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UNEMPLOYMENT VERSUS IN-WORK BENEFITS WITH SEARCH UNEMPLOYMENT AND OBSERVABLE ABILITIES

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ABSTRACT

Unemployment versus In-work Benefits with Search Unemployment and Observable Abilities

This Paper explores the optimal interaction between the tax system and unemployment compensation in insuring people against the risks of involuntary unemployment and low ability. To that end, we introduce search unemployment in a model of optimal non-linear income taxation. We find that the optimal search subsidy (i.e. the difference between the in-work benefit and the unemployment benefit) increases if, for efficient agents, the participation constraint (governing job search) becomes relatively more important than the incentive compatibility constraint (determining hours worked). The relation between unemployment benefits and the optimal level of in-work benefits (the number of people exerting positive work effort) is U (inversely U) shaped.

JEL Classification: H21, J64 and J65
Keywords: in-work tax benefits, search and unemployment compensation

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Unemployment vs. in-work benefits with search unemployment and observable abilities

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Abstract

This paper explores the optimal interaction between the tax system and unemployment compensation in insuring people against the risks of involuntary unemployment and low ability. To that end, we introduce search unemployment in a model of optimal non-linear income taxation. We find that the optimal search subsidy (i.e. the difference between the in-work benefit and the unemployment benefit) increases if, for efficient agents, the participation constraint (governing job search) becomes relatively more important than the incentive compatibility constraint (determining hours worked). The relation between unemployment benefits and the optimal level of in-work benefits (the number of people exerting positive work effort) is U (inversely U) shaped.

Key words: search, in-work tax benefits, unemployment compensation, redistribution, risk aversion.

JEL codes: H21, J64, J65

1 Introduction

In recent years many industrialized countries have employed tax policies to encourage unemployed persons to seek work, thereby reducing expenditures on welfare and unemployment benefits. Following the example of the United States, several European countries have introduced or are considering in-work tax benefits in the form of an Earned Income Tax Credit (EITC). More generally, by lowering taxes on low skilled work, governments increasingly encourage low skilled workers to look for jobs. These policies are part of the so-called strategy

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of 'welfare to work,' where governments fight poverty by raising employment for low skilled workers rather than by increasing welfare benefits for these workers.

To investigate the optimal interaction between tax policies and welfare payments, we introduce unemployment risk due to search unemployment in a model of optimal non-linear income taxation in which agents feature heterogeneous abilities (similar to Mirrlees (1971)). In such a setting, unemployment benefits are required to insure risk-averse agents against the risk of becoming unemployed. At the same time, by redistributing between high skilled and low skilled workers, a non-linear income tax protects the same risk-averse agents against the risk of being born without many skills. Indeed, inequities originate in differences in not only abilities (as in Mirrlees (1971)) but also employment status.

In such a setting, we explore how the government can best address these two dimensions of poverty: low skills and involuntary unemployment. Should the government rely mainly on unemployment benefits rather than on in-work benefits or may it be optimal to offer in-work benefits that exceed unemployment benefits? Generous unemployment compensation helps those who are poorest (i.e. the unemployed) but harms incentives to look for a job. In-work benefits do not suffer from this latter drawback, but are less well targeted at those most in need.

In investigating this trade off, this paper arrives at the following main insights. Whereas unemployment benefits distort agents' job search (i.e. the participation or extensive margin), in-work benefits for low-ability agents distort the hours that agents choose to work (i.e. incentive compatibility or intensive margin) by making it more attractive for high-ability agents to mimic lower ability agents. If the government can observe an agent's ability, the incentive compatibility constraint (i.e. the intensive margin) can be ignored and the participation constraint (i.e. extensive margin) is binding also for high-ability types. In that case, in-work benefits tend to be generous. In fact, the government may find it optimal to offer in-work benefits that exceed optimal unemployment benefits, even though the unemployed are poorer than agents with a job. The reason is that higher unemployment compensation is a relatively ineffective way to fight poverty because it benefits not only the poorest but also the richest agents. In particular, if unemployment benefits are increased, the government must reduce taxes on the most efficient agents in order to prevent these high-ability workers from leaving the labor market. The associated adverse distributional effect of reducing taxes for the richest agents may outweigh the benefit of reducing poverty among the unemployed.

If the government cannot observe ability (so that it has to meet the incentive compatibility constraint), in-work benefits should be smaller than optimal unemployment compensation. Intuitively, the incentive compatibility constraint rather than the participation constraint is binding for high-ability agents. Indeed, the incentive compatibility constraint implies that utility increases with ability so that the participation constraint is not binding for the richest agents.[1]

[1] This assumes that search costs are uniform for all agents. If search costs would increase in ability, the participation constraint could be binding for high ability agents. We do not

Accordingly, in order to prevent richer agents from mimicking poorer workers, generous in-work benefits for poor workers force the government to reduce taxes on richer workers. In this way, in-work benefits for poor agents also benefit richer agents and thus are a relatively ineffective way to fight poverty. Rather than using in-work benefits, the government therefore relies more on unemployment benefits, which benefit the poorest agents and do not directly harm the incentives facing the richest and most efficient workers but only damage the search incentives facing marginal workers.

We also explore the case in which the government can optimize only the tax system and has to take unemployment compensation as given. This allows us to investigate the interaction between the tax and social insurance systems by considering the impact of an exogenously given unemployment benefit on the optimal tax system. The relationship between unemployment benefits and the level of in-work benefits appears to be U-shaped. As unemployment benefits are raised from a low initial level, unemployment benefits absorb the budgetary room for generous in-work benefits as an instrument to fight poverty. At low unemployment compensation, unemployment benefits and in-work benefits are thus substitutes in fighting poverty. As unemployment benefits are increased further, however, the participation constraint for marginal workers becomes binding and the government needs to raise in-work benefits to draw people out of unemployment into work. At high levels of unemployment benefits, therefore, in-work benefits and unemployment compensation become complements: in-work benefits help to offset the impact of higher unemployment benefits on the participation constraint. This U-shaped relationship between in-work benefits and unemployment compensation reveals that generous in-work benefits are called for in both countries with low and high unemployment benefits, but for different reasons. In countries with low unemployment compensation (such as the United States), in-work benefits are aimed at poverty alleviation. In countries with high unemployment compensation (such as most Western European countries), in contrast, in-work benefits protect the incentives to participate in the labor market.

In introducing search unemployment in an optimal tax model, we focus on the case in which the government can verify a worker's ability. We adopt this informational assumption for four main reasons. First, it allows us to explore the arguments for in-work tax benefits in circumstances that are favorable to these tax subsidies. As argued above, in-work benefits are most attractive if the participation rather than the incentive compatibility constraint is binding for high-ability agents. The second reason for our informational assumption is that this allows us to clearly identify how search incentives restrict the ability of the government to redistribute resources away from the most efficient workers towards agents with lower consumption levels. Without the intensive decision margin (i.e. the selection of work effort after having found a job) limiting redistribution, only the search margin (i.e. the extensive decision margin or

consider this latter case because there is not much empirical support for search costs rising with ability.

participation constraint) constrains redistribution. A third reason for assuming that types are observable is that it allows us to interpret high benefits for low-ability households as disability benefits where the government tags low-ability agents (see Akerlof (1978)). In this interpretation, the search costs can be interpreted as the information costs associated with the government verifying the ability level of these low-ability agents. The final reason for our informational assumption is that it simplifies the analysis and is a first step towards exploring a more complex model in which the intensive and extensive margins interact (see Boone and Bovenberg (2001)).

Our analysis is based on the special utility function studied by Ebert (1992). In Ebert's model, work effort enters private utility in a linear fashion. This implies that income effects on consumption are absent and that the government cares about the distribution of consumption rather than the distribution of work effort. Our approach is thus close to Kanbur, Keen and Tuomala (1994), who note that policy debates focus on raising consumption rather than reducing the work effort (or increasing leisure) of the poor. Kanbur, Keen and Tuomala (1994), however, adopt a non-welfarist social welfare function. We, in contrast, continue to follow the welfarist tradition, but assume a special, quasi-linear utility function. Another reason for adopting this particular utility function is that it clearly illustrates the importance of the search margin in constraining the redistributive powers of the government. In the absence of the search margin, the government engages in extreme redistribution if it can observe worker's ability: only the most efficient worker exerts work effort, as the distribution of work is determined by efficiency rather than equity considerations. We show how the search margin prevents this extremely redistributive policy.

We study issues similar to those explored by Saez (2000), who incorporates two labor-supply margins in an optimal tax model, namely not only hours worked (as in the traditional model) but also the participation decision. He calls these two margins the intensive and extensive margins, respectively. Although the search margin in our model resembles the extensive margin in the model of Saez, our model is quite different from Saez' approach. First of all, in the Saez 'model, agents voluntarily choose not to participate in the labor market while in our model, in contrast, agents are exposed to the risk of being involuntarily unemployed. Indeed, whereas in our model the unemployed feature the same tastes as employed agents, Saez (2000) assumes that the agents who are not employed exhibit a higher preference for leisure than the agents who are employed. Hence, within the Saez framework with voluntary unemployment, the arguments for providing a benefit to agents without employment are considerably weaker than in our setting with involuntary unemployment; in Saez' model, a person's unemployment status is entirely his or her own responsibility, while in our model it may be the result of bad luck. Our framework thus allows us to explore the optimal interaction between categorical unemployment benefits and non-linear income taxes as instruments to insure agents against the risk of not only low ability but also involuntary unemployment.

The rest of this paper is structured as follows. After Section 2 formulates the model, section 3 sets up the optimization problem for the government and discusses the optimality conditions. Subsequently, section 4 explores optimal tax policy if welfare benefits b are exogenously set. We can interpret this case as the tax authorities optimizing the tax system, taking the welfare system as given. The case in which the government can simultaneously optimize the tax and welfare systems is investigated in Section 5. In this section we also show that in-work benefits are less attractive if the government cannot observe workers' type. Section 6 concludes. All proofs of the results are in the appendix.

2 Model

Consider an economy with agents who feature homogenous preferences but heterogeneous skills. A worker of ability (or skill or efficiency level) n working y hours (or providing y units of work effort) supplies ny efficiency units of homogeneous labor. With a linear production function featuring a constant unitary labor productivity, these efficiency units are transformed in the same number of units of output. We select output as the numeraire. Hence, the before-tax wage per hour is given by the exogenous parameter n. Overall gross output (or gross income) $z(n)$ amounts to $z(n) \equiv ny(n)$. The density of agents of ability n is denoted by $f(n)$, while $F(n)$ represents the corresponding cumulative distribution function. The support of the distribution of abilities is given by $[n_0, n_1]$.

Workers feature homogeneous tastes. In particular, they share the following quasi-linear utility function over consumption x and hours worked (or work effort) y

$$u(x, y) = v(x) - y,$$

where $v'(x) > 0, v''(x) < 0$ for all $x \geq 0$ while $\lim_{x \downarrow 0} v'(x) = \infty$ and $\lim_{x \longrightarrow +\infty} v'(x) = 0$. The concavity of $v(.)$ implies that agents are risk averse and thus want to obtain insurance against the risks of unemployment and low earning capacity n. The specific cardinalization of the utility function affects the distributional preferences of a utilitarian government. In particular, the social planner wants to insure agents both against the risk of becoming unemployed and against the risk of being born with low ability. In other words, the concavity of $v(.)$ implies that a utilitarian government aims to fight poverty of both unemployed and low-ability agents.

As in Lollivier and Rochet (1983), Weymark (1987) and the example in Ebert (1992), work effort y enters the utility function in a linear fashion. This has two major consequences. First, consumption x is not affected by income effects. A higher average tax rate thus induces households to work more rather than facing a lower level of consumption. Second, a utilitarian government cares only about overall work effort and not about the distribution of that work effort over the various agents. Such a government thus aims at an equal distribution of consumption (i.e. the alleviation of poverty) rather than an equal distribution of welfare.

Instead of working with work effort $y(n)$ and consumption $x(n)$ as the instruments of the worker, we find it more convenient to write the utility function in terms of gross income (or output) $z(n) \equiv ny(n)$ and net income (or consumption) $x(n)$. Utility of type n is then written as $u(n) \equiv v(x(n)) - z(n)/n$.

Agents have to search for a job. In particular, by searching with intensity $s \in [0, 1]$, agents find a job with probability s.[2] Search costs $\gamma(s)$ are given by

$$\gamma(s) = \begin{cases} \gamma s & \text{if} \quad s \in [0, \bar{s}] \\ +\infty & \text{otherwise,} \end{cases}$$

where $\bar{s} < 1$ captures the idea that agents can fail to find a job, even if they search at full capacity $\bar{s}$. Hence, in contrast to Saez (2000), agents are exposed to the risk of becoming involuntarily unemployed. If an agent does not succeed in finding a job in our one-period model, (s)he receives an unemployment benefit $b \geq 0$, which the agent takes as given. Since the government cannot observe the ability of unemployed agents, the unemployment benefit does not depend on n. An agent of ability n thus selects search intensity s to maximize expected utility

$$U(n) = \max_{s} \{-\gamma(s) + su(n) + (1 - s) v(b)\}.$$

Substituting in here the search cost function $\gamma(s)$ introduced above, one can easily verify that the optimal choice of s for type n amounts to

$$s(n) = \begin{cases} 0 & \text{if} \quad u(n) < \gamma + v(b) \\ \bar{s} & \text{if} \quad u(n) \geq \gamma + v(b). \end{cases} \tag{1}$$

The government has to meet its budget constraint

$$\int_{n_0}^{n_1} f(n) s(n) [b + T(n)] dn = g + b, \tag{2}$$

where g represents the exogenously given government expenditure and $T(n) = z(n) - x(n)$ denotes the tax paid by type n. The utilitarian government maximizes ex-ante expected utility (i.e. expected utility before ability and labor market status have been revealed)

$$\max_{s(.), z(.), x(.)} \int_{n_0}^{n_1} f(n) [-\gamma s(n) + s(n) u(n) + (1 - s(n)) v(b)] dn. \tag{3}$$

In optimizing the objective function, the government is able to verify a person's ability n after that person has found a job. As long as a person remains unemployed, however, the government cannot observe the ability of a person. By finding a job, a person thus reveals his ability. The government can observe the productivity of an agent only if that agent participates in the production process. Hence, in contrast to unemployment insurance, the tax system can discriminate across types. Since a worker's ability n is verifiable, high-ability workers cannot mimic low-ability workers. The government therefore does not

[2] This formalization of the search margin in a static framework is similar to the one-period search model in Hosios (1990).

have to meet the incentive compatibility constraint associated with the intensive margin of labor supply (i.e. work effort or hours worked). This increases the scope for redistribution within the group of workers: taxes on labor income can be progressive without hurting the incentives of high-ability workers to exert effort in their jobs.

Without the intensive decision margin (i.e. the selection of work effort y after having found a job) constraining redistribution, only the search margin (i.e. the extensive decision margin) limits the ability of the government to redistribute resources away from the most efficient workers towards agents with lower consumption levels. Whereas high-ability workers are not able to mimic the low-ability workers by reducing their work effort y, they can mimic the low-skilled unemployed by reducing search effort s and remaining unemployed. Hence, the optimal unemployment insurance and tax systems have to meet the participation constraint $u(n) \geq v(b) + \gamma$ in order to induce type n to look for a job. High-ability types search for work only if they can find a job (which reveals their identity) yielding a utility level that is at least as large as that enjoyed when refraining from search and remaining unemployed (and thus saving on search costs and hiding ability). With the government able to verify the ability of workers, the extensive rather than the intensive margin binds for high-ability agents. This contrasts with the case in which the government cannot observe a worker's type (see below). In that case, the search constraint is binding only for the marginal (i.e. the least efficient worker), n_w. With non-verifiable types, if the participation constraint is met for the marginal type n_w, the incentive compatibility constraint (which implies that the utility of workers rises with type) ensures that the participation constraint is met also for more efficient types $n > n_w$.[3]

Summarizing, compared to the unemployment compensation, the tax system is a relatively efficient instrument to redistribute resources to low-ability workers because it can employ more information, namely the worker's type. Whereas unemployment compensation cannot be differentiated across various types (i.e. b cannot depend on type n), the tax authorities observe a worker's type and can thus differentiate in-work benefits across types (i.e. the average tax rate $T(n)$ may depend on n).

3 The optimal tax problem

This section introduces the main ingredients for characterizing the optimal tax schedule if the government cannot observe agents' search effort but does observe the ability of workers. We first rewrite the government's optimization problem by using two observations. First, due to the linearity of the cost function $\gamma(s)$,

[3] With verifiable worker's types, applying a work test (by observing search) is most valuable for the most efficient types. Indeed, relaxing the participation constraint of these types yields the largest social benefits. With non-observable worker's types, in contrast, the government would like to apply the work test (and thus observe search) only for the marginal worker n_w because this is the only worker for which the search margin binds.

an agent should either search full time $\bar{s}$ or not at all (i.e. $s = 0$). Second, the highest ability types should work and therefore search for a job because these types are most efficient and thus feature the lowest labor costs. These two observations imply that the government selects a marginal type $n_w \in [n_0, n_1]$ such that types $n < n_w$ do not search, while all types $n \geq n_w$ search at full capacity. We can thus formulate the optimization problem of the utilitarian government as follows

$$\max_{n_w, x(.), z(.)} F(n_w) v(b) + [1 - F(n_w)] (-\gamma \bar{s} + (1 - \bar{s}) v(b)) + \tag{4}$$
$$+ \int_{n_w}^{n_1} \left\{ \begin{array}{c} \bar{s} \left(v(x(n)) - \frac{z(n)}{n} \right) f(n) + \lambda_E f(n) \bar{s} [z(n) - x(n)] \\ -\eta(n) \left(\gamma - v(x(n)) + \frac{z(n)}{n} + v(b) \right) \end{array} \right\} dn$$
$$-\lambda_E \{ b [F(n_w) + (1 - F(n_w)) (1 - \bar{s})] + g \}$$

with complementary slackness conditions for the participation constraint

$$\begin{aligned} v(x(n)) - \frac{z(n)}{n} &\geq v(b) + \gamma \text{ and } \eta(n) \geq 0 \\ \eta(n) \left(\gamma - v(x(n)) + \frac{z(n)}{n} + v(b) \right) &= 0 \end{aligned}$$

for all types $n \geq n_w$ whom the government wants to search and

$$\begin{aligned} \int_{n_w}^{n_1} f(n) \bar{s} [z(n) - x(n) + b] \, dn &= b + g \text{ and } \lambda_E \geq 0 \\ \lambda_E \left(\int_{n_w}^{n_1} f(n) \bar{s} [z(n) - x(n) + b] \, dn - b - g \right) &= 0 \end{aligned}$$

for the government budget constraint, where $\eta(n)$ and λ_E represent the Lagrange multipliers for the participation constraint and the government budget constraint, respectively.

The first-order condition for maximizing (4) with respect to consumption $x(.)$ is given by

$$\eta(n) = \bar{s} f(n) \left[\frac{\lambda_E}{v'(x(n))} - 1 \right] \tag{5}$$

for types $n \geq n_w$. The shadow value $\eta(n)$ on the left-hand side of this expression captures the gain from raising consumption of type n in terms of relaxing the participation constraint of that type. The right-hand side stands for the direct distributional loss of raising consumption of type n. This loss is directly related to the difference between the marginal cost of producing additional resources (i.e. the marginal utility value of government revenue) λ_E and the marginal benefit of shifting resources to type n in terms of marginal utility of consumption of that type, $v'(x(n))$. This expression thus formalizes the trade off between efficiency considerations (i.e. providing incentives to search for jobs) and equity considerations (i.e. redistributing resources away from those with a relatively high consumption level (so that $v'(x(n)) < \lambda_E$) to those with lower consumption levels).

Turning to the optimal level of before-tax income $z(n)$, we observe that the optimization problem (4) is linear in $z(n)$ with coefficient $\left\{-\bar{s}f(n)\frac{1}{n} + \lambda_E f(n)\bar{s} - \eta(n)\frac{1}{n}\right\}$. The optimal $z(n)$ is therefore determined as follows[4]

$$
\begin{aligned}
z(n) &= n\left(v(x(n)) - \gamma - v(b)\right) \text{ only if } \left\{-\bar{s}f(n)\frac{1}{n} + \lambda_E f(n)\bar{s} - \eta(n)\frac{1}{n}\right\} = 0 \\
z(n) &= 0 \text{ if } \bar{s}f(n) + \eta(n) > \lambda_E f(n)\bar{s}n. \qquad (6)
\end{aligned}
$$

In order to interpret these results, we rewrite the condition $\bar{s}f(n) + \eta(n) > \lambda_E f(n)\bar{s}n$ as $\frac{1}{n} > v'(x(n))$ (by using expression (5) to eliminate $\eta(n)$). Agent n exerting more work effort yields additional consumption for agent n, which is valued at $v'(x(n))$, but costs $\frac{1}{n}$ in terms of disutility of work effort. Hence, $\frac{1}{n} > v'(x(n))$ implies that the costs of work exceed the marginal benefits so that work effort should be reduced to its minimum level of zero.

The first-order condition for maximizing (4) with respect to the least efficient type that searches for a job (the so-called marginal searcher), n_w, amounts to

$$
\bar{s}f(n_w)\left\{v(b) + \gamma - v(x(n_w)) + \frac{z(n_w)}{n_w} - \lambda_E\left(z(n_w) - x(n_w) + b\right)\right\} = 0, \qquad (7)
$$

where we have assumed that the optimal n_w is not a corner solution (i.e. $n_w > n_0$; we return to possibility of a corner solution below). The direct utility gain from having the marginal type search for a job (i.e. $v(x(n_w)) - \frac{z(n_w)}{n_w} - v(b) - \gamma$) should equal the costs in terms of government revenue (i.e. the net search subsidy $-T(n) - b$, which is defined as the in-work tax benefit $x(n_w) - z(n_w) = -T(n_w)$ net of the unemployment benefit b). In other words, at the marginal searching type n_w, the private benefit of search (i.e. $v(x(n_w)) - \frac{z(n_w)}{n_w} - v(b) - \gamma = u(n_w) - v(b) - \gamma$) should equal the social cost of the search subsidy $-(T(n_w) + b)$ (or the net subsidy on work) in terms of the government budget constraint.

3.1 The optimal tax system without search

In order to understand the impact of the search margin on the optimal tax system, this sub-section explores the case in which the search margin is absent (i.e. $\eta(n) = 0$). In that case, expressions (5) and (6) imply that all work should be performed by the most efficient type n_1 so that marginal production costs λ_E are determined by the marginal labor costs of this type, i.e. $\lambda_E = 1/n_1$. All

[4] We cannot have $-\bar{s}f(n)\frac{1}{n} + \lambda_E f(n)\bar{s} - \eta(n)\frac{1}{n} > 0$ because that would imply (by the linearity of $z(.)$ in the optimization problem) $z(n) = +\infty$ so that $z(n) > n\left(v(x(n)) - \gamma - v(b)\right)$. This, however, would violate the participation constraint. see also sub-section 3.1.

types feature the same consumption level $\bar{x}$ determined by $v'(\bar{x}) = \lambda_E = 1/n_1$. Intuitively, given the linear specification of the disutility of work effort $c(y) = y$, efficiency rather than equity considerations determine the distribution of work effort. Hence, all labor should be performed by the most efficient workers, i.e. the workers of type n_1. In contrast to the distribution of work effort, the distribution of consumption affects social utility in view of the concave nature of the utility of consumption $v(x)$. In particular, the government prefers an equal distribution of consumption, with the uniform consumption level being determined by the costs of supplying labor by the most efficient type n_1 (i.e $v'(\bar{x}) = 1/n_1$). Whereas the distribution of consumption is thus equal, the distribution of utility is highly unequal as the most efficient workers perform all work. Indeed, the government levies a highly progressive tax $T(n_1)$ on these types, inducing these workers to produce sufficient resources for providing a consumption level $\bar{x}$ to all other agents. The non-negative tax level $T(n_1)$ is determined (from the government budget constraint (2)) by the costs of granting tax subsidies (i.e. $-T(n) = \bar{x}$) to all types $n < n_1$. The tax level $T(n_1)$ thus declines with the share of most efficient types in the population.

This paper assumes that the government cannot verify search. In that case, the search margin prevents the government from shifting all work effort to the most efficient type. In particular, having the most efficient type perform all work effort would make it unattractive for this type to look for such a demanding job. In this way, the participation constraint $u(n) \geq v(b) + \gamma$ prevents the government from exploiting the most efficient types through an extremely redistributive tax system. Hence, work effort must be distributed more equally over the population.

4 The optimal tax system with search

In characterizing the optimal tax system with endogenous search, we distinguish three separate cases. To do so, we define $\hat{n}$ as the least efficient type that searches for a job (i.e. $u(\hat{n}) \geq v(b)+\gamma$) without a net search subsidy (or subsidy on work). The appendix shows that $\hat{n}$ is the biggest root to the equation $\zeta_\gamma(.,b) = 0$, that is,

$$\hat{n} \equiv \max\left\{n \geq 0 | \zeta^*_\gamma(n,b) = 0\right\} \tag{8}$$

with[5]

$$\zeta^*_\gamma(n,b) \equiv v(x(n)) - v(b) - \gamma + \frac{1}{n}(-x(n) + b) \tag{9}$$

where $x(n)$ is determined by $v'(x(n)) = \frac{1}{n}$. To see that $\hat{n}$ is just productive enough to search for a job without government help in the form of a work subsidy

[5] This expression follows from expression (7) and a number of additional results that are established in the appendix (e.g., $z(n_w) = 0$ and $\lambda_E = 1/\hat{n}$).

$-(T(\hat{n})+b)>0$, we note that $v(x(\hat{n}))-v(b)-\gamma+\frac{1}{\hat{n}}(-x(\hat{n})+b)=0$ can be written as (using $T(\hat{n})=z(\hat{n})-x(\hat{n})$)

$$v(b)+\gamma=v(x(\hat{n}))-\frac{z(\hat{n})-T(\hat{n})-b}{\hat{n}}$$

An in-work benefit $-T(\hat{n})=b$ that exactly offsets the impact of the unemployment benefit b on search incentives makes this type $\hat{n}$ indifferent about searching

$$v(b)+\gamma=v(x(\hat{n}))-\frac{z(\hat{n})}{\hat{n}}=u(\hat{n})$$

Accordingly, types $n>\hat{n}$ search for a job even if the government does not subsidize search. Less efficient types $n<\hat{n}$, in contrast, must be paid a search subsidy to induce them to look for a job.

In order to define the three separate cases, we relate $\hat{n}$, which does not depend on the density function $f(.)$ and its support $[n_0,n_1]$, to the distribution of skills in the following definition.

Definition 1 *For given b and γ, an economy is characterized by the density function $f(.)$ and its support $[n_0,n_1]$. We define a rich [R], a poor [P] and a normal [N] economy as follows:*
[R] $\int_{\hat{n}}^{n_1} f(n)\,\bar{s}\,(n(v(x(n))-\gamma-v(b))-x(n)+b)\,dn+\int_{n_0}^{\hat{n}} f(n)\,\bar{s}\,(-x(\hat{n})+b)\,dn>b+g$;
[P] $\int_{\hat{n}}^{n_1} f(n)\,\bar{s}\,(n(v(x(n))-\gamma-v(b))-x(n)+b)\,dn<b+g$ *and*
[N] there exists $n_w\in[n_0,\hat{n}]$ *such that* $\int_{\hat{n}}^{n_1} f(n)\,\bar{s}\,(n(v(x(n))-\gamma-v(b))-x(n)+b)\,dn+\int_{n_w}^{\hat{n}} f(n)\,\bar{s}\,(-x(\hat{n})+b)\,dn=b+g$,
where $x(n)$ is determined by $v'(x(n))=\frac{1}{n}$ and $\hat{n}$ is defined in equation (8).

In the rich economy, the government enjoys a budget surplus if the government taxes all workers of type $n>\hat{n}$ to the maximum amount without discouraging them from finding a job $(z(n)=n(v(x(n))-\gamma-v(b)))$ and using the proceeds for three purposes, namely, to pay, first, exogenous government spending g; second, unemployment benefits to the part of the population $1-\bar{s}$ that remains unemployed even though it has searched for a job; and third, in-work tax benefits to all other workers $n<\hat{n}$ so that these workers enjoy the same consumption level as the marginal worker of type $\hat{n}$ without having to exert any work effort (i.e. $-T(n)=x(\hat{n})>b$ for $n<\hat{n}$).

In the poor economy, in contrast, the maximum amount of tax from all types $n>\hat{n}$ yields less revenue than needed to finance, in addition to government spending g, a consumption level of only b (rather than $\bar{x}>b$) to all other agents. The normal economy occupies an intermediate position between the rich and the poor economies. In this economy, the maximum tax revenue that can be extracted from the types $n>\hat{n}$ allows the government to provide a relatively high consumption level $x(\hat{n})=T(n)>b$ to only a subset of types $[n_0,\hat{n}]$ while paying a lower consumption level b to the remaining types.

In order to establish the link between the unemployment benefit b and the three cases defined above, we first need to rule out the uninteresting case in which government spending g is so high that the economy is poor even without unemployment benefits (i.e. $b = 0$).

Definition 2 *For given $f(.), n_0$ and n_1 define $g^*(\gamma)$ as*

$$g^*(\gamma) = \int_{\hat{n}}^{n_1} f(n)\,\bar{s}\,(n(v(x(n)) - \gamma) - x(n))\,dn,$$

where $\hat{n}$ is defined in equation (8).

Using this definition of the maximal government expenditure that can be financed in the absence of unemployment benefits, we can prove the following result.

Lemma 3 *Assume that $g < g^*(\gamma)$ then there exist values $\underline{b}, \bar{b}$ satisfying $0 \leq \underline{b} \leq \bar{b} < +\infty$ such that*
$b \in [0, \underline{b}\rangle$ implies that the economy is rich,
$b \in [\underline{b}, \bar{b}]$ implies that the economy is normal,
$b > \bar{b}$ implies that the economy is poor.

At high levels of unemployment compensation, the economy is poor. Lowering unemployment benefits eventually turns the economy into a normal one and a large enough cut in unemployment compensation makes the economy rich.

We can now formulate the solution to the optimization problem (4) with an exogenous level of unemployment compensation b as follows.

Proposition 4 *The optimal solution has the following form for the three cases [R], [P] and [N]:*
[R] *there exist $n_z \in \langle n_0, n_1\rangle$ such that*
for $n \in [n_0, n_z\rangle$, it is the case that $s(n) = \bar{s}$, $z(n) = 0$, $x(n) = \bar{x}$, and $v'(\bar{x}) = \frac{1}{n_z}$,
for $n \in \langle n_z, n_1]$, it is the case that $s(n) = \bar{s}$, $z(n) = n(v(x(n)) - \gamma - v(b))$, and $v'(x(n)) = \frac{1}{n}$,
where n_z is determined by the government budget constraint

$$\begin{aligned} & \int_{n_z}^{n_1} f(n)\,\bar{s}\,(n(v(x(n)) - \gamma - v(b)) - x(n) + b)\,dn + \int_{n_0}^{n_z} f(n)\,\bar{s}\,(-x(n_z) + b)\,dn \\ = \; & b + g \end{aligned}$$

and

$$\lambda_E = \frac{1}{n_z}.$$

[P] *this economy unravels as the government cannot pay the unemployment benefits b and its exogenous revenue requirement g;*
[N] *there exist $n_z \geq n_w \geq n_0$ such that*
for $n \in [n_0, n_w\rangle$, it is the case that $s(n) = 0$,

for $n \in [n_w, n_z\rangle$*, it is the case that* $s(n) = \bar{s}$*,* $z(n) = 0, x(n) = \bar{x}$ *and* $v'(\bar{x}) = \frac{1}{n_z}$*,*
for $n \in \langle n_z, n_1]$*, it is the case that* $s(n) = \bar{s}$*,* $z(n) = n(v(x(n)) - \gamma - v(b))$*,* $v'(x(n)) = \frac{1}{n}$*,*
where n_z *is determined by*

$$n_z = \hat{n}$$

with $\hat{n}$ *defined in (8),* n_w *is determined by the government budget constraint*

$$\int_{n_z}^{n_1} f(n)\,\bar{s}\,(z(n) - x(n) + b)\,dn + \int_{n_w}^{n_z} f(n)\,\bar{s}\,(-\bar{x} + b)\,dn = b + g,$$

and

$$\lambda_E = \frac{1}{n_z}.$$

Remark 5 *The choice of the interval* $[n_w, n_z\rangle$ *in the solution for the normal economy above is somewhat arbitrary. Any other (set of) subinterval(s) of* $[n_0, n_z\rangle$ *with the same mass of agents is a solution as well. However, the solution above is the most natural choice.*

Corollary 6 *In the normal economy* $T(n) + b > 0$ *for all* $n \in \langle n_z, n_1]$*,* $\lim_{n \downarrow n_z} [T(n) + b] = 0$[6] *and* $T(n) + b = -\bar{x} + b < 0$ *for all* $n \in [n_w, n_z\rangle$*;*
In the rich economy $T(n) + b > 0$ *for all* $n \in \langle n_z, n_1]$*,* $\lim_{n \downarrow n_z} [T(n) + b] > 0$ *and* $T(n) + b = -\bar{x} + b < 0$ *for all* $n \in [n_0, n_z\rangle$

We first discuss the properties of the solution that hold in both the rich and normal economies before we turn to the properties that differ across these two economies. In both the rich and normal economies, a set of agents optimally search for a job (and pay search costs in the process) even though they do not exert any effort in their jobs (i.e. $z(n) = 0$)[7]. The reason why costly search for a job is optimal even though workers do not produce anything is that workers reveal their ability if they find a job. This additional information has social value because it allows the government to better target its policy instruments aimed at poverty alleviation at the less efficient, most deserving, types by providing in-work benefits that are explicitly aimed at these types. In-work benefits are a relatively efficient means to fight poverty because, in contrast to unemployment compensation, these benefits do not harm the search incentives of the more efficient types and therefore do not reduce the rents that can be extracted from these latter types. Jobs are thus an effective way to get less able people out of poverty even though these agents do not produce anything in their jobs. The

[6] We consider the limit $n \downarrow n_z$ here because with an atomless distribution of types, $z(n_z)$ and $T(n_z)$ are not uniquely determined.

[7] The reader may wonder why the government cannot observe the ability of unemployed agents but does observe the ability of employed agents working zero hours. The point is that these agents work $\varepsilon > 0$ hours but that the government minimizes ε so that it can just uncover a worker's type. We in fact assume that ε is infinitely small.

relative efficacy of in-work benefits as a poverty fighting instrument compared to unemployment benefits explains why a search subsidy (*i.e.* $T(n) + b < 0$) is optimal for some people (although these people do not provide work effort if their job search is successful).

We can interpret the jobs in which agents do not produce anything (i.e. $z(n) = 0$) as disability insurance.[8] In this interpretation, the search costs correspond to the costs of uncovering the information about the ability level of low-ability agents $n < n_z$. Indeed, it does not matter for the optimal allocation whether the costs of tagging these agents are paid by the government directly or indirectly (i.e. by having to pay agents a sufficiently high in-work benefit to induce these agents to search for a job). We thus can interpret the search subsidies as disability benefits. In that interpretation, it seems most natural to provide these benefits to the agents with the lowest ability rather than the more able agents $[n_w, n_z\rangle$.

The marginal (labor) cost of production λ_E is determined by the cost of labor of the marginal worker n_z, i.e. by the least efficient type that works positive hours so that $\lambda_E = 1/n_z$. The cost of additional resources thus exceeds that in an economy without a search margin where $\lambda_E = \frac{1}{n_1}$ (see the discussion in sub-section 3.1). Intuitively, in the presence of a search margin, the government can less easily exploit the most efficient types. It therefore has to rely on less efficient types to supply marginal output so that marginal labor costs increase.

The distributional wedge (i.e. the difference between the marginal utility of additional consumption and the marginal direct resource cost of that consumption, $v'(x(n)) - \lambda_E$) is negative for all workers who work positive hours (since $v'(x(n)) = 1/n < \lambda_E = 1/n_z$ for $n > n_z$). The government would thus like to take resources away from the more efficient workers who feature a relatively high consumption level but does not do so because this would harm the incentives of these efficient workers to look for a job.[9]

The agents who collect search subsidies $-(T(n)+b) > 0$ feature a zero distributional wedge (i.e. $v'(x(n)) = v'(\bar{x}) = \lambda_E$) because providing in-work benefits do not imply any direct distortions on the decision of the more efficient types. Since the government does not have to worry about the more efficient workers mimicing these less efficient workers, it can provide generous in-work benefits to less able workers that makes these workers better off than more able workers. Hence, the tax system can be rather redistributive. The absence of the intensive margin explains also why the work effort decision is not distorted: workers do not exert any work effort if and only if the direct costs of additional production in terms of work effort $1/n$ exceed the benefits of additional production λ_E (and the benefits of addition production in terms of additional consumption

[8] Another interpretation of these jobs is workfare for agents who are not productive enough to earn high wages in the private sector. High ability agents are not eligible for these jobs.

[9] Whereas the distributional wedge for those who provide positive work effort is negative, the corresponding wedge for the unemployed is positive (since $v(\bar{x}) - z_z = v(b) + \gamma$ (from the participation constraint of the marginal worker n_z) implies that $b < \bar{x}$ and hence $v'(b) > v'(\bar{x}) = \lambda_E$).

$v'(x(n))$.[10]

4.1 The normal economy

We now turn to results that are specific to the cases distinguished in definition 1, starting with the results for the normal economy. The normal economy cannot afford to pay search subsidies to all types $[n_0, \hat{n}\rangle$ that are not productive enough to look for a job without search subsidies. As a result, a subset of these types $[n_0, n_w\rangle$ is not paid in-work tax benefits so they do not search and remain unemployed.[11] In a normal economy, therefore, the tax system (by paying in-work tax benefits) cannot fully protect less able agents against poverty. Also unemployment compensation plays a role in insuring agents against the risk of being born with low ability n. The utility level of the low-ability agents who do not search $[n_0, n_w\rangle$ corresponds to that enjoyed by the most productive types $\langle n_z, n_1]$ who have found jobs in which they exert positive work effort.[12] For these most efficient types, the average tax level $T(n)$ (which determines the production level $z(n)$) is selected so that these types are indifferent between searching for job and staying unemployed (i.e. $u(n) - \gamma = v(b)$).

The subset $[n_w, n_z\rangle$ who are paid in-work benefits $-T(n) > b$ without having to exert any effort in their jobs enjoy the highest utility level of all types.[13] Corollary 6 shows that $x(n) - b > 0$ for $n \in [n_w, n_z\rangle$. Substituting the Taylor expansion $v(b) = v(\bar{x}) + v'(\bar{x})(b - \bar{x}) + 1/2v''(\xi)(b - \bar{x})^2$ (and noting that $v'(\bar{x}) = 1/\hat{n}$)) into (9), we find that $\gamma = -1/2v''(\xi)(\bar{x} - b)^2$. Hence, in deciding whether to give one more search subsidy and thus relying more on the tax system rather than unemployment benefits in fighting poverty, the government faces a trade off between saving on the costs of search γ and fighting poverty by reducing the cost of unequal consumption level (which are directly related to concavity of utility $v(.)$). Large search costs γ imply that the agents have to pay relatively large costs to reveal their type by finding a job.[14] This makes the tax system a relatively expensive instrument to redistribute resources: relatively large search

[10] The extreme result that work effort rises discontinuously at type n_z is due to the linear specification of the disutility of work effort $c(y) = y$. If $c(.)$ were convex, it would be optimal to allocate work effort more equally over the population without discontinuous jumps in work effort.

[11] The optimal n_w in a normal economy is not the corner solution n_0 so that (7) holds with equality.

[12] The consumption level of the unemployed, however, is lower than that of the more productive types who have found a job. In terms of utility, the higher consumption level of these latter types is balanced by higher work effort and search costs. Since the government is interested in fighting poverty (i.e. inequality in consumption levels), it would like to redistribute resources from the more productive types to the unemployed.

[13] This is an ex-post utility level. The proportion $1 - \bar{s}$ of these types who do not find a job even though they have searched (and thus have paid search costs) enjoy lower utility than the types $[n_w, n_z\rangle$ who remain unemployed without having searched. The same holds true for the most productive types $\langle n_z, n_1]$ who have the bad luck that they have searched for a job without finding one.

[14] Lemma 20 in the appendix shows that, in the absence of search costs (i.e. $\gamma = 0$), all agents should search (i.e. $n_w = n_0$). Accordingly, the tax system rather than unemployment benefits insure people against the risk of being born with low ability.

subsidies are required to induce agents to reveal their type. Encouraging more agents to search thus becomes less attractive. A concave utility function, in contrast, implies that unequal consumption levels $\bar{x}$ and b become relatively costly. Hence, the government finds it relatively attractive to fight poverty by taking people out of unemployment by paying them a relatively high in-work tax benefit $\bar{x} = -T(n) > b$.

At the marginal worker n_z, the in-work benefit corresponds to the unemployment benefit so that the search subsidy goes to zero (i.e. $\lim_{n \downarrow n_z} [T(n) + b] = 0$). Search thus does not impose any first-order welfare effects as the government pays this worker the same resources in unemployment and work: search is neither taxed nor subsidized. Accordingly, the search margin is not distorted: the government can afford to eliminate the search distortion for the lowest type that provides positive work effort $z(n) > 0$ through generous in-work benefits because it does not have to worry about such generous in-work benefits for less efficient workers distorting the intensive margin of more efficient workers.

For the non-marginal workers $n > n_z$, the government extracts positive rents by fully taxing away the rents on search through a net tax on search (i.e. $T(n) + b > 0$). The government would like to extract more rents from these types since the marginal value of production exceeds the direct marginal resource (i.e. labor) costs for these types, implying a negative distributional wedge (i.e. $\lambda_E > 1/n = v'(x(n))$). The government cannot do this, however, because the search constraint is binding for these types (i.e. $\eta(n) > 0$): taxing search more heavily by extracting more rents from high-ability workers would discourage these high-ability types from looking for a job.

4.1.1 Comparative statics with respect to unemployment benefits

This sub-section explores how the optimal tax system responds to an exogenous change in the unemployment benefit b. This analysis thus assumes that b is exogenously given rather than optimally set.

Lemma 7 $\frac{dn_z}{db} > 0, \frac{dn_w}{db} > 0, \frac{dT(n)}{db} < -1$ *for* $n > n_z$, $\frac{dT(n_z)}{db} = -1$, $\frac{d\lambda_E}{db} < 0$, $-\frac{dT(n)}{db} = \frac{d\bar{x}}{db} > 0$ *for* $n_w < n < n_z$.

Higher unemployment compensation reduces the number of agents who work positive hours (i.e.$\frac{dn_z}{db} > 0$) . Intuitively, a higher unemployment benefit raises the productivity requirements for workers who search without a search subsidy: with a better alternative option (namely not searching and collecting a higher unemployment benefit), less workers are inclined to search for a job in the absence of search subsidies $-T(n) + b > 0$. The lowest type that searches for a job without a work subsidy thus becomes more efficient. Since the marginal worker is more productive, marginal resource costs decline (i.e. $\frac{d\lambda_E}{db} < 0$). These lower resource costs raise in-work benefits of those who collect search subsidies (i.e. $\frac{d\bar{x}}{db} = \frac{dx(n_z)}{db} > 0$).

The distributional implications of a higher unemployment benefit are as follows. Higher unemployment compensation benefits all people who do not change their employment position. In particular, the consumption levels (and thus welfare) increase not only for the unemployed but also for all types $n < n_z$ who keep collecting in-work benefits: not only unemployment compensation but also in-work benefits are increased (i.e. $\frac{d\bar{x}}{db} > 0$). Moreover, also the more efficient workers who exert positive work effort improve their position, albeit not by increasing their consumption level but by decreasing their work effort. The reason is that the government reduces the average tax rate $T(n) = n(v(x(n) - v(b) - \gamma) - x(n)$ on these types in order to continue to encourage them to search for a job now that they have a better alternative option of not searching and staying unemployed (i.e. $dT(n)/db = -nv'(b) < -1$ as $v'(b) > 1/n_z > 1/n$). The binding participation constraint thus implies that higher unemployment compensation is shifted to the high-ability workers. Hence, the government can extract less rents from the most efficient types. Indeed, although the marginal worker becomes more productive, the government provides more in-work benefits to the marginal worker (i.e. $dT(n_z)/db = -1$). The most efficient types respond to the higher average tax burden by exerting less work effort. Their consumption, in contrast, remains the same as income effects do not impact consumption.

The cost of higher unemployment benefits is a higher unemployment rate. Accordingly, a larger part of the population has to rely on relatively low unemployment benefits rather than more generous in-work benefits. In particular, the government budget constraint implies that the higher unemployment benefits for the unemployed and the higher in-work benefits for (and lower tax revenues from) the employed reduce the budgetary room to pay in-work benefits $\bar{x}$ exceeding the unemployment benefit b to those who do not search without a search subsidy. The government thus can afford to pay these generous in-work benefits to less agents, thereby reducing the group of fortunate types who benefit from search subsidies. At the same time, more people become dependent on unemployment benefits (i.e. $dn_w/db > 0$). Intuitively, higher unemployment compensation reduces the redistribution that can be carried out through the tax system by tightening the participation constraint that constrains the taxation of the more able. Accordingly, social insurance plays a more prominent role in protecting people against the risk of being born with little skills.

We summarize the distributional implications as follows. Only the types who no longer obtain the in-work benefit (and thus stop searching for a job) are worse off. A coalition of the most able and the least able who do not collect in-work benefits thus push for a more important role for unemployment benefits. Those who risk losing their generous in-work benefits, in contrast, resist such a policy by supporting tax benefits rather than unemployment benefits as an instrument to alleviate poverty.

4.2 Rich economy

In a rich economy, the government can afford to pay relatively generous search subsidies to all agents who do not search for a job without such a subsidy. Rather than unemployment benefits, therefore, in-work benefits insure agents against the risk of being born with low ability.[15] Jobs (with in-work benefits) rather than unemployment benefits are thus the preferred route out of poverty due to low productivity. The unemployment system insures people only against the risk of remaining unemployed after having actively searched for a job. This risk of involuntary unemployment (i.e. $1-\bar{s}$) is the same for all skills. Indeed, unemployment compensation is paid only to the population share $1-\bar{s}$ that is involuntarily unemployed, i.e. those unfortunate agents who looked for a job (and thus paid search costs) but nevertheless did not find one.

In the rich economy, the government budget enjoys a surplus if the government extracts the participation rents from all types that search without a net subsidy $n > \hat{n}$ $(z(n) = n(v(x(n)) - \gamma - v(b)))$ while at the same time providing not only unemployment benefits to the part of the population $1-\bar{s}$ that has not found work but also tax subsidies that allow all other types who found a job to enjoy the same consumption level as the marginal worker $\hat{n}$ without having to exert any work effort (i.e. $-T(n) = x(\hat{n}) > b$ for $n < \hat{n}$). What is the optimal strategy for spending this surplus at a given level of the unemployment compensation b? In view of the concavity of $v(.)$, the marginal utility of consumption is highest for the marginal worker $\hat{n}$ (and for all the types $n < \hat{n}$ who benefit from net search subsidies $-(T(n) + b) > 0)$). The government thus would like to employ the surplus to raise the consumption level of this worker. However, providing the marginal worker $\hat{n}$ with more consumption reduces the marginal utility of consumption $v'(x(\hat{n}))$ below the marginal utility costs of providing labor by this type, $1/\hat{n}$. Accordingly, the marginal worker $\hat{n}$ is better off if the government reduces his work time $z(\hat{n})$ to zero instead of giving him more consumption. The government therefore utilizes the surplus to increase not only the consumption level $\bar{x}$ for all types that exert no work effort (by raising in-work benefits and thus the search subsidy $-(T(n) + b)$) but also the number of fortunate types that do not have to exert effort in their jobs. Hence, the surplus is spent by both raising the level of the search subsidy and the number of people who are eligible for it. The government thus pays search subsidies also to people who would search for a job even without search subsidies. As a direct consequence, the marginal worker is taxed on search (i.e. $\lim_{n \downarrow n_z} [T(n) + b] > 0$ as shown in corollary 6). Intuitively, the government can afford to distort the search margin for the marginal worker n_z because all agents search (i.e. n_w has a corner solution).

The government continues to spend its surplus in this way until the government budget constraint holds with equality. When spending the surplus, the government ensures that the marginal utility of consumption equals the marginal labor costs for the lowest type that exerts effort (i.e, $v'(\bar{x}) = \frac{1}{n_z}$ where n_z is the marginal worker). How much of the surplus goes to additional consumption and how much goes to less work effort depends on the concavity of

[15] In a rich economy, those born with low ability are better off (in terms of utility) than the most efficient types but enjoy lower consumption levels. In a normal economy, in contrast, insurance against low ability is less good since some agents with low ability do not benefit from generous in-work benefits and have to rely on relatively low unemployment benefits. This still makes them as well off (in terms of utility), however, as the most efficient types.

$v(.)$ and the density function $f(.)$. The faster marginal utility of consumption declines with consumption (as reflected in the concavity of $v(.)$) and the thicker is the density $f(.)$ just above the marginal worker $\hat{n}$, the larger is the share of the surplus that is spent on reducing work effort by marginal workers rather than on raising search subsidies for those who exert no work effort.[16]

4.2.1 Comparative statics with respect to unemployment benefits

An exogenous increase in unemployment compensation yields different employment impacts in the rich economy as compared to the normal economy.

Lemma 8 $\frac{dn_z}{db} < 0$, $\frac{dT(n)}{db} < -1$ *for* $n > n_z$, $\frac{d\lambda_E}{db} > 0$ *and* $-\frac{dT(n)}{db} = \frac{d\bar{x}}{db} < 0$ *for* $n \in [n_0, n_z\rangle$.

Whereas in a normal economy a higher unemployment benefit decreases the number of workers who exert work effort (i.e. $\frac{dn_z}{db} > 0$), in a rich economy such a higher benefit increases this number ($\frac{dn_z}{db} < 0$). The reason for this is as follows. Higher unemployment benefits impose two additional burdens on the government budget. First, they raise the expenditures on unemployment compensation for the share of the population $(1 - \bar{s})$ that is involuntarily unemployed. Second, they reduce the tax revenues that can be extracted from the most efficient types as these types now find it more attractive to rely on the higher unemployment benefits rather than searching for a job. These two additional budgetary burdens reduce the resources that can be spend on search subsidies. Hence, higher unemployment compensation reduces both the level of these search subsidies (thereby reducing the consumption of the least efficient agents) and the number of people who are eligible for these search subsidies (thereby increasing work effort for marginal workers). At the same time, in response to their lower average tax burden, the more efficient types who already exerted work effort reduce that work effort. Work effort is thus spread out more equally over various workers as the more generous unemployment benefits, by tightening the participation constraint of the most efficient workers, allows for less redistribution through the tax system. Indeed, a higher unemployment benefit pushes the rich economy away from the economy without a search margin (discussed in sub-section 3.1) in which only the most efficient worker provides positive work effort.

The distributional effects of higher unemployment compensation are now as follows. Higher unemployment compensation benefits not only the unemployed (a proportion $1-\bar{s}$ of the population) by raising their benefit level and thus their consumption but also the efficient types by reducing their tax burden and thus

[16] Lemma 14 shows formally that $v'(\bar{x}) = \frac{1}{n_z}$ is optimal. In particular, if too much of the surplus is spend on consumption (i.e. $v'(\bar{x}) < \frac{1}{n_z}$), marginal labor costs exceed marginal benefits from consumption for type n_z. Hence, the government can raise social welfare by reducing both $\bar{x}$ and $z(n_z)$. If too much of the surplus is spend on leisure (i.e. $v'(\bar{x}) > \frac{1}{n_z}$), in contrast, some types $n < n_z$ could raise their utility by working more $z(n) > 0$ to raise their own consumption because the own labor costs are lower than the benefits from consumption.

their work effort. All other types (who all have searched for a job and found one) lose as the marginal worker becomes less productive so that the opportunity costs of consumption increases. Also in the rich economy, therefore, a higher unemployment benefit makes the tax system a less effective redistributional device as the inequality of consumption increases between the most efficient types who exert positive work effort (and whose consumption is not affected by the higher unemployment compensation) and the types who work zero hours and rely on in-work benefits to finance their consumption. A coalition of the most able and the unemployed therefore pushes for higher unemployment benefits while people who risk losing generous in-work benefits resist such a policy. In a normal economy, these latter agents are hurt by becoming unemployed while in a rich economy they are harmed by receiving less generous in-work benefits.

Combining the comparative static results on higher unemployment compensation for the rich and the normal economies, we find that the population that exerts work effort is largest at the point where a rich economy turns into a normal economy. Starting from a rich economy, a higher unemployment benefit first reduces the population that can be allowed to enjoy leisure (by paying them generous in-work benefits). Eventually, a higher unemployment benefit exhausts the surplus that can be spend on providing search subsidies to agents who would search without being paid such a search subsidy. At that point, the rich economy turns into a normal economy. An even higher unemployment benefit raises the productivity requirements for the worker that is indifferent about searching for a job in the absence of search subsidy. This causes the number of workers who exert positive work effort to decline. Accordingly, the relationship between the unemployment benefit and the number of people who do exert work effort is U shaped.

Also the relationship between the in-work benefits and unemployment benefits is U shaped. At low unemployment compensation, unemployment benefits and in-work benefits are substitutes in fighting poverty because higher unemployment benefits absorb the budgetary room for generous in-work benefits as an instrument to fight poverty. At high unemployment benefits, however, the participation constraint for marginal workers is binding and higher unemployment benefits induce government to raise in-work benefits on marginal workers and reduce taxes on more efficient workers in order to induce these agents to continue to look for work. In-work benefits thus help to offset the impact of higher unemployment benefits on the participation constraint so that in-work benefits and unemployment compensation are complements. The relationship between in-work benefits and unemployment compensation explains why generous in-work benefits are called for in countries that grant both low and high unemployment benefits. In countries with low unemployment compensation (such as the United States), in-work benefits are used to alleviate poverty. In countries with high unemployment compensation (such as most Western European countries), in contrast, in-work benefits protect the incentives to participate in the labor market.

5 Optimal unemployment benefits[17]

This section establishes that, even if unemployment benefits are optimally set[18], a search subsidy ($T(n) + b < 0$) can be optimal if the participation constraints for high types are binding. These participation constraints are binding because the government can observe the ability of working agents. Relaxing this latter assumption implies that high-ability types enjoy a positive surplus so that their participation constraints are no longer binding. In these circumstances, the incentive compatibility constraint binds for higher types and a search subsidy ($T(n) + b < 0$) cannot be optimal.

We determine the optimal level of unemployment compensation by taking the first order condition of (4) with respect to b. This yields (using (5) to eliminate $\eta(n)$)

$$(v'(b) - \lambda_E)\left[F(n_w) + (1-\bar{s})(1-F(n_w))\right] - \bar{s}v'(b)\int_{n_z}^{n_1} f(n)\left[\frac{\lambda_E}{v'(x(n))} - 1\right]dn = 0 \tag{10}$$

Employing the results from proposition 4 (i.e. $v'(x(n)) = 1/n$ for $n > n_z$ and $\lambda_E = 1/n_z$), we can write (10) as

$$v'(b) = \lambda_E + \frac{v'(b)\bar{s}\int_{n_z}^{n_1} f(n)\left[\frac{n}{n_z} - 1\right]dn}{\left[F(n_w) + (1-\bar{s})(1-F(n_w))\right]} \tag{11}$$

The left-hand side of this expression stands for the direct marginal benefits of more generous unemployment compensation in terms of a higher utility level for the unemployed. The right-hand side represents the marginal social costs, consisting of the direct marginal resource (i.e. labor) costs λ_E and the indirect adverse impact on the participation margin (and therefore the search incentives) of the more efficient types who exert positive work effort. These latter distortionary effects of higher unemployment compensation on search incentives give rise to a positive distributional wedge for the unemployed (i.e. $v'(b) - \lambda_E > 0$). In this way, the participation margin produces a trade off between equity (increasing the consumption level of the unemployed) and efficiency (stimulating the most efficient types to search). Indeed, rewriting expression (10) as

$$(1 - \frac{\lambda_E}{v'(b)})\left[F(n_w) + (1-\bar{s})(1-F(n_w))\right] = \bar{s}\int_{n_z}^{n_1} f(n)\left[\frac{\lambda_E}{v'(x(n))} - 1\right]dn, \tag{12}$$

we observe that positive distributional wedges for the unemployed (i.e. the left-hand side of (12)) contrast with negative[19] distributional wedges for the workers

[17] This section assumes $g < g^*(\gamma)$ so that government spending can be financed with $b = 0$.

[18] If unemployment benefits are not optimally set, it is obvious that $T(n) + b < 0$ can be optimal also if workers' types cannot be observed. To see this, consider the case where $b = 0$. Then we are left with a standard Mirrlees (1971) problem in which the government redistributes from high ability types to low ability types. Thus $T(n) < 0$ for low ability types and hence $T(n) + b = T(n) + 0 < 0$ for low ability types.

[19] This follows from $\lambda_E/v'(x(n)) = n/n_z > 1$ for $n > n_z$ (see proposition 4).

who work positive hours. The government thus wants to redistribute resources from the latter to the former group but cannot do so because of the binding participation constraint of the latter group.

The major distributional problem differs between the case in which the worker's type is verifiable and the case in which search is verifiable (see Boone and Bovenberg (2001)). If search rather than the worker's type is verifiable, the intensive rather than the extensive margin is relevant. In that case, the distributional wedge is positive (i.e. $v'(x(n)) > \lambda_E$) for the least efficient workers as the government would like to redistribute resources to the less efficient workers but cannot do so because doing so would induce the more efficient workers to mimic the less efficient workers by reducing their work effort. Indeed, in the presence of the intensive margin, the redistribution between high skill and low skill workers who have found a job and exert positive effort is problematic. Without the extensive margin, the government can redistribute from workers towards the unemployed without harming search incentives. In the presence of the extensive margin, in contrast, the distribution between those who supply positive work effort and the unemployed is problematic.

Whereas unemployment benefits are a relatively efficient redistributive instrument in the absence of the extensive margin, in-work tax benefits are relatively efficient in the absence of the intensive margin. Indeed, in the absence of the extensive margin, the marginal utility of consumption of the unemployed equals the direct resource costs so that distributional wedge for the unemployed is zero (i.e. $v'(b) = \lambda_E$, see Boone and Bovenberg (2001)). In the absence of the intensive margin, in contrast, the distributional wedge for the least efficient workers who rely on in-work benefits is zero (i.e. $v'(-T(n)) = v'(\bar{x}) = \lambda_E$ for $n_w < n < n_z$). The reason is that the more efficient types cannot mimic (by providing less work effort) the less efficient working types who benefit from generous in-work benefits (i.e. $-T(n) = \bar{x}$). These in-work benefits therefore do not directly distort the incentives of the more efficient workers who supply positive work effort. Whereas the more efficient workers cannot mimic the less efficient workers, they can still mimic the unemployed by not searching for a job. In contrast to in-work benefits, unemployment compensation therefore distorts the decisions of the most efficient workers. This gives rise to a trade off between equity and efficiency in that the unemployed feature a positive distributional wedge.

The relative efficiency of in-work benefits compared to unemployment benefits explains why a search subsidy (i.e. $T(n) + b < 0$) is optimal for some types even though the unemployment benefits are optimally set. The general insight is that a search subsidy is optimal only if the tax system is a more effective distributional instrument than the social insurance system. This can be the case because the participation constraint is binding for high types. The following simple example shows that under these conditions the government may not want to use unemployment compensation at all if search is effective (i.e. if $\bar{s}$ large enough).

Example 9 *Consider an economy with only two types of agents:* $n_0 = 0$ *and*

$n_1 > 0$ with proportions f_0 and f_1 ($f_0 + f_1 = 1$). Assume that the search cost γ satisfies the following inequalities

$$v(x) - \frac{x}{n_1 f_1} < \gamma < v(x)$$

where x is determined by $v'(x) = \frac{1}{n_1}$. In this economy it is clear that $z_0 = y_0 = 0$ because production by type n_0 does not yield any output. Clearly, the government would like to redistribute from the n_1 type to the n_0 type. The question is: does this take the form of unemployment benefits b or in work benefits x_0. To determine this, the social planner solves

$$\begin{aligned}
&\max_{\substack{x_0, x_1, z_1, \\ 0 \leq s_0, s_1 \leq \bar{s} \\ b \geq 0}} \left\{ \begin{array}{c} f_0 \left(-\gamma s_0 + s_0 v(x_0) + (1 - s_0) v(b)\right) + \\ f_1 \left(-\gamma s_1 + s_1 \left[v(x_1) - \frac{z_1}{n_1}\right] + (1 - s_1) v(b)\right) \end{array} \right\} + \\
&+\lambda_E \left\{-f_0 s_0 x_0 + f_1 s_1 (z_1 - x_1) - \left[f_0 (1 - s_0) + f_1 (1 - s_1)\right] b\right\} \\
&+\eta_0 \left\{v(x_0) - \gamma - v(b)\right\} \\
&+\eta_1 \left\{v(x_1) - \frac{z_1}{n_1} - \gamma - v(b)\right\}
\end{aligned}$$

We derive conditions here under which the optimal unemployment benefit b equals zero, while the in work benefit x_0 is positive. For this we only need the first order conditions for x_1 and z_1:

$$\begin{aligned}
f_1 s_1 v'(x_1) - \lambda_E f_1 s_1 + \eta_1 v'(x_1) &= 0 \\
-f_1 s_1 \frac{1}{n_1} + \lambda_E f_1 s_1 - \frac{\eta_1}{n_1} &= 0
\end{aligned}$$

From these two equations we derive that

$$\begin{aligned}
\lambda_E &= \frac{1}{n_1} + \frac{\eta_1}{n_1 f_1 s_1} \\
v'(x_1) &= \frac{1}{n_1}
\end{aligned}$$

Since we want to redistribute as much consumption from type n_1 to type n_0 as possible, type n_1 has to work as much hours as possible.[20] *The maximal amount is determined by the search constraint $v(x_1) - \frac{z_1}{n_1} \geq v(b) + \gamma$. Hence we find*

$$z_1 = n_1 \left(v(x_1) - v(b) - \gamma\right)$$

Now if b is raised, the social gains equal

$$\left[f_0 (1 - s_0) + f_1 (1 - s_1)\right] v'(b)$$

[20] The planner, however, should not redistribute so much that $x_0 > x_1$. To avoid this, the following condition must hold (note that z_1 is maximal if $b = 0$): $x_0 = \frac{f_1 \bar{s} (z_1 - x_1)}{f_0 s_0} \leq \frac{f_1 (n_1 v(x_1) - n_1 \gamma - x_1)}{f_0} < x_1$. The last inequality is met due to the assumptions made on γ.

while the social costs equal the cost of additional expenses on b plus the loss in tax revenue (which equals $f_1 s_1 n_1 v'(b)$ because the expression for z_1 above implies that $\frac{dz_1}{db} = -n_1 v'(b)$)

$$\lambda_E \{f_0 (1 - s_0) + f_1 (1 - s_1) + f_1 s_1 n_1 v'(b)\}$$

If at $b = 0$ the costs exceed the gains, the optimal unemployment benefit level equals 0:

$$\lambda_E f_1 s_1 n_1 v'(0) + [f_0 (1 - s_0) + f_1 (1 - s_1)] (\lambda_E - v'(0)) > 0$$

or equivalently (using $v'(0) = +\infty$ and $\lambda_E \geq \frac{1}{n_1}$ because $\eta_1 \geq 0$)

$$\bar{s} > \frac{1}{(1 + f_1)}$$

The interpretation of this condition is that search should be relatively effective.

We can approximate the magnitude of the optimal search subsidy $-T(n) - b = \bar{x} - b$ by using $v'(\bar{x}) - v'(b) = v''(\xi)(\bar{x} - b)$ (with $b < \xi < \bar{x}$) and substituting $v'(\bar{x}) = \lambda_E$ into (11) to arrive at

$$\frac{\bar{x} - b}{\xi} = \frac{v'(b)}{-\xi v''(\xi)} \frac{\bar{s} \int_{n_z}^{n_1} f(n) \left[\frac{n}{n_z} - 1\right] dn}{[F(n_w) + (1 - \bar{s})(1 - F(n_w))]}. \tag{13}$$

The inequality in consumption levels between the fortunate types who benefit from in-work benefits and the unfortunate ones who have to rely on lower unemployment compensation increases in the distortionary costs of unemployment benefits on the search margin of the more efficient types (as reflected in the term $\bar{s} \int_{n_z}^{n_1} f(n) \left[\frac{n}{n_z} - 1\right] dn$). The inequality in consumption levels declines, however, in the aversion against inequality in consumption (as reflected in the inverse of the coefficient of relative risk aversion $\frac{-v'(b)}{\xi v''(\xi)}$) and in the number of people who rely on unemployment benefits for their consumption (as reflected in the unemployment rate $F(n_w) + (1 - \bar{s})(1 - F(n_w))$).

The following proposition shows that the result above that in-work benefits can exceed b crucially depends on the assumption that the government can observe workers' ability.

Proposition 10 *If the government cannot observe workers' ability and b is optimally set, then $n_w > n_0$ implies that*

$$T(n) + b \geq 0$$

for each $n \geq n_w$.

If the government cannot observe workers' ability, any feasible tax system has the property that in-work utility rises with ability (i.e. $u'(n) > 0$). For all non-marginal workers $n > n_w$, $u(n_w) \geq \gamma + v(b)$ together with $u'(n) > 0$ implies that $u(n) > \gamma + v(b)$ for $n > n_w$ so that the participation constraint is binding only for the marginal worker n_w. For the non-marginal workers, the incentive compatibility constraint rather than the participation constraint is binding. Higher unemployment compensation therefore damages the search incentives of only the marginal worker n_w and do not directly harm the tax revenues that can be extracted from more efficient types. In-work benefits for poor workers (which raise $u(n)$ for low types), in contrast, harm the incentives facing more efficient types, thereby forcing the government to reduce the tax level on more efficient types so as to prevent high-ability workers from mimicing low-ability types. Indeed, compared to the case with observable abilities considered in the rest of this paper, in-work benefits become a less efficient instrument to fight poverty while unemployment benefits become more efficient. This provides some intuition for why in-work benefits $-T(n)$ can exceed unemployment benefits only if ability is verifiable.

The reason for why work should be taxed (i.e. $T(n)+b \geq 0$) can alternatively be stated as follows. The participation constraint $u(n) \geq \gamma + v(b)$ implies that consumption of workers exceed the consumption of the unemployed. Hence, unemployment compensation is more effective way to fight poverty than in-work benefits are because unemployment benefits accrue to the poorest agents. Since the participation constraint is binding only for the marginal worker n_w, the only efficiency cost of higher unemployment compensation is that it harms the incentives of marginal workers to search for a job. In the optimum, therefore, the government balances the marginal distributional benefits of unemployment compensation with its marginal efficiency costs. Accordingly, the marginal external effect on the government budget constraint of the marginal worker n_w looking for a job should be positive, i.e. $T(n_w) + b \geq 0$. Non-marginal workers are taxed even heavier (i.e. $T(n) > T(n_w)$ for $n > n_w$) in view of the redistributive preferences of the government so that the tax on search, $T(n) + b$, is positive for all workers.

5.1 Rich versus normal economies

This subsection explores the conditions for the optimal allocation being either a rich or a normal economy. Lemma 21 in the Appendix shows that with zero search costs ($\gamma = 0$) the optimal tax system implies a rich economy in which the unemployed (a proportion $1 - \bar{s}$ of the population) are involuntarily unemployed in the sense that they searched to full capacity for a job but could not find one. By having all types search for a job, the government relies maximally on in-work benefits to relieve poverty. Intuitively, the advantage of in-work benefits over unemployment compensation as a redistributive device is that these benefits can better target the less efficient types by using more information. This information, however, is not costless: agents can reveal this information only after having searched for jobs, thereby paying search costs $\gamma > 0$. Without these search costs, however, in-work benefits only have advantages so that the government maximally relies on in-work benefits. By searching and finding a job, people costlessly reveal their ability.

The comparative static results on raising unemployment compensation in a rich economy showed that raising unemployment benefit eventually turns a rich economy into a normal economy as the government no longer has enough funds to pay generous search subsidies to all types that would not search without a subsidy. Hence, at given positive search costs $\gamma > 0$, a rich economy is more likely to be optimal if the optimal unemployment benefit b is relatively low. In that case, the government can extract substantial rents from the most efficient types because these types find it relatively unattractive to mimic the unemployed. Expression (13) indicates that the optimal unemployment benefits are not generous if search is effective (as indicated by a high level of $\bar{s}$) and the agents are not particularly risk averse (as indicated by a low coefficient of relative risk aversion $\frac{-v'(b)}{\xi v''(\xi)}$). Intuitively, if not many people are involuntarily unemployed and the government does not care much about inequality in consumption, the government sets unemployment compensation b at a relatively low level.[21]

We thus conclude that a rich economy, which minimizes the dependency on the social insurance system and employs jobs (and in-work benefits) rather than benefit dependence as a anti-poverty device, is more likely to be observed if search is cheap (i.e. γ is low) and effective (i.e. $\bar{s}$ is large), agents are productive (so that $\hat{n}$ is close to n_0 and the government does not need to pay many agents a search subsidy to induce them to search for a job) and agents are not particularly risk averse so that they do not mind poor insurance against unemployment risk. In these circumstances, unemployment risk is low and people do not mind this risk.

The unemployment system continues to play an important role in protecting agents against lack of skills if search is expensive and ineffective and agents are not productive (so that using the tax system is expensive), especially if agents are risk averse. In that case, high unemployment benefits reduce the budgetary room for paying in-work benefits to a large population that is not productive enough to search for a job without these generous in-work benefits. In particular, people must be paid high in-work benefits in order to overcome the search costs for revealing their type. Moreover, in-work benefits cannot reach the involuntary unemployed (i.e. those who search but still remain unemployed). Intuitively, jobs (and the associated in-work benefits) are expensive and difficult to get. Work is an expensive and ineffective way out of poverty because in-work benefits are expensive (as search costs are high) and fail to reach the involuntary unemployed.

6 Conclusions

This paper has explored the interaction between the tax system and unemployment insurance in insuring people against the risks of involuntary unemployment

[21] In the limit with $\bar{s} = 1$ and $\gamma = 0$, we are back in the model without a search margin. In that case $b = -\infty$ and only type n_1 works.

and low ability. Our results suggest that the optimal search subsidy (i.e. the difference between the in-work benefit and the unemployment benefit) falls with the importance of the incentive compatibility constraint (work effort) relative to that of the individual rationality constraint (search effort). The more elastic search effort and the less well the government can enforce the work test by observing search effort, the more in-work benefits should be employed to fight poverty of low-ability agents and protect the search incentives of workers. This holds true especially if ability can be observed or work effort is relatively inelastic so that the incentive compatibility constraint plays no or only a minor role in constraining redistribution within the group of workers.

By assuming that the government can observe a worker's type, we have stacked the cards in favor of in-work tax benefits as an instrument to fight poverty. We showed, however, that even with this strong informational assumption, the redistributive power of the tax system is constrained if the government cannot verify job search. In particular, the financing of in-work benefits can be problematic if search costs are high and agents are not productive. This is especially so if high unemployment benefits constrain the ability of the government to extract taxes from the more efficient workers. In this way, unemployment compensation limits the ability of the tax system to finance in-work benefits.

If the government can optimally set both the tax system and unemployment compensation, the government faces a trade off in deciding on the relative importance of in-work benefits and unemployment compensation. Relying on in-work benefits allows effective targeting of benefits at workers with low ability without directly distorting the search incentives of more able workers. However, in-work benefits are expensive if search costs are high. Moreover, they do not reach individuals who are suffer from involuntary search unemployment. Clearly, unemployment insurance remains important in insuring agents against unemployment risk. The unemployment system continues to play an important role also in protecting agents against lack of skills if search is expensive and ineffective and agents are not productive. In that case, work is an expensive and ineffective way out of poverty. Interpreting the in-work benefits to agents who do not exert any work effort as disability benefits, the government trades off the benefits of better targeting of disability benefits against the costs of tagging. In particular, the government relies on disability rather than unemployment benefits to protect agents against the risk being born with low ability if tagging is cheap and effective (i.e. γ is low and $\bar{s}$ is high) and agents are productive so that only a few agents must be granted disability benefits.

References

Akerlof. G., 1978, 'The economics of 'tagging' as applied to the optimal income tax, welfare programs, and manpower planning,' American Economic Review, Vol. 68, pp. 8-19.

Boone, J, and A.L. Bovenberg, 2001, 'Unemployment vs in-work benefits: When is an EITC optimal?'

Ebert, U., 1992, 'A reexamination of the optimal nonlinear income tax,' Journal of Public Economics, Vol. 49, pp. 47-73.

Hosios, A.J., 1990, 'On the efficiency of matching and related models of search and unemployment', Review of Economic Studies, Vol. 57, pp. 279-298.

Kamien, M.I. and N.L. Schwartz, 1981, Dynamic optimization: The calculus of variations and optimal control in economics and management, North-Holland.

Kanbur, R., M. Keen, and M. Tuomala, 'Optimal Non-Linear Income Taxation for the Alleviation of Income-Poverty,' European Economic Review, Vol. 38, pp. 1613-1632.

Mirrlees, J.A., 1971, An Exploration in the Theory of Optimal Income Taxation,' Review of Economic Studies, Vol. 38, pp. 175-208.

Saez, E., 2000, Optimal Income Transfer Programs: Intensive Versus Extensive Labor Supply Responses, NBER Discussion Paper No. 7708.

Appendix: Proofs of results

Proof of lemma 3

Define $\underline{b}$ as the solution in b of the following equation

$$\int_{\hat{n}(b)}^{n_1} f(n)\,\bar{s}\left(n\left[v(x(n))-\gamma-v(b)\right]-x(n)\right)dn+F(\hat{n}(b))\,\bar{s}\left(-x(\hat{n}(b))\right)=b(1-\bar{s})+g$$

and if there is no solution $b \geq 0$ to this equation, define $\underline{b}=0$. The left-hand side of this equation is decreasing in b (where $\hat{n}$ is defined in equation (8) and we use $\frac{d\hat{n}(b)}{db}>0$ as derived in the proof of lemma 7 below) while the right-hand side is increasing in b. Hence, if there is a solution to the equation, it is unique. Note that if $\underline{b}>0$,then a value $b<\underline{b}$ implies that the economy is rich.

Define $\bar{b}$ as the solution in b of the following equation

$$\int_{\hat{n}(b)}^{n_1} f(n)\,\bar{s}\left(n\left[v(x(n))-\gamma-v(b)\right]-x(n)\right)dn=b\left(1-\bar{s}\left[1-F(\hat{n}(b))\right]\right)+g$$

This value of $\bar{b}$ is always well defined because (i) the left-hand side of this equation is decreasing in b (use $\frac{d\hat{n}(b)}{db}>0$ as derived in the proof of lemma 7 below), (ii) the right-hand side is increasing in b, (iii) at $b=0$ the left-hand side exceeds the right-hand side by the assumption that $g<g^*(\gamma)$ and (iv) the right-hand side exceeds the left hand side at a value of b satisfying $\hat{n}(b)=n_1$. Finally, note that for $b\in[\underline{b},\bar{b}]$ we have the normal economy and for $b>\bar{b}$ we have a poor economy. QED

Proof of proposition 4

The proposition is proved for the normal economy, using a number of lemmas derived below. Once the solution for the normal economy is proved, the extension to the rich economy is straightforward.

Lemma 11 *If $z(n)>0$ for some $n\in[n_0,n_1]$ then $z(n')>0$ for each $n'>n$.*

Proof. Suppose not, that is (from (6))

$$\left\{-\bar{s}f(n)\frac{1}{n}+\lambda_E f(n)\,\bar{s}-\eta(n)\frac{1}{n}\right\}=0$$

and there exists $n' > n$ such that

$$\left\{-\bar{s}f(n')\frac{1}{n'} + \lambda_E f(n')\bar{s} - \eta(n')\frac{1}{n'}\right\} < 0 \tag{14}$$

Using equation (5) these two equations can be rewritten as

$$\begin{aligned} 1 - \frac{1}{nv'(x(n))} &= 0 \\ 1 - \frac{1}{n'v'(x(n'))} &< 0 \end{aligned}$$

which implies that $x(n') > x(n)$ because $n' > n$. By assumption $z(n') = 0$, hence we get

$$v(x(n')) > v(x(n)) - \frac{z(n)}{n} \geq v(b) + \gamma$$

and thus $\eta(n') = 0$, while $\eta(n) \geq 0$. However, this implies

$$-\frac{1}{n'} + \lambda_E - \eta(n')\frac{1}{\bar{s}f(n')n'} = -\frac{1}{n'} + \lambda_E > -\frac{1}{n} + \lambda_E - \eta(n)\frac{1}{\bar{s}f(n)n} = 0$$

which contradicts inequality (14). ■

Lemma 12 *If $z(n) > 0$ for some n then it is the case that*

$$z(n') = n'(v(x(n')) - \gamma - v(b))$$

for each $n' > n$.

Proof. Suppose not, that is assume that for some $n' > n$ we have

$$z(n') < n'(v(x(n')) - \gamma - v(b))$$

Then there exists $\varepsilon > 0$ such that the alternative function $\tilde{z}(n)$ which equals $z(n)$ everywhere except for the points n and n' where $\tilde{z}$ is determined by

$$\begin{aligned} \tilde{z}(n') &= z(n') + \varepsilon \\ \tilde{z}(n) &= z(n) - \frac{f(n')}{f(n)}\varepsilon \end{aligned}$$

For ε small enough this function $\tilde{z}$ satisfies all the constraints and $\tilde{z}$ is budget neutral by construction. It is routine to verify that $\tilde{z}$ yields higher welfare than z. ■

Now define $\tilde{n}_z$ as follows

$$\tilde{n}_z \equiv \inf\{n \geq n_0 | z(n) > 0\}$$

Then we find from the results above, (5), and (6) that for each $n > \tilde{n}_z$, it is the case that

$$\begin{aligned} \eta(n) &> 0 \\ z(n) &= n(v(x(n)) - \gamma - v(b)) \\ v'(x(n)) &= \frac{1}{n} \end{aligned}$$

Now turn to the types below $\tilde{n}_z$. Note that the following result does not assume that the set $[n_w, \tilde{n}_z\rangle$ is non-empty. We will prove below that the set is in fact non-empty.

Lemma 13 *For each $n \in [n_w, \tilde{n}_z\rangle$ we have that $z(n) = 0$ and $x(n) = \tilde{x}$ for some $\tilde{x} > 0$.*

Proof. $z(n) = 0$ follows from the definition of $\tilde{n}_z$. The result that all these agents get the same consumption level follows from the concavity of $v(.)$. ■

Lemma 14 $v'(\tilde{x}) = \frac{1}{\tilde{n}_z}$.

Proof. Suppose not (in two parts):

(i) suppose (by contradiction) that $v'(\tilde{x}) < \frac{1}{\tilde{n}_z}$. This implies that $\tilde{x} > x(\tilde{n}_z)$ and hence

$$v(\tilde{x}) > v(x(\tilde{n}_z)) - \frac{z(\tilde{n}_z)}{\tilde{n}_z} \geq \gamma + v(b)$$

Then we can construct new functions $\tilde{x}(.)$ and $\tilde{z}(.)$ which are identical to $x(.)$ and $z(.)$ with the following exceptions

$$\begin{aligned} \tilde{x}(n) &= \tilde{x} - \varepsilon \\ \tilde{z}(\tilde{n}_z) &= z(\tilde{n}_z) - \varepsilon \frac{f(n)}{f(\tilde{n}_z)} \end{aligned}$$

for some type $n \in [n_w, \tilde{n}_z\rangle$ and $\varepsilon > 0$ small enough. These functions $\tilde{z}$ and $\tilde{x}$ are budget neutral and do not violate any constraint (as long as ε small enough) and raise aggregate welfare.

(ii) suppose (by contradiction) that $v'(\tilde{x}) > \frac{1}{\tilde{n}_z}$. Then there exists a type $n < \tilde{n}_z$ such that for some $\varepsilon > 0$ we have

$$v(\tilde{x} + \varepsilon) - \frac{\varepsilon}{n} > v(\tilde{x})$$

That is the government can raise the utility of this type n without making anyone worse off. ■

Lemma 15 *Assume that $n_w < \tilde{n}_z$ then $v'(\tilde{x}) = \lambda_E$.*

Proof. $\eta(n) \geq 0$ implies (from (5)) that $v'(\tilde{x}) \leq \lambda_E$. So the question is whether $v'(\tilde{x}) < \lambda_E$ is possible. Assume (by contradiction) that $v'(\tilde{x}) < \lambda_E$ is indeed possible. Then we have $\eta(n) > 0$ and thus (by complementary slackness and $z(n) = 0$) $v(\tilde{x}) = \gamma + v(b)$ so that $\tilde{x} > b$. Now consider equation (7) using $v(\tilde{x}) = \gamma + v(b)$:

$$0 - \lambda_E (z(\tilde{n}_w) - \tilde{x} + b) = 0$$

However, this implies that $z(\tilde{n}_w) = \tilde{x} - b > 0$ which contradicts $n_w < \tilde{n}_z$. ■

Corollary 16 $\eta(n) = 0$ *for all* $n \in [n_w, \tilde{n}_z\rangle$.

Lemma 17 $\lambda_E = \frac{1}{\tilde{n}_z}$.

Proof. Suppose not (in two parts)

(i) suppose (by contradiction) that $\lambda_E > \frac{1}{\tilde{n}_z}$. The equation for $z(\tilde{n}_z) > 0$ can be written as

$$-\frac{1}{\tilde{n}_z} + \lambda_E - \eta(\tilde{n}_z) \frac{1}{\bar{s} f(\tilde{n}_z) \tilde{n}_z} = 0$$

But then there exists $n < \tilde{n}_z$ such that $-\frac{1}{n} + \lambda_E - 0 > 0$ which contradicts $z(n) = 0$.

(ii) suppose (by contradiction) that $\lambda_E < \frac{1}{\tilde{n}_z}$. Then there exists $n' > \tilde{n}_z$ such that $\lambda_E n' < 1$ which contradicts $\eta(n') = f(n') \bar{s} (\lambda_E n' - 1) \geq 0$ for this type $n' > \tilde{n}_z$. ■

Lemma 18 $\tilde{n}_z = \hat{n}$ *where* $\hat{n}$ *is defined in equation (8).*

Proof. Substituting $z(n_w) = 0$, $x(n_w) = \tilde{x}$ (with $v'(\tilde{x}) = \frac{1}{\tilde{n}_z}$), and $\lambda_E = \frac{1}{\tilde{n}_z}$ into equation (7), we arrive at

$$v(b) + \gamma - v(\tilde{x}) - \frac{1}{\tilde{n}_z}(-\tilde{x} + b) = 0. \tag{15}$$

To analyze the solution to this equation, we write the function $\zeta_\gamma : \Re_+ \times \Re_+ \longrightarrow \Re$ as defined in equation (9) as

$$\begin{aligned} \zeta_\gamma(n, b) &= n(v(x(n)) - v(b) - \gamma) - x(n) + b \\ \text{where } x(n) \text{ is defined by } v'(x(n)) &= \frac{1}{n} \end{aligned}$$

Thus equation (15) boils down to

$$\zeta_\gamma(n, b) = 0$$

for exogenously given b.

As the next lemma shows this equation has at most two solutions.

Lemma 19 *For each $b \geq 0$, it is the case that*
(i) the equation $\zeta_0(n,b) = 0$ has a unique solution $\hat{n} \geq 0$ with $x(\hat{n}) = b$;
(ii) the equation $\zeta_\gamma(n,b) = 0$ with $\gamma > 0$ has exactly two solutions $0 < \hat{n}_1 < \hat{n}_2$ with

$$\begin{aligned} v(x(\hat{n}_1)) &< v(b) + \gamma \\ v(x(\hat{n}_2)) &> v(b) + \gamma \end{aligned}$$

Proof. First, note that ζ_γ is strictly convex in n. This follows from

$$\begin{aligned} \frac{\partial \zeta_\gamma(n,b)}{\partial n} &= v(x(n)) - v(b) - \gamma \\ \frac{\partial^2 \zeta_\gamma(n,b)}{\partial n^2} &= v'(x(n)) \frac{dx(n)}{dn} > 0 \end{aligned}$$

because $v''(x(n)) \frac{dx(n)}{dn} = \frac{-1}{n^2}$ implies that $\frac{dx(n)}{dn} > 0$.

Second, note that $\zeta_\gamma(0,b) = b > 0$ (here we use the assumption that $\lim_{x \downarrow 0} v'(x) = +\infty$), $\lim_{n \to +\infty} \zeta_\gamma(n,b) = +\infty$ and that

$$\min_{n \geq 0} \zeta_\gamma(n,b) = b - v^{-1}(v(b) + \gamma)$$

Now consider the cases $\gamma = 0$ and $\gamma > 0$ in turn. If $\gamma = 0$, we see that $\min_{n \geq 0} \zeta_0(n,b) = 0$. So by the strict convexity of ζ_0, the value $\hat{n}$ for which this minimum is reached is the unique solution to $\zeta_0(n,b) = 0$. Further, using the first order condition $\frac{\partial \zeta_0(n,b)}{\partial n} = 0$ for this minimum yields immediately that $v(x(\hat{n})) - v(b) = 0$ or equivalently $x(\hat{n}) = b$.

Next, consider the case where $\gamma > 0$. Then clearly $\min_{n \geq 0} \zeta_\gamma(n,b) < 0$, so (using $\zeta_\gamma(0,b) = b > 0$ and $\lim_{n \to +\infty} \zeta_\gamma(n,b) = +\infty$) we have two solutions to the equation $\zeta_\gamma(n,b) = 0$. At the smallest ($\hat{n}_1$) of these solutions, we have that ζ_γ is downward sloping (see figure 1):

$$\left. \frac{\partial \zeta_\gamma(n,b)}{\partial n} \right|_{n = \hat{n}_1} = v(x(\hat{n}_1)) - v(b) - \gamma < 0$$

and thus $v(x(\hat{n}_1)) < v(b) + \gamma$. Similarly, at $\hat{n}_2$ the function ζ_γ is upward sloping (again see figure 1) and we have $v(x(\hat{n}_2)) > v(b) + \gamma$. ■

Using this result, we continue the proof that $\tilde{n}_z = n_z = \hat{n}$. If $\gamma = 0$, equation (15) has only one solution and hence it is indeed the case that $\tilde{n}_z = \max\{n \geq 0 | \zeta_0(n,b) = 0\}$. If $\gamma > 0$, we need the solution to equation (15) which features $v(x(n)) - \gamma \geq v(b)$. That is, we need the solution $\hat{n}_2$ in the lemma above. So again we have $\tilde{n}_z = \max\{n \geq 0 | \zeta_\gamma(n,b) = 0\}$. ■

This finishes the proof of the proposition for the normal economy case.

Section 4.2 discusses how the solution for the normal economy is adapted for the rich economy case. QED.

Proof of Corollary 6

Because $n_z = \hat{n}$ in a normal economy, we know for $n > n_z$ that

$$v(x(n)) - v(b) - \gamma + \frac{1}{n}(-x(n) + b) > 0 \tag{16}$$

where $v'(x(n)) = \frac{1}{n}$. Combining this with $z(n) = n(v(x(n)) - \gamma - v(b))$ for $n > n_z$ we find

$$\begin{aligned} T(n) + b &= z(n) - x(n) + b \\ &= n\left\{v(x(n)) - v(b) - \gamma + \frac{1}{n}(-x(n) + b)\right\} > 0 \end{aligned}$$

because of equation (16). Similarly, because by definition of $\hat{n}$ we have

$$v(x(\hat{n})) - v(b) - \gamma + \frac{1}{\hat{n}}(-x(\hat{n}) + b) = 0$$

and $n_z = \hat{n}$, we have that $\lim_{n \downarrow n_z}[T(n) + b] = 0$. Finally, for types $n \in [n_w, n_z\rangle$ we have $x(n) = \bar{x} = x(n_z) > 0$ and $z(n) = 0$. Thus $v(\bar{x}) \geq v(x(n_z)) - \frac{z(n_z)}{n_z} \geq v(b) + \gamma$. Thus $\gamma > 0$ implies $\bar{x} > b$ or equivalently $-\bar{x} + b < 0$.

In the rich economy, $n_z > \hat{n}$ and $z(n) = n(v(x(n)) - v(b) - \gamma)$ imply $T(n) + b > 0$ for all $n \geq n_z$. The argument that $T(n) + b = -\bar{x} + b < 0$ is the same as in the normal economy.■

Proof of Lemma 7

In normal economy $n_z = \hat{n}$ and n_w is determined by government budget constraint. Hence $\frac{dn_z}{db} = \frac{d\hat{n}}{db}$ where $\hat{n}$ is defined in equation (8). To find the effect of b on $\hat{n}$ we differentiate the equation $\zeta_\gamma(n, b)$ with respect to n and b. This yields

$$(v(x(\hat{n})) - v(b) - \gamma)\frac{d\hat{n}}{db} - (\hat{n}v'(b) - 1) = 0$$

Therefore $\frac{d\hat{n}}{db} > 0$ because $v(x(\hat{n})) - v(b) - \gamma > 0$ and $v'(b) > \frac{1}{\hat{n}}$ (the last inequality follows from $b < x(\hat{n})$). This is illustrated in figure 1 where b is raised from b to $b' > b$.

The effect that $\frac{dn_w}{db} > 0$ follows from government budget constraint as follows.

$$\begin{aligned} &\left[-\underbrace{f(\hat{n})\bar{s}(\hat{n}(v(x(\hat{n})) - \gamma - v(b)))}_{>0} - \underbrace{\bar{s}(F(\hat{n}) - F(n_w))\frac{dx(\hat{n})}{d\hat{n}}}_{>0}\right]\underbrace{\frac{d\hat{n}}{db}}_{>0} + \\ &+\underbrace{f(n_w)\bar{s}(x(n_w) - b)}_{>0}\frac{dn_w}{db} \\ = \;&\underbrace{1 - \bar{s}[1 - F(n_w)] + v'(b)\bar{s}\int_{\hat{n}}^{n_1} nf(n)\,dn}_{>0} \end{aligned}$$

Hence we find that $\frac{dn_w}{db} > 0$.

Using the fact that $b < x(\hat{n})$ and thus that $v'(b) > v'(x(\hat{n})) = \frac{1}{\hat{n}}$ we find that

$$\frac{dT(n)}{db} = \frac{d(z(n) - x(n))}{db} = -nv'(b) < -\frac{n}{\hat{n}} < -1$$

for all $n > \hat{n}$.

By definition $T(n_z) + b = 0$ (because $n_z = \hat{n}$ is the least efficient type that can work without subsidy), therefore we have

$$\frac{dT(n_z)}{db} = -1$$

To find $\frac{d\lambda_E}{db}$, recall that $\lambda_E = \frac{1}{n_z}$ and that $\frac{dn_z}{db} > 0$ (as shown above). Hence we get

$$\frac{d\lambda_E}{db} = \frac{d\lambda_E}{dn_z}\frac{dn_z}{db} = -\frac{1}{n_z^2}\frac{dn_z}{db} < 0$$

Finally, using similar arguments, one can derive that

$$-\frac{dT(n)}{db} = \frac{dx(n_z)}{db} = \frac{dx(n_z)}{dn_z}\frac{dn_z}{db} > 0$$

since $\frac{dx(n_z)}{dn_z} > 0$ (from $v'(x(n_z)) = 1/n_z$).

Proof of Lemma 8

To find $\frac{dn_z}{db}$ differentiate the government budget constraint in the rich economy with respect to b and n_z. This can be written as

$$-f(n_z)\bar{s}n_z[v(x(n_z)) - \gamma - v(b)]\frac{dn_z}{db} = \left[1 - \bar{s}F(n_z) - \int_{n_z}^{n_1} f(n)\bar{s}(-nv'(b) + 1)\right]$$

Using the results that $[v(x(n_z)) - \gamma - v(b)] > 0$ and $-nv'(b) + 1 < 0$ (because $b < x(n)$ implies $v'(b) > v'(x(n)) = \frac{1}{n}$ for $n > n_z$) we find $\frac{dn_z}{db} < 0$.

The proof that $\frac{dT(n)}{db} < -1$ for $n > n_z$ is the same as the analogous proof for the normal economy in lemma 7 above.

Using the results that $\frac{d\lambda_E}{dn_z} = -\frac{1}{n_z^2} < 0$ and $\frac{dn_z}{db} < 0$ (derived above), we find

$$\frac{d\lambda_E}{db} = \frac{d\lambda_E}{dn_z}\frac{dn_z}{db} > 0$$

Using similar arguments, we find

$$-\frac{dT(n)}{db} = \frac{d\bar{x}}{db} = \frac{dx(n_z)}{dn_z}\frac{dn_z}{db} < 0$$

for $n \in [n_0, n_z\rangle$.

Proof of Proposition 10

Here we no longer assume that the government can observe workers' ability. Hence in our optimization problem we need to take into account that the proposed tax schedule is incentive compatible. In other words, given a certain tax schedule $\tilde{T}(z)$ as a function of gross income, workers choose their production to maximize in-work utility $u(x,z) = v\left(z - \tilde{T}(z)\right) - \frac{z}{n}$. Using, as above, $u(n)$ to denote type n's maximized utility level, we have

$$u(n) = \max_z \left\{ v\left(z - \tilde{T}(z)\right) - \frac{z}{n} \right\}$$

Using the envelope theorem, we find

$$u'(n) = \frac{z(n)}{n^2} \tag{17}$$

To derive this we have used the first order condition for z.[22] To facilitate the inclusion of equation (17) into our optimization problem, we use $u(n)$ as variable instead of $x(n)$ as we did in (4). Thus the optimization problem now becomes

$$\begin{aligned} &\max_{n_w, u(.), z(.)} F(n_w)\, v(b) + [1 - F(n_w)]\left(-\gamma\bar{s} + (1-\bar{s})\, v(b)\right) + \\ &+ \int_{n_w}^{n_1} \left\{ \bar{s}u(n)f(n) - \lambda_u(n)\left[u'(n) - \frac{z(n)}{n^2}\right] + \lambda_E\left[f(n)\bar{s}T(n)\right] \right\} dn \\ &-\lambda_E \left\{ b\left[F(n_w) + (1 - F(n_w))(1-\bar{s})\right] + g \right\} \\ &-\eta_w\left(\gamma - u(n_w) + v(b)\right) \end{aligned}$$

where $T(n) = z(n) - x(n) = z(n) - v^{-1}\left(u(n) + \frac{z(n)}{n}\right)$. Furthermore, since $z(n) \geq 0$ for each type, equation (17) implies that $u'(n) \geq 0$ so that there is only one type n_w for which the restriction $\gamma - u(n_w) + v(b) \leq 0$ is binding. In fact, if $n_w > n_0$ then it is the case that

$$u(n_w) = \gamma + v(b)$$

To see this, note first that $u(n_w) < \gamma + v(b)$ is not possible as type n_w will not exert effort s to find a job. Also $u(n_w) > \gamma + v(b)$ is not possible because that gives an incentive to a type $n < n_w$ (but close to n_w) to mimic type n_w and find a job. This violates incentive compatibility in terms of search. The first-order

[22] Ebert (1992) shows that one can derive an optimal tax system which is in fact not incentive compatible if one ignores the second order condition for $z(.)$. To avoid this, one should add the restriction that $z'(n) \geq 0$ but this is beyond the scope of this paper. The interested reader is referred to Boone and Bovenberg (2001).

conditions for $n_w, u(.)$ and $z(.)$ can be written as

$$\eta_w u'(n_w) = f(n_w)\left\{\bar{s}\left[\underbrace{-\gamma - v(b) + u(n_w)}_{=0} + \lambda_E(b + T(n_w))\right]\right\} \quad (18)$$

$$\lambda_u'(n) = \bar{s}f(n)\left(\frac{\lambda_E}{v'(x(n))} - 1\right) \quad (19)$$

$$0 = -\frac{\lambda_u(n)}{n^2} + \lambda_E f(n)\bar{s}\left(\frac{1}{nv'(x(n))} - 1\right) \quad (20)$$

The transversality conditions are[23]

$$\lambda_u(n_w) + \eta_w = 0 \quad (21)$$

$$\lambda_u(n_1) = 0, \quad (22)$$

and the government budget constraint amounts to

$$\int_{n_w}^{n_1} f(n)\bar{s}\left[z(n) - v^{-1}\left(u(n) + \frac{z(n)}{n}\right)\right]dn = [F(n_w) + (1 - F(n_w))(1 - \bar{s})]\,b + g \quad (23)$$

Equation (20) can be written as

$$\frac{1}{v'(x(n))} = n + \frac{1}{n}\frac{\lambda_u(n)}{\lambda_E f(n)\bar{s}}. \quad (24)$$

Substituting this into (19) to eliminate $v'(x(n))$, we arrive at

$$\lambda_u'(n) = \frac{\lambda_u(n)}{n} + \bar{s}f(n)(\lambda_E n - 1). \quad (25)$$

We are now set to prove that $b + T(n_w) \geq 0$. Suppose not. That is, assume (by contradiction) that $b + T(n_w) < 0$. Then equation (18) implies that $\eta_w < 0$ and thus we find from (21) that

$$\lambda_u(n_w) > 0$$

We now consider two subcases: (i) $\lambda_E n_w - 1 \geq 0$ and (ii) $\lambda_E n_w - 1 < 0$ and show that the result $\lambda_u(n_w) > 0$ leads to a contradiction in each case.

(i) If $\lambda_E n_w - 1 \geq 0$, then $\lambda_u(n_w) > 0$ together with equation (25) implies that $\lambda_u(n) > 0$ for each $n > n_w$. However, this contradicts the second transversality condition (22) that $\lambda_u(n_1) = 0$.

[23] Kamien and Schwartz (1981: 208, 209) derive the transversality condition for the case where the end value of a state variable can be freely chosen but has to satisfy an inequality constraint. Our condition above is the equivalent of that condition for a free starting point under an inequality constraint. The intuition is the following. If the constraint is binding, the Lagrange multiplier is strictly positive $\eta_w > 0$ and hence the shadow value $\lambda_u(n_w)$ is negative. In other words, one would like to reduce $u(n_w)$ (as $\lambda_u(n_w) < 0$) but cannot do so because of the constraint $u(n_w) \geq \gamma + v(b)$.

(ii) If $\lambda_E n_w - 1 < 0$, then using $\lambda_u(n_w) > 0$ together with (22) it must be the case that there exists $\tilde{n} \leq n_1$ such that $\lambda_u(\tilde{n}) = 0$ and $\lambda_u'(\tilde{n}) \leq 0$. The combination of $\lambda_u(\tilde{n}) = 0$ with $\lambda_u'(\tilde{n}) \leq 0$ implies that

$$\lambda_E \tilde{n} - 1 \leq 0 \tag{26}$$

Now we write the first-order condition for b as

$$[F(n_w) + (1 - F(n_w))(1 - \bar{s})] v'(b) - \lambda_E [F(n_w) + (1 - F(n_w))(1 - \bar{s})] = \eta_w v'(b)$$

Since our assumption (to be contradicted) that $T(n_w) + b < 0$ implies that $\eta_w < 0$, this equation implies $\lambda_E > v'(b)$. Furthermore, the participation constraint (with $\gamma > 0$) implies that $x(n) > b$ for $n \geq n_w$ and thus we must have $v'(b) > v'(x(n))$ for all $n \geq n_w$. Now using the characterization of $\tilde{n} > n_w$ that says $\lambda_u(\tilde{n}) = 0$ together with equation (24), we establish

$$\lambda_E > v'(b) > v'(x(\tilde{n})) = \frac{1}{\tilde{n}}.$$

This, however, contradicts equation (26) above. QED

Special case: $\gamma = 0$ For the case where $\gamma = 0$ we can derive the following results:

Lemma 20 *For given $g < g^*(0), f(.), n_0, n_1$ and $\gamma = 0$, there exists a benchmark value $\bar{b} > 0$ such that*

$$\begin{aligned} \text{for } b &< \bar{b} \text{ we have } R \text{ economy} \\ \text{for } b &> \bar{b} \text{ we have } P \text{ economy} \\ \text{for } b &= \bar{b} \text{ we have } N \text{ economy with } n_w = n_0 \end{aligned}$$

In other words, with $\gamma = 0$ it is the case that $\underline{b} = \bar{b}$ in lemma 3.

Proof. Since the normal economy is the benchmark case in this result, start by considering this economy. In the normal economy, everyone works who does not a search subsidy: $n_z = \hat{n}$ where $v'(x(\hat{n})) = v'(b) = \frac{1}{\hat{n}}$. Note that this $(x(\hat{n}) = b)$ together with $\gamma = 0$ implies that $n_w = n_0$. Now write the government budget constraint as

$$\int_{\hat{n}(b)}^{n_1} f(n) \bar{s} (n (v(x(n)) - v(b)) - x(n)) \, dn + \int_{n_0}^{\hat{n}(b)} f(n) \bar{s} (-x(\hat{n})) \, dn = (1 - \bar{s}) b + g$$

Note that the left hand side is decreasing in b because $\hat{n}'(b) > 0$. Further the right hand side is increasing in b. Next by the assumption that $g < g^*(0)$ we see that for $b = 0$ the left hand side exceeds the right hand side. Further, if b satisfies $v'(b) = \frac{1}{n_1}$ we see that the left hand side is negative. Hence we have a unique point $\bar{b} > 0$ where the equality above holds, namely a normal economy with $n_w = n_0$. If we increase b above $\bar{b}$ we can no longer afford such value of b and we are in a poor economy. If $b < \bar{b}$ we are in a rich economy. ■

Lemma 21 *Assume $g < g^*(0), \gamma = 0$ and $n_0 \geq 0$ then the optimal b implies a rich economy.*

Proof. The assumption $g < g^*(0)$ implies that with $b = 0$ the poor economy can be ruled out. To prove the theorem we have to show that a normal economy is impossible. We prove this by contradiction. Suppose that with optimal b we are in a normal conomy. Then the following equalities hold:

$$\begin{aligned} \hat{b} &= \bar{b} \\ n_w &= n_0 \\ x(\hat{n}) &= b \\ \lambda_E &= \frac{1}{\hat{n}} \\ \hat{n} &= \frac{1}{v'(b)} \end{aligned}$$

where $\bar{b}$ is the benchmark value defined in the lemma above, the other equations follow from the results on the normal economy. Substituting $\lambda_E = v'(b)$ in the equation (11) for optimal b yields

$$v'(b) = \frac{1 - \bar{s} + \bar{s}F(\hat{n})}{(1-\bar{s})\frac{1}{v'(b)} + \bar{s}\int_{\hat{n}}^{n_1} f(n)\frac{1}{v'(x(n))}dn}$$

Because $v'(x(n)) < v'(b)$ for $n > \hat{n}$ we have a contradiction. Hence a normal economy is ruled out. ■

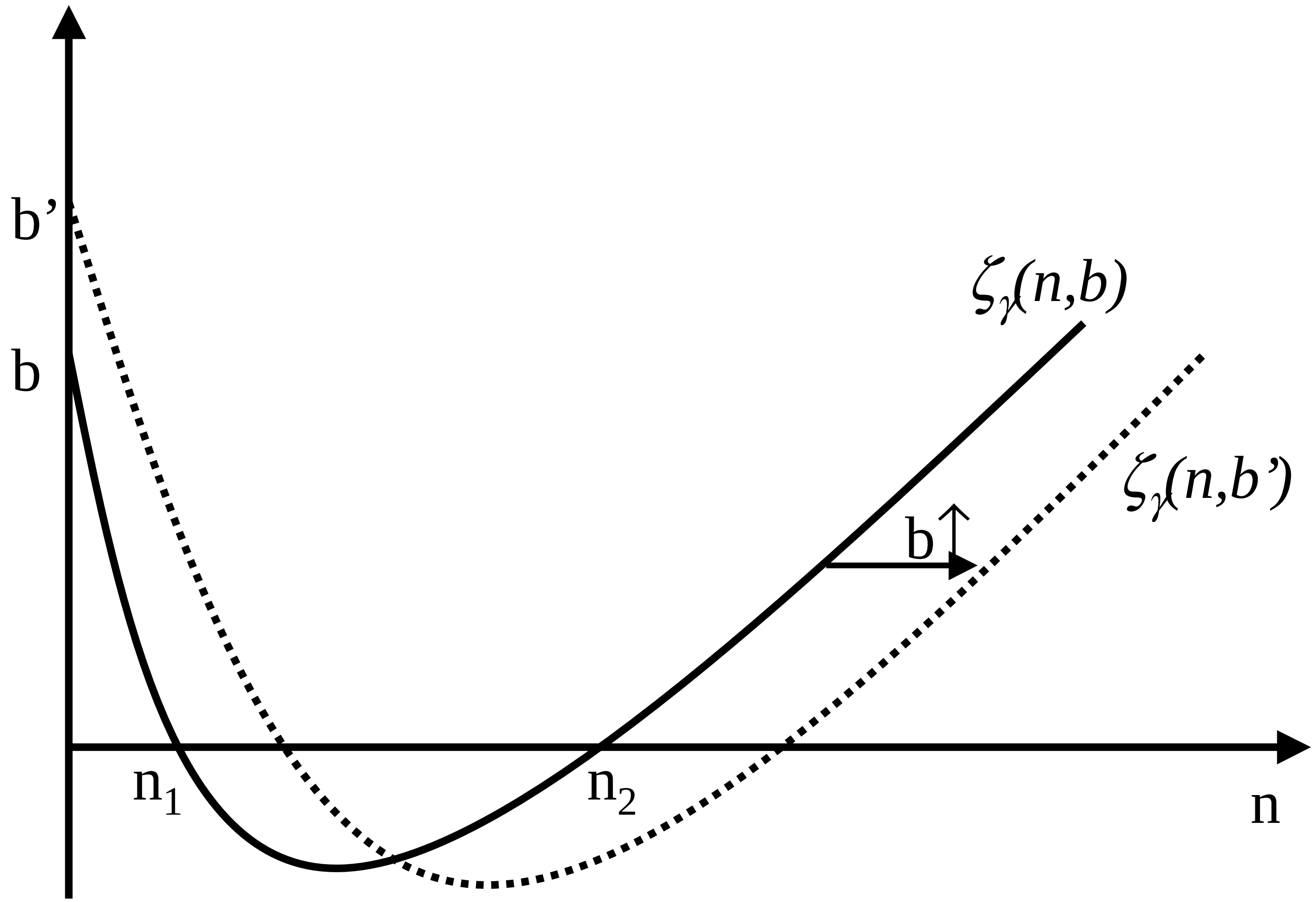

Figure 1: $\zeta_\gamma(n,b)$ as a function of *n*.