK). This test is chi-square distributed with two degrees of freedom. Once again, the asymptotic p-value is reported below the value of the statistic.

Hypotheses tested	USA	Germany	Japan	UK
		Hypothesis (1): A one-country model is correct		
	$c^2(3)$: 4.11 p-value: 0.249	$c^2(3)$: 8.69 p-value: 0.051	$c^2(3)$: 23.86 p-value: < 0.000	$c^2(3)$: 6.54 p-value: 0.088
		Hypothesis (2): A two-country model is correct		
Second country: US		$c^2(2)$: 6.32 p-value: 0.042	$c^2(2)$: 12.64 p-value: 0.002	$c^2(2)$: 1.38 p-value: 0.501
Second country: GR	$c^2(2)$: 0.49 p-value: 0.783		$c^2(2)$: 20.39 p-value: < 0.000	$c^2(2)$: 2.55 p-value: 0.279
Second country: JP	$c^2(2)$: 1.64 p-value: 0.440	$c^2(2)$: 3.41 p-value: 0.181		$c^2(2)$: 5.86 P-value: 0.053
Second country: UK	$c^2(2)$: 1.07 p-value: 0.586	$c^2(2)$: 0.24 p-value: 0.886	$c^2(2)$: 6.69 p-value: 0.035	

Table 5: Second Moments for the Short-Term Interest Rates

The dependent variable in these regressions is $(\Delta r_{t+1})^2$. The letter *r* refers to interest rates, and the subscript *i* to the home country. Germany is denoted by GR, Japan by JP, United Kingdom by UK, and United States by US. The monthly data are expressed in percentage terms. Newey-West (1987) t-values corrected for heteroskedasticity and serial correlation up to lag two appear in parentheses under the coefficient estimates. The *R*-squares are adjusted for degrees of freedom.

Home country	Constant (t-value)	$r_{t,i} - r_{t,US}$ (t-value)	$r_{t,i} - r_{t,GR}$ (t-value)	$r_{t,i} - r_{t,JP}$ (t-value)	$r_{t,i} - r_{t,UK}$ (t-value)	$r_{t,i}$ (t-value)	$adj. R^2$
USA	$-1.09*10^{-4}$		$-1.69*10^{-4}$	$1.12*10^{-4}$	$2.42*10^{-5}$	$2.11*10^{-4}$	31.0%
	(-3.13)		(1.03)	(1.67)	(0.50)	(5.19)	
Germany	$-6.75*10^{-5}$	$-1.58*10^{-4}$		$1.10*10^{-6}$	$5.03*10^{-5}$	$1.68*10^{-4}$	4.8%
	(-1.11)	(-1.07)		(0.02)	(0.77)	(1.24)	
Japan	$7.43*10^{-7}$	$-4.12*10^{-6}$	$5.76*10^{-6}$		$5.33*10^{-6}$	$2.70*10^{-5}$	10.6%
	(0.15)	(-0.81)	(0.55)		(0.59)	(3.92)	
UK	$1.66*10^{-5}$	$-9.94*10^{-5}$	$6.26*10^{-6}$	$-1.01*10^{-4}$		$8.83*10^{-5}$	6.0%
	(0.62)	(-2.50)	(0.16)	(-1.30)		(3.04)	

Table 6: Hypotheses tests on the specification for the second moments of the one-month interest rates

Two types of hypotheses are tested. The first hypothesis is that a one-country model is sufficient to explain the second moments of the one-month interest rates. This amounts to testing whether parameters other than the constant and the coefficient on the lagged domestic one-month interest rate are zero. This test is chi-square distributed with three degrees of freedom. The asymptotic p-value is reported under the value of the statistic. The second hypothesis tested is that a two-country model is correct. This tests the parameter restriction that the coefficients other than the constant, the lagged one-month domestic interest rate and the lagged one-month interest rate differential between the domestic and the second country's interest rate are zero. We test whether the second country is the USA (US), Germany (GR), Japan (JP), or the United Kingdom (UK). This test is chi-square distributed with two degrees of freedom. Once again, the asymptotic p-value is reported below the value of the statistic.

Hypotheses tested	USA	Germany	Japan	UK
Hypothesis (1): A one-country model is correct				
	$\chi^2(3)$: 22.42 p-value<0.000	$\chi^2(3)$: 1.36 p-value: 0.715	$\chi^2(3)$: 1.81 p-value: 0.612	$\chi^2(3)$: 6.48 p-value: 0.090
Hypothesis (2): A two-country model is correct				
Second country: US		$\chi^2(2)$: 0.65 p-value: 0.722	$\chi^2(2)$: 0.62 p-value: 0.735	$\chi^2(2)$: 3.62 p-value: 0.163
Second country: GR	$\chi^2(2)$: 14.91 p-value: 0.001		$\chi^2(2)$: 0.70 p-value: 0.705	$\chi^2(2)$: 6.27 p-value: 0.044
Second country: JP	$\chi^2(2)$: 22.13 p-value<0.000	$\chi^2(2)$: 1.24 p-value: 0.537		$\chi^2(2)$: 6.33 P-value: 0.042
Second country: UK	$\chi^2(2)$: 22.26 p-value<0.000	$\chi^2(2)$: 1.14 p-value: 0.564	$\chi^2(2)$: 1.76 p-value: 0.415	

Table 7: The dynamics of the first moments of excess bond returns

The dependent variable in these regressions is $12R_{t+12,j} - 11R_{t+11,j}$. The letter *r* refers to the one-month interest rates and the subscript *i* to the home country. We denote by R the 12-month interest rate. Germany is denoted by GR, Japan by JP, United Kingdom by UK, and United States by US. The monthly data are expressed in percentage terms. Newey- West (1987) t-values corrected for heteroskedasticity and serial correlation up to lag two appear in parentheses under the coefficient estimates. The *R*-squares are adjusted for degrees of freedom.

Home country	Constant (t-value)	$r_{t,i} - r_{t,US}$ (t-value)	$r_{t,i} - r_{t,GR}$ (t-value)	$r_{t,i} - r_{t,JP}$ (t-value)	$r_{t,i} - r_{t,UK}$ (t-value)	$r_{t,i}$ (t-value)	$adj. R^2$
USA	-0.178		-0.017	-0.063	-0.147	0.399	1.5%
	(-1.09)		(-0.06)	(-0.18)	(-0.45)	(1.41)	
Germany	-0.016	-0.059		0.061	0.084	0.132	-0.3%
	(-0.14)	(-0.34)		(0.28)	(0.42)	(0.68)	
Japan	0.071	-0.018	-0.179		0.259	0.143	2.7%
	(0.92)	(-0.25)	(-1.59)		(1.91)	(1.71)	
UK	-0.196	0.308	-0.221	0.333		0.105	2.3%
	(-1.48)	(1.51)	(-0.87)	(0.97)		(0.59)	

Table 8: Hypotheses tests on the first moments of excess bond returns

Two types of null hypotheses are tested. The first hypothesis is that a one-country model is sufficient to explain the dynamics of the first moments in excess bond returns. This amounts to testing whether parameters other than the constant and the coefficient on the lagged domestic one-month interest rate are zero. This test is chi-square distributed with three degrees of freedom. The asymptotic p-value is reported under the value of the statistic. The second hypothesis tested is that a two-country model is correct. This tests the parameter restriction that the coefficients other than the constant, the lagged domestic one-month interest rate and the lagged one-month interest rate differential between the domestic and the second country's interest rate are zero. We test whether the second country is the USA (US), Germany (GR), Japan (JP), or the United Kingdom (UK). This test is chi-square distributed with two degrees of freedom. Once again, the asymptotic p-value is reported below the value of the statistic.

Hypotheses tested	USA	Germany	Japan	UK
Hypothesis (1): A one-country model is correct				
	$\chi^2(3)$: 2.44 p-value: 0.486	$\chi^2(3)$: 1.12 p-value: 0.772	$\chi^2(3)$: 6.31 p-value: 0.097	$\chi^2(3)$: 3.01 p-value: 0.389
Hypothesis (2): A two-country model is correct				
Second country: US		$\chi^2(2)$: 0.92 p-value: 0.632	$\chi^2(2)$: 6.22 p-value: 0.045	$\chi^2(2)$: 1.08 p-value: 0.583
Second country: GR	$\chi^2(2)$: 0.77 p-value: 0.680		$\chi^2(2)$: 3.64 p-value: 0.161	$\chi^2(2)$: 2.82 p-value: 0.245
Second country: JP	$\chi^2(2)$: 0.23 p-value: 0.891	$\chi^2(2)$: 0.21 p-value: 0.901		$\chi^2(2)$: 2.91 P-value: 0.233
Second country: UK	$\chi^2(2)$: 0.05 p-value: 0.975	$\chi^2(2)$: 0.19 p-value: 0.907	$\chi^2(2)$: 2.54 p-value: 0.281	

Table 9: Second moments of excess bond returns

The dependent variable in these regressions is $(12R_{t+12,j} - 11R_{t+11,j})^2$. The letter *r* refers to interest rates and the subscript *i* to the home country. Germany is denoted by GR, Japan by JP, United Kingdom by UK, and United States by US. The monthly data are expressed in percentage terms. Newey- West (1987) t-values corrected for heteroskedasticity and serial correlation up to lag two appear in parentheses under the coefficient estimates. The *R*-squares are adjusted for degrees of freedom.

Home country	Constant (t-value)	$r_{t,i} - r_{t,US}$ (t-value)	$r_{t,i} - r_{t,GR}$ (t-value)	$r_{t,i} - r_{t,JP}$ (t-value)	$r_{t,i} - r_{t,UK}$ (t-value)	$r_{t,i}$ (t-value)	$adj. R^2$
USA	-0.004		-0.006	0.000	0.004	0.012	23.3%
	(-2.10)		(-2.13)	(0.03)	(1.61)	(4.34)	
Germany	$-1.03*10^{-3}$	$-2.35*10^{-3}$		$7.93*10^{-4}$	$6.97*10^{-4}$	$3.00*10^{-4}$	12.9%
	(-1.97)	(-2.09)		(0.68)	(0.07)	(3.16)	
Japan	$-2.08*10^{-4}$	$2.91*10^{-4}$	$-1.17*10^{-4}$		$-9.62*10^{-4}$	$8.15*10^{-4}$	4.3%
	(-0.63)	(0.93)	(-0.22)		(-2.17)	(2.29)	
UK	0.000	-0.001	-0.002	-0.002		0.005	3.4%
	(-0.45)	(-0.55)	(-0.48)	(-0.65)		(3.11)	

Table 10: Hypotheses tests on the specification for the second moments of excess bond returns

Two types of null hypotheses are tested. The first hypothesis is that a one-country model is sufficient to explain the second moments of the excess bond returns. This amounts to testing whether parameters other than the constant and the coefficient on the lagged domestic one-month interest rate are zero. This test is chi-square distributed with three degrees of freedom. The asymptotic p-value is reported under the value of the statistic. The second hypothesis tested is that a two-country model is correct. This tests the parameter restriction that the coefficients other than the constant, the lagged domestic one-month interest rate and the lagged one-month interest rate differential between the domestic and the second country's interest rate are zero. We test whether the second country is the USA (US), Germany (GR), Japan (JP), or the United Kingdom (UK). This test is chi-square distributed with two degrees of freedom. Once again, the asymptotic p-value is reported below the value of the statistic.

Hypotheses tested	USA	Germany	Japan	UK
		Hypothesis (1): A one-country model is correct		
	$\chi^2(3)$: 8.93 p-value: 0.030	$\chi^2(3)$: 7.73 p-value: 0.050	$\chi^2(3)$: 6.64 p-value: 0.084	$\chi^2(3)$: 3.39 p-value: 0.335
		Hypothesis (2): A two-country model is correct		
Second country: US		$\chi^2(2)$: 1.10 p-value: 0.577	$\chi^2(2)$: 4.76 p-value: 0.093	$\chi^2(2)$: 2.67 p-value: 0.263
Second country: GR	$\chi^2(2)$: 6.15 p-value: 0.046		$\chi^2(2)$: 6.52 p-value: 0.038	$\chi^2(2)$: 0.56 p-value: 0.754
Second country: JP	$\chi^2(2)$: 8.59 p-value: 0.013	$\chi^2(2)$: 7.43 p-value: 0.024		$\chi^2(2)$: 0.90 P-value: 0.637
Second country: UK	$\chi^2(2)$: 8.91 p-value: 0.011	$\chi^2(2)$: 5.54 p-value: 0.063	$\chi^2(2)$: 1.24 p-value: 0.538	

Table 11: Parameter values used in the simulated economy.

In the simulated economy, the interest rate of country j follows the process:

$$r_{t+1,j} = (1-\varphi_j)\mu_j + \varphi_j r_{t,j} + \sigma_j r_{t,j}^{1/2} \varepsilon_{t+1,j}$$

Furthermore, the natural logarithms of the currency j pricing kernel is:

$$-m_{t+1,j} = \frac{1}{2}\theta_j^2 + (1+\frac{1}{2}\lambda_j^2) r_{t,j} + \lambda_j r_{t,j}^{1/2}\varepsilon_{t+1,j} + \theta_j \nu_{t+1,j}$$

The values for the unconditional means, μ_j are reported in Table 1. Note that the correlations of the ε's are denoted by ρ's. We assume that θ_j's are equal to a common θ, and set θ^2 to be equal to ten percent of the average variance of the rates of appreciations of the three currencies relative to the dollar. The resulting value for θ is 0.0108 or 1.08%. We also assume that the λ for the US is equal to the average of the λ's of the three other countries.

Parameter	USA	Germany	Japan	UK
σ	0.966	0.670	0.434	0.625
λ	-0.413	-0.234	-0.577	-0.428
φ	0.961	0.972	0.991	0.975
		Values for ρ's		
USA		0.434	0.757	0.654
Germany			0.724	0.553
Japan				0.931

Table 12: P-values of hypotheses tests based on the small-sample distributions: First moments of exchange rates.

The table reports simulated p-values for the chi-square tests with two degrees of freedom whose asymptotic distributions are reported in Table 2. The hypothesis tested is that a two-country model is sufficient to explain the exchange rate dynamics. The second country is always that of the reference currency. The first row of the table below gives the reference currencies. The results should be read as follows. For instance, 0.0840 is the p-value of the $\chi^2(2)$ test that a two-country model is sufficient to explain the dynamics of the German Mark - US dollar exchange rate.

	US dollar	British Pound	Japanese Yen
German Mark	0.084	0.527	0.196
Japanese Yen	0.547	0.951	
British Pound	0.193		

Table 13: P-values of hypotheses tests based on the small-sample distributions: First and second moments of the one-month interest rates.

The table reports simulated p-values on two types of null hypotheses. The first type of hypothesis is that a one-country model is sufficient to explain the first and second moments of the one-month interest rates. This amounts to testing whether parameters other than the constant and the coefficient on the lagged domestic one-month interest rate are zero. This test is chi-square distributed with three degrees of freedom. The first panel reports the simulated p-values for the first moments of the one-month interest rates, whereas the third panel reports the simulated p-values for the second moments of the one-month interest rates. The second hypothesis is that a two-country model is correct. This tests the parameter restriction that the coefficients other than the constant, the lagged domestic one-month interest rate and the lagged one-month interest rate differential between the domestic and the second country's interest rate are zero. We test whether the second country is the USA (US), Germany (GR), Japan (JP), or the United Kingdom (UK). This test is chi-square distributed with two degrees of freedom. The second panel reports the simulated p-values for the first moments of the one-month interest rates and the forth panel reports the simulated p-values for the second moments of the one-month interest rates.

Hypotheses tested	USA	Germany	Japan	UK
Panel 1. First Moments: Hypothesis (1): A one-country model is correct - $\chi^2(3)$ test				
	0.558	0.192	0.003	0.325
Panel 2. First Moments: Hypothesis (2): A two-country model is correct $\chi^2(2)$ test				
Second country: US		0.205	0.023	0.735
Second country: GR	0.896		0.002	0.551
Second country: JP	0.659	0.403		0.178
Second country: UK	0.793	0.945	0.135	
Panel 3. Second Moments: Hypothesis (1): A one-country model is correct - $\chi^2(3)$ test				
	0.000	0.748	0.651	0.107
Panel 4. Second Moments: Hypothesis (2): A two-country model is correct $\chi^2(2)$ test				
Second country: US		0.749	0.752	0.184
Second country: GR	0.000		0.734	0.046
Second country: JP	0.000	0.569		0.049
Second country: UK	0.000	0.598	0.446	

Table 14: P-values of hypotheses tests based on the small-sample distributions: First and second moments of the 12-month excess bond returns.

The table reports simulated p-values on two types of null hypotheses. The first type of hypothesis is that a one-country model is sufficient to explain the first and second moments of the 12-month excess bond returns. This amounts to testing whether parameters other than the constant and the coefficient on the lagged domestic one-month interest rate are zero. This test is chi-square distributed with three degrees of freedom. The first panel reports the simulated p-values for the first moments of the 12-month excess bond returns, whereas the third panel reports the simulated p-values for the second moments of the 12-month excess bond returns. The second hypothesis is that a two-country model is correct. This tests the parameter restriction that the coefficients other than the constant, the lagged domestic one-month interest rate and the lagged one-month interest rate differential between the domestic and the second country's interest rate are zero. We test whether the second country is the USA (US), Germany (GR), Japan (JP), or the United Kingdom (UK). This test is chi-square distributed with two degrees of freedom. The second panel reports the simulated p-values for the first moments of the 12-month excess bond returns and the forth panel reports the simulated p-values for the second moments of the 12-month excess bond returns.

Hypotheses tested	USA	Germany	Japan	UK
Panel 1. First Moments: Hypothesis (1): A one-country model is correct - $\chi^2(3)$ test				
	0.753	0.914	0.308	0.685
Panel 2. First Moments: Hypothesis (2): A two-country model is correct $\chi^2(2)$ test				
Second country: US		0.817	0.176	0.783
Second country: GR	0.839		0.354	0.503
Second country: JP	0.944	0.951		0.440
Second country: UK	0.989	0.958	0.476	
Panel 3. Second Moments: Hypothesis (1): A one-country model is correct - $\chi^2(3)$ test				
	0.109	0.174	0.233	0.601
Panel 4. Second Moments: Hypothesis (2): A two-country model is correct $\chi^2(2)$ test				
Second country: US		0.744	0.245	0.498
Second country: GR	0.144		0.119	0.872
Second country: JP	0.049	0.078		0.765
Second country: UK	0.050	0.184	0.684	