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ABSTRACT

Optimal Majority Rules and Enhanced Cooperation*

The decision-making rules of the European Union (EU) are defined in an incomplete contract signed by 15 national governments. The design of the contract defines the set of policy issues where it applies – in decision-making rules i.e. the majority rules and the division of powers among the actors involved. The treaty also gives competence to the Commission in terms of making legislative proposals.

In this Paper, our objective is to study how decision-making rules affect expected outcomes in the above-described hierarchy. Our aim is to derive *ex ante* optimal voting rules in the setting where decision-making can be modelled as a spatial voting game. We thus assume that the players have spatial preferences but the rules are designed behind the veil of ignorance. The designers only know how member states' ideal policies are distributed in the policy space but as the rules are later applied in day-to-day decision-making, preferences are known by all players.

We also analyse two types of designers, benevolent and self-interested. Benevolent designers take the whole range of national preferences into account in their design. Self-interested government designs the rules by taking only the preferences of the same type of government into account but assesses the consequences of increased status quo risk if an anti-integrationist government is in power.

The Paper shows that when the probability of 'no gains from integration' is positive it is, in general, impossible to design majority rules efficiently. This holds as far as the player set is finite and countable. The welfare loss is not, however, necessarily very big if the likelihood of potential integration gains is sufficiently high. Then, the optimal (i.e. as close to efficient as possible) design leads to outcomes that have an expectation close to the expected ideal policies of member states. When design completely disregards anti-integrationist views an increase in status quo risk makes the optimal majority threshold lower. If anti-integrationist views are taken into account an increase in status quo risk has a smaller and potentially non-monotonic impact. If the likelihood of no gains from integration increases enough the discrepancy between the two designs becomes wider, which makes the trade-off more difficult to solve. The compromise solution is closer to a purely self-interested solution, which makes it possible that the countries with less likely gains are better off when they stay out. The Paper demonstrates, however, that when the likelihood of no gains from integration is relatively small the discrepancy between the two designs is rather small as well.

The Nice Treaty of the European Union, signed in February 2001, defines explicit rules for so-called enhanced cooperation. It allows sufficient numbers of member states to proceed in integration to areas where not necessarily all member states are likely to gain equally. In this Paper, we have demonstrated how the need for enhanced cooperation might emerge. According to the Nice Treaty, enhanced cooperation projects make decisions in a similar fashion as the decisions are made in common policies but among a smaller group. If these rules are a result of self-interested or average loss minimizing design and the general design is a result of small likelihood of 'no gains from integration' this is what the results of this paper suggest to be a plausible solution.

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1. Introduction

The decision-making rules of the European Union (EU) are defined in an incomplete contract signed by 15 national governments. The design of the contract (henceforth the treaty) covers the set of policy issues where it applies, decision-making rules, i.e. the majority rules and the division of powers among the actors involved. The treaty also gives competence to the Commission in terms of making legislative proposals. Institutionally, the treaty thus establishes a two-level hierarchy where supranational views are represented by the Commission and the national interests are represented by the Council. In issues that are covered by the treaty, agenda-setting has been devoted to the former and decision-making in the form of accepting or rejecting proposals to the latter.[1]

In this paper, our objective is to study how the decision-making rules affect to expected decisions of the above-described hierarchy. Our aim is to derive ex ante optimal voting rules in the setting where decision-making can be modelled as a spatial voting game. We thus assume that the players have spatial preferences but the rules are designed behind the veil of ignorance. Only the probability distributions of member states' ideal policies and the policy space are known. When rules are later applied in day-to-day decision-making the preferences of players are assumed to be known by all players.

The treaty gives the agenda-setter the right to represent supranational views as an independent self-interested player. We assume that the Commission tries to minimise the distance between its own ideal policy and the outcome. The treaty's nature as an incomplete contract and monopolised agenda-setting make this plausible. We also analyse two types of designers, benevolent and self-interested. The former take the whole range of national preferences into account in their design. In the case of the latter, we distinguish between two types of governments in each country and assume that the other type (pro-integrationist) designs the rules without taking into account the views of the other type (anti-integrationist) of government but is able to assess the consequences of increased status quo risk if an anti-integrationist government is in power.

In our analysis, we ignore the question why political constitutions are structured as incomplete contracts. Actual real life contracts appear, however, quite incomplete. For example political organisations tend to have rather loose objec-

[1] The third player in EU legislation is the European Parliament. In this paper, we concentrate on the relationship between national and supranational interests and, therefore, disregard the role of the parliament.

tives without making explicit mention of how decisions should react to possible contingencies that determine the desirable choices. Furthermore, political constitutions only specify who has the right to make proposals, who accepts or rejects them, how this is done and under what circumstances.[2] Consequently, we assume that the set of rules of a two-level hierarchy forms an incomplete contract and the main emphasis of the paper is to analyse how to design such a contract optimally.

To participate in cooperation without knowing future states of nature, a country must expect to gain from that cooperation on average. This has been taken into account by assuming that at the national level the expected gain from membership must exceed the expected losses. If this does not hold the country will not sign the treaty.

Although there are examples of different levels of participation in the EU, the bulk of legislation is common for all current member states without exceptions. An alternative way to organise cooperation would be a less restrictive institutional structure.[3] At the extreme, this would mean issue-by-issue contracting among member states. Then, if information is perfect and participation in this partnership is open for all countries, there is no role for agenda-setting as the equilibrium outcome is determined by unanimous decision of participating countries. This would indicate that the extreme views would determine outcomes. Another way to introduce a less rigid institutional structure is to allow for different levels of participation. In this paper, we demonstrate how differences in member states' potential gains from integration work as a source of the need for more flexible design.

The existing literature on EU decision-making, can be divided into two categories.[4] First, there is literature on voting power within European institutions and on the distribution of power among the three main legislative bodies in the EU. This literature usually draws on Shapley-Shubik or Banzhaf indices of power in simple cooperative voting games (e.g. Widgrén 1994 and references therein). One of the key questions in this literature is how the EU voting rules distribute power among member states (see Laruelle and Widgrén 1998 and references therein).

[2]See Persson et al. (1997). More generally, incomplete contracts are often explained by unforeseen contingencies, costs of writing complete contracts or costs of enforcing contracts. At least to some extent one can find all these elements in political organisations. (For a recent discussion on incomplete contracts and their foundations, see Tirole 1999 or Maskin and Tirole 1999).

[3]For a discussion on overlapping political jurisdictions, see Casella and Frey (1992). A more flexible organisation of European integration is discussed in Dewatripont et al. (1995).

[4]For a more detailed discussion see Nurmi (1998).

The institutional reform that was decided in Nice in December 2000 made the topic even more relevant (see Baldwin et al 2001, Felsenthal and Machover 2001).

Power indices do not, however, take into account players' policy preferences, strategic agenda-setting nor inter-institutional relations.[5] These issues are usually analysed using spatial voting games. In spatial models, players' preferences are presented as points in a policy space, which represent their most preferred policy (ideal point). Each conceivable policy alternative is then also modelled as a point in policy space. Players' pay-offs are determined by distances between ideal points and conceivable policy points. In the case of the EU, the Commission is usually modelled as a unitary actor, hence having a single ideal policy. In terms of agenda-setting, member states' various preference configurations can be seen as constraints of the Commission's utility maximisation problem.[6]

As tools for analysing EU decision making, spatial voting games and voting power models differ since agenda-setting is strategic in the former whereas it is not in the latter. Moreover, member states' preferences are almost like observed variables in the former whereas they are not in the latter. Steunenberg et al. (1999) add strategic aspects of agenda-setting into power analysis by defining power as the difference that a player makes to an equilibrium outcome compared to the outcome in the case where the player would be a dummy-player. As their analysis contains a random element it lies in a grey area between pure spatial modelling and power analysis although their model basically stems from spatial voting analysis.

This paper has its essential emphasis on spatial modelling as well but, like Steunenberg et al. (1999), it can be seen as an intermediate example. The paper contributes to the existing literature on EU institutions by adding the institutional design phase to a spatial voting model. To our knowledge this has not been done before. Our work is directly related to spatial models since the legislative stage of the game is modelled as a simple spatial voting game. Those who design the rules know neither the state of nature nor the ideal points of the players' at the legislation stage. In principle, this puts the first part of the game closer to probabilistic voting power analysis but when the two parts are merged into one strategic aspects at the legislation phase affect directly to the design.[7] The main

[5] To some extent these questions are tackled in Laruelle and Widgrén (2001), Widgrén (1995) and Kirman and Widgren (1995). For a critical view and discussion, see Garrett and Tsebelis (1999a, 1999b), Berg and Lane (1999), Holler and Widgrén (1999) and Steunenberg et al. (1999).

[6] For applications to the EU, see Crombez (1996, 1997), Steunenberg (1994), Tsebelis (1994), Tsebelis and Garrett (1997), Garrett and Tsebelis (1999a, 1999b).

[7] This, in fact, means that the designers have some information about the future as they

purpose of the paper is to find out how the decision-making rules work as a control mechanism for the Commission's agenda-setting and, in particular, how they can be designed optimally. As the legislation part of the model follows standard ways of spatial voting games the contribution of this paper lies in the treaty design and its combination with the legislation game.

The paper shows that when the probability of 'no gains from integration' is positive it is, in general, impossible to choose an efficient majority rule. This holds as far as the player set is finite and countable. The welfare loss is not, however, necessarily very big if the likelihood of potential integration gains is sufficiently high. Then the optimal, i.e. as close to efficient as possible, design leads to outcomes that have an expectation close to the expected ideal policies of member states. If the designers favour only pro-integrationist views an increase in status quo risk makes optimal majority threshold lower. If anti-integrationist views are taken into account an increase in status quo risk has smaller and potentially non-monotonic impact. If the likelihood of no gains from integration increases enough the discrepancy between the two designs becomes wider. A compromise solution, which minimises the average discrepancy, tends to be closer to self-interested optimum than to benevolent optimum. This means that if the participating countries differ in terms of their probabilities of obtaining gains from integration there might be a need for a solution where those who expect to gain more likely proceed without those whose gains are more uncertain.

The rest of the paper is organized as follows. Section 2 gives the basic definitions and describes the model. Section 3 summarises the spatial voting part and section 4 describes the treaty design. Section 5 derives the results and, finally, section 6 concludes and discusses some ideas for future research.

2. The model

Consider a $2n+1$ player game with a player set $N = \{N_0, C, N\}$, where C is the Commission, N_0 the set of n national governments in period zero and the set N forms the Council formed by the future governments of the same n nations. In period zero, the governments of N_0 design the voting rules for day-to-day legislation, give the Commission the competence to make legislative proposals and

are able to compute the expected equilibrium and the distribution of this expectation is not uniform. Note that the model of this paper can be used for the purposes of assessing actors' a priori influence in legislation (see Widgrén and Napel 2001).

the future governments to accept or reject the proposals. The treaty designers do not observe future governments' preferences at the legislation phase.

In each issue of vote, national governments and the Commission are supposed to have an ideal policy, which is which is expressed as a point on a policy space I. The policy space is defined as an interval $I = [\alpha, \beta] \in R$. We interpret I in terms of attitude towards integration, α representing the extreme of anti-integrationist views and β the extreme of pro-integrationist views. In each issue, we normalize the current state of affairs, i.e. status quo, to zero and assume that $\alpha \leq 0$ and $\beta > 0$. The sub-interval $[0, \beta] := I_+$ thus consists of outcomes with more integration in the respective policy area than in status quo and the sub-interval $[\alpha, 0) := I_-$ consists of outcomes with less integration. The set of ideal points of national governments can be expressed as a random vector $\tilde{\Lambda} = \{\tilde{\lambda}_1, \tilde{\lambda}_2, ...\tilde{\lambda}_n\}$ and the ideal point of the Commission by a random variable $\tilde{\sigma}$. All players know the distributions $F_{\tilde{\Lambda}}$ and $F_{\tilde{\sigma}}$ of voters' random ideal points, $\tilde{\lambda}_i$, $i \in N$, and the Commission's random ideal point, $\tilde{\sigma}$ but after the state of nature has been observed also the realizations Λ and σ of ideal points are known.

Let us call governments having their ideal point on I_+ pro-integrationist and those having their ideal point on I_- anti-integrationist. The ideal points of national governments are assumed to be identically uniformly distributed on I and the ideal policy of the Commission is assumed to be uniformly distributed on I_+. Anti-integrationist governments are supposed to have $\lambda_i^A \sim U(\alpha, 0)$ for all i and pro-integrationist governments $\lambda_i^P \sim U(0, \beta)$ for all i where superscripts A and P refer to anti-integrationist and pro-integrationist respectively.

At the third phase of the game the Commission and the national governments play a simple spatial voting game where players have observed the realisations of the ideal points. After the state of nature has been observed information concerning players' ideal points is assumed to be perfect. The Commission makes a take-it-or-leave-it proposal $\chi \in I$ to the Council, which either accepts or rejects it. Let us denote the outcome of the vote by $\Omega \in I$.

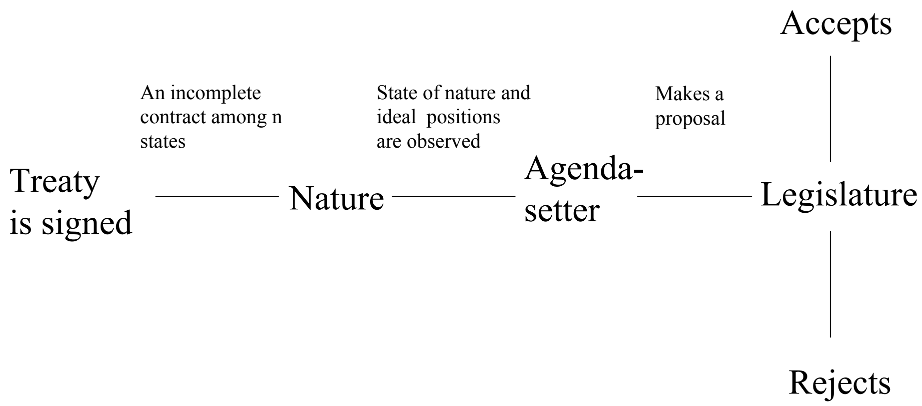

Fig. 1. The sequence of moves

We analyse two altenative designs. First, we assume that there is either anti- or pro-integrationist governments in power but in their design they take the whole range $[\alpha, \beta]$ of ideal points' into account. Let us refer to such designers as *benevolent*. Second, we assume that the pro-integrationist types of governments are in power and they consider only pro-integrationist views in their design. Let us refer to such designers as *self-interested*.[8] In the first case, the vector of relevant expected ideal positions can be expressed as $E\tilde{\Lambda}_B^* = \{E\tilde{\lambda}_1, E\tilde{\lambda}_2, ...E\tilde{\lambda}_n\} = \left\{\frac{\alpha+\beta}{2}, ..., \frac{\alpha+\beta}{2}\right\}$ and in the second as $E\tilde{\Lambda}_S^* = \{E\left(\tilde{\lambda}_1 \middle| \tilde{\lambda}_1 \geq 0\right), E\left(\tilde{\lambda}_2 \middle| \tilde{\lambda}_2 \geq 0\right), ... E\left(\tilde{\lambda}_n \middle| \tilde{\lambda}_n \geq 0\right)\} = \left\{\frac{\beta}{2}, ..., \frac{\beta}{2}\right\}$ where the superscripts B and S denote benevolent and self-interested respectively. If the government is anti-integrationist it signs the treaty only if it is benevolent and there are expected gains from integration.

[8] We disregard the case where a government is anti-integrationist and self interested. A government like that would not sign the treaty regarless of future expected gains from integration. The emphasis of this paper is on decision-making rules.

A pro-integrationist government does sign the treaty in any case. We assume that if a pro-integrationist government signs the treaty an anti-integrationist governments are not able to resign afterwards.

Let us define players' pay-offs as squares of euclidian distances between their ideal policies and the outcome Ω. Hence

$$\begin{aligned} \Pi &= \left(-E\left(\tilde{\lambda}_1-\tilde{\Omega}\right)^2, ..., -E\left(\tilde{\lambda}_n-\tilde{\Omega}\right)^2, -\left(\sigma-\Omega^*\right)^2, \right. \\ &\quad \left. -\left(\lambda_1-\Omega^*\right)^2, ..., -\left(\lambda_n-\Omega^*\right)^2\right) \end{aligned} \tag{2.1}$$

where σ and λ_i, $i = 1, ..., n$ denote the realisations of $\tilde{\sigma}$ and $\tilde{\lambda}_i$ respectively and Ω^* the equilibrium outcome at the legislation phase.

From the Treaty designers' point of view their expected cost of integration is then $E\left(\lambda_i-\Omega\right)^2$. If a proposal lapses, we simply have $\Omega = 0$, which is also the status quo point. In sum, we assume that treaty designers do not evaluate integration on issue-by-issue basis and consequently their participation is based on expected outcomes. Figure 1 summarises how the game proceeds.

3. Legislation stage

At the legislation stage, the Council members form their opinion in each issue after observing the state of nature. We assume simple uni-dimensional spatial preferences. From the viewpoint of one government the acceptance of a proposal is simply determined by the individual rationality constraints (IR_i), that can be written

$$\left(\lambda_i-\Omega\right)^2 \leq \lambda_i^2. \tag{IR_i}$$

This gives

$$0 \leq \Omega < 2\lambda_i \qquad \vee \qquad 2\lambda_i < \Omega < 0.$$

The Commission's individual rationality (IR_σ) constraint is

$$\left(\sigma-\Omega\right)^2 \leq \sigma^2 \tag{IR_σ}$$

where σ^2 gives the opposite of its reservation utility. Since it was assumed that $\sigma \geq 0$ this yields

$$0 \leq \Omega \leq \min \{2\sigma, \beta\} .$$

We make the tie-breaking assumption that player i votes for χ if he is indifferent. Note that IR-conditions imply that only connected coalitions will form. Let (i) denote the player j whose ideal point, λ_j, turns out to be the i-th smallest of all voters so that $\lambda_{(1)} \leq \ldots \leq \lambda_{(n)}$. Then

$$(i) \in S \wedge (i+l) \in S \;\Rightarrow\; (i+1), \ldots, (i+l-1) \in S, \qquad l \geq 2,$$

where S denotes a coalition. From the agenda setter's point of view she only needs the acceptance of a minimal winning coalition to pass a proposal. Given a fixed (observed) preference configuration a proposal χ (almost always) induces a unique pivot. Among the players in a minimal winning coalition, it is reasonable to take the swing of that member $j \in S$ most seriously who is most reluctant to accept any χ given his ideal point λ_j. This is precisely the *pivotal voter* $(n-m+1)$. This voter's ideal point, $\lambda_{(n-m+1)}$, is the only one which needs to be taken into account by the Commission when it deliberates its proposal χ.

Next, write the following well-known result from spatial voting games. In the case of perfect information, we get the following subgame perfect Nash equilibrium proposals[9]

$$\chi^*(\Lambda) = \chi^*\left(\lambda_{(n-m+1)}\right) = \begin{cases} \sigma & \text{if } \lambda_{(n-m+1)} > \frac{1}{2}\sigma \\ 2\lambda_{(n-m+1)} & \text{if } \lambda_{(n-m+1)} \in (0, \frac{1}{2}\sigma] \\ 0 & \text{if } \lambda_{(n-m+1)} \leq 0 \end{cases} \tag{3.1}$$

which is accepted by players $(n), \ldots, (n-m+1)$ and by any $(n-m), \ldots, (l)$, $n-m \geq l \geq 1$, for whom $(\lambda_i - \chi^*)^2 \leq \lambda_i^2$ holds. Hence $\Omega^*(\Lambda, \sigma) = \chi^*\left(\lambda_{(n-m+1)}\right)$.

4. Treaty design

Let us next turn to treaty design stage of the game. The designers know the equilibrium but they are able to use it only as an expectation. Suppose that

[9] One may assume small costs of being rejected for the Commission to ensure uniqueness of Commission's proposal in the last sub-case. There are, depending on Λ, multiple subgame perfect equilibria corresponding to the same unique equilibrium proposal by agenda setter A. We focus on (χ^*, Λ)-IR coalitions.

voters' ideal points λ_i are a priori uniformly distributed on $[\alpha, \beta]$ where $\alpha \leq 0$, $\beta > 0$. Agenda setter's ideal point σ is supposed to lie in $[0, \beta]$ and a priori it is assumed to have uniform distribution. This gives the following cumulative distributions functions

$$F_{\tilde{\lambda}_i}(x) = \begin{cases} 0, & \text{if } x \leq \alpha \\ \frac{x-\alpha}{\beta-\alpha}, & \text{if } \alpha < x \leq \beta \\ 1, & \text{if } x > \beta \end{cases}$$

and

$$F_{\tilde{\sigma}}(x) = \begin{cases} 0, & \text{if } x \leq 0 \\ \frac{x}{\beta}, & \text{if } 0 < x \leq \beta \\ 1, & \text{if } x > \beta. \end{cases}$$

For computational purposes let us re-scale the policy space such a way that the ideal points of the national governments vary between zero and one, i.e such that re-scaling yields $\hat{\alpha} = \frac{\alpha-\alpha}{\beta-\alpha} = 0$ and $\hat{\beta} = \frac{\beta-\alpha}{\beta-\alpha} = 1$. Hence

$$\hat{\lambda}_i \equiv \frac{\tilde{\lambda}_i - \alpha}{\beta - \alpha} \sim U(0, 1)$$

$\hat{\lambda}_i$ is the ideal point of government i in a re-scaled space. For the commission we get

$$\hat{\sigma} \equiv \frac{\tilde{\sigma} - \alpha}{\beta - \alpha} \sim U\left(\frac{-\alpha}{\beta - \alpha}, 1\right).$$

Let us also define the following

$$\hat{\Pi}(\sigma) \equiv \frac{\frac{1}{2}\sigma - \alpha}{\beta - \alpha}.$$

The location of $\Pi(\sigma)$ relative to $\lambda_{(n-m+1)}$ is crucial for the location of the outcome. We get

$$\hat{\Pi} \equiv \frac{\frac{1}{2}\tilde{\sigma} - \alpha}{\beta - \alpha} \sim U\left(\frac{-\alpha}{\beta - \alpha}, \frac{\frac{1}{2}\beta - \alpha}{\beta - \alpha}\right).$$

For the location of the pivotal player in the re-scaled space we have

$$\hat{\lambda}_{(n-m+1)} \sim beta\,(n-m+1,m)$$

In the re-scaled space, the equilibrium of the legislation game depends on the position of $\hat{\lambda}_{(n-m+1)}$ relative to $\hat{\Pi}$ and status quo $\frac{-\alpha}{\beta-\alpha}$. Let us denote re-scaled status quo by $\hat{Q}$.

To find the expected outcome we need to calculate the following probabilities $P\left(\hat{\lambda}_{(n-m+1)} > \hat{\Pi}\right)$, $P\left(\hat{Q} < \hat{\lambda}_{(n-m+1)} \leq \hat{\Pi}\right)$ and $P\left(\hat{\lambda}_{(n-m+1)} \leq \hat{Q}\right)$. The expected outcome of the legislation game ca be written

$$\begin{aligned} E\Omega^* &= P\left(\hat{\lambda}_{(n-m+1)} > \hat{\Pi}\right)\tilde{\sigma} \\ &\quad +2\left[P\left(\hat{Q} < \hat{\lambda}_{(n-m+1)} \leq \hat{\Pi}\right)\tilde{\lambda}_{(n-m+1)}\right] \\ &\quad +\left[P\left(\hat{\lambda}_{(n-m+1)} \leq \hat{Q}\right)\cdot 0\right] \end{aligned} \tag{4.1}$$

This can be written explicitly as follows (for derivation see appendix 1)

$$\begin{aligned} E\hat{\Omega}^* &= 2\left(\frac{n-m+1}{n+1}\right)\int_{\frac{-\alpha}{\beta-\alpha}}^{\frac{\frac{1}{2}\beta-\alpha}{\beta-\alpha}} \frac{2(\beta-\alpha)}{\beta}G\left(x\right)\,dx \\ &\quad -2\left(\frac{n-m+1}{n+1}\right)G\left(\frac{-\alpha}{\beta-\alpha}\right) \\ &\quad +\left(\frac{\frac{1}{2}\beta}{\beta-\alpha}\right)\left[1-\int_{\frac{-\alpha}{\beta-\alpha}}^{\frac{\frac{1}{2}\beta-\alpha}{\beta-\alpha}} \frac{2(\beta-\alpha)}{\beta}G\left(x\right)\,dx\right] \end{aligned} \tag{4.2}$$

where $G\left(x\right)$ is the cumulative distribution function of $beta\,(n-m+1,m)$. Using shorter notation for the following concepts

$$\begin{cases} \int_{\frac{-\alpha}{\beta-\alpha}}^{\frac{\frac{1}{2}\beta-\alpha}{\beta-\alpha}} \frac{2(\beta-\alpha)}{\beta}G\left(x\right)\,dx := \Gamma\left(\hat{\Pi}\right) \\ G\left(\frac{-\alpha}{\beta-\alpha}\right) := \Gamma\left(\hat{Q}\right) \end{cases}$$

simplifies the expected outcome to

$$E\Omega^* = 2\beta\left(\frac{n-m+1}{n+1}\right)\left[\Gamma\left(\hat{\Pi}\right)-\Gamma\left(\hat{Q}\right)\right]+\left(\frac{1}{2}\beta\right)\left[1-\Gamma\left(\hat{\Pi}\right)\right]\left[1-\Gamma\left(\hat{Q}\right)\right] \tag{4.3}$$

where $\Gamma\left(\hat{Q}\right)$ can be interpreted as the probability of status quo bias and $\Gamma\left(\hat{\Pi}\right)$ gives the pivotal player's conditional probability of affecting the oucome given that the ideal point of the pivotal player $\lambda_{(n-m+1)} \in (0, \beta]$. The passage probability of the agenda setter's ideal point is thus $P\left(\Omega^* = \sigma\right) = \left[1-\Gamma\left(\hat{\Pi}\right)\right]\left[1-\Gamma\left(\hat{Q}\right)\right]$.

Let us next derive a general criterion for optimal voting rule. We assume that the designers aim for optimality by minimising the expected distance between an ideal point of a future government and the outcome. Formally we have

$$Min\left\{E\left(\tilde{\lambda}-\tilde{\Omega}\right)^2\right\}$$

subject to IR-contraints above. Let us denote the variance of $\tilde{\Omega}$ by $D^2\tilde{\Omega}$. As we get

$$\begin{aligned}
E\left(\tilde{\lambda}-\tilde{\Omega}\right)^2 &= E\left[\left(\tilde{\Omega}-E\tilde{\Omega}\right)+\left(E\tilde{\Omega}-\tilde{\lambda}\right)\right]^2 \\
&= E\left[\left(\tilde{\Omega}-E\tilde{\Omega}\right)^2-2\left(\tilde{\Omega}-E\tilde{\Omega}\right)\left(E\tilde{\Omega}-\tilde{\lambda}\right)+\left(E\tilde{\Omega}-\tilde{\lambda}\right)^2\right] \\
&= D^2\tilde{\Omega}+E\left(\tilde{\lambda}-E\tilde{\Omega}\right)^2
\end{aligned}$$

this is equivalent to minimising $E\left(\lambda-E\Omega\right)^2$. This in turn implies

$$\begin{aligned}
E\left(\tilde{\lambda}-E\tilde{\Omega}\right)^2 &= E\left[\left(\tilde{\lambda}-E\tilde{\lambda}\right)+\left(E\tilde{\lambda}-E\tilde{\Omega}\right)\right]^2 \\
&= E\left[\left(\tilde{\lambda}-E\tilde{\lambda}\right)^2-2\left(\tilde{\lambda}-E\tilde{\lambda}\right)\left(E\tilde{\lambda}-E\tilde{\Omega}\right)+\left(E\tilde{\lambda}-E\tilde{\Omega}\right)^2\right] \\
&= D^2\tilde{\lambda}+\left(E\tilde{\lambda}-E\tilde{\Omega}\right)^2.
\end{aligned}$$

where $D^2\lambda$ denotes the variance of λ. Hence

$$E\left(\tilde{\lambda}-\tilde{\Omega}\right)^2 = D^2\tilde{\Omega} + D^2\tilde{\lambda} + \left(E\tilde{\lambda}-E\tilde{\Omega}\right)^2. \tag{4.4}$$

Suppose that the designers are risk neutral. Then efficient majority rule requires $E\tilde{\lambda} = E\tilde{\Omega}$.

Since all participating countries are a priori similar, i.e. their ideal points have the same distribution, we write the following efficiency criterion.

Definition 1. *For benevolent designers a majority rule* $\frac{m}{n}$ *is* ex ante efficient *if* $E\Omega^* = E\lambda_i = \frac{\beta+\alpha}{2}\ \forall i$. *For self-interested designers a majority rule* $\frac{m}{n}$ *is ex-ante efficient if* $E\Omega^* = E(\lambda_i|\lambda_i \geq 0) = \frac{\beta}{2}\ \forall i$.

In other words, ex ante efficiency requires that the expected outcome equals the national governments' expected ideal point. Note, however, that it might be impossible to satisfy this requirement. This is due to the fact that the majority rule when defined as the ratio $\frac{m}{n}$ indicates that there is a finite countable number of potential expected outcomes. It is thus possible that there exist no majority rules such that $E\tilde{\lambda} = E\tilde{\Omega}$. Let us, therefore, define

Definition 2. *A majority rule* m *is* ex ante optimal *if*

$$\frac{m}{n} = \arg\min\left(E\Omega\left(\frac{m}{n}\right)^* - E\tilde{\lambda}_i\right)^2 \forall i.$$

Let us next analyse efficiency and optimality more in detail.

5. Results

Let us start the analysis with a special case $\alpha = 0$. In this case, there is no difference between benevolent and self-ineterested designers. Using equation (4.2) above the expected outcome can be written

$$
\begin{aligned}
E\hat{\Omega}^{*} &= 4\left(\frac{n-m+1}{n+1}\right)\int_{0}^{\frac{1}{2}} G(x)\,dx \\
&\quad +\frac{1}{2}\left[1-\int_{0}^{\frac{1}{2}} 2G(x)\,dx\right] \\
&= \frac{1}{2}+4\left(\frac{n-m+1}{n+1}\right)\int_{0}^{\frac{1}{2}} G(x)\,dx-\int_{0}^{\frac{1}{2}} G(x)\,dx
\end{aligned}
$$

Let us write

Proposition 1. *Suppose that the agenda-setter's and national governments' preferred points are a priori identically uniformly distributed on* $[0,\beta]$ *. Then ex ante efficient majority rule is achieved if the following holds* $q_0^* = \frac{m^*}{n} = \frac{3}{4} + \frac{3}{4n}$.

Proof. We need to show that $E\hat{\Omega}^* = E\hat{\lambda} \Leftrightarrow q = \frac{3}{4} + \frac{3}{4n}$. As $\hat{\lambda} \sim U(0,1)$ and hence $E\hat{\lambda} = \frac{1}{2}$ we get

$$
E\hat{\Omega}^{*} = \frac{1}{2}+4\left(\frac{n-m+1}{n+1}\right)\int_{0}^{\frac{1}{2}} G(x)\,dx-\int_{0}^{\frac{1}{2}} G(x)\,dx = \frac{1}{2}.
$$

This holds iff $\frac{n-m+1}{n+1} = \frac{1}{4}$, which gives the claim and completes the proof.

A closer look at the proposition shows that m^* is not necessarily an integer. We get the following.

Corollary 1. *If* $\alpha = 0$ *there exists an efficient majority rule if there exists an integer* m *such that* $m = \frac{3n+3}{4}$.

Proof. Follows directly from the proposition above. The smallest possible number of players is three. Then unanimity is the efficient voting rule.

The corollary above implies that ex ante efficiency can be achieved if $n = 3, 7, 11, 15, ...$, hence n can be expressed as $n = 3k + 3$ where $k = 0, 1, 2, ...$ If n is none of the above then ex ante optimal majority rule is either $q^- = \frac{m^-}{n}$ or $q^+ = \frac{m^+}{n}$ where m^- is the largest integer smaller than or equal to m^* and m^+ is the smallest integer greater than or equal to m^*.

Let us next turn to the following more general considerations.

Proposition 2. *Suppose benevolent designers and $\alpha \leq 0$. Ex ante efficient majority rule q_B^* requires that*

$$q_B^* = \frac{3}{4} + \frac{3}{4n} - \frac{\frac{\alpha}{\beta} + \Gamma\left(\hat{Q}\left(q, \alpha\right)\right)}{\Gamma\left(\hat{\Pi}\left(q, \alpha\right)\right) - \Gamma\left(\hat{Q}\left(q, \alpha\right)\right)}\left(\frac{n+1}{n}\right) \tag{5.1}$$

Proof. See Appendix 2.

Proposition 3. *Suppose self-interested designers and $\alpha \leq 0$. Ex ante efficient majority rule q_S^* requires that*

$$q_S^* = \frac{3}{4} + \frac{3}{4n} - \frac{\Gamma\left(\hat{Q}\left(q, \alpha\right)\right)}{\Gamma\left(\hat{\Pi}\left(q, \alpha\right)\right) - \Gamma\left(\hat{Q}\left(q, \alpha\right)\right)}\left(\frac{n+1}{n}\right) \tag{5.2}$$

Proof. See Appendix 3.

Note that, as before, the existence of an efficient voting rule depends now on n but also on stochastic status quo bias and pivotal players probability of affecting the outcome. These in turn depend on m and α. Suppose that n and α are known at the time of design and β is normalized to one as above. Re-arranging the efficiency conditions yields

$$m^* = \frac{3n+3}{4} - \frac{I_B\left(\frac{\alpha}{\beta}\right) + \Gamma\left(\hat{Q}\left(q, \alpha\right)\right)}{\Gamma\left(\hat{\Pi}\left(q, \alpha\right)\right) - \Gamma\left(\hat{Q}\left(q, \alpha\right)\right)}\left(n+1\right) \tag{5.3}$$

where I_B is an indicator variable and $I_B = 1$ if the designers are benevolent and zero otherwise. If α, β and n are known this can be expressed as

$$m^* = \delta(m)$$

where $\delta : IN_+ \rightarrow IR$ is the function on the right-hand side of the re-arranged effciency condition. Now, it is easy to see that as long as m is an integer the fixed point, hence the efficient majority rule, i.e. the fixed point, does not necessarily exist. In fact, if we let α vary we can argue that $\delta(m)$ is continuous random variable and then $P(\delta(m) = m^*) = 0$. Thus, if $\alpha < 0$ any majority rule is almost surely inefficient.

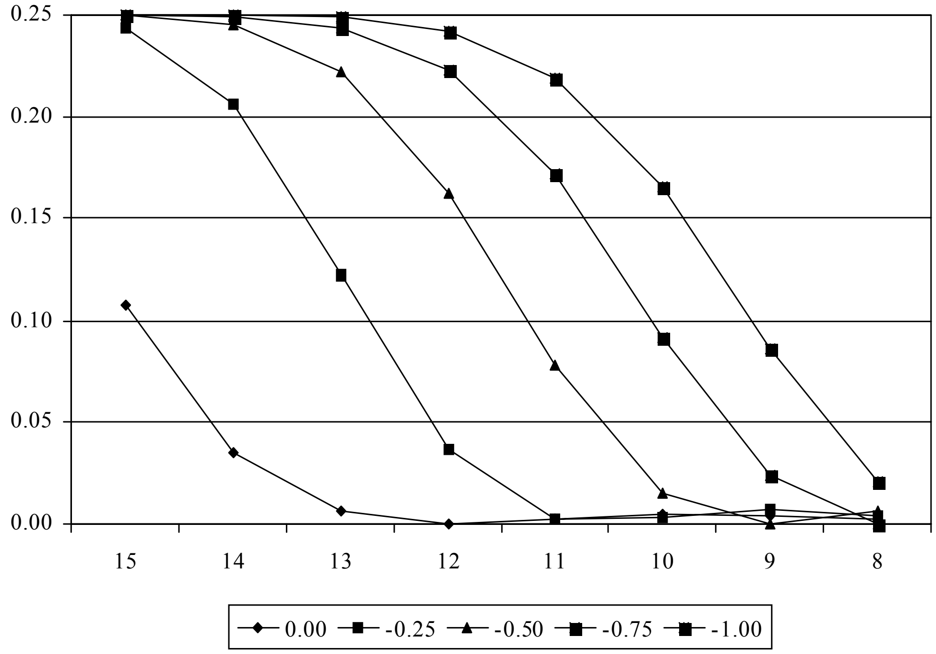

Fig. 2. Self interested design: expected loss under different majority rules and α-values, $n = 15$

To find an optimal majority rule we need to minimise $E\Omega^* - E\lambda_i$. Using arbitrary n and m this is impossible and, therefore, we illustrate optimality with an example and concentrate on the impact of α. However, from the equations above we see that when α decreases efficient majority rules of benevolent and self-interested design diverge. In self-interested design, the nominator of the third term clearly increases as α decreases. Status quo bias becomes more likely as α decreases. In self-interested design, the nominator is positive for any $\alpha < 0$. In benevolent design, $\frac{\alpha}{\beta}$ is like a stabiliser since it offsets the impact of status quo bias which works towards a lower majority threshold. It also makes it possible that the nominator turns negative. The denominator of the third term is positive. The first term $\Gamma\left(\hat{\Pi}\left(q,\alpha\right)\right)$ gives a probability of member states' influence on Commission proposal on condition that the ideal point of the pivotal player is on $(0,\beta]$ and the latter $\Gamma\left(\hat{Q}\left(q,\alpha\right)\right)$ is the status quo bias.

Let us next analyse how α affects the optimal majority rule. Figs 2 and 3 give the squares of the difference between the expected outcome and expected ideal point, $\left(E\Omega^{*}-E\lambda_{i}\right)^{2}:=L^{2}\left(m\right)$ of a representative member state i as functions of α. We have chosen five values of α between zero and -1 on the intervals of one quarter. The maximum value of $L^{2}\left(m\right)$ is 0.25β. In figures 2 and 3, β is normalised to one. Intuitively, there is a trade-off between decision-making efficacy, i.e. smooth passage of proposals, and the discrepancy between the outcome and a single player's ideal policy. Basically, the former can be improved by lowering the majority rule but the latter is minimised by using unanimity. It turns out that regardless of the type of the designers we obtain a u-shaped relationship between chosen majority rule and the expected loss. In self-interested design, the turning point (minimum loss) is not, however, necessarily obtained in majority rules when α is small enough. This means that the efficacy effect tends to dominate when the risk of status quo increses. The optimal self-interested majority rules are $12, 11, 9, 8, 8$ when a decreases from zero to -1 respectively. In Fig 2, the shapes of L^{2} are monotonically decreasing when $\alpha=-0.75$ or $\alpha=-1$. Moreover, the relationship is rather skewed: majority rules that are higher than the self-interested optimum create great losses but majority rules that are lower than the optimum do not.

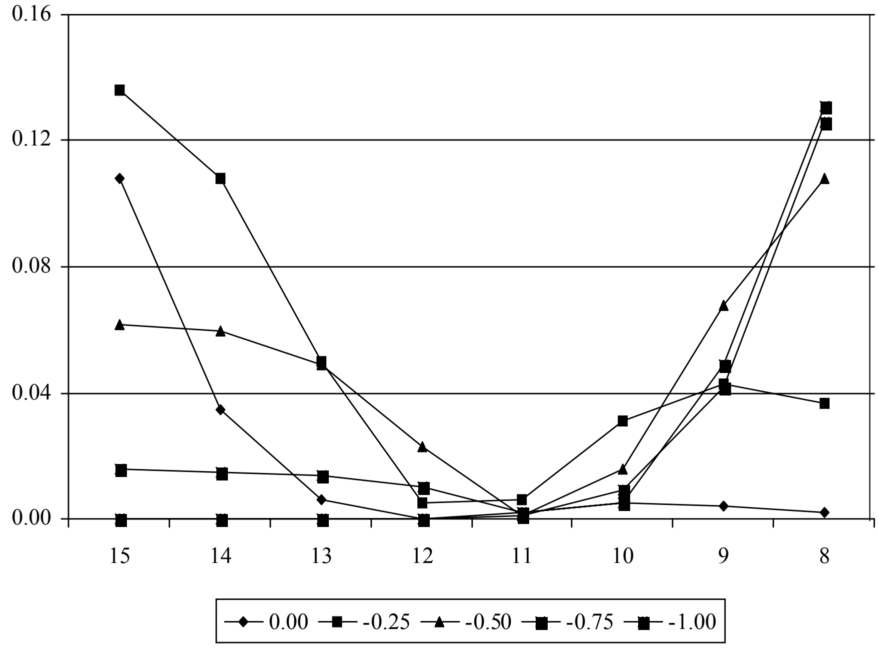

Fig. 3. Benevolent design: expected loss under different majority rules and α-values, $n = 15$

In benevolent design, the u-shaped relationship can be seen more clearly (see Fig. 3). The $\frac{\alpha}{\beta}$ term in the nominator also stabilises the optimum. In benevolent design, optimal majority rules are $12, 12, 11, 11, 15$ when a decreases from zero to -1 respectively. The optimum is thus not monotonically decreasing when the risk of 'no gains from any proposal' increases. Fig. 3 shows that the out of optimum losses are more symmetric in benevolent than in self-interested design but also here we obtain somewhat skewed shapes of L^2. The losses due to too high majority thresholds tend to be smaller than the losses due to too low majority thresholds. It is also interesting the the overall losses tend to be smaller for benevolent designers than for self-interested designers. Even the extreme case of an absolute majority and $\alpha = -1$ yields L^2 value less than 0.15 whereas for benevolent designers unanimity or close to unanimity rules give L^2 values more than 0.2 for most α-values.

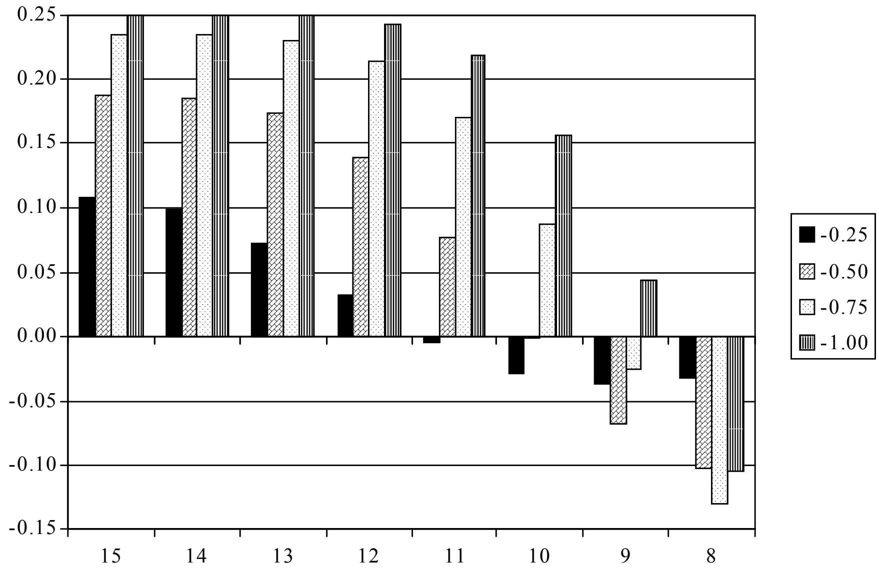

Fig. 4. Benevolent and self interested design: expected average loss under different majority rules and α-values, $n = 15$

As can be seen from the efficiency requirements in propositions 2 and 3, the discrepancy between self-interested and benevolent design becomes more severe when α decreases. This has been illutrated in Fig 4, which gives the aritmethic averages $\frac{L_S^2+L_B^2}{2}$ using the same five values of α as in Figs 2 and 3. Subscript S denotes self-interested and B benevolent. Figs. 2 and 3 clearly demonstrate that for majority thresholds 11 or higher the losses of self-interested designers are significantly bigger than for benevolent designers. In this example, 8 is the only majority rule that yields greates losses for benevolent designers regardless of α. Table 1 summarises the results of Figs. 2,3 and 4. It gives the five α-values as above, optimal majority rules in benevolent and self interested designs and the majority rules m_A^* that minimise the average loss.

Table 1 shows an interesting pattern. If $\alpha = 0$ the designs are the same. Given that $n = 15$, then 12 is the optimal and efficient majority threshold. Thereafter, when α decreases the compromise which minimises the average loss is closer to self-interested optimum than to benevolent optimum. On average it seems that the efficacy effect dominates. When the risk of status quo increses it is on average optimal to lower the threshold. Interpreted in slightly different manner, suppose that all member states aim for benevolent design but the expected gains from integration vary. Assume that there is a core group of member states having $\alpha = 0$ and the rest of the member states having some $\alpha < 0$. Then to minimise the average loss, optimal majority threshold would be closer to the core countries than to the countries having $\alpha < 0$. Despite the fact that the expected loss that is due to a lower than optimal majority rule is likely to be rather small the difference between the optimal and average loss minimising thresholds might be large enough to not to be acceptable. For instance, if $\alpha = -0.75$ optimal $\frac{m}{n}$ is 60 per cent in average loss minimising design and approximately 73 per cent in benevolent design.

Table 1.
Optimal majority thresholds and majority rules that minimise average loss

α	m_B^*	m_S^*	m_A^*
-0.00	12	12	12
-0.25	12	11	11
-0.50	11	9	10
-0.75	11	8	9
-1.00	15	8	9

6. Conclusions

This paper has dealt with optimal design of voting rules in an integration treaty. The main emphasis of the paper is on majority rules and on how different goals of design affect optimality. One of the main contributions of the paper is that it combines treaty design and legislative procedure using the rules designed in the treaty. Technically, the paper contributes to the grey area research between spatial voting games and a priori voting power analysis that are the major lines of research in EU decision making.

The paper shows that when the probability of 'no gains from integration' is positive it is, in general, impossible to choose an efficient majority rule. This holds as far as the player set is finite and countable. The welfare loss is not, however, necessarily very big if the likelihood of potential integration gains is sufficiently high. Then the optimal, i.e. as close to efficient as possible, design leads to outcomes that have an expectation close to the expected ideal policies of member states. If the designers favour only pro-integrationist views an increase in status quo risk makes optimal majority threshold lower. If anti-integrationist views are taken into account an increase in status quo risk has a smaller and potentially non-monotonic impact. If the likelihood of no gains from integration increases enough the discrepancy between the two designs becomes wider.

If member states differ in terms of their likely gains from integration the optimal designs of voting rules are different regarding whether the anti-integrationist views are or are not taken into account.In the former case optimal majority rule tends to be rather stable but in the latter case there is a tendency to offset increased status quo risk by setting a lower majority threshold. The compromise solution is closer to the latter, which makes it possible that the countries with less likely gains are better-off when they stay out. The paper demonstrates, however, that when the likelihood of no gains from integration is relatively small the discrepancy between the two designs is rather small as well.

The Nice Treaty of the European Union that was signed in February 2001 defines explicit rules for the so called enhanced cooperation. It allows sufficient number of member states to proceed in integration to areas where not necessarily all member states are likely to gain equally. In this paper, we have demonstrated how the need for enhanced cooperation might emerge. According to the Nice Treary enhanced cooperation projects make decisions like the decisions are made in common policies but among a smaller group. If these rules are a result of self-interested or average loss minimising design and the general design is a result of small likelihood of 'no gains from integration' this is what the results of this paper suggest.

The model of this paper has several assumptions that can be extended within the context of this model. Member states that sign the integration treaty are assumed to be a priori similar and only the state of nature makes differences between their preferences. Generally countries may have different attitudes also towards the treaty design, which would make them a priori different. This can be done by introducing differentiated a priori preference distributions, which would certainly affect to treaty design part of the model. At legislation level, the model has at least two assumptions that can be extended. They are perfect information and the fact that decision-making rule is closed, hence proposals cannot be amended. Both aspects would change the division of power between the agenda-setter and the decision-maker. Another extension would be to derive explicitly the conditions for participation in sub-optimal integration arrangement. However, a more careful analysis of these questions is left for future research.

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Appendix 1

Derivation of equation (4.2)
In general, we can write the following probability

$$\begin{aligned} P\left(\lambda_{(n-m+1)} \leq x\right) &= \int_0^x n \binom{n-1}{n-m} F_{\hat{\lambda}}(s)^{n-m} \left[1 - F_{\hat{\lambda}(s)}\right]^{m-1} f_{\hat{\lambda}}(s)\, ds \\ &= \frac{1}{B\left(n-m+1, m\right)} \int_0^x F_{\hat{\lambda}}(s)^{n-m} \left[1 - F_{\hat{\lambda}(s)}\right]^{m-1} f_{\hat{\lambda}}(s)\, ds. \end{aligned}$$

Specifically, let us consider i.i.d. U(0,1) random variables $\hat{\lambda}_1, \ldots, \hat{\lambda}_n$, their $n-m+1$ order statistic $\hat{\lambda}_{(n-m+1)}$, and $\hat{\Pi}$ which is independently $U(\hat{Q}, \frac{\frac{1}{2}\beta-\alpha}{\beta-\alpha})$-distributed with density $f_{\hat{\Pi}}$. We get

$$\begin{aligned} & P(\hat{\lambda}_{(n-m+1)} \leq \hat{\Pi}\ ) \\ = & \int_{-\infty}^{\infty} P(\hat{\lambda}_{(n-m+1)} \leq x)\, f_{\hat{\Pi}}(x)\, dx \\ = & \int_{-\infty}^{\frac{-\alpha}{\beta-\alpha}} 0 \cdot f_{\hat{\Pi}}(x)\, dx + \int_{\frac{-\alpha}{\beta-\alpha}}^{\frac{\frac{1}{2}\beta-\alpha}{\beta-\alpha}} P(\hat{\lambda}_{(n-m+1)} \leq x)\, f_{\hat{\Pi}}(x)\, dx + \int_{\frac{\frac{1}{2}\beta-\alpha}{\beta-\alpha}}^{\infty} 1 \cdot 0\, dx \\ = & \int_{\frac{-\alpha}{\beta-\alpha}}^{\frac{\frac{1}{2}\beta-\alpha}{\beta-\alpha}} \left[\int_0^x n \binom{n-1}{p-1} s^{n-m} \left[1-s\right]^{m-1} ds\right] \cdot \frac{2(\beta-\alpha)}{\beta}\, dx \end{aligned}$$

Pivotal player has an effect on outcome when $\hat{\lambda}_{(n-m+1)} < \hat{\Pi}$. The probability of this event's occurrence is $P\left(\hat{\lambda}_{(n-m+1)} < \frac{1}{2}\hat{\sigma}\right)$ where $\hat{\sigma} \sim U\left(\hat{Q}, 1\right)$, hence $\frac{1}{2}\hat{\sigma}$ is $U(\frac{-\alpha}{2(\beta-\alpha)}, \frac{1}{2})$-distributed with density $f_{\frac{1}{2}\hat{\sigma}}$. Using this we can write the expected outcome as follows

$$
\begin{aligned}
E\hat{\Omega} &= 2EP\left\{0 < \hat{\lambda}_{(n-m+1)} < \hat{\Pi}\right\}\tilde{\lambda}_{(n-m+1)} + \left[1 - P\left\{\hat{\lambda}_{(n-m+1)} < \hat{\Pi}\right\}\right] E\hat{\sigma} \\
&= 2\int\limits_{-\infty}^{\infty} P\left\{\hat{\lambda}_{(n-m+1)} < x\right\} f_{\hat{\Pi}}(x)\,dx - P\left\{\hat{\lambda}_{(n-m+1)} < \frac{-\alpha}{(\beta-\alpha)}\right\} E\hat{\lambda}_{(n-m+1)} \\
&\quad + \left[1 - \int\limits_{-\infty}^{\infty} P\left\{\hat{\lambda}_{(n-m+1)} < x\right\} f_{\hat{\Pi}}(x)\,dx\right] E\hat{\sigma} \\
&= 2\left(\frac{n-m+1}{n+1}\right)\int\limits_{\frac{-\alpha}{\beta-\alpha}}^{\frac{\frac{1}{2}\beta-\alpha}{\beta-\alpha}} \left[\int_0^x n\binom{n-1}{n-m} s^{p-1}\left[1-s\right]^{n-p} ds\right] \cdot \frac{2(\beta-\alpha)}{\beta}\,dx \\
&\quad -2\left(\frac{n-m+1}{n+1}\right)\int_0^{\frac{-\alpha}{(\beta-\alpha)}} n\binom{n-1}{n-m} s^{p-1}\left[1-s\right]^{n-p} ds \\
&\quad + \left(\frac{\frac{1}{2}\beta}{\beta-\alpha}\right)\left[1 - \int\limits_{\frac{-\alpha}{\beta-\alpha}}^{\frac{\frac{1}{2}\beta-\alpha}{\beta-\alpha}} \left[\int_0^x n\binom{n-1}{n-m} s^{p-1}\left[1-s\right]^{n-p} ds\right] \cdot \frac{2(\beta-\alpha)}{\beta}\,dx\right]
\end{aligned}
$$

Suppose that the number of countries that participate in integration is at least three. Let us denote the cumulative distribution function of *beta* $(n-m+1, m)$ by

$$
\begin{aligned}
G(x) &= \frac{1}{B(n-m+1,m)}\int_0^x s^{n-m}(1-s)^{m-1}\,ds \\
&= \int_0^x n\binom{n-1}{n-m} s^{p-1}\left[1-s\right]^{n-p} ds
\end{aligned}
$$

where $B(n-m+1,m) = \frac{\Gamma(n+1)}{\Gamma(n-m+1)\Gamma(m)}$ and $\Gamma(k) = (k-1)!$, *the gamma-*

function. The expected equilibrium outcome can be written explicitly as follows

$$
\begin{aligned}
E\hat{\Omega}^{*} &= 2\left(\frac{n-m+1}{n+1}\right)\int\limits_{\frac{-\alpha}{\beta-\alpha}}^{\frac{\frac{1}{2}\beta-\alpha}{\beta-\alpha}} \frac{2(\beta-\alpha)}{\beta}G\left(x\right)\,dx \\
&\quad -2\left(\frac{n-m+1}{n+1}\right)G\left(\frac{-\alpha}{\beta-\alpha}\right) \\
&\quad +\left(\frac{\frac{1}{2}\beta}{\beta-\alpha}\right)\left[1-\int\limits_{\frac{-\alpha}{\beta-\alpha}}^{\frac{\frac{1}{2}\beta-\alpha}{\beta-\alpha}} \frac{2(\beta-\alpha)}{\beta}G\left(x\right)\,dx\right].
\end{aligned}
$$

which is equation (4.2). In general, to compute the first and the third term of this expectation we need to integrate the cumulative distribution function $G\left(x\right)$.

Appendix 2

Ex ante efficiency requires $E\Omega^* = \frac{\alpha+\beta}{2}$. This implies

$$
\begin{aligned}
E\hat{\Omega}^* &= 2\beta\left(\frac{n-m+1}{n+1}\right)\left[\Gamma\left(\hat{\Pi}\right)-\Gamma\left(\hat{Q}\right)\right]+\left(\frac{1}{2}\beta\right)\left[1-\Gamma\left(\hat{\Pi}\right)\right]=\frac{\alpha+\beta}{2} \\
&\Leftrightarrow \\
&\left(\frac{n-m+1}{n+1}\right)\left[\Gamma\left(\hat{\Pi}\right)-\Gamma\left(\hat{Q}\right)\right]=\frac{\alpha}{4\beta}+\frac{1}{4}\Gamma\left(\hat{\Pi}\right) \\
&\Leftrightarrow \\
&\left(\frac{n-m+1}{n+1}\right)=\frac{1}{4}\left[\frac{\alpha+\beta\Gamma\left(\hat{\Pi}\right)}{\beta}\right]\left[\Gamma\left(\hat{\Pi}\right)-\Gamma\left(Q\right)\right]^{-1} \\
&\Leftrightarrow \\
&\frac{n(n+1)}{n(n+1)}-\frac{nm}{n(n+1)}=\frac{1}{4}\left[\frac{\alpha+\beta\Gamma\left(\hat{\Pi}\right)}{\beta}\right]\left[\Gamma\left(\hat{\Pi}\right)-\Gamma\left(\hat{Q}\right)\right]^{-1} \\
&\Leftrightarrow \\
&\frac{n+1}{n}-\frac{m}{n}=\frac{1}{4}\left(\frac{n+1}{n}\right)\left[\frac{\alpha+\beta\Gamma\left(\hat{\Pi}\right)}{\beta}\right]\left[\Gamma\left(\hat{\Pi}\right)-\Gamma\left(\hat{Q}\right)\right]^{-1} \\
&\Leftrightarrow \\
\frac{m}{n} &= \left(\frac{n+1}{n}\right)\left[1-\frac{1}{4}\frac{\left[\frac{\alpha+\beta\Gamma\left(\hat{\Pi}\right)}{\beta}\right]}{\left[\Gamma\left(\hat{\Pi}\right)-\Gamma\left(\hat{Q}\right)\right]}\right] \\
&\Leftrightarrow \\
\frac{m}{n} &= \frac{3}{4}+\frac{3}{4n}-\frac{\left[\frac{\alpha}{\beta}+\Gamma\left(\hat{Q}\right)\right]}{\left[\Gamma\left(\hat{\Pi}\right)-\Gamma\left(\hat{Q}\right)\right]}\left(\frac{n+1}{n}\right)
\end{aligned}
$$

which completes the proof.

Appendix 3

Now, ex ante efficiency requires $E\Omega^* = \frac{\beta}{2}$. In re-scaled policy space we get

$$\begin{aligned} E\hat{\Omega}^* &= 2\beta\left(\frac{n-m+1}{n+1}\right)\left[\Gamma\left(\hat{\Pi}\right)-\Gamma\left(\hat{Q}\right)\right]+\frac{1}{2}\beta\left[1-\Gamma\left(\hat{\Pi}\right)\right]=\frac{\beta}{2} \\ &\Leftrightarrow \\ &\left(\frac{n-m+1}{n+1}\right)\left[\Gamma\left(\hat{\Pi}\right)-\Gamma\left(\hat{Q}\right)\right]=\frac{1}{4}\Gamma\left(\hat{\Pi}\right) \end{aligned}$$

Using the proof in appendix 1 we get

$$\begin{aligned} \frac{n+1}{n}-\frac{m}{n} &= \frac{1}{4}\left(\frac{n+1}{n}\right)\left[\frac{\beta\Gamma\left(\hat{\Pi}\right)}{\beta}\right]\left[\Gamma\left(\hat{\Pi}\right)-\Gamma\left(\hat{Q}\right)\right]^{-1} \\ &\Leftrightarrow \\ \frac{m}{n} &= \left(\frac{n+1}{n}\right)\left[1-\frac{1}{4}\frac{\Gamma\left(\hat{\Pi}\right)}{\left[\Gamma\left(\hat{\Pi}\right)-\Gamma\left(\hat{Q}\right)\right]}\right] \\ &\Leftrightarrow \\ \frac{m}{n} &= \frac{3}{4}+\frac{3}{4n}-\frac{\Gamma\left(\hat{Q}\right)}{\left[\Gamma\left(\hat{\Pi}\right)-\Gamma\left(\hat{Q}\right)\right]}\left(\frac{n+1}{n}\right) \end{aligned}$$

which completes the proof.