DISCUSSION PAPER SERIES

No. 3038

NO PAIN, NO GAIN? THE SIMPLE ANALYTICS OF EFFICIENT DISINFLATION IN OPEN ECONOMIES

Willem H Buiter and Clemens Grafe

INTERNATIONAL MACROECONOMICS

Centre for Economic Policy Research

www.cepr.org

Available online at: **www.cepr.org/pubs/dps/DP3038.asp**

ISSN 0265-8003

NO PAIN, NO GAIN? THE SIMPLE ANALYTICS OF EFFICIENT DISINFLATION IN OPEN ECONOMIES

Willem H Buiter, EBRD and CEPR
Clemens Grafe, EBRD

Discussion Paper No. 3038
November 2001

Centre for Economic Policy Research
90–98 Goswell Rd, London EC1V 7RR, UK
Tel: (44 20) 7878 2900, Fax: (44 20) 7878 2999
Email: cepr@cepr.org, Website: www.cepr.org

CEPR Discussion Paper No. 3038

November 2001

ABSTRACT

No Pain, No Gain? The Simple Analytics of Efficient Disinflation in Open Economies*

This Paper studies the design of efficient disinflation programmes in open economies using the sacrifice ratio; that is, the cumulative additional unemployment or cumulative lost output required to achieve a 1% sustained reduction in the rate of inflation, as the metric of efficiency. The 'new Keynesian' Phillips curve first proposed by Calvo has a zero sacrifice ratio: costless disinflation is possible, because the inflation process is purely forward-looking. There is inertia or rigidity in the price level but not in the rate of inflation. More interesting inflation kernels for which current inflation is partly forward-looking and partly backward-looking have a positive sacrifice ratio. Real exchange rate appreciation early in the disinflation process may raise the sacrifice ratio relative to a policy that keeps the real exchange rate constant. The sacrifice ratio is lower under gradualism than under 'cold turkey'. Efficient disinflation policies may, however, be time-inconsistent and therefore not credible.

JEL Classification: E31 and F41
Keywords: disinflation, new Keynesian Phillips curve and sacrifice ratio

Willem H Buiter
Chief Economist
EBRD
One Exchange Square
LONDON
EC2A 2JN
Tel: (44 20) 7338 6805
Fax: (44 20) 7338 6111
Email: buiterw@ebrd.com

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=100351

Clemens Grafe
Economist
EBRD
One Exchange Square
LONDON
EC2A 2JN
Tel: (44 20) 7338 6072
Fax: (44 20) 7338 6110
Email: grafec@ebrd.com

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=138749

* The views and opinions expressed in this Paper are those of the authors. They do not necessarily represent the views of opinions of the European Bank for Reconstruction and Development.

Submitted 04 October 2001

1 Introduction

The purpose of this paper is to explore what the literature of the past 40-odd years can tell us about the design of efficient disinflation programmes in open economies suffering from moderate inflation. The analysis is restricted to those implications that can be drawn solely from a consideration of the wage and price inflation 'block'- what we shall call the *inflation kernel*. Surprisingly, it turns out that it is possible to say quite a bit about the cost of disinflation, and about ways of minimising that cost, simply by considering the inflation kernel. Other conclusions, for instance those related to the role of the exchange rate in the disinflation process can be reached on the basis of the inflation kernel plus a rather uncontroversial assumption about the long-run relationship between inflation and the value of the real exchange rate.

Because of this focus on the inflation kernel, there is no scope for considering such key issues as the institutional arrangement for the conduct of monetary and fiscal policy - central bank operational and/or target independence, budgetary rules etc. Nevertheless, our analysis does point to possible problems, when the inflation process is at least in part forward-looking, with the credibility of policy announcements and with the ability to commitment to effective policy rules.

The paper studies how different kinds of nominal rigidities (price level inertia and inflation inertia) should affect the design of efficient disinflation strategies. Efficiency is measured by the sacrifice ratio, the undiscounted cumulative additional unemployment incurred (or output lost) in order to achieve a given sustained reduction in the rate of inflation. The various models of the inflation process considered in what follows are assumed to be structurally invariant under the kind of disinflation policies (or policy rules) considered in the paper. This assumption of a constant structure means that we must limit, in our interpretation of the models' properties, the range of inflationary experiences that we can hope to shed light on. With the possible exception of the New Classical model (and possibly of the New Keynesian model as well), the models studies here are only useful for the study of low and moderate inflations or deflations. For practical purposes, low inflation means any rate below 5% per annum, and moderate anything below, say, 20% or 25% per annum. High inflation, let alone hyperinflation, would destroy the nominal rigidities in the wage and price contracting processes that are central in what follows. During periods of high inflation or hyperinflation, the coefficients representing the structural dependence of current inflation on past inflation would turn out to be endogenous variables rather than constant parameters. If there are fixed costs associated with changing the frequency with which wage and price contracts are revised, or with changing the way prices and wages are mapped from the economic environment within which firms and workers operate, the constancy assumed for the parameters in our models can be assumed to reflect threshold effects: unless inflation is assumed to rise sufficiently high for a sufficiently long period of time, the contracting structure remains constant.

The paper considers a sequence of models, differentiated only by the nature of the core inflation process, that is, by the specification of the augmentation term in a conventional-looking price Phillips curve. First, we consider a number of backward-looking core inflation processes: exogenous, static, adaptive and generalised adaptive. Next comes the New Classical Phillips curve or supply function. Here the gap between the actual and the natural rate of unemployment depends on the current price surprise. Expectations are rational. Finally we consider core inflation processes which imply that current inflation is a function both of past inflation and of anticipated future inflation. We shall call these 'mixed models'. This includes the New Keynesian Phillips curve as the special case for which current inflation depends on anticipated future inflation but not on lagged inflation. There are interesting differences within the class of mixed models between those for which the weight on future anticipated inflation exceeds that on past inflation ('mainly forward-looking mixed models) and those for which the opposite holds. For the mainly backward-looking mixed model - the model we consider to be the most relevant and interesting for policy purposes - there is an interesting conflict between efficient and credible disinflation strategies. Finally, we consider the effect on the sacrifice ratio of hysteresis in the natural rate of unemployment.

2 Wage and Price Dynamics in an Open Economy

We base our analysis on a *prima-facie* old-fashioned augmented Phillips curve model of an open economy. It turns out that both the New-Classical and the New-Keynesian inflation kernels are special cases of the final, quasi-reduced form of our model. We start with a conventional wage Phillips curve. Money wage inflation depends inversely on the unemployment rate, u, our index of deflationary pressure in the labour market. Money wage inflation also changes one-for-one with 'core inflation'. A vast range of alternative views on the nature of the wage-price process can be characterised transparently through different interpretations and specifications of 'core inflation'. 'Core inflation' refers to persistence or momentum in the inflationary process. This can be due to a variety of behavioural (e.g. expectational) and institutional (e.g. contracting) features of the wage-price setting process. Formally, until the next section, 'core inflation' is simply part of the drift parameter in the wage inflation equation. We view the wage inflation equation as representing the behaviour of workers searching and negotiating in the labour market. The core inflation term in the open economy wage Phillips curve should therefore be interpreted as the core inflation rate of a cost -of-living index, that is something like the consumer price index (CPI) or the retail price index (RPI). The rate of inflation of the consumer price index is denoted $\tilde{\pi}(t) \equiv \Delta\tilde{p}(t)$, where $\tilde{p}$ is the (logarithm of the) cost-of living index. Core CPI inflation is denoted $\tilde{\pi}_c$. The rate of inflation of the 'gross' price of domestic output (not the GDP deflator, because domestic output may be produced using imported intermediate and raw materials inputs) is denoted $\pi \equiv \Delta\tilde{p}$, where p is the (logarithm of the) domestic producer price level. The rate of inflation of world prices (in foreign currency) is denoted $\pi^* \equiv \Delta p^*$ where p^* is the (logarithm of the) foreign price level; $\pi_w \equiv \Delta w$, where w is the (logarithm of the) nominal wage, denotes the growth rate of money wages; the depreciation rate of the nominal exchange rate is denoted $\varepsilon \equiv \Delta e$ where e is the (logarithm of the) nominal spot exchange rate (the number of units of home currency per unit of foreign currency). The coefficient γ (the drift term in the wage Phillips curve), can be interpreted as the target growth of real wages pursued by workers.

$$\begin{aligned} \pi_w &= \gamma - \alpha u + \tilde{\pi}_c \\ \alpha &> 0 \end{aligned} \tag{1}$$

$$\begin{gathered} \tilde{\pi} = \omega\pi + (1-\omega)(\pi^* + \varepsilon) \\ 1 \geq \omega \geq 0 \end{gathered} \tag{2}$$

The share of imports in the consumption bundle is denoted ω.

An empirically plausible price equation makes the domestic producer price a constant proportional mark-up on 'normal unit variable cost'. Unit variable cost is the sum of unit labour cost and unit import cost per unit of domestic gross output. Variables with *overbars* denote the normal, that is, cyclically adjusted, values of the corresponding variable. The share of labour cost in total variable cost is λ, the growth rate of average labour productivity is γ_L and the growth rate of average import productivity is γ_N The markup pricing model implies the following equation for the rate of inflation of domestic producer prices:

$$\pi = \lambda(\bar{\pi}_w - \bar{\gamma}_L) + (1-\lambda)(\bar{\pi}^* + \bar{\varepsilon} - \bar{\gamma}_N) \tag{3}$$

The open economy Phillips curve is therefore given by:

$$\begin{array}{l} \tilde{\pi} = \tilde{\pi}_c - \alpha u + \gamma - \gamma_L - \frac{\omega(1-\lambda)}{\omega\lambda}\gamma_N + \frac{(1-\omega\lambda)}{\omega\lambda}(\pi^* + \varepsilon - \tilde{\pi}) \\ +\bar{\pi}_w - \bar{\gamma}_L - (\pi_w - \gamma_L) + \frac{\omega(1-\lambda)}{\omega\lambda}\left[\bar{\pi}^* + \bar{\varepsilon} - \bar{\gamma}_N - (\pi^* + \varepsilon - \gamma_N)\right] \end{array} \tag{4}$$

Note that, *cet. par.*, the rate of inflation is higher when normal unit labour cost inflation exceeds current unit labour cost inflation and/or when normal unit import cost inflation exceeds current unit import cost inflation.

Inflationary pressures are also lower if the country's terms of trade are improving (that is, if its price competitiveness is worsening). When domestic inflation exceeds world inflation $\tilde{\pi} > \pi^* + \varepsilon$, the worker's real consumption wage (which is defined in terms of the cost-of-living index $\tilde{p}$, rises relative to the real product wage that determines the demand for labour by firms, which is defined in terms of the domestic producer price index p. Workers' real wage aspirations can be satisfied to a greater extent through relatively cheaper import prices rather than through higher money wages. This has a dampening effect on inflation, that is of course only present as long as the terms of trade are improving. If and when the terms of trade improvement unwinds and $\tilde{\pi} < \pi^* + \varepsilon$, inflationary pressures are enhanced.

The (long-run) natural rate u_N, is the unemployment rate that prevails when actual inflation equals core inflation, normal unit variable cost inflation equals actual unit variable cost inflation *and* the real exchange rate is constant. That is, $\tilde{\pi} = \tilde{\pi}_c$, $\gamma_L = \bar{\gamma}_L$, $\bar{\pi}_w = \pi_w$, $\bar{\pi}^* = \pi^*$, $\bar{\varepsilon} = \varepsilon$, $\bar{\gamma}_N = \gamma_N$,and $\pi^* + \varepsilon = \pi$. It follows that

$$u_N \equiv \alpha^{-1}\left[\gamma - \gamma_L - (1-\lambda)\lambda^{-1}\gamma_N\right] \tag{5}$$

The natural rate of unemployment will be lower the greater the value of α,,the responsiveness of wage inflation to unemployment, the lower the target growth rate of real wages, γ, and the higher the growth rates of labour productivity, γ_L and import productivity, γ_N.

The open economy price Phillips curve can now be written as

$$\begin{array}{l} \tilde{\pi} = \tilde{\pi}_c - \alpha(u - u_N) + \frac{(1-\omega\lambda)}{\omega\lambda}(\pi^* + \varepsilon - \tilde{\pi}) \\ + \left[\bar{\pi}_w - \bar{\gamma}_L - (\pi_w - \gamma_L) + \frac{\omega(1-\lambda)}{\omega\lambda}\left[\bar{\pi}^* + \bar{\varepsilon} - \bar{\gamma}_N - (\pi^* + \varepsilon - \gamma_N)\right]\right] \end{array} \tag{6}$$

In what follows, we work with a slightly stripped-down version of 6. The expression $\left[\bar{\pi}_w - \bar{\gamma}_L - (\pi_w - \gamma_L) + \frac{\omega(1-\lambda)}{\omega\lambda}\left[\bar{\pi}^* + \bar{\varepsilon} - \bar{\gamma}_N - (\pi^* + \varepsilon - \gamma_N)\right]\right]$ is, like the deviation of the actual from the natural rate of unemployment, a cyclical variable (when the economy is in a downturn, normal unit variable costs increases are likely to be below actual unit variable cost increases (actual profit mark-ups decline). We can capture this as a weakening (but not a reversal) of the negative effect on inflation of a larger gap between the actual and the natural unemployment rate. Let $\rho \equiv e + p^* - \tilde{p}$ be the (logarithm of the) real exchange rate, a measure of competitiveness or the reciprocal of the terms of trade, and $\pi_\rho \equiv \varepsilon + \pi^* - \tilde{\pi}$ the growth rate of competitiveness or the growth rate of the real exchange rate. We therefore simplify 6 to

$$\tilde{\pi} = \tilde{\pi}_c - \beta(u - u_N) + \delta\pi_\rho \qquad (7)$$
$$\beta,\ \delta > 0$$

3 The Sacrifice Ratio and the core inflation process

One useful, operational measure of the output or employment cost of reducing inflation is the *sacrifice ratio.* This is defined as the *cumulative* increase in the unemployment rate (or cumulative reduction in output) required to achieve a one percentage point *sustained* reduction in the rate of inflation. Changes in unemployment at different dates are summed without any discounting. The principal reason for the absence of discounting is that, for the class of models we shall consider, it is simpler to calculate the undiscounted cumulative total. The approach says nothing about the nature or magnitude of the gains from achieving a sustained reduction in inflation. It only studies the cumulative employment or output foregone in order to achieve such as sustained reduction.

For our augmented open economy Phillips curve model, the sacrifice ratio depends on four key features. First, the responsiveness of wage inflation to unemployment. Second, the determinants of the core inflation rate, and in particular the degree of inertia or persistence in core inflation and the extent to which core inflation is forward-looking. Third, the behaviour of the real exchange rate from the beginning till the end of the disinflation process. Fourth, the determinants of the natural rate of unemployment, and especially the question as to whether the disinflation process itself can affect the natural rate, that is, whether the natural rate is *hysteretic.*

3.1 An Exogenous Natural Rate

In this subsection, the natural rate is treated as exogenous and, for simplicity, constant. Note that a permanent reduction in the actual rate of inflation has been achieved when the core rate of inflation equals the actual rate of inflation at the new lower level. When that happens, the actual rate of unemployment can be set equal to the natural rate again and the lower actual and core rates of inflation will persist forever.

3.1.1 Exogenous core inflation

The review of disinflation in open economy models with backward-looking core inflation found in the next subsections is essentially a restatement of Buiter and Miller [7]. The simplest approach to core inflation is to treat it as exogenous and, for simplicity, constant:

$$\tilde{\pi}_c = \bar{\tilde{\pi}}_c \tag{8}$$

This includes the original non-augmented Phillips curve as a special case with $\tilde{\pi}_c = 0$. In a closed economy ($\delta = 0$), when core inflation does not respond at all to past, present or anticipated future inflation or policy actions, a permanent increase in unemployment is required to ensure a permanent reduction in inflation. In this case, to get inflation lower by 1% for one period, unemployment that period will have to rise by β^{-1}%. To keep inflation at the new lower level, unemployment will have to stay at that higher level. The sacrifice ratio, σ, is therefore infinite in this case. In an open economy ($\delta > 0$), the same result holds, if the process of achieving a sustained reduction in the rate of inflation has no *permanent* effect on the level of the real exchange rate (that is, if $\sum_{t=1}^{\infty} \pi_\rho(t) = 0$). If, on the other hand, the process of achieving a sustained reduction in the rate of inflation is associated with an permanent appreciation of the real exchange rate (that is, if $\sum_{t=1}^{\infty} \pi_\rho(t) < 0$), the sacrifice ratio will be lower. Note, however, that only a process of permanent further depreciation (an infinite real appreciation of the level of the real exchange rate) is required to keep the sacrifice ratio finite, if core inflation is exogenous. Equation 9 provides the general expression for the sacrifice ratio when core inflation is exogenous. Assume the reference path for unemployment is $u = u_N$. The initial period is $t = 1$. Note that

$$\sum_{t=1}^{\infty}(u(t) - u_N) = -\beta^{-1} \sum_{t=1}^{\infty} \left[\tilde{\pi}(t) - \bar{\tilde{\pi}}_c\right] + \frac{\delta}{\beta} \sum_{t=1}^{\infty} \pi_\rho(t) \tag{9}$$

The sacrifice ratio is therefore given by

$$\sigma = -\beta^{-1} \lim_{j \to \infty} j + \frac{\delta}{\beta} \sum_{t=1}^{\infty} \pi_\rho(t) \tag{10}$$

We will maintain in what follows that the process of achieving a sustained reduction in the rate of inflation has no permanent effect on the level of the real exchange rate, that is

$$\sum_{t=1}^{\infty} \pi_\rho(t) = 0 \tag{11}$$

Assumption 11 asserts that, in the long run, our economy has classical features. In the short run, with a floating nominal exchange rate, a process of disinflation is likely to be associated with an appreciation of the real exchange rate. Many models, including the 'overshooting' models have this property.[1] What 11 asserts is that such short-run anti-inflationary benefits

[1] We can provide a quick-and-dirty simple macromodel that supports these conclusions. Let m be the logarithm of the nominal money stock, i the domestic short nominal interest rate, i^* the exogenous and constant world short nominal interest rate, y the logarithm of real GDP, and f an exogenous demand shifter, which could represent fiscal shocks or shocks to external demand.

$$\begin{aligned} m - \tilde{p} &= k(y + p - \tilde{p}) - \xi i \\ k, \xi &> 0 \end{aligned} \tag{LM}$$

$$\begin{aligned} y &= -\zeta_0(i - \tilde{\pi}) + \zeta_1 \rho + f \\ \zeta_0, \zeta_1 &> 0 \end{aligned} \tag{IS}$$

$$\begin{aligned} u &= \phi_0 - \phi_1 y \\ \phi_1 &> 0 \end{aligned} \tag{Okun's Law}$$

$$i(t) = i^* + E_t \varepsilon(t+1) \tag{UIP}$$

$$\Delta m \equiv \mu$$

The foreign price level is exogenous. For simplicity the world rate of inflation can be takes as constant. The growth rate of the nominal money stock is the policy instrument. Assume that, at least in the long run, the growth rate of the nominal money stock is constant.

of real exchange rate appreciation have to be handed back in at some time later during the disinflation process. It does not affect the (undiscounted) cumulative unemployment cost of achieving a sustained reduction in the rate of inflation.

With exogenous core inflation and an exogenous natural rate, the sacrifice ratio is infinite

$$\sigma = \infty$$

3.1.2 Static core inflation

We next consider the case where this period's core rate of inflation equals last period's actual rate of inflation:[2]

$$\tilde{\pi}_c(t) = \tilde{\pi}(t-1) \tag{12}$$

In this case

$$u(t) - u_N = -\frac{1}{\beta}\left[\tilde{\pi}(t) - \tilde{\pi}(t-1)\right] + \frac{\delta}{\beta}\pi_\rho(t) \tag{13}$$

Now we only need a temporary increase in unemployment in order to achieve a permanent reduction in inflation. In fact, by raising employment by β^{-1} for one period only (say during period 1), we can reduce inflation that period by 1%. The next period, (period 2) we can reduce unemployment back to its initial level (u_N). Core inflation in period 2 is the actual level of inflation during period 1. Inflation from period 2 on can therefore be kept permanently at the new lower level even though unemployment from period 2 on is back at the natural rate. From 13 it follows that.

Together with 42 and 43, and a rule for the nominal money stock, this constitutes a complete model that produces Ball's result. Note that, at the announcement date of the future monetary contraction (a permanent reduction in the future rate of growth of the nominal money stock, say), both m and p are predetermined. The initial stock of real mone balances is therefore. Lower future monetary growth increases future unemployment (temporarily). This reduces current inflation and increases the demand for real money balances. With the real money stock given, output has to go up and unemployment falls. The real exchange rate is unaffected. The closed economy version of the model has $\zeta_1 = 0$ and drops the UIP condition.

[2] One possible interpretation of core inflation is *expected* inflation.

$$\begin{aligned}\sum_{t=1}^{\infty}[u(t)-u_N] &= \beta^{-1}\sum_{t=1}^{\infty}[\tilde{\pi}(t)-\tilde{\pi}(t-1)]+\delta\beta^{-1}\sum_{t=1}^{\infty}\pi_\rho(t)\\ &= \beta^{-1}[\tilde{\pi}(\infty)-\tilde{\pi}(0)]+\delta\beta^{-1}\sum_{t=1}^{\infty}\pi_\rho(t)\end{aligned}$$

Assumption 11, that the disinflation process has no permanent effect on the level of the real exchange rate, then implies that the sacrifice ratio for this case is given by:

$$\sigma=\beta^{-1} \tag{14}$$

The sacrifice ratio will be lower the greater the responsiveness of inflation to the unemployment rate, β. As regards the sacrifice ratio, there are no gains to or losses from concentrating the unemployment in one period or spreading it out over many periods. Nor does it make any difference whether the unemployment is earlier or later. Of course, the benefits from lower inflation will accrue earlier if the unemployment costs are incurred earlier.

3.1.3 Adaptive core inflation

In the adaptive core inflation process, current core inflation is a convex combination of lagged core inflation and lagged actual inflation.

$$\tilde{\pi}_c(t)=\eta\tilde{\pi}(t-1)+(1-\eta)\tilde{\pi}_c(t-1), 0\leq\eta\leq 1 \tag{15}$$

Using 7 to eliminate the actual inflation rate from 15, we get

$$\tilde{\pi}_c(t+1)-\tilde{\pi}_c(t)=-\eta\beta\left[u(t)-u_N\right]+\eta\delta\pi_\rho(t)$$

Remember that a permanent 1% point change in the actual rate of inflation is achieved once there has been a change of 1% point in the core rate of inflation. A 1 point change in the core rate of inflation can, for instance, be achieved through a one-period increase in unemployment of $(\beta\eta)^{-1}$. More generally,

$$\begin{aligned}\sum_{t=1}^{\infty}[u(t)-u_N] &= -(\eta\beta)^{-1}\sum_{t=1}^{\infty}[\tilde{\pi}_c(t+1)-\tilde{\pi}_c(t)]+\frac{\delta}{\beta}\sum_{t=1}^{\infty}\pi_\rho(t)\\ &= -(\eta\beta)^{-1}[\tilde{\pi}(\infty)-\tilde{\pi}(0)]+\frac{\delta}{\beta}\sum_{t=1}^{\infty}\pi_\rho(t)\end{aligned}$$

Again making the assumption, given in 11, that a sustained reduction in the rate of inflation is not associated with any permanent change in the level of the real exchange rate, the sacrifice ratio for the case of adaptive core inflation is given by:

$$\sigma = (\beta\eta)^{-1} \tag{16}$$

The sacrifice ratio is lower the greater β, the responsiveness of inflation to unemployment and the greater η, the speed of adaptation of core inflation to past inflation. Again, neither the timing (sooner vs. later), nor the concentration of the unemployment (bunched in a few or even in a single period vs. spread out over many periods) affects the sacrifice ratio. Note that the two previous models of core inflation are special cases of the adaptive case.

3.1.4 Generalised adaptive core inflation

Now consider the case where core inflation is a more general distributed lag function of past actual rates of inflation

$$\tilde{\pi}_c(t) = A(L)\tilde{\pi}(t-1) = \sum_{i=0}^{\infty} a_i \tilde{\pi}(t-1-i) \tag{17}$$

We want the augmented Phillips curve to have the (long-run) *natural rate property*, that is,

$$A(1) \equiv \sum_{i=0}^{\infty} a_i = 1$$

This means that, if the actual rate of inflation is constant forever, the core rate of inflation will equal the actual rate of inflation. The long-run Phillips curve is vertical.

>From 7 and 17 it follows that

$$\tilde{\pi}(t) = A(L)\tilde{\pi}(t-1) - \beta\left[u(t) - u_N\right] + \delta\pi_\rho(t) \tag{18}$$

and that

$$\tilde{\pi}_c(t+1) = A(L)\tilde{\pi}_c(t) - A(L)\beta[u(t) - u_N] + A(L)\delta\pi_\rho(t) \tag{19}$$

It follows that

$$
\begin{aligned}
\sum_{t=1}^{\infty}[u(t)-u_N] &= -\beta^{-1}\sum_{t=1}^{\infty}\left[A(L)^{-1}\tilde{\pi}_c(t+1)-\tilde{\pi}_c(t)\right]+\frac{\delta}{\beta}\sum_{t=1}^{\infty}\pi_\rho(t) \qquad (20)\\
&= -\beta^{-1}\sum_{t=1}^{\infty}A(L)^{-1}[1-LA(L)]\tilde{\pi}_c(t+1)+\frac{\delta}{\beta}\sum_{t=1}^{\infty}\pi_\rho(t)
\end{aligned}
$$

Note again that, assuming 11 holds, the sacrifice ratio will be independent of the behaviour of the real exchange rate during the disinflation process. For these general adaptive core inflation processes, for which current core inflation can depend on inflation arbitrarily far in the past, the sacrifice ratio, that is, the cumulative unemployment cost of achieving a *sustained* reduction in the rate of inflation, can be approached quite intuitively by characterising the properties of sequences of unemployment that can bring the inflation rate from a given constant level π^* for $t < 0$ to another constant level π^{**} for $t \geq 0$. Tedious calculation shows that for this case (as indeed in general (from 20)), the sacrifice ratio is given by

$$
\sigma \equiv -\left(\frac{\sum_{t=0}^{\infty}[u(t)-u_N]}{\pi^{**}-\pi^*}\right)=\beta^{-1}\sum_{j=0}^{\infty}(1+j)a_j \qquad (21)
$$

$\sum_{j=0}^{\infty}(1+j)a_j$ is the mean lag of the lag operator $1-LA(L)$.

The three previous core inflation processes are special cases of 17. Exogenous core inflation is the special case where $a_i = 0$ for all i. Static core inflation is the special case where $a_0 = 1$ and $a_i = 0$ for $i > 0$. The adaptive core inflation process is the special case where $a_i = \eta(1-\eta)^i$. As expected, the sacrifice ratio is lower the lower β and the shorter the mean lag in the core inflation process.

Conclusion 1 *When the core inflation process is backward-looking, the sacrifice ratio is independent of the behaviour of the real exchange rate during the disinflation process. The sacrifice ratio depends only on the responsiveness of wage inflation to pressure in the labour market and on the mean lag in the inflation process.*

Conclusion 2 *Because the sacrifice ratio is independent of the speed and timing of the disinflation process when the core inflation process is backward-looking, it is efficient to disinflate immediately and speedily, as in that case the gains from lower inflation come sooner.*

Non-linearities Thus far, the effect of the gap between the actual and the natural unemployment rates on the rate of inflation (holding constant core inflation and the growth rate of the real exchange rate) has been assumed to be linear. The early work of Phillips and others postulated a strictly convex relationship like the one shown in 22 below

$$\begin{aligned} \tilde{\pi} &= \tilde{\pi}_c + f(u - u_N) + \delta\pi_\rho \\ f(0) &= 0 \\ f' &< 0 \\ f'' &> 0 \end{aligned} \tag{22}$$

The strict convexity of f provides a familiar argument for gradualism: disinflation is subject to diminishing returns. From the point of view of lowering the sacrifice ratio, it is therefore better to spread it out over multiple periods rather than concentrating it all in a single period of high unemployment. The empirical evidence on the importance and statistical significance of non-linearities in the (conditional) unemployment-inflation trade-off is ambiguous. From here on we restrict the analysis to the linear case.

3.1.5 Costless disinflation 1: the New Classical wonderland

The first example of a wage-price process for which costless disinflation is possible, is the open economy version of the 'surprise supply function' cum rational expectations, popularised by the New Classical macroeconomics literature of the 1970s and 1980s. In our open economy Phillips curve model, we obtain the New Classical variant by equating core inflation in period t with the rate of inflation expected, in period $t-1$, to prevail in period t, that is

$$\tilde{\pi}_c(t) = E_{t-1}\tilde{\pi}(t) \tag{23}$$

where E_{t-1} is the conditional expectation operator for expectations formed at time $t-1$. Rational, or model-consistent expectations imply that

$$\tilde{\pi}(t) = E_{t-1}\tilde{\pi}(t) + \varepsilon(t-1,t)$$

where $\varepsilon(t-1,t)$ is the rational forecast error in period t for forecasts made in period $t-1$. Since $\varepsilon(t-1,t)$ is a rational forecast error,

$$E_{t-1}\varepsilon(t-1,t)=0$$

Inflation is no longer a predetermined state variable. There is no structural inflation persistence or inertia. Inflation can adjust instantaneously and costlessly to credible announcements about future policy. It follows that the sacrifice ratio is zero: costless disinflation only requires credibility:

$$\sigma = 0$$

In a closed economy ($\delta = 0$), with a surprise supply function and rational expectations, policy cannot influence the first moment of the distribution of real output, employment or unemployment.[3] In an open economy, equilibrium unemployment can be affected by policy to the extent that policy (whether anticipated or unanticipated) can affect the rate of depreciation or appreciation of the real exchange rate.

$$u(t) = u_N - \beta^{-1}\varepsilon(t-1,t) + \beta^{-1}\delta\pi_\rho(t)$$

3.1.6 Mixed backward-looking and forward-looking core inflation

Neither the old-style, backward-looking core inflation models nor the New Classical wonderland model are satisfactory vehicles for analysing the inflation processes and the design of efficient disinflation policies in advanced industrial countries or in the advanced transition countries that stand on the threshold of EU accession. Policy-relevant models should incorporate the following two key features of the inflation process in these countries. First, the inflation rate is, in part, anchored in the past. For low and moderate of inflation rates, there is a structurally invariant degree of persistence or inertia in the inflation process. Second, the inflation process is in part forward-looking. Anticipations of future policy actions do influence current wage and price setting. Credibility matters for the sacrifice ratio but is not in general sufficient to achieve costless disinflation.

Equation 24 below makes current core inflation a convex combination of last period's inflation and of current expectations of next period's inflation. This specification is in the spirit, if not the letter, of Taylor-style staggered,

[3] Even with a surprise supply function, and rational expectations, it may still be possible to influence higher moments of the conditional and unconditional distribution of real variables. See e.g. Buiter [4].

overlapping contract models (see Taylor [13], [14]) The model of equation 24 is closest to the staggered overlapping *real* wage contracting variant of the Taylor model proposed by Buiter and Jewitt [6] and popularised by Fuhrer and Moore [9].

$$\begin{gathered}\tilde{\pi}_c(t) = (1-\theta)E_t\tilde{\pi}(t+1) + \theta\tilde{\pi}(t-1) \\ 0 \leq \theta \leq 1\end{gathered} \tag{24}$$

Substituting this into 7 yields

$$\tilde{\pi}(t) = (1-\theta)E_t\tilde{\pi}(t+1) + \theta\tilde{\pi}(t-1) - \beta[u(t) - u_N] + \delta\pi_\rho(t) \tag{25}$$

We will refer to the model of equation 25 as the *mixed* model. The version with $0 \leq \theta < \frac{1}{2}$ will be called the *mainly forward-looking* mixed model; the version with $\frac{1}{2} < \theta \leq 1$ will be referred to as the *mainly backward-looking* mixed model.

$>$From 25 it is clear that, since $\tilde{\pi}(t+1)$ will depend on $u(t+1)$ and on period $t+1$ expectations of $\tilde{\pi}(t+2)$, it is reasonable to postulate the following form for a solution that is both mathematically acceptable and economically meaningful:

$$\begin{aligned}\tilde{\pi}(t) \quad = \quad & A_0\tilde{\pi}(t-1) + B_0\left[u(t) - u_N\right] + \sum_{i=1}^{\infty} B_i\left[E_t u(t+i) - u_N\right] \\ & + C_0\pi_\rho(t) + \sum_{i=1}^{\infty} C_i E_t\pi_\rho(t+i) + S(t)\end{aligned} \tag{26}$$

The coefficients A_0, B_i, C_i, $i \geq 0$ and the process $S(t)$ are found by substituting the proposed solution into the model and equating coefficients between the resulting equation and the proposed solution.[4]. Everything on the right-hand-side of 26 except for $S(t)$ constitutes the 'minimal state, fundamental solution'. The term $S(t)$ denotes the remainder of the solution. It can either involve non-fundamental or 'sunspot' variables (that is, variables not present in 25) or non-minimal state representations involving the fundamental variables.

[4] Use is made of the 'Law of Iterated Projections', $E_t E_{t+j}(x) = E_t(x)$, $j \geq 0$, (the earlier expectation of a later expectation is the earlier expectation). This is an immediate application of a fundamental property of conditional expectations, provided that the information set conditioning expectations at an earlier date is not richer than the information set at a later date. We assume for simplicity that the current state is known, that is, $E_t\tilde{\pi}(t) = \tilde{\pi}(t)$.

The undetermined coefficients are solved for from:

$$A_0 = [1 - (1 - \theta)A_0]^{-1}\theta \tag{27}$$

$$B_j = -\left\{[1 - (1 - \theta)A_0]^{-1}(1 - \theta)\right\}^j [1 - (1 - \theta)A_0]^{-1}\beta, \quad j \geq 0 \tag{28}$$

$$C_j = \left\{[1 - (1 - \theta)A_0]^{-1}(1 - \theta)\right\}^j [1 - (1 - \theta)A_0]^{-1}\delta, \quad , j \geq 0 \tag{29}$$

$$E_t S_{t+1} = [1 - (1 - \theta)A_0](1 - \theta)^{-1}S_t \tag{30}$$

The two solutions for A_0 and the corresponding solutions for the other coefficients and processes are given in 31 and 32:[5]

$A_0 = \frac{\theta}{1-\theta}$	
$B_i = -(1-\theta)^{-1}\beta, \quad i \geq 0$	(31)
$C_i = (1-\theta)^{-1}\delta, \quad i \geq 0$	
$E_t S_{t+1} = S_t$	

$A_0 = 1$	
$B_i = -\left(\frac{1-\theta}{\theta}\right)^i \theta^{-1}\beta, \quad i \geq 0$	(32)
$C_i = (\frac{1-\theta}{\theta})^i \theta^{-1}\delta, \quad i \geq 0$	
$E_t S_{t+1} = \left(\frac{\theta}{1-\theta}\right) S_t$	

We have a linear rational expectations models with constant coefficients whose homogeneous part is a second-degree difference equation. When there are two roots, one of which is weakly stable (modulus less than or equal to 1) and one of which is unstable (modulus greater than 1), we would choose the stable one to drive the predetermined state variable. In equation 25, except when $\theta = 0,$ the rate of inflation, $\tilde{\pi}$, is a predetermined state variable. We want the homogeneous part of our proposed solution in equation 26, that is, $\tilde{\pi}(t) = A_0\tilde{\pi}(t-1)$, to be non-explosive.

[5] $\frac{\theta}{1-\theta}$ and 1 are the two roots of the homogenous equation of 25 $(1-\theta)E_t\tilde{\pi}(t+1) - \tilde{\pi}(t) + \theta\tilde{\pi}(t-1) = 0$.

For $A_0 = \frac{\theta}{1-\theta}$, A_0 increases monotonically with θ over the interval $0 \leq \theta \leq 1$. Also, $\theta > \frac{1}{2}$ implies $A_0 > 1$, and indeed $\lim_{\theta \uparrow 1} \left(\frac{\theta}{1-\theta}\right) = \infty$. We therefore use $A_0 = \frac{\theta}{1-\theta}$ for $\theta \leq \frac{1}{2}$ and $A_0 = 1$ for $\theta \geq \frac{1}{2}$.

For $\theta \leq \frac{1}{2}$, that is, for the mainly forward-looking model, the solution looks as follows:

$$\begin{aligned} \tilde{\pi}(t) &= \left[\frac{\theta}{1-\theta}\right] \tilde{\pi}(t-1) - (1-\theta)^{-1}\beta \sum_{i=0}^{\infty} E_t[u(t+i) - u_N] \\ &+ (1-\theta)^{-1}\delta \sum_{i=0}^{\infty} E_t \pi_\rho(t+i) + S_t \end{aligned} \tag{33}$$

$$S(t) = E_t S(t+1) \tag{34}$$

For $\theta \geq \frac{1}{2}$, that is, for mainly backward-looking model, the solution looks as follows:

$$\begin{aligned} \tilde{\pi}(t) &= \tilde{\pi}(t-1) - \theta^{-1}\beta \sum_{i=0}^{\infty} \left[\frac{1-\theta}{\theta}\right]^i E_t[u(t+i) - u_N] \\ &+ \theta^{-1}\delta \sum_{i=0}^{\infty} \left[\frac{1-\theta}{\theta}\right]^i E_t \pi_\rho(t+i) + S(t) \end{aligned} \tag{35}$$

$$\begin{aligned} S(t) &\equiv 0 \text{ for } \theta > \frac{1}{2} \\ &= E_t S(t+1) \text{ for } \theta = \frac{1}{2} \end{aligned} \tag{36}$$

Note that, when $\theta = \frac{1}{2}$, that is, when future expected inflation has the same weight as past inflation, the solution given by 33 and 34 is the same as the solution given by 35 and 36. When $\theta > \frac{1}{2}$, that is, when the model is mainly backward-looking, the sunspot component of the solution has to equal zero if the sunspot is not to blow up in expectation (see 32).

Disinflation when the inflation process is 'mainly forward-looking' Consider first the case where $\theta \leq \frac{1}{2}$ and the inflation process is mainly forward-looking. Assume that condition 11 holds and the disinflation process does not permanently affect the real exchange rate.

The solution for the inflation rate becomes

$$\tilde{\pi}(t) = \left[\frac{\theta}{1-\theta}\right] \tilde{\pi}(t-1) - (1-\theta)^{-1}\beta \lim_{\ell\to\infty} \sum_{i=0}^{\ell} E_t[u(t+i) - u_N] + S_t \quad (37)$$

$$S(t) = E_t S(t+1) \quad (38)$$

Solving 37 forward we find

$$\begin{aligned}
\lim_{t\to\infty} \tilde{\pi}(t) &= \lim_{t\to\infty} \left[\frac{\theta}{1-\theta}\right]^t \tilde{\pi}(0) \quad (39)\\
&+(1-\theta)^{-1}\beta \lim_{t\to\infty} \sum_{k=0}^{t} \left[\frac{\theta}{1-\theta}\right]^k \lim_{\ell\to\infty} \sum_{i=0}^{\ell} E_t[u(t-k+i) - u_N]\\
&+ \lim_{t\to\infty} \sum_{k=0}^{t} \left(\frac{\theta}{1-\theta}\right)^k S_{t-k}
\end{aligned}$$

The first term on the RHS of 39 vanishes because $0 \leq \theta \leq \frac{1}{2}$. Equation 39 has to hold for all processes driving the gap between the actual and natural unemployment rates. We assume that, in the long-run, the actual and natural unemployment rates are the same and that there is a constant long-run rate of inflation. The second term on the RHS of 39 therefore also vanishes

Using 38, this implies

$$S_t = \left(\frac{1-2\theta}{1-\theta}\right) \lim_{i\to\infty} E_t \tilde{\pi}(t+i) \quad (40)$$

Before considering the mainly forward-looking case in detail, we consider the special case of the purely forward-looking process, $\theta = 0$. This is of interest because it is prominent in the New Keynesian literature.

Costless disinflation 2: the New Keynesian wonderland When $\theta = 0$, that is, when only expected future inflation affects current inflation and past inflation has no weight at all, the model reduces to the canonical time-contingent New Keynesian model developed by Calvo [8] (see also Ball [1], Ball, Mankiw and Romer [2] and Mankiw [12]). The model simplifies to

$$\tilde{\pi}(t) = E_t\tilde{\pi}(t+1) - \beta[u(t) - u_N] + \delta\pi_\rho(t) \tag{41}$$

Its solution is

$$\tilde{\pi}(t) = -\beta\sum_{i=0}^{\infty} E_t[u(t+i) - u_N] + \delta\sum_{i=0}^{\infty} E_t\pi_\rho(t+i) + S_t \tag{42}$$

$$S(t) = \lim_{i\to\infty} E_t\tilde{\pi}(t+i) \tag{43}$$

While it is not apparent from 41, the Calvo [8] model that generates the (closed economy version of) 41, does have nominal price *level* rigidity or inertia. In any given period, the initial price level is predetermined. It does not, however, have *inflation* rigidity or inertia in the rate of inflation. Given the appropriate monetary policy support therefore, costless disinflation can be achieved. The title of Mankiw's [12] paper: "The Inexorable and Mysterious Tradeoff Between Inflation and Unemployment", does therefore not adequately characterise the (closed economy) version of 41 which figures prominently in that paper. Given the proper monetary policy, there is no trade-off between inflation and unemployment in the New Keynesian universe characterised by equation 41, or, equivalently, by equations 42 and 43.[6] Note that, if 11 holds and the disinflation policy does not have a permanent effect on the real exchange rate, the solution to the New Keynesian inflation equation can be written as:

$$\sum_{i=0}^{\infty} E_t[u(t+i) - u_N] = \beta^{-1}\left[\lim_{i\to\infty} E_t\tilde{\pi}(t+i) - \tilde{\pi}(t)\right] \tag{44}$$

Credible policies and policy announcements can lower both the long-run equilibrium rate of inflation, $\lim_{i\to\infty} E_t\tilde{\pi}(t+i)$ and the current rate of inflation $\tilde{\pi}(t)$ by equal amounts without the need to go through any transitional (let alone permanent) unemployment. In Buiter and Miller [7] (for the case of a closed economy) and in Buiter [5] (for the case of the open economy), inflation processes like 41 are embedded in simple but complete macroeconomic

[6] In general, the implementation of the appropriate monetary policies may also require suitably supportive fiscal policies, owing to the inextricable interconnection between intertemporal monetary and fiscal policy through the government solvency constraint.

models. The kind of monetary policy that would support costless disinflation in the Calvo model of equation 41 for which the price *level* is predetermined, would be an unanticipated immediate and permanent reduction in the *rate of growth* of the nominal money stock, accompanied by an immediate (unanticipated) one-off increase in the *level* of the nominal money stock of just the right magnitude to accommodate the increased demand for real money balances at the lower rate of inflation (and lower nominal interest rates) associated with the successful implementation of the policy.

Ball [1] got close to diagnosing the absence of any trade-off between unemployment and inflation in New Keynesian models à la Calvo, when he noted that, with the inflation equation given in 41, the unexpected, credible announcement of a future permanent reduction in the growth rate of the nominal money stock could cause a boom rather than a slump, between the announcement date and the date the reduction in the growth rate of the nominal money stock is actually implemented. Given 41, the anticipated future reduction in monetary growth would, because it raises the future unemployment rate (after the implementation date), reduce the rate of inflation immediately (that is, at the announcement date). Since (by assumption) the level of the nominal money stock path is unchanged until the implementation date, the lower inflation rate will gradually increase the real money stock. This supports a higher level of economic activity between the announcement date and the implementation date.

Ball's analysis and his conclusion are correct. They fail to point, however, to the key structural feature of the New Keynesian inflation equation - the absence of inflation inertia. This means that - far from being inexorable, let alone mysterious - the trade-off between inflation and unemployment is absent in the New Keynesian models considered by Calvo, Ball and Mankiw, as long as monetary policy is designed intelligently.

Strictly speaking, the conclusion that the sacrifice ratio is zero in the purely forward-looking model carries over to the entire class of 'mainly forward-looking' mixed models ($\frac{1}{2} \leq \theta < 1$), .although in this case costless disinflation cannot be achieved in finite time, but only asymptotically.

This can be seen from equations 37 and 40. These imply

$$\begin{aligned} \tilde{\pi}(t) &= \left[\frac{\theta}{1-\theta}\right]\tilde{\pi}(t-1) - (1-\theta)^{-1}\beta\sum_{i=0}^{\infty}E_t[u(t+i)-u_N] \qquad (45) \\ &+ \left(\frac{1-2\theta}{1-\theta}\right)\lim_{i\to\infty}E_t\tilde{\pi}(t+i) \end{aligned}$$

Assume that the government targets an inflation rate of $\tilde{\pi}^*$. We determine whether, for the inflation process given in 45, the transition from $\tilde{\pi}(t-1)$ to $\tilde{\pi}^*$ can be achieved with the unemployment rate always at the natural rate. From 45, this would require that the following process converges to $\tilde{\pi}^*$:

$$\tilde{\pi}(t) = \left[\frac{\theta}{1-\theta}\right]\tilde{\pi}(t-1) + \left(\frac{1-2\theta}{1-\theta}\right)\tilde{\pi}^* \tag{46}$$

Therefore

$$\lim_{t\to\infty}\tilde{\pi}(t) = \lim_{t\to\infty}\left[\frac{\theta}{1-\theta}\right]^t\tilde{\pi}(0) + \left(\frac{1-2\theta}{1-\theta}\right)\tilde{\pi}^*\lim_{t\to\infty}\sum_{i=0}^{t}\left[\frac{\theta}{1-\theta}\right]^i \tag{47}$$

Since $0 \leq \theta < \frac{1}{2}$,

$$\lim_{t\to\infty}\tilde{\pi}(t) = \tilde{\pi}^* \tag{48}$$

Thus, asymptotically, the inflation rate can be brought to any level, without incurring any unemployment rate different from the natural rate. Unlike the fully forward-looking New Keynesian case, however, for which convergence to the new inflation rate can be instantaneous, convergence with $u = u_N$ will only be gradual (indeed asymptotic) when $\theta > 0$.[7]

Conclusion 3 *In both the New Keynesian (purely forward-looking) model and in the mainly forward-looking mixed model, the sacrifice ratio is zero. Achieving any desired change in the rate of inflation can be instantaneous in the New Keynesian model, but only asymptotic in the mainly forward-looking mixed model.*

For a sustained reduction in the rate of inflation to be achieved in finite time in this model will require a positive sacrifice ratio. For instance, consider 45 when a sustained reduction to $\tilde{\pi}^*$ is to be achieved in one period, t. The required unemployment rate in period t satisfies

$$\tilde{\pi}^* = \left(\frac{\theta}{1-\theta}\right)\tilde{\pi}_{t-1} - (1-\theta)^{-1}\beta[u(t) - u_N] + \left(\frac{1-2\theta}{1-\theta}\right)\tilde{\pi}^*$$

or

$$u(t) - u_N = \beta^{-1}[\tilde{\pi}(t-1) - \tilde{\pi}^*]$$

Note that $\pi(t+i) = \pi^*$ and $u(t+i) = u_N$ for $i \geq 1$, as required.

[7]The simple macromodel of Footnote 1 can be used to demonstrate the validity of the zero sacrifice ratio and asymptotic convergence to the new inflation rate. One policy experiment that supports this is the unanticipated announcement of an immediate and permanent reduction in the proportional growth rate of the nominal money stock.

Disinflation when the inflation process is 'mainly backward looking' The previous subsection showed that, given intelligently designed monetary policy, the sacrifice ratio is zero in the mainly forward-looking mixed model $(0 \leq \theta \leq \frac{1}{2})$. When the inflation process is completely backward-looking $(\theta = 1)$, we are back to the case with static core inflation and $\sigma = \beta^{-1}$, analysed earlier. We now focus on the most interesting case - that of an inflation process - the mainly backward-looking mixed process $(\frac{1}{2} < \theta \leq 1)$. It is obvious from 41 and 35 and 36, that the sacrifice ratio is finite: it takes (at most) a temporary increase in unemployment to achieve a permanent reduction in inflation. It is also clear that the sacrifice ratio depends only on β and θ, and, when $\theta = \frac{1}{2}$, on $\lim_{i \to \infty} E_t S(t+i)$.

When $\theta > \frac{1}{2}$ (which implies $A_0 = 1$), it is always possible, since there only is one lag in the inflation process, and the coefficient on lagged inflation in 35 is unity, to achieve a permanently lower rate of inflation by increasing (expected) unemployment for just one period. Consider the case where unemployment is raised only in period t, that is, $u(t+i) = u_N\ ,\ i \geq 1$. Assume also, until further notice, that the current and expected future real exchange rate depreciation rates are held constant. From 35 it then follows that

$$\sigma = \theta \beta^{-1} \tag{49}$$

As expected, the sacrifice ratio is smaller when the responsiveness of inflation to unemployment is higher and when the relative weight of past inflation is lower. Note that the desired full reduction in the rate of inflation is achieved immediately in this case, that is, in period t. Note that, the path of inflation over time is given by

$$\tilde{\pi}_{t+i} = \tilde{\pi}_{t-1} - 1 \quad , i \geq 0 \tag{50}$$

Forward-looking expectations and the case for 'delay in implementation' Again considering the case where $\theta > \frac{1}{2}$ and where the path of the real exchange rate is held constant. We can determine the sacrifice ratio when unemployment is increased by the same amount in two successive periods, t and $t+1$, say. For all subsequent periods, unemployment is kept at the natural rate. This means

$$\tilde{\pi}(t) = \tilde{\pi}(t-1) - \frac{\beta}{\theta}\left[u(t) - u_N\right] - \frac{\beta}{\theta}\left(\frac{1-\theta}{\theta}\right)\left[u(t+1) - u_N\right]$$

$$\begin{aligned}\tilde{\pi}(t+1) &= \tilde{\pi}(t) - \frac{\beta}{\theta}\left[u(t+1) - u_N\right] \\ &= \tilde{\pi}(t-1) - \frac{\beta}{\theta}\left[u(t) - u_N\right] - \frac{\beta}{\theta^2}\left[u(t+1) - u_N\right]\end{aligned}$$

Since unemployment is, by assumption, the same in periods t and $t+1$,

$$u(t) - u_N = u(t+1) - u_N = \bar{u} - u_N$$

Therefore,

$$\tilde{\pi}(t+1) = \tilde{\pi}(t-1) - \frac{\beta}{\theta^2}(1+\theta)\left(\bar{u} - u_N\right)$$

The unemployment gap we need to maintain for *2* periods in order to reduce inflation by 1% in two periods (and keep it at that lower level forever after) is therefore given by

$$\bar{u} - u_N = \frac{\theta^2}{\beta(1+\theta)}$$

The sacrifice ratio is

$$\sigma = \left(\frac{2\theta}{1+\theta}\right)\frac{\theta}{\beta} < \frac{\theta}{\beta} \text{ for } \theta < 1$$

This is intuitively plausible. Unemployment in period $t+1$ works twice. First, directly, in period $t+1$. Second, during period t, when the expectation of period $t+1$ unemployment depresses period t inflation. Since policy has both *direct effects* and *announcement effects* when expectations are forward-looking, there is a case for a disinflationary policy taking the form of the *immediate, credible announcement* of *future* contractionary policy measures that will result in *future* unemployment. Thus the *credible* policy announcement should be forthcoming immediately, but the recession it announces should be postponed into the future, in order for the total, cumulative unemployment cost of achieving a sustained reduction in the rate of inflation to be as low as possible. Note that this constitutes an argument for *gradualism* over *cold turkey,* provided announcements concerning future policy are credible, that is, provided the policy maker is able to commit to future policy actions or policy rules. Gradualism means that the restrictive policy measures and their effect on unemployment and inflation are delayed and spread out over a number of periods. Cold turkey means that implementation is immediate and concentrated in a short period of time.

The inflation profile is of course not the same when part of the contractionary package required to achieve a given reduction in inflation (permanently but by some unspecified date) is postponed. When unemployment is increased by equal amounts in periods t and $t+1$, the profile of inflation is as follows

$$\begin{aligned} \tilde{\pi}(t) &= \tilde{\pi}(t-1) - \frac{1}{1+\theta} \\ \tilde{\pi}(t+i) &= \tilde{\pi}(t-1) - 1 \ , \quad i \geq 1 \end{aligned} \tag{51}$$

Comparing 50 and 51, we note that, while the inflation reduction in period $t+1$ and beyond is the same for both unemployment profiles, the reduction in inflation in period t is larger when all the unemployment in concentrated in period t rather than being spread equally over periods t and $t+1$.

Starting in period t, the authorities could also achieve a permanent reduction in inflation starting in period $t+1$ by just increasing unemployment in period $t+1$, and making a credible announcement to that effect in period t. In this case:

$$\tilde{\pi}(t) = \tilde{\pi}(t-1) - \frac{\beta}{\theta}\left(\frac{1-\theta}{\theta}\right)[u(t+1) - u_N]$$

$$\begin{aligned} \tilde{\pi}(t+1) &= \tilde{\pi}(t) - \frac{\beta}{\theta}[u(t+1) - u_N] \\ &= \tilde{\pi}(t-1) - \frac{\beta}{\theta^2}[u(t+1) - u_N] \end{aligned}$$

In this case the sacrifice ratio is

$$\sigma = \frac{\theta^2}{\beta} < \left(\frac{2\theta}{1+\theta}\right)\frac{\theta}{\beta} < \frac{\theta}{\beta} \text{ for } \theta < 1$$

As expected, concentrating the necessary unemployment increase completely in period $t+1$ results in an even lower sacrifice ratio than spreading it evenly over periods t and $t+1$. The sequence of inflation rates in this case is

$$\begin{aligned}\tilde{\pi}(t) &= \tilde{\pi}(t-1) - (1-\theta) \\ \tilde{\pi}(t+i) &= \tilde{\pi}(t-1) - 1 \ , \ \ i \geq 1\end{aligned} \tag{52}$$

Again, while the same inflation rate is ultimately achieved and sustained, period t inflation is highest when all the unemployment is concentrated in the later period.

What the logic of the model therefore calls for, if the purpose were to minimise the sacrifice ratio, is the immediate, credible announcement of (and commitment to), a delayed implementation of policy measures some time in the future (the further in the future, the better for the sacrifice ratio).

Note that there is likely to be a credibility problem or commitment problem here. Once the (credible) announcement (in period t) of higher unemployment in period $t+1$ (or even further into the future) has had its desired announcement effect on inflation in period t, the authorities may no longer wish to incur the higher unemployment when period $t+1$ arrives, just to ensure that their period t commitments are honoured. How would the public respond to the following policy statement?

> "Dear electorate. Several years ago my government announced that we would create a recession, starting today, in order to achieve a lasting reduction in the rate of inflation. You believed us. Your wage and price setting practices moderated and the desired reduction in inflation has now been achieved without, thus far, any increase in unemployment. There is therefore no reason, from the point of view of our inflation performance, to go ahead now and implement the measures that will produce the recession we promised you all those years ago. However, I cannot tell a lie. My credibility matters to me. We'll have the recession anyway. I look forward with confidence to your support at the next general elections"

Clearly, the policy announcement of a future policy-induced recession, with the costs postponed until after the gains have been reaped, would not in fact be credible. It is true that politics is a repeated game. Therefore, considerations of repetition and reputation can, in principle, overcome time inconsistency problems of the kind that arise here, or indeed whenever policy has announcement effects. It seems questionable, however, that a government would create a recession when there is no longer any use for it from the point of view of reducing inflation, simply to preserve its credibility for possible future policy announcements.

There are two further qualifications to the proposition that the postponement of disinflationary pain may be good for you. First, the sacrifice ratio sums the *undiscounted* increases in unemployment. The authorities may not in fact be indifferent about the timing of unemployment.[8] Second, the inflation profiles are different for the three cases, before period $t+1$. Inflation in period t is lowest when the unemployment is all concentrated in period t and highest when it is all concentrated in period $t+1$.

Can the behaviour of the real exchange rate make a difference to the sacrifice ratio in the mainly backward-looking model? When $\theta > \frac{1}{2}$, and therefore $S(t) = 0$, the inflation process looks as follows:

$$\tilde{\pi}(t) = \tilde{\pi}(t-1) - \theta^{-1}\beta \sum_{i=0}^{\infty} \left[\frac{1-\theta}{\theta}\right]^{i} E_t[u(t+i) - u_N] + \theta^{-1}\delta \sum_{i=0}^{\infty} \left[\frac{1-\theta}{\theta}\right]^{i} E_t\pi_\rho(t+i) \tag{53}$$

Even granting the assumption that the disinflation process does not have a lasting effect on the real exchange rate, that is $\sum_{i=0}^{\infty} E_t\pi_\rho(t+i) = 0$, a given real exchange rate appreciation at time t followed by a later depreciation of the same magnitude will have a lasting effect on the inflation rate. Indeed, an initial real appreciation followed by a later real depreciation that restores the original real exchange rate will raise inflation and the sacrifice ratio. For instance, in the Dornbusch 'overshooting' model of Footnote 1, when 25(with $\theta > \frac{1}{2}$) is substituted for the conventional Phillips curve, the real exchange rate 'overshooting' that occurs in response to the unanticipated, immediate and permanent reduction in the growth rate of the nominal money stock, raises the sacrifice ratio relative to a policy that pursues a constant real exchange rate throughout the disinflation process.[9] The real overshooting does, of course, provide the anti-inflationary gains earlier. For instance, a 1% real appreciation in period t followed by the expectation of a 1% real depreciation in period $t+1$ (and the actual occurrence of that depreciation in period $t+1$) would *reduce* inflation in period t by $\delta(2\theta-1)\theta^{-2}\%$, but would also *raise* it during period $t+1$ by $\delta(1-\theta)\theta^{-2}\%$ relative to where it would have been with a constant real exchange rate. The reason is that, from 53, the future real exchange rate depreciation works twice, once when it actually occurs, during period $t+1$, and once when it is anticipated, in period t. The earlier appreciation does not have these announcement effects.

[8] If the subjective discount rate of the authorities is positive, this would further favour postponing the unemployment.

[9] This would require the use of time-varying fiscal policy.

Conclusion 4 *In the mainly backward-looking model, costless disinflation is impossible. The sacrifice ratio is always positive (but finite).*

Conclusion 5 *In the mainly backward-looking model, the unexpected, credible announcement of a given cumulative increase in current or future unemployment has a stronger sustained effect on the rate of inflation the further in the future it is applied. Equivalently, a given sustained reduction in inflation can be achieved with a lower sacrifice ratio if the unemployment is postponed further into the future. However, postponing the unemployment also delays the achievement of the full reduction in inflation.*

Conclusion 6 *Technically more efficient disinflation plans that involve the achievement of some or all of the full anti-inflationary gains through announcement effects of unemployment to be incurred after the anti-inflationary gains have been achieved, are unlikely to be credible.*

Conclusion 7 *In the mainly backward-looking model, temporary real exchange rate changes can affect the sacrifice ratio. An early real appreciation followed by a later real depreciation that takes the real exchange rate back to its initial level will raise the sacrifice ratio.*

4 Hysteresis in the Natural Rate of Unemployment

Hysteresis, path dependence or history dependence is a property of dynamical systems that have multiple stationary (steady-state) or long-run equilibria. The long-run behaviour of a dynamical system with multiple steady states depends on the initial conditions and on transitory shocks along the adjustment path to the steady state. The example given below has a continuum of steady states, but hysteresis can be present also if there are only a discrete number of steady states.

The natural rate of unemployment is hysteretic if this period's value of the natural rate of unemployment depends on the past history of the actual rate of unemployment. A simple linear process is shown in 54 below:

$$\begin{gathered} u_N(t) = \zeta u(t-1) + (1-\zeta)u_N(t-1) \\ 0 \leq \zeta \leq 1 \end{gathered} \tag{54}$$

This means that the current value of the natural rate depends on the entire past history of actual rates:

$$u_N(t) = \zeta \sum_{i=0}^{T-1} (1-\zeta)^i u(t-1-i) + (1-\zeta)^T u_N(t-T)$$

There are several theories that support the view that the natural rate may be history-dependent. The first is the *human capital decumulation* explanation of hysteresis in the natural rate. This holds that the experience of actual unemployment hurts both the aptitude for work (loss of skills) and the attitude towards work (demotivation, discouragement). The human capital, that is, the efficiency of the unemployed component of the labour force therefore declines when unemployment is high. This may be especially relevant for the long-term unemployed and for young labour force entrants who miss out on labour market experience during the formative years of their lives. While labour supply in physical units may remain the same, labour supply in units of efficiency or quality falls. The employed become a progressively smaller influence on the wage bargain.

The second explanation is *insider-outsider theory*, due to Gregory [10, Boo], elaborated by Snower and Lindbeck [11] and popularised by Blanchard and Summers [3]. This views the labour force as segmented between insiders and outsiders. The insiders are the employed workers, who negotiate with the employers without taking account of the implications of their wage bargain for the employment prospects of the outsiders. The outsiders are assumed to be completely disenfranchised in the wage and employment negotiations among the insiders. Effective barriers to entry (both by workers and by new firms employing only outsiders) are required for this to be a robust equilibrium. When workers become unemployed, they cease to be insiders and become outsiders. They no longer exercise any restraining influence on the wage and employment negotiations of the remaining insiders. Again, the unemployment cease to represent an effective supply of labour.

An explanation in the same spirit as the human capital theory of hysteresis in the natural rate is the *physical capital stock theory* of hysteresis. It notes that most episodes of disinflation and high unemployment are recessions with low levels of capital formation. If labour and capital are complements in the production function, this will reduce the demand for labour. The full employment level of employment therefore falls or the natural rate of unemployment rises.

For illustrative purposes, we shall combine 54 with 7, and static core inflation, $\tilde{\pi}_c(t) = \tilde{\pi}(t-1)$. This yields

$$\tilde{\pi}(t) = -\beta \left[u(t) - u_N(t) \right] + \delta \pi_\rho(t) + \tilde{\pi}(t-1) \tag{55}$$

Equations 54 and 55 imply:

$$\Delta\tilde{\pi}(t) = (1-\zeta)\Delta\tilde{\pi}(t-1) - \beta\Delta u(t) + \delta\Delta\pi_{\rho}(t) + \delta\zeta\pi_{\rho}(t-1) \qquad (56)$$

Note that the *change* in the inflation rate in period t depends only on the *change* in the unemployment rate and not on the *level* of the unemployment rate. Consider the case where the real exchange rate is constant. Equation 56 implies that, with hysteresis in the unemployment rate, the sacrifice ratio is infinite. In the simple case where $\zeta = 1$, and the natural rate this period equals last period's actual rate: $u_N(t) = u(t-1)$, .for the inflation rate in period t to fall by 1%, the unemployment rate in period t has to rise by β^{-1}%. If the natural rate were constant, the unemployment rate could be reduced back to its original level in period $t+1$, and inflation in periods $t+1$ and beyond would remain at the new 1% lower level, since core inflation in period $t+1$ equals the lower actual rate of inflation in period t. However, because of the hysteresis in the natural rate, $u_N(t+1) = u(t)$ and in order to stop the inflation rate from rising back to its original level, the unemployment rate will have to stay permanently at its new higher level. While the long-run Phillips curve is still vertical, it can be vertical at any level of the unemployment rate, depending on the past history of the unemployment rate. The actual rate of unemployment drags the natural rate of unemployment with it as time passes. The restraining effect on inflation from a higher unemployment rate therefore wears off over time and ultimately disappears altogether (in our simple example it is gone after one period). Hysteresis in the natural rate of unemployment therefore has important implications for the cost-benefit analysis of anti-inflationary policy: the sacrifice ratio is infinite.[10][11]

---

[10] This approach to hysteresis in the natural rate of unemployment is very similar in spirit to models of wage and price inflation that make the rate of inflation a function of the change in unemployment rather than its level. A simple example with static core inflation is given below:

$$\Delta\tilde{\pi}(t) = -\beta\Delta u(t) + \delta\pi_{\rho}(t)$$

This specification implies that as soon as unemployment stabilizes, no matter at what level, inflation stops rising or falling. The sacrifice ratio is again infinite.

[11] Many empirical Phillips curves that do have the long-run natural rate property nevertheless have the change in the unemployment rate as a determinant of short-run inflation dynamics. An example (in a closed economy setting) is the following

$$\begin{aligned} \tilde{\pi}(t) &= \tilde{\pi}_c(t) - \beta_0\left[u(t) - u_N(t)\right] - \beta_1\Delta u(t) \\ \beta_0, \beta_1 &> 0 \end{aligned}$$

With hysteresis in the natural rate of unemployment, the sacrifice ratio will be infinite for all models considered earlier, except for the New Classical and New Keynesian models, for which the sacrifice ratio is zero and any desired change in the rate of inflation can be achieved instantaneously. For the other mainly forward-looking mixed models (those for which $0 < \theta \leq \frac{1}{2}$), the sacrifice ratio is also zero, but any desired change in the rate of inflation can only be achieved asymptotically if no unemployment is to be incurred.

One of the reasons why some version of the hysteresis hypothesis continues to find support is the (casual) empirical observation that estimates of the natural rate of unemployment have, empirically, tended to track the actual rate of unemployment quite closely. The fact that the natural and the actual rates of unemployment tend to move together empirically is consistent with the hysteresis hypothesis, but can also be rationalised with approaches that view the natural rate as causally independent of the current and past behaviour of actual unemployment, and of stabilisation policy generally. The natural rate could, for instance, be driven by structural factors such as normal productivity growth, the balance of bargaining power in the labour market, direct and indirect tax wedges between the marginal cost of labour to the employer and the after-tax wage of the worker, unemployment benefits, skills mismatch, minimum wage legislation, employment 'protection' legislation etc. The actual unemployment rate could track this exogenous natural rate quite closely either because the servo-mechanism of the market economy sooner or later drives the actual and the natural closer together and/or because policy actions in support of stable inflation keep the actual unemployment rate quite close to the natural rate.

If the hysteresis hypothesis were to be valid, the benefit from unconventional approaches to inflation control would be greatly increased. Even a temporary incomes policy (or price and wage freeze) could have a permanent effect on the inflation rate. Equivalently, the ability to override the core inflation process through some kind of incomes policy could reduce the sacrifice ratio to a finite level or, in the best case, reduce it to zero. Formally, in our models, such in incomes policy would have to be modeled as a *deus ex machina* that permits one to override the 'normal' inflation kernel at least temporarily. Whether such a mechanism exists in the real world is doubtful. Past experience with incomes policy in industrial countries like the UK and the US have been disasters. There is some evidence that in the Nordic countries and in the Netherlands, social compacts of this kind may have been

---

The natural rate is exogenous in this example. The contribution of $\Delta u(t)$ to the short-run inflation dynamics is sometimes referred to as a 'speed limit effect'.

successful. For this to work, a culture of cohesion, consensus and basic trust between the 'social partners' and the government must be firmly embedded.

5 Conclusion

The weight-watchers nostrum: "No gain without pain" applies to the disinflation process as well as to dieting. The benefits from eliminating moderate inflation cannot be enjoyed without incurring the pain of increased unemployment and lost production. All but two of the models considered in this review share this property. The exceptions are the New Classical and New Keynesian models.[12] All but two of the models have a finite sacrifice ratio: a permanent reduction in inflation can be achieved with a temporary increase in unemployment. The exceptions are the exogenous core inflation model and the model with hysteresis in the natural rate of unemployment. The richest model, and the one we consider most appropriate as a framework for monetary policy evaluation and design in low and moderate inflation countries is the mainly backward-looking mixed model with an exogenous natural rate of unemployment - our *preferred* model. Because it incorporates forward-looking behaviour as well as inflation inertia, it permits the explicit consideration of key issues faced by central bankers throughout the industrial world and in many candidate countries for accession.

The preferred model has the feature that painless disinflation is impossible. It also has the property that policies that reduce the sacrifice ratio by relying on the announcement effects on inflation today of policies that produce unemployment in the future, have two weaknesses. First, they may be time-inconsistent. A commitment to inflict pain (increased unemployment) in the future, after the anti-inflationary benefits have been achieved in full (making the increase in unemployment unnecessary), is not credible. Second, even if credible, such policies would delay the achievement of the anti-inflationary gains relative to a policy that imposes unemployment earlier (and which therefore has a higher sacrifice ratio). Finally, the preferred model highlights the role of changes in the real exchange rate in the disinflation process. Even if successful disinflation does not alter the long-run real exchange rate, a policy of disinflation under a floating exchange rate and with perfect international financial capital mobility is likely to be associated with an initial sharp appreciation of the real exchange rate followed by a more gradual real depreciation back to the invariant long-run real exchange

[12] The mainly forward-looking mixed model with $0 < \theta \leq \frac{1}{2}$ qualifies for a zero sacrifice ratio only on a technicality: costless disinflation is not possible in finite time, but only asymptotically.

rate. Such a pattern of real exchange rate variations raises the sacrifice ratio relative to a policy that keeps the real exchange rate constant. The policy also speeds up the achievement of the lower inflation target. The preferred model permits a transparent characterisation of the policy maker's menu of choice.

References

[1] Ball, Laurence [1994], "What Determines the Sacrifice Ratio?", in *Monetary Policy*, N. G. Mankiw, ed., Chicago, University of Chicago Press, pp. 155-82.

[2] Ball, Laurence, Mankiw, N. Gregory and Romer, David [1988], "The New Keynesian Economics and the Output-Inflation Trade-Off", *Brookings Papers on Economic Activity*, Vol. 1, pp. 1-65.

[3] Blanchard, Olivier J. and Summers, Lawrence [1986], "Hysteresis and the European Unemployment Model", *NBER Macroeconomics Annual 1*: pp. 15-78.

[4] Buiter, Willem H. [1981], "The Superiority of Contingent Rules over Fixed Rules in Models with Rational Expectations", *The Economic Journal*, 91, September, pp. 647-70. Reprinted in Willem H. Buiter, *Macroeconomic theory and stabilization policy*, Manchester University Press, 1989, pp. 77-104.

[5] Buiter, Willem H. [1985], "International Monetary Policy to Promote Economic Recovery", in C. van Ewijk and J.J. Klant (eds.), *Monetary Conditions for Economic Recovery*, Martinus Nijhof, Amsterdam. Reprinted as Chapter 6 in Willem H. Buiter, *International Macroeconomics*, Clarendon Press, Oxford, 1990, pp. 171-194.

[6] Buiter, Willem H. and Jewitt, Ian [1981], "Staggered wage setting with real wage relativities: variations on a theme of Taylor", *The Manchester School*, Vol. 49, pp. 211-28. Reprinted in Willem H. Buiter, *Macroeconomic theory and stabilization policy*, Manchester University Press, 1989, pp. 183-199.

[7] Buiter, Willem H. and Marcus H. Miller [1985], "Costs and benefits of an antiinflationary policy: questions and issues", in A. Argy and J. Neville, (eds), *Inflation and Unemployment: Theory, Experience and Policy Making*, London, George Allen and Unwin, reprinted in Willem

H. Buiter, *Macroeconomic theory and stabilization policy*, Manchester University Press, 1989, pp. 200-227.

[8] Calvo, Guillermo A. [1983], "Staggered Prices in a Utility Maximising Framework", *Journal of Monetary Economics*, Vol 12, September, pp. 383-98.

[9] Fuhrer, Jeffrey and Moore, George [1995], "Inflation Persistence", *Quarterly Journal of Economics*, Vol. 110(1), February, pp. 127-60.

[10] Gregory, R.G. [1986], "Wages Policy and Unemployment in Australia", *Economica*, 53, (Supplement), pp. S53-S74.

[11] Lindbeck, Assar and Snower, Dennis [1988], *The Insider-Outsider Theory of Employment and Unemployment*. Cambridge: MIT Press.

[12] Mankiw, N. Gregory [2001], "The Inexorable and Mysterious Tradeoff Between Inflation and Unemployment", *The Economic Journal*, 111, May, pp. C45-C61.

[13] Taylor, John B. [1980], "Aggregate Dynamics and Staggered Contracts", *Journal of Political Economy*, Vol. 88, pp. 1-22.

[14] Taylor, John B. [1998], "Staggered Price and Wage Setting in Macroeconomics", NBER Working Paper No. 6754.